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ERRATA

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Page 593, second paragraph, and page 600, line 2, change "Plithalate" to "Phthalate."

Transactions, Vol. 110, 1945

Page 1823, sentence beginning on line 31 should be changed to read: "He also served from May 1, 1935, until his death as a consulting engineer on the staff of the Commissioner of Public Buildings, Federal Works Agency, Washington, D. C."

Page 1899, line 31, change "1876" to "1879."

Page 1899, line 33, change "the public schools and later entered" to "Knox Academy and * * *."

Page 1901, line 12, add "He is also survived by a sister, Harriette (Mrs. James C. Simpson)."

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AMERICAN SOCIETY OF CIVIL ENGINEERS
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Paper No. 2266

EVAPORATION FROM A FREE WATER SURFACE

By G. H. Hickox,[1] M. Am. Soc. C. E.

With Discussion by Messrs. Carl Rohwer, C. W. Thornthwaite, Harry F. Blaney, Arthur A. Young, A. A. Kalinske, Adolph F. Meyer, Maurice L. Albertson, and G. H. Hickox.

Synopsis

Experiments on an evaporation pan, 1 ft in diameter, in still air and under controlled conditions, are described in this paper. An analogy is drawn between heat transfer and mass transfer as they occur in evaporation, and evaporation rates are expressed by means of appropriate dimensionless groups.

The results of the experiments are correlated by the use of these dimensionless groups, and the effects of the variables—vapor-pressure difference, wind velocity, pan diameter, water temperature, air pressure, rim height, color of pan, and depth of pan—are discussed and evaluated.

Suggestions are made for the further study of evaporation records and for obtaining field data.

Introduction

Evaporation records in the United States have been obtained by observations on small pans, the purpose being generally to provide a basis for estimating the evaporation from larger bodies of water such as lakes and reservoirs. Direct measurements on large bodies of water are extremely hard to obtain because of the difficulty of measuring inflow, outflow, and leakage with sufficient accuracy.

It is well known that evaporation from the small pans in use is not the same as the evaporation from lakes and reservoirs and considerable effort has been made to determine the relationship between pan evaporation and lake evaporation. The pans most commonly used are the exposed land pan, the buried land pan, and the floating pan (1).[2] Carl Rohwer, M. Am. Soc. C. E. (1), has assembled data comparing the evaporation from various types of pans and reservoirs, and has computed the coefficients necessary to convert pan

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Note.—Published in October, 1944, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

[1] Senior Hydr. Engr., TVA, Hydraulic Laboratory, Norris, Tenn.

[2] Numerals in parentheses, thus: (1), refer to corresponding items in the Bibliography (see Appendix I).

evaporation to reservoir evaporation. The coefficients vary with the type of pan, the locality, and the season of the year. The final report (2) of the Sub-Committee on Evaporation of the Special Committee on Irrigation Hydraulics of the Society has adopted 0.70 as the value of the coefficient to reduce annual evaporation from the U. S. Weather Bureau Standard Class A evaporation pan to reservoir evaporation. This coefficient is based largely on Mr. Rohwer's data and it is stated that a reasonable range of the coefficient is from 0.60 to 0.82. This represents a variation of 31%.

The variation can be accounted for by taking into consideration the various factors that affect the transfer of heat energy to and from the evaporation pan. The difference between the energy supplied by radiation, and the energy lost by back radiation, conduction, and convection, must be utilized either to heat the water or to evaporate it. If the differences in radiation, conduction, and convection can be established for two pans under similar conditions, it should be possible to compute the difference in evaporation. It should also be possible to apply similar calculations to determine the evaporation from a reservoir.

If such calculations can be made satisfactorily, it will be possible to utilize to much better advantage the large mass of evaporation data that have been collected and summarized (3). Robert Follansbee, M. Am. Soc. C. E. (4), has made an attempt to reduce a great many existing evaporation pan records to evaporation from reservoirs. In doing so, he has used coefficients for each pan which are average annual values and has applied them to monthly and seasonal evaporation. The application of average annual coefficients to monthly records is open to question, particularly since many of the data supporting these annual coefficients were obtained in localities where there is little evaporation in the winter months. An annual coefficient based on such data would not be expected to be quite applicable to Southern California, for example.

Until it has been established that calculations, as outlined, can be made with sufficient accuracy, considerable progress can be made by analyzing the factors that affect evaporation. An understanding of the physical occurrences of the evaporation process would lead to observations of those elements of the problem which would make the transfer of pan records to reservoirs more reliable.

The purpose of this paper is to describe the mechanism of evaporation, to draw an analogy between heat transfer and evaporation, to define the relationship between evaporation and certain basic factors, and to show how the results obtained by a number of different experimenters can be correlated.

HOW EVAPORATION OCCURS

When water changes its state from a liquid to a vapor it is said to evaporate. The transformation requires energy that is known as the latent heat of evaporation. In order for a given mass of water to evaporate and at the same time to maintain its temperature, energy must be supplied to replace that required for evaporation. If there is no loss of heat across the boundary of the mass other than that due to evaporation, the rate of supply of energy is a measure of the rate of evaporation. This is the basis of one method of calculating evaporation. The actual calculation is difficult because there are several ways in which energy may be added to and removed from a mass of water, not all of which are susceptible of easy evaluation.

From the kinetic standpoint, evaporation occurs when more molecules pass through the surface from the liquid to the surrounding gas than pass from the gas into the liquid. The molecules composing a liquid are in constant motion. Those near the surface are somewhat restrained in their motion by the fact that they are acted on more strongly by the greater number of molecules below them. Nevertheless, a few of these molecules will have energy enough to break through the surface and escape as vapor. In the space immediately above the liquid, there are molecules of the gas which are also in violent motion. A number of these molecules will strike the surface and occasionally will pass through it into the liquid. The rate at which molecules leave the surface depends on the energy with which these molecules move about in the liquid; that is, on the temperature, or the vapor pressure, of the liquid. Similarly, if the space above the liquid contains only its own vapor, the rate at which molecules enter the liquid depends on the number of molecules present and on the energy, or temperature, of the gas, or its vapor pressure. The net loss of molecules from the liquid, or rate of evaporation, thus depends on the difference between the vapor pressure of the liquid at its temperature and the vapor pressure of the gas above it. The rate of evaporation is somewhat reduced by the presence of the gases above the surface of the evaporating liquid (5).

For evaporation to continue, the molecules leaving the surface must be removed from the immediate vicinity. If they were not so removed, their number would increase to the point where the number of molecules entering the surface would equal the number leaving. The liquid then would be in equilibrium with its own vapor and evaporation would cease. The pressure of the gas immediately above the surface of the liquid would be equal to the vapor pressure of the liquid at that temperature. Ordinarily, the molecules are removed by two processes—diffusion and turbulence. The molecules leaving the surface must first pass by diffusion through a thin layer of gas, known as the boundary layer. The laws governing the diffusion of a gas are well known and if it were possible to measure the vapor concentration on both sides of the boundary layer, as well as the thickness of the layer, the rate of passage of molecules, and hence the evaporation, could be calculated easily. Unfortunately, there is no existing technique for making these measurements. After passing through the boundary layer, the vapor molecules are removed more completely by convection in which turbulence is frequently a governing factor. The laws governing transport of mass by turbulence are fairly well understood, but there are certain fundamental constants entering into the equations which have not yet been sufficiently well established for all conditions of atmospheric turbulence. However, an attempt has been made to calculate evaporation by means of the vapor concentration and wind velocity at two different levels above an evaporating surface with fair results (6).

The process of heat transfer from a solid to a gas is analogous to the removal of vapor particles from an evaporating water surface. A boundary layer of stagnant gas surrounds the solid through which heat is transferred by conduction alone. Outside the boundary layer, the heat is transferred by convection, frequently aided by turbulence. Heat transfer has been computed successfully by the use of an over-all transfer coefficient, combining the coefficients of conduction and convection, known as a film coefficient. The temperature difference used with this coefficient is the wall temperature of the solid boundary and the temperature of the surrounding gas at a considerable distance. There seems to be no reason why the rate of transfer of vapor particles cannot be calculated in an entirely similar fashion. The potential difference is the difference in vapor concentration, or vapor pressure, between the water-air interface and some far distant location. The evaporation coefficient, of course, must be determined experimentally. Heat-transfer coefficients are correlated by means of dimensionless groups. Similar groups may be used to correlate evaporation coefficients.

Evaporation as a Mass Transfer Process

Evaporation Into Still Air.—According to the kinetic theory, evaporation from a water surface is a function of the difference in water-vapor concentration at the water surface and at a distance, and of the nature of movement of the gas into which evaporation occurs. It is a process of mass transfer which is analogous to heat transfer.

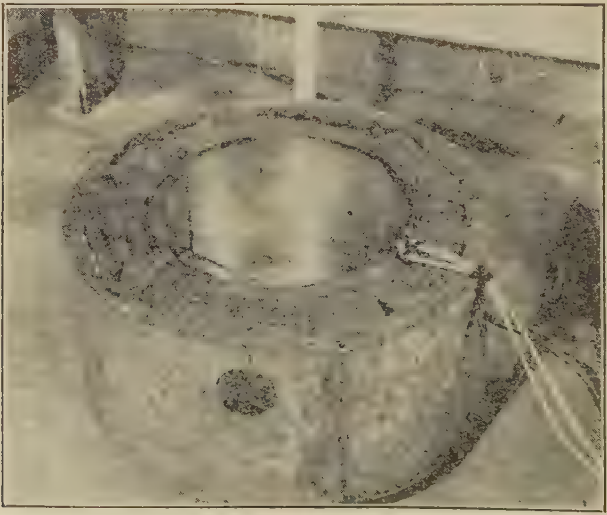


Fig. 1.—Evaporation Pan, Showing Outer Shell, Insulation, Thermocouples for Measuring Temperatures, and Heater Leads. False Floor of Quieting Chamber Removed

Experiments to study evaporation into still air from a circular water surface 1 ft in diameter were undertaken at the University of California at Berkeley. Space for conducting the experiments was made available by the Civil Engineering Department in one of its concrete curing rooms where the temperature and relative humidity were maintained during the course of the experiments at average values of 71.0° F and 53.1%, respectively. The evaporation pan was made of tin-plated copper, 12 in. in diameter and 6 in. deep. It was insulated by a 4-in. layer of hair felt. Thermocouples were placed at various points within the pan to measure water temperatures. Heat was supplied by an electric heating element immediately below the pan which was enclosed by a quieting chamber 5 ft square and 7.5 ft high in order to reduce stray air currents. A false floor was built and the pan was mounted so that the water surface was flush with it. A few tests were made with the rim of the pan projecting 0.5 in. and 1.5 in. above the false floor. All operations, including filling the pan and measuring evaporation rates and temperatures, were conducted from outside the chamber. Water and air temperatures inside the chamber were measured by thermocouples. Air temperature outside the chamber was measured with a thermometer. Relative humidity was measured with a standard sling psychrometer. The heating element was supplied with direct current power, controlled by rheostat, and measured with voltmeter and ammeter. Atmospheric pressure was measured by a recording barograph. Fig. 1 illustrates the evaporation pan (7*b*) and Fig. 2 shows the relative arrangement of apparatus.

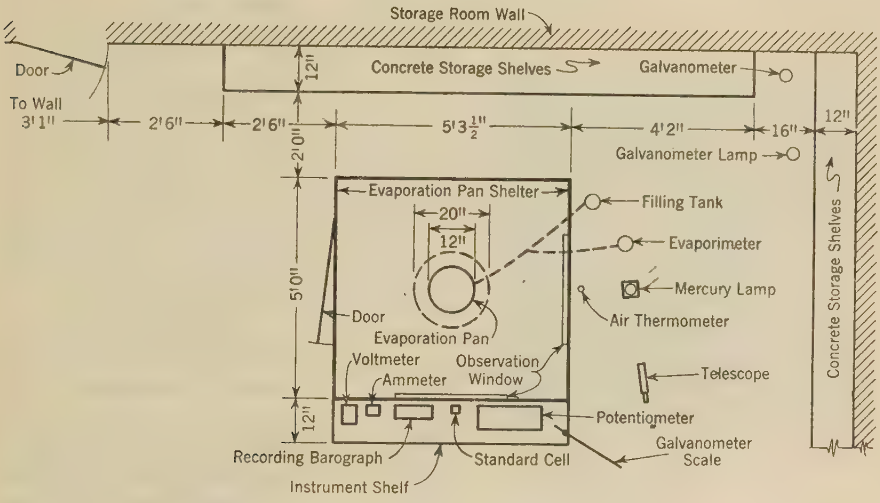
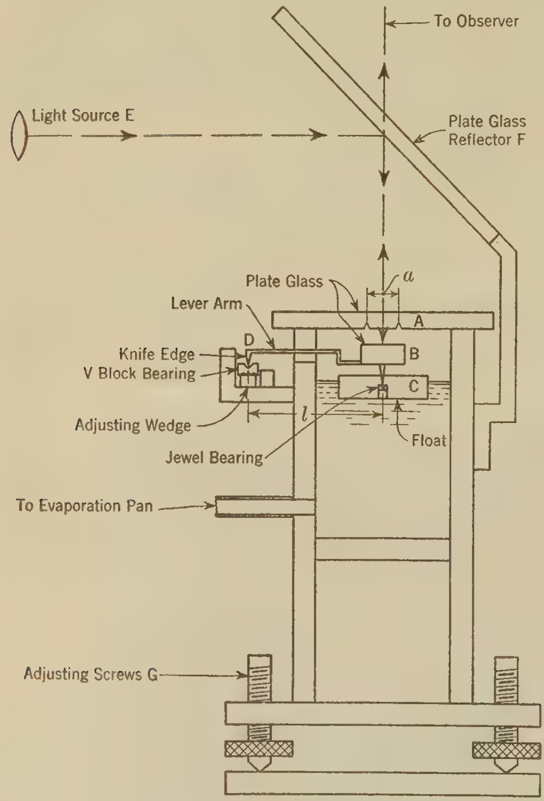

Storage Room Wall
Door
To Wall 3'1"
2'6"
2'6"
12"
Concrete Storage Shelves
Galvanometer
5'3½"
4'2"
16"
12"
2'0"
Galvanometer Lamp
Evaporation Pan Shelter
20"
12"
Filling Tank
Evaporimeter
Mercury Lamp
Air Thermometer
5'0"
Door
Evaporation Pan
Voltmeter
Ammeter
Observation Window
Telescope
Concrete Storage Shelves
12"
Potentiometer
Galvanometer Scale
Recording Barograph
Standard Cell
Instrument Shelf

Fig. 2.—Arrangement of Evaporation Pan, Quieting Chamber, and Instruments

The rate of evaporation was measured by an optical interferometer especially designed for the purpose. It consisted of a tilting mirror B and a glass plate A (Fig. 3). Monochromatic light from a source E was reflected toward the plate and mirror, producing interference bands which were then observed through a telescope. The tilting mirror was actuated by a float in a cylinder connected with the evaporation pan. As the water level changed, the number of interference bands visible between two lines etched on the glass plate changed. The change ΔN in the number of bands that occurred in the space a between the lines is a measure of the depth of evaporation E, which can be expressed by the equation

$$E = \frac{l}{a}\frac{\lambda}{2}\Delta N \quad \ldots\ldots (1)$$

In Eq. 1, l = length of lever arm; a = the space between two etched lines on a glass plate; and λ = the wave length of light. All letter symbols used in this paper have been collected and defined in Appendix II. Fig. 3 is a schematic diagram of the evaporimeter and Fig. 4 shows the type of interference pattern produced (7*c*). This meter was so sensitive that a determination of evaporation rate could be made in from 5 to 15 min. In fact, for high water temperatures the number of bands changed so rapidly that it was difficult to count them.



To Observer
Light Source E
Plate Glass Reflector F
Plate Glass
a
Lever Arm
A
D
B
Knife Edge
V Block Bearing
C
l
Adjusting Wedge
Float
Jewel Bearing
To Evaporation Pan
Adjusting Screws G

FIG. 3.—SCHEMATIC DRAWING OF EVAPORIMETER

Before making any observations, the temperatures and evaporation rate were allowed to become steady, and were maintained as nearly constant as possible throughout the test.

The basic equation for conduction of heat may be written

$$dq = k\frac{\partial t}{\partial x}\,dA \quad \ldots (2a)$$

in which: A = area in square feet; q = rate of heat transmission in British Thermal Units (Btu) per second; k = conductivity, in Btu per second-degree-foot; t = thermometric temperature; x = length along the path of heat flow; and $\frac{\partial t}{\partial x}$ = the temperature gradient in degrees per foot.

The analogous equation for mass transfer is

$$dw = D_v\frac{\partial c}{\partial x}\,dA \quad \ldots\ldots (2b)$$

in which, corresponding to Eq. 2*a*: w = rate of flow of vapor in pounds per second; D_v = diffusivity in feet^2 per second; c = unit vapor concentration, in pounds per cubic foot; and $\frac{\partial c}{\partial x}$ = unit concentration gradient in pounds per cubic foot-foot.

Experimental Data.—The results obtained have been plotted in terms of concentration difference and evaporation rate in Fig. 5. Because of the scattering of the points, they have been represented arbitrarily by two straight lines. The equations of these lines are, for $\Delta c < 4.2 \times 10^{-4}$,

$$w_A = 0.0045\,\Delta c \quad (3a)$$

and, for $\Delta c > 4.2 \times 10^{-4}$,

$$w_A = -\,0.0000059 + 0.0183\,\Delta c \quad (3b)$$

In Eqs. 3, w_A is the weight rate of evaporation in pounds per (square foot)-second and Δc is the concentration difference in pounds per cubic foot.

When water evaporates from a circular surface into still air so that the vapor is removed by diffusion alone, the rate of evaporation can be calculated with the aid of the following equation developed by H. Gröber and Sigmund Erk (8) for calculating the transmission of heat q through a circular area of radius r into a semi-infinite solid:

$$q = 4\,k\,\Delta t\,r \quad (4)$$

Applying the analogy between heat transfer and diffusion, the evaporation w_A may be written as

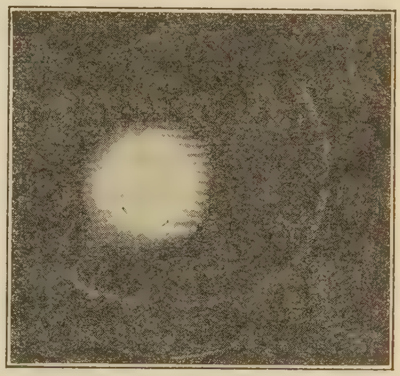


Fig. 4.—Typical Interference Bands Produced by Interferometer

$$w_A = \frac{4\,D_v\,\Delta c}{\pi\,r} \quad (5)$$

This equation is represented in Fig. 5 by the dashed line. The difference between this equation and the experimental values needs explanation. Eq. 5 applies only when the diffusing gas is moving through another stationary gas, which was not the case in these experiments. The movement of water vapor was aided materially by convection currents caused by the heated air next to the water surface, thus increasing the rate of evaporation considerably over that governed by diffusion alone. It is not necessary for the water surface to be warmer than the air, however, in order for convection currents to exist. Since water vapor is lighter than air, convection currents may be set up due to differences in density even when the temperatures of the water and the air at a distance are the same.

For this series of experiments there was a critical temperature of the water surface at which the layer of the air and water vapor immediately above it had the same density as at a distance. When these densities were equal, there should be no convection currents, and evaporation should be governed by diffusion alone. Below this critical temperature, the layer next to the water surface is cooler than the air above and does not rise. It probably increases in thickness until a slow movement occurs toward the edges of the pan. The evaporation rate for this condition should be slightly greater than that for diffusion due to the movement of the air in removing saturated vapor. Fig. 5 shows that a break in the curve, with a change in the rate of evaporation, actually occurs at a concentration difference of about 4.2 × 10^{-4} lb per cu ft. Under the conditions of these tests, the weight of a cubic foot of air and water vapor at a distance was 0.0743 lb per cu ft. Calculation shows that a mixture of saturated water vapor and air has the same unit weight at a temperature of 67.7° F and the unit weight of the water vapor will be 10.7 × 10^{-4} lb per cu ft. An average of test data for eight runs in the immediate vicinity of the break in the curve of Fig. 5 gave the water temperature as 68.2° F and the unit weight of water vapor at the water surface as 10.77 × 10^{-4} lb per cu ft. This is excellent agreement.

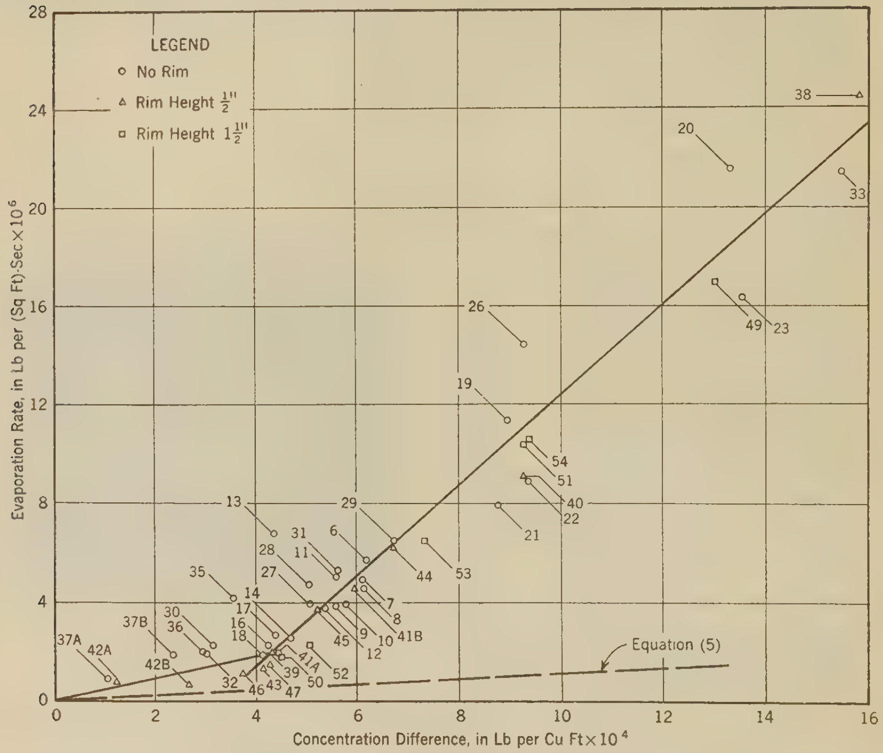


Fig. 5.—Evaporation Rate Versus Concentration Difference

Examination of Fig. 5 shows that Eq. 5 does not apply even when there seems to be no convection. This is probably due to the fact that it is practically impossible to bring the air absolutely to rest. Under the conditions given, the rate of evaporation given by Eq. 5 is 0.278 × 10^{-6} lb per (sq ft)-sec, and the corresponding velocity of the saturated vapor diffusing through air at rest is $V = \frac{0.278 \times 10^{-6}}{10.7 \times 10^{-4}} = 0.00026$ ft per sec. It is evident that a slight movement of air over the water surface would increase the removal of water vapor materially.

Dimensionless Representation.—The data of Fig. 5 for vapor transfer can be represented by dimensionless moduli in a manner similar to that for heat transfer by convection. In general, for heat transfer by free convection, experimental data may be represented by

$$N_{Nu} = f\,(N_{Gr}, N_{St}) \qquad \text{(6a)}$$

in which (see Eqs. 22*a*, 21*a*, and 24*a*, Appendix II): N_{Nu} = Nusselt's number; N_{Gr} = Grashof's number; and N_{St} = Stanton's number. Following the analogy between heat transfer and mass transfer, experimental data on evaporation in free convection may be represented by

$$(N_{Nu})' = f\,[(N_{Gr})', (N_{St})'] \qquad \text{(6b)}$$

in which (see Eqs. 22*b*, 21*b*, and 24*b*, Appendix II): $(N_{Nu})'$ = Nusselt's number primed; $(N_{Gr})'$ = Grashof's number primed; and $(N_{St})'$ = Stanton's number primed. Eq. 6*b* is commonly written in the form

$$(N_{Nu})' = K' \left[\frac{(N_{Gr})'}{(N_{St})'} \right]^{n} \qquad \text{(6c)}$$

in which K' = a coefficient and n = an exponent. Values of Nusselt's number primed, Grashof's number primed, and Stanton's number primed, and values of the Grashof-prime Stanton-prime ratio were computed from the experimental results for all the tests shown in Fig. 5. Nusselt's number primed has been plotted against the Grashof-prime Stanton-prime ratio in Fig. 6. The best straight line through the points can be represented by the

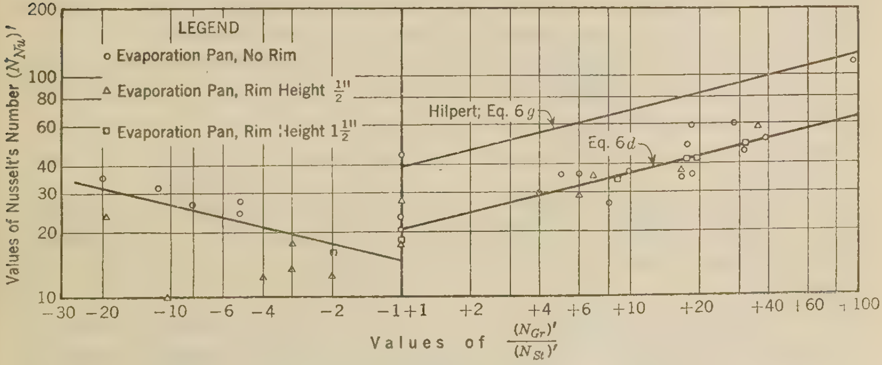


FIG. 6.—NUSSELT'S NUMBER $(N_{Nu})'$ VERSUS THE RATIO $\frac{(N_{Gr})'}{(N_{St})'}$

equation

$$(N_{Nu})' = 0.645 \left[\frac{(N_{Gr})'}{(N_{St})'} \right]^{0.25} \qquad \text{(6d)}$$

Here K' and n of Eq. 6*c* have values of 0.645 and 0.25, respectively.

The only experimental data available for comparison are those of R. Hilpert (9) on moist vertical plates which have been expressed by V. H. Cherry (10) as

$$(N_{Nu})' = 0.94\, (N_{Gr})' f\, (N_{St})' \qquad \text{(6e)}$$

If it be assumed that this can be written as

$$(N_{Nu})' = 0.94 \left[\frac{(N_{Gr})'}{(N_{St})'} \right]^{0.25} \qquad \text{(6f)}$$

and that $(N_{Nu})'$ should be increased 30% for the upper face of a horizontal surface as suggested by W. J. King (11),

$$(N_{Nu})' = 1.22 \left[\frac{(N_{Gr})'}{(N_{St})'} \right]^{0.25} \qquad \text{(6g)}$$

This equation has been plotted in Fig. 6.

Evaporation Into Air in Motion.—The general equations for diffusion of one gas through another gas in motion have not as yet yielded a direct solution to the problem of evaporation into atmosphere. F. Graham Millar (12) has succeeded in approximating actual evaporation records by calculations based on turbulence and eddy diffusivity, but is able to correlate Mr. Rohwer's work only by assuming Mr. Rohwer's anemometer to be in error. However, the diffusion equations have pointed the way to a correlation of experimental results by means of dimensionless moduli. The analogy between mass transfer and heat transfer by free convection has already been shown. A similar analogy for air in motion has been shown by T. K. Sherwood (13).

No experiments were performed to measure the evaporation into moving air, but it is possible to utilize the data of other experimenters. A satisfactory relationship can be expressed by an equation of the form

$$(N_{Nu})' = K\, (N_{Re})^{n} \qquad \text{(7a)}$$

n which N_{Re} = Reynolds' number. Eq. 7*a* can be valid only for values of $(N_{Nu})'$ greater than those for diffusion into still air since, for a given concentration difference, there is a minimum rate of evaporation which is governed by diffusion.

B. C. Shepherd, C. Hadlock, and R. C. Brewer (14) made experiments on evaporation from a free water surface and from saturated sand in pans 1 ft square in a wind tunnel. Water temperatures ranged from 86.4 to 132.8° F, vapor-concentration differences from 0.5×10^{-4} to 16.5×10^{-4} lb per cu ft, and wind velocities from 2.08 to 22.0 ft per sec. Although the data are slightly scattered, they may be represented by the equation

$$(N_{Nu})' = 0.103\, (N_{Re})^{0.75} \qquad \text{(7b)}$$

This equation is plotted in Fig. 7.

The inclusion of the data on evaporation from saturated sand is justified because no significant difference from evaporation from a water surface is evident. This is further substantiated by R. W. Powell (15), who performed experiments on evaporation from a free water surface and water retained by linen, filter paper, and gelatin.

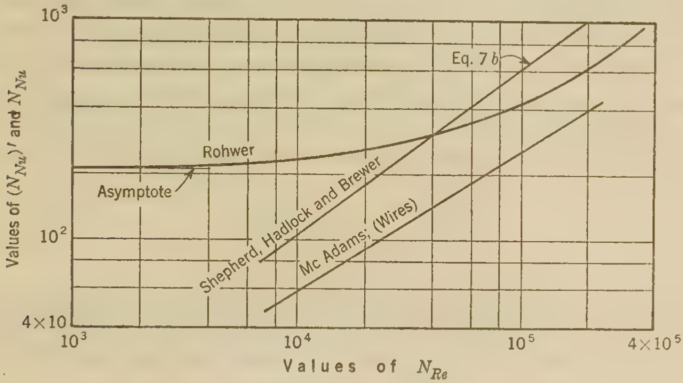


FIG. 7.—NUSSELT'S NUMBERS N_{Nu} AND $(N_{Nu})'$ VERSUS N_{Re}

Mr. Rohwer (16) made experiments on the rate of evaporation E from a 3-ft square pan in a tunnel with controlled wind velocities up to 17.5 ft per sec. He expressed his results in the equation

$$E = C\,(p_S - p_H) \qquad (8)$$

in which C is a constant coefficient and p_S and p_H, respectively, are the pressures of saturated vapor at the temperature of the water surface and the dewpoint. With an assumed average temperature for his experiments, it was possible to compute Nusselt's number primed and Reynolds' number. A curve based on Mr. Rohwer's experiments is plotted in Fig. 7.

These data, although not in sufficient detail to permit an accurate estimate of evaporation, tend to substantiate the analogy between heat and mass transfer. Further experimentation is needed in order to define the curve of $(N_{Nu})'$ versus N_{Re}. At present it may be assumed that $(N_{Nu})'$ varies approximately as the 0.75 power of N_{Re}, with a coefficient of about 0.1 or (compare Eq. 7b):

$$(N_{Nu})' = 0.10\,(N_{Re})^{0.75} \qquad (9)$$

CORRELATION OF RESULTS OF OTHER EXPERIMENTERS

Evaporation Into Still Air.—It has been shown that evaporation into still air probably can be expressed in the form

$$(N_{Nu})' = C\left[\frac{(N_{Gr})'}{(N_{St})'}\right]^{0.25} \qquad (10)$$

which is analogous to the equation used for heat transfer. For atmospheric conditions, $(N_{St})'$ is nearly constant, and may be neglected. Eq. 10 may then be written

$$\frac{h' L}{D_v} = C \left(\frac{L^3 \gamma \, \Delta\gamma}{g \, \mu^3} \right)^{0.25} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (11)$$

in which h' = coefficient of mass (vapor) transfer; L = a characteristic dimension of the surface; γ = unit weight of water vapor and air; g = acceleration of gravity; and μ = coefficient of absolute viscosity. Eq. 11 may be transformed by a number of substitutions and assumptions. The evaporation E is proportional to $h' \, \Delta c$. Also, $(N_{St})' = \frac{D_v \gamma}{g \mu}$ = constant, and g is constant. It may be assumed, furthermore, that $\Delta\gamma$ is proportional to Δc. Hence

$$E \propto \Delta c \left(\frac{\mu \, \Delta c}{\gamma^3 L} \right)^{0.25} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (12a)$$

The viscosity varies approximately as $T^{0.50}$ and the unit weight as $\frac{p}{T}$, so that Eq. 12*a* can be written in terms of unit pressure p and temperature,

$$E \propto \Delta c^{1.25} \frac{T^{0.875}}{L^{0.25} \, p^{0.75}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (12b)$$

The most important variable affecting the rate of evaporation is the difference in vapor concentration. This variable enters to the 1.25 power because of its effect on convection. The experiments on the 1-ft pan described in this paper showed that evaporation into still air varies as the 1.72 power of the concentration difference. J. W. Hinchley (17) found that the exponent was 1.2. J. R. Griffin (18) reports that his experiments combined with those of B. F. Sharpley (7) indicate an exponent of 1.25. Mr. Rohwer's data (16) give an exponent between 1.4 and 2.0. His apparatus was not completely shielded. Also he determined relative humidity at a distance from air which was drawn across the water surface before passing the psychrometer. Undoubtedly, the effect was a reduction in the apparent concentration difference. It also seems probable that the exponent is not constant because of the assumption made in deriving the equation. It was assumed that the difference in vapor concentration was proportional to the difference in unit weight. This would be true if the system were at one temperature throughout It was stated previously, however, that a vapor-concentration difference could exist without any difference in the unit weights because of the temperature difference. The vapor-concentration difference for which the difference in unit weight is zero depends on both the vapor concentration and the temperature of the air. This may explain the different results obtained by the various experimenters.

According to Eq. 12*b* the evaporation into still air may be expected to increase with the 0.875 power of the air temperature, and inversely as the 0.75 power of the air pressure. There are very few data available, however, on evaporation into still air and it is not possible to check these conclusions.

Most experimenters have developed formulas which express the evaporation in terms of vapor-concentration difference and wind movement in such a way that, for zero wind velocity, the evaporation is directly proportional to the concentration difference. The evaporation rate may be expected to vary inversely as the 0.25 power of the pan diameter. This effect is discussed in the next section. Because of the difficulty of producing absolutely quiet air, and the effect of even slight air movement, as shown previously, the discussion probably has more academic than real value.

Evaporation Into Air in Motion.—Following the analogy between heat transfer and mass transfer by free convection, if it is assumed that evaporation into wind can be expressed by

$$(N_{Nu})' = K\,(N_{Re})^{0.75} \qquad (13)$$

then

$$\frac{h'\,d}{D_v} = K\left(\frac{V\,\rho\,L}{\mu}\right)^{0.75} \qquad (14)$$

Making the same substitutions as before,

$$E \propto \Delta c\left(\frac{V^3\,\mu}{\gamma\,L}\right)^{0.25} \qquad (15a)$$

In terms of temperature and pressure,

$$E \propto \Delta c\,\frac{V^{0.75}\,T^{0.375}}{L^{0.25}\,p^{0.25}} \qquad (15b)$$

It will be noted that the effects of temperature and pressure are different in still and moving air. For high velocities, the exponents given by Eq. 15*b* may be expected to govern, and, for low velocities, they will tend to the value of Eq. 12*b*. It is interesting to examine the results of various experimenters in the light of Eq. 15*b*.

Practically all recent investigators find that evaporation is directly proportional to the difference in vapor concentration at the water surface and at a distance. This is the conclusion reached by Messrs. Rohwer (16), Shepherd, Hadlock, and Brewer (14), G. W. Himus (19), and Millar (12) among recent investigators and by the late D. FitzGerald, Hon. M. Am. Soc. C. E. (20), and J. A. Folse (21) earlier. The later experiments were conducted in wind tunnels with controlled velocities. For any velocity, it was found that evaporation was proportional to the vapor-concentration difference.

Recent investigators who disagree with this conclusion are H. C. Hickman, Assoc. M. Am. Soc. C. E. (22), G. N. Cox, M. Am. Soc. C. E. (23), and C. W. Thornthwaite and B. Holzman (6). Mr. Hickman found evaporation on the Great Lakes for zero vapor-concentration difference. His determination of vapor-pressure difference was based on but one observation of relative humidity per day which he applied to the average of the maximum and minimum air temperatures to obtain the average vapor concentration in the atmosphere. This is entirely in error since the relative humidity varies widely throughout the day and no single observation can be considered representative. It is quite probable that a vapor-concentration difference existed at the time the evaporation occurred. A more detailed discussion of Mr. Hickman's work, by the writer, has been published elsewhere (24).

Mr. Cox assumed "that the evaporation is a function of the difference between the vapor pressure as determined by the relative humidity and that which would exist for saturated vapor." This conclusion does not seem to be supported by other experimenters or to be justified by the kinetic theory. In fact, for a floating pan, Mr. Cox finds it necessary to introduce the difference of temperature between air and water in order to account for the variable effect of the vapor pressure at the water surface.

Messrs. Thornthwaite and Holzman point out that the movement of water vapor from water surface to air at a distance takes place by means of two entirely different processes—namely, by diffusion through a laminar boundary layer for a short distance and then by turbulence through the overlying atmosphere. They claim that no over-all transfer coefficient can account for the variations that occur in the boundary layer and the turbulent mass because of the varying degrees of roughness which are encountered and which affect both the thickness of the boundary layer and the degree of turbulence. They propose to measure evaporation by means of simultaneous readings of vapor-pressure concentration and wind velocity V at two different elevations H above the laminar layer. The roughness coefficient may be determined from the relationship

$$\log_e Z = \frac{V_2 \log_e H_1 - V_1 \log_e H_2}{V_2 - V_1} \quad \ldots\ldots\ldots\ldots\ldots (16)$$

Subscripts denote location of observation, 1 being the lower and 2 the upper location. The rate of vapor transfer, which is assumed to be equal to the rate of evaporation or condensation, is given by

$$E = \frac{(K_0)^2 (c_1 - c_2) V_2}{\log_e \frac{H_2}{H_1} \log_e \frac{H_2}{Z}} \quad \ldots\ldots\ldots\ldots\ldots (17)$$

This method is correct only for an adiabatic atmosphere and is subject to difficulties in the way of determination of constants; but it offers promise as a possible means of determining transpiration and evaporation from soils.

Effect of Wind Velocity.—According to Eq. 15*b*, the evaporation should vary as the 0.75 power of the wind velocity, other things being equal. Mr. Millar (12) found this to be the case for his experiments, and the data of Messrs. Shepherd, Hadlock, and Brewer (14) also conform to this criterion. Mr. Himus (19) found that evaporation varied approximately as the 0.77 power, and Mr. Powell and E. Griffiths' work (25) indicates a variation with the 0.85 power. The conclusions of Mr. Hickman (22) on this point are influenced by his erroneous determination of vapor concentration.

A number of investigators have used wind correction factors of the form $a + b\,V$, in which a and b are constants. A few are listed as follows:

Authority	Factor F
F. H. Bigelow (26)	$1 + 0.07\,V$
D. FitzGerald (20)	$1 + 0.5\,V$
A. F. Meyer (27)	$1 + 0.1\,V$
Carl Rohwer (16)	$1 + 0.268\,V$

Mr. Folse (21) proposed the following correction:

$$F = 0.358\,(V - 10.8) \qquad (18)$$

and R. E. Horton, M. Am. Soc. C. E. (28), gave a curve for velocities as great as 30 miles per hr. All these expressions except that of Mr. Folse are in a form which permits computation of evaporation at zero wind. Mr. Rohwer's experiments, however, indicated that evaporation into still air followed a different law. This is in accordance with the present analysis. Furthermore, zero wind does not mean the same for Mr. Rohwer's work as for that of the other investigators. Mr. Rohwer took considerable pains to see that the air surrounding his pan was quiet, whereas the other experiments were made outdoors where zero wind is that movement which is not recorded by an anemometer. The effect of even a slight air movement on evaporation has been discussed previously. For practical purposes, however, absolutely still air seldom exists and the form of the correction is suitable over an appropriate range of wind

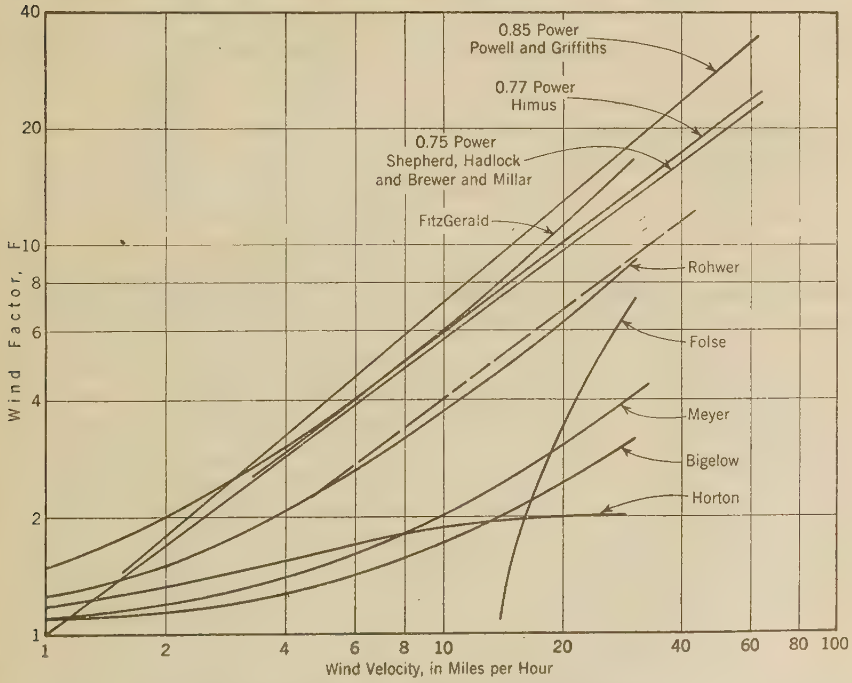


FIG. 8.—RELATION OF EVAPORATION TO WIND VELOCITY

velocity. Mr. Rohwer's correction, for example, conforms closely to the 0.75 power law between 5 and 30 miles per hr, whereas Mr. Bigelow's curve has the same slope between about 20 and 100 miles per hr. The exponential law has better theoretical and experimental support for velocities above about 2 miles per hr but cannot be used for extrapolation to zero wind. It can be very useful, however, in correlating results and in moderate extrapolation. The various correction factors are plotted in Fig. 8. The exponential forms have arbitrarily been given a value of unity at 1 mile per hr. For the purposes of this paper, the dashed line of Fig. 8 has been adopted as the best available wind correction factor.

Effect of Pan Diameter.—The effect of pan size has been studied by a number of investigators and the results are summarized herein.

TABLE 1.—EFFECT OF PAN SIZE ON EVAPORATION

No.	Description	Square pan	Circular pan	Reservoir
	Dimension	3 ft by 3 ft	Diameter, 4 ft	1,800 acres
1	Difference in vapor pressure (inches of mercury)	0.640	0.639	0.672
2	Wind velocity (miles per hour)	1.94	1.90	1.68
	Relative Evaporation:			
3	Observed	1.33	1.28	1.00
4	Corrected	1.32	1.29	1.00

Mr. Rohwer (1) compared evaporation from a 3-ft square pan and a 4-ft circular pan with that from a 1,800-acre reservoir with the results given in lines 1, 2, and 3, Table 1. Correcting the evaporation from the pans to the same vapor-pressure difference and wind velocity as that for the reservoir on the assumption that the evaporation varies directly with the vapor-pressure difference and with the wind as shown by the dashed curve in Fig. 8, the

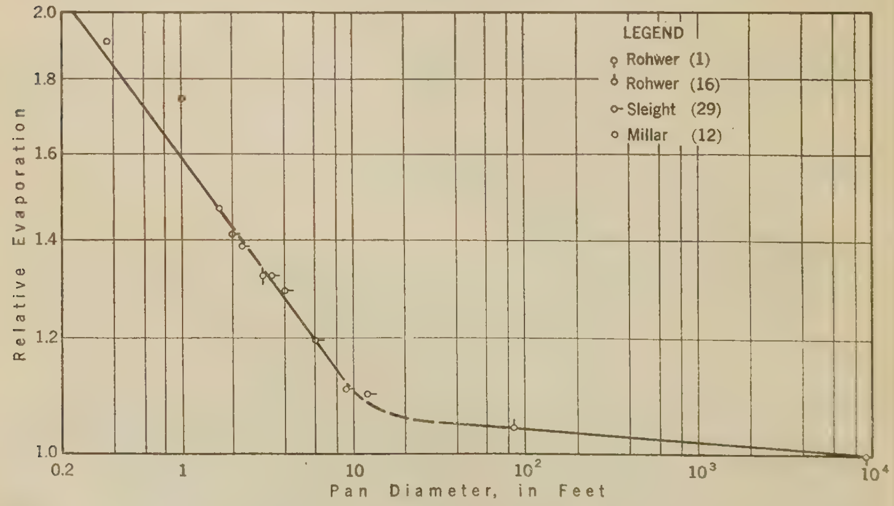


FIG. 9.—RELATION OF EVAPORATION TO DIAMETER OF SURFACE

relative evaporation rates are as shown in line 4, Table 1. The shape of the reservoir is not known, but if it is circular, its diameter is approximately 9,000 ft. These points are plotted in Fig. 9.

Mr. Rohwer (16) also compared evaporation from three different types of pans with that from an 85-ft reservoir. Only one of the pans was comparable, however, the others having different exposure. He found that evaporation from a 3-ft square pan was 26.6% greater than from the reservoir. Assigning the previous value of 1.32 to the 3-ft pan, the relative evaporation from the 85-ft reservoir is 1.043. This value is plotted in Fig. 9.

The late R. B. Sleight, Assoc. M. Am. Soc. C. E. (29), gave the relative rates of evaporation from circular pans of different diameters as indicated in Table 2. The last line of Table 2 gives the rate of evaporation relative to the 1,800-acre reservoir, assuming that the 4-ft pans of Messrs. Sleight and Rohwer were comparable. These data are also plotted in Fig. 9.

TABLE 2.—RELATIVE RATES OF EVAPORATION SHOWING EFFECT OF PAN SIZE ON EVAPORATION

Description	DIAMETER OF PAN, IN FEET							
	12	9	6	4	3.39	2.26	2	1
Compiled by R. B. Sleight	1.00	1.009	1.089	1.175	1.202	1.260	1.284	1.589
Compared with 1,800-acre reservoir as unity	1.099	1.108	1.196	1.290	1.320	1.383	1.410	1.745

Mr. Millar (12) found that the evaporation from a 50.8-cm pan was only 0.77 as great as from an 11.2-cm pan. Fig. 9 shows that the evaporation from a 50.8-cm (1.67-ft) pan relative to a large reservoir is about 1.47. The corresponding evaporation from the 11.2-cm (0.37-ft) pan is 1.91. This point is also plotted in Fig. 9. Two straight lines have been drawn through the points to represent the effect of pan size on evaporation. For pans smaller than, say, 10 ft, the evaporation varies as the − 0.16 power of the diameter, and for areas larger than 50 ft in diameter about as the − 0.01 power. It is doubtful if the transition occurs abruptly; it may be more gradual, as indicated by the dashed line.

The results are not in excellent agreement with Eq. 15*b*, but are close enough to indicate that the derivation of the equation may be valid.

Effect of Water Temperature.—According to Eq. 15*b*, the rate of evaporation should vary approximately with the 0.375 power of the absolute water temperature. The effect is difficult to find as a variation of 30° at a temperature of 70° F indicates a change in evaporation rate of only 2.1%. This is probably negligible in view of the other uncertainties.

Effect of Air Pressure.—A number of investigators have included a pressure term in their expression for evaporation. T. Russell (30) decided that evaporation varied inversely as the pressure, and Mr. Millar (12) decided it varied as the − 0.25 power. On the basis of scanty data, Mr. Sleight (29) gave values which indicated that the evaporation varied inversely as the square of the pressure. Mr. Rohwer corrected the evaporation computed for Fort Collins, Colo., to other elevations by use of the factor

$$G = 1.465 - 0.0186\,B \qquad (19)$$

in which B is the barometric pressure in inches of mercury. Mr. Millar found that Mr. Rohwer's data were consistent with the -0.25 power of the pressure.

TABLE 3.—EFFECT OF PRESSURE ON EVAPORATION

No.	Location (1)	Elevation (2)	Barometric pressure[a] (3)	Wind velocity[a] (4)	Air temperature[a] (5)	Daily evaporation[a] (6)	Vapor pressure[a] (7)	Wind factor $f(V)$ (8)	$f(B)$ (9)
1	Imperial, Calif......	−68	29.86	1.93	81.7	0.388	0.641	1.52	0.398
2	Fort Calhoun, Nebr.	1,160	28.77	1.35	67.1	0.175	0.300	1.36	0.428
3	Logan, Utah.......	4,778	25.07	1.89	68.0	0.291	0.470	1.50	0.412
4	Fort Collins, Colo...	5,000	25.03	1.01	55.0	0.149	0.268	1.27	0.430
5	Lake Tahoe, Calif...	6,300	23.76	1.02	55.1	0.222	0.366	1.27	0.478
6	Victor, Colo........	10,089	20.43[b]	3.06	50.5	0.161	0.186	1.81	0.478
7	Pikes Peak, Colo....	14,109	18.08	3.98	40.1	0.108	0.112	2.06	0.468
			18.04	10.9	41.4	0.202	0.123	4.23	0.388

[a] All mean values, with units as follows: Col. 3, inches of mercury; Col. 4, miles per hour; Col. 5, degrees Fahrenheit; Col. 6, inches; and Col. 7, vapor-pressure differences in inches of mercury. [b] Estimated from altitude.

This may be confirmed by a brief analysis of Mr. Rohwer's data on the basis of the previous discussion. His results are summarized in Cols. 3 to 7, Table 3. In accordance with the conclusions drawn regarding the variation of evaporation with vapor-concentration (vapor-pressure) difference, wind velocity, and temperature (neglected), the rate of evaporation can be expressed as

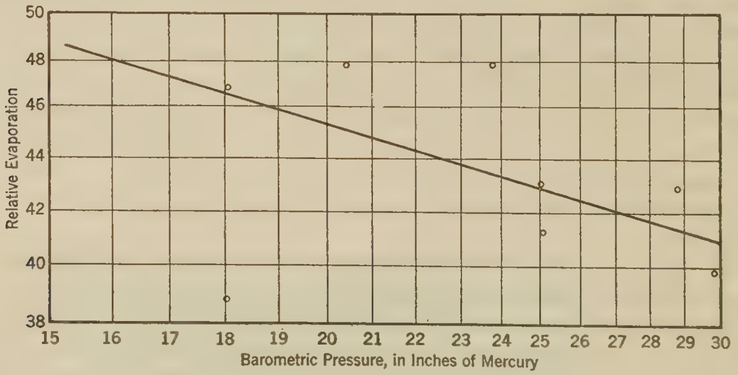


FIG. 10.—RELATION OF EVAPORATION TO BAROMETRIC PRESSURE

$$E \propto \Delta c\, f(V)\, f(B) \qquad (20)$$

in which $f(V)$ is a wind factor taken from Fig. 8. Values of $f(V)$ are given in Col. 8, Table 3.

Values of $f(B)$ obtained by dividing the observed evaporation by the vapor-pressure difference and the wind factor are given in Col. 9, Table 3. They are plotted against the pressure in Fig. 10. The line on this plate has been drawn arbitrarily with a slope of -0.25 and seems to represent the data as well as any other line that could be drawn through the points.

Eq. 15*b* is seen to be justified for representation of the effects on evaporation of vapor-pressure concentration difference, temperature, size of pan, air pressure, and wind velocities greater than about 5 miles per hr. It may be used for the correlation of experimental results and for moderate extrapolation.

Effect of Rim Height.—Three other variables have been observed to have an effect on evaporation from a pan although they have not been included in Eq. 15*b*. These are the height of rim above the water surface and the color and depth of the pan. R. W. Revell, Jun. Am. C. E. (31), attempted to evaluate the effect of rim height on evaporation from a Class A pan. He made experiments on pans having rim heights of 2 in. and 3 in. and found that more evaporation occurred into a wind from the pan with the 3-in. rim height during periods of moderate to high winds, but that less evaporation occurred from this pan during periods of low wind velocity. He suggested that the explanation of this difference in behavior lay in the difference in the size of the turbulent eddies caused by the passage of air over the rim of the pan. His data are so scattered that it is difficult to determine the effect quantitatively. He noted a higher water temperature in the pan with the 3-in. rim height, however, and attributed it in part to the lesser mass of water in the shallower pan. This assumption seems to be reasonable and is consistent with greater evaporation from this pan.

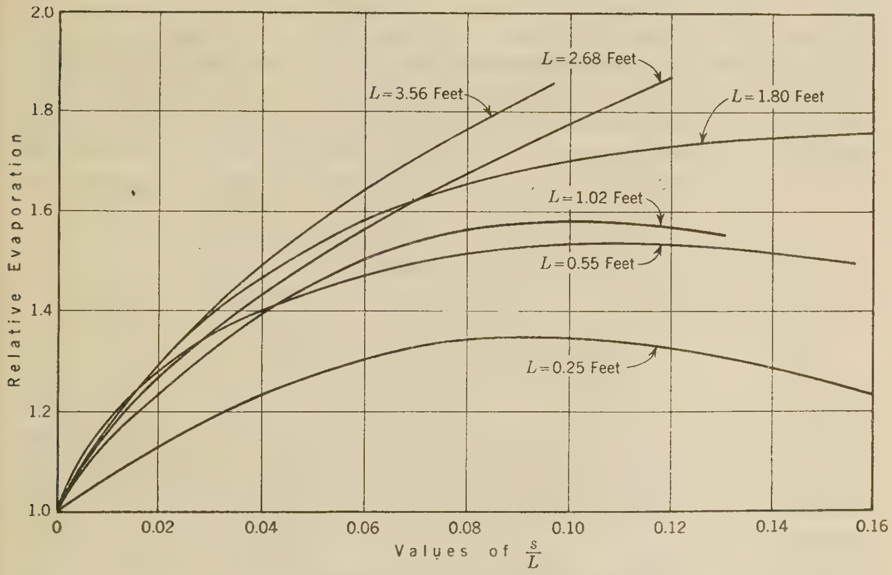


Fig. 11.—Effect of Rim Height on Evaporation

Mr. Powell (15) measured the evaporation from a cylindrical saturated surface in an air stream parallel to its axis. He found that the rate of evaporation was increased when a collar was placed around the cylinder at its leading end. He attributed the increase in evaporation to the increased turbulence of the air stream caused by the projecting collar. In Fig. 11, prepared from his data, the relative evaporation rate has been plotted against the ratio of height of collar to length of evaporating surface. It may be seen that the data for lengths greater than 2 ft and ratios of s/L less than about 0.08 may be represented with fair accuracy by a single curve. This is the range in which the Weather Bureau Standard Class A pan operates. The data by Mr. Powell are probably not strictly applicable to flat surfaces but the curve indicates that, if the rim height of a standard 4-ft pan be increased from 2 in. to 3 in. by evaporation or other loss, the relative evaporation rate in a wind of 6.7 miles per hr increases from 1.47 to 1.61, an increase of about 10%.

Effect of Pan Color.—The color of an evaporation pan has been observed to have an effect on the evaporation rate. This is undoubtedly due to the difference in absorption of solar and sky radiation which affects the water temperature and consequently the evaporation. A series of observations was made by the writer for one year, at Berkeley, on four pans of identical size, shape, and exposure. The pans were of galvanized steel, 1 ft square and 0.5 ft deep, and were buried in the ground so that the upper edge was just above the ground surface. They differed only in their color. Three were painted white, gray, and black, respectively, using lead base paint, and the fourth was left unpainted. Table 4(*a*) shows the results obtained. The

TABLE 4.—EFFECTS OF PAN COLOR AND DEPTH ON RATE OF EVAPORATION

(Inches per Month)

Month	(*a*) COLOR OF PAN:				(*b*) DEPTH OF PAN (FEET):			
	White	Gray	Galvanized	Black	0.25	0.50	1.00	2.00
1933								
May[a]	3.42	3.79	3.72	4.14	4.06	3.72	3.74	3.92
June	7.20	8.10	8.12	8.52	8.43	8.12	8.26	8.43
July	8.69	9.13	8.94	9.85	9.14	8.94	9.07	9.68
August	7.27	7.41	7.57	7.89	7.42	7.57	7.85	8.38
September	5.77	5.90	6.09	5.94	6.01	6.09	6.31	7.04
Sub-total	....	....	....	....	35.06	34.44	35.23	37.45
October	4.09	4.08	4.12	4.04	3.36	4.12	4.33	4.52
November	2.19	2.08	2.29	2.21	1.95	2.29	2.62	3.10
December[b]	0.69	0.61	0.59	0.68	0.52	0.59	0.73	0.85
1934								
January	1.22	1.25	1.25	1.29	1.06	1.25	1.34	1.54
February	1.31	1.21	1.07	1.35	1.42	1.07	1.25	1.27
March	2.50	2.79	2.46	3.00	2.74	2.46	2.50	2.65
April	5.28	5.51	5.50	5.96	5.26	5.50	5.47	5.74
Total	49.63	51.86	51.72	54.87	51.37	51.72	53.47	57.12

[a] Sixteen days only.

[b] Twenty-three days only.

black pan which should have had the greatest absorptivity showed 10.8% more evaporation for the year than the white pan and 5.8% more than the gray pan. The gray and galvanized pans showed almost the same results. The differences were not constant throughout the year, however, and, in some months, the annual trend was reversed. For example, from November to February, the evaporation from the white pan was greater than that from the gray pan. Significantly, the greatest differences were found during the months of maximum solar radiation. Similar effects have been noted by other investigators. Mr. Cox (23) found that, when a galvanized pan was painted with an asphaltic paint, the evaporation rate was increased about 10%. Mr. Rohwer (16) found a difference of 4% between two different brands of asphaltic paint.

Effect of Pan Depth.—The depth of an evaporation pan also has an effect on the evaporation rate. Very few data are available for exposed land pans but a few comparisons have been made for buried pans. The writer measured evaporation from four galvanized pans 1 ft square with depths of 3, 6, 12, and 24 in., respectively. The data summarized in Table 4(*b*) indicate that evaporation increased as the pan depth increased. This is not borne out by Mr. Sleight's results (29), which are as follows:

Depth of pan (ft)	Relative evaporation
0.25	127.9
0.75	126.8
1.25	127.9
1.75	127.9
2.75	128.4
5.75	127.4

The lack of agreement may be explained in part by the fact that Mr. Sleight's observations, with the exception of those on the 2.75-ft pan, were made during the period, May to September, whereas the Berkeley tests extended throughout the year.

Inspection of Table 4(*b*) shows that the relative evaporation for the Berkeley tests varies with the season. The totals for the shorter period are more nearly in agreement with Mr. Sleight's results in that the evaporation from the 3-in. pan is seen to be greater than that from either the 6-in. or the 9-in. pan. The temperature of the deeper pans appeared to be less affected by radiation than that of the shallowest pan because their masses were proportionately greater relative to the surface which received and emitted radiant energy. This led to higher temperature in summer and lower in winter, with a corresponding effect on the relative evaporation rates. Also, the deeper pans remained at a more nearly constant temperature because of the stabilizing effect of the earth with which they were in contact.

Correlation by Other Investigators.—Other investigators have also attempted to correlate their own results with those of others. Mr. Millar (12) was able to explain Mr. Rohwer's results with a fair degree of satisfaction, although he found what seemed to be a discrepancy in the wind velocities observed by Mr. Rohwer.

Mr. Powell (15) plotted $\frac{E\,L}{p_S - p_H}$ against $V\,L$ and was able to obtain a fair correlation of the results reported by Messrs. Millar (12), S. K. Banerji and H. M. Wadia (32), FitzGerald (20), T. B. Hine (33), Rohwer (16), H. Thiesenhusen (34), M. Lurie and N. Michailoff (35), and Shepherd, Hadlock, and Brewer (14). This excellent contribution should be read by all who are interested in this phase of the subject.

The Energy Balance Method of Calculating Evaporation.—Evaporation has been computed by several investigators by means of an energy balance. In this method, the total energy input (solar radiation), less the energy losses (back radiation, conduction, convection, increase of water temperature, and evaporation) must equal zero. If all quantities are known except the evaporation, this is determinable as a residual. The method has been described by Wilhelm Schmidt (36) and A. Angstrom (37) and later by N. W. Cummings (38)(39), Burt Richardson (39)(40), and others. These writers applied the method to lakes and oceans so that conduction through the walls was not a problem as with a pan. Messrs. Richardson and Cummings also used insulated pans.

In all cases the heat lost by convection has been an uncertain quantity. It was assumed by Mr. Schmidt that there was a ratio between the heat lost by convection and that used in evaporation since the processes of heat transfer by convection are analogous. I. S. Bowen (41) made the same assumption later and wrote at length to substantiate it, although in the meantime Mr. Angstrom (37) had shown the impossibility of such a ratio, pointing out that, if it existed, there would be, for a temperature gradient of zero, either no convection and no evaporation, or convection and a corresponding evaporation. It is obvious that both conditions are absurd. The first condition might be satisfied if the atmosphere were saturated with water vapor, but the second condition is impossible. Furthermore, Mr. Bowen assumed equilibrium conditions which seldom exist. Water temperatures are constantly rising and falling, so that the storage of heat is a considerable item in the energy balance, at times amounting to many times the energy used for evaporation. Under these conditions, the ratio varies considerably and, in fact, loses its significance entirely. Mr. Bowen's ratio can be significant only if applied to average conditions over a period of time long enough so that storage of heat is not a factor.

EVAPORATION MEASUREMENT IN THE FIELD

Instrumental Observation.—The factors entering Eq. 15*b* are the evaporation rate, the vapor-pressure difference, the wind velocity, the air temperature, the pan diameter, and the barometric pressure. The determination of these factors and their relationships requires field measurements of water surface temperature, air temperature, relative humidity, wind velocity, pan diameter, and barometric pressure. The methods of making these measurements depend somewhat on the purpose for which they are desired. Daily averages may be suitable for comparing evaporation at two stations which have similar climatic conditions, but are valueless for purposes of careful investigation. The distribution and range of temperature and humidity, for example, throughout the day, have a marked effect on the total evaporation. Most of the day's evaporation occurs when the water temperature is higher than the average, and the vapor pressure at the water surface is high. If the temperature were uniform throughout the day, the higher vapor pressure causing evaporation would not exist. Hence the average temperature has no particular significance with respect to daily evaporation, unless the range of temperature is also specified. Similar considerations apply to the measurement of air temperature and relative humidity since they govern the vapor pressure of the atmosphere. Probably much of the difficulty that is experienced in correlating field observations of evaporation lies in the indiscriminate use of daily averages for temperature and humidity.

A suitable method of measuring the evaporation rate depends on the purpose for which the information is desired. A careful study of evaporation requires that its rate be determined over short periods while the accompanying conditions are sensibly constant. For field data, daily totals will often be sufficient. The optical interferometer described in this paper is sensitive to half the wave length of the light used (about 0.00001 in. for green light), and is quite satisfactory where evaporation rates are desired over very short periods and where the instrument can be set on a very rigid support. It is suitable for experimentation under controlled conditions. Mr. Rohwer (1) described an optical lever arrangement that gave good results in the laboratory. The Tennessee Valley Authority secures a continuous record of evaporation from a standard Class A pan by mounting it on a balance with an attached device that prints the total weight of platform, pan, and water every 15 min. The standard hook gage is quite suitable for measuring daily totals, but is not accurate enough for short-period measurements.

The most important variable in Eq. 15*b*, after the evaporation rate, is the vapor-concentration difference. Its evaluation requires measurement of water temperature, air temperature, and relative humidity. The mechanics of the kinetic theory indicates that the water temperature should be measured as close to the surface as possible. The vapor concentration immediately above the water surface is dependent on the activity (temperature) of the molecules which pass through the surface. The temperature of a layer of water having a thickness equal to the mean free length of path of the molecules next to the surface is the temperature which governs the vapor concentration immediately above the water surface, and the rate of evaporation. This layer is very thin, probably only a fraction of a millimeter. Normally the water temperature increases from sunrise until about 2 p.m. by reason of solar radiation. During this period, the water temperature is highest at the surface. This is explained by Fig. 12, which shows that about 18% of the solar energy incident on the water surface is absorbed in the first millimeter of depth, causing a localized heating at the surface. The surface water, being warmer, does not sink, but remains on top. Typical measurements made in the middle vertical of a standard Class A evaporation pan with a mercury thermometer showed a temperature of 69° F with the bulb just under the surface, 67° one inch down, $65\frac{1}{2}$° at mid-depth, and $64\frac{1}{2}$° at the bottom. Since the bulb of the thermometer was about 4 mm in diameter, the temperature in the top millimeter was undoubtedly still higher. A small thermocouple placed just under the surface will give good results in laboratory work if properly shielded. It is not convenient for most installations since suitable potentiometers and recording equipment are expensive. Measurement of temperature with a copper-constantan thermocouple to the nearest tenth degree Fahrenheit requires measurement of potential with an accuracy of 2 microvolts. Satisfactory correlation of field observations probably can be obtained by the use of a thermometer with a cylindrical bulb placed horizontally in the upper quarter inch of water. It may be necessary to support it by means of a float where evaporation exceeds 0.1 in. per day to insure that the bulb is always submerged. Of course, the instrument should be shielded.

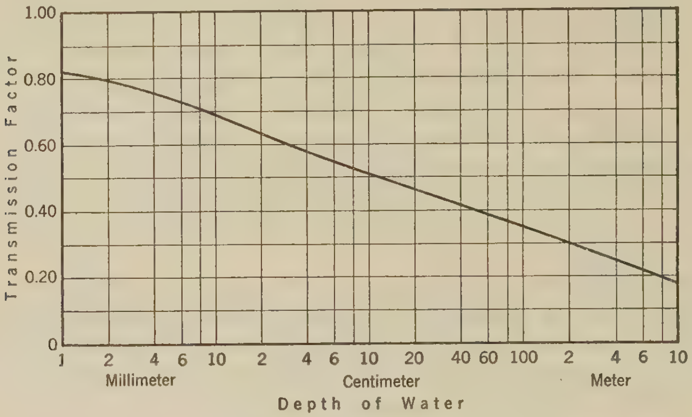


FIG. 12.—TRANSMISSIVITY OF WATER FOR SOLAR RADIATION

The most consistent correlation of evaporation rate with vapor-concentration difference has been found in the work of those experimenters who used pans heated from beneath rather than pans heated at the surface by solar radiation. Undoubtedly this is due largely to the fact that the experimenters on outdoor pans used average daily temperature, but may also be due, in part at least, to the difficulty of measuring the actual surface temperature.

Measurement of air temperature and relative humidity offers no particular difficulty. Suitable recording instruments are standard meteorological equipment. Care should be taken, however, that the measurements are made at a sufficient distance from the pan to be representative of conditions at a distance.

It is desirable to measure the wind velocity close to the surface of the pan, since practically all field installations as well as experimental laboratory tests have been made this way. The standard Weather Bureau anemometer as installed at Class A evaporation pans should be satisfactory.

The daily fluctuations of barometric pressure have a very slight effect on the evaporation rate. Barometric pressure needs to be considered only when comparing results obtained from installations at different altitudes. For this purpose, the average pressure corresponding to the altitude is sufficiently accurate.

Type of Evaporation Pan.—The pan itself has considerable effect on the evaporation and it should be standardized more carefully than has been common in past practice. It should be as large as can be handled conveniently since the change of evaporation rate with change of diameter becomes less as the diameter increases, making the results more reliable. The evaporation from a large pan also approaches more closely the evaporation from a lake, although this is not necessarily an advantage.

Mr. Powell's experiments show that the ratio of rim height to diameter has considerable effect on the evaporation rate. Fig. 11 indicates that a change in rim height for a given diameter has least effect on the evaporation rate when the ratio of rim height to diameter is about 0.1. Greater evaporation with an increased height of rim has been noted but has been variously attributed to the greater amount of solar radiation intercepted, and to evaporation from the water film drawn up the side of the pan by capillarity. The effect of rim height for a floating pan may be different from that for an exposed land pan. Further experimentation on this subject appears to be required. Until better information is available, it would seem desirable to maintain the rim height, or the depth of water in the pan, as nearly constant as possible.

The color and surface characteristics of the pan walls have an effect on the evaporation rate because of the variation in the absorptivity. The effect is not constant, but varies seasonally, although over a year's time a black pan will evaporate more water than a white one. For comparable results, pans should have similar surfaces which should be carefully maintained. The natural oxidation of a galvanized surface changes its absorptivity. Failure to clean pans frequently results in the growth of algae and a change in the characteristics of the surface.

Considerable amounts of heat are transferred through the sides and bottom of an uninsulated evaporation pan. It is evident that the ratio of wall area to surface area will affect the evaporation rate. Similar pans exposed under identical atmospheric conditions, but at different latitudes, will show differences in evaporation rate caused by the change in the angle of incidence of solar radiation on water surface and pan walls. The differences found between floating and buried pans as compared with an exposed pan can be accounted for by the difference in the amount of heat transferred through the pan walls. The amount of heat transferred can be calculated with approximate accuracy for the exposed and floating pans where the air and water temperatures, respectively, are known. It is much more difficult for the buried pan where the thermal conductivity of the soil is uncertain, varying with the moisture content, and the soil temperature is constantly fluctuating. The evaporation from a uninsulated pan is also affected by changes in the absorptivity of the outer wall of the pan. These difficulties can be most easily avoided by the use of an insulated pan. Further, the insulated pan has characteristics similar in many ways to a large body of water. Since heat transfer through the walls and bottom is minimized, the boundary or edge effect from this cause is made very small and the evaporation from the pan area approaches that from the larger body. The corrections depend largely on the pan geometry rather than on the meteorological conditions, which vary both locally and seasonally.

There seems to be no argument in favor of an uninsulated pan except cheapness. The cost of insulating a pan is not great. It may be argued that records from the standard Class A pan have the advantage of being comparable with each other. In view of the manner in which the data accompanying evaporation are obtained and recorded, it is doubtful if comparisons of results from Class A pans have much value.

In this connection, the writer wishes to call attention to an admirable discussion by Charles H. Lee, M. Am. Soc. C. E. (42), of the most important factors affecting evaporation, the shortcomings of the Class A pan, and the general confusion attending the subject of evaporation. Although first published in August, 1926, it appears not to have attracted the attention it deserves. Two quotations may not be out of place:

> "The writer's experience has led to the conclusion that in order to use data obtained from the Class A land pan it is necessary to determine a convection factor, which will differ for every locality or meteorological condition."
>
> "* * * much of the confusion and uncertainty which exists regarding evaporation rates is due to a failure to consider the fundamental factors in choosing observational methods."

Suggestions for Further Study

It is suggested that further study of the subject of evaporation from the standpoint of practical use be along the following lines:

1. The establishment of a standard evaporation station with accompanying measuring instruments designed to recognize the fundamental principle of evaporation and to measure the significant variable factors in such a way that they may be useful in correlating the observational results.
2. A study of the probable heat-transfer losses through the walls of uninsulated pans for an average day for each month of the year at all stations where there are significant changes in air temperature, humidity, or insolation. Such a study would make it possible to recompute the recorded evaporation in terms of that from an insulated pan. Although the original data are not in the best form, it is possible that a much better correlation of existing data could be obtained in this manner than is now available.
3. A study of the relationships between the existing data and those obtained from the new standard pan with the purpose of correlating all records as far as possible.

Acknowledgment

This paper is a condensation and revision of a dissertation by the writer entitled "Evaporation from a Limited Free Water Surface." It was submitted in July, 1939, in partial fulfilment of the requirements for the degree of Doctor of Philosophy in Mechanical Engineering in the Graduate Division of the University of California.

The writer wishes particularly to acknowledge the interest, encouragement, and advice offered during the study by Professors L. M. K. Boelter, M. P. O'Brien, M. Am. Soc. C. E., and L. B. Loeb of the University of California, and the cooperation of the Tennessee Valley Authority through A. S. Fry, M. Am. Soc. C. E., Head Hydraulic Engineer, in the use of its experimental evaporation apparatus.

A copy of the dissertation, containing observational data, details of calculation, and additional technical discussions, has been placed on file in the Engineering Societies Library for further reference.

APPENDIX I

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APPENDIX II

NOMENCLATURE

In dealing with material drawn from many sources, the meanings of symbols inevitably become confused necessitating changes in the notation used by the original authors. The following symbols, defined where they first appear, conform essentially with American Standard Letter Symbols for Heat and Thermodynamics Including Heat Flow,[3] prepared by a Committee of the American Standards Association, with Society participation, and approved by the Association in 1943:

A = area of water surface;
a = spacing of etched reference lines on a glass plate (see also b);
B = barometric pressure;
b = a constant expressing the form of a wind correction factor; as $a + b\,V$;
C = coefficient in Eqs. 10 and 11;
C_R = coefficient;
c = unit vapor concentration; Δc = concentration difference;
c_p = specific heat at constant pressure;
D_v = diffusivity;
E = depth of evaporation;
e = base of natural logarithms;
F = a wind correction factor (Eq. 18);
G = a barometric pressure correction factor (Eq. 19);
g = acceleration of gravity;
H = elevation above ground surface (Eq. 16);
h = coefficient of heat transfer;
h' = coefficient of mass (vapor) transfer;
K = coefficient in Eqs. 7, 13, and 14;
K' = coefficient in Eq. 6c;
K_0 = von Kármán's universal roughness coefficient;
k = thermal conductivity;
L = characteristic dimension of a surface; diameter of an evaporation pan; length of a cylinder;
l = length of lever arm of interferometer evaporimeter;
N = number of interference bands; also a dimensionless number (Eqs. 6); when primed, these dimensionless numbers characterize mass transfer;
ΔN = change in number of interference bands (Eq. 1);
N_{Nu} = Nusselt's number;
N_{Gr} = Grashof's number;
N_{Re} = Reynolds' number;
N_{St} = Stanton's number;
n = exponent in Eqs. 6 and 7;
p = unit pressure;
p_S = pressure of saturated vapor at the temperature of the water surface;
p_H = pressure of saturated vapor at the temperature of the dewpoint;
q = rate of heat transmission;
r = radius;
s = height of a collar on a cylinder;
T = absolute temperature;
t = thermometric temperature;

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[3] ASA — Z10.4 — 1943.

V = wind velocity;
w = flow rate of vapor;
w_A = flow rate of vapor per unit area;
x = length along path of heat flow or vapor transfer;
Z = coefficient of roughness in Eq. 16;
β = coefficient of expansion;
γ = unit weight of mixture of air and water vapor; $\Delta\gamma$ = difference in unit weights;
λ = wave length of light;
μ = coefficient of absolute viscosity;
ρ = mass density.

Dimensionless Moduli.—Dimensionless groups have been used freely throughout the paper. Because of their usefulness in classifying physical phenomena and their independence of dimensions, they are widely used for correlation of experimental data. In applying these dimensionless groups, physical properties were evaluated at the arithmetic mean temperature of the fluid.

Grashof's number—

$$N_{Gr} = \frac{L^3 \, \Delta t \, \beta \, \gamma^2}{g \, \mu^3} \quad \text{(21a)}$$

—characterizes the movement of a gas or liquid under the influence of inertia forces and the buoyancy caused by temperature differences. It is associated with heat transfer by convection. Grashof primed—

$$(N_{Gr})' = \frac{L^3 \, \gamma \, \Delta\gamma}{g \, \mu^3} \quad \text{(21b)}$$

—is similar to the Grashof number but characterizes convection associated with mass transfer.

Nusselt's number—

$$N_{Nu} = \frac{h \, L}{k} \quad \text{(22a)}$$

—characterizes the phenomenon of heat conduction between a solid and a gas or liquid. Nusselt primed—

$$(N_{Nu})' = \frac{h' \, L}{k} \quad \text{(22b)}$$

—is similar to Nusselt's number but characterizes mass transfer between a solid and a gas, as in evaporation.

Reynolds' number—

$$N_{Re} = \frac{V \, \rho \, L}{\mu} \quad \text{(23)}$$

—characterizes the movement of a gas or liquid under the influence of inertia and viscous forces.

Stanton's number—

$$N_{St} = \frac{k}{\mu\, c_p\, g} \quad \text{(24a)}$$

—characterizes the conduction of heat into a viscous fluid which is in motion. Stanton primed—

$$(N_{St})' = \frac{D_v \gamma}{\mu\, g} \quad \text{(24b)}$$

—is similar to Stanton's number but is applicable to mass transfer into a moving viscous fluid, as evaporation into air.

DISCUSSION

Carl Rohwer,[4] M. Am. Soc. C. E.—Observations on the evaporation from free water surfaces in pans are usually made either to discover the fundamental laws showing the relation between the meteorological data and evaporation or to make available information from which it will be possible to estimate the loss by evaporation from large bodies of water such as lakes, reservoirs, and streams.

Because of the complex nature of evaporation phenomena and the many dependent variables involved, investigators have resorted to various expedients when developing evaporation formulas to simplify the analysis of the effect of the influencing factors. Since the laws governing certain similar physical phenomena such as turbulence, heat conduction, and mass transfer have been fairly well established, the analogies between these phenomena and evaporation are sometimes used as a basis for attacking the problem of determining the laws of evaporation. This is the method adopted by the author. However, evaporation laws cannot be developed by analogy alone because there are certain constants involved which must be determined experimentally, and for which carefully controlled evaporation observations are necessary.

The evaporation formula (Eq. 5) based on analogy between heat transfer and diffusion developed by the author is represented by the dashed line in Fig. 5. Evaporation data obtained by the author, by means of the optical interferometer designed by him, when plotted on the same figure, result in the two full lines that meet at the point where the vapor-concentration difference is 4.2×10^{-4} lb per cu ft. Since the line representing Eq. 5 and the lower part of the line representing the experimental data both pass through the origin, Eq. 5 could be made to conform to the experimental data by multiplying by a constant. However, Eq. 5 cannot be made to conform to the remainder of the experimental data by a simple transformation. The fact that there is a break in the line indicates that a different law applies beyond this point.

As stated by the author, Eq. 5 applies only to still air conditions which probably are never obtained when conducting evaporation experiments even under controlled laboratory conditions. Although all external disturbances are excluded, any difference in temperature or density will cause the air to move. This is confirmed by the experience of the writer in conducting evaporation experiments under controlled conditions in the laboratory at Fort Collins (16*a*), although the exact significance of the phenomenon was not recognized at the time. The evaporation was at a maximum when the water was warmer than the air. When this occurred, the air in contact with the water would be warmer than the surrounding air and would rise, carrying with it a certain amount of water vapor which would have had to be removed by diffusion if the air were absolutely still. When the water was colder than the air, the density gradient was reversed and the air would remain in contact with the water surface because it was held there by the rim of the tank. When this occurred the water vapor could escape only by diffusion, which is a slow process. These explanations, however, do not account for the fact that the observed evaporation below the break in the curve (Fig. 5) is about four times that computed by Eq. 5. In fact, the discrepancy beyond the break in the curve is so great that it seems doubtful whether the analogy of mass transfer is valid.

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[4] Irrig. Engr., Div. of Irrig., SCS, U. S. Dept. of Agriculture, Colorado Experiment Station, Fort Collins, Colo.

According to the author's analysis the evaporation should vary as the 0.75 power of the wind velocity. The results obtained by several investigators are shown plotted in Fig. 8. Although the total effect of wind found by these investigators varies considerably, with one exception the results conform to the 0.75 power at the higher velocities. Whether this is a fundamental difference or whether it is due to the uncertainty of wind measurements at low velocities is not known. The author states that F. Graham Millar was able to approximate actual evaporation records on the basis of turbulence and eddy diffusivity, but could correlate the writer's data only on the assumption that the anemometer was in error. Cup anemometers probably are not the most accurate devices for measuring wind velocity, but they are almost universally used and for this reason they were adopted for the experiments at Fort Collins. Although the indicated velocities differ from the true velocities, particularly at high velocities, similar differences would occur whenever the anemometers were used and consequently comparable results would be obtained.

The effect of pan diameter on evaporation has been plotted by the author in Fig. 9, showing that beyond 10 ft the diameter has very little effect on the evaporation. This is the generally accepted conclusion. However, the author's formula for evaporation into air in motion, Eq. 15*b*, indicates that the evaporation is inversely proportional to the fourth root of the diameter of the pan. According to this formula, the evaporation from a 10-ft pan would be more than five times as great as that from a pan 9,000 ft in diameter under similar conditions instead of about 10% greater as indicated in Fig. 9.

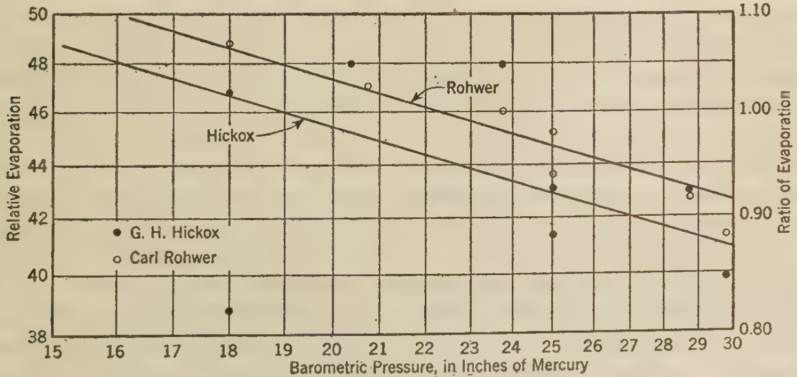

FIG. 13.—RELATION OF EVAPORATION TO BAROMETRIC PRESSURE BASED ON DIFFERENT ANALYSES OF THE SAME DATA

Fig. 10 shows the relation between the barometric pressure and the relative evaporation at different altitudes based on data obtained by the writer, recomputed in accordance with Eq. 20. Fig. 13 shows the same data and in addition the relative evaporation as computed by the writer (16*b*). It will be observed that the points fall much closer to a line with a slope of −0.25 than do those computed by the author. This indicates that the empirical analysis used by the writer gives results that are more nearly in accord with the facts than the analysis based on the analogy between evaporation and other physical phenomena.

The Sub-Committee on Evaporation of the Special Committee on Irrigation Hydraulics of the Society (2) adopted 0.70 as the ratio of the annual evaporation from a Weather Bureau pan to that from a reservoir. The author states (see heading, "Introduction") that the ratio is probably not constant throughout the year. Although this view is proved by the writer's experiments at Fort Collins (16*c*), it is doubtful whether it is worth while to use a different ratio for each month because conditions vary from year to year on account of seasonal differences. Table 5 (computed by the writer from data collected by A. A. Young, Assoc. M. Am. Soc. C. E. (43)(44), for the Division of Irrigation, Soil Conservation Service, U. S. Department of Agriculture) shows that the ratio reached a maximum during the cold months in the fall and winter at Elsinore, Calif., whereas at Fullerton, Calif., the maximum ratio occurred during the warm months of the year. The mean values for these two ratios for the entire year are, respectively, 0.77 and 0.78 instead of 0.70 as recommended by the writer and by the Sub-Committee on Evaporation of the Special Committee on Irrigation Hydraulics of the Society. There was little change in the ratios from year to year. These results support the author's contention that the ratios would not be the same under Southern California conditions as they would be in areas where the evaporation is very small during the winter months.

TABLE 5.—MONTHLY RATIOS FOR THE CONVERSION OF THE EVAPORATION FROM A WEATHER BUREAU PAN TO THE EVAPORATION FROM OTHER WATER BODIES

Month	(*a*) LAKE ELSINORE, CALIFORNIA				(*b*) 12-FT SUNKEN PAN AT FULLERTON, CALIF.				
	1939	1940	1941	Mean	1936	1937	1938	1939	Mean
January	0.90	0.85	0.65	0.80	....	0.65	0.65	0.66	0.65
February	0.69	0.66	0.51	0.62	....	....	0.67	0.71	0.69
March	0.79	0.58	0.68	0.68	0.77	0.73	0.75	0.80	0.76
April	0.59	0.68	0.74	0.67	0.79	0.79	0.79	0.81	0.80
May	0.72	0.66	0.66	0.68	0.80	0.82	0.78	0.83	0.81
June	0.78	0.77	0.75	0.77	0.81	0.79	0.85	0.82	0.82
July	0.75	0.72	0.75	0.74	0.79	0.81	0.81	0.83	0.81
August	0.78	0.69	0.87	0.78	0.79	0.83	0.79	0.81	0.80
September	0.84	0.84	0.92	0.87	0.79	0.78	0.78	0.71	0.76
October	0.94	0.90	0.97	0.94	0.75	0.74	0.78	0.74	0.75
November	0.90	1.08	0.91	0.96	0.68	0.75	0.72	0.76	0.73
December	0.93	1.11	0.78	0.94	0.57	0.72	0.65	0.74	0.67
Mean (weighted)	(0.78)	(0.76)	(0.78)	(0.77)	(0.77)	(0.78)	(0.77)	(0.78)	(0.78)

The optical interferometer developed by the author is an ingenious device for measuring very small rates of evaporation with precision. It is unfortunate that the experiments with this device were limited to a narrow range of conditions. A series of observations under controlled conditions in the laboratory on the evaporation from pans of different sizes into air moving at definite velocities would have made it possible for Mr. Hickox to check the accuracy of Eq. 15*b* without making assumptions as to the conditions under which the tests were made.

A study of the device raises the questions of whether the plate glass reflector F, in Fig. 3, would not reflect the beam of light from both surfaces of the plate glass and of whether, as a result, two sets of interference bands would appear to the observer. The one from the interior face of the glass plate would be less distinct, but, when the dark bands of one set were superimposed over the light band of the other, the difficulty of counting the bands would be increased. This possibility is mentioned because of a similar difficulty experienced by the writer in making evaporation observations by the use of the optical lever. Until the plate glass mirror first used was replaced by a metal mirror with only one reflecting surface, two distinct images of the scale on which the evaporation was read appeared in the telescope. The use of a metal mirror completely eliminated the second image.

After studying the paper the reader cannot help but feel that there is considerable doubt as to whether the analogy used is suitable for interpreting evaporation phenomena. At least the available data do not fit the theory very well. The writer does not wish to leave the impression that he is attempting to discredit Mr. Hickox's work even though the results are not conclusive. Much of the work on evaporation has been on an empirical basis and any attempt at a rational analysis of the problem should be commended.

C. W. Thornthwaite,[5] Esq. —Although the author is reporting original observations of evaporation, the most valuable contribution of his paper is probably in showing the possibility of developing an evaporation equation in terms of mass transfer of moisture through a layer of the atmosphere. For more than two decades meteorologists have been investigating turbulence in the lower atmosphere for the purpose of determining the mechanism of mass transfer of momentum, heat, moisture, and dust. The literature on the subject is already very extensive; but few engineers have given much attention to this development.

The author compares the transfer of moisture from an evaporating water surface to the process of heat transfer from a solid to a gas. He states (see heading, "How Evaporation Occurs"):

> "* * * A boundary layer of stagnant gas surrounds the solid through which heat is transferred by conduction alone. Outside the boundary layer, the heat is transferred by convection, frequently aided by turbulence. Heat transfer has been computed successfully by the use of an over-all transfer coefficient, combining the coefficients of conduction and convection, known as a film coefficient."

The temperature difference used is that between the wall temperature of the solid boundary and the temperature of the gas at a considerable distance. The author believes that the rate of transfer of water vapor through the atmosphere could be calculated in a similar manner. Whether or not it has been possible to combine the coefficients of conduction and convection in the computation of heat transfer need not be considered at this time. It is important to understand, however, that the successful determination of the flow of heat through a layer would not mean that moisture transfer could be computed similarly.

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[5] Principal Climatologist, SCS, U. S. Dept. of Agriculture, Washington, D. C.

John Leighly (45) has outlined clearly the difficulty of computing evaporation in this way, and the discussion immediately following is adapted from his paper. Conditions for the transfer of matter or energy within the boundary layer are totally different from those which prevail in the zone of turbulence above that layer (see Fig. 14). When air is turbulent, its properties are transported rapidly between layers by the actual exchange of bodies or masses of air, and except under special conditions steep gradients of velocity, temperature, or admixtures cannot exist. In the laminar boundary layer, however, where no mixing by movement normal to the boundary surface takes place, heat may be transferred only by molecular conduction; momentum may be transferred only by molecular friction; and gaseous admixtures, such as water vapor, may be transferred only by molecular diffusion. Therefore, gradients are characteristically steeper in the boundary layer than in turbulent air. Rates of molecular transfer—coefficients of heat conduction, molecular friction, and diffusion—are nearly constant within the thickness of the boundary layer, and this constancy of rate of transfer produces sensibly linear gradients of temperature, velocity, and humidity from the bounding surface out to the limit of the boundary layer. Outside the boundary layer, in the region of turbulent mixing, molecular transfer is negligible in comparison with molar transfer through turbulence. The intensity of turbulent mixing varies greatly with distance from the bounding surface. Hence, gradients are variable in

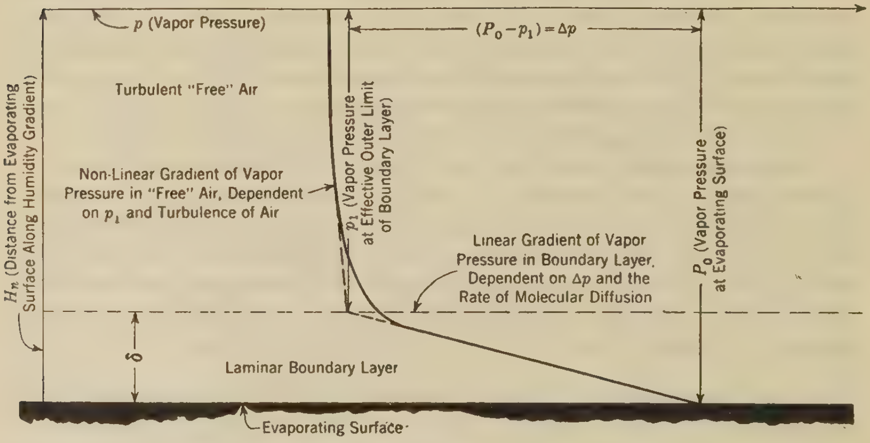
p (Vapor Pressure)
$(P_0 - p_1) = \Delta p$
Turbulent "Free" Air
Non-Linear Gradient of Vapor Pressure in "Free" Air, Dependent on p_1 and Turbulence of Air
p_1 (Vapor Pressure at Effective Outer Limit of Boundary Layer)
P_0 (Vapor Pressure at Evaporating Surface)
Linear Gradient of Vapor Pressure in Boundary Layer, Dependent on Δp and the Rate of Molecular Diffusion
H_n (Distance from Evaporating Surface Along Humidity Gradient)
δ
Laminar Boundary Layer
Evaporating Surface

FIG. 14.—CHARACTERISTIC DISTRIBUTION OF HUMIDITY IN THE LAMINAR BOUNDARY LAYER NEXT TO AN EVAPORATING SURFACE AND IN THE ADJACENT PARTS OF THE OVERLYING TURBULENT LAYER

the turbulent parts of the air, normally decreasing with increasing distance from a bounding surface, as the intensity of turbulent interchange increases.

When considering the proportionality of rate of evaporation to the difference in vapor concentration at the water surface and at a distance, Mr. Leighly showed that the simplest expression of such proportionality is

$$E = C\,(P_o - p_1) \qquad (25)$$

in which E denotes mass of water lost (or gained) by unit area in unit time; C is a proportionality coefficient; P denotes vapor pressure at saturation; p denotes actual vapor pressure; and the subscripts o and 1 denote location at an evaporating surface and in the air, respectively. The equation would be complete if humidity difference were the only variable. This simple equation for E has the familiar form of many equations that express rate of transfer of matter or energy through space—conduction of heat, diffusion of solutes, etc.—along a gradient; but the difference $(P_o - p_1)$ can define a gradient only when it is divided by a term defining the distance between the points of observation of moisture concentration. If this distance is designated z, and the change in moisture concentration along it is linear, the gradient is defined by $(P_o - p_1)/z$. Therefore, the vapor-pressure difference alone can be used as the basis of an evaporation formula only if z is constant for all evaporating surfaces. Since z cannot be constant, it must be included in any evaporation formula that uses the difference $(P_o - p_1)$. Even then, the expression $(P_o - p_1)/z$ accurately describes the gradient only if change in vapor pressure normal to the evaporating surface is linear. Within the turbulent layer and thus for unlimited distances from the evaporating surface, the gradient is not constant. The only satisfactory general expression of the gradient is therefore dp/dz, which is the change in moisture concentration with respect to height.

To determine the rate of flow of heat through unit cross section of a window pane, not only must one know the coefficient of thermal conductivity of the glass and the difference in temperature between the inner and outer surfaces, but also the thickness of the glass. This is easily understood; yet from the time of John Dalton, evaporation formulas have given the rate of flow of water vapor in terms of a difference in vapor concentration at the water surface and at an undetermined distance.

One of the finest investigations of evaporation ever conducted in the United States was that by Carl Rohwer, M. Am. Soc. C. E., reported in 1931 (16). All observations used in the derivation of his formula were carefully made under rigorously controlled conditions. He used an aspiration psychrometer to determine the vapor pressure of the air within an inch of the water surface. His laboratory experiments were conducted in still air and with controlled wind. The general form of his evaporation formula is given in Eq. 25. He found that the coefficient C increased linearly with increasing wind velocity according to the relation,

$$C = (0.44 + 0.118\,V) \qquad (26)$$

in which V = wind velocity, in miles per hour. Actually the coefficient C contains both the coefficient of moisture transfer and the distance term that must be associated with difference of moisture concentration in order to define the moisture gradient. Mr. Rohwer's measurements of moisture concentration in the air, although made within an inch of the water surface, were well within the turbulent layer; but, since they were made under controlled conditions, the fact that two layers with very different transfer coefficients were involved does not vitiate the results. With increased wind velocity the coefficient of turbulent exchange becomes larger and the laminar layer decreases in thickness. Thus both factors work together to increase the value of the coefficient of proportionality *C*.

With outdoor observations of evaporation made under uncontrolled conditions, Mr. Rohwer found his formula to be less satisfactory. He recognized that both wind velocity and atmospheric moisture are highly variable in the free air and he rightly attributed some of the inconsistencies in the data to this variability.

The author presented a general formula (Eq. 2*b*) for mass transfer of water vapor which uses a gradient of vapor concentration rather than a difference. Nevertheless, there is no indication that he measured a gradient in the course of his experiments. He refers to vapor concentration "at a distance" (see heading, "Evaporation as a Mass Transfer Process: Experimental Data"). He measured the relative humidity of the air with a standard sling psychrometer. When using this instrument, the height of the observation cannot be specified nor the moisture gradient determined with any degree of accuracy. Furthermore, it is not possible to use a sling psychrometer without disturbing still air and profoundly modifying its moisture structure. Apparently all other measurements were made with great care. Considering the crudeness of the one, such accuracy in the others was scarcely justified.

The author has stated in the paragraphs following Eq. 15 that the writer and his associates do not feel that it is possible to develop a satisfactory equation for evaporation in terms of the "difference in vapor concentration at the water surface and at a distance." As a matter of fact, the evaporation equation introduced by the writer and his associates does employ a difference (46*a*). Both observations of moisture concentration were made within the turbulent layer, and the distance element of the moisture gradient was incorporated into a coefficient of proportionality.

The author greatly underestimates the difficulty of the problem when he states (see heading, "How Evaporation Occurs") that the rate of moisture transport can be determined by use of an over-all transfer coefficient, combining the coefficients of conduction and convection. In the first place, the coefficient of turbulent exchange increases with height in the free air and varies not only with wind velocity but also with its stability structure. In the second place, the thickness of the laminar layer, in which the coefficient of molecular diffusion applies, becomes less as the intensity of turbulence in the turbulent layer above it increases.

The most difficult problem is to obtain the turbulent transfer coefficient in terms of some measurable quantity such as wind velocity or temperature. This has been accomplished for atmospheric conditions when the wind velocity varies as the logarithm of height (46*b*). The writer and his associates (47)(48)(49), however, have presented measurements of wind velocity at different heights to show that the logarithmic law of wind distribution is of restricted validity. The data show that the law applies very well when the temperature lapse rate is adiabatic (or, within a shallow layer, isothermal), but that it fails when the lapse rate is either superadiabatic or stable. When plotted on a semilogarithmic scale, only the late morning and early evening observations fall along a straight line. The daytime observations plot along lines having concave curvature; those made at night show convex curvature.

The writer concluded that when the thermal lapse rate is adiabatic, as for brief periods in the morning and again in the evening, the logarithmic law is fulfilled and the coefficient of turbulent mixing derived from it is valid. At this time turbulent mixing is mechanical only.

In the middle of the day, when the ground becomes heated and superadiabatic lapse rates may become very large and thermal convection is actively induced, convective turbulence is superimposed upon mechanical turbulence and the turbulent exchange coefficient becomes greater, perhaps much greater, than the logarithmic law would indicate. The concave curvature of the wind gradients is a consequence of this increase in the coefficient of turbulent mixing. As momentum is transported downward more actively, the tendency for differences in velocity in adjacent layers to be equalized increases, and the wind profile becomes more nearly vertical except very close to the ground surface.

At night when the ground, having been cooled by radiation, cools the adjacent air, the lowest layer of air becomes cooler than that above it and a stable lapse rate is created. The air next to the ground, being cooler, is denser than that above and, if moved mechanically upward into a faster moving layer, would tend to move down again instead of mixing with the air in the new environment. In this instance the coefficient of turbulent transfer is less than that specified by the logarithmic law. Momentum is transported downward less actively; the tendency for velocity differences in adjacent layers to be equalized diminishes; and the velocity profile departs more than ever from the vertical except within a foot or two of the ground.

The fact that the logarithmic law of wind structure is valid only when an adiabatic temperature structure prevails means that the problem of determining the coefficient of turbulent transfer is much more complicated than had been anticipated. First, the wind velocity gradients themselves must be determined more accurately than is possible with ordinary anemometers. In addition, it will probably be necessary to make measurements of temperature at various heights in order to establish the degree of stability of the atmospheric layer through which the water vapor is moving.

The writer has not seen the literature referred to by the author in his statement (see heading, "How Evaporation Occurs"), "Heat transfer has been computed successfully by the use of an over-all transfer coefficient, combining the coefficients of conduction and convection * * *." He does not share the author's view, however, that "There seems to be no reason why the rate of transfer of vapor particles cannot be calculated in an entirely similar fashion." If it is possible to determine an over-all coefficient of heat transfer through both laminar and turbulent layers, it is only because the temperature structure directly controls the intensity of turbulent transfer and indirectly the thickness of the laminar layer. The temperature difference term of the equation of heat transfer, therefore, may contain a part of the transfer coefficient. Atmospheric moisture concentration does not share with heat the property of controlling the magnitude of the coefficient of turbulent transfer, however. Consequently, it appears to the writer that it will not be possible to integrate across the boundary layer. In computing the rate of transfer of water vapor from a natural surface there seem to be only two possibilities that are scientifically defensible—either determine the moisture gradient wholly within the laminar layer where the coefficient of diffusion applies, or wholly within the turbulent layer, where it will be necessary to utilize a coefficient of turbulent transfer (50*a*).

HARRY F. BLANEY,[6] M. AM. SOC. C. E.—Experiments on evaporation, described in this paper, are opportune since they afford engineers, physicists, and hydrologists another opportunity to discuss the long-standing controversy as to which is the best device for measuring evaporation from a free water surface.

When estimating evaporation from lakes or reservoirs, engineers usually cannot wait years or even months for the results of experiments. Thus, they use the best data available. Engineers will continue to use evaporation pan records and empirical formulas until the time when research workers can demonstrate the practicability of estimating evaporation from large bodies of water by the energy balance, by the vapor-pressure gradient, or by other methods discussed by the author. When long-period evaporation pan records are not available, then, from necessity, the engineer may base his estimates on climatological data.

The author has mentioned some of the many formulas developed for determining evaporation from meteorological observations. For the most part, such formulas are based on short-period observations and require data that are not usually available in many localities. In such instances, these formulas are of little practical use to engineers in estimating average annual lake evaporation.

Several years ago, in connection with water supply and irrigation investigations made by the Division of Irrigation, Soil Conservation Service, the writer was confronted with the problem of determining, within a limited time, probable past and future evaporation and evapo-transpiration losses in valley areas along the Pecos River in New Mexico and Texas (51). Formulas were developed for this purpose. Long-period records of evaporation, temperature, and humidity were available for a few sections of the Pecos River basin and adjacent watersheds. These data, together with the records published by the Texas Agricultural Experiment Station (52) in College Station were used to develop an empirical formula for computing annual evaporation from a lake surface, where temperature and humidity data are available. The relation between temperature, humidity, and evaporation was found by plotting average annual observed data for some fifteen stations having records varying from 14 to 24 years (53). A formula was developed as

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$$E_L = 0.0167\,T\,(114 - H) \qquad (27)$$

in which E_L = annual lake evaporation, in inches; T = annual mean temperature, in degrees Fahrenheit; and H = annual average percentage of relative humidity. Eq. 27 has proved useful in estimating evaporation losses from water surfaces in the Lower Pecos River basin, California coastal valleys, and other areas within the limits of the temperature, humidity, and evaporation pan records from which the formula was evolved. For example, using the average mean annual temperature and humidity for a 10-yr period (1932 to 1941), the evaporation from a lake surface in San Francisco, Calif., is computed as 41.2 in., and the evaporation for San Diego, Calif., is computed as 45.7 in. These estimates compare favorably with estimates made from nearest Weather Bureau Standard Class A pan evaporation records at Alvarado and Chula Vista, Calif., respectively, when the appropriate conversion coefficient is used.

The writer agrees with the author that the annual conversion coefficient for a Weather Bureau Standard Class A pan of 0.70, adopted by the Sub-Committee on Evaporation of the Special Committee on Irrigation Hydraulics of the Society, is not applicable to all areas. Experiments (43) by A. A. Young, Assoc. M. Am. Soc. C. E., indicate that the coefficient for Southern California is 0.77. The writer is inclined to believe that 0.75 would be a better average annual value for areas where pan records are available for the entire year.

There are many types of pans and evaporimeters used in the United States to measure evaporation. There is a great need for standardization, particularly in Southern California where at least six types of pans—Weather Bureau Standard Class A, Colorado land, Los Angeles County Flood Control District, Bureau of Plant Industry, and insulated pans—are being used and sponsored by various agencies.

The author suggests the establishment of a standard evaporation station and (see "Suggestions for Further Study") "3. A study of the relationships between the existing data and those obtained from the new standard pan with the purpose of correlating all records as far as possible." This is an excellent proposal and it is hoped that detailed specifications for such a station will be submitted by the author in the closing discussion.

ARTHUR A. YOUNG,[7] ASSOC. M. AM. SOC. C. E.—Theoretical and practical aspects of evaporation from pans, and the relation of this evaporation to evaporation from lakes and reservoirs, are presented in this paper. The results are particularly valuable to the student of evaporation and are of practical use to those men who determine evaporation losses from water in storage. The following considerations, usually not considered in evaporation measurements, will add to the general knowledge of the subject and should result in a better evaluation of evaporation rates.

Effect of Rim Height.—For most evaporation studies, pan exposure and limitations of water surface variations have been standardized in the interest of comparable results although such precautions are not always followed. Any departure from a standard method of obtaining data will be reflected in the results. Rim height varies with evaporation and replenishment, and the rate of evaporation is affected by each changing level. The immediate effect will be small but the ultimate total effect may be considerable. Investigations generally have neglected to determine, quantitatively, evaporation totals as affected by changes in the level of water surfaces.

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In the attempt to show the effect of rim height on evaporation, the author has been handicapped by a lack of data directly related to evaporation pans. The data compiled by R. W. Revell, Jun. Am. Soc. C. E. (31), for a Weather Bureau Standard Class A pan were limited to a change in water surface only from 2 in. to 3 in. below the rim—the standard variation between replenishments of this pan. This study shows an increase in evaporation with increased rim height. Further information would be desirable for other levels since carelessness of observers often permits greater variations.

The study of evaporation from a saturated cylinder having a collar on the leading edge (15) showed increased evaporation with an increase in the height of the collar. The collar on the cylinder may be compared with the leading edge of the pan rim at which wind turbulence occurs. However, turbulence must also occur at the rim on the opposite side after the wind has passed over the water surface. Whatever the effect of turbulence, it would seem to be greater within the boundary of the water surface in a pan than within a cylinder with a single collar. These examples show an increase in evaporation due to greater rim heights. The following example shows the opposite effect.

In 1935 the writer made some tests to determine the effect of rim height on evaporation for the Division of Irrigation, Soil Conservation Service, at the Fullerton station near Los Angeles, Calif., using three pans, each 2 ft in diameter by 10 in. deep, set about 2 in. above the ground surface. Measurements were made continuously for five months under outdoor conditions, thus providing data for average temperature, wind, and humidity. The results are shown in Table 6. Water levels in each pan were as follows:

Pan No.	Depth of water surface below the rim (in.)	Depth of water in the pan (in.)
1	8	2
2	4	6
3	2	8

Water temperatures were obtained only at the time of observation in midmorning, and no maximum, minimum, or mean water temperatures are available. Wind movement was recorded a few inches above the top of the pans.

Least evaporation occurred from the pan having the greatest rim height, and most evaporation occurred from the pan having the least rim height. These data are contrary to findings reported by the author (15) (31), but the variations possibly are due to the different conditions under which the experiments were conducted—particularly to length of observations, variations between laboratory and outdoor conditions, size of pans, and differences in the evaporating surfaces. As depth to water increased, more of the water was shaded, greater wind protection was afforded by the sides of the pan, and there should have been greater moisture content in the air at the water surface. Each of these conditions tends to reduce the evaporation. Under such influences, water temperatures increase, and, between the 4-in. and 8-in. rim, there was an average monthly difference of 5.2° F, which was insufficient to overcome the decrease in evaporation caused by other factors.

TABLE 6.—EFFECT OF RIM HEIGHT ON EVAPORATION FROM PANS, 2 FT IN DIAMETER

Month (1935)	PAN NO. 1 (8 IN.)[a]		PAN NO. 2 (4 IN.)[a]			PAN NO. 3 (2 IN.)[a]			Wind velocity (miles per hr)
	Evaporation (in.)	Water temperature[b] (degrees F)	Evaporation (in.)	Water temperature[b] (degrees F)	Ratio, Col. 4 / Col. 2	Evaporation (in.)	Water temperature[b] (degrees F)	Ratio, Col. 7 / Col. 2	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
August....	7.72	88.1	8.61	82.8	1.11	9.92	82.1	1.28	2.4
September.	6.08	85.0	6.78	80.3	1.12	7.82	80.0	1.29	2.4
October...	5.42	73.1	6.21	69.2	1.15	6.88	69.0	1.27	2.7
November.	3.27	69.4	3.96	63.8	1.21	4.41	63.2	1.35	2.0
December.	2.91	65.5	3.49	59.0	1.20	4.53	59.8	1.56	2.3
Total[c]....	25.40	(76.2)	29.05	(71.0)	....	33.56	(70.8)	....	(2.4)
Relative ..	1.00	....	....	....	1.14	....	....	1.32	....

[a] Depth of water surface below the rim. [b] Spot readings of water temperature made only at the time of observations; daily maximum and minimum temperatures were not obtained. [c] Mean values are shown in parentheses; thus: (76.2).

The Division of Irrigation also has records of a 2-ft ground pan, 3 ft deep, in which, for several years, the index point for the water surface was 1 in. higher than that for the ground. However, the index point later was lowered to ground level, thus increasing the rim height above the water surface from 2 in. to 3 in. During the first period evaporation from the ground pan exceeded that from an adjacent Weather Bureau pan by 3.90 in. annually, and the coefficient for conversion of evaporation from the ground pan to evaporation from Weather Bureau pan was 0.94. For the second period, loss from both pans was practically the same, and the corresponding coefficient was 1.01. The decrease in evaporation in the ground pan was due to increased rim height as well as to lowering the water to ground surface level. Practically, water in a ground pan should not stand above the ground level as, in so doing, it has less protection than when the water volume is entirely below ground.

Effect of Pan Color.—The effect of the color of evaporation pans depends upon the reflective and absorptive capacities of the color to light and heat. Dark pans absorb more heat than galvanized pans and their evaporation rate is greater. White pans, or those painted with aluminum paint, absorb less heat than standard galvanized pans and their evaporation loss is less.

Several tests made by the writer for the Division of Irrigation show the effect of color on evaporation pans. Evaporation for one year from a pan painted inside, below the water surface, with flat black paint was 99.2% of that from an unpainted galvanized pan of the same size and with the same exposure. For a similar pan, completely painted inside and outside with the same kind of paint, evaporation was 5% greater than that from the untreated pan. This value agrees closely with the author's data. The darkened surface beneath the water had practically no effect on evaporation, but, with the entire surface painted, evaporation increased.

Further tests were made at the Fullerton station, using nine new pans, 2 ft in diameter by 10 in. deep, set just above the ground surface to allow ventilation below. Depth of water varied from 7 in. to 8 in. A check pan retained its initial galvanized surface; the other pans were painted inside and outside with automobile enamel in various colors, with the exception that one pan was coated with aluminum paint and another was made of copper. The tests were continued from February through December, 1939. The paint below the water surface eventually became stained with lime, and, on the outside, the pans lost some of their initial brightness although not sufficiently to change the rate of evaporation materially. Results of the test are shown in Table 7. The difference in relative evaporation of 0.83 for the pan painted with white enamel (see last line of Table 7) and a value of 0.96 for the pan with white, lead-base paint, computed from Table 4, may be due to the kind of paint applied. Enamel, with its smooth surface, seems to have lower heat-absorptive qualities, which would account for some difference in evaporation. It is not claimed that the relative evaporation values as recorded for enamel paint will hold for other brands; but these values show, in general, that different colors affect evaporation in different degrees. In other tests by the writer, using the same kinds of pans at different times, but with different brands of aluminum paint, reduction in evaporation varied from 2% to 9%. This result agrees with the findings of Carl Rohwer, M. Am. Soc. C. E. (16), that different brands give different results. Evaporation pans should not be painted, as coloration always affects the rate of evaporation. This point is recognized by most operators but a few instances have appeared in the literature on evaporation where black or copper pans have been used. More often, pans are painted with one of the brands of aluminum paint to prevent discoloration or rusting of the gal-

TABLE 7.—COMPARISON OF EVAPORATION FROM PAINTED PANS WITH THAT FROM A GALVANIZED IRON AND A COPPER PAN (IN INCHES)

Month (1939)	Galvanized untreated	White enamel	Orange enamel	Light yellow enamel	Aluminum paint	Dark blue enamel	Dark green enamel	Black enamel	Copper untreated
February..	3.66	3.09	3.28	3.37	3.58	3.83	3.70	3.91	3.90
March.....	3.68	2.86	3.30	3.27	3.59	3.91	3.91	4.02	4.05
April......	6.04	4.98	5.53	5.54	5.87	6.22	6.15	6.37	6.28
May......	7.65	6.45	7.25	7.20	7.50	8.08	8.25	8.27	8.30
June......	9.16	7.75	8.85	8.64	9.10	9.59	9.77	9.94	9.90
July.......	9.71	8.16	9.17	9.12	9.44	10.02	10.04	10.34	10.50
August....	9.39	7.90	8.74	8.89	9.16	9.60	9.62	10.00	10.10
September.	9.05	7.76	8.46	8.71	8.90	8.95	9.26	9.51	9.71
October...	7.01	5.63	6.27	6.44	6.80	6.76	6.99	7.25	7.37
November.	3.83	2.88	3.19	3.51	3.64	3.60	3.59	3.79	3.95
December .	2.83	2.08	2.24	2.49	2.69	2.58	2.51	2.73	2.77
Total....	72.01	59.54	66.28	67.18	70.27	73.14	73.79	76.13	76.83
Relative...	1.00	0.83	0.92	0.93	0.98	1.02	1.02	1.06	1.07

vanized surface. Standard galvanized pans darken with age, and, if similar surfaces are maintained, comparable records are more likely to be obtained.

Evaporation from Screened Pans.—At the Fullerton station and elsewhere in Southern California, efforts have been made to obtain a pan that would have an annual loss sufficiently close to reservoir evaporation so that a reduction coefficient would be unnecessary. This result was obtained by using a ground pan shaded by a 0.25-in. mesh galvanized iron screen suspended horizontally midway between the top of the pan and the water surface. The pan was 2 ft in diameter by 3 ft deep, set 33 in. in the ground. The mean annual coefficient for eleven pan-years was 0.98 within ranges varying from 0.95 to 1.00 at the Fullerton station and from 0.97 to 1.00 at the Lake Elsinore (California) station of the Division of Irrigation. The method used in screening the water surface is shown in Fig. 15.

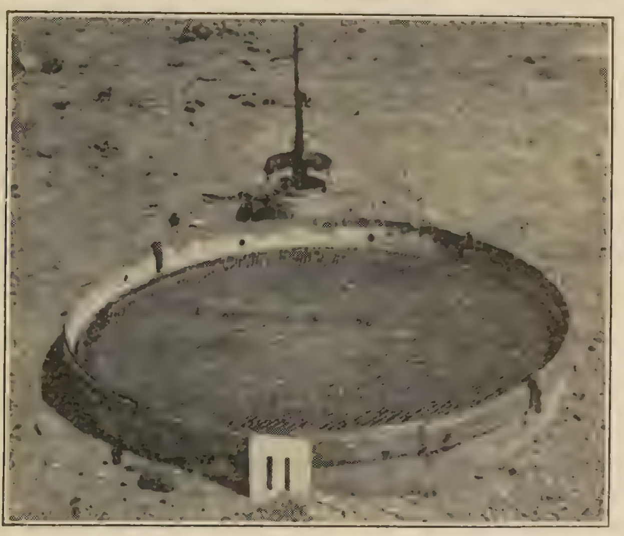

Fig. 15.—Method Used in Screening the Water Surface

Temperatures of water in the screened pan are more uniform than those in most other pans, holding up well during the night and maintaining a reasonably uniform rate of evaporation that is less than the rate from other known types. Monthly coefficients are variables. Comparison of evaporation from a screened pan with that from a 12-ft pan at the Fullerton station resulted in high coefficients during the early part of the year; but, when compared with evaporation from Lake Elsinore, coefficients were highest during the fall months. The screened pan has had a limited use in Texas where the same coefficients appear to hold true.

Evaporation Coefficients.—Coefficients for the conversion of evaporation from a pan to equivalent evaporation from a lake or a reservoir, as discussed in the paper, refer to the Weather Bureau pan for which an annual coefficient of 0.70 has been recommended (2), based upon separate experiments in Colorado by the late R. B. Sleight, Assoc. M. Am. Soc. C. E. (54), and by Mr. Rohwer (16). Both these men concluded that evaporation from a lake or a reservoir was substantially the same as that from either a ground pan, 12 ft in diameter, or a shallow reservoir, 85 ft in diameter. The coefficient of 0.70 is applicable to the general climatic region of Colorado where evaporation is limited to a part of the year because of freezing weather, but does not apply exactly to the lower altitudes of the Southwest where evaporation occurs throughout the year.

Monthly coefficients vary according to temperature; they are higher than the annual mean in summer and lower in winter. Usually, the annual coefficient is used in estimating reservoir evaporation. At times, however, monthly coefficients would be convenient for use by reservoir operators—particularly in estimating evaporation losses for snow survey forecasts (in cases where the losses are proportioned between interested users during the months of the irrigation season) and for reservoirs that contain water only during a part of the year.

Evaporation experiments by the Division of Irrigation in Southern California indicate an average annual coefficient of 0.77 for the Weather Bureau pan, based on evaporation from a 12-ft diameter pan, 36 in. deep. The coefficients were consistent from year to year, varying only from 0.76 to 0.78, and the average may be used safely for similar climates in southwestern states.

The value of 0.77 also has been obtained by a similar study in which measured evaporation from a Weather Bureau pan was compared with computed evaporation from Lake Elsinore, which has an area of 5,500 acres and is some 30 miles from the Fullerton station. The maximum depth of the lake was about 30 ft, but large areas were less than 10 ft deep. In this respect, the lake was more like a large evaporation pan than a deep reservoir. Inflow to the lake was from the San Jacinto River, which is subject to brief floods and to long-continued periods of no flow. Since 1917 there has been little or no outflow from the lake because of a deficiency in water supply. The only change in water levels has been caused by loss through evaporation since the lake bed is composed of silt through which seepage does not occur in sufficient quantities to change the values of computed evaporation.

The Lake Elsinore study was for a 3-yr period during which a Weather Bureau pan was placed immediately above the shore line, and lake evaporation was computed from inflow, rainfall on the lake surface, and change in water levels. Because of small uncertainties in computed evaporation, there were slightly greater differences in annual coefficients—from 0.76 to 0.80—than at the Fullerton station, but the average annual coefficient was 0.77 for both the Lake Elsinore and Fullerton stations.

Effect of Heat Storage in Water.—Because of the small volume of water in evaporation pans, heat storage available for the process of evaporation varies during the day, and much of the heat gained in the daytime is lost at night. Temperatures of large volumes of water are more uniform; they change little daily and their seasonal changes are slow. A part of the heat received by the reservoirs in spring and summer is used in creating evaporation, and another part is absorbed in warming the water to a considerable depth. Later in the year heat storage becomes available for evaporation during cooler months when pan evaporation is approaching a minimum. During spring and summer, pan evaporation may exceed lake or reservoir evaporation; but, because of heat in storage, evaporation from larger volumes of water in the fall may equal or exceed evaporation from a near-by pan. The annual heat energy received from the sun may be the same for pan and reservoir alike; hence, actual annual reservoir evaporation should be substantially the same as that computed by means of annual coefficients.

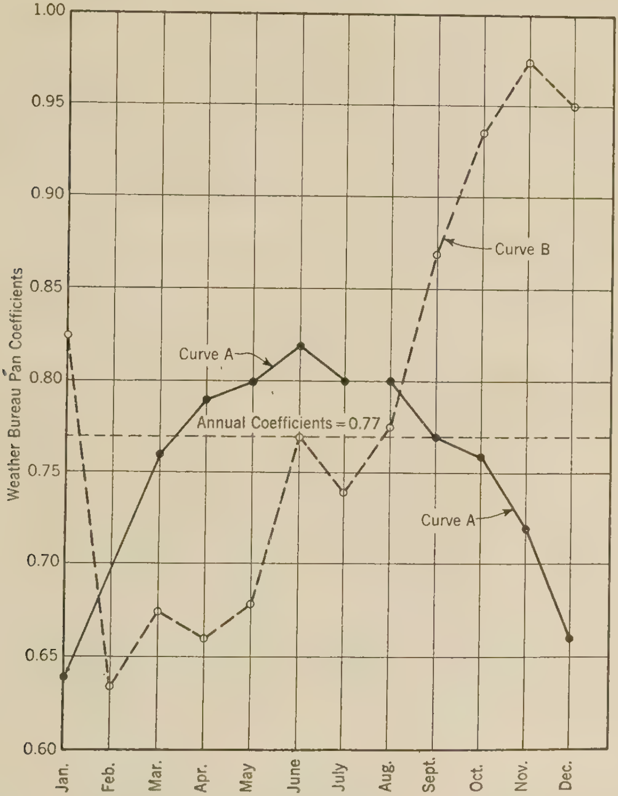

FIG. 16.—VARIATIONS IN MONTHLY WEATHER BUREAU PAN COEFFICIENTS

Actual monthly reservoir evaporation, however, differs from that computed from pan evaporation by reduction coefficients. In Fig. 16, two curves are shown in which plotted coefficients indicate the different effects of heat storage capacity. Both are for a Weather Bureau pan—one at the Fullerton station (curve A) where coefficients were based on comparison with a 12-ft diameter pan, the other at Lake Elsinore (curve B) where comparison was made with evaporation from the lake. Between the two the relation is not constant. The Fullerton coefficients rose to a maximum during the warmest months because pan temperatures change according to air temperatures. The lake coefficients were minimum during the early part of the year when the lake was storing heat, and maximum during fall months when for much of the time mean water temperatures were greater than mean air temperatures. The annual coefficient is 0.77 for each condition. Since evaporation is small during spring and fall months, the difference in computed evaporation, regardless of which set of coefficients is used, will be less important than is indicated by Fig. 16.

A. A. Kalinske,[8] Assoc. M. Am. Soc. C. E.—Evaporation from water surfaces, and also from land areas, is an important hydrological problem on which fundamental investigative work is very scarce indeed. The author reviews, quite completely, past theoretical and experimental studies, and presents some laboratory data of his own which he correlates by use of dimensionless parameters—a method that has been quite successful in analyzing heat-transfer data. Since the analogy between heat transfer and vapor transfer in gases is very apt, the author is undoubtedly on safe ground in using this analogy in his studies.

Although mentioned by the author in connection with his discussion of the influence of pan diameter on evaporation rate, a fact that many engineers sometimes overlook is that, for evaporation from a finite area into a moving air stream, the rate of evaporation varies with down-wind distance from the windward edge. Certain theoretical analyses have been made by H. Jeffreys (55), M. A. Giblett (56), O. G. Sutton (57), and F. Graham Millar (12) which indicate how the rate of evaporation varies with down-wind distance; however, the mathematical formulations are somewhat complex and not very convenient for practical use. Since this particular phenomenon is of considerable importance in connection with measuring and predicting evaporation from various areas, it might be of interest to present briefly the results of the theoretical analyses showing at least the approximate variation of evaporation with down-wind distance. The writer does not recall any such analyses in engineering literature. The analyses are based on the turbulence-diffusion theory and several simplifying assumptions must, of necessity, be made.

For the steady, two-dimensional case the general differential equation for diffusion is:

$$\frac{V\,\partial c}{\partial x} = \frac{\partial}{\partial x}\left(D_x \frac{\partial c}{\partial x}\right) + \frac{\partial}{\partial y}\left(D_y \frac{\partial c}{\partial y}\right) \ldots\ldots\ldots\ldots\ldots (28)$$

in which, corresponding to the notation of the paper, V is the wind speed in the x-direction (horizontal wind); D_y and D_x are the eddy diffusion coefficients in the y-direction and x-direction, respectively; and c is the vapor concentration per unit volume. The change in the value of $\partial c/\partial x$ with x is negligible compared with $\partial^2 c/\partial y^2$; so the first term on the right side of Eq. 28 will be dropped. The simplest possible solution of Eq. 28 is obtained when V and D_y are assumed independent of y—that is, when they are constant. The diffusion equation then becomes:

$$\frac{\partial c}{\partial x} = \frac{D_y}{V}\frac{\partial^2 c}{\partial y^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (29)$$

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[8] Associate Prof., Univ. of Iowa, Iowa Inst. for Hydr. Research, Iowa City, Iowa.

The solution of Eq. 29 is:

$$c = c_0 \left(1 - \phi_e \frac{V\, y}{2\, D_y \sqrt{x}} \right) \qquad (30)$$

in which c_0 = vapor concentration at $y = 0$, and

$$\phi_e = \frac{2}{\sqrt{\pi}} \int_0^s e^{-s^2}\, ds \qquad (31a)$$

and

$$s = \frac{V\, y}{2\, D_y \sqrt{x}} \qquad (31b)$$

The normal error function ϕ_e can be obtained by reference to statistical tables. The rate at which water vapor is transferred upward is then equal to $D_y\,(dc/dy)$, according to the diffusion theory, and therefore the rate of evaporation per unit area E at any point can be found by evaluating dc/dy at $y = 0$ from Eq. 30; thus:

$$E = c_0 \sqrt{\frac{D_y\, V}{\pi\, x}} \qquad (32)$$

The rate of evaporation from 0 to x from a strip Δz wide is:

$$E_x = 2\, c_0 \left(\frac{D_y\, V\, x}{\pi} \right)^{0.5} \Delta z \qquad (33a)$$

or, in general, over a strip L feet long:

$$E_L = 2\, c_0 \left(\frac{D_y\, V}{\pi} \right)^{0.5} \int L^{0.5}\, dz \qquad (33b)$$

For a circular area of radius r, Eq. 33b becomes:

$$E_r = 3.95\, c_0\, (D_y\, V\, r^3)^{0.5} \qquad (33c)$$

This analysis indicates that the unit rate of evaporation varies inversely as the downward distance, x, to the 0.5 power, and directly as the square root of the wind speed. The total rate of evaporation, over the entire area, varies as the area to the 0.75 power.

Mr. Sutton derived expressions such as Eqs. 33, taking into account the variation of V and D_y upward from the evaporating surface. He assumed a velocity distribution such that:

$$V = V_1 \left(\frac{y}{y_1} \right)^{n/(2-n)} \qquad (34)$$

in which V_1 is the velocity at same height y_1 and in which n can vary from 0 to 1.0, its value being somewhat dependent on the turbulence present. Mr. Sutton's expressions were quite complex. He found, nevertheless, that the total mean evaporation rate, E_L, was related to velocity and area dimensions as follows:

$$E_L \propto V_1^{(2-n)/(2+n)}\, L^{2/(2+n)}\, b \qquad (35)$$

in which b is the area width. Experiments have shown that the quantity $n/(2 - n)$, for fully developed turbulent flow, is about 1/7, from which $n = 1/4$. From Eq. 35, it is apparent that E_L varies as the down-wind linear dimension of the area to the 0.89 power, and as the air velocity to the 0.78 power. The average evaporation per unit area would then be proportional to the $-$ 0.11 power of L. Mr. Sutton states that data reported by G. W. Himus (19) seem to verify these results. Mr. Millar's derivations for finite areas indicated that the average unit rate of evaporation would be proportional to $V^{0.75}$ and $L^{-0.14}$. Since Mr. Millar's method of attacking the problem was somewhat different from that of Mr. Sutton, it is significant that there is such good agreement.

The foregoing analyses indicate that the rate of evaporation from any area into a moving air stream is dependent on the distance down wind from the windward edge of that area. This item should be kept in mind when measurements of evaporation are made at some point on an area, either with pans or by the Thornthwaite-Holzman method (6). The writer believes that, in the derivation and use of the latter method, C. W. Thornthwaite and B. Holzman did not emphasize strongly enough the fact that their equation and data did not necessarily indicate the evaporation rate over the entirety of any particular area, but rather the value of this rate at some particular point.

The theory of evaporation as based on the turbulence-diffusivity-coefficient concept has advanced to a point where there is need for some extensive and carefully controlled experiments, both in the laboratory and field—to verify directly certain assumptions made in developing this theory and to evaluate certain constants present in the equations. It was with this idea in mind that, several years ago, the Iowa Institute of Hydraulic Research, at Iowa City, Iowa, with hopes of obtaining the cooperation of the U. S. Weather Bureau, became interested in an experimental program on evaporation. The writer at that time outlined the direction to be taken by the proposed laboratory and field experiments in order that direct confirmation might be obtained as to whether or not the Thornthwaite-Holzman method of evaporation measurement actually gave values of evaporation from water surfaces. So far as the writer is aware, no such direct confirmation exists; at least he has not come across any in the literature on evaporation. It is important that direct evidence be made available before this particular method of evaporation measurement (which can have such extensive usefulness) is used too widely and confidently for obtaining evaporation data from water and land areas. The laboratory data were to be obtained in a wind tunnel some 90 ft long with a cross section 4 ft by 2 ft. In the design of this tunnel, provision was made for varying and controlling the temperature and humidity of the approaching air, temperature of evaporating water, and air velocity. Air velocity, vapor concentration, and air temperature were to be measured at various points above the evaporating surface. Turbulence measurements were to be made with a hot-wire anemometer. Field studies were planned to check and supplement the laboratory observations. It was proposed to make these studies on Spirit Lake in Iowa, which has many desirable features for conducting such a study. Actual evaporation at various points on the lake in the wind direction was to be measured with floating pans, and data were to be compared with evaporation calculations made from wind speed and humidity determinations above the lake surface.

The experiments indicated should make it possible to determine the following items definitely: (*a*) Feasibility of using the turbulence-diffusion-theory equations for computing actual evaporation, and the accuracy to be obtained from this approach; (*b*) accuracy and limitations of the method of computing evaporation from wind speed and humidity measurements at two points above a surface; and (*c*) data necessary for making predictions of evaporation from nonexistent water areas, such as proposed reservoirs.

An experimental program like the one outlined would help to provide some very important missing links between evaporation theories and actual evaporation determinations such as are needed in hydrological studies.

Because evaporation phenomena are of interest to so many different groups of scientists, the literature on evaporation is very widely scattered. In addition to the papers previously mentioned by the author and writer, some papers by R. B. Montgomery (58), H. V. Sverdrup (59), and Mr. Holzman (60) might be added to the "Bibliography." Mr. Montgomery gives extensive data on humidity distribution above an ocean surface, and in his analyses indicates that Mr. Millar (12) made some errors in his derivations. Mr. Sverdrup discusses the so-called "Bowen ratio" and indicates that, since this ratio is only approximately correct for atmospheric conditions, meteorologists are reluctant to use it. Mr. Holzman analyzes the influence of stability in the atmosphere on evaporation, and concludes that it is significant only for cold, smooth surfaces, such as snow or ice, or for low air speeds.

ADOLPH F. MEYER,[9] M. AM. SOC. C. E.—In the "Introduction," the author states,

> "An understanding of the physical occurrences of the evaporation process would lead to observations of those elements of the problem which would make the transfer of pan records to reservoirs more reliable."

Although the writer has read the paper with considerable care and has reread many parts of it, he has no definite recollection of emphasis being placed on what observations are needed and on how the transfer from pan records to reservoirs is to be made. Perhaps in the closing discussion this matter will be amplified.

From the practicing engineer's viewpoint, the following useful information developed by Mr. Hickox may well be further emphasized:

1. The color of the evaporation pan is important as it affects heat absorption, which is one of the elements influencing evaporation. The black evaporation pan used by the author consistently showed higher evaporation than the white pan. As would be expected, the difference was small during the winter months. On an average, the black pan showed 10% more evaporation than the white pan. All pans were buried in the ground.

2. Another important fact evidenced by the author's records is the higher evaporation from the deeper pan. The pan 2 ft deep showed about 10%

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greater evaporation than the pans 3 in. and 6 in. deep and about 7% greater evaporation than the pan 1 ft deep. Heat absorption by the water during the daytime and less radiation at night appear to be the causes. Is it fair to conclude, in consequence, that a Weather Bureau Standard Class A evaporation pan, if 2 ft deep, would give substantially greater evaporation when similarly exposed than the present standard Class A pan?

The writer has frequently warned against the acceptance, without reservation, of a coefficient of 0.70 for reducing Class A evaporation to reservoir evaporation. In California, Arthur A. Young, Assoc. M. Am. Soc. C. E., found that a coefficient of 0.77 or 0.78 should be applied to the records of Class A evaporation pans to reduce them to reservoir evaporation. Most of the data on which the coefficient of 0.70 is based were limited to summer evaporation. However, it is a well-known fact that the difference between pan evaporation and reservoir evaporation is less during the winter months. It is also a well-known fact that reservoir evaporation is relatively much higher in fall and lower in spring. An average coefficient is meaningless in the solution of many problems faced by the practicing engineer.

The author also calls attention to the fact, not always recognized, that evaporation occurs even when air and water temperatures, some distance apart in elevation, are equal, because water vapor is lighter than the dry gases of the atmosphere and not only diffuses but sets up convection currents of its own accord.

Mr. Hickox tabulates the wind correction factors (see heading, "Correlation of Results of Other Experimenters: Effect of Wind Velocity") proposed by five different investigators. In most instances, the correction factor is to be applied to the wind velocity observed near the water level whereas the writer's factor, $1 + 0.1\ V$, refers to wind velocity measured about 25 ft to 30 ft above the ground or water surface. When that factor was first published in 1915 (61), the writer stated that the wind velocity at 25 ft or 30 ft above the ground was two and one-half to three times the velocity at about 1 ft above the ground. When this correction is applied, the writer's factor for wind effect becomes practically identical with that used by Carl Rohwer, M. Am. Soc. C. E., and stated by Mr. Hickox in the list preceding Eq. 18, namely, "$1 + 0.268\ V$."

Studies like those of Mr. Hickox are valuable and deserve encouragement. The practicing engineer, however, is sorely in need of records of air temperatures above lakes and reservoirs and of the temperature of the water in lakes and reservoirs about 1 ft below the surface, under the diverse climatic conditions that exist even within the borders of the United States. The equations for evaporation available to the practicing engineer are better than the basic data which he is forced to use in those equations. Little is known about relative air and water temperatures in the arid regions where evaporation losses are extremely high. Available records for northern lakes indicate that those of ordinary depth have summer water temperatures in excess of air temperatures. Some scattered records for the arid southwestern states indicate that lakes and reservoirs of ordinary depth in that region (probably because of the high heat loss during the night by radiation and through high daily evaporation) reach temperatures relatively lower in comparison with air temperatures than do lakes and reservoirs in the northern states where the heat received from the sun is relatively greater during the summer months than it is in the south. Water is a good absorber of heat.

The rather comprehensive study directed by the writer and published in 1942 (62) would have been more reliable in regions where unusual climatic conditions prevail if satisfactory records of comparative air and water temperatures of such regions had been available. After the work was nearly completed and it was apparent from the scattered records available that the estimated water temperatures for stations in the arid region were a little too high, corrections were made for Yuma and Phoenix, Ariz.; El Paso, Tex.; Roswell, N. Mex.; and Fresno, Calif.; but neither time nor available staff permitted corrections at other stations somewhat similarly located at which the computed evaporation appeared to be somewhat high when compared with pan evaporation records. No data are available on the basis of which a gradual reduction from about 10% in the regions of highest evaporation to zero in about the central portion of western United States can be made. The writer again cautions, however, against the use of 70% of Class A evaporation records in estimating the evaporation from lakes and reservoirs of ordinary depth. Lakes like Lake Superior obviously must be excepted. They constitute a problem in themselves. Evaporation from Lake Superior is probably not greater than 18 in. per yr (62).

MAURICE L. ALBERTSON,[10] JUN. AM. SOC. C. E.—Many civil engineers think of the theory of evaporation as applying only to water and of interest to the hydrologist alone. The chemical engineer, however, must understand the vaporization of liquids in drying and evaporating processes and, consequently, has conducted considerable research using other liquids as well as water. Moreover, the phenomenon of turbulence, which plays such an important rôle as the prime mover in the process of evaporation, has been studied in detail by the aeronautical engineer and the meteorologist.

In the section, "How Evaporation Occurs," the author describes briefly the kinetic interpretation of evaporation as used by the chemist. Any molecule that is to leave the surface by overcoming the attraction of the surrounding liquid must possess a minimum upward velocity component, v_o. It follows, therefore, that, as a molecule escapes, the over-all kinetic energy is diminished by $\frac{m\,(v_o)^2}{2}$ so that, when n molecules have departed, the loss of kinetic energy of the system is

$$E_k = \frac{n\,m\,(v_o)^2}{2} \qquad (36)$$

in which m is the mass of one molecule. This loss is equivalent to the heat of vaporization. The rate n_E at which these molecules are leaving the surface may be determined when the system is in equilibrium, because at that time the rate of return of the molecules n_R (max) is equal to the rate of exit n_E—that is, $n_E = n_R$ (max). According to the kinetic theory (63), the number of molecules striking a square centimeter of surface per second is

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$$n_R = \frac{p_v}{(2\,\pi\,m\,N_{Bo}\,T)^{0.5}} \qquad (37)$$

in which p_v is the partial pressure of the water vapor above the surface in dynes per square centimeter at absolute temperature T in degrees centigrade; n_R is the rate of return of molecules per square centimeter per second; n_R (max) is the limit of n_R when p_v is numerically equal to the vapor pressure p_S of the water; and N_{Bo} is the Boltsmann constant. Therefore, when the system is not in equilibrium—while evaporation is occurring—the net loss of molecules n_L becomes

$$n_L = n_E - n_R = \frac{p_S - p_v}{(2\,\pi\,m\,N_{Bo}\,T)^{0.5}} \qquad (38)$$

in which p_S is the vapor pressure of the water at its surface in dynes per square centimeter; and the temperature of the water surface and that of the boundary layer immediately above it are assumed to be the same. It is possible, from Eq. 38, to determine the actual weight rate of evaporation E_w in grams per second per square centimeter by multiplying by m so that

$$E_w = m\,n_L = m\,\frac{p_S - p_v}{(2\,\pi\,m\,N_{Bo}\,T)^{0.5}} \qquad (39)$$

Upon applying this relationship specifically to water, cubic centimeters may be conveniently substituted for grams, making E_w numerically equal to centimeters per second. This, in turn, may be converted into inches per day giving

$$E = 2.88 \times 10^5\,\frac{p_S - p_v}{T^{0.5}} \qquad (40a)$$

in which T is now in degrees Fahrenheit, E is evaporation in inches per day, and p_S and p_v are in inches of mercury. Eq. 40*a* demonstrates that the net rate of water transport through the boundary layer varies directly with the difference between the vapor pressure of the water and the partial pressure of the water vapor above it and inversely with the square root of the absolute temperature. As the author has shown, the change in $T^{0.5}$ is relatively small over the temperature range of interest to the hydrologist and, therefore, has little effect on the evaporation rate except as it changes the vapor pressure and partial pressure. One of the limits of Eq. 40*a* (the case of equilibrium) exists when $p_v = p_S$. Furthermore, were it possible to sweep away all water molecules as they leave the surface, allowing none to return, $p_v = 0$ and Eq. 40*a* would reduce to

$$E = 2.88 \times 10^5\,\frac{p_S}{T^{0.5}} \qquad (40b)$$

which is the limit of Eq. 40*a*, giving the greatest rate of evaporation possible at a given temperature. Evaporation as determined by this relation (see Fig. 17)

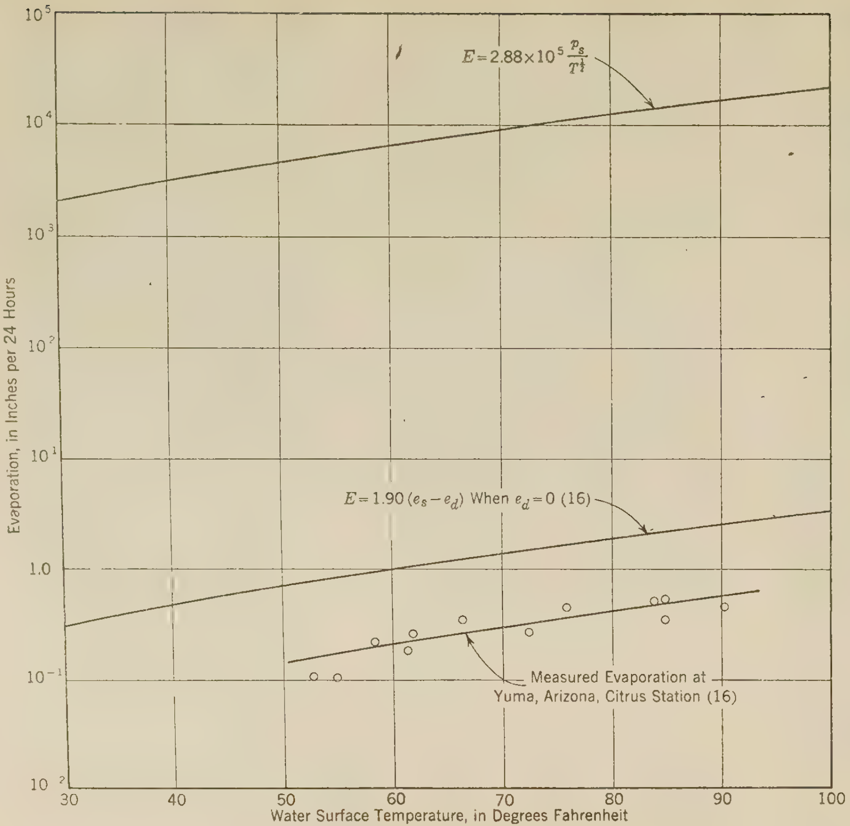

FIG. 17.—RELATION OF MAXIMUM EVAPORATION TO SURFACE TEMPERATURE

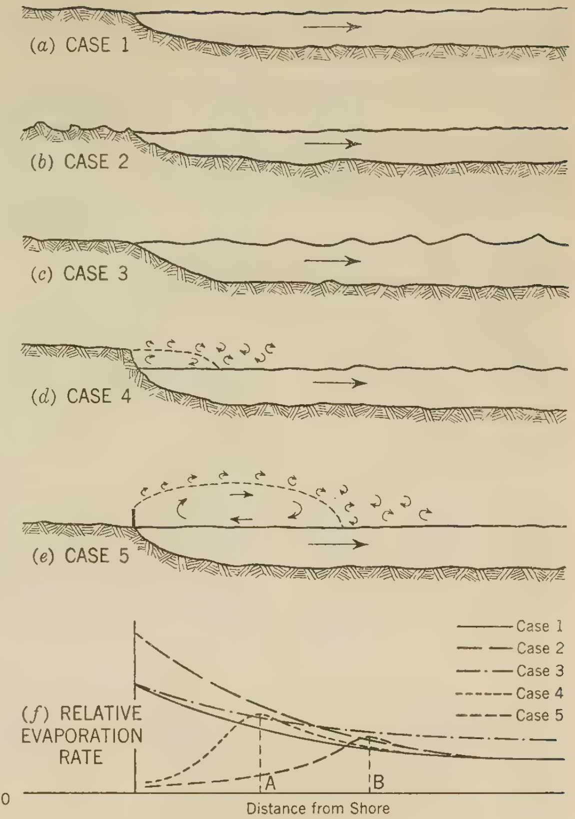
(a) CASE 1
(b) CASE 2
(c) CASE 3
(d) CASE 4
(e) CASE 5
(f) RELATIVE EVAPORATION RATE
Case 1
Case 2
Case 3
Case 4
Case 5
A
B
Distance from Shore

FIG. 18.—RELATIVE EVAPORATION RATES FOR VARIOUS SURFACE CONDITIONS

is many times as great as any actually observed. Furthermore, it is much greater than the evaporation computed with the largest wind coefficient given by Carl Rohwer, M. Am. Soc. C. E. (16*d*). Although this tremendous difference is in part caused by the existence of water vapor in the ambient air, it is primarily explained by assuming, as the author does, that the molecules travel through the boundary layer by molecular diffusion and, therefore, that only a very small percentage of them escape to the turbulent air above without returning to the liquid. Evidently, the natural turbulence over the water surface is never sufficiently great even to approach the elimination of this boundary layer.

There is little doubt that the processes of heat transfer and evaporation have many similarities as described by the author. However, it is well to remember that, as ∂x approaches zero for a constant value of ∂t in Eq. 2*a*, the rate of heat flow will approach infinity; but, according to the kinetic theory as discussed herein, the rate of evaporation can only approach some maximum finite value which is a function of the temperature of the water surface. This discrepancy, on the other hand, is explained by the fact that another form of the heat transfer equation must be used when the heat flow from a surface is being considered. If H is the rate of heat transmission per unit area and T_s and T_v are, respectively, the absolute temperatures of the surface and the air immediately above it, then the equation

$$H = k' (T_s - T_v) \dots\dots\dots\dots (41)$$

shows that, as T_v approaches zero, H approaches a maximum finite value, thereby completing the heat transfer analogy to follow Eq. 40*a*.

In the foregoing paragraphs consideration has been given to the mechanics of molecular movement from the liquid into, and through, the boundary layer. Of equal importance, although much less understood, is the transfer of vapor from the boundary layer to the air immediately above it. On the one hand, if absolutely stagnant conditions prevail, the boundary layer is infinitely high, and the resultant mixing is by molecular diffusion only. On the other hand, if conditions arise to make the flow unstable, turbulence will develop and the degree of mixing will increase tremendously. At any given point unstable conditions may be obtained by decreasing the viscosity or by increasing the velocity gradient through increasing the ambient velocity or the boundary roughness.

Although the foregoing conditions lead to the perpetual formation of turbulence, there are factors, such as viscosity, that lead to its decay. If the turbulence is being damped (that is, decaying) as rapidly as it is being created, a state of equilibrium prevails. However, if the creative forces increase, the turbulence becomes greater and the damping forces must then become equally large to reestablish equilibrium. If, instead, the creative forces diminish, the damping forces will decrease the prevailing turbulence to a magnitude that the creative forces are able to support.

Consideration of the terms in Eq. 40*a* reveals that p_s and T are dependent only upon the temperature of the water, whereas p_v is a function of the overhead turbulence and the vapor concentration and temperature of the ambient air. The turbulence is dependent upon the wind velocity, the viscosity of the air, and the surrounding roughness, such as, for example, the geometry of the evaporation pan, and is therefore dependent upon a characteristic Reynolds number of the flow over the pan. Hence, $p_v = \phi(\mathbf{R}, c_A)$ and Eq. 40*a* becomes

$$E = 2.88 \times 10^5\, T^{-0.5}\, p_S - 2.88 \times 10^5\, T^{-0.5}\phi(\mathbf{R}, c_A) \quad \text{.......}(42)$$

in which **R** is the Reynolds number; and c_A describes the temperature and vapor concentration of the ambient air. In Eq. 42, the first parameter 11,330 $T^{-0.5}\, p_S$ is the same as that in Eq. 40*b* and has been determined as in Fig. 17. The second parameter, however, must be determined empirically. The following typical cases serve to illustrate the foregoing principles.

Case 1.—First consider the case of a large lake, bounded on the windward side by a broad smooth plain, with very little difference in the shore and lake elevations. Assume that the surface temperature of the lake is constant, that the general wind velocity distribution over the plain and lake is uniform, and that the ripples on the lake give it the same roughness as that of the land. With these conditions prevailing, the turbulence of the wind near the water surface will likewise be uniform and the only variable factor influencing the rate of evaporation at various points across the lake is the vertical vapor-concentration gradient $\partial c/\partial x$. As shown in Fig. 18, the maximum rate of evaporation will occur at the water's edge; then, as the air immediately above the surface develops a partial pressure of water vapor, the gradient becomes smaller; and, hence, the rate of evaporation asymptotically approaches a minimum value. The total evaporation per unit width across the lake is represented by the area under the curve.

Case 2.—If the land on the windward side is not smooth but extremely rough, the turbulence of the oncoming air will be higher and at the shore the initial rate of evaporation will also be higher than in case 1. However, the turbulence will be partly damped across the lake by the viscosity of the air (the roughness of the lake not being great enough to maintain the initial turbulence) until the turbulence and resulting rate of evaporation approach the same limit as that of case 1—although the total evaporation across the lake is obviously greater.

Case 3.—If the windward land conditions are the same as those in case 1 except that there are waves on the lake, the initial rate of evaporation will be the same as that in case 1 but will approach some higher minimum value as the turbulence reaches equilibrium for the given surface (lake) roughness.

Case 4.—If the conditions are again the same as those in case 1, except that the land is considerably higher than the water surface because of a cliff at the shore, then a large eddy will result at the down-wind side of the cliff; and, along the upper edge of the eddy, considerable turbulence will be created and carried into the main wind stream. Because the turbulence is greater at point A than in case 1, the rate of evaporation tends to be greater; however, this is partly offset by the fact that the vapor concentration has also increased. Hence, the resulting evaporation rate may be greater or smaller than that

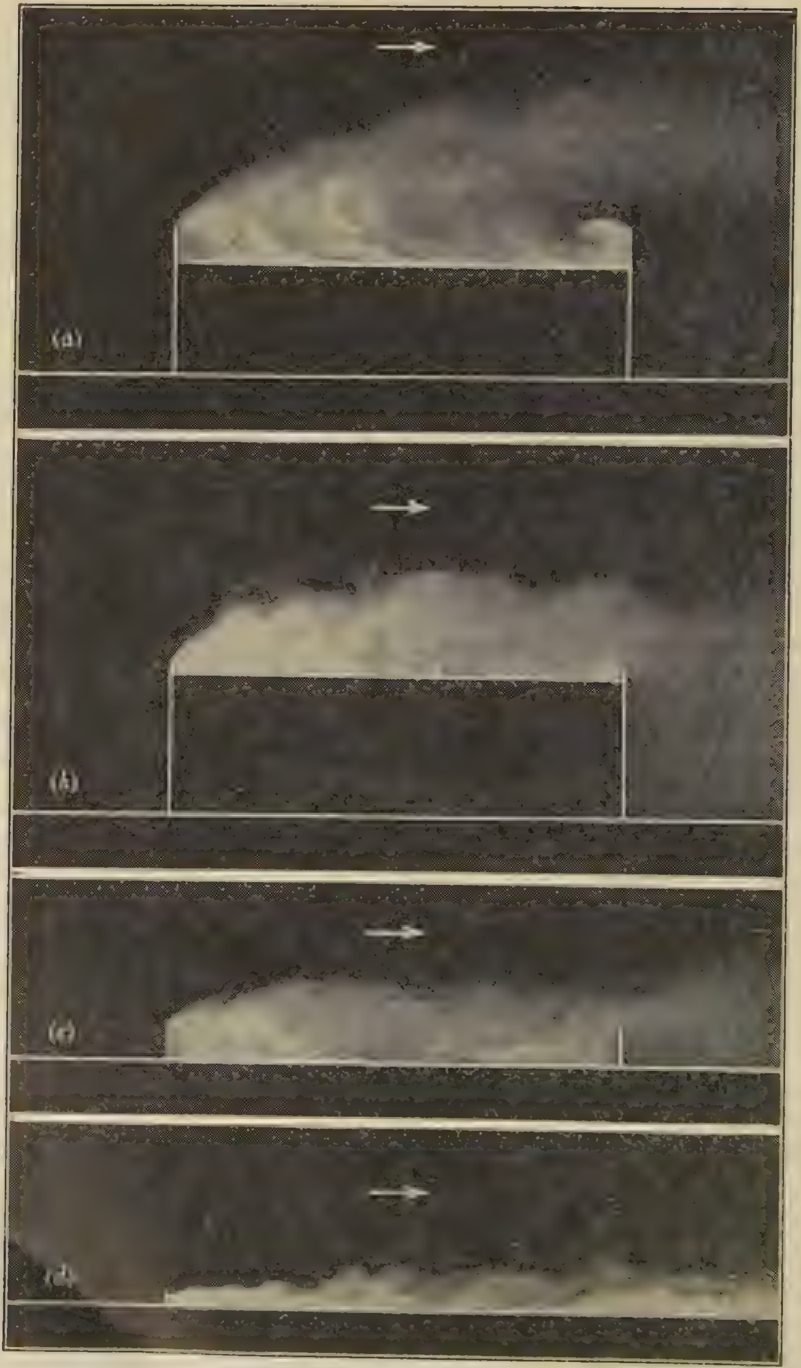

Fig. 19.—Concentration of Smoke over Variously Shaped Schematic Evaporation Pans with the Same Oncoming Wind Velocity

indicated at point A but will approach the minimum value of case 1. Within the primary eddy zone the rate of evaporation is obviously much less than in the previous cases because the eddy tends to recirculate the same air, thereby maintaining a high vapor concentration over this region.

Case 5.—Finally, assume for the sake of argument that the conditions are the same as those in case 1 but that a high wall is at or near the shore; turbulence will be created in much the same manner as in case 4 except that the eddy will extend higher and farther from the shore, causing the greatest rate of evaporation at point B. Again, the effect of increased turbulence is offset partly by a high vapor concentration at point B, and the evaporation curve may lie above or below that shown except that it approaches the same minimum as that in cases 1, 2, and 4.

Of particular concern to the hydrologist is the evaporation from a standard evaporation pan which may be considered as a slightly modified miniature of one or another of the foregoing cases. The turbulence at ground level is determined by the roughness of the terrain and objects immediately upwind from the point in question; and in many cases, such as the Weather Bureau Standard Class A pan, the governing turbulence over the pan is determined by the shape of the pan itself. This is quite graphically seen in Figs. 19(*a*), 19(*b*), 19(*c*), and 19(*d*) which are photographs of the diffusion of smoke over the surface of schematic two-dimensional pans—the concentration of smoke being analogous to the concentration of water vapor over an evaporation pan. To obtain these photographs, titanium tetrachloride was placed on the surface of the pans in a glass-walled air tunnel, 2 in. wide by 22 in. high and 36 in. long. For the range of conditions investigated, the geometry of the pans fixed the scale of the turbulence and the eddy pattern whereas the wind velocity determined only the violence of the turbulence.

As the wall in case 5 creates an eddy, so the side of the evaporation pan creates a similar eddy in miniature. At the left edge of the water surface in Fig. 19(*a*) is a relatively stagnant zone where, as described in cases 4 and 5, there is less evaporation than farther to the right where the concentration gradient $\partial c/\partial x$ is greater. When the freeboard is reduced, as in Fig. 19(*b*), the primary eddy (in these cases extending to the other edge of the pan) is changed only slightly—its size being determined mainly by the height of the pan above ground level. When the pan height is reduced, however (as in Figs. 19(*c*) and 19(*d*) which simulate buried pans), the primary eddy is so small that it covers only a fraction of the pan width, thereby leaving a considerable portion of the surface exposed to the small but violent eddies shown in the photographs. From these photographs it is readily seen that, when the pan is so placed that the turbulence of the air immediately above it is different from that of the surrounding atmosphere, an abnormal evaporation rate will result. Examples of this condition are obtained from placing a standard Class A pan on the surface of level ground or among scattered clumps of underbrush or trees. Evaporation in the first example will be more, and that in the second will be less, than when the turbulence of the air along the ground remains relatively unchanged as it passes over the water surface. Before deciding upon the shape and position of a pan to be used at a given location, consideration should be given to the general roughness and other features of the surrounding terrain which create turbulence so that a pan which will not cause abnormal turbulence may be chosen.

Many investigators in the past have studied evaporation of different liquids from various surfaces using wind tunnels to control the wind velocity. However, the writer could find no place in the literature where adequate measurement of the turbulence or the velocity gradient above the liquid surface has been described. These variables obviously must be determined for a complete analysis and understanding of evaporation. Thus, there is an excellent opportunity for much desired research in a wind tunnel, designed and operated so that the turbulence of the oncoming air and the eddy form over the liquid may be controlled and measured. There is also a need for wind-tunnel studies of the flow patterns over standard evaporation pans and actual measurement of the evaporation from them.

G. H. Hickox,[11] M. Am. Soc. C. E.—The discussion evoked by this paper is gratifying. Many valuable additional data have been submitted, particularly in the discussions by Messrs. Young and Albertson.

Mr. Albertson's amplification of the kinetic theory of evaporation is welcome. A detailed mathematical treatment was omitted from the original paper to save space. It is urgently recommended that every engineer who undertakes to study evaporation familiarize himself with the applicable kinetic theory. An excellent treatment by L. B. Loeb (5) is available. If more of the investigators of evaporation phenomena had recognized the validity of the kinetic theory, much more rapid progress would have been made. It should be noted that the equations apply only to momentary or equilibrium conditions and that agreement need not necessarily be expected by trying to use figures based on daily averages.

Mr. Thornthwaite questions the applicability of the analogy between heat transfer and mass transfer. He seems to be of the opinion that turbulent transfer cannot be predicted unless the mass concentration controls the coefficient of turbulent transfer. This objection does not seem to have interfered with the successful prediction of the vertical distribution of silt in a turbulent stream in which the silt concentration has no appreciable effect on the turbulence. The applicability of the analogy can only be judged from the results obtained by its use.

The literature on heat transfer in which over-all transfer coefficients, or film coefficients, have been used is very extensive and forms the basis of present-day studies of heat transfer. Reference is made to such standard texts as those by W. H. McAdams (64) and W. J. King (11). The mass transfer analogy has been used successfully in diffusion problems (13).

Mr. Thornthwaite refers to the observations by Carl Rohwer, M. Am. Soc. C. E. (16), in which vapor pressure of the air was determined within an inch of the water surface by means of an aspiration psychrometer; and he refers to the writer's measurement of relative humidity by means of a sling psychrometer as "crude." It has not been generally recognized that the use of the aspiration psychrometer by Mr. Rohwer required a volume of about 9 cu ft of air per minute, which is the rate necessary to maintain the required velocity past the wet and dry bulb thermometers. During an observation period of one minute, therefore, a volume of 9 cu ft of air was drawn through the instrument. This scarcely can be considered as representing a sample one inch from the surface. The writer agrees that the use of a sling psychrometer disturbs still air but he is not sure that the disturbance is much greater than that caused by removing a volume of 9 cu ft from the vicinity of the tests. In the experiments described under the heading, "Evaporation as a Mass Transfer Process: Evaporation into Still Air," the chamber containing the evaporation pan was placed in a room in which the humidity was controlled rather closely and in which the air was continually circulated to maintain a uniform distribution. All measurements within the chamber were made with thermocouples and the air within the chamber was not disturbed except by the evaporation process. Provision was made for circulation of air through the chamber as required by natural convection processes within the chamber. The false floor was placed one-quarter inch from the wall to allow entrance of air and the top of the chamber was open. It was provided with vertical baffles, however, so that the circulation of air in the room outside would not set up disturbances within the chamber itself. The relative humidity was measured outside the chamber and the air within was not disturbed. Under these circumstances the determination of relative humidity can scarcely be considered crude.

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[11] Senior Hydr. Engr., TVA, Hydraulic Laboratory, Norris, Tenn.

Mr. Rohwer questions whether two sets of interference bands might not be obtained from the two surfaces of the mirror A shown in Fig. 3. The production of two sets of interference bands requires much greater optical accuracy than does the reflection of two images. Two sets of bands could be produced only if both the surfaces of mirror A were optically flat and nearly enough parallel to produce interference bands either with respect to each other or mirror B. The interferometer constructed by the author utilized plate glass from a broken automobile windshield. Considerable selection was necessary to find a piece having one surface flat enough to serve. It would be extremely unusual to find the two sides of one piece of glass both flat and parallel. This is a refinement that has not yet been reached in the manufacture of automobile windshields. No difficulty from this source was experienced.

Mr. Rohwer observes that the evaporation below the break in the curve of Fig. 5 is about four times that computed by Eq. 5 and considers that this is an indictment of the equation, because when the water is colder than the air, the cold air above the evaporation pan is stationary and water vapor can escape by diffusion alone. This is not quite correct. Eq. 5 was obtained by analogy with heat transfer from a circular surface into a semi-infinite solid. The analogy is not complete for two reasons. First, the condition of a semi-infinite solid is not fulfilled, as it would require the presence of an infinite mass of cold, perfectly quiet air above the surface of the water. It is clear that the mass of cold air is not infinite. Second, the air is not quiet, as Mr. Rohwer has assumed, but is in motion. Most of the experiments plotted in Fig. 5 were made on a pan with zero rim height. Smoke injected into the chamber showed that the air mass above the pan settled slowly and spread out on to the false floor around the pan. The result was that the air above the pan was moving slowly. This accounts fully for the increase in observed evaporation over that given by the theoretical equation.

Mr. Meyer's correction regarding the point of measurement of wind velocity used in his formula is gratefully acknowledged.

Mr. Rohwer comments on the application of Eq. 15*b* to diameters greater than 10 ft. Eq. 15*b*, showing the effect of pan diameter on the evaporation rate, was based on heat transfer experiments made on areas having a maximum diameter of probably 3 or 4 ft. Since the value of the exponent of Reynolds' number in Eq. 14 is empirical, one should not expect to find agreement outside the range of experiment. It is perhaps surprising to find that the agreement is as good as it is for diameters up to 10 or 12 ft.

Professor Kalinske concludes that the effect of pan diameter on evaporation varies as the diameter to the -0.125 power. The experimental data available appear to indicate that the exponent is somewhat less than the -0.25 predicted from the mass transfer analogy. Various experimenters have found exponents varying from -0.11 to -0.19.

Mr. Young's observations on the effect of rim height are interesting. They are in direct contradiction to those offered by the writer, but may be explained by the observations reported by Mr. Albertson.

Mr. Young presents additional data on the effect of pan color on evaporation. The effect of pan color appears to be well known but does not appear to be reflected in very many of the published data. The standard galvanized pan changes its surface characteristics very rapidly in summertime because of the growth of algae. It would seem that standard operating instructions for any evaporation pan installation should stress the necessity for frequent cleaning and restoration of the pan surface to a standard condition.

The effect of a screen over a pan in reducing temperature fluctuations is what might be expected. The screen intercepts incoming radiation from the sun during the day and also shields the pan from the low effective space temperatures at night. The result is less absorption of heat in the daytime and a lower maximum temperature, and a reduced back radiation at night with less cooling and a resulting higher minimum temperature.

The photographs of Fig. 19 are valuable additions to current knowledge as to the effect of turbulence. Study of these photographs leads to the conclusion expressed by Mr. Albertson that the surroundings should be taken into consideration in the mounting of a pan. It should be noted also that any pan with a projecting rim possesses inherent boundary effects that should be taken into consideration when an attempt is made to apply the results of pan measurements to large reservoirs where the boundary effects may be insignificant.

Mr. Meyer's statement regarding the absorptive capacity of water is substantiated by Fig. 12, in which the transmission factor is equal to one minus the absorptivity. Fig. 12 indicates that 18% of the incident solar energy is absorbed in the first millimeter of depth and 31% in the first centimeter. This is a convincing argument for the necessity of measuring water temperatures as near to the surface as possible.

The answer to Mr. Meyer's question about the relative evaporation of a Weather Bureau Standard Class A pan 2 ft deep compared to the 12-in. depth now in use is intimately related to the problem of applying the results of pan observations to reservoirs. The various factors affecting evaporation are so interrelated that it is scarcely possible to change one without affecting all the others. For this reason the writer is not able to give a categorical answer to this question.

The writer regrets that the scope of the paper prevents a discussion of the transfer from pan records to reservoirs. The dissertation filed in the Engineering Societies Library contains additional material which bears on the subject. The problem is complicated and lengthy and has not been worked out in detail. Much additional research will be necessary before it can be solved satisfactorily.

Mr. Blaney asks for detailed specifications of a standard evaporation station. The writer wishes that he was able to present these specifications. Although he has some ideas of what might constitute a suitable installation, it is recognized that they are based largely on incomplete data. The first objective for further study is the establishment of a suitable standard station. Without specifying the details of such an installation it is felt that it should satisfy at least the following six requirements:

1. Water temperatures should be measured as close to the surface as practicable and the measuring device should be sheltered from the direct rays of the sun so that the recorded temperature will be that of the water very close to the surface. The temperature gradient in still water is very high near the surface in daytime and it is the surface temperature that controls evaporation. In large bodies of water that are exposed to wind and wave action the temperature is practically uniform throughout the upper 2 to 5 ft and measurements made a few inches from the surface may be satisfactory. This is not the case with still water pans and this difference should be recognized in the installation.
2. It is probably desirable that the pan be insulated to minimize the boundary effects which do not exist or are negligible in reservoirs. The Weather Bureau Standard Class A pan probably departs more widely from reservoir or lake conditions than any other pan.
3. The pan should be as deep as feasible in order to simulate more closely lake or reservoir conditions.
4. The exposure should undoubtedly be governed by criteria similar to those outlined by Mr. Albertson in his excellent discussion of the effect of exposure on turbulence.
5. Care should be taken to maintain the interior surface so that large variations in rate will not occur.
6. The effect of the fences and instrument shelters in producing turbulence should be investigated and evaluated. It may be found desirable to enlarge the enclosure and to place instruments with regard to the direction of the prevailing winds in so far as this can be done without shading the pan.

Further research is indicated to establish the type of installation that will be most useful both in furthering the knowledge of evaporation and in supplying useful data to the practicing engineer.

It is hoped that Professor Kalinske and Mr. Albertson will have the opportunity to complete the research outlined in their discussions. If the studies are carefully made and properly interpreted, they will lead to a much better understanding of the problem than has been possible with the type of analyses heretofore made.

No attempt was made to submit a complete bibliography with this paper, and many valuable references were omitted simply because it was not necessary to refer to them. A complete bibliography, existing as it does, in the fields of civil, mechanical, and chemical engineering, meteorology, hydrology, oceanography, biology, and plant ecology, would occupy an entire volume. Two publications, by Mr. Meyer (62) and by A. A. Young, Assoc. M. Am. Soc. C. E. (65), however, contain so many data that they should be included in any comprehensive list.

In conclusion, the writer wishes to express his thanks to those who took time to read the paper and to submit criticism, discussion, and additional data.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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TRANSACTIONS

Paper No. 2267

TRANSPORTATION OF SUSPENDED SEDIMENT BY WATER

By Vito A. Vanoni,[1] Assoc. M. Am. Soc. C. E.

With Discussion by Messrs. Ralph W. Powell, E. R. Van Driest, Weston Gavett, Berard J. Witzig, A. A. Kalinske, Paul F. Nemenyi, and Vito A. Vanoni.

Synopsis

Measurements of the sediment and velocity distributions in a laboratory flume were made for various values of rate of flow, slope of channel, and size and amount of suspended load. The experimental sediment-distribution measurements were compared with the theoretical distribution derived on the assumption that the coefficient for the turbulent transfer of suspended sediment is the same as the coefficient for the turbulent transfer of momentum.

The following are the main findings:

(1) The measured sediment distributions have the same form as the theoretical distribution but do not agree quantitatively with them. This shows how invalid is the basic assumption that the transfer coefficient for sediment was equal to that for momentum. In general, the measured relative concentrations were larger than those given by the theory, the difference being greatest for the fine sediment and diminishing as the suspended load became coarser.

(2) Suspended sediment in a flow tends to reduce appreciably the turbulent transfer of momentum and hence the resistance to flow, allowing the sediment-laden water to flow more rapidly than a comparable clear water flow. This effect varies with the total sediment load and the grain size of the sediment.

(3) Suspended load in a flow may be a factor in causing the secondary circulation observed in rivers.

Introduction

Many important practical problems of today involve the movement of sediments by water, and their complete solution requires more knowledge of

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Note.—Published in June, 1944, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

[1] Project Supervisor, Cooperative Laboratory, U. S. Dept. of Agriculture, SCS, Asst. Prof. of Hydraulics, California Inst. of Technology, Pasadena, Calif.

this subject than now exists. Flood control and channel maintenance on streams, both large and small, are just as much problems of sediment movement as of water flow. Established practices, in fact, are successful in handling the water flow of floods, but the problem of passing the sediment load is still unsolved. In recent years the work of the United States Department of Agriculture in developing methods of reducing accelerated erosion on agricultural lands has encountered many new sediment-transportation problems, which have shown the urgent need for additional fundamental knowledge of this subject. It is in connection with this need that this study was undertaken.

Sediment may be transported by flowing water in two essentially different ways—by rolling or sliding along the bed of the stream channel or in suspension in the body of the fluid. The material transported is called, respectively, the "bed load of the stream," or simply "bed load," and the "suspended load." Just above the bed a continual interchange of material is occurring between the bed and the overlying fluid and in this locality it is obviously difficult to distinguish between the bed load and suspended load. The two types of transportation are by no means independent. They are separated only for convenience in studying them and referring to them.[2] This paper deals only with suspended load.

PREVIOUS WORK

Theories.—Modern theories of suspended-load transportation are based on developments in the mechanics of turbulent flow of fluids. Consider a two-dimensional turbulent flow in the x-direction with a velocity $u(y)$, in which y is normal to x. Then, according to Osborne Reynolds,[3] the shear parallel to x on a plane normal to y may be expressed by

$$\tau = -\rho\,\overline{u'\,v'} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1a)$$

in which ρ is the mass density of the fluid, and u' and v' are the turbulent velocity fluctuations in the x and y directions, respectively, and the bar denotes mean value.

M. J. Boussinesq[4] introduced an expression for the shear of the form,

$$\tau = \rho\,\epsilon_m \frac{du}{dy} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1b)$$

in which the quantity $\rho\,\epsilon_m$ (often called the eddy viscosity) is analogous to the coefficient of viscosity in the expression for viscous shear; and u is the mean velocity in the x-direction. The quantity ϵ_m has the dimensions of a kinematic viscosity ν, but unlike its counterpart it is a function of flow conditions and, therefore, varies from point to point. The quantity ϵ_m is a coefficient expressing the exchange of momentum between neighboring filaments.

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[2] "A Distinction Between Bed-Load and Suspended Load in Natural Streams," by H. A. Einstein, A. G. Anderson, and J. W. Johnson, *Transactions*, Am. Geophysical Union, Pt. II, 1940, pp. 628–633.

[3] "On the Dynamical Theory of Incompressible Viscous Fluids and the Determination of the Criterion," by Osborne Reynolds, *Catalogue of Scientific Papers (1800–1900)*, compiled by the Royal Soc. of London, Cambridge Univ. Press, Vol. 2, pp. 535–577; also *Philosophical Transactions*, Royal Soc. of London, Series A, Vol. 186, 1895, p. 123.

[4] "Essai sur la Theorie des Eaux Courantes," by M. J. Boussinesq, *Mémoires, Presenté par divers Savants*, L'Académie de l'Institut de France, Vol. 23, 1877, pp. 1–680.

This quantity is called the turbulent momentum transfer coefficient. Obviously it has the dimensions, length × velocity.

Theodor von Kármán,[5] M. Am. Soc. C. E., introduced the equation,

$$\tau = \beta \, \rho \, v' \, l \frac{du}{dy} \quad \text{(2)}$$

in which β is a correlation coefficient, v' is the average of the absolute values of the fluctuations normal to the main flow, and l is the mixing length. From Eq. 2 it is seen that the turbulent momentum transfer coefficient, ϵ_m, has the value,

$$\epsilon_m = \beta \, v' \, l \quad \text{(3)}$$

and the dimensions, velocity × length.

Just as Eq. 2 expresses the rate of transfer of momentum due to turbulent mixing, it is possible to write by a similar equation the transfer of suspended particles, or any other property that is carried by the fluid. Thus, the rate of transfer of mass of suspended particles per unit area is

$$G = - \beta \, v' \, l \frac{dC}{dy} \quad \text{(4)}$$

in which C is the concentration of suspended material in mass per unit volume. Professor von Kármán[5] pointed out that the values of β, v', and l in Eqs. 2, 3, and 4 need not necessarily be the same.

For steady conditions the upward transport of material due to turbulence is balanced by the settling due to the force of gravity. This is expressed by

$$- \beta \, v' \, l \frac{dC}{dy} = w \, C \quad \text{(5)}$$

in which C is now the concentration of the fraction of the sediment with a settling velocity w. Transposing, the differential equation for suspended sediment becomes,

$$w \, C + \epsilon_s \frac{dC}{dy} = 0 \quad \text{(6)}$$

in which ϵ_s is the sediment transfer coefficient that is similar to ϵ_m but not necessarily equal to it. Solution of Eq. 6 gives,

$$\log_e \frac{C}{C_a} = - w \int_a^y \frac{dy}{\epsilon_s} \quad \text{(7)}$$

in which C_a is the concentration at an arbitrary reference level $y = a$.

If ϵ_s is constant over the depth, that is, $\epsilon_s(y) =$ constant, Eq. 7 may be integrated to give

$$\frac{C}{C_a} = e^{-i} \quad \text{(8)}$$

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[5] "Some Aspects of the Turbulence Problem," by Theodor von Kármán, *Proceedings*, 4th International Cong. of Applied Mechanics, 1934, pp. 54–91.

in which e is the base of the natural logarithms; and, to simplify typography,

$$i = \frac{w}{\epsilon_s}(y - a) \qquad (9)$$

Eq. 8 was apparently first given by Wilhelm Schmidt,[6] who applied it to the study of suspension of dust particles in the atmosphere. Eq. 6 was derived by a formal procedure by Morrough P. O'Brien,[7] M. Am. Soc. C. E. The more direct derivation is the work of Professor von Kármán.[5]

In general, ϵ_s is a function of y and must be so expressed before Eq. 7 can be integrated. Since the expression for ϵ_s is not known, the assumption is made that $\epsilon_s = \epsilon_m$ and Eq. 7 is integrated on this basis. From Eqs. 1b and 2,

$$\epsilon_m = \beta\, v'\, l = \frac{\tau}{\rho \dfrac{du}{dy}} \qquad (10)$$

Introducing Eq. 10 into Eq. 7 yields the relation

$$\log_e \frac{C}{C_a} = -\rho\, w \int_a^y \frac{\dfrac{du}{dy}}{\tau}\, dy \qquad (11)$$

which was first presented by Professor von Kármán[5] in 1934.

In a uniform open-channel flow whose ratio of width to depth is large, these relations will apply. The expression for the shear stress is given by

$$\tau = \tau_0 \frac{y_m - y}{y_m} \qquad (12a)$$

in which τ_0 is the shear at the bottom, or,

$$\tau_0 = \gamma\, y_m\, S \qquad (12b)$$

Introducing Eq. 12a into Eq. 11,

$$\log_e \frac{C}{C_a} = \frac{w}{\dfrac{\tau_0}{\rho}} \int_a^y \frac{\dfrac{du}{dy}}{\dfrac{y_m - y}{y_m}}\, dy \qquad (13)$$

An expression for the velocity gradient is given by the von Kármán[8] universal velocity defect law,

$$\frac{u - u_{\max}}{\sqrt{\dfrac{\tau_0}{\rho}}} = \frac{1}{k} \log_e \frac{y}{r_0} \qquad (14)$$

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[6] "Der Massenaustauch in freier Luft und verwandte Erscheinungen," by Wilhelm Schmidt, *Probleme der kosmischen Physik*, 1925, Pt. 7.

[7] "Review of the Theory of Turbulent Flow and Its Relation to Sediment-Transportation," by M. P. O'Brien, *Transactions*, Am. Geophysical Union, Section of Hydrology, 1933, pp. 487–491.

[8] "Turbulence and Skin Friction," by Theodor von Kármán, *Journal of the Aeronautical Sciences*, Vol. 1, No. 1, 1934 pp. 1–20.

which has been shown by J. Nikuradse[9,10] to apply to pipes and by G. H. Keulegan[11] to apply also to open channels. In Eq. 14, k is a universal constant (0.40 for pipes) and y is the distance from the wall of the pipe of radius r_0. For wide open channels, as discussed here, r_0 in Eq. 14 is replaced with y_m, the depth of the flow. The derivative of the velocity then becomes,

$$\frac{du}{dy} = \frac{1}{k}\sqrt{\frac{\tau_0}{\rho}}\,\frac{1}{y}\ldots\ldots\ldots\ldots(15a)$$

which, when introduced into Eq. 13, gives

$$\log\frac{C}{C_a} = -\frac{w}{k\sqrt{\frac{\tau_0}{\rho}}}\int_a^y \frac{dy}{y\left(\frac{y_m - y}{y_m}\right)}\ldots\ldots\ldots\ldots(15b)$$

Integrating Eq. 15*b*,

$$\frac{C}{C_a} = \left(\frac{y_m - y}{y} \times \frac{a}{y_m - a}\right)^z = H^z\ldots\ldots\ldots\ldots(16)$$

in which, to simplify typography,

$$z = \frac{w}{k\sqrt{\frac{\tau_0}{\rho}}} = \frac{w}{k\sqrt{g\,y_m\,S}}\ldots\ldots\ldots\ldots(17)$$

From Eq. 16 it is possible to plot curves of relative sediment distribution as was first done about 1936 by Arthur T. Ippen, Assoc. M. Am. Soc. C. E., at the suggestion of Professor von Kármán.[12]

Eq. 16, as well as the equations preceding it, gives only a relative concentration and therefore does not make possible the calculation of the total transport of the stream. To make this possible a value for C_a is needed, and the problem of obtaining the transporting capacity of a flow may be simply stated as one of determining this value. In discussing the magnitude of C_a, Professor von Kármán[5] states that it probably depends on the size of the sediment and on the magnitude of the shearing stress acting at the ground. Hunter Rouse[13] has suggested that the limiting value of a in Eq. 16 be made equal to the height of the roughness elements h_s. He further suggests that the corresponding concentration C_k for this level be calculated from Eq. 8, assuming that in the interval $y = 0$ to $y = h_s$, $\epsilon(y) = \epsilon(h_s) =$ constant, and that the concentration at $y = 0$ be set equal to that of the bed material. E. W. Lane, M. Am. Soc. C. E., and A. A. Kalinske, Assoc. M. Am. Soc. C. E.,

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[9] "Gesetzmässigkeiten der turbulenten Strömung in Glatten Rohren," by J. Nikuradse, *Forschungsheft 356*, Vereines Deutscher Ingenieure, Vol. 3, 1932, p. 36.

[10] "Strömungsgesetze in Rauhen Rohren," by J. Nikuradse, *Forschungsheft 361*, Vereines Deutscher Ingenieure, Vol. 4, 1933, pp. 1–22.

[11] "Laws of Turbulent Flow in Open Channels," by Garbis H. Keulegan, *Journal of Research*, National Bureau of Standards, U. S. Dept. of Commerce, Vol. 21, 1938, pp. 707–741.

[12] "Modern Conceptions of the Mechanics of Fluid Turbulence," by Hunter Rouse, *Transactions*, Am. Soc. C. E., Vol. 102 (1937), Fig. 30, p. 536.

[13] "An Analysis of Sediment Transportation in the Light of Fluid Turbulence," by Hunter Rouse, *Mimeograph Publication SCS-TP-25*, U.S.D.A., 1939.

approaching the problem statistically, argue[14] that only those particles having settling velocities w less than the vertical fluctuations v' of the bed velocity will be picked up. By assuming a normal distribution of v' and setting the root mean-square fluctuation $\sqrt{\overline{(v')^2}}$ proportional to the friction velocity $\sqrt{\frac{\tau_0}{\rho}}$, they proceed to calculate the probable rate of suspension of material.

As a result of an analysis of the energy balance of a stream carrying suspended load, Robert T. Knapp,[15] M. Am. Soc. C. E., advanced the hypothesis that the capacity of a stream is limited only if the settling velocity w of the sediment is greater than the rate of fall of the stream $\bar{u}\,S$, in which $\bar{u}$ is the average velocity of flow and S is the slope of the channel. Furthermore, he deduced that when the settling velocity is less than $\bar{u}\,S$ the material contributes energy to the flow, thus making it possible to pick up additional load until extremely high concentrations are reached.

Verification of Theories.—Although no comprehensive suspended-load experiments have been made in which the important variables have been investigated systematically, some noteworthy experimental work has been done.

Mr. Schmidt[6] appears to have first applied the turbulence theory to the problem of suspended load in his studies of the suspension of dust particles in air. He obtained Eq. 8, which assumes a constant value for the exchange coefficient.

In 1929 H. E. Hurst,[16] reported a series of experiments on the suspension of sediment in a cylindrical column of water agitated uniformly by a series of propellers much the same as those used by Joule in his experiments on the mechanical equivalent of heat. At various levels in his apparatus he measured the concentrations of three well-graded sands (mean size 0.9 mm, 0.4 mm, and 0.2 mm) for several propeller speeds and found that the distribution of sediment followed an exponential relationship of the form of Eq. 8. Mr. Hurst was also able to derive his relationship from an analysis based on the kinetic theory of gases.

Professor Rouse[17] performed some experiments similar to those by Mr. Hurst using an agitator of coarse mesh bars that was oscillated in the vertical direction with simple harmonic motion. The results of Professor Rouse's work indicate that Eq. 8 is valid, and verify the theory. Professor Rouse observed a systematic deviation of the value of ϵ obtained for the coarse sand from that obtained from fine sand. The deviation was attributed to a difference in the correlation factor β of Eq. 3 for mixing of the fluid and of the sediment.

In 1932 John B. Leighly[18] outlined a method of calculating the exchange coefficient ϵ for natural streams and discussed its effect in suspending sediment in the body of the flow. In a later paper Mr. Leighly[19] calculated the exchange coefficient from some sediment measurements on natural streams by using Eq. 8. He also calculated the sediment distribution from the exchange coefficient by step integration.

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[14] "The Relation of Suspended to Bed Material in Rivers," by A. A. Kalinske and E. W. Lane, *Transactions*, Am. Geophysical Union, Pt. 4, 1939, pp. 637–641.

[15] "Energy-Balance in Stream-Flows Carrying Suspended Load," by Robert T. Knapp, *ibid.*, Pt. 1, 1938, pp. 501–505.

[16] "The Suspension of Sand in Water," by H. E. Hurst, *Proceedings*, Royal Soc. of London, Series A, Vol. 124, 1929, pp. 196–201.

[17] "Experiments on the Mechanics of Sediment Suspension," by Hunter Rouse, *Proceedings*, 5th International Cong. for Applied Mechanics, 1938, pp. 550–554.

[18] "Toward a Theory of the Morphologic Significance of Turbulence in the Flow of Water in Streams," by John B. Leighly, *University of California Publications in Geography*, Vol. 6, No. 1, 1932, pp. 1–22.

In 1935 J. E. Christiansen,[20] Assoc. M. Am. Soc. C. E., attempted to check Eq. 7 by applying it to sediment measurements made by Harry F. Blaney and the late Samuel Fortier, Members, Am. Soc. C. E., in the Colorado River and Imperial Canals.[21] In general the observations agreed with Eq. 7, but the results were not entirely conclusive since complete data were not available and it was necessary to estimate such quantities as the slope of the canals and the settling velocity of the sediment before calculations could be made.

E. G. Richardson[22] made some simultaneous sediment and velocity measurements in a flume 5 cm wide, 8 cm deep, and 150 cm long which led him to conclude that the vertical distribution of sediment is nearly exponential. This agrees with Eq. 8 and would indicate that the mixing coefficient had approximately a constant value over the depth. Such a condition could be approximated in a channel downstream from a disturbance such as might occur in the inlet to a flume. In a later paper[23] this same author reports the results of experiments in a somewhat larger channel (1 ft by 1 ft by 6 ft) and concludes that in the boundary region the concentration of suspended sediment is inversely proportional to y and that in the open stream it varies exponentially, much as expressed by Eq. 8.

APPARATUS AND PROCEDURE

Main Flume.—The experiments were performed in a flume 33$\frac{1}{4}$ in. wide by 60 ft long, arranged so the slope and discharge could be varied. The slope was adjusted by means of four pairs of screw jacks, and the discharge was varied from 1 cu ft per sec to about 5.5 cu ft per sec by changing the speed of the pump in the system through a variable-ratio transmission in the electric motor drive. The sides of the channel were made of 12-in. structural channels lined with $\frac{1}{8}$-in. rubber. The bottom was a $\frac{5}{16}$-in. steel plate artificially roughened with sand. By making the circulation system closed circuit with the minimum velocity in the flume, it was possible to recirculate the suspended sediment with the water without danger of depositing material in the piping system.

The flume as first constructed was described in considerable detail[24] by Robert T. Knapp and Robert M. Oaks and is shown diagrammatically in Fig. 1(a). Experiments with this circuit yielded fair results but indicated that

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[19] "Turbulence and the Transportation of Rock Debris by Streams," by John B. Leighly, *The Geographical Review*, Vol. 24, No. 3, 1934, pp. 453–464.

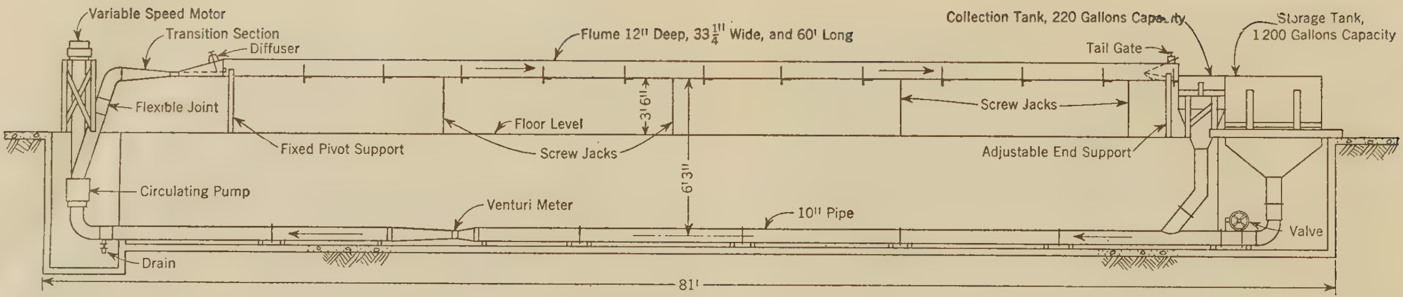
[20] "Distribution of Silt in Open Channels," by J. E. Christiansen, *Transactions*, Am. Geophysical Union, Pt. 2, 1935, pp. 478–485.

[21] "Silt in the Colorado River and Its Relation to Irrigation," by Harry F. Blaney and Samuel Fortier, *Technical Bulletin No. 67*, U.S.D.A., 1928.

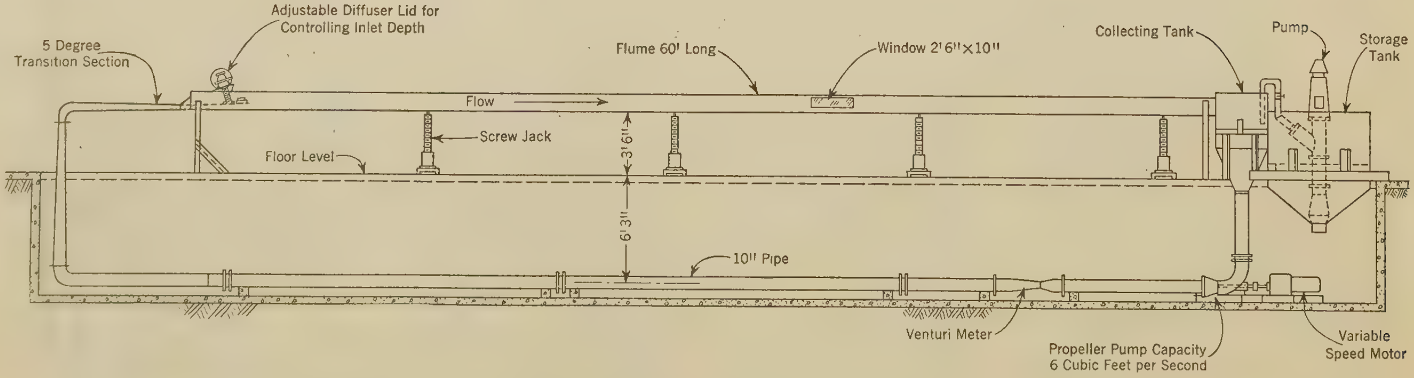
[22] "The Transport of Silt by a Stream," by E. G. Richardson, *Philosophical Magazine and Journal of Science*, Vol. 17, No. 7, 1934, pp. 769–783.

[23] "The Suspension of Solids in a Turbulent Stream," by E. G. Richardson, *Proceedings*, Royal Soc. of London, Series A, Vol. 162, 1937, pp. 583–597.

[24] "A 'Closed-Circuit' Flume for Suspended-Load Studies," by Robert M. Oaks and Robert T. Knapp, *Transactions*, Am. Geophysical Union, Pt. 2, 1936, pp. 525–528.


Variable Speed Motor
Transition Section
Diffuser
Flume 12'' Deep, 33$\frac{1}{4}$'' Wide, and 60' Long
Collection Tank, 220 Gallons Capacity
Storage Tank, 1 200 Gallons Capacity
Tail Gate
Flexible Joint
Screw Jacks
Floor Level
3'6''
Fixed Pivot Support
Screw Jacks
Adjustable End Support
Circulating Pump
Venturi Meter
6'3''
10'' Pipe
Valve
Drain
81'

(*a*) FIRST EXPERIMENTS, SERIES 1, RUNS 1 TO 13


Adjustable Diffuser Lid for Controlling Inlet Depth
5 Degree Transition Section
Flume 60' Long
Window 2'6''×10''
Collecting Tank
Pump
Storage Tank
Flow
Screw Jack
Floor Level
3'6''
6'3''
10'' Pipe
Venturi Meter
Propeller Pump Capacity 6 Cubic Feet per Second
Variable Speed Motor

(*b*) FINAL EXPERIMENTS, SERIES 2, RUNS 14 TO 22

FIG. 1.—FLUME CIRCUITS USED FOR EXPERIMENTS

the disturbances due to the pump and inlet were exerting an appreciable effect on flow in the flume.

The flow was improved by revising the circuit as shown in Figs. 1(*b*) and 2. Moving the pump and venturi meter to the discharge end of the flume provided a run of about 50 ft of 10-in. pipe to "equalize" the flow before it entered the transition to the 33¼-in. flume section. To eliminate the possibility of separation the included angle between the sides of the transition was made 5°.

Fig. 3 shows a general view of the flume from the inlet end. The beam spanning the channel at about midlength carries the instruments used in making measurements. The diffuser lid at the flume inlet is shown at the left.

Adjustment of Flow.—The flow in the flume was readily adjusted to a given depth by pumping the correct volume of water into the system and varying the discharge until uniform flow was reached. Uniform flow for a given discharge was obtained easily (once the flow was set with the water manometer on the venturi meter) by adjusting the amount of water in the closed system. No difficulty was ever experienced with the closed system because of pendulation or any other instability. The flow was adiusted easily and remained steady for indefinite lengths of time.

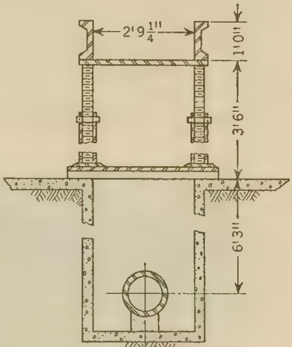


FIG. 2.—SECTION THROUGH A-A, FIG. 1(*b*)

Depth Measurements.—Water depths were measured with a point gage which can be read to 0.001 ft by means of a vernier scale. Depth measurements were made by first adjusting the elevation of the bracket holding the instrument so that the scale read zero when the point was resting on the bottom and then lifting the point so that it just came in contact with the surface. The average depth for the channel was obtained from measurements of at least 10 or 15 points.

Velocity Measurements.—Velocities were measured with a pitot static tube (diameter, $\frac{3}{16}$ in.) of the standard Prandtl design. The differential pressure on the tube was read to 0.001 ft on a water manometer and converted to velocity by assuming a coefficient of unity for the tube. (That this assumption is justified is shown by H. Peters.[25])

Sediment Sampler.—The distribution of sediment was determined from samples siphoned from the flume through a brass-tube pipette (outside diameter, $\frac{5}{16}$ in.) shaped much like a pitot tube. The tip of the sampler was flattened so its inside dimensions were 0.490 in. by 0.139 in. The average velocity at which the sediment-laden fluid entered the flat-tip section (sampling velocity) was made equal to the stream velocity at the sampling point by adjusting the head on the siphon.

The time to remove a 1-liter sample was calculated for each point from the measured stream velocity and the area of the flat-tip sampler (0.064 sq in.), the head on the siphon required for the rate of flow was determined by trial, and three 1-liter samples were taken. The sediment was removed from the sample by filtering and was dried and weighed. The average weight of the three samples was taken as the concentration at the point. Sample weights of more than 1 g were determined to 1 cg, and the smaller quantities were determined to 1 mg.

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[25] "Druckmessung," by H. Peters, *Handbuch der Experimentalphysik*, Leipzig, Vol. 4, Pt. 1, 1931, pp. 489–510.

Water removed from the flume in sampling was replaced in order to keep the depth uniform. It was found that removing or adding as little as 3 gal of water made a measurable difference in the tailwater depth.

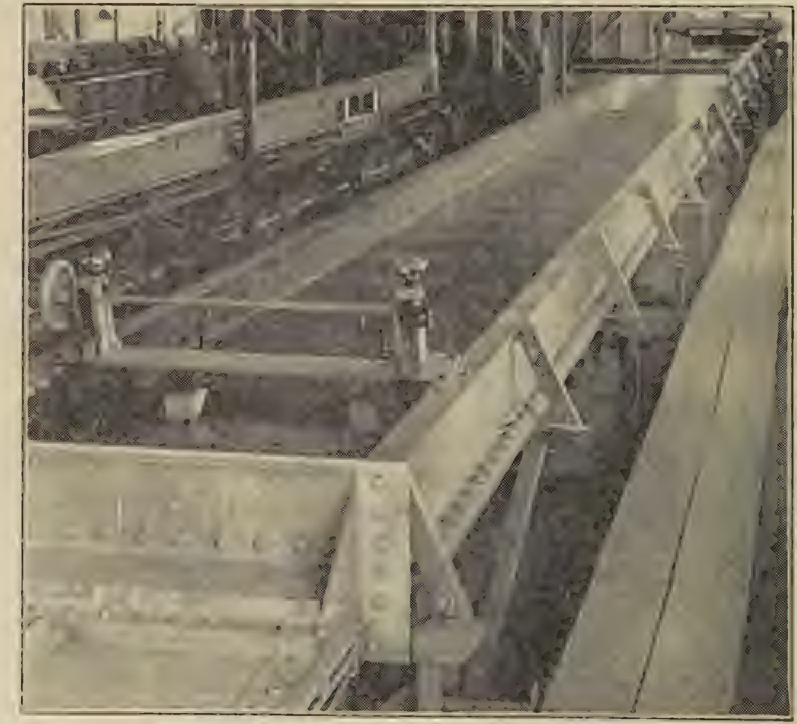

Fig. 3.—General View of Flume from Inlet End

The effect of the sampling velocity on the amount of sediment removed with the sample was studied experimentally. Several 1-liter samples were removed from a point at sampling velocities equal to 1.00, 0.86, and 0.75 times the local stream velocity at the sampling point. The sampling point was 0.5 ft from the center of the flume, 44 ft from the inlet (that is, at station 44), and 0.1 ft from the bottom. The local velocity was 3.19 ft per sec, the flume was set to a slope of 0.0025, and the water was flowing at an equilibrium depth of 0.495 ft. The suspended load had a mean settling rate in still water of 0.058 ft per sec.

The results of this experiment, presented in Table 1, show that, as the sampling velocity decreases, the quantity of sediment collected per given volume of fluid increases. When the sampling velocity is lower than the local stream velocity, the fluid streamlines curve away from the sampler tip.

TABLE 1.—EXPERIMENTAL DETERMINATION OF THE EFFECT OF THE SAMPLING VELOCITY ON THE AMOUNT OF SEDIMENT COLLECTED

No.	Observations[a]	$\frac{u_s}{u} = 1.00$	$\frac{u_s}{u} = 0.86$	$\frac{u_s}{u} = 0.75$	Definitions
1	$\bar{C}$	1.03	1.08	1.12	
2	$\frac{\bar{C}}{\bar{C}_s}$	1.00	1.05	1.09	[a] u_s/u is the ratio of sampling velocity to the velocity of the stream at the sampling point; $\bar{C}$ is the average sediment concentration, in grams per liter, for the series of samples; $\bar{C}/\bar{C}_s$ is the ratio of $\bar{C}$ to the average concentration obtained when the sampling velocity is equal to the stream velocity; and $C/\bar{C}$ is the ratio of the sediment concentration for individual sample to $\bar{C}$.
3	$\frac{C}{\bar{C}}$	0.99 1.04 0.99 0.95 0.97 1.06	0.98 0.94 1.08 0.90 1.00 0.99 1.05 0.91 1.10	1.13 0.99 0.95 0.99 1.00 0.97	

Because of their greater inertia the sediment particles follow paths of less curvature than the fluid, with the result that sediment is transferred to the fluid entering the sampler and the apparent concentration is increased. When the sampling velocity is greater than the stream velocity the curvature is toward the sampler, and a tendency to decrease the amount of sediment collected would therefore be expected.

The values of $C/\bar{C}$ in line 3, Table 1, give some idea of the fluctuation in concentration at a point in the flow. Although the deviation of the weights of individual samples from the mean was as high as 10%, it is interesting to note that the deviation of the average weight of three consecutive samples from the mean does not exceed 3%. Furthermore, the length of a filament of fluid having a volume of 1 liter and an area equal to that of the sampler tip is well in excess of the length of the flume, which is 60 ft.

Sediment Used as Suspended Load.—Sediments used as suspended load were prepared by grading a silica sand in a wind-tunnel classifier constructed[26] by George H. Otto and Professor Rouse. Fig. 4(*a*) shows a logarithmic probability chart of the size distributions of the three sands used as suspended load. The parameters that describe size and grading of these sands are listed in Fig. 4(*b*).

The quantity D_m is the geometric mean sieve diameter and is read directly from the chart. For a normal distribution (that is, when the curve is a straight line), the geometric mean and median diameters are equal. The quantity σ_g (Fig. 4(*b*)) is known as the geometric standard deviation of the sieve diameters. It is the ratio of the geometric mean size D_m to the size corresponding to the ordinate value 15.9%, which in turn is also equal to the ratio of the size corresponding to the ordinate value 84.1% to D_m. The method of plotting the size distribution and of calculating these parameters was shown by Mr. Otto.[27]

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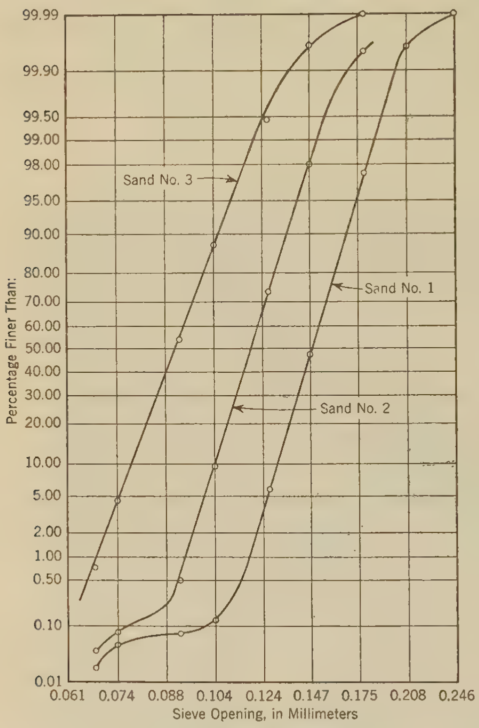
[26] "Wind-Tunnel Classifier for Sand and Silt," by George H. Otto and Hunter Rouse, *Civil Engineering*, July, 1939, pp. 414–415.

The quantity $\bar{D}_s$ is a weighted mean sedimentation diameter, the sedimentation diameter of a particle being defined as the diameter of a sphere having the same density and the same settling velocity as the particle. The mean settling velocity $\bar{w}$ of the center of mass of a group of particles was determined from observed settling times of individual particles of a representative sample of the sediment. The mean sedimentation diameter $\bar{D}_s$ is then the diameter of the quartz sphere having the settling velocity $\bar{w}$ in water. Calculation of the sedimentation diameter was based on the resistance coefficients for spheres as outlined by Professor Rouse.[28] The mean sedimentation diameter was also determined from sieve analyses by taking a weighted mean of the sedimentation diameters of the material held in each sieve, which were determined from actual settling-velocity measurements of samples taken from the sieve separates.

The average sediment concentration C_m for the entire flow was determined from samples siphoned from the collection tank through a $\frac{3}{4}$-in. pipe and rubber hose. The entrance of the siphon was directed vertically upward so the downward flow in the tank could enter without changing direction. During sampling the pipe was moved around to obtain fluid from all parts of the cross section.



(a) SIZE DISTRIBUTION

(b) SEDIMENTATION DIAMETERS IN MILLIMETERS

Sand No.	Geometric mean sieve size D_m	Geometric standard deviation σ_g	Mean sediment diameter $\bar{D}_s$
1	0.147	1.10	0.160
2	0.120	1.11	0.133
3	0.091	1.13	0.100

FIG. 4.—LOGARITHMIC PROBABILITY GRAPH OF SIZE DISTRIBUTION OF SANDS USED AS SUSPENDED LOAD

Bottom Roughness.—As mentioned previously, the sides of the flume were lined with rubber, and the bottom was roughened with sand. The two bottom roughnesses used in the experiments were designated as sands 4 and 5. The mean diameters $\bar{D}_s$ for sands 4 and 5 were, respectively, 0.47 mm and 0.88 mm, and the corresponding standard deviations, σ_g, were 1.12 and 1.10. These sands were graded in the wind-tunnel classifier described previously. The sand was stuck to the bed of the flume with a water-resistant bitumastic paint. The metal surface was first painted and the sand immediately applied in sufficient quantity to cover the surface one layer deep. Any excess material was swept off after the paint was dry.

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[27] "A Modified Logarithmic Probability Graph for the Interpretation of Mechanical Analyses of Sediments," by George H. Otto, *Journal of Sedimentary Petrology*, Vol. 9, No. 2, 1939, pp. 62–76.

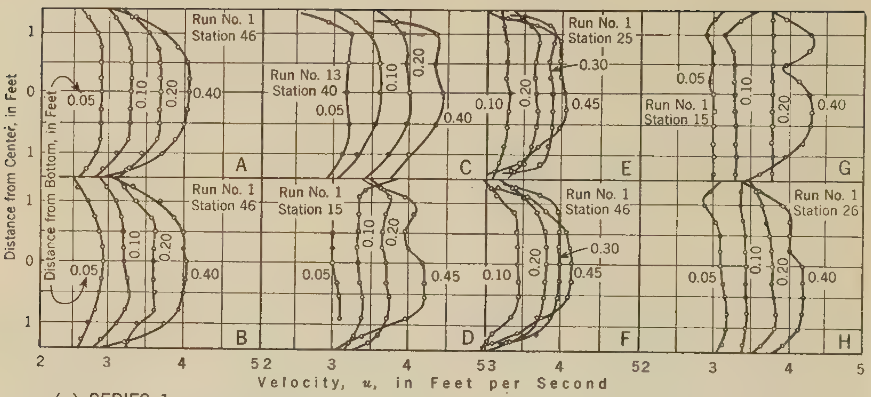
[28] "Nomogram for the Settling Velocity of Spheres," by Hunter Rouse, Exhibit D of the Annual Report of the Committee on Sedimentation, National Research Council, Div. of Geology and Geography, Washington, D. C., 1936–1937, pp. 57–64.

RESULTS

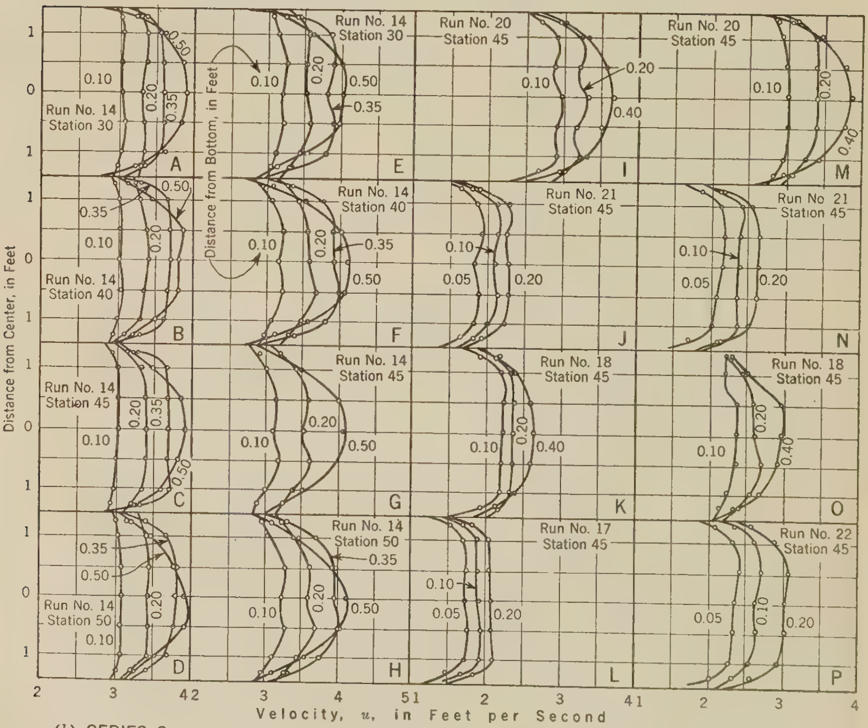
General Outline of Experiments.—The experiments in this study may be divided into two series. In the first series, in the flume circuit shown in Fig. 1(*a*), the slope of the flume ($S = 0.0025$) was kept constant and only one bottom roughness (sand 4) and one suspended load (sand 1) were used. Only the rate at which water was discharged and the amount of sediment in the system were varied. As mentioned under the heading, "Apparatus and Procedure," this circuit gave a disturbed condition at the entrance and was not considered entirely satisfactory. The disturbances were due to the pump, which was near the inlet to the flume, and to separation of the flow in the transition from the pipe to the flume. The latter separation of the flow was remedied, at least in part, by baffles, which, of course, introduced considerable resistance and, therefore, turbulence. A good index of the performance of the system is given by the head loss or the head developed by the pump, which amounted to about 5 ft for a discharge of 5 cu ft per sec.

The second series of experiments was made in the revised circuit shown in Fig. 1(*b*). No separation occurred in the transition section, and the velocity distribution was uniform and symmetrical. Only about 3 ft of head was required to circulate a flow of 5 cu ft per sec in the revised system. In this series, complete measurements were made with one bottom roughness (sand 5), three sizes of suspended load, and two depths at each of two slopes.

Velocity Distribution.—Eq. 16 for the distribution of sediment over a vertical section of a stream was derived for a flow with a large width-depth ratio—that is, two-dimensional conditions—and for the case of logarithmic velocity distribution. Before the relation can be applied to the present experiments it is necessary to show that the assumed conditions obtain. Fig. 5(*a*), with Table 2(*a*), shows the velocity distribution across the flume for the first series of experiments (runs 1 to 13). Typical velocity distribution curves for series 2 (runs 14 to 22) are shown in Fig. 5(*b*), with Table 2(*b*). Comparison of Fig. 5(*b*), distributions A to D, with Fig. 5(*a*), distributions D to F, shows at once the improvement in the flow effected in series 2 by changing the system from that shown in Fig. 1(*a*) to the one in Fig. 1(*b*). These velocity distributions also show that the influence of the walls of the channel does not extend to the center of the flume and that the assumption of two-dimensional conditions near the center of the flume is justified. The only way the resistance of the walls can be transferred to the center of the channel is by turbulent shear or momentum transfer, which can exist only if there is a horizontal velocity



(a) SERIES 1



(b) SERIES 2

Fig. 5.—Velocity Distribution Over Cross Section of Flume (see Fig. 1 for Circulation Systems)

gradient. The fact that this gradient is zero or very small permits the conclusion that near the center of the flume the influence of the walls is negligible.

Fig. 5(*a*), distributions D to F, and Fig. 5(*b*), distributions A to D, give the velocity at various distances from the inlet, and show the gradual stabilization of the velocity. Here again may be noted the improved flow conditions in series 2 (Fig. 5(*b*)). Despite the higher discharge, the equilibrium velocity is established in a short distance and obtains over the remaining length of the flume. The distortion of the velocity at depth 0.5 ft at station 50 (Fig. 5(*b*), distribution D) is due to backwater from the discharge end, which is at station 57. Fig. 5 also shows the effect of the sediment on the flow, discussed subsequently.

TABLE 2.—DATA FOR VELOCITY DISTRIBUTIONS IN FIG. 5

Distribution	Run	Station	Depth y_m	Flow q	Velocity $\bar{u}$	Concentration C_m
(*a*) SERIES 1; CIRCULATION SYSTEM, FIG. 1(*a*)						
A	1	46	0.481	4.80	3.60	0
B	1	46	0.484	4.75	3.54	0.75
C	13	40	0.473	4.87	3.72	3.2
D	1	15	0.485	4.85	3.61	0
E	1	25	0.485	4.85	3.61	0
F	1	46	0.485	4.85	3.61	0
G	1	15	0.480	4.87	3.66	0.75
H	1	26	0.480	4.87	3.66	0.75
I	1	46	0.480	4.87	3.66	0.75
(*b*) SERIES 2; CIRCULATION SYSTEM, FIG. 1(*b*)						
A	14	30	0.554	5.24	3.42	0
B	14	40	....	....	....	....
C	14	45	....	....	....	....
D	14	50	....	....	....	....
E	14	30	0.538	5.24	3.51	1.3
F	14	40	....	....	....	....
G	14	45	....	....	....	....
H	14	50	....	....	....	....
I	20	45	0.46	3.91	2.96	0
J	21	45	0.234	1.29	1.98	0
K	18	45	0.462	2.82	2.20	0
L	17	45	0.269	1.31	1.76	0.05
M	20	45	0.462	4.07	3.18	1.2
N	21	45	0.234	1.42	2.19	1.2
O	18	45	0.462	3.11	2.43	1.2
P	22	45	0.295	2.13	2.60	1.3

Figs. 6 and 7 show dimensionless semilogarithmic graphs of velocity profiles for series 1 and 2, respectively. The relative distance from the bottom $\frac{y}{y_m}$ is plotted on the logarithmic ordinate scale against the dimensionless velocity $\frac{u}{\sqrt{g\, y_m\, S}}$ on the abscissa scale. The quantity $\sqrt{g\, y_m\, S}$ is the so-called friction velocity, commonly written $\sqrt{\frac{\tau_0}{\rho}}$. The abscissa scale is shifted for each curve, the abscissa value 15 being indicated for each curve. The curves in Fig. 6 are arranged in four groups according to the average sediment content of the

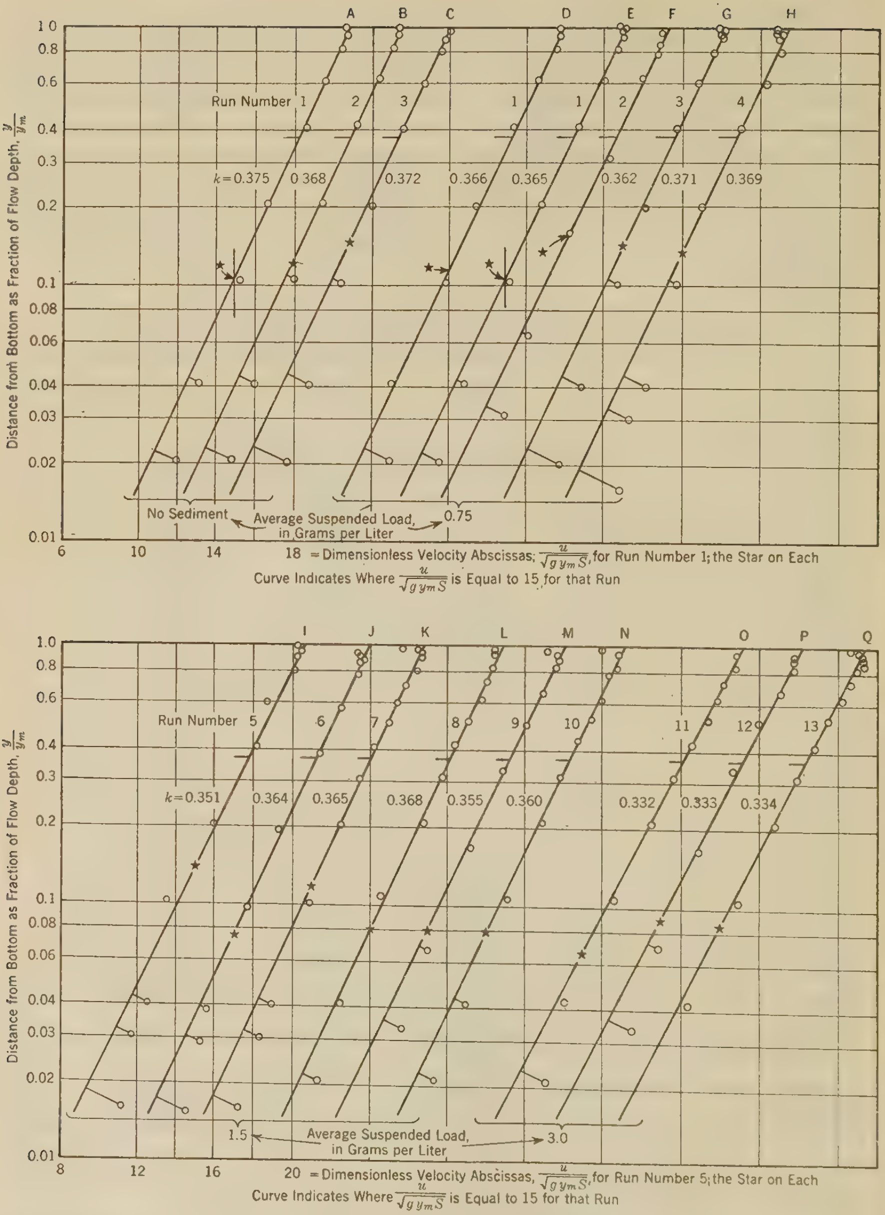


Fig. 6.—Series 1, Runs 1 to 13; Dimensionless Velocity Profiles on Center of Flume

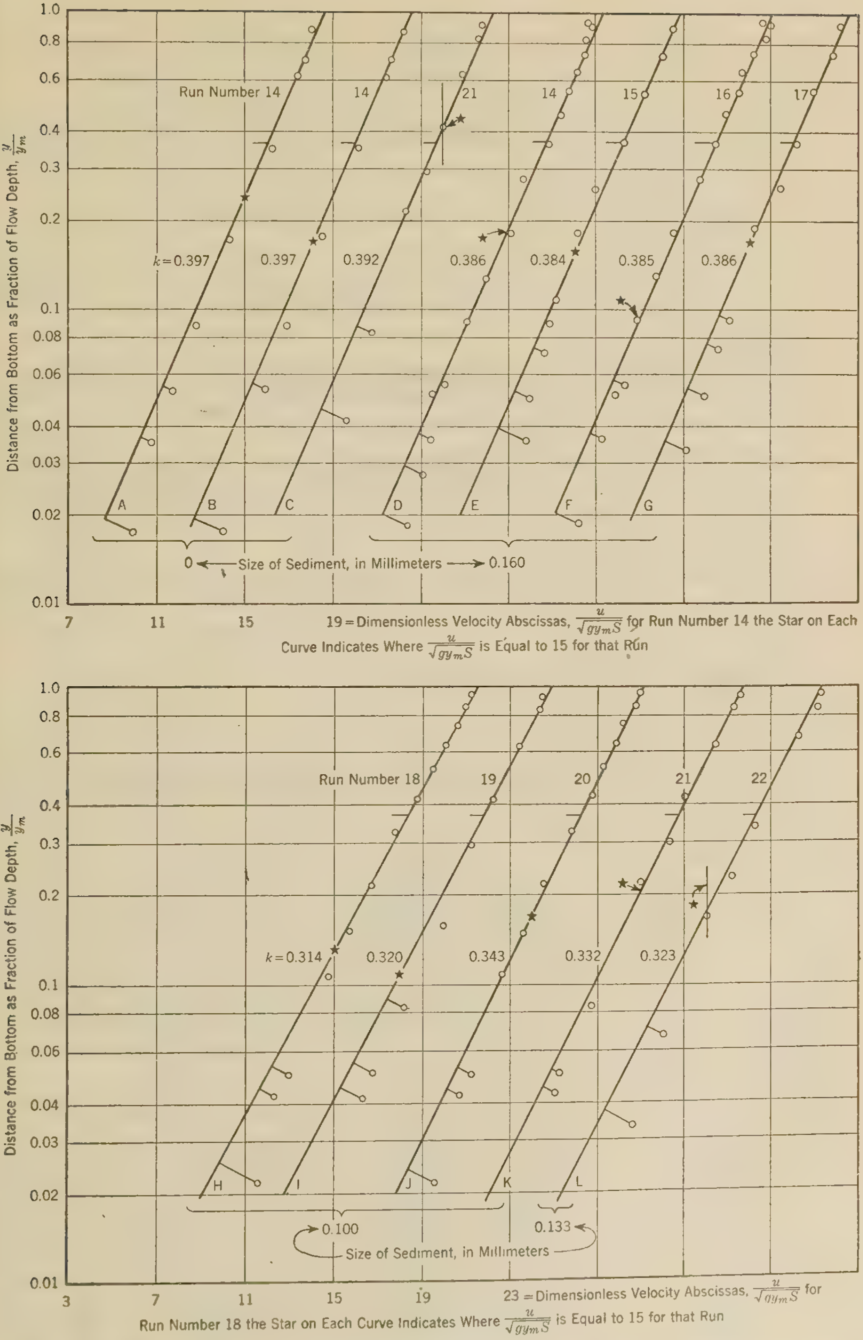


FIG. 7.—SERIES 2, RUNS 14 TO 22; DIMENSIONLESS VELOCITY PROFILES ON CENTER OF FLUME

TABLE 3.—SUMMARY OF EXPERIMENTS

Run No.	Slope S, in ft per ft	Depth y_m, in ft	Friction velocity $\sqrt{g\, y_m\, S}$ (ft per sec)	MEASURING SECTION; DISTANCE, IN FT FROM:		UNIVERSAL CONSTANT, k		Average velocity $\bar{u}_1$, at profile section	Velocity distribution curve[a] (Fig. 6 or 7):	Water temperature, in degrees centigrade	Settling velocity w, in ft per sec	EXPONENT IN EQ. 16		SEDIMENT CONCENTRATION		
				Flume inlet	Center line	Individual	Average					Theoretical z (Eq. 17)	Experimental z_1 (Eq. 18)	Distribution curve[b] (Fig. 9 or 10):	At reference level, C_a	Mean value for flow, C_m
(a) SERIES 1 (SEE FIGS. 6 AND 9)																
1	0.0025	0.481	0.197	46	0.0	0.375	0.372	3.62	A	....	0.0	0.0	0.0	....	0.0	0.0
2	0.0025	0.496	0.200	44		0.368		3.60	B	....						
3	0.0025	0.481	0.197	44		0.372		3.45	C	....						
1	0.0025	0.484	0.197	46	0.0	0.366	0.370	3.60	D	23.6	0.0522	0.723	....	....	....	0.72
					0.0	0.365		3.63	E	21.8	0.0485	0.672	0.67	A	2.60	
					0.5L	0.374		3.63	....	22.6	0.0492	0.667	0.67	B	2.80	
					0.5R	0.374		3.63	....	21.2	0.0478	0.650	0.67	C	2.62	
2	0.0025	0.314	0.159	46	0.0	0.362	0.362	2.80	F	19.0	0.0450	0.782	0.67	D	2.33	0.35
3	0.0025	0.496	0.200	44	0.0	0.371	0.371	3.52	G	23.3	0.0590	0.795	0.80	E	5.80	0.85
					0.5L	0.371		3.52	....	24.4	0.0602	0.810	0.82	F	5.20	
4	0.0025	0.492	0.199	44	0.0	0.408	0.369	3.63	....	20.8	0.0564	0.767	0.77	H	5.18	0.79
					0.5L	0.369		3.54	H	23.3	0.0590	0.804	0.97	G	5.20	
5	0.0025	0.496	0.200	44	0.0	0.351	0.351	3.57	I	23.1	0.0590	0.840	0.83	I	8.30	1.59
					0.5L	0.351		3.48	....	23.8	0.0596	0.850	0.86	J	6.95	
					1.0L	0.359		3.31	....	24.4	0.0603	0.840	0.86	K	7.70	
6	0.0025	0.518	0.204	44	0.0	0.373	0.369	3.97	J	23.8	0.0596	0.784	0.81	L	9.25	1.78
					0.5L	0.364		3.94	....	26.3	0.0626	0.844	0.83	M	7.35	
7	0.0025	0.498	0.200	44	0.0	0.365	0.368	3.82	K	25.1	0.0610	0.835	0.85	N	8.40	1.61
					0.5L	0.371		3.63	....	26.5	0.0626	0.845	0.91	O	7.00	
					1.0L	0.418		3.43	....	27.4	0.0636	0.761	0.91	P	7.30	
8	0.0025	0.483	0.197	40	0.0	0.363	0.365	3.79	L	26.6	0.0626	0.875	0.87	Q	8.35	1.34
					0.5L	0.368		3.75	....	27.8	0.0636	0.877	0.97	R	8.65	
9	0.0025	0.303	0.156	40	0.0	0.355	0.355	3.03	M	23.7	0.0541	0.977	0.96	S	7.80	1.41
					0.5L	0.355		3.00	....	24.7	0.0541	0.977	0.97	T	7.45	
10	0.0025	0.478	0.196	40	0.0	0.360	0.360	3.82	N	26.3	0.0626	0.887	....	....	....	....
					0.5L	0.360		3.76	....	26.8	0.0626	0.887	....	....	....	....
					1.0L	0.360		3.64	....	27.7	0.0636	0.901	....	....	....	....
					1.0R	0.372		3.50	....	28.5	0.0643	0.882	....	....	....	....
11	0.0025	0.474	0.195	40	0.0	0.332	0.332	3.97	O	27.0	0.0630	0.973	0.97	V	19.6	3.36
					0.5L	0.332		3.85	....	28.2	0.0643	0.993	1.01	W	18.0	
					1.0L	0.332		3.73	....	25.8	0.0620	0.957	1.10	U	22.9	
12	0.0025	0.299	0.155	40	0.0	0.333	0.333	2.97	P	24.8	0.0568	1.10	1.04	X	12.9	2.56
					0.5L	0.333		2.97	....	25.7	0.0578	1.12	1.02	Z	11.9	
					1.0L	0.353		2.85	....	26.2	0.0585	1.07	1.06	Y	11.7	
13	0.0025	0.478	0.196	45	0.0	0.334	0.334	3.81	Q	26.9	0.0580	0.886	0.85	AA	14.0	3.19

TABLE 3.—SUMMARY OF EXPERIMENTS (*Continued*)

Run No.	Slope S, in ft per ft	Depth y_m, in ft	Friction velocity $\sqrt{g\, y_m S}$ (ft per sec)	MEASURING SECTION; DISTANCE, IN FT FROM:		UNIVERSAL CONSTANT, k		Average velocity $\bar{u}_1$, at profile section	Velocity distribution curve[a] (Fig. 6 or 7):	Water temperature, in degrees centigrade	Settling velocity w, in ft per sec	EXPONENT IN EQ. 16		SEDIMENT CONCENTRATION		
				Flume inlet	Center line	Individual	Average					Theoretical z (Eq. 17)	Experimental z_1 (Eq. 18)	Distribution curve[b] (Fig. 9 or 10):	At reference level, C_a	Mean value for flow, C_m
(*b*) SERIES 2 (SEE FIGS. 7 AND 10)																
14	0.0025	0.554	0.211	40	0.0	0.397	0.394	3.38	A	....	0.0	0.0	0.0	....	0.0	0.0
14	0.0025	0.554	0.211	45		0.397		3.39	B	....				....		
20	0.0025	0.462	0.193	45		0.389		3.21	....	....				....		
21	0.0025	0.234	0.137	45		0.392		1.99	C	....				....		
14	0.0025	0.538	0.208	45	0.0	....	0.386	3.46	D	24.0	0.0590	0.735	0.89	M	9.09	1.33
15	0.0025	0.274	0.148	45	0.0	0.384	0.384	2.54	E	21.0	0.0577	1.02	1.02	N	5.25	0.59
16	0.00125	0.537	0.147	45	0.0	0.385	0.385	2.70	F	21.5	0.0577	1.02	1.05	O	3.02	0.59
17	0.00125	0.269	0.104	45	0.0	....	0.386	....	....	20.0	0.0571	1.42	1.62	S	0.535	0.048
						0.386		1.77	G	20.0	0.0571	1.42	1.08	P	0.036	
						0.386		1.82	....	19.3	0.0567	1.41	1.45	R	0.995	
						....		....	....	19.6	0.0567	1.41	1.47	Q	0.416	
18	0.00125	0.462	0.136	45	0.0	0.314	0.317	2.48	H	19.0	0.0269	0.629	0.53	D	4.79	1.18
						0.314		2.57	....	20.0	0.0275	0.644	0.54	E	4.89	
						0.317		2.49	....	22.0	0.0285	0.660	0.48	C	3.54	
						0.324		2.45	....	21.6	0.0285	0.646	0.54	F	4.25	
19	0.00125	0.236	0.0975	45	0.0	0.334	0.330	1.78	....	23.2	0.0292	0.897	0.63	G	0.949	0.17
				40		0.334		1.82	....	21.1	0.0282	0.866	0.64	H	0.999	
				40		0.334		1.78	....	19.2	0.0272	0.835	0.69	I	1.20	
				45		0.320		1.82	....	20.3	0.0275	0.881	0.68	J	1.44	
				45		0.320		1.82	I	20.5	0.0278	0.891	0.68	K	1.29	
20	0.0025	0.461	0.193	45	0.0	....	0.349	3.24	J	22.0	0.0285	0.423	0.34	A	4.27	....
21	0.0025	0.234	0.136	45	0.0	0.332	0.332	2.28	K	19.7	0.0275	0.609	0.42	B	3.42	....
22	0.0025	0.295	0.154	45	0.0	....	0.341	2.60	L	17.8	0.0407	0.775	0.66	L	6.75	....

[a] See the lettered curves in Fig. 6 for series 1 and Fig. 7 for series 2. [b] See the lettered curves in Fig. 9 for series 1 and Fig. 10 for series 2.

flow. Those in Fig. 7 are grouped according to size of the sediment being transported.

In all runs the velocity distribution fits the logarithmic law very well, although near the bottom there is a systematic deviation from the law, which is not understood. A departure is expected near the bottom since the viscous forces are no longer negligible and the conditions for which the logarithmic law was derived no longer obtain. The viscosity tends to suppress the turbulence and to reduce the exchange coefficient ϵ_m in Eq. 1*b*, and, in order to develop the shear, the velocity gradient must increase. This would cause the velocity to decrease more rapidly than is indicated by the logarithmic law, and the result is opposite to the results shown in Figs. 6 and 7. In the present experiments, the origin of the depth coordinate was assumed to be at the top of the sand grains forming the bottom roughness. An inspection of the velocity profile curves will show that the fit could be improved by assuming the origin below this level although the justification for doing so is not clear. The deviation from the law was first attributed to errors in measurement due to interference between the pitot static tube (diameter, $\frac{3}{16}$ in.) and the bottom. Tubes as small as $\frac{1}{16}$ in. in diameter gave substantially the same readings as the larger instruments and showed that the measurements were reliable. An apparent deviation from the law is also to be noted at the free surface. However, because of surface waves and the disturbance of the pitot tube itself, it is felt that these measurements are not too reliable and that conclusions based on them must be drawn with reservation.

The fact that the velocity profiles fit the logarithmic law so well means that the average velocity will always occur at the relative depth $\frac{y}{y_m} = 0.368$.[29] This point is marked on the curves of Figs. 6 and 7 by a short dash. The average velocities determined by this method agree within 1% or 2% with those determined by graphical integration. The velocities $\bar{U}_1$ in Table 3 were determined by this method.

The velocity curves of Figs. 6 and 7 are of the form,

$$\frac{u}{\sqrt{g\,y_m\,S}} = \frac{u_{\max}}{\sqrt{g\,y_m\,S}} + \frac{2.3}{k}\log\frac{y}{y_m} \qquad (18)$$

in which k is a universal constant. The quantity $2.3/k$ is the slope of the curve on the semilogarithmic graph, and should be constant if k does not vary. A close inspection of the curves will show that the slopes do vary, as the size and concentration of sediment are varied. The values of k for each curve are shown in Figs. 6 and 7, and in Table 3. In general, k decreases as the concentration is increased or as the size of sediment in suspension is decreased.

Fig. 8 shows the velocity profiles for series 1 and 2, respectively, plotted according to the von Kármán velocity defect law (Eq. 14). In Fig. 8(*a*), the data of Fig. 6 have been combined according to the sediment content of the flow and show clearly the variation of k. In Fig. 8(*b*), the data of Fig. 7

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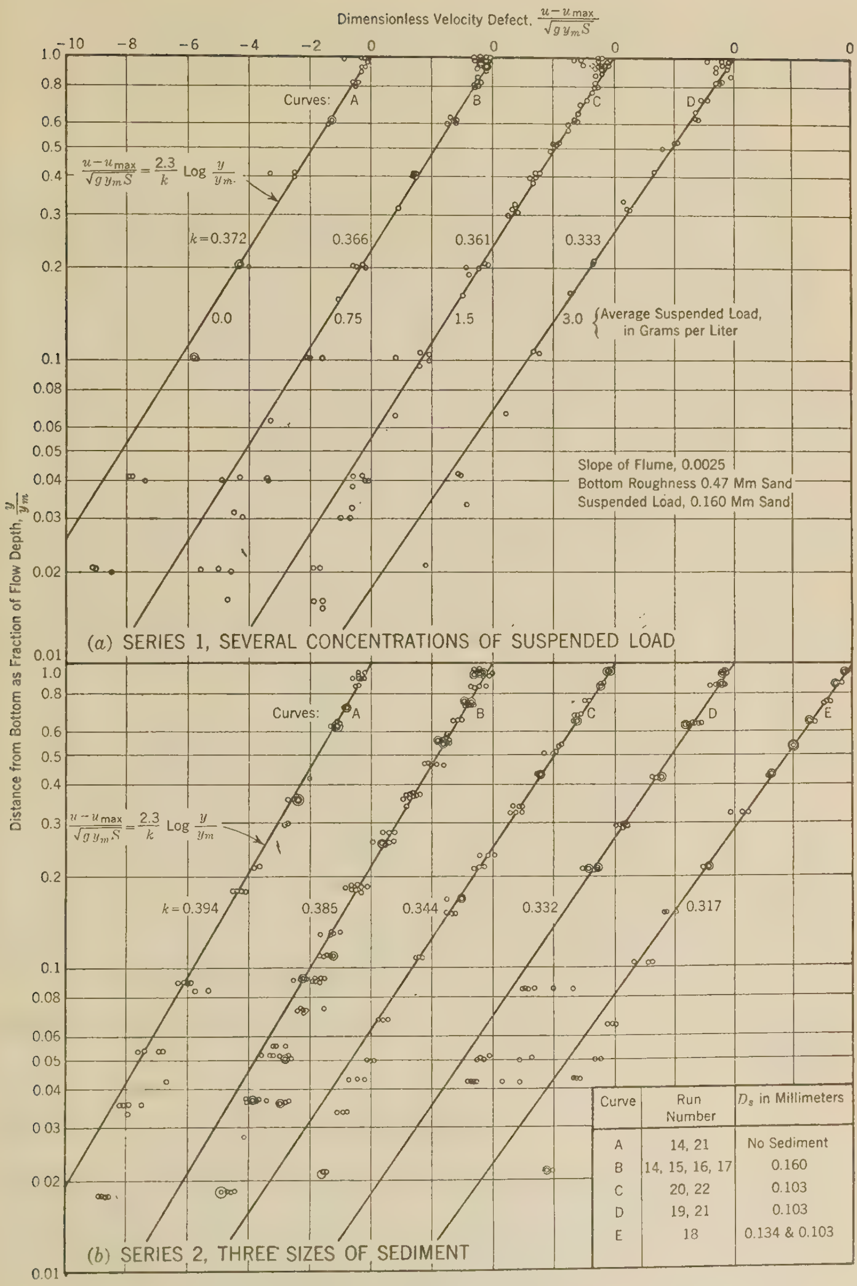
[29] "Velocity Distribution in Open Channels," by Vito A. Vanoni, *Civil Engineering*, June, 1941, pp. 356–357.



Curve	Run Number	D_s in Millimeters
A	14, 21	No Sediment
B	14, 15, 16, 17	0.160
C	20, 22	0.103
D	19, 21	0.103
E	18	0.134 & 0.103

FIG. 8.—VELOCITY PROFILES

have been grouped according to the values of k. The curves show that k is decreased by the presence of the sediment, but the relation between size of sediment and the universal constant k is not clear. Fig. 8 does show, however, that the logarithmic velocity law is valid and that its use is justified in developing Eq. 16 for suspended load distribution.

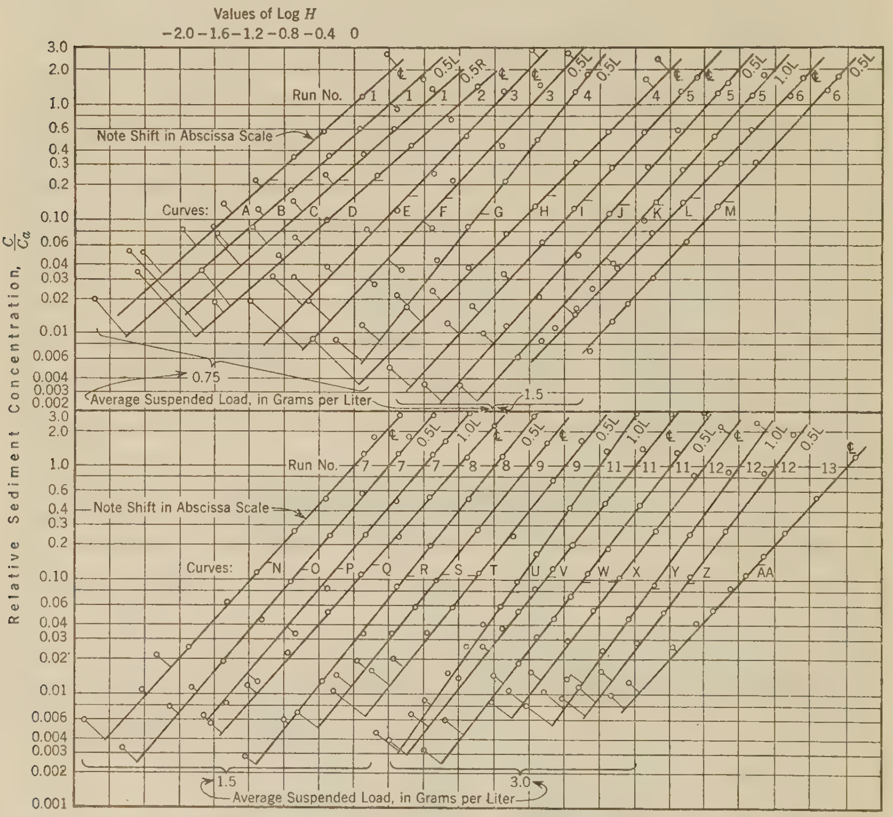


FIG. 9.—SERIES 1; MEASURED SEDIMENT DISTRIBUTION PLOTTED AGAINST THE DISTRIBUTION FUNCTION, H

Distribution of Suspended Load.—Eq. 16 plotted logarithmically is a straight line with a slope,

$$z_1 = \frac{\log \frac{C}{C_a}}{\log H} \dots\dots\dots\dots\dots\dots\dots\dots (19)$$

which may be read directly from the graph. Figs. 9 and 10 show the sediment distribution data for series 1 and 2, respectively, plotted logarithmically; that is, $\log C/C_a$ is plotted against $\log H$. The origin of the abscissa scale ($\log H$), which occurs at $C/C_a = 1$, is shifted for each curve. (The results of experiment series 2 were first reported by the writer in 1941.[30])

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[30] "Some Experiments on the Transportation of Suspended Load," by Vito A. Vanoni, *Transactions*, Am. Geophysical Union, Pt. III, 1941, pp. 608–621.

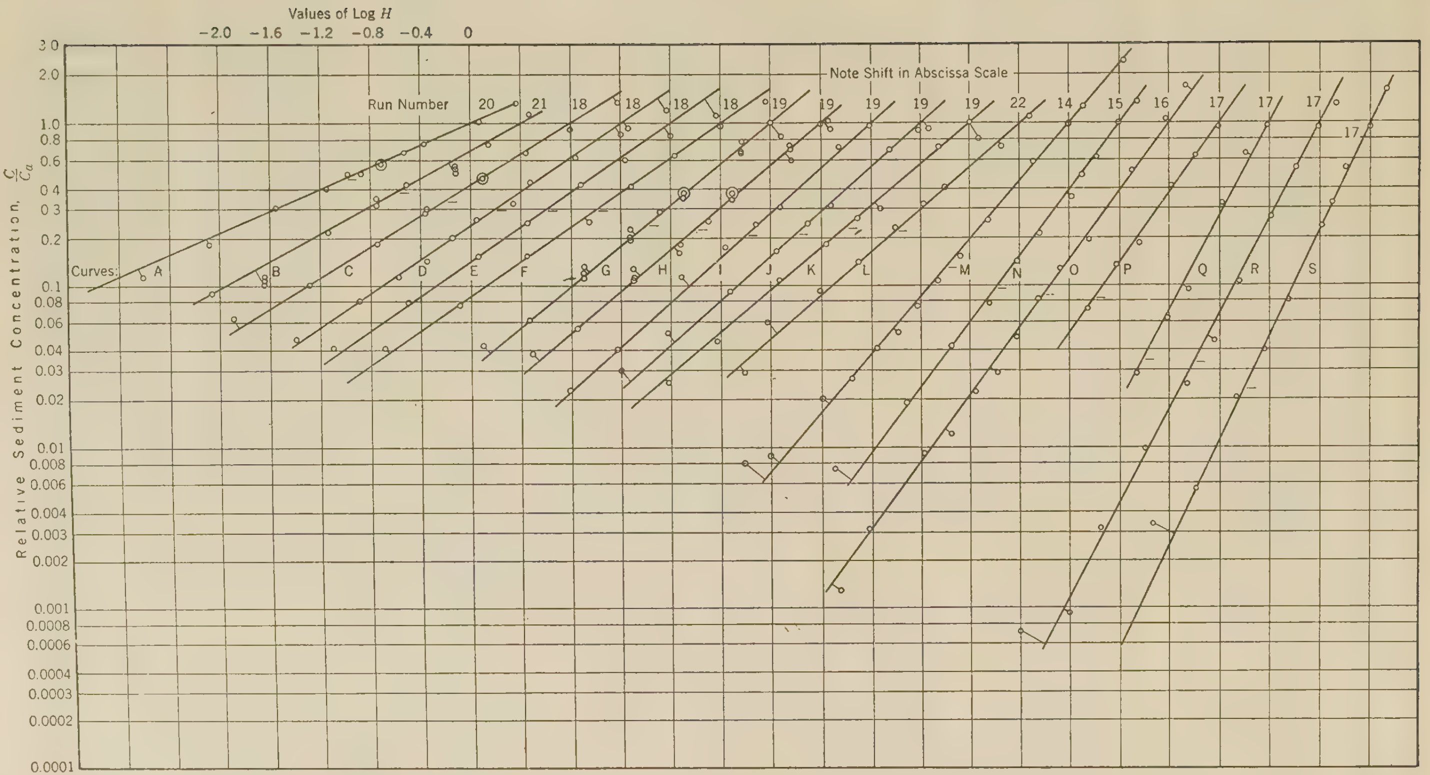


FIG. 10.—SERIES 2; MEASURED SEDIMENT DISTRIBUTION PLOTTED AGAINST THE DISTRIBUTION FUNCTION, H

The value of z may also be calculated from Eq. 17, in which w is the settling velocity of the sediment, k is the von Kármán universal constant, and $\sqrt{\frac{\tau_0}{\rho}}$ is the friction velocity. (In calculating z the value $\bar{w}$ of the settling velocity of the center of mass of a group of particles representative of the suspended load was used.) To distinguish between the value obtained by Eq. 17 and that obtained by fitting the experimental data in Figs. 9 and 10, the latter value is denoted by z_1. Obviously, if the theory is correct, the two values will agree.

The intersection of each curve with the line $\log H = 1.0$ is indicated by a short horizontal dash. The vertical distance from the origin ($C/C_a = 1$) to the dash, measured in cycles on the logarithmic scale is the value of z_1. Values of both z and z_1 are listed in Table 3. The two values agree well for runs 1 to 17, which were all made with the coarsest suspended load (sand 1). For all the runs with finer sand the value of z is from about 20% to 40% greater than z_1.

From inspection of Figs. 9 and 10, it is at once apparent that the results of series 2 are in much closer agreement with the distribution law (Eq. 16) than those of series 1. The disagreement occurs in the upper levels of the flow—that is, for small values of H—where more sediment is found than would be expected from the law. Results of the two series may be compared only for those runs made under comparable conditions with sediment of the same size. Runs 14 to 17 of series 2 were made with the same sediment as the runs of series 1 and may therefore be compared with runs of that series. These runs (14 to 17) also show more sediment in the upper levels than is given by Eq. 16, but the difference is slight. The rather large departures from the distribution law in series 1 are no doubt due to disturbances in the flow at the entrance, which, as stated previously, were appreciable compared with those occurring in the revised circulation system (Fig. 1(*b*)), used in series 2.

Figs. 9 and 10 show that the departures of the measured relative concentrations from the distribution law are small, and the curves also show conclusively that at least the form of the distribution function is correct. This conclusion is confirmed by Fig. 11, which shows $z_1 \log H$ plotted against $\log C/C_a$ for series 1 and 2. Here again the disagreement in the upper levels between measurement and the distribution function is noticeable, as is also the better agreement of the results of series 2.

Fig. 12 shows the relative sediment distribution as a function of the relative depth measured upward from the reference level $a = 0.05\, y_m$. The measurements are indicated by circles and the theoretical distribution is shown as a solid line. The agreement for series 2 (Fig. 12(*b*)), as previously seen, is good over the entire depth, whereas, series 1 (Fig. 12(*a*)) shows the departure in the higher levels noted before.

It is well to note that, in calculating the sediment distribution, the experimental values, z_1, of the exponents were used. Table 3 shows agreement between z and z_1 for only one sediment so it is not possible to say that the distribution law (Eq. 16) gives the correct values of the relative concentration, C/C_a. What can be said is that the distribution function is of the correct form.

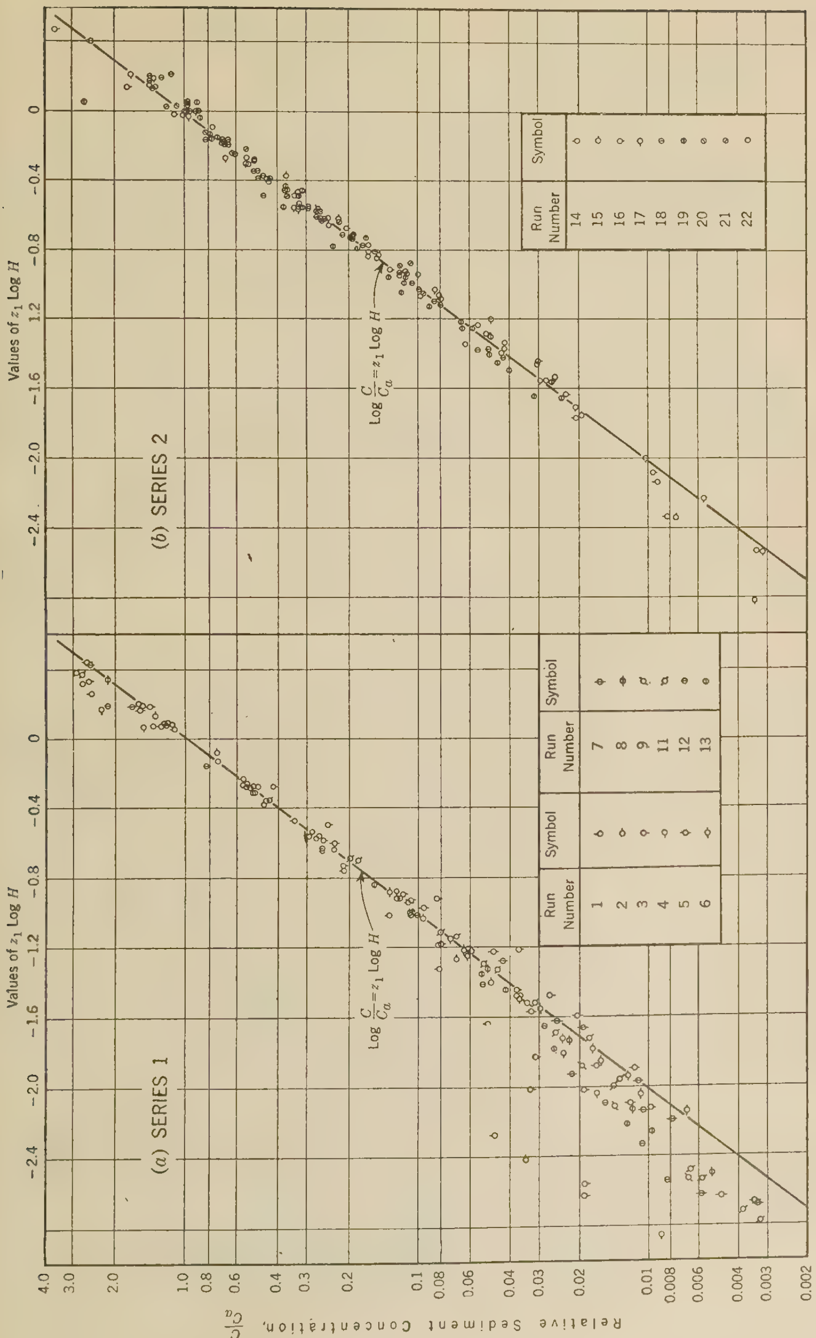


FIG. 11.—MEASURED RELATIVE SEDIMENT CONCENTRATION PLOTTED AGAINST CALCULATED SEDIMENT CONCENTRATION

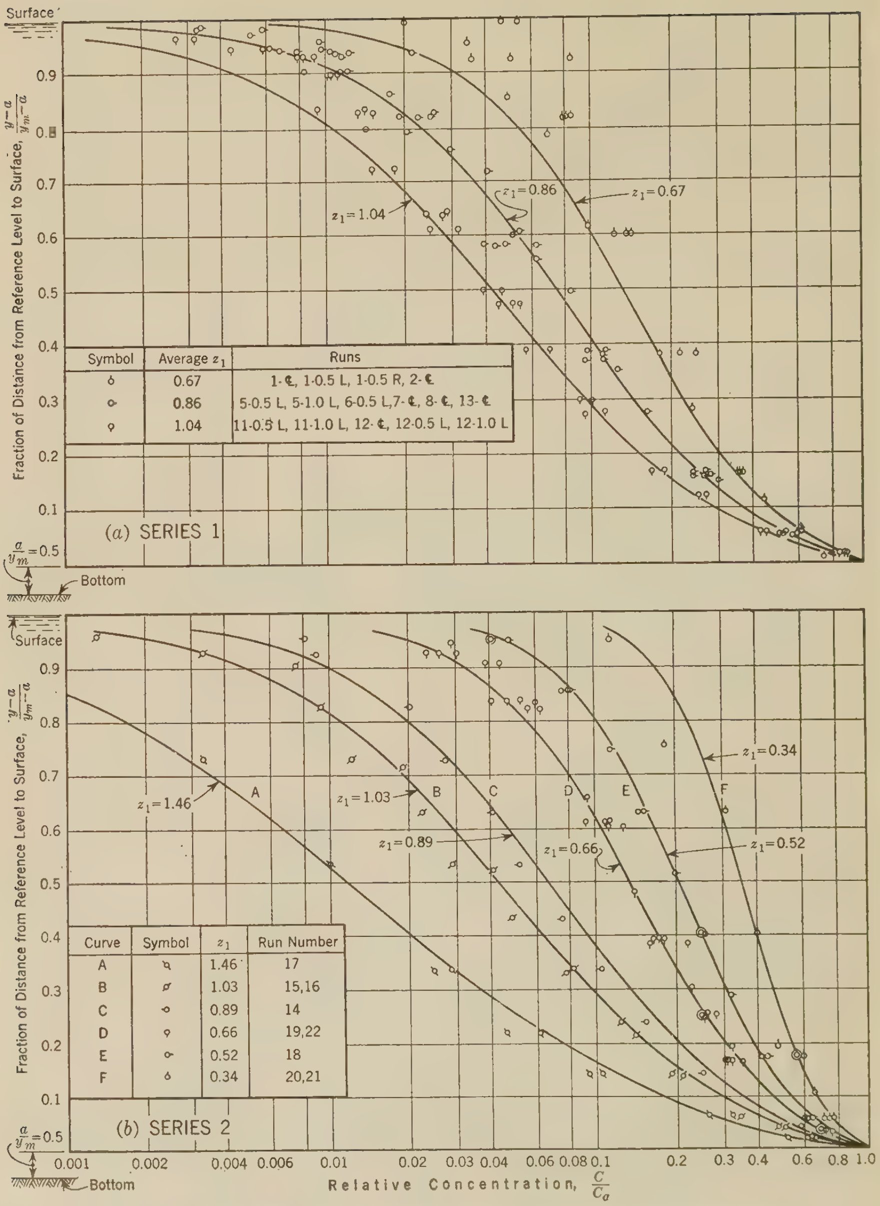


FIG. 12.—MEASURED AND CALCULATED RELATIVE SEDIMENT CONCENTRATIONS PLOTTED AS A FUNCTION OF RELATIVE DEPTH

Turbulent Transfer Coefficients.—The turbulent transfer coefficient for momentum is given by Eq. 10. Introducing Eqs. 12*b* and 15*a* into Eq. 10 gives

$$\epsilon_m = \beta v'_{\cdot} l = k \sqrt{\frac{\tau_0}{\rho}} \left(1 - \frac{y}{y_m} \right) y \ldots\ldots\ldots\ldots (20a)$$

Eq. 20*a* assumes logarithmic velocity distribution and linear distribution of shear and gives the coefficient ϵ_m for momentum transfer. In deriving Eq.16, the assumption was made that $\epsilon_m = \epsilon_s$. The measured sediment distribution now makes it possible to calculate the coefficient ϵ_s for the sediment by applying Eq. 6, which gives

$$\epsilon_s = \frac{w\,C}{\frac{dC}{dy}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (20b)$$

The transfer coefficients may be reduced to a dimensionless form by dividing by the depth times the friction velocity as follows:

$$\frac{\epsilon_m}{y_m \sqrt{y_m\,g\,S}} = k \left(1 - \frac{y}{y_m} \right) \frac{y}{y_m} \ldots\ldots\ldots\ldots\ldots (21a)$$

and

$$\frac{\epsilon_s}{y_m \sqrt{g\,y_m\,S}} = \frac{w}{\sqrt{g\,y_m\,S}} \times \frac{C}{y_m \frac{dC}{dy}} \ldots\ldots\ldots\ldots\ldots (21b)$$

Fig. 13 shows both the coefficient for momentum (Eq. 21*a*) and for sediment (Eq. 21*b*) for four flow conditions. Figs. 13(*a*) and 13(*b*) are for series 1 with average suspended loads of 1.34 and 3.36 g per liter (Table 3), respectively, and in general are typical of this series. Figs. 13(*c*) and 13(*d*) are typical of results obtained in series 2, the former being for the coarsest sediment (sand 1).

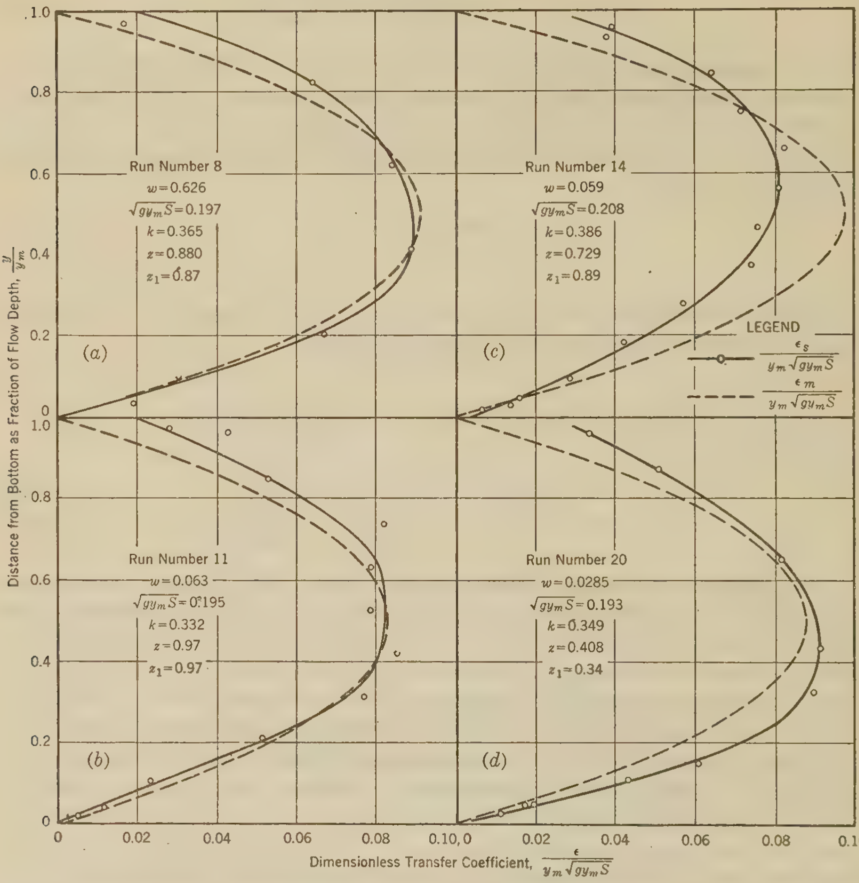
The agreement between the two mixing coefficients for series 1 (Figs. 13(*a*) and 13(*b*)) is rather good. For the coarse sediment in series 2, the sediment coefficient is smaller than the momentum coefficient, whereas the opposite is true for the fine sediment. Eq. 7 shows immediately that small values of ϵ_s give small values of the concentration, C. It is also apparent from Eq. 16 that increasing ϵ_s, and therefore C, will tend to make the concentration more uniform, that is, it will decrease z. These effects may be noted in Figs. 13(*c*) and 13(*d*). In Fig. 13(*c*), ϵ_s is less than ϵ_m and the measured exponent z_1 is greater than z; whereas in Fig. 13(*d*) the opposite is true—z_1 is less than z.

DISCUSSION OF RESULTS

Effect of Suspended Load on Velocity Distribution.—The von Kármán universal constant k (Eqs. 14 and 18) depends only on the mechanism of turbulent mixing, and a change in k means that the mixing mechanism has in some way been altered. From Eqs. 1*b* and 15*a*,

$$\epsilon_m = k\,y \frac{\tau}{\rho \sqrt{\frac{\tau_0}{\rho}}} \ldots\ldots\ldots\ldots\ldots\ldots (22)$$

or the momentum transfer coefficient is directly proportional to k. A decrease in k decreases ϵ_m, and if the shear (depth and slope) is kept constant the velocity gradient will have to increase to compensate for the reduced ϵ_m.



(e) FLOW CHARACTERISTICS

Fig.	Series	Run	w	$\sqrt{g\,y_m\,S}$	k	z	z_1
a	1	8	0.0626	0.197	0.365	0.880	0.87
b	1	11	0.063	0.195	0.332	0.97	0.97
c	2	14	0.059	0.208	0.386	0.729	0.89
d	2	20	0.0285	0.193	0.349	0.408	0.34

FIG. 13.—COEFFICIENTS FOR MOMENTUM TRANSFER AND SEDIMENT TRANSFER

This will result in an increase in the velocity of the flow. Thus it is seen that k is also a factor in establishing the velocity distribution.

The values of k obtained for various sediments and flow conditions are shown in Table 3. The effect upon k of increasing the sediment load is shown by series 1, runs 1 to 13. As the load, C_m, was increased, the k-values decreased. The results of series 2 show that the coarse sediment (0.160 mm), although reducing k somewhat, has less effect than the finer sediment. In general, k also appears to vary systematically with the friction velocity $\sqrt{g\, y_m\, S}$. However, there appears to be no simple relationship between the several factors. From these results one would expect k to be a function of w, the settling velocity of the sediment, the average sediment load, and the friction velocity $\sqrt{g\, y_m\, S}$.

Attention is called to the lack of agreement between the values of k obtained in series 1 and 2 for similar flows. For instance, the average k for clear water for series 1 was 0.372 whereas for series 2 it was 0.394. No straightforward explanation of these results can be offered. However, the difference is most likely due to the difference in the inlet conditions for the two series of experiments. The k-value obtained for series 2, which was run with very good flow conditions, checks very well with that obtained for pipes, and it is reasonable to assume that the results of this series are more reliable than those of series 1.

The Effect of Suspended Load on the Resistance to Flow.—As stated, a reduction in k results in an increased velocity, which will also give a reduction in the resistance coefficient of the channel. The Chézy formula for flow in conduits is,

$$\bar{u} = c\sqrt{R\,S} \quad \text{(23)}$$

in which $\bar{u}$ is the mean velocity in the cross section, c is the Chézy coefficient, R is the hydraulic mean depth (area of cross section divided by wetted perimeter), and S is the slope or pressure gradient. For open channels, c is given by the Manning formula,

$$c = \frac{1.486}{n} R^{1/6} \quad \text{(24a)}$$

in which n is a roughness coefficient. The Chézy coefficient can also be expressed in terms of f, the resistance coefficient,

$$c = \sqrt{\frac{8\,g}{f}} \quad \text{(24b)}$$

Eqs. 23 and 24 can be solved for n and f with the following result:

$$n = \frac{1.486}{\bar{u}} R^{2/3} S^{1/2} \quad \text{(25a)}$$

and

$$f = \frac{8\,g}{\bar{u}^2} R\,S \quad \text{(25b)}$$

Several values of n and f for flows with and without sediment are shown in Table 4.

The effect of the suspended load in reducing the resistance is seen clearly. The reduction in both n and f appears to be about the same for the two sediments but is greater for the low flows than for the high flows. The first three lines of Table 4 indicate that the resistance decreases as the sediment load increases.

The increase in velocity or decrease in channel resistance, as suspended load is added to the flow, can be explained by the effect of the sediment in reducing the turbulence. To keep the sediment from settling, work must be done on it. The energy for this work can be provided only by the turbulence which is damped and reduced in intensity when it gives up energy. This means that the momentum transfer coefficient is also decreased thus allowing the velocity and velocity gradient to increase according to Eq. 1*b*. (The late Henry M. Eakin, formerly head of the Sedimentation Section, Division of Research, Soil Conservation Service, had anticipated the effect of the sediment on the flow and advanced his idea on the subject to the writer in 1936 during a discussion of the suspended-load problem.)

TABLE 4.—RESISTANCE AND ROUGHNESS COEFFICIENTS OF FLUME WITH AND WITHOUT SUSPENDED LOAD IN FLOW

Run No.	Flow depth, y_m (ft)	Average velocity,[a] $\bar{u}$	Manning's n	Ratio, n: $\frac{\text{clear flow}}{\text{silt flow}}$	Resistance coefficient, f	Ratio, f: $\frac{\text{clear flow}}{\text{silt flow}}$	Mean diameter, $\bar{D}_s$ (mm)	Average concentration,[b] C_m
....	0.496	3.49	0.0108	....	0.0192	....	....	0
8, 10	0.484	3.59	0.0104	1.04	0.0179	1.07	0.160	1.6
11, 13	0.475	3.64	0.0102	1.06	0.0172	1.12	0.160	3.3
14	0.554	3.42	0.0117	1.04	0.0217	1.08	0.160	0
14	0.538	3.52	0.0112		0.0201			1.33
18	0.462	2.20	0.0118	1.10	0.0231	1.22	0.100	0
18	0.462	2.43	0.0107		0.0189			1.18
20	0.461	3.05	0.0121	1.04	0.0240	1.09	0.100	0
20	0.462	3.18	0.0116		0.0221			1.2
21	0.234	1.99	0.0128	1.10	0.0326	1.22	0.100	0
21	0.234	2.19	0.0116		0.0268			1.2

[a] Feet per second. [b] Grams per liter.

A. B. Buckley[31] observed, from measurements on the Nile River in Egypt, that suspended load decreased the roughness coefficient of the channel. The reduction in resistance that he obtained for sediment loads of from 1.0 to 2.0 g per liter was in general greater than that obtained in the experiments described in this paper. Mr. Buckley attributed the reduction in resistance to the damping effect upon the turbulence of a heavily loaded layer of water near the bed of the stream. He expected the greatest reduction when a stream was depositing, for then the layer near the bottom would be most dense.

Distribution of Suspended Load.—All the measurements of suspended-load distribution followed very closely the form of the sediment distribution law, Eq. 16, but agreement between the calculated and measured value of the exponent z occurred only for the coarsest suspended load ($\bar{D}_s = 0.160$ mm). For the finer sediments the value z_1 of the exponent that fits the data, or the measured value, was about 20% less than the value calculated from the theory. (Measurements of suspended load in a natural stream made recently by Alvin G. Anderson,[32] Assoc. M. Am. Soc. C. E., show that z_1 was about half as large as z.) As noted previously, the smaller the value of z the more uniformly the sediment is distributed over the depth.

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[31] "The Influence of Silt on the Velocity of Flowing Water in Open Channels," by A. B. Buckley, *Minutes of Proceedings*, Inst. C. E., Vol. 216, 1922–1923, pp. 183–211.

[32] "Distribution of Suspended Sediment in a Natural Stream," by Alvin G. Anderson, *Transactions*, Am. Geophysical Union, Pt. II, 1942, pp. 678–683.

The mixing coefficient ϵ is given by Eq. 3 and has the same form for both momentum transfer and sediment transfer but not necessarily the same value.[5] For a fine sediment the mixing coefficient for momentum was smaller than the corresponding coefficient for sediment transfer. It can readily be seen that increasing ϵ_s will tend to produce a more uniform sediment distribution, which results in z_1 being smaller than z.

The fact that the coefficients of sediment transfer and of momentum transfer are not the same may be explained by two differences in the transfer mechanisms for sediment and momentum. The first difference occurs because, for momentum transfer, a correlation is required between the horizontal and vertical turbulent velocity fluctuations and uncorrelated or random fluctuations do not contribute to the momentum transfer. For sediment transfer this kind of correlation is not necessary and the random fluctuations also transfer sediment and tend to make ϵ_s larger than ϵ_m. The second difference exists because of the "slip" between the fluid and the sediment. The theory assumes that the instantaneous vertical velocity of the sediment relative to the fluid is always equal to its settling velocity w in still fluid. This is true only for steady conditions and during acceleration the relative velocity must be greater than w and the sediment is carried a shorter distance than assumed. This will tend to make ϵ_s less than ϵ_m.

The experiments show that in flows carrying the coarsest sediment the two transfer coefficients are about equal and that, as finer sediment is introduced, ϵ_s becomes increasingly larger relative to ϵ_m. Apparently for the fine sediment the effect of the random turbulence in increasing ϵ_s is greater than the effect of the "slip" between the particles and fluid in decreasing ϵ_s. For the coarse sediment the two effects are about balanced and if still coarser sediment were used a resultant effect of opposite sign is expected and ϵ_s should be less than ϵ_m. The effect of the variation of the transfer coefficients relative to one another is reflected in the relative values of z and z_1 which may be noted in Table 1, For flows carrying the fine sediment where ϵ_s is greater than ϵ_m, z_1 is less than z. and when the two transfer coefficients are equal, z and z_1 are also equal.

The tendency for the sediment content near the surface of the flow to exceed that indicated by the distribution formula, Eq. 16, can be explained by the action of the random turbulence. Although the mixing coefficient for momentum goes to zero at the surface, the random velocity fluctuations carry sediment right up to the very surface, thus giving the mixing coefficient for sediment a finite value at the surface. This can be noted in Fig. 13.

Attention is called to the wide disagreement between the values of z and z_1 in the first two sediment profiles of run 17 (Table 3). Considerable difficulty was experienced in this run because the sediment was not distributed in a regular manner over the cross section. Since the flow was very low and the sand coarse, considerable material was moving along the bottom. Any irregularity in the distribution was therefore easily noticed. The first measurement (Table 3 and curve S, Fig. 10) was made over a point where considerable sediment was in motion in a narrow band on the bottom. The abnormally large value of z_1 could be accounted for by the occurrence of a transfer of sediment in the horizontal direction away from the measuring point, brought about by the existence of a concentration gradient across the flow. In the same way it is possible to account for the low value of z_1 obtained in the second measurement of run 17. This measurement was taken over a point where very little sediment was in motion on the bed but a few inches to one side of the section considerable sand was being carried. The transverse transfer of sediment into the measuring section would decrease the vertical concentration gradient and therefore the value of z_1. It is surprising that even under such conditions the distribution should follow the law expressed by Eq. 7.

Secondary Flow Due to Suspended Load.—The effect of the suspended load was first noticed in measuring the velocity distribution over cross sections of the flume. Symmetrical velocity distributions in clear flows became distorted by the addition of very small amounts of sand. This effect may be noted in Fig. 5(*a*), distributions A, B, and C, which show the velocity distributions for clear water and for average sediment concentrations of 0.75 and 3.0 g per liter, respectively. To give an idea how small the concentrations actually were, it might be well to remark that the first concentration ($C_m = 0.75$) was obtained by adding only 12.5 lb of sand to the entire system. The effect of this small concentration on the flow may also be seen by comparing the center column of Fig. 5(*a*) with the right column. The effect is small for this small concentration, but it is noticeable. Incidentally, attention is called to the uniform spacing of the velocity lines in Fig. 5(*a*), distribution A. This shows at once that the velocity distribution is logarithmic since the distances y from the bottom of the flume are spaced geometrically.

The disturbing effect of the sediment is also shown in Fig. 5(*b*). Distributions A to D, Fig. 5(*b*), for clear water, may be compared with distributions E to H, Fig. 5(*b*), which show the velocities under like conditions except that there is sediment in the flow. Distributions I and M, Fig. 5(*b*), show a case in which the sediment apparently has a stabilizing effect. In this one instance, the conditions were apparently just right to establish good flow at this particular station. The fact that the distorted velocity distribution with clear water was made more uniform by the addition of a small amount of sediment serves to illustrate again that the sediment has considerable influence on the flow.

The sediment appeared to be transported in clouds or ribbons parallel to the flow. At low flows when appreciable material was moving along the bottom, these ribbons appeared as bands of sand on the bottom. When proper flow conditions had been attained, these ribbons were symmetrically spaced about the center line and were stable regardless of the flow rate. Attempts to break them up artificially were only partly successful. Fig. 14 is a photograph of these bands of sediment taken after run 15, looking upstream. The photograph was taken after the flow had been stopped and the water slowly drained off. Material in suspension settled to the bottom, thus showing how it was distributed across the flow. Three main streaks occur in the center portion of the flume and two less pronounced streaks occur at the walls. This pattern, which was symmetrical, occurred over the entire length of the

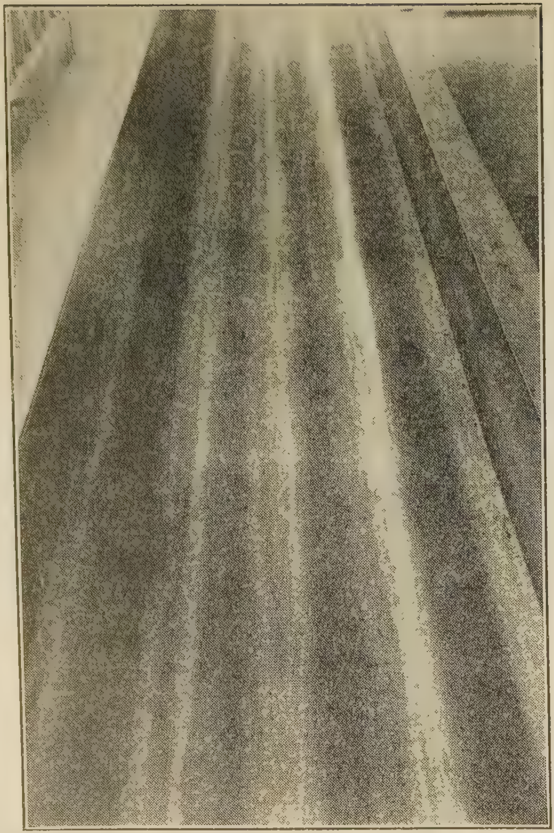
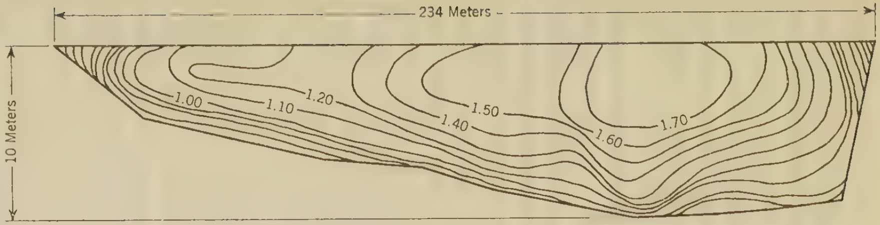
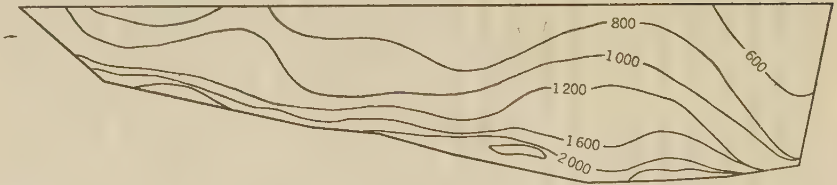


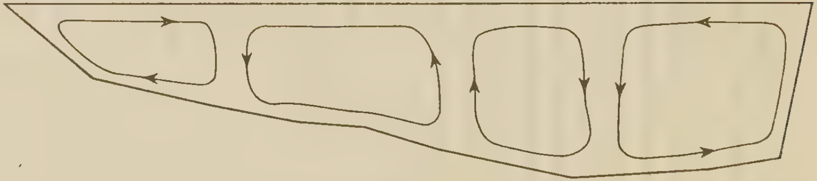
FIG. 14.—SEDIMENT ON FLOOR OF FLUME AS DEPOSITED BY THE FLOW OF RUN NO. 15



(*a*) VELOCITY IN METERS PER SECOND



(*b*) SUSPENDED LOAD IN GRAMS PER CUBIC METER



(*c*) CIRCULATION

FIG. 15.—FLOW CHARACTERISTICS, RIVER PO IN ITALY

flume. Measurement of suspended load at a distance of 0.02 ft from the bottom showed that the sediment was very nearly uniformly distributed over the center half of the flume, that it then suddenly dropped to about 25% of the center concentration as the clear areas near the walls (Fig. 14) were approached, and finally increased to about 50% of the center concentration near the walls. The concentration for the center 6 in. was about 5% less than that over the concentrated streaks at the side. (The flume is 33¼ in. wide by 12 in. deep.)

The cause of these longitudinal bands of sediment is not known, although it is believed that they are the result of secondary circulation and disturbances due to the presence of suspended load. Even the slightest departure from a uniform sediment distribution will cause horizontal density gradients that can set up secondary flows. This distribution shows a concentration gradient across the flow, and one could then expect a circulation with an upward flow at the points of low concentration and a downward flow where the load was high. Since there appear to be four points of low concentration, four pairs of circulations might be expected. No direct observations were made to attempt to check this hypothesis, and it is doubtful that this could be done without considerable work. However, this is a possible explanation of the action of the sediment in causing secondary circulations in the flow. Further investigation will show whether or not this idea is valid.

If circulation could be caused by sediment in a uniform and straight laboratory channel, there is a very good possibility that it will also occur in natural streams where irregularities and disturbances always exist. Therefore some measurements of suspended load in rivers were examined. Figs. 15(*a*) and 15(*b*) show measurements of velocity and suspended load at a cross section of the River Po in Italy, reported by Giandotti.[33]

Note the high concentration of suspended load at about the one-quarter points of the section. The circulation expected from this distribution is shown in Fig. 15(*c*). The writer is not able to state whether this distribution is typical of rivers. However, these measurements in the River Po are evidence that a circulation may be caused by the sediment. The circulation that would be induced is the same as that advanced as an explanation for the depression of the maximum velocity in natural streams. Although this circulation explains the velocity distribution, no satisfactory explanation was ever found for the circulation itself. The action of sediment on river flow may well prove to be an important factor in determining the velocity distribution.

Sediment-Transporting Capacity.—The idea that sediment in suspension reduces the turbulence furnishes a possible mechanism for establishing the capacity of a stream. Since the turbulence supports the sediment a reduction in the turbulence by adding sediment means that the stream then is less capable of supporting sediment than before a load was added. The capacity load is probably determined by continually decreasing the turbulence as more sediment is taken into suspension. The fact that the addition of sediment to a flow increases the velocity, therefore, does not mean that the sediment transporting capacity is also increased. On the contrary the capacity is actually decreased. The increase in velocity also means that the depth and, hence, the boundary shear is decreased and that particles in the bed are probably subjected to less entrainment force than when the velocity was lower.

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[33] "Les Déplacements du Lit du Pô," by Mario Giandotti, Le Service Hydrographique Italien, XV Congres International De Navigation, Venice, September, 1931, pp. 297–322.

It is seen not only that turbulence is involved in the sediment transportation problem, but that the sediment changes the turbulence itself, the amount of change apparently depending upon the amount and size of the suspended load. In a clear flow the velocity can be expressed in terms of certain constants which characterize the turbulence mechanism. In sediment-laden flows these constants are now functions of the sediment load and sediment characteristics.

CONCLUSIONS

1. The distribution of the relative concentration of suspended load has the form of the derived distribution law (Eq. 16) but does not agree quantitatively with it; that is, the value of the exponent z given by the theory does not agree with z_1, the exponent that fits the experimental results. When the suspended sediment is fine, z_1 is less than z. This shows that the relative concentration is more uniformly distributed and has a greater average value than that indicated by the theory. In other words, a flow can support and, therefore, transport fine sediment more effectively than the theory indicates. As the size of the sediment is increased, the agreement between the two exponents improves up to a certain point, beyond which the value of z_1 may be expected to exceed z.

2. The disagreement between the calculated and measured distributions cited in conclusion 1 is attributed to (*a*) the action of the random turbulent fluctuations in suspending sediment, and (*b*) the "slip" between the fluid and the sediment as the sediment is accelerated. The theory is based on the assumption that the coefficient of sediment transfer is equal to the coefficient of momentum transfer, and therefore does not account for the action of the purely random turbulence and of the slip. Such disagreement was anticipated, aud the present results confirm it.

3. For fine material the coefficient of sediment transfer tends to exceed the coefficient of momentum transfer; for coarser sediment the opposite tendency is found. Conclusion 3 may be considered a corollary of conclusion 1, the physical reason for the conclusions being set forth in conclusion 2.

4. Suspended load decreases the coefficient of momentum transfer, the reduction varying with the amount of the load and being largest for fine sediment.

5. Suspended load reduces the value of the von Kármán universal constant of turbulent exchange k, which characterizes the effectiveness of the turbulence in transferring momentum. Reduction of k means that the mixing is less effective and would indicate that the sediment tends to suppress or damp out the turbulence. This conclusion follows directly from conclusion 4.

6. Suspended load reduces the resistance to flow, thus causing sediment-laden water to flow faster than clear water. This follows directly from con-

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clusions 4 and 5. In the laboratory channel an average suspended load of 1.2 g per liter reduced the friction factor, *f*, as much as 20%, and the corresponding Manning roughness coefficient was reduced as much as 10%.

7. Conclusions 1 to 6 can all be related to essentially three effects that occur in the flow in the presence of the sediment: (1) The sediment appears to damp out the turbulence in such a way that the momentum transfer is reduced, (2) random turbulence, which is not a factor in the transfer of momentum, contributes to the transfer of sediment, and (3) the "slip" between the fluid and the sediment tends to make the sediment transfer coefficient less than the momentum transfer coefficient.

8. The idea that suspended sediment damps the turbulence suggests that the transporting capacity of a stream is determined through the reduction of the turbulence by the sediment.

9. The increase in velocity, due to the addition of sediment to a clear stream, probably does not give rise to an increased sediment-transporting capacity since it is accompanied by a reduction in turbulence and in the entrainment force at the stream bed.

10. Suspended load tends to cause a flow to become unevenly distributed. Unsymmetrical sediment distribution within the flow may cause secondary circulation.

11. In rivers, density gradients due to suspended load may be an important cause of the secondary circulation, which has the effect of causing the maximum velocity to occur below the water surface.

12. The velocity distribution near the center of the flume used in these experiments was of the logarithmic form and followed the von Kármán universal velocity defect law.

ACKNOWLEDGMENTS

The writer wishes to acknowledge the friendly assistance of Professors von Kármán and Knapp and of members of the staff of the Cooperative Laboratory of the Soil Conservation Service and the California Institute of Technology in which the experiments reported herein were performed. Acknowledgment is due also to Roy Peterson, Milton Wood, and Robert Wise for their assistance in collecting and analyzing the data.

Assistance in the preparation of these data was furnished by the personnel of Work Projects Administration, Official Project No. 65–2–07–58, Work Projects No. PS–11496.

DISCUSSION

RALPH W. POWELL,[34] M. AM. SOC. C. E.—The subjects covered in this paper have recently attracted much attention, and the quite complete treatment the author has given is most welcome. It is hoped that in his closure he will explain the relation of his results to those of A. A. Kalinske and C. L. Pien,[35] Assoc. Members, Am. Soc. C. E., which seem to be founded on an entirely different approach, but which also lead to laws of logarithmic vertical distribution of both longitudinal velocity and sediment concentration. It may be stated, however, that G. I. Taylor,[36] in his "modified vorticity transport theory," deduces that velocity distribution for two-dimensional flow is cycloidal (except for the laminar boundary layer at the bottom). Although the cycloidal equation is entirely different from the author's logarithmic one, the numerical values differ hardly enough to be distinguished experimentally. The writer once amused himself by deriving a simpler distribution by thinking of the laminar two-dimensional flow of oil of high viscosity with a thin layer of water between it and the bottom. The velocity profile, of course, proves to be the parts of two parabolas, the lower one in the water being practically a straight line, and the upper one with its vertex at the upper surface of the oil. This gives an approximate picture of two-dimensional turbulent flow, the water representing the laminar boundary layer, and the oil the turbulent part with its high (mechanical) viscosity. It is only approximate because the mechanical viscosity is not uniform, but is a function of the depth.

Neither this rough approximation nor the more accurate cycloidal or logarithmic equations fit the case where the flow is not two-dimensional, however—as, for example, in narrow rectangular channels or rivers of nonuniform depth. As is well known, in these cases the maximum velocity occurs some distance below the surface, and Eq. 15*a* could not possibly apply. The only satisfactory explanation of this depression, which the writer has seen, was

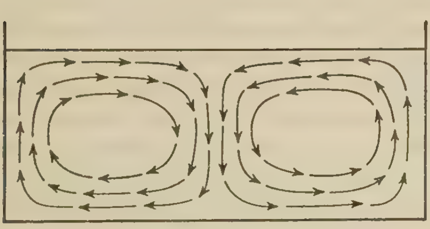

FIG. 16.—SECONDARY FLOW AS DESCRIBED BY STEARNS, MÖLLER, AND GIBSON

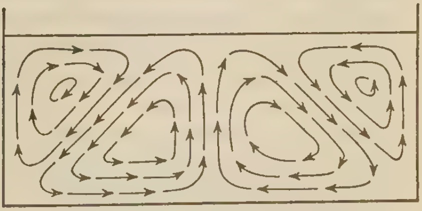

FIG. 17.—HYPOTHETICAL SECONDARY FLOW SUGGESTED BY THE WRITER

developed independently in 1882 by the late F. P. Stearns,[37] M. Am. Soc. C. E., in America and Max Möller[38] in Germany, although Mr. Stearns acknowledges that the explanation had been suggested to him some six years earlier by the late Hiram F. Mills, M. Am. Soc. C. E. The explanation is that in straight channels there is a double spiral flow as shown in Fig. 16, superimposed on the forward flow, which carries the slower moving water from the bottom up along the sides and out along the top to the center. At the center it mixes with what would otherwise be the fastest moving water, thus slowing the rapid water to a velocity somewhat less than that in the region just below.

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[34] Associate Prof. of Mechanics, Ohio State Univ., Columbus, Ohio.

[35] "Experiments on Eddy-Diffusion and Suspended-Material Transportation in Open Channels," by A. A. Kalinske and C. L. Pien, *Transactions*, Am. Geophysical Union, Pt. II, 1943, pp. 530–536.

[36] "Flow in Pipes and Between Parallel Planes," by G. I. Taylor, *Proceedings*, Royal Soc. of London, Series A, Vol. 159, 1937, pp. 496–506.

Those who inquire as to what causes these spirals discover a very amusing situation. A. H. Gibson[39] follows out the thinking of Mr. Stearns and states:

> "Frictional losses at the sides reduce the energy and thus the head of the water in their neighborhood, with the result that the surface level at the sides is lower than near the center of the stream, and the cross-sectional profile of the water surface is a curve concave to the bed. Owing to this superelevation of the water near the center and its tendency to find its own level, transverse currents are set up which travel downwards near the center of the stream; outwards along the bottom to either bank, upwards along the sides, and, for permanence of *régimé*, inwards along and near the surface."

Mr. Möller,[38] however, takes just the opposite view:

> "Each filament of water that approaches the bank * * * suffers retardation, and hence requires greater cross section, whereas the conditions in the center of the stream are reversed, water filaments there being accelerated and requiring less cross section. Both actions lead to the production of a slight transverse drop in the water surface * * * and in the formation of spiral motion on either side."

Mr. Möller then shows an illustration with the water surface highest at the sides; but all three agree that the spirals are as shown in Fig. 16. Hydrodynamics would seem to the writer to show that these spirals would cause the water surface to be slightly higher at both the sides and the center, and lower at the quarter points, but, since the velocity of the spirals is very small, the additional head due to curvature would be extremely minute, and at any rate it would be an effect, rather than a cause, of the spirals. The writer has the same feeling in regard to the author's conclusion (10), and believes that the double spiral flow is a cause of the unequal distribution of suspended load, rather than a result of it.

The cause of these double spirals is still obscure to the writer. L. Prandtl[40] has shown why there will be secondary flow toward the corners and away from the sides in the case of turbulent flow in rectangular pipes. Similar reasoning applied to a rectangular channel might explain spirals somewhat as shown in Fig. 17, but this is a pure flight of fancy on the part of the writer and he has no proof that it actually exists. There is just the possibility that such spirals might have been observed by Messrs. Stearns, Möller, and Gibson, and have been mistaken for the form shown in Fig. 16, but it does not seem very probable.

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[37] "On the Current-Meter, Together with a Reason Why the Maximum Velocity of Water Flowing in Open Channels Is Below the Surface," by F. P. Stearns, *Transactions*, Am. Soc. C. E., Vol. XII (1883), pp. 331–338.

[38] "Studien über die Bewegung des Wassers in Flüssen mit Bezugnahme auf die Ausbildung des Flussprofiles," by Max Möller, *Zeitschrift für Bauwesen*, 1883, p. 201. A summary in English is given in *Hydraulic Laboratory Practice*, edited by J. R. Freeman, A.S.M.E., 1929, p. 70.

[39] "Hydraulics and Its Applications," by A. H. Gibson, D. Van Nostrand Co., 3d Ed., 1925, p. 331, following the article "On the Depression of the Filament of Maximum Velocity in a Stream Flowing Through an Open Channel," by A. H. Gibson, *Proceedings*, Royal Soc. of London, Series A, Vol. 82, 1909, pp. 149–159.

[40] "Über die ausgebildete Turbulenz," by L. Prandtl, *Proceedings*, 2d International Cong. for Applied Mechanics, Zurich, 1927, p. 62. The essence is given in English in "Fluid Mechanics for Hydraulic Engineers," by Hunter Rouse, McGraw-Hill Book Co., Inc., New York and London, 1938, pp. 267–268.

TABLE 5.—"EQUIVALENT ROUGHNESS" OF FLUME WITH VARIOUS SEDIMENT CONCENTRATIONS

Run No.	Number of values averaged	Average concentration, C_m (g per liter)	Mean diameter, $\overline{D}_s$ (mm)	Ratio of equivalent roughness to size of bottom sand
1, 2, 3	3	0	0	0.89
14, 20, 21	4	0	0	0.86
17	2	0.048	0.160	0.34
19	5	0.17	0.103	0.19
2	1	0.35	0.160	0.65
15, 16	2	0.59	0.160	0.40
1	4	0.72	0.160	0.78
4	2	0.79	0.160	0.89
3	2	0.85	0.160	1.02
18	4	1.18	0.103	0.40
20, 21	2	1.2	0.103	0.51
22	1	1.3	0.134	0.42
8, 14	3	1.34	0.160	0.68
9	2	1.41	0.160	0.36
5, 7	6	1.60	0.160	1.07
6	2	1.78	0.160	0.60
12	3	2.56	0.160	0.41
13	1	3.19	0.160	0.54
11	3	3.36	0.160	0.50

The other point which the writer wishes to discuss is the reduction in resistance to flow which the author finds accompanies the presence of sediment. The writer has suggested,[41] as a formula for resistance to flow in rough channels,

$$c = 40 \log \frac{R}{\epsilon} + c_s \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (26a)$$

in which c is the Chézy coefficient, R is the hydraulic radius, ϵ is the equivalent roughness of the channel bottom (in the same units as R), and c_s is the shape correction coefficient proposed by G. H. Keulegan[42] to correct the Chézy coefficient to a corresponding channel of infinite width; that is, to eliminate the effect of the side-walls. (The value of c_s is 14.18 times the value of β given by Mr. Keulegan.) In the author's experiments the side-walls were smooth and relatively far away from the sections studied, so that c_s may be taken as zero and R as y_m, the depth; thus:

$$c = 40 \log \frac{y_m}{\epsilon} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (26b)$$

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[41] "Flow in a Channel of Definite Roughness," by Ralph W. Powell, *Proceedings*, Am. Soc. C. E., December, 1944, p. 1521.

[42] "Laws of Turbulent Flow in Open Channels," by Garbis H. Keulegan, *Journal of Research*, National Bureau of Standards, Vol. 21 (December, 1938), p. 719.

Values of c were computed for all the data of Table 3, and from these the value of ϵ, the equivalent roughness, given by Eq. 26b, was determined. These values were then divided by the diameter (in feet) of the sand forming the bottom roughness. For water without sediment it might be expected that this ratio would be 1, but the scale of roughness used in Eqs. 26 is not quite the same as that of Nikuradse, values of the writer's ϵ being somewhat smaller than Nikuradse's sand size.[41] All runs with approximately the same concentration were grouped together and the ratios of ϵ to size of bottom sand were averaged. Table 5 shows the results, arranged in ascending order of sediment size. The general weighted average for each size of sediment, irrespective of concentration, was as follows:

Mean diameter, $\bar{D}_s$ in millimeters	Ratio of equivalent roughness to size of bottom sand
0	0.87
0.103	0.32
0.134	0.42
0.160	0.69
All runs with sediment	0.59

Although it is evident that in general the equivalent roughness is reduced by the presence of sediment, there is great variation in the effect and there are even a few cases in which there seem to be an increase (runs 3, 5, and 7, Table 5). There also seems to be no relationship between the amount of sediment and the reduction, run 17 with only 0.048 g per liter giving the largest reduction of any concentration of the 0.160 mm sediment. The only consistent variation is that shown by the average for each size, which indicates that the reduction is greater for the smallest size and least for the largest. This might be interpreted as meaning that the reduction in resistance is a function of the number of particles present rather than of their total mass. It should perhaps be pointed out that the variation is not quite as great as might be thought at first. If the bottom sand were 0.47 mm in diameter and the depth 0.50 ft in both cases, the values of c corresponding to the ratios of 0.34 and 1.07 (the smallest and largest shown in Table 5 for 0.160 mm sediment) are 119.2 and 99.3, a variation of only 9.1% each way from the mean.

The writer has not tried to touch on some of the most interesting points in this valuable paper, which can better be discussed by those more familiar with the details of this subject, but has tried to indicate a few directions in which he hopes that further investigation will be made.

E. R. Van Driest,[43] Assoc. M. Am. Soc. C. E.—Experimental investigations conducted with the accuracy and thoroughness that is evidenced by this paper are of fundamental importance in dealing with problems of applied fluid mechanics. Particularly is this true in the field of sedimentation in flowing water where the number of variables encountered is great. The paper depicts clearly the trials and tribulations of an experimenter (1) in building apparatus for the proper control of conditions and (2) in bringing theory and data together.

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[43] Asst. Prof., Mech. Eng., Mass. Inst. Tech., Cambridge, Mass.

In the presentation of the theory, it might add to the paper to point out the fact that the problem of suspension of sediment by flowing water is but one of the three important turbulence transfer problems usually encountered in engineering. The most common is that of internal friction or momentum transfer expressed by Eq. 2. Another such problem which has received considerable attention is that of heat transfer, the formula for which is

$$q = -\rho\, c_h\, \epsilon_h \frac{dT}{dy} \dots\dots\dots\dots (27)$$

in which, in addition to the notation of the paper, q is the rate of heat transfer across a unit area normal to the y-axis; c_h is the specific heat of the fluid; T is the temperature; and ϵ_h is the heat transfer coefficient. Through the assumption that $\epsilon_h = \epsilon_m$ (the basic assumption of the Reynolds analogy between heat transfer and skin friction), great progress has been made in heat transfer for flow in pipes. The third transfer problem—that of sediment transfer—has for its basic relation Eq. 4. The paper proceeds to show that progress is now being made in the sedimentation field when ϵ_s is assumed equal to ϵ_m or when the mechanism for the transfer of sediment is assumed the same as the mechanism for the transfer of momentum. Since these three phenomena are of the same nature, they can be derived from one general transfer formula,

$$Q_\theta \doteq -\epsilon \frac{d\theta}{dy} \dots\dots\dots\dots (28)$$

in which Q_θ is the mean rate of transfer of any property θ per unit volume across a unit area perpendicular to y. By merely substituting any of the foregoing properties per unit volume—that is, momentum $\rho\, u$, heat content $\rho\, c_h\, T$, or sediment concentration C—into Eq. 28, the aforementioned equations will result. Indeed, any other properties may be assumed to be transferable in the same sense.

The author shows some concern about the fact that the velocity distribution data do not fall entirely on a straight line when plotted on semilogarithmic paper. Furthermore, he states that this departure is due to viscous action, but finds that such an assumption does not explain the results. The truth of the matter is that Nikuradse's data do not fall exactly on a straight line either.[44] Neither do his data for smooth pipes[45] fall precisely on a straight line when one plots the data separately for each run. All Nikuradse's data for smooth and rough pipes, except those for the lowest three Reynolds numbers in the smooth pipes when the viscous layer does show its presence, have a characteristic S-shape. A straight-line plot does appear to exist when all the data are plotted on the $\left(\frac{u}{\sqrt{\frac{\tau_0}{\rho}}}, \log \frac{y}{e}\right)$-scales, in[46] which e is the linear dimension characterizing roughness; however, even here the prevailing S-pattern can be observed for each run. That the viscous layer does not affect the author's data is evidenced by a simple computation which will show that the position of the data taken nearest the bottom is at least ten times as far from the bottom of the roughness grains as the edge of the laminar boundary layer which is at the most one half as thick as the sand roughness grains. Measuring the distance y from the bottom of the grains makes the measurements consistent with those of Nikuradse who measured his distances from the wall or, rather, from the base of the sand grains, and this does have a decided effect upon the plotting of the data very close to the wall. In Nikuradse's plots for individual runs, the straight line drawn for $k = 0.40$ passes through the data as a mean line. It appears that such a line for $k = 0.40$ has the same mean relation to the data of Fig. 8 for no sediment added, even after the data are shifted to conform with measurements from the bottom of the grains.

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[44] "Strömungsgesetze in Rauhen Rohren," by J. Nikuradse, *Forschungsheft 361*, Vereines Deutscher Ingenieure, Vol. 4, 1933, Figs. 15*a*, 15*b*, 15*c*, 15*d*, 15*e*, and 15*f*, p. 9.

[45] "Gesetzmässigkeiten der turbulenten Strömung in Glatten Rohren," by J. Nikuradse, *Forschungsheft 356*, Vereines Deutscher Ingenieure, Vol. 3, 1932, pp. 1–36.

[46] "Strömungsgesetze in Rauhen Rohren," by J. Nikuradse, *Forschungsheft 361*, Vereines Deutscher Ingenieure, Vol. 4, 1933, Fig. 14.

The data of Fig. 8 seem to prove quite conclusively that the value of k is decreased by the presence of sediment, and it appears logical that this should be so, because of the dampening effect of the sediment. However, the absolute effects of size and quantity are still uncertain, thus showing the need for further controlled experimentation. If the concentration has a decided effect upon k, then it should be logical to assume that the velocity distribution curve should be distorted considerably because of the large variation of concentration over the depth. This effect does not appear to exist.

An intriguing observation is that the exponent z_1 suddenly changes from the theoretical value when the size, and not the amount of sediment, is changed. The only explanation that the writer can give for this disagreement follows from the apparent circulation of sediment. Since the value of ϵ in any vertical line is more uniform near the wall than in the central part of the channel, the sediment transfer theory requires the sediment concentration to be more uniform vertically near the wall than farther in the channel thereby causing the concentration to be greater at the water surface near the wall than in the channel. This variation along the surface then causes a horizontal flow of sediment toward the center of the channel. In this process, however, all the sediment at the bottom corner is picked up, leaving a sediment gradient along the bottom decreasing toward the corner, which in turn results in a flow of sediment along the bottom toward the corner. The entire process obviously results in a circulation of sediment, not necessarily water, which may develop into a complex circulation pattern. It follows that heavier or larger grains should not be diffused too far into the flow and therefore that such sediment should be more uniformly distributed horizontally in the central part of the channel. On the other hand, the lighter or smaller grains would be diffused farther into the flow, thus easily causing higher concentrations near the surface at the central part of the flow and therefore producing values of z_1 smaller than the theoretical values. It is concluded, therefore, that complete sediment traverses throughout the stream are necessary to interpret, fully, the results obtained.

As mentioned by the author, sediment appears at the stream surface even though the momentum transfer coefficient there must become zero if the logarithmic velocity distribution law is assumed to hold all the way to the surface. This inconsistency promptly disappears when it is noted that the logarithmic law does not hold at the surface if the velocity gradient there is to be zero because of a presumably zero shear. Although one would expect the transfer coefficient to approach zero at the surface as a result of a zero approaching velocity gradient, yet some agitation should be expected because of the migration of eddies from the adjacent lower layer of turbulent fluid. Evidence of this diffusion can be observed readily on the surface of turbulent rivers. A more simple case of such diffusion is demonstrated by the movement of eddies from a point of disturbance at the bottom of a jar of water into the overlying quiet fluid.

The writer wishes to re-emphasize the need and importance of such basic experimental work in sediment transportation. The paper unquestionably represents considerable progress in this field.

WESTON GAVETT,[47] ASSOC. M. AM. SOC. C. E.—Velocity distribution profiles, obtained from careful observations of velocities in a wide open channel with known bottom roughness, are an important part of this comprehensive paper.

The author states (see heading, "Results: Velocity Distribution") that "In all runs the velocity distribution fits the logarithmic law very well, although near the bottom there is a systematic deviation from the law, which is not understood." Figs. 6 and 7 are plotted with semilogarithmic coordinates in accordance with the von Kármán velocity defect law (Eq. 14), which the author states (see heading, "Previous Work") "* * * has been shown by J. Nikuradse[9,10] to apply to pipes and by G. H. Keulegan[11] to apply also to open channels."

The general acceptance of the von Kármán law which the author endorses in this and in an earlier paper[29] may discourage interest in the exponential formula of the form,

$$u = u_{\max}\left(\frac{y}{y_{\max}}\right)^{n} \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(29)$$

which, although not universal, seems to give better agreement with observations.

Nikuradse[45] showed that his velocity distribution curves for smooth pipe followed the exponential law and that n varied with the Reynolds number. Prandtl[48] has mentioned the seventh-power law based on Blasius' experimental law of pressure drop and has shown that, although the seventh-power law breaks down for Reynolds' numbers above about 50,000, similar derivations may be made for other powers. The writer has plotted a number of velocity profiles from various sources on log-log paper and has found that the greater number show good agreement with the exponential formula.

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[47] Asst. Engr., Clyde Potts, New York, N. Y.

[48] "Applied Hydro- and Aeromechanics," based on lectures of L. Prandtl, by O. G. Tietjens, McGraw-Hill Book Co., Inc., New York, N. Y., 1934, p. 70.

The velocity defect law has the advantages of being universal and of locating the approximate velocity profile when the friction velocity is known, without regard to roughness.

The exponential formula also has merits: (1) In giving a more accurate expression for velocity distribution and (2) in providing a simple expression for the velocity distribution in a pipe or channel and perhaps also for that in a plate or side of a ship. To apply the exponential formula, it is necessary to know the appropriate value of n, which appears to vary in accordance with definite laws that should be determinable from sufficient experimental data. For smooth pipe, n varies with the Reynolds number and decreases as $\mathbf{R}$ increases. It also varies in smooth pipe with δ, the laminar thickness, showing an exponential relationship. If n is plotted against δ, the laminar thickness for smooth pipe, and ϵ, the equivalent sand size for rough pipe, on log-log paper, the (n-ϵ)-curves for rough pipe appear to intercept the (n-δ)-curve for smooth pipe at points determined by a value of r/ϵ and to branch off with slopes greater than the smooth pipe curve.

The author states (see heading, "Results: Velocity Distribution") that "The fact that the velocity profiles fit the logarithmic law so well means that the average velocity will always occur at the relative depth $\frac{y}{y_m} = 0.368$.[29]" If the exponential law is valid, there will be some variation of y/y_{max} with n because

$$\frac{u_{avg}}{u_{max}} = \frac{1}{n+1} \qquad \text{(30a)}$$

and

$$\frac{y_{avg}}{y_{max}} = \left(\frac{1}{n+1}\right)^{1/n} \qquad \text{(30b)}$$

By replotting the points from Figs. 6 and 7 on log-log paper, the writer obtained a mean value of n of 0.145 for runs 1A, 2B, and 3C with a bottom roughness of 0.47-mm sand and no suspended load. For bottom sand size of 0.88 mm and no suspended load (runs 14A, 14B, and 21C), the value of n was 0.175. For these values of n, the mean velocity depths from Eq. 30b are as follows:

Description	Runs 1A, 2B, and 3C	Runs 14A, 14B, and 21C
Bottom sand size, in millimeters	0.47	0.88
Roughness coefficient n	0.145	0.175
Relative depth ratio $\frac{y_{avg}}{y_{max}}$	0.393	0.398

The variation in y_{avg}/y_{max} is not great, but, for the extreme values of n for smooth and rough surfaces of about 0.10 and 0.36, the corresponding values of y_{avg}/y_{max} would be 0.385 and 0.425.

For pipe, the author[29] has given a constant mean distance of 0.223 r_0. The following expressions are derived from the exponential formula for pipe:

$$\frac{u_{avg}}{u_{max}} = \frac{2}{(n+1)(n+2)} \qquad \text{(31a)}$$

and

$$\frac{y_{\text{avg}}}{r_0} = \left[\frac{2}{(n+1)\,(n+2)}\right]^{1/n} \ldots\ldots\ldots\ldots\ldots\ldots (31b)$$

For values of n of 0.1 and 0.36, the corresponding values of y_{avg} would be $0.237\ r_0$ and $0.269\ r_0$.

The author's data indicate that the bottom sand size, equivalent to that used by Nikuradse, is smaller than the actual size of sand used, possibly because a part of the sand was embedded in the coating.

The average velocities computed by Eq. 30*a* check closely with the average velocities given by the author as computed from the velocity defect law, indicating that the von Kármán law may be correct for the average velocity. If so, the formula of B. A. Bakhmeteff,[49] M. Am. Soc. C. E., is simple and convenient:

$$\frac{u_{\max} - u_{\text{avg}}}{\sqrt{\frac{\tau_0}{\rho}}} = \frac{3}{2\,k} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (32)$$

Substituting $k = 0.4$ in Eq. 32, $\dfrac{u_{\max} - u_{\text{avg}}}{\sqrt{\frac{\tau_0}{\rho}}} = \dfrac{3}{2} \times \dfrac{1}{0.4} = 3.75$. The same value of 3.75 results from substituting 0.223 for y/r_0 and $k = 0.4$ in Eq. 14. Prandtl's value for this constant according to Professor Bakhmeteff is 4.07, which agrees better with values computed from Nikuradse's data.

BERARD J. WITZIG,[50] ASSOC. M. AM. SOC. C. E.—An excellent service to the science of hydraulics has been rendered by Professor Vanoni in enlarging and completing his previous paper on sediment transportation,[30] and he is to be commended for a careful exposition and presentation of details leading to logical conclusions. The problem of sediment suspension and transportation by water involves fundamental principles that are only imperfectly understood, and only by painstaking research will limited existing knowledge of the phenomena be expanded.

In the "Introduction," the author mentions two essentially different modes of sediment transportation—"bed load" and "suspended load." As long as it is remembered that the distinction between these modes is merely arbitrary[51] because of the insurmountable difficulties of treating the sediment load of a stream as a whole, there should be no slighting of the possibility that a much larger quantity of sediment in some streams may move as bed load than as suspended load.

The relations and the results given in this paper should not be applied indiscriminately to the total suspended load. The exponential vertical distribution of sediment applies only to the coarse suspended load. However, in some streams, a large part of the total suspended load consists of very fine silt, termed the "wash load,"[52] and is very nearly uniformly distributed over the stream cross section. Any conclusions based on a logarithmic or exponential distribution of the wash load would be misleading or erroneous. Apparently an appreciable wash load did not occur in the author's experiments, in which a carefully graded sand was used. It would be of interest, however, to know Mr. Vanoni's views on the probable influence of a uniformly distributed wash load, in combination with a logarithmically distributed coarse suspended load, on the sediment transfer coefficient.

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[49] "The Mechanics of Turbulent Flow," by Boris A. Bakhmeteff, Princeton Univ. Press, Princeton, N. J., 2d Ed., 1941, Eq. 116, p. 70.

[50] Engr. (Civ.), U. S. Engr. Office, Buffalo, N. Y.

[51] "Fluid Mechanics for Hydraulic Engineers," by Hunter Rouse, McGraw-Hill Book Co., Inc., New York, N. Y., 1938, p. 331.

Although probably of merely academic interest, it seems appropriate to point out that a transitional mode of sediment transportation exists, known as "saltation." In saltation, grains of bed material are lifted momentarily into the moving fluid, then bouncing back on to the bed, possibly dislodging other particles. When the turbulence of the stream exceeds a certain level, these particles move into the suspended load zone. Quantitatively, the saltation load is probably insignificant. Prof. A. A. Kalinske,[53] Assoc. M. Am. Soc. C. E., concluded that, when the water velocity is great enough to cause saltation, the turbulence level is also sufficient to obscure the saltation effect entirely, by placing the material lifted from the stream bed into suspension.

The paper succinctly summarizes modern fluid turbulence theory as applied to sediment suspension. The present state of knowledge and research necessitated the author's initial assumptions that the sediment transfer coefficient, ϵ_s, is equal to the momentum transfer coefficient, ϵ_m, and that these coefficients are constant over the depth. The experimental results described demonstrate the falsity of the former assumption, but it is not apparent whether they either prove or disprove the latter. The writer considers it probable that some of the apparent discrepancies in the published data, such as the large variation in the values of z and z_1, run 17, Table 3, might be due to some unaccountable deviation in the $(\epsilon - y)$-relation.

The author explains that the laboratory flume used in his experiments had rubber sides, with apparently a very small roughness coefficient. A narrow flume, however, is bound to exert a pronounced "side-wall effect," especially where the bottom itself is comparatively smooth, as was the case here and as is shown in Fig. 5. Although Fig. 5 demonstrates that the side-wall effect of the experimental flume extended only about one sixth of the flume width from each side, the writer is of the opinion that such an extensive series of tests should have been made in a wider flume. A fundamental assumption for the two-dimensional analysis of the problem is that the theoretical equations for the vertical distribution of velocity and of sediment are valid only in "wide" channels. In this case, the ratio of depth to width was about 1:5.5, and its recognized effects on the experimental observations are indicated by the fact that all measurements in Table 3 were confined either to the center line of the flume, or to a vertical not more than 0.5 ft from the center line. Laboratory limitations, of course, specify the practicable width of flume, but some increase in the ratio of width to depth should be desirable in future experiments.

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[52] Discussion by Joe W. Johnson of "Sedimentation in Reservoirs," by Berard J. Witzig, *Transactions*, Am. Soc. C. E., Vol. 109 (1944), p. 1072.

[53] "Criteria for Determining Sand-Transport by Surface Creep and Saltation," by A. A. Kalinske, *Transactions*, Am. Geophysical Union, Pt. II, 1942, pp. 639–643.

The effect of the depth-width ratio in a turbulent and rapidly flowing natural stream is illustrated by discharge measurements made in 1943 under the writer's direction near the head of the Cascades, in the American channel of the Niagara River at Niagara Falls, N. Y. At the time of the measurements, the average depth was about 6 ft; the width at the gaging section, about 480 ft; the measured discharge, 16,150 cu ft per sec; and the mean velocity, 7.5 ft per sec. Vertical velocity traverses made with a current meter showed that a generally exponential gradient existed, with the maximum velocities occurring at the surface. These measurements also indicated that the mean velocity in a vertical occurred, on the average, at the 0.55 depth from the surface, instead of at the 0.6 depth commonly assumed in discharge measurements. The logarithmic law places the mean at the 0.632 depth from the surface. It seems probable that such variation in field observations from theoretical assumptions and laboratory results must place in doubt the validity of generally applying the latter to the large-scale phenomena existing in natural streams.

The writer was particularly interested in the analysis of the effect of sediment loads on the roughness coefficient of the flume. Runs 18, 20, and 21, Table 4, however, apparently indicate some inconsistencies. For the clear water run in each case, Manning's n was 0.0118, 0.0121, and 0.0128, respectively; whereas, with a sediment concentration of 1.2 g per liter, n was 0.0107, 0.0116, and 0.0116, respectively. Runs 18 and 21 showed a ratio of 1.10 for clear flows compared with silt flows, but run 20, under practically identical conditions, gave a ratio of 1.04. Can this apparent inconsistency be explained by the change in slope and velocity, or should it be ascribed to the difficulties of laboratory measurement?

It would be of interest to find some explanation of the mechanism whereby the channel roughness is decreased by the presence of sediment in the fluid. May not the lamina of high concentration fill the rough projections of the channel bed, thereby acting as a lubricating film and decreasing the apparent roughness coefficient? Merely as a speculation, it is also suggested that an increase in silt load may decrease the apparent roughness at a diminishing rate to a certain point, and that thereafter a further increase in silt content may produce a result similar to an increase in viscosity, with greater resistance to flow.

The author suggests that one of the reasons for the difference between the momentum and sediment transfer coefficients is the "slip" between the fluid and the sediment particles. The writer considers that this "slip" is due in part to the inertia and the "drag" of the particles. The difference in the effect of fine and coarse sediments on ϵ_s compared with ϵ_m might reasonably be expected. Small particles, with a relatively larger surface area per unit of mass as compared with larger particles, should naturally be more readily moved or kept in suspension by the random turbulence; the "slip" effect also should be less because of the smaller mass and drag; and molecular impact should be more pronounced.

It is suggested that discrepancies between z and z_1, as well as the consistent variation actually demonstrated, might be caused by secondary, or more properly "tertiary," circulation in the third (horizontal) dimension. If such variations appear in carefully controlled laboratory tests, even wider variations may be expected in natural streams, where complex circulations are induced by winds at the surface, bends in alinement, tributary and surface inflow, and irregularities of the sides and bed.

The description of the horizontal distribution of sediment in segregated bands and clouds omits reference to any observed vertical segregation. Did such vertical separation exist, and, if so, did the sediment tend to follow the fluid filaments of highest velocity? The writer's observations of silt-laden natural streams indicate that sediment flows appear to move in "slugs" or clouds, but he has not noticed band-like movements. It is probable that these may not occur in natural streams because of the large degree of turbulence. With reference to Fig. 15(*c*), it is not readily apparent how the secondary circulation is derived from the velocity and sediment diagrams.

It is questionable whether the increase in velocity accompanying silt flows results in an actual decrease in the entrainment forces exerted on bed particles. An increase in stream velocity causes increased erosion when the stream is not burdened to capacity with its sediment load. When the stream is fully charged with sediment, it seems probable that the concentration near the bottom may be so great that no net increase in load may result, although a continual interchange takes place. Such a stable condition may be due, not so much to the theoretical decrease in shear accompanying the smaller depth and greater velocity, as to the fact that the full transporting capacity is already in use.

The writer considers that the author has ably and amply demonstrated previously anticipated effects of sediment loads on the transfer coefficient, and that his experiments indicate the relative magnitude of these effects for the range of concentrations and sizes of sediments studied. The apparent effect on channel roughness or resistance suggests that further intensive research on this particular aspect of the problem would be of interest economically, as it may appreciably affect practical design of pipe lines or flumes for commercial or industrial transportation of materials in suspension.

The author has shown that a very complex problem in hydraulics is susceptible, at least in part, to rational and experimental analysis. Other experimenters should find encouragement in his report of progress.

A. A. KALINSKE,[54] ASSOC. M. AM. SOC. C. E.—Data from an extensive series of measurements of the transportation of suspended material by water in an open channel are presented in this paper. The writer does not in any way question the value or accuracy of these data; however, he does seriously disagree with many of the conclusions drawn from the data, especially with the conclusions in regard to the effect of suspended sediment on the turbulence and hydraulic characteristics of the water flow. Specifically, the paper contains no real concrete evidence to prove the validity of the conclusions relating to: (1) The action of random velocity fluctuations on suspended sediment; (2) the effect of suspended sediment on the von Kármán k-values; and (3) the hypothesis that suspended load reduces resistance to flow.

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[54] Associate Prof., Univ. of Iowa, and Associate Director, Iowa Inst. for Hydr. Research, Iowa City, Iowa.

So far as similarity of the transfer of sediment by turbulence to the transfer of momentum in an open water channel is concerned, the writer does not believe any too definite statements can be made as yet—for the one important reason that it is difficult to calculate the correct value of the momentum-exchange coefficient. All such computations must be based on some assumption as to the value and distribution of the shear. It is doubtful if, in a channel such as that used by the author, the shear is distributed linearly, due to the side and free surface effects and secondary currents. The author calculated the value of ϵ_m using the logarithmic velocity distribution law, which is not quite correct either near the water surface or near the bottom. Thus, the value of ϵ_m in the paper may not necessarily be the true value of the coefficient of momentum transfer. Any rigid comparison between ϵ_m and ϵ_s, therefore is not really warranted.

The author's remarks regarding the presence or absence of correlation between the turbulent velocity components are somewhat confusing. The correlation that exists when momentum is exchanged in two-dimensional flow is the relationship between the vertical component v' and the horizontal component u'. The presence or absence of any such correlation has no relation to the transfer of sediment since the horizontal component u' has nothing to do with keeping sediment in suspension. If the shear is zero, the correlation between u' and v' is zero, since, according to Reynolds, the shear in the turbulent flow can be expressed as

$$\tau = \rho \, \overline{u' \, v'} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (33)$$

However, this does not mean that the turbulent exchange coefficient is zero. Thus, in the center of a pipe the shear is zero, but ϵ has a finite value since heat or mass can be transferred across the center of a pipe even though momentum cannot be transferred. So far as the writer knows, the only way to obtain an accurate idea of the transfer coefficient in a region of flow where the shear is difficult to estimate is by direct measurement using diffusion experiments. The writer has reported such measurements elsewhere.[35]

The writer's chief criticism of the author's laboratory procedure is that the experiments were not made with the channel bed completely covered with the sand being transported in suspension. If the bed is not completely covered with unconsolidated sand, a particular flow is not transporting all the sediment in suspension that it is capable of transporting. Of course, there is always some loose sand on the bottom; however, the patches of sand are constantly changing and, of course, are influencing the bed roughness. Thus, the bed roughness for one discharge may be different from that for another discharge. In other words, the bed undoubtedly differed for the various runs, thus very likely producing the effects on velocity, friction factor, and k that the author attributes to changes in concentration or size of material in suspension.

For the study of the transportation of sediment in suspension and of the effects that this suspended sediment may have on the hydraulic characteristics of the water flow, it is necessary that the flow take place over a bed entirely covered with the material being transported in suspension and that the suspended matter be in equilibrium with such a bed. Unless this is done, the data obtained will be useful only for the study and analysis of the vertical distribution of suspended sediment in turbulent flow.

In similar experimental studies made by the writer, the channel bottom was always covered with the material being moved in suspension and the results from these studies do not in any way confirm the author's conclusions. In 1943 an extensive series of experiments was made at the Iowa Hydraulics Laboratory, University of Iowa, Iowa City, on the transportation of a fine sediment (median diameter 0.011 mm) in a flume 2¼ ft wide with about ½ ft of water.[55] Runs were made with water velocities varying from 0.84 to 2.73 ft per sec and with sediment concentrations varying from 0.64% to 11.1% by weight. Some of the conclusions drawn from these studies were as follows:

(1) The velocity distribution in the center of the channel followed the logarithmic law very well and no difference was noted even when the sediment concentration was 11.1%;

(2) Manning's *n* was influenced by the form of the bed, which varied with the bottom shear, and not by sediment concentration; and

(3) The value of the von Kármán *k*-factor changed from 0.32 to 0.44 without any regular variation with respect to sediment concentration.

The sediment concentrations in the studies at the Iowa Hydraulics Laboratory were considerably higher than those in the author's experiments and also had a wider range. However, the size of the material used was very much smaller. The writer is forced to state that the results of the studies at the Iowa Hydraulics Laboratory do not seem in any way to confirm the author's conclusions which apparently were intended to be quite general in scope.

PAUL F. NEMENYI,[56] ESQ.—The investigations made by Professor Vanoni represent a considerable advance in the study of sediment dynamics and of the relation of sediment dynamics to turbulence. Professor Vanoni's analysis of the deviations between the distribution of momentum transfer and sediment transfer in the cross section of a conduit seems particularly valuable. The highly controversial subject of secondary currents, too, has been advanced considerably by the author (see heading, "Discussion of Results: Secondary Flow Due to Suspended Load), by the phenomenon observed in connection with run 15.

The first observations on this subject were made by the German geophysicist, Max Möller,[38] and the American hydraulic engineer, the late F. P. Stearns,[37] M. Am. Soc. C. E., simultaneously and independently of each other in 1882. Both writers agreed that secondary circulation consists of two perfectly symmetrical parts tending toward the middle of the open conduit at the top and tending outward at the bottom of the open conduit. Fig. 18(*a*) represents the Möller interpretation of this flow. The phenomenon was rediscovered, independently of both Messrs. Möller and Stearns, by A. H. Gibson,[39] in 1909 (see Fig. 18(*b*)), who found the order of magnitude of the cross-velocity component to be 5% of the longitudinal main velocity. Mr. Gibson's investigations seem to improve upon those of Messrs. Möller and Stearns both as to exactness and range of observations. Mr. Gibson found the surface convex whereas Mr. Möller thought it to be somewhat concave. There is a good theoretical reason to believe that in this point Mr. Gibson's observation is correct, because roughness protuberances, like any other obstacles, tend to produce a locally lowered average pressure. The theoretical explanations offered by Mr. Möller are "begging the question," whereas those by Mr. Gibson appear to be obscure.

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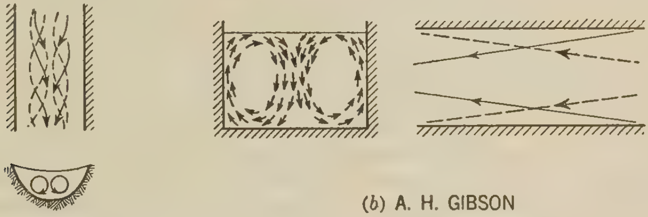
[55] "Study of Transportation of Fine Sediments by Flowing Water," by A. A. Kalinske and C. H. Hsia, *Bulletin No. 29*, Studies in Engineering, Univ. of Iowa, Iowa City, 1945.

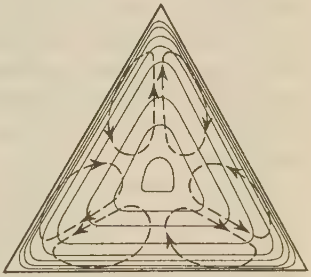
[56] Asst. Prof., Mathematics Dept., The State College of Washington, Pullman, Wash.


(*a*) MAX MOLLER

(*b*) A. H. GIBSON

Fig. 18.—Original Observations on Secondary Currents

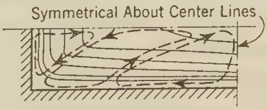
The three independent investigators agree remarkably well, in assuming the simplest imaginable general pattern of secondary circulation. This very reassuring finding, which is generally accepted by most hydraulic engineers, is challenged by the more elaborate research of L. Prandtl[40] concerning turbulent flow in conduits. Although Mr. Prandtl's investigations refer mainly to closed conduits (see Fig. 19), their implications for open channels and rivers are obvious. Mr. Prandtl's starting point is the surprising shape of the "isovels" in a closed conduit with turbulent flow. The following translation (by the writer) will give an idea of the experimental facts and the hypothetical conclusions based upon them by Mr. Prandtl—


(*a*) TRIANGULAR SECTION

LEGEND

——— Isovels by Prandtl

– – – – Secondary Currents Sketched by (*a*) Prandtl and (*b*) the Writer



(*b*) RECTANGULAR SECTION

Fig. 19.—Secondary Currents in Closed Conduits, According to the Prandtl Hypothesis

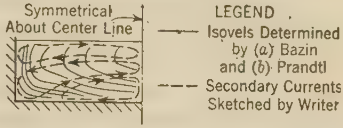
> "Instead of a velocity distribution having increasingly rounded (convex) isovels proceeding inward, as they are known in laminar flow, remarkable types of velocity distributions are found, as shown by the particularly

> surprising illustrations for the rectangle. For a long time no reasonable explanation was attainable. A paper concerning some old observations on helicoidal movements in straight channels gave a clue which led to an adequate explanation. The water in any conduit with a noncircular cross section has such a secondary movement that in any corner the secondary movement is directed along the bisectrix of the angle toward the corner, whereas, along the adjoining sides, the secondary movement is away from the corner. These secondary streams, together with the turbulent pulsations, explain the observations well. The secondary streams are always carrying longitudinal momentum toward the corner, thus causing the unexpectedly high longitudinal velocities found there. However, how can the existence of the secondary streams be explained? There is no explanation unless the pulsatory, mixing movement along the isovels of the cross section—the 'asimutal' pulsation—is assumed to be more vigorous than that perpendicular to it along the velocity gradient. Indeed, if this is true, a simple consideration of the momentum distribution shows that the type of pulsations assumed to be particularly vigorous is connected of necessity with forces pointing toward the convex side of the isovels—forces which are particularly strong where the isovels have sharp curvatures. The question of why the pulsations are of the particular type assumed, however, is part of the 'great problem of turbulence' for which even a tentative solution, for the time being, is unattainable."

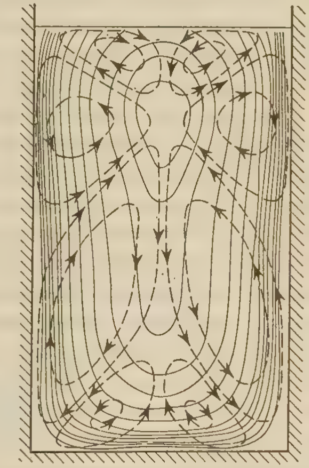
It is obviously impossible to draw a system of secondary streamlines for any cross section, such that each line is closed and, at the same time, is an orthogonal trajectory to the isovels. The best procedure to follow, therefore, is to start by tracing orthogonals to the system of isovels in the region where they have very sharp curvature and to consider these as part of the secondary streamlines. The remainder of the secondary streamlines, even if Mr. Prandtl's basic assumption is accepted as correct, will be based more or less upon conjecture.

Some uncertainty, however, is inherent in every theory that is merely qualitative. More serious than these uncertainties in practical application is the question as to the acceptability of the fundamental conceptions underlying the Prandtl hypotheses. For a statistically steady and permanent flow it is strictly meaningless to consider the steady secondary streams as caused by certain pulsations, and the shape of the isovels as caused by the secondary streams, because all are present at the same time and identically present in any cross section. The basic idea of causation, the explanation of the future from the present (or the present from the past), is inapplicable here. Therefore, in the background of Mr. Prandtl's too schematically expressed hypotheses one must seek an explanation as to the gradual development of the statistically steady, fully turbulent flow from laminar flow. If the investigator begins with laminar flow and assumes that with increasing average velocity the flow becomes turbulent gradually (an assumption which is by no means always justified) he can evolve a rational form to the Prandtl hypotheses. As soon as turbulence develops, comparatively strong pulsations along the originally laminar isovels may be assumed, which would indeed generate the secondary stream. The secondary stream, in its turn, starts to alter the isovels. With increasing turbulence, the "asimutal" pulsations (that is, pulsations favoring the isovels as orbits), and with them the secondary streams, increase and modify the isovels so long as, with fully developed turbulence, a statistical equilibrium is established. Stated in this way, the Prandtl hypothetical secondary currents, and the hypothetical prevalence of asimutal pulsations, would give a probable explanation for the surprising shape of the isovels, provided the turbulent régime develops gradually from the laminar régime. To the writer's knowledge, however, no direct experimental verification, even of the existence of secondary currents, much less of the particular patterns assumed by Mr. Prandtl, is as yet available for any closed conduit.

For open channels, Mr. Prandtl did not indicate the pattern of secondary currents. The uncertainty of the practical application of his hypothesis to open channels is particularly great. In Fig. 20, the writer has applied the hypothesis tentatively to a broad, very rough channel and to a somewhat narrow channel—in one case tracing the hypothetical secondary circulations on the basis of the velocity measurements by H. Bazin and, in the other case, on the basis of Mr. Prandtl's isovels. In each case, there is a rather complicated pattern of circulation which is thoroughly at variance with the simple results obtained by Mr. Gibson and his precursors. However, in the latter example on the surface (Fig. 20(*b*)), and in the former example (Fig. 20(*a*)), both on the surface and at the bottom, the secondary currents are qualitatively the same according to both assumptions, so that only a more exact investigation can provide sufficient information for a decision between the conflicting possibilities.


Symmetrical About Center Line
LEGEND
Isovels Determined by (a) Bazin and (b) Prandtl
Secondary Currents Sketched by Writer

(*a*) WIDE, ROUGH CHANNEL



(*b*) NARROW, SMOOTH CHANNEL

Fig. 20.—Secondary Currents in Open Rectangular Channels, According to the Prandtl Hypothesis

The latest information concerning secondary currents in a straight conduit containing homogeneous water comes from a very stimulating paper[57] by M. A. Welikanov, published in 1936. This work is a survey of fairly recent contributions of hydrologists and hydrodynamists of the U.S.S.R. to the understanding of river forms on a fluid dynamic basis. A considerable part of the paper deals with secondary currents in straight open channels.

Apart from somewhat questionable theoretical arguments Mr. Welikanov adduces the experimental results of Mr. Lossievsky in favor of the presence of steady secondary currents in any straight conduits with turbulent flow. Mr. Lossievsky found:

(1) For a broad rectangular cross section, the same result as Mr. Gibson;

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[57] "Flussbettbewegungen und Geschiebefuhrung," by M. A. Welikanov, *Bulletin No. 6, D*, Fünfte Hydrologische Konferenz der Baltischen Staten, Helsinki, Finland, 1936.

(2) For rectangular channels with large ratio of depth to width, two secondary currents but with circulation opposite to the Gibson pattern;

(3) For intermediate depth-width ratio, a more complicated and rather unstable pattern of secondary currents, and

(4) For a strongly unsymmetrical cross section, one single stable secondary current.

Only item (2) is in direct contradiction to the expectation based on Mr. Gibson's results, whereas all four items (with the possible, but doubtful, exception of item (3)) are at variance with what one would expect on the basis of the Prandtl hypothesis.

A possible explanation of these contradictions is that Mr. Lossievsky failed to observe the entire pattern of secondary currents in the cross section and, on the basis of observations at the surface and the bottom, reached unwarranted conclusions; or that his results are thoroughly vitiated by the use of insufficiently long channels. Of course, it is also possible that Mr. Lossievsky's results are reasonably exact and that Mr. Prandtl's work, as well as that of Mr. Gibson, needs to be revised. Mr. Lossievsky's item (3) would be probable enough, provided that items (1) and (2) were correct. Indeed, an unstable, shifting pattern of flow can be expected under transitional conditions between two substantially different, distinct types of flow.

The great number of contradictions and discrepancies among the various investigations available on this subject suggests also the possibility that all the secondary currents are a mere consequence either of an insufficiently permanent régime of flow or of temperature influences or other nonhomogeneities in the force field. Professor Vanoni seems to be precisely of this opinion when he ascribes the interesting secondary circulations observed by him entirely to the presence of sediment. However, a simple theoretical consideration shows that, even in a perfectly homogeneous fluid, if the flow is turbulent, secondary currents of appreciable intensity will normally be present. Indeed, however questionable may be Mr. Prandtl's assumption that the pulsations favor everywhere the isovels as orbits, it is perfectly rational to assume that at most points of the cross section there will normally be a favored circle of curvature for the pulsations just as there is a favored direction of pulsations. A strictly isotropic pattern of turbulence is an exceptional (actually only an artificially produced) type of turbulence. On the average, straight pulsations are less exceptional; but still it is reasonable to assume that, unless the cross section is circular, in most points of the cross section there will be a favored circle of curvature for the pulsations. Wherever there is such a favored finite circle of curvature for the pulsations, Mr. Prandtl has shown convincingly that a permanent, steady momentum tending to generate secondary currents is present. The writer is convinced, therefore, that nonhomogeneities in the force field are not necessary conditions for the presence of secondary currents.

It must be admitted, however, that nonhomogeneity of the fluid can create substantially stronger secondary currents, at the same time that their pattern may be transformed. The first to observe this new type of secondary currents

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[58] "Report of the Aeronautical Research Institute," by Terada, Tokyo Imperial Univ., Tokyo, Japan, Vol. III, 1928, and Vol. IV, 1929.

was Mr. Terada,[58] who showed that the particles of a fluid flowing in a very wide open flume, slightly and uniformly heated from below, move along a helicoidal curve (Fig. 21) so that the movement can be resolved into a straight flow along the channel axis, with secondary circulatory motions in the cross section. The regular striped clouds (popularly called "mackerel sky") were formerly considered "ripple" clouds and were attributed, in accordance with G. H. Darwin, to the rippling influence of a wind perpendicular to the stripes. Many hydrodynamists and meteorologists are of the opinion that this phenomenon may in most cases be due more probably to forced convection streams connected with wind parallel to the stripes—that is, to vortices with a longitudinal axis similar to those which were produced under laboratory conditions by Mr. Terada.

Although Mr. Terada's paper[58] is not accessible to the writer, analogy with other convection phenomena makes it possible to infer that a vorticity with a longitudinal axis of this kind can be generated by convection in the nonturbulent as well as in the turbulent régime of flow.

In their brilliant vortex study,[59] M. Luntz and P. Schwartz remarked aptly that the Terada phenomenon is closely analogous to the vortices[60] in the fluid between two coaxial rotating cylinders discovered by G. I. Taylor—the nonhomogeneity of the centrifugal force field replaces the nonhomogeneity of the gravity force field conditioned by temperature differences in the Terada experiment.

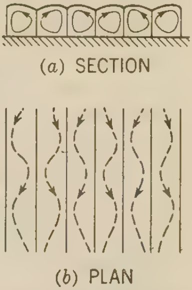

FIG. 21.—OBSERVATIONS OF SECONDARY CURRENTS, BY TERADA, IN AN OPEN, WIDE, AND STEEP CHANNEL HEATED FROM BELOW

In a paper by Hugh Casey,[61] there is a photograph which was made in a channel with turbulent flow and movable bed. No temperature influence was present in this case (at least not intentionally); yet the pattern of the secondary currents which is assumed by Mr. Casey (see Fig. 22) shows a surprising likeness to the convective secondary currents in the Terada study, of which Mr. Casey had no knowledge. Equally striking is the qualitative resemblance between these studies and Professor Vanoni's new observations. At the same time, it is important to observe that the number of the stripes is very different in the various cases; it is probably controlled primarily by the depth-width ratio of the flow.

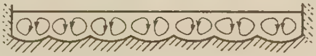

FIG. 22.—SECONDARY CURRENTS IN THE CROSS SECTION OF A BROAD CHANNEL, WITH MOVABLE BED, AS OBSERVED BY HUGH CASEY

Related to these phenomena, although somewhat less closely, are the longitudinal ridges observed in the African desert by R. A. Bagnold.[62] They differ from the stripes observed by Messrs. Casey and Vanoni in the essential fact that they are interrupted; the interruptions in the ridges observed by Mr. Bagnold are arranged in a more or less regular echelon-like pattern.

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[59] "Hydrodynamique du Cylindre Tournant," by M. Luntz and P. Schwartz, Paris, France, 1935.

[60] "Stability of a Viscous Liquid Contained Between Two Rotating Cylinders," by G. I. Taylor, *Philosophical Transactions*, Royal Soc. of London, Series A, Vol. 223, 1923, pp. 289–343.

[61] "Über Geschiebebewegung Mitteilung der Preussischen Versuchsanstaltfür Wasserbau und Schiffbau," by Hugh Casey, Heft 19, 1935, Fig. 8.

[62] "The Physics of Blown Sand and Desert Dunes," by R. A. Bagnold, William Morrow & Co., New York, N. Y., 1942.

The foregoing leads to the decisive question of the rôle played by the secondary currents in the distribution of sediments in suspension and on the bottom. Mr. Möller was the first to demonstrate the rôle played by secondary currents in the distribution of sediment, in the sorting of sediment in the cross section, and in the gradual alteration of the shape of the bed. There is much valuable material in Mr. Möller's observations and ideas; but it will have to be analyzed, in the light of modern concepts concerning turbulent flow, when knowledge of the secondary currents has become more accurate.

The effect of secondary currents in sediment flow has a twofold significance. First, if the magnitude of these secondary currents reaches a few per cent of the main longitudinal flow, these currents are capable, in their upward reaches, of carrying the finer particles of sediment directly although they may not be able to move the coarser sediment particles. It is necessary to emphasize this point, because John B. Leighly[63] denies the competence of the secondary currents to carry material even in curved rivers where the circulatory movement is very much stronger. Of course, the final decision concerning the relative importance of this effect must be postponed until more exact information is available, but it is obvious that the steady secondary currents constitute a ponderable factor in the analysis of suspension and distribution of finer particles. Second and even more important than this direct effect may be the indirect influence of the secondary currents on sediment distribution. If it is true, as Mr. Prandtl's studies indicate, that the longitudinal velocity distribution and the distribution of pulsations are inseparably interconnected with the steady secondary currents, then the distributions of the shearing stresses and of the Austausch coefficient are also linked, and thus even the phenomena at the bottom itself cannot be properly understood and analyzed without reference to the secondary currents.

For this reason, the writer believes that—in explaining the deviation of sediment distribution from the results of the simple theory—in addition to the two factors considered by Professor Vanoni, as a third, equally important factor the secondary currents (and, with them, the essentially three-dimensional character of the entire flow) should be taken into account.

This substantial deviation from a two-dimensional pattern of flow is probably the main reason also for the unsatisfactory agreement of the velocity distribution observed by Professor Vanoni with the theory advanced by Theodor von Kármán, M. Am. Soc. C. E. It would be quite justifiable to put the boundary of the profile a little below the top of the sand particles; but, even so, an appreciable discrepancy between theoretical and experimental velocity distribution would remain. It appears that there is no way to account for this discrepancy, except by studying the influence of the secondary currents and the entire essentially three-dimensional nature of turbulent flow in any open channel.

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[63] "Turbulence and the Transportation of Rock Debris by Streams," by John B. Leighly, *The Geographical Review*, Vol. 24, No. 3, 1934, p. 461.

That such an approach is necessary follows immediately from scrutiny of Fig. 3. The substantial inadequacy of the assumption of a two-dimensional flow can be made particularly clear by drawing the distribution of velocities at the mid-channel vertical and at some near-by verticals—for example, at one sixth of the channel width from the middle on both sides.

If the foregoing reasoning is correct, the modification of any factor—the isovels, the pulsations, or the secondary currents—would influence all other factors and with them the sediment distribution and the bed shape. If the tentative experiments of Potapow can be accepted, it appears that the secondary currents are those elements that are most easily subject to deliberate modification; and thus the study of secondary currents may reveal a new method for the regulation of rivers. In the paper by Mr. Welikanov,[57] there is a short abstract of Mr. Potapow's experimental research which may be translated as follows:

> "The principle of the method is to arrange fixed leading walls in the rivers. To create a clockwise helicoidal movement, it is sufficient to divert the surface stream by the aid of surface screens toward the right. The bottom stream will then be diverted to the left, and a circulation pattern is produced which is maintained for a fairly long distance downstream, where it disintegrates gradually. Instead of surface screens, bottom screens may be used, arranged in the opposite direction; or both devices may be combined. The experiments of Mr. Potapow have established the following applications of these devices: (*a*) Through production of a double circulation [probably Mr. Welikanov means a reinforcement of the circulation according to Mr. Lossievsky's type (1) currents, previously mentioned herein], the axial strip of the bed can be deepened; (*b*) in a curve the existing circulation pattern can be reversed, bringing the scour to the convex side and making the concave side, on which the 'talweg' of the natural river was situated, shallower; (*c*) the increase of sand bars can be prevented through artificial diversion of the bed load from the bar threshold; and (*d*) in case of bifurcation the sediment can be directed, according to wish, into either branch. These four examples suffice to indicate the great importance of Mr. Potapow's results in various fields of hydraulic engineering."

To summarize:

1. The presence of nonhomogeneity in the fluid (caused either by temperature influence or by the presence of a dissolved or granular material) is by no means a necessary condition for the generation of steady secondary currents. The occurrence of such currents is normal, rather than exceptional, in turbulent flow.
2. The distributions of the longitudinal velocities, of the pulsations, of the steady secondary currents, and of suspended sediment (if suspendible material is present) are mutually and inseparably interconnected, so that an artificial change of any one of these factors would inevitably tend to modify all others.
3. Observations by Mr. Potapow suggest that an artificial modification of the secondary currents is easily practicable and that therefore these might be used as a steering effect to modify, or to stabilize, sediment distribution and bed formation.

4. From items 1 and 2, it follows that the three-dimensional nature of turbulent flow in an open channel is a major factor—probably the main factor—required to explain deviations of experimental findings from the two-dimensional von Kármán theory of velocity distribution and sediment suspension.

On the basis of the summary items 1 to 4, and supporting comment, the writer ventures to suggest that the greatest progress in turbulence research in the near future is not to be expected from a more accurate and more detailed investigation of the turbulence tensor in some exceptionally simple cases (for example, the flow in a circular pipe). Still less can be expected from further study of such artificial cases as isotropic turbulence. Rather, the greatest progress will result from an improved and broadened study of the actual flow conditions in a variety of straight open channels and in a few straight closed conduits of noncircular cross section.

Even if, in the first stages, such a study were restricted to the investigation of the exact shape of the isovels and of the secondary currents (that is, even if no measurements of fluctuations are made at first), by implication it would afford a deepened insight into the three-dimensional mechanism of uniform turbulent flow, making possible a better hypothesis concerning the nature of pulsations than Mr. Prandtl could put forward without any reliable experimental facts concerning the secondary currents. Later, more exact pulsation studies and studies on the distribution of salinity, fine sediment, and other quantities subject to Austausch will make possible the checking and, if necessary, the correction of the hypothetical conclusions drawn from the first stage of the research.

As a beginning, therefore, the problem is to find a reliable picture of the three-dimensional distribution of the time averages of the velocity vectors. Since the secondary currents are comparatively small, this problem is rather difficult, challenging the ability of the best experimenter. Indeed, the contradictions presented by the various writers—all of whom were, in their time, able observers and experimenters—indicate the difficulty of obtaining more exact and more conclusive results concerning the steady secondary currents. The difficulties, of course, are mainly due to the fact that the turbulent pulsations disturb the recording of the steady components. Another difficulty is that, since the angle of the actual time-average velocity vector with the channel axis is rather small, any measuring device introduced into the liquid may vitiate the result. For these and other reasons, the writer's opinion is that the investigation in question will depend essentially upon the further development of purely optical techniques related to the one suggested by A. Fage and H. C. H. Townend.[64] In this method the extremely fine submicroscopic particles, which are present even in the purest drinking water, are observed by the aid of an ultramicroscope. If this or a similar optical method could be combined with a photographic technique to record time averages of velocity vectors, a basis for the first stage of the investigation would be presented.

Another method, simpler but probably less accurate, could perhaps be developed through improvements of Mr. Gibson's technique of working with weighted wax pellets. If very small (preferably streamlined) solid bodies can be made of exactly the same weight as the water displaced by them, and if their orbit and speed are recorded photographically, kinematographically, or by aid of Charles Camichel's[65] "chronophotographic" process, the solution of the problem may be possible.

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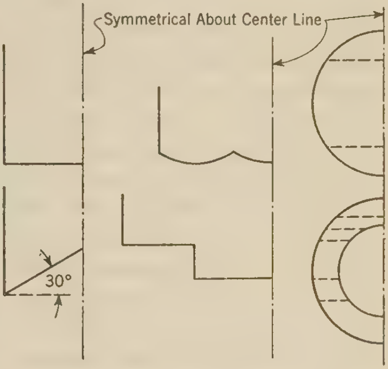
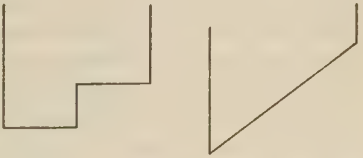
[64] "An Examination of Turbulent Flow with an Ultramicroscope," by A. Fage and H. C. H. Townend, *Proceedings*, Royal Soc. of London, Series A, Vol. 135, 1932, p. 656.

Those methods which restrict the study to the surface and the bottom (for example, the conventional use of silk threads and surface floats) have been tried and found insufficient. Only a study covering the entire cross section can lead to clarification of controversial points and to the attainment of new insight.

Of course, the greatest care is necessary in such investigations to establish a completely uniform régime of flow. The investigation of the fundamental questions of turbulent flow in an open channel presents a certain amount of additional difficulty compared to investigations in closed conduits. However, open channels have also certain practical advantages. The triangular cross section of an open channel affords excellent opportunities for the study of scale effects by the very simple device of varying the stages. On the other hand, the variation of stages in a nontriangular channel model makes possible the study of a large variety of geometrically dissimilar and, sometimes, even qualitatively very different cross sections. Thus studies of the few channel models indicated in Fig. 23 would give a very wide and highly instructive variety of flow patterns.



(*a*) SYMMETRICAL



(*b*) NONSYMMETRICAL

FIG. 23.—CROSS SECTIONS OF CHANNEL MODELS FOR PROPOSED INVESTIGATION

It is the writer's conviction that a research project along these or similar lines would clarify many problems which are essential to the understanding of turbulent flow in its relation to sediment propulsion, bed formation, and bed stabilization, and thus vital for river engineering. In addition, it may prove important for the most general and fundamental aspects of turbulence research.

The writer wishes to express his thanks to E. W. Lane, M. Am. Soc. C. E., of the Iowa Institute of Hydraulic Research, Iowa City, Iowa, for permission to use the results of an unpublished study that the writer made while he was a member of the staff of the Iowa Institute of Hydraulic Research.

VITO A. VANONI,[66] ASSOC. M. AM. SOC. C. E.—The subject of secondary circulation was discussed at considerable length by several individuals, although it was not considered of primary importance in the paper and little space was devoted to it. The writer especially appreciates the scholarly presentation of the subject by Professor Nemenyi, who covered a tremendous amount of material, some of which is not readily available to American engineers.

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[65] "Rapport des oeuvres effectuées dans les années 1932–35," by C. Camichel, Toulouse, France, 1935.

[66] Project Supervisor, U. S. Dept. of Agriculture, SCS, California Inst. of Technology, Pasadena, Calif.

Professor Nemenyi advances the opinion that the problem of sediment suspension is essentially three dimensional and that secondary circulation is a factor of prime importance and therefore one which must be studied to insure sound progress in solving the sediment problem. There is no doubt that secondary circulations do exist in sediment-laden flows, and there is also every reason to expect that circulations will affect the sediment suspension process. The important question at this stage of the development of the science is the extent to which circulation affects the phenomenon. If the circulation is of first-order importance, even approximate results probably cannot be obtained without taking the circulation into consideration. On the other hand, there is always the possibility that the effect is relatively minor and that good, workable answers can be obtained from theories that do not give any consideration to the circulation. The engineer who is interested in obtaining results that can be applied to field problems probably assumes that the circulation is of second-order importance and neglects it until experience shows that circulation must be considered. On the other hand, the problem is intriguing from a purely scientific point of view, and one may be tempted to devote considerable time to it. The writer is inclined to adopt the point of view of the engineer and to neglect circulation, at least temporarily. Channel resistance to flow, like sediment suspension, also depends on the process of turbulence exchange; yet usable, practical friction formulas, like the Manning formula, have been obtained without considering circulations. Because one flow formula can be used for all cross sections by considering the hydraulic radius as the depth parameter of the flow thus far, at least, it has been possible to neglect the shape factors and resulting differences in circulation and still obtain usable results. This does not mean that in investigations of sediment transportation careful attention should not be given to secondary circulation and to the possibility that it may be a factor of the first-order importance. The attitude is merely that this factor may be neglected until there is proof that it is not negligible. The writer's interpretation is also accompanied by a sincere hope that secondary circulation does not prove to be an important factor. Investigation of this factor would be a very tedious, time-consuming task accompanied by the same difficulties as were the investigations of turbulence which have occupied so many engineers during the past several decades.

Professors Nemenyi and Powell appear to agree that the nonuniform distribution of sediment across the flow is the result, rather than the cause, of circulation, as suggested by the writer. The capacity of a stream to suspend sediment is not uniform across its width. Therefore, the amount suspended at various points in a cross section will be different. Thus, density gradients or forces are produced that can be of appreciable magnitude and will tend to cause correspondingly appreciable cross flows or circulations. These forces will be added to those which normally cause circulation in clear flows and the result will be a modified circulation pattern. The writer's objective in mentioning the subject of secondary circulations in the paper was to call attention to the fact that such density gradients occurred in streams carrying material in suspension and that these gradients could contribute to the circulation. The writer believes that the gradients set up by uneven distribution of sediment may cause circulations even greater than those occurring in clear flows, although this is merely an opinion without any quantitative data to substantiate it.

Professor Van Driest suggests that the discrepancy between the theoretical exponent z, in the sediment distribution equation, and the empirical exponent z_1 may be due, in part, to the transfer of sediment across the flow caused by a concentration gradient in that direction. This is undoubtedly a factor. However, the writer does not believe that the effect would be at all important in measurements made near the center of the flume, since the flume width was almost five times the maximum depth of flow. Professor Kalinske and Mr. Witzig believe that the discrepancy between exponents z and z_1 can be explained by secondary circulations. The writer believes that the discrepancies can be explained in terms of other factors that are more important and that have more effect on the flow. These ideas are outlined in conclusions 1, 2, and 3. Mr. Witzig questions the method of deducing the circulations presented in Fig. 15(c). It will be noted from Fig. 15(b) that the sediment concentrations at about the quarter points are higher than at other parts of the section. Because of the higher sediment concentrations, the fluid at these sections would be heavier than the fluid at other sections and, therefore, it would tend to sink and start the circulations indicated.

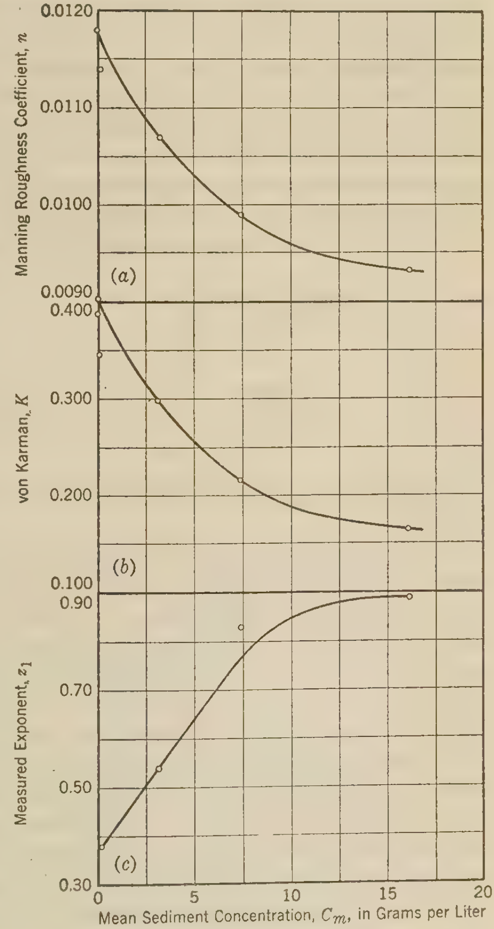

FIG. 24.—EFFECT OF CONCENTRATION OF SUSPENDED LOAD ON FLOW CHARACTERISTICS

From an analysis of the data in the paper, Professor Powell concludes that sediment in the flow tends to reduce the roughness, although no relationship is evident between roughness and sediment load. This conclusion agrees with the ones advanced by the writer. Fig. 24 shows additional data on the effect of sediment on the flow obtained by the writer from experiments made subsequent to those reported in the paper. These measurements were made in the flume with a slope of 0.0025, a bottom roughness of 0.88-mm sand, flow depth y_{max} of 0.295 ft, and varying amounts of suspended load, with a sedimentation diameter, $\bar{D}_s$, of 0.100 mm. Fig. 24(*a*), giving the Manning roughness factor, n, as a function of the average sediment concentration, C_m, shows clearly that the roughness decreases as the load increases. The experiments reported in the paper were made with sediment loads corresponding to the lower concentrations shown in the curve. The roughness factors for these rather small concentrations appear to vary considerably; and, for these curves, it will be noted that the point of the smallest concentration does not fit the general relationship very well. This phenomenon may account for the fact that a relationship between the roughness and the sediment concentration was not apparent from the original experiments.

The ratios of equivalent roughness to size of bottom sand calculated from Eq. 26*b* for the conditions shown in Fig. 24 are as follows:

Average sediment load C_m in grams per liter	Ratio of equivalent roughness to size of bottom sand
0	0.328
0.17	0.282
3.21	0.190
7.36	0.110
16.2	0.072

The relative roughness decreases very appreciably as the sediment load is increased.

Mr. Witzig suggests that fine sediment deposited between the sand roughnesses cemented to the bottom may account for the decrease in the roughness observed with suspended load. This explanation was considered during the experiments, but it was discarded after further investigation. A careful examination of the flow showed that the sediment load tended to decrease the roughness and to increase the velocity even when very minute quantities of suspended material were deposited on the bed. A more tenable explanation, as outlined in conclusion 6, attributes the decrease in the resistance to the effect of the sediment in damping out the turbulence and thus reducing the eddy viscosity of the flow and the universal constant k in the universal logarithmic velocity distribution formula. This effect is expressed as a reduction in the boundary roughness, although, in a strict sense, the boundary roughness does not change. Instead the change occurs within the body of the fluid. By reducing the intensity of the turbulence, the capability of the flow to transfer the boundary shear to the body of the flow is reduced, with the result that equilibrium between the gravity force tending to cause flow and the shear force resisting the flow is reached at a higher velocity.

Professor Van Driest's observation of the similarity between the velocity profile curves measured by Nikuradse and those reported in the paper is significant. The fact that the semilogarithmic velocity profile graphs do not follow precisely the logarithmic formula means that some judgment must be used in fitting a curve to the data. This may account for the variation in the values of von Kármán's universal constant, k, for clear flows, which has been reported in the literature. The values reported vary from 0.36 to 0.40 and it is easy to see that variations of this magnitude could occur between determinations by different workers.

Mr. Gavett has called attention to the interesting fact that a power law fits the measured velocity profiles better than the logarithmic law. The power law may be advantageous for certain problems but, in general, it is less desirable than the universal logarithmic law. Examination of the measured velocity profiles of Figs. 6 and 7 will show that the logarithmic law fits the measurements for more than 90% of the depth and that the most serious departures occur in the lower 7% or 8% of the depth. The error in calculating the position of the average velocity, due to this departure, is very small, probably much less than errors of measurements; and therefore it is not serious. The difference in velocity between relative depths of 0.368 and 0.393, which are the depths to the average velocity given by the logarithmic law and the one-seventh power law, is only about 1%.

The rather simple looking power law for velocity distribution, when substituted into formulas such as those for sediment suspension, may produce results that are much more complicated than those given by the more general logarithmic law. This is soon realized if an attempt is made to determine the distribution of suspended load, using the power law. The resulting expressions contain the maximum velocity, U_{max}, and the exponent, n, for which analytical expressions do not exist, whereas the logarithmic law gives expressions containing the friction velocity and k, both of which can be evaluated easily. Actually, the relationships between U_{max} and n are contained in the friction formulas which are expressed very conveniently by the more general laws and there is no need to develop the relationships between U_{max} and n.

Mr. Witzig's experience in measuring velocity profiles in the Niagara River cannot be taken as proof that the logarithmic distribution equation does not apply in nature. His experience is contrary to that of John C. Hoyt and Nathan C. Grover, Members, Am. Soc. C. E., who found[67] that the mean depth to the average velocity for 476 measurements in rivers was 0.62 y_{max}. It is possible that Mr. Witzig's measurements were made under conditions of nonuniform flow where the velocity profile could be distorted considerably from the profile expected for uniform flow.

The writer does not agree with Mr. Witzig's statement that the distribution law does not apply to the very fine sediments which are known as the wash load. The distribution of these sediments follows the general trend of the distribution equation, even though quantitatively the equation and the measurements do not agree. This disagreement also exists for the coarser sediments, as has been shown by the writer's experiments and by the experiments of Alvin G. Anderson,[32] Assoc. M. Am. Soc. C. E. In general, for fine sediments the actual distribution is more uniform than that indicated by the equations.

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[67] "River Discharge," by John C. Hoyt and Nathan C. Grover, John Wiley & Sons, New York, N. Y., 1912.

In the present experiments, all the suspended load could be considered as wash load because very little of it was to be found in the bed; yet the distribution followed the law very well.

The writer has been asked to comment on the probable effect of uniformly distributed fine sediment in a flow. The principal effect of such a sediment is an increase in the effective specific gravity of the flow. Since the sediment is practically uniformly distributed, its settling velocity or its tendency to settle under gravitational forces is very slight compared to the lifting forces of the turbulence. Because of this fact there will be very little damping effect of the sediment on the turbulence, and in such cases the writer would expect that effects, such as reduction in friction factor and in the von Kármán k, would also be small. Experiments by Professor Kalinske and C. H. Hsia[55] seem to confirm this idea.

It is undesirable to classify the various modes of transportation into bed load, saltation load, and suspended load, because such a classification tends to build up the impression that the modes are fundamentally different. Actually, they are fundamentally the same and are caused by the same kind of fluid forces acting on the sediment. Therefore, the writer prefers to think of the modes as parts of one phenomenon, the only difference being in the intensity of transportation. As has been shown by H. A. Einstein,[68] Assoc. M. Am. Soc. C. E., movement of material on the bed is caused by turbulence fluctuations, which move sediment particles in steps or jumps. As these fluctuations increase in intensity with changing flow conditions, the steps or orbits of the particles increase in length and the sediment is said to "saltate." In the suspension processes, particles that have been lifted off the bed are given further impulses by turbulence in the body of the fluid and they may migrate well into the cross section before they are brought back to the bed. From this point of view, the transportation process is the same throughout its range, the only difference being in the intensity of the transporting forces and their result on the length of step taken by the particles.

Mr. Witzig's statement that the momentum and sediment transfer coefficients are constant over the depth of flow is incorrect. This is evident from Eqs. 21 and Fig. 13, which show that the coefficients have minimum values at the top and bottom and approach maximum values near mid-depth. Fig. 13 also shows that the two coefficients are not equal. In the discussion of Eqs. 2, 3, and 4, the coefficients were expected to differ, although in the development of the theory they were assumed to be the same in order to derive a working theory. In the discussion of the results, the writer has tried to show why the coefficients would not be the same and how this difference would be reflected in the theoretical exponent z and in the measured exponent z_1 for the sediment distribution equation.

Mr. Witzig asks if there was any vertical segregation of the sediment corresponding to the horizontal segregation represented by the sediment clouds or bands. All vertical sediment distribution measurements followed rather closely the form of the distribution formula, Eq. 16, and no discontinuities were evidenced. Since observing these bands of horizontal segregation in the flume, the writer has looked for them in natural streams and canals and has actually seen some bands in flows where the sediment load and depth of flow were small enough to permit seeing the bottom of the flume through the flow. The writer has also seen bands of this kind in dust storms. His observations appear to be contrary to those reported by Mr. Witzig.

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[68] "Formulas for the Transportation of Bed Load," by H. A. Einstein, *Transactions*, Am. Soc. C. E., Vol. 107 (1942), pp. 561–597.

The writer agrees with Mr. Witzig's statement that an increase in stream velocity caused by the addition of a sediment load will not result in an increase in sediment-transporting capacity. In discussing this point, the writer called attention to the fact that the depth of flow (and hence the average boundary shear) would decrease as the velocity is increased; and he presented this as another argument to indicate that the sediment-transporting power would not increase with the velocity. The fact that the boundary shear does diminish indicates the probability that the entrainment force will also diminish.

The writer disagrees with Professor Kalinske's statement that no concrete evidence was presented to support the conclusions regarding (1) the effect of suspended load on k and (2) the effect of suspended load in reducing resistance to flow. If Professor Kalinske accepts the measurements as being accurate, then the conclusions drawn by the writer will follow. Admittedly, no evidence is presented to support the conclusion regarding the effect of random fluctuations on suspended sediment. This point was presented to explain the discrepancy between the measurement and the theory and it is believed that the ideas are physically sound and explain the discrepancies qualitatively. The theory outlined in the paper has certain limitations, but the fact that it gives the correct form of the distribution equation is certainly evidence of correctness. This is also ample proof of the accepted theory that sediment and momentum transfer are similar, although Professor Kalinske apparently does not accept this theory.

The correlation between the horizontal velocity fluctuations U' and the vertical fluctuations V' enters into sediment distribution because in deriving the theory it was assumed that the momentum and sediment transfer coefficients were the same. The expression for the momentum transfer coefficient includes the correlation between U' and V'. Inspection of Eq. 1a will show that, to produce a shear, the fluctuations must be of opposite signs; or, in other words, a positive value of U' must be associated with a negative value of V' and vice versa. It is easy to imagine a positive V' occurring when U' is zero. This condition will not contribute to the shear, but it still may cause transfer of sediment. On the other hand, a situation can occur in which momentum is transferred by a V'-fluctuation without transferring sediment because the instantaneous concentration gradient is zero. In this way, one can reason that the coefficients for momentum transfer and sediment transfer need not be the same. Professor Kalinske's example of the condition at the center of the pipe illustrates this fact. Here the shear is zero and hence the correlation between U' and V' is also zero; but the sediment transfer coefficient has a finite value. The idea that the transfer coefficients are not equal was expressed by Theodor von Kármán,[5] M. Am. Soc. C. E., when he remarked that the quantities β, v', and l in Eqs. 2 and 4 which go to make up the turbulent transfer coefficient need not be the same for sediment transfer and momentum transfer.

The writer agrees that, to obtain data on the sediment-transporting capacity of a stream, the bed must be covered with loose material. However, he disagrees with Professor Kalinske's statement that, unless the bed is covered with sediment, the effect of sediment on the hydraulic characteristics cannot be studied. As a matter of fact, having a fixed bed has some definite advantages over having a movable bed, especially in making measurements near the bed. For instance, with a fixed bed the extremely important slope measurement can be made especially well. In this connection, it is possible that the inherent difficulty of measuring the slope with a movable bed may account for fluctuations in k obtained by Professor Kalinske.[55] The total drop in elevation of the bottom of the 80-ft flume, used in Professor Kalinske's experiments, varied from a minimum of $\frac{1}{4}$ in. to a maximum of about 1 in. Accurate measurements of such small slopes with a movable bed would be extremely difficult.

Deposits of some sediment on the fixed bottom of the flume used by the writer may have influenced the bottom roughness, but the amount was never great enough to do more than fill a small part of the space between the fixed sediment, and it is not believed to be an important factor. The strongest evidence of the effect of the sediment on the flow characteristics comes from the measured variations in the von Kármán k in the logarithmic velocity distribution formula. Fig. 24(*b*) offers additional evidence on this point. It will be noted that, as the load increases, k decreases; it is difficult to understand how any sediment deposited on the bottom will affect the value of k. This value can be changed only by modifying the turbulence in the interior of the flow and cannot be changed by merely altering the boundary roughness. The effect is caused by a damping of the turbulence since the turbulence must suspend the sediment against settling. This tends to reduce the momentum transfer coefficients as well as the sediment transfer coefficients. The reduction of the sediment transfer coefficients is reflected in an increase of exponent z_1, as shown in Fig. 24(*c*).

In comparing the results from Professor Kalinske's experiments[55] with those reported by the writer, the important differences between the two sets of experiments must be borne in mind. The principal differences were:

1. In Professor Kalinske's experiments, the average size of the suspended load was only 0.01 mm, or one tenth the size of the smallest material in the writer's experiments; and
2. In Professor Kalinske's experiments, the bed was always covered with sediment, whereas, in the writer's experiment, the bed was fixed.

With a very fine material, the distribution is practically uniform and the settling velocity of the material is small compared to the lifting ability of the turbulence. Therefore, one would expect that the damping effect on the turbulence would be minor. From this point of view the results of Professor Kalinske's experiments are in agreement with the deductions made by the writer rather than in conflict with them as Professor Kalinske reports. The fact that the von Kármán k, measured by Professor Kalinske, varies as much as it did may be attributed to the difficulties in measuring the slope of the flume with the movable bed.

Professor Powell's questions, regarding the adaptation of Professor Kalinske's turbulence measurements to the problem of sediment transportation, are answered in part in the discussion by Professor Kalinske. Two difficulties occur in connection with using these data: (*a*) Once these coefficients are measured in a flow, it is still necessary to determine the correlation coefficient β in Eq. 4 before the measured coefficients can be used for calculations; and (*b*) no readily available analytical expression exists for the transfer coefficient and therefore this coefficient cannot yet be used for the development of a quantitative theory.

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ANALYSIS OF STATICALLY INDETERMINATE STRUCTURES USING REDUCED EQUATIONS

By Lee H. Johnson, Jr.,[1] M. Am. Soc. C. E.

With Discussion by Messrs. Josef Sorkin, I. Oesterblom, Leon Beskin, Jaroslav Polivka, and Lee H. Johnson, Jr.

Synopsis

The method presented in this paper is useful in simplifying the analysis of certain types of statically indeterminate structures and is based upon a simple arithmetical principle. The principle is explained first and then applied for purposes of illustration to the solution of two-hinged arches. Practical data to aid the designer of two-hinged arches are derived. Additional applications of the method are mentioned in the conclusion.

The method may be applied either to framed or to solid-type structures. It permits a close approximation of the reactions of certain statically indeterminate structures solely from the geometry of the structure—that is, without having to assume the sizes of members or sections. This method does not replace the methods in current use; nor does it apply to all structures. It does serve, in certain instances, to eliminate most of the preliminary computations and to make the solutions short and direct.

The writer emphasizes that the paper explains and illustrates the method, but does not intend to present an exhaustive study of its applications. It is presumed that the reader is familiar with the current methods for solving statically indeterminate structures.

Notation

The letter symbols in this paper are defined where they first appear and are assembled for reference in the Appendix.

Basic Principle

The basic principle is one of simple arithmetic which applies to a fraction of the type, $\frac{a_1 k_1 + a_2 k_2}{b_1 k_1 + b_2 k_2}$, in which corresponding terms in numerator and

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Note.—Published in November, 1944, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

[1] Dean and Prof. of Civ. Eng., School of Eng., Univ. of Mississippi, University, Miss.

denominator have the same multiplier; that is, a_1 and b_1 are both multiplied by k_1. Stated simply, the principle is that

$$\frac{a_1 k_1 + a_2 k_2}{b_1 k_1 + b_2 k_2} = \frac{a_1 + a_2}{b_1 + b_2} \quad \text{(1)}$$

provided either that $k_1 = k_2$ or that $a_1/b_1 = a_2/b_2$. This is obvious from an inspection of the difference of the two fractions, which in this paper is called the "difference term"; or, for brevity, G:

$$\frac{a_1 + a_2}{b_1 + b_2} - \frac{a_1 k_1 + a_2 k_2}{b_1 k_1 + b_2 k_2} = \frac{(a_1 b_2 - a_2 b_1)(k_2 - k_1)}{(b_1 + b_2)(b_1 k_1 + b_2 k_2)} = G \quad \text{(2)}$$

The two fractions on the left side are equal when G is equal to zero—that is, when $a_1/b_1 = a_2/b_2$; or when $k_1 = k_2$. Extending this principle for expressions with any number of terms, n:

$$\frac{a_1 + a_2 + \cdots + a_n}{b_1 + b_2 + \cdots + b_n} - \frac{a_1 k_1 + a_2 k_2 + \cdots + a_n k_n}{b_1 k_1 + b_2 k_2 + \cdots + b_n k_n}$$
$$= \frac{(a_1 b_2 - a_2 b_1)(k_2 - k_1) + \cdots + (a_{n-1} b_n - a_n b_{n-1})(k_n - k_{n-1})}{(b_1 + b_2 + \cdots b_n)(k_1 b_1 + k_2 b_2 + \cdots + k_n b_n)}$$
$$= G = 0 \quad \text{(3)}$$

provided either that $k_1 = k_2 = \cdots = k_n$, or that $a_1/b_1 = a_2/b_2 = \cdots = a_n/b_n$.

If the k-values are not all equal and the (a/b)-ratios are not all equal, the difference term will not be zero. The magnitude of G depends upon the magnitude of the variations between the k-values and between the (a/b)-values. For small variations in these values and for finite values of n, the difference term G is very small. The numerator will be small since it consists of small terms, some + and some −, formed by the products of small differences, but the denominator will be correspondingly large since it consists of the product of two sums of comparatively large terms. Expressed in terms of integrals, the principle is that—

$$\frac{\int f_1(x)\,dx}{\int f_2(x)\,dx} = \frac{\int f_1(x)\,k(x)\,dx}{\int f_2(x)\,k(x)\,dx} \quad \text{(4)}$$

—provided that $k(x) = a$, in which a is an arbitrary constant; or, that $f_2(x) = b\,f_1(x)$, in which b is an arbitrary constant. This is true for definite integrals provided that all integrals have the same limits of integration.

The function $k(x)$ may not be constant, although nearly so—for example, when

$$k(x) = a + \epsilon(x) \quad \text{(5a)}$$

in which $\epsilon(x)$ is a function whose absolute value can be made arbitrarily small. Furthermore, $f_1(x)$ may not have a constant coefficient as in the expression

$$f_2(x) = [b + \eta(x)]\,f_1(x) \quad \text{(5b)}$$

in which $\eta(x)$, like $\epsilon(x)$, is a function whose absolute value can be made arbitrarily small. Under the conditions expressed by Eqs. 5, the difference between the two fractions in Eq. 4 will not be zero, but can be expressed in the "difference term"—

$$G = \frac{\int f_1(x)\,dx \int \epsilon(x)\,\eta(x)\,f_1(x)\,dx - \int \epsilon(x)\,f_1(x)\,dx \int \eta(x)\,f_1(x)\,dx}{\int [b + \eta(x)]\,f_1(x)\,dx \int [b + \eta(x)]\,[a + \epsilon(x)]\,f_1(x)\,dx} \quad \ldots (6)$$

—simply by putting the difference of the two fractions over a common denominator.

Assume that all the integrals in the difference term are definite integrals with the same limits of integration. Also, let $f_1(x)$ be a continuous function in the region of integration, $c \leqq x \leqq d$, and let $\left| \int_c^d f_1(x)\,dx \right| \geqq B$, in which B is an arbitrary constant. These conditions exist in many engineering problems involving such equations. The difference term will approach the quantity,

$$G = \frac{0}{b^2\,a \int_c^d f_1(x)\,dx \int_c^d f_1(x)\,dx} \quad \ldots\ldots\ldots\ldots\ldots\ldots (7)$$

as $|\epsilon(x)| \rightarrow 0$, and as $|\eta(x)| \rightarrow 0$. If $|\epsilon(x)|$ and $|\eta(x)|$ are not equal to zero but are arbitrarily small, the numerator of the difference term will also be arbitrarily small. The denominator, however, will remain comparatively large since $\left| b^2\,a \int_c^d f_1(x)\,dx \int_c^d f_1(x)\,dx \right| \geqq b^2\,a\,B^2$, regardless of the arbitrarily small terms involving $\epsilon(x)$ and $\eta(x)$. Hence the difference term will be small for small values of $\epsilon(x)$ and $\eta(x)$; that is, for small variations in $k(x)$ and in the ratio of $f_2(x)$ to $f_1(x)$. A formal mathematical proof of the foregoing statements is not offered in this paper, which is intended primarily for engineers. The conclusions can be verified by well-known mathematical procedures.

CLASSIC EQUATIONS APPLIED TO THE ANALYSIS OF TWO-HINGED ARCHES

The general equation for the horizontal reaction H of a solid, two-hinged arch neglecting rib shortening and temperature effects is

$$H = \frac{\int M\,y\,\frac{ds}{E\,I}}{\int y^2\,\frac{ds}{E\,I}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8a)$$

in which M = bending moment at any section due to given loads and vertical reactions only; y = ordinate of the arch at any section; ds = differential length of an element measured along the center line of the arch; E = modulus of elasticity; and I = rectangular moment of inertia of the arch section. In actual design, Eq. 8a is usually expressed as a summation, and the arch rib is divided into a finite number of segments of length Δs:

$$H = \frac{\sum M\,y\,\frac{\Delta s}{E\,I}}{\sum y^2\,\frac{\Delta s}{E\,I}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8b)$$

For two-hinged arch trusses:

$$H = \frac{\sum \frac{\sigma\, u\, L}{A\, E}}{\sum \frac{u^2\, L}{A\, E}} \qquad \text{(8c)}$$

in which σ is the total stress on the section due to given loads and vertical reactions only; A is its area; L is the length of the member; and u is the total stress in the section due to a horizontal force of unity applied at the reaction.

It is presumed that Eqs. 8 are familiar to the reader. Their derivation can be found in texts on the theory of statically indeterminate structures.

The horizontal reaction is dependent upon the sizes of the sections or members of the arch. However, before the arch can be designed (that is, before the stresses that determine the size of sections or members can be computed) it is necessary to compute the horizontal reactions under various loads. Hence the sizes of sections or members must be assumed at the start and consequently the design is a process of successive approximations.

METHOD OF REDUCED EQUATIONS APPLIED TO TWO-HINGED ARCHES

The purpose of the "method of reduced equations" is to eliminate as many steps as possible in the process of successive approximations and to provide a more direct solution. The method is so named because the application of the basic principle to the solution of two-hinged arches, and other structures, eliminates several quantities from the classic equations, thereby reducing them. In particular, the sizes of sections or members are eliminated so that the solution by this method depends only upon the center-line geometry of the structures.

Definitions.—Four new terms are introduced in this paper, as follows—(*a*) "difference term," (*b*) "reduced equation," (*c*) "basic influence line," and (*d*) "correspondence of ordinates."

(*a*) The "difference term" is the analytical expression of the difference between two fractions of the types shown in Eqs. 2 and 4.

(*b*) In the classical equation, s is not necessarily the most convenient independent variable. Choosing x as the independent variable, Eq. 8*a* can be written in terms of x:

$$\frac{\int \frac{M\, y\, ds}{E\, I}}{\int \frac{y^2\, ds}{E\, I}} = \frac{\int \frac{M\, y}{E\, I} \frac{ds}{dx}\, dx}{\int \frac{y^2}{E\, I} \frac{ds}{dx}\, dx} \qquad \text{(9)}$$

In general, $ds/dx = \sqrt{1 + (dy/dx)^2}$, which depends upon the form of the arch axis. If the cross section of the arch is so chosen as to make $E\, I$ proportional to $\sqrt{1 + (dy/dx)^2}$—then the quantity $\left(\frac{1}{E\, I} \frac{ds}{dx}\right)$ will be constant and can be factored from Eq. 9.

A "reduced equation" is the classic equation (Eq. 8*a*) with the quantity $\left(\frac{1}{E\, I} \frac{ds}{dx}\right)$, or $\frac{L}{A\, E}$, assumed constant and factored from the equation, thus:

For the solid type of arch,

$$H = \frac{\int M\, y\, dx}{\int y^2\, dx} \qquad \text{(10a)}$$

or, if Δx is constant,

$$H = \frac{\sum M\, y}{\sum y^2} \qquad \text{(10b)}$$

and, for the arch truss,

$$H = \frac{\sum \sigma\, u}{\sum u^2} \qquad \text{(11)}$$

(*c*) A "basic influence line" is the one determined from the reduced equation of an arch.

(*d*) Two curves have "correspondence of ordinates" if the ratio of their ordinates at corresponding points on each curve remains approximately constant. For example, in Fig. 1(*a*), curves A and B have perfect correspondence of ordinates for their entire length; in Fig. 1(*b*), curves C and D have correspondence of ordinates, not perfect however, for their entire length; in Fig. 1(*c*), curves E and F have correspondence of ordinates only between 0 and point a; and, in Fig. 1(*d*), curves G and H have no correspondence of ordinates.

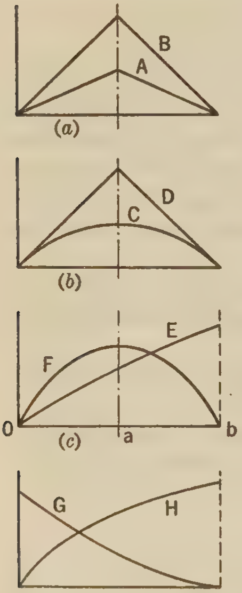


FIG. 1

The words "influence lines" and "influence values" throughout the discussion of two-hinged arches are understood to refer always to the horizontal reactions.

Procedure.—The use of the "reduced equations" (Eqs. 10 or 11) in the design of two-hinged arches is worth while when these equations give values sufficiently close for practical purposes to the final correct values determined from the classic equations (Eqs. 8).

The following procedure will be followed in applying the method of reduced equations to the solution of solid-type, two-hinged arches: First, the effect of variations in cross section upon the basic influence line will be studied for parabolic, circular, and semi-elliptic arches successively; and second, the effect of variation in curve or geometry of the arch upon the influence lines will be studied.

EFFECT OF VARIATION IN CROSS SECTION

General.—The difference between the two fractions in Eq. 2 is zero when $\frac{a_1}{b_1} = \frac{a_2}{b_2}$, regardless of the values of k_1 and k_2. Similarly (compare Eq. 9), the difference between $\dfrac{\sum_1^n M\,y}{\sum_1^n y^2} - \dfrac{\sum_1^n M\,y\,\dfrac{\Delta s}{E\,I}}{\sum_1^n y^2\,\dfrac{\Delta s}{E\,I}}$ is zero, when $\dfrac{M_1\,y_1}{y^2_1} = \dfrac{M_2\,y_2}{y^2_2} = \cdots$ $= \dfrac{M_n\,y_n}{y^2_n}$, regardless of the $\left(\dfrac{\Delta s}{E\,I}\right)$-values.

If the $\left(\dfrac{M\,y}{y^2}\right)$-values are not all equal, but nearly so, then the difference between the two fractions will be very small, regardless of the $\left(\dfrac{\Delta s}{E\,I}\right)$-values.

In terms of the integral equations, the difference $\dfrac{\int M\,y\,dx}{\int y^2\,dx} - \dfrac{\int M\,y\,\dfrac{ds}{E\,I}}{\int y^2\,\dfrac{ds}{E\,I}}$ will be zero or very small, if the ratio $\dfrac{M\,y}{y^2}$ is constant, or nearly so, for all values of x.

Parabolic Arch.—The equation of a parabolic arch with the origin at the left end, with span, L, and rise, h, is

$$y = \frac{4\,h}{L^2}\,(x\,L - x^2)\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(12a)$$

and letting $l = L/2$,

$$y = \frac{h}{l^2}\,(2\,l\,x - x^2)\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(12b)$$

For a parabolic arch with a unit load at the center, and $\dfrac{L}{h} = 4$, characteristic curves for M-values, y-values, $(M\,y)$-values, and y^2-values are shown in Fig. 2. For different values of x, the moment M is as follows: When $0 \leqq x \leqq l$,

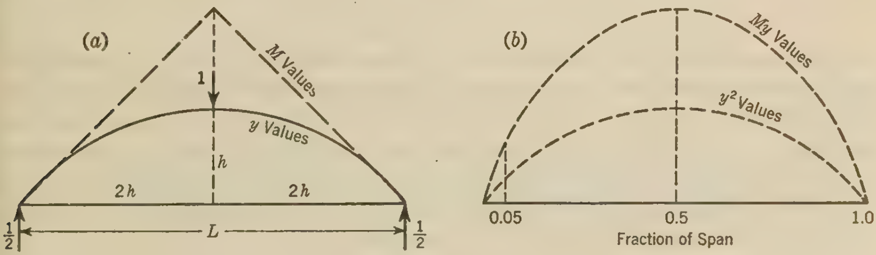


FIG. 2

$$M = 0.5\,x\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(13a)$$

and, when $l \leqq x \leqq L$,

$$M = l - 0.5\,x\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(13b)$$

Values of $H = \dfrac{M\,y}{y^2}$ (Eq. 10b) are given in Table 1 for l/h = 2, 4, and 8.

From Fig. 2, it is seen that there is "correspondence of ordinates" between the curves representing ($M\,y$)-values and y^2-values and that the difference term for a parabolic arch with center load will be small, regardless of the variations in cross sections or sizes of members.

TABLE 1.—VALUES OF $\frac{M\,y}{y^2}$ (Eq. 10*b*)

Fraction of span[a]	Values of L/h		
	2	4	8
0.05	0.26	0.52	1.04
0.50	0.50	1.00	2.00

[a] See Fig. 2(*a*).

For intermediate positions of the unit load, there is not correspondence of ordinates for the entire span length. In Fig. 3, the curves of M-values and y-values do not correspond between points B and C. However, the error introduced by the small lack of correspondence between points B and C is of opposite sign from the error introduced by imperfection of correspondence between points A and B and between points C and D, so that at one position of the intermediate load, there is no error. It must be kept in mind that the errors mentioned are those between the basic influence line determined from Eq. 10*a* and the actual influence line determined from Eq. 8*a*.

The correctness of the conclusion that variations in cross section will have but little effect on the influence values because of the correspondence of ordinates is shown by the following computations of the basic influence line and influence lines for several types of cross section. These computations also reveal the fact that the basic influence lines for the parabolic arch are directly proportional to the span: rise ratio. The basic influence line for the parabola is computed from Eq. 10*a* between the limits of 0 and L. By using a double unit load, placed symmetrically, it is necessary to integrate only over one half of the span, since both the arch and the moment diagram are symmetrical about the center line. Thus (see Fig. 4):

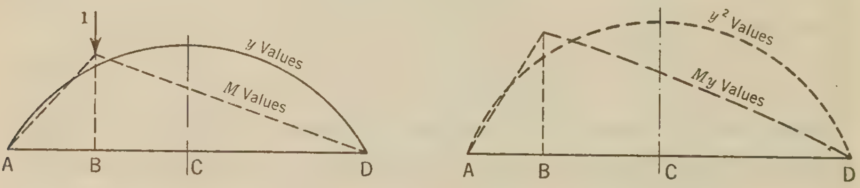


FIG. 3

$$2\,H = \frac{\int_0^a x\,y\,dx + \int_a^l a\,y\,dx}{\int_0^l y^2\,dx} \quad \text{(14a)}$$

or

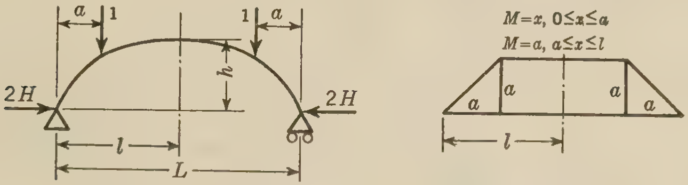
$$H = \frac{\int_0^a x\,y\,dx + \int_a^l a\,y\,dx}{\int_0^L y^2\,dx} \quad \text{(14b)}$$

In Eq. 14*b*, substituting the value of y from Eq. 12*b*:

$$H = \frac{\dfrac{h}{l^2}\displaystyle\int_0^a (2\,l\,x^2 - x^3)\,dx + \dfrac{a\,h}{l^2}\displaystyle\int_a^l (2\,l\,x - x^2)\,dx}{\dfrac{h^2}{l^4}\displaystyle\int_0^L (2\,l\,x - x^2)^2\,dx} \quad \ldots\ldots(15a)$$

which becomes

$$H = \frac{\dfrac{h}{l^2}\left[\dfrac{2\,l\,x^3}{3} - \dfrac{x^4}{4}\right]_0^a + \dfrac{a\,h}{l^2}\left[l\,x^2 - \dfrac{x^3}{3}\right]_a^l}{\dfrac{h^2}{l^4}\left[\dfrac{4\,l^2\,x^3}{3} - \dfrac{4\,l\,x^4}{4} + \dfrac{x^5}{5}\right]_0^L} = \frac{\dfrac{l^2}{h}\left[\dfrac{2\,l^3\,a}{3} - \dfrac{l\,a^3}{3} + \dfrac{a^4}{12}\right]}{\dfrac{4\,l^2\,L^3}{3} - l\,L^4 + \dfrac{L^5}{5}} \quad \ldots(15b)$$



(*a*) LOAD DIAGRAM (*b*) MOMENT DIAGRAM

FIG. 4

Let j = a fraction of the half span, and $a = j\,l = 0.5\,j\,L$; then Eq. 15*b* becomes

$$H = \frac{\dfrac{1}{h}\left(\dfrac{L}{2}\right)^6\left[\dfrac{2\,j}{3} - \dfrac{j^3}{3} + \dfrac{j^4}{12}\right]}{\dfrac{L^5}{3} - \dfrac{L^5}{2} + \dfrac{L^5}{5}} = \frac{5\,L}{128\,h}(8\,j - 4\,j^3 + j^4) \quad \ldots(16a)$$

For a unit load at the center, $j = 1$ and

$$H = \frac{25\,L}{128\,h} = 0.1953\,\frac{L}{h} \quad \ldots\ldots\ldots\ldots(16b)$$

It is observed that the basic influence line is a direct function of the (L/h)-ratio. Consequently, for basic influence lines, one curve can be plotted for all parabolic arches, as a function of that ratio. Influence lines for non-constant $\left(\dfrac{1}{E\,I}\dfrac{ds}{dx}\right)$-values are computed from Eq. 8*a* (between the limits 0 and l). In this paper, influence lines are computed first for six types of arch rib which were selected primarily because of their mathematical simplicity. These are shown in Fig. 5(*b*). Also shown in Fig. 5(*b*) is the diagram for an arch with a constant quantity $\left(\dfrac{1}{E\,I}\dfrac{ds}{dx}\right)$. For each type the quantity $\left(\dfrac{1}{E\,I}\dfrac{ds}{dx}\right)$ is expressed as a function of x in Col. 2, Table 2. Types 1, 2, 4, and 5 represent functions which are not symmetrical about the center line of the arch, and can

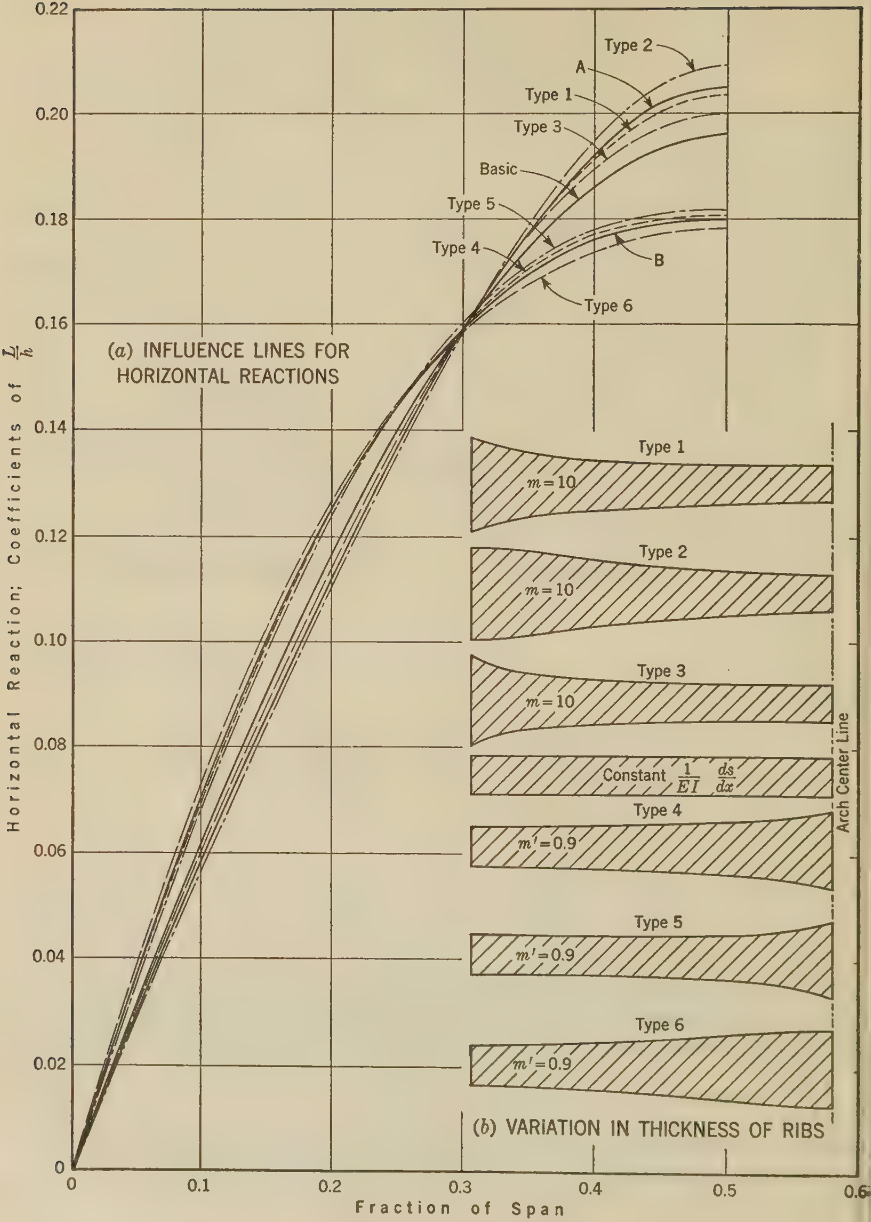


FIG. 5.—PARABOLIC ARCH RIBS

be applied only in integrations over the left half of an arch. A double unit load is used as in Eq. 14*a*.

TABLE 2.—COMPUTATION OF INFLUENCE LINES, PARABOLIC ARCH

Type (see Fig. 5) (1)	$\frac{1}{EI}\frac{ds}{dx}$ (2)	Difference term (3)
1	$\left[1 + m\frac{x}{l}\right]$	$\frac{m}{128\,(16 + 11\,m)}\,\frac{L}{h}\,(55\,g_1 - 16\,g_2)$
2	$\left[1 + m\left(\frac{x}{l}\right)^2\right]$	$\frac{m}{128\,(56 + 29\,m)}\,\frac{L}{h}\,(145\,g_1 - 112\,g_3)$
3	$\left[1 + m\frac{(2\,l\,x - x^2)}{l^2}\right]$	$\frac{m}{64\,(7 + 6\,m)}\,\frac{L}{h}\,(15\,g_1 - 7\,g_4)$
4	$\left[1 - m'\frac{x}{l}\right]$	$\frac{m'}{128\,(16 - 11\,m')}\,\frac{L}{h}\,(16\,g_2 - 55\,g_1)$
5	$\left[1 - m'\left(\frac{x}{l}\right)^2\right]$	$\frac{m'}{128\,(56 - 29\,m')}\,\frac{L}{h}\,(112\,g_3 - 145\,g_1)$
6	$\left[1 - m'\frac{(2\,l\,x - x^2)}{l^2}\right]$	$\frac{m'}{64\,(7 - 6\,m')}\,\frac{L}{h}\,(7\,g_4 - 15\,g_1)$

The quantities m and m' in Col. 2 are arbitrary constants such that: $0 \leqq m < \infty$ and $0 \leqq m' < 1$. The $\left(\frac{1}{E\,I}\frac{ds}{dx}\right)$-values are constant when m and m' equal 0. As m and m' increase, the difference between the $\left(\frac{1}{E\,I}\frac{ds}{dx}\right)$-values at the haunch and at the crown increases. The difference term for each type is given in Col. 3, Table 2.

Typical Derivation of Difference Term, Parabolic Arch.—A computation of the difference term for type 1 will serve to demonstrate the general procedure For this type (see Col. 2, Table 2),

$$\frac{1}{E\,I}\frac{ds}{dx} = \left(1 + \frac{m\,x}{l}\right) \qquad \text{.....................(17)}$$

The difference term is:

$$\frac{\int_0^a x\,y\,dx + \int_a^l a\,y\,dx}{\int_0^L y^2\,dx}$$

$$-\frac{\int_0^a x\,y\,dx + \int_a^l a\,y\,dx + \int_0^a \frac{m\,x}{l}\,x\,y\,dx + \int_a^l \frac{m\,x}{l}\,a\,y\,dx}{\int_0^L y^2\,dx + 2\int_0^l \frac{m\,x}{l}\,y^2\,dx} = G\,..(18)$$

From the foregoing derivation (Eqs. 14*a* and 15*b*):

$$\int_0^a x\,y\,dx + \int_a^l a\,y\,dx = \frac{h\,l^2}{12}\,(8\,j - 4\,j^3 + j^4) \ldots\ldots\ldots (19a)$$

and, letting $g_1 = 8\,j - 4\,j^3 + j^4$,

$$\int_0^a x\,y\,dx + \int_0^l a\,y\,dx = \frac{h\,l^2}{12}\,g_1 = \frac{h\,L^2}{48}\,g_1 \ldots\ldots\ldots\ldots (19b)$$

Also,

$$\int_0^L y^2\,dx = \frac{h^2}{l^4} \times \frac{L^5}{30} = \frac{16}{30}\,h^2\,L \ldots\ldots\ldots\ldots\ldots\ldots (20)$$

The two expressions involving m in the numerator of Eq. 18 are now evaluated:

$$\int_0^a \frac{m\,x}{l}\,x\,y\,dx + \int_a^l \frac{m\,x}{l}\,a\,y\,dx = \int_0^a \frac{m\,h}{l^3}\,(2\,l\,x^3 - x^4)\,dx$$

$$+ \int_a^l \frac{m\,h\,a}{l^3}\,(2\,l\,x^2 - x^3)\,dx = \frac{m\,h}{l^3}\left[\frac{2\,l\,x^4}{4} - \frac{x^5}{5}\right]_0^a \frac{m\,h\,a}{l^3}\left[\frac{2\,l\,x^3}{3} - \frac{x^4}{4}\right]_a^l$$

$$= \frac{m\,h}{l^3}\left[\frac{l\,a^4}{2} - \frac{a^5}{5}\right] + \frac{m\,h}{l^3}\left[\frac{2\,l^4\,a}{3} - \frac{l^4\,a}{4} - \frac{2\,l\,a^4}{3} + \frac{a^5}{4}\right] \ldots\ldots (21)$$

Substituting $a = j\,l$, in Eq. 21:

$$\int_0^a \frac{m\,x}{l}\,x\,y\,dx + \int_a^l \frac{m\,x}{l}\,a\,y\,dx = m\,h\,l^2\left[\frac{j^4}{2} - \frac{j^5}{5} + \frac{2\,j}{3} - \frac{j}{4} - \frac{2\,j^4}{3} + \frac{j^5}{4}\right]$$

$$= \frac{m\,h\,l^2}{60}\,(25\,j - 10\,j^4 + 3\,j^5) = \frac{m\,h\,L^2}{240}\,g_2 \ldots\ldots\ldots\ldots (22)$$

in which $g_2 = 25\,j - 10\,j^4 + 3\,j^5$. The term involving m in the denominator of Eq. 18 is next evaluated; thus:

$$2\int_0^l \frac{m\,x}{l}\,y^2\,dx = \frac{2\,m}{l}\int_0^l x\,\frac{h^2}{l^4}\,(2\,l\,x - x^2)^2\,dx$$

$$= \frac{2\,m\,h^2}{l^5}\int_0^l (4\,l^2\,x^3 - 4\,l\,x^4 + x^5)\,dx = \frac{11}{30}\,m\,h^2\,L \ldots\ldots\ldots (23)$$

Evaluating the difference term (see Eq. 6) gives

$$G = \frac{\dfrac{h\,L^2\,g_1}{48}\,\dfrac{11\,m\,h^2\,L}{30} - \dfrac{m\,h\,L^2\,g_2}{240}\,\dfrac{16\,h^2\,L}{30}}{\dfrac{16\,h^2\,L}{30}\left(\dfrac{16\,h^2\,L}{30} + \dfrac{11\,m\,h^2\,L}{30}\right)}$$

$$= \frac{m}{128\,(16 + 11\,m)}\,\frac{L}{h}\,(55\,g_1 - 16\,g_2) \ldots\ldots\ldots\ldots\ldots (24)$$

The substitution constants g in Col. 3, Table 2, are as follows:

$$g_1 = (8\,j - 4\,j^3 + j^4) \ldots\ldots\ldots\ldots\ldots\ldots (25a)$$

$$g_2 = (25\,j - 10\,j^4 + 3\,j^5) \ldots\ldots\ldots\ldots\ldots\ldots (25b)$$

$$g_3 = (9\,j - 3\,j^5 + j^6) \ldots\ldots\ldots\ldots\ldots\ldots (25c)$$

$$g_4 = (16\,j - 10\,j^4 + 6\,j^5 - j^6) \ldots\ldots\ldots\ldots\ldots\ldots (25d)$$

An examination of the difference terms shows that the errors are also a direct function of the (L/h)-ratios. Consequently, the influence lines for any parabolic arch of any of these types of $\left(\frac{1}{E\,I}\frac{ds}{dx}\right)$-values can be obtained from one curve of that type, plotted in terms of L/h. The influence lines for these six types of arch sections are plotted with the basic influence line in Fig. 5(*a*). Values of $m = 10$ and $m' = 0.9$ were used.

The influence values are not greatly affected by variations in the ratio of haunch thickness to crown thickness. Different values of this ratio are obtained by using different values of m or m' in the expressions for the cross sections of types 1 to 6 (Col. 2, Table 2). Since the influence values are proportional to the (L/h)-ratio, illustrative computations are shown only for a specific arch of $L/h = 4$, the computations for any other parabolic arch being directly proportional to these. The maximum influence ordinate at the crown for the basic influence line of a parabolic arch of $L/h = 4$ is 0.781. Tables 3 and 4 show the amounts by which the maximum influence ordinates for the six types of cross sections differ from this value. It is observed that increasing the haunch thickness from twice to four times the crown thickness produces a maximum change in ordinate, for types 1, 2, and 3, of $0.061 - 0.046 = 0.015$. Increasing the crown thickness from one and a half to three times the haunch thickness produces a maximum change, for types 4, 5, and 6, of $0.106 - 0.045 = 0.061$. These effects are shown graphically in Fig. 7.

TABLE 3.—Influence Ordinates for Parabolic Arch Types 1 to 3—Cross Sections Decreasing from the End to the Center

$\left(\text{Using } m\text{-Constants and } \frac{L}{h} = 4\right)$ with a Unit Load at the Crown

Constant m	Ratio of haunch thickness to crown thickness	Type 1		Type 2		Type 3	
		Difference term	Ordinate	Difference term	Ordinate	Difference term	Ordinate
5	2.0 : 1.0	+0.029	0.810	+0.046	0.827	+0.017	0.798
10	2.5 : 1.0	+0.032	0.813	+0.053	0.834	+0.019	0.800
20	3.1 : 1.0	+0.034	0.815	+0.058	0.839	+0.020	0.801
40	3.9 : 1.0	+0.036	0.817	+0.061	0.842	+0.020	0.801

Influence values for a unit load at the center only were computed for several types of cross-section distributions, with thicknesses selected arbitrarily and shown in Fig. 6 as types 7 to 11. These types were selected as distributions not represented in types 1 to 6. The column analogy method was used for these computations, since the mathematical expressions of $\frac{1}{EI}\frac{ds}{dx}$ for these types are complicated. Results are shown in Fig. 7.

TABLE 4.—INFLUENCE ORDINATES FOR PARABOLIC ARCH TYPES 4 TO 6—CROSS SECTIONS INCREASING FROM THE END TO THE CENTER $\left(\text{USING } m'\text{-CONSTANTS AND } \frac{L}{h} = 4\right)$ WITH A UNIT LOAD AT THE CROWN

Constant m'	Ratio of haunch thickness to crown thickness	Type 4		Type 5		Type 6	
		Difference term	Ordinate	Difference term	Ordinate	Difference term	Ordinate
0.8	1.0 : 1.5	−0.045	0.736	−0.045	0.736	−0.045	0.736
0.9	1.0 : 1.9	−0.060	0.721	−0.055	0.726	−0.070	0.711
0.95	1.0 : 2.4	−0.070	0.711	−0.062	0.719	−0.091	0.690
0.975	1.0 : 3.1	−0.075	0.706	−0.065	0.716	−0.106	0.675

Inspection of the data shows that the influence lines fall in three groups—the basic influence line, a group of influence lines for diminishing cross section from end to center of arch, and a group for increasing cross section from end to center of arch.

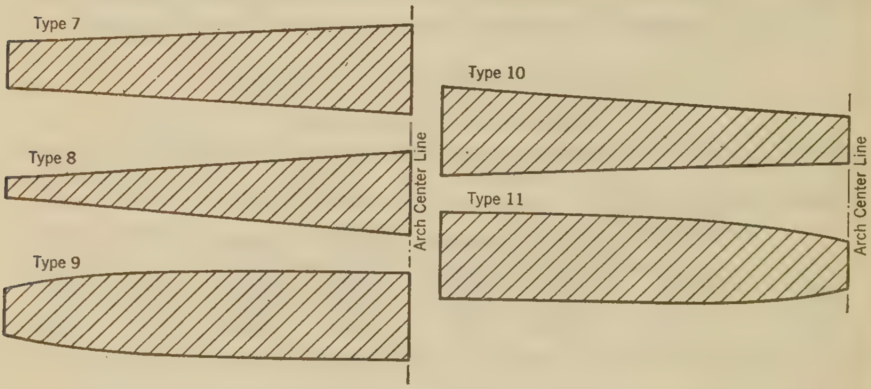


FIG. 6.—VARIATION IN THICKNESS OF PARABOLIC ARCH RIBS

Influence lines for the parabola can be generalized by plotting an average curve A for the group of influence lines above the basic influence line and an average curve B for the group below the basic influence line, as are shown in Fig. 5(*a*). The maximum percentage error encountered in using the average curve in each group is approximately 2% for the top group (cross section diminishing toward center) and approximately 6% for the bottom group (cross section increasing toward center), for a maximum ratio between thicknesses at crown and haunch of about 4.5:1, which is very high. Thus, if the arch is to have haunches, influence line A, Fig. 5(*a*), can be used to make a direct solution for the reactions and required cross-section dimensions. If the arch is to have practically constant cross section, the basic influence line can be used. If the arch is to taper from the center to the ends, influence line B, Fig. 5(*a*), can be used.

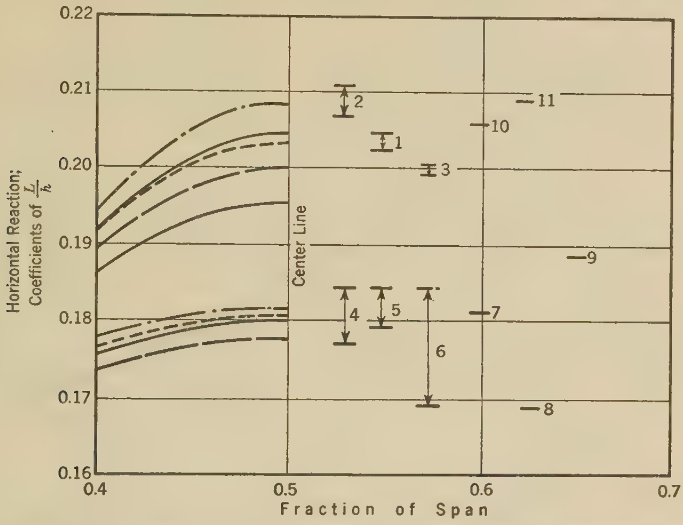


FIG. 7.—RANGES OF INFLUENCE VALUES FOR TYPES 1 TO 6 AND MAXIMUM VALUES FOR TYPES 7 TO 11

Circular Arch.—The equation of a circular arch of span L, radius r, and rise h, with the origin at the left end (Fig. 8), is

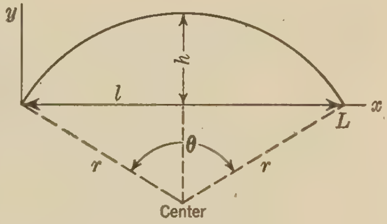


FIG. 8

$$y = h - r + \sqrt{r^2 - (x - l)^2} \qquad (26)$$

The basic influence value for the unit load at the crown is:

$$H = \frac{\int_0^l x\,y\,dx}{\int_0^L y^2\,dx} = \frac{3\,l\,r^2\,\theta + 6\,h\,r\,(h - r) - 2\,h^3}{12\,(h - r)\,r^2\,\theta + 12\,l\,r^2 - 4\,l^3} \qquad (27)$$

The influence values (see Table 5, lines 1 and 2) are not a linear function of L/h, as in the case of the parabola.

Basic influence lines for (L/h)-ratios of 8, 4, and 2 were computed by column analogy. Influence lines for variations in $\frac{1}{E\,I}\frac{ds}{dx}$ of types 1 to 6 were

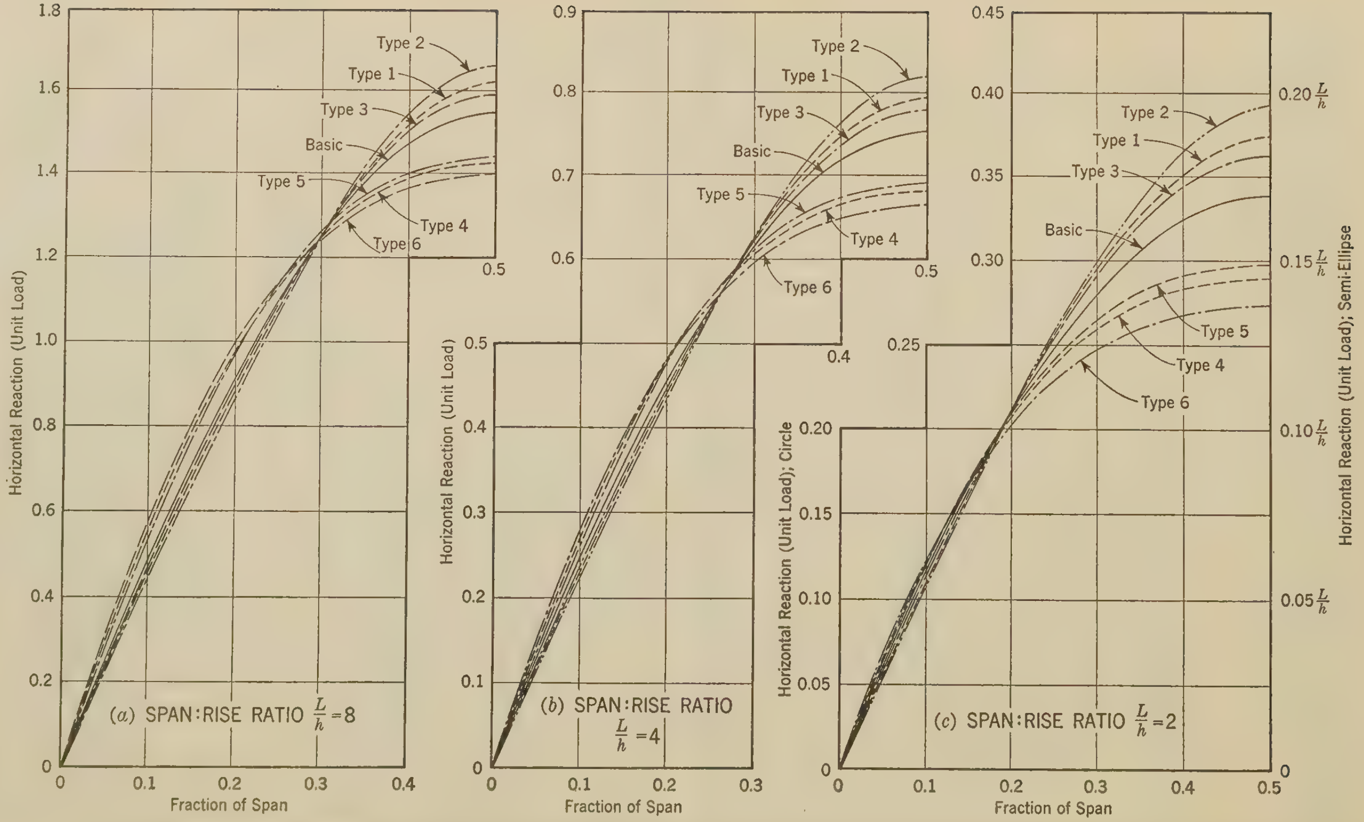


Fig. 9.—Influence Lines for Horizontal Reaction, Circular Arch Ribs, When a Unit Load Is at the Center of the Span

also computed by this method. The results are shown in Fig. 9. The variations from the basic influence line increase as the (L/h)-ratio decreases—that is, as the curve of the arch departs more and more from a straight line.

TABLE 5.—BASIC INFLUENCE VALUES AT THE CENTER OF CIRCULAR ARCHES, WITH A UNIT LOAD AT THE CROWN

No.	Description	RATIO OF SPAN LENGTH L TO RISE h								
		14 : 1	12 : 1	10 : 1	8 : 1	6 : 1	4 : 1	3 : 1	2.5 : 1	2 : 1[a]
	Basic Values:									
1	Actual	2.7240	2.3337	1.9404	1.5466	1.1512	0.752	0.548	0.4439	0.339
2	In terms of $L : h$	0.1946	0.1945	0.1940	0.1933	0.1919	0.1880	0.1828	0.1775	0.1695
3	Ratio[b]	0.9964	0.9959	0.9933	0.9898	0.9826	0.9626	0.9360	0.9089	0.8679

[a] Semicircle or semi-ellipse.

[b] $\frac{\text{Maximum coefficient for a circle}}{\text{Maximum coefficient for a parabola (that is, 0.1953)}}$.

Semi-Ellipse.—The equation of the semi-ellipse with the origin at the left end is

$$y = \frac{h}{l}\sqrt{2\,l\,x - x^2} \qquad (28)$$

For a unit load at the crown, the basic influence ordinate is

$$H = \frac{\int_0^l x\,y\,dx}{\int_0^L y^2\,dx} = \frac{3\,\pi - 4}{32}\,\frac{L}{h} = 0.1695\,\frac{L}{h} \qquad (29)$$

In the case of the semi-ellipse, as for the parabola, the influence ordinates are found to be a linear function of the (L/h)-ratio. The basic influence values are also proportional to those for a semicircle $(L/h = 2)$. The basic influence line for the semi-ellipse and influence lines for types 1 to 6 were computed by column analogy and are similar to those for a circle of $L/h = 2$ (see Fig. 9(*c*)).

EFFECT OF VARIATIONS IN CURVE

The comparative curves of the parabolic, circular, and semi-elliptic arches are shown in Fig. 10. An examination of these curves, together with the respective influence lines in Fig. 9 and the data in Table 5, shows that the more the curve departs from a straight line—(1) the lower are the influence values, and (2) the greater is the effect of variation in cross section upon these values.

GENERALIZED CURVES FOR TWO-HINGED ARCHES

Fig. 11 is a comparative diagram of the basic influence lines for the parabolic, circular, and semi-elliptical arch, plotted as fractions of the maximum values. It shows that, for practical purposes, all the basic influence lines have the same relative shape. This is also true for the influence lines for different $\left(\frac{1}{E\,I}\frac{ds}{dx}\right)$-variations. Consequently, using the parabola as a standard, an influence line for any circular or semi-elliptic arch can be determined by multiplying the influence values for the corresponding parabola (basic, A, or B in Fig. 5(*a*)) by the ratio of the respective (L/h)-coefficients. Table 5, line 3, gives the ratios of coefficients for basic influence values.

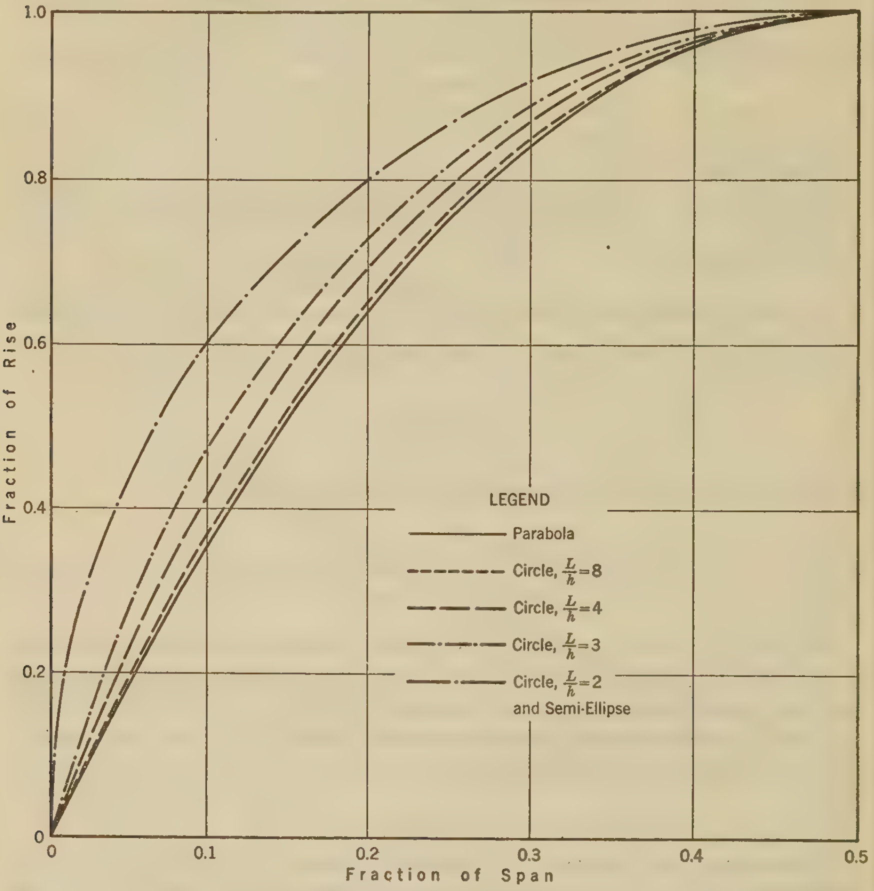


FIG. 10.—COMPARISON OF CURVE SHAPES

Plotting these points gives a reference curve for general use in the solid line in Fig. 12. This is used in conjunction with the basic values for the parabola in Fig. 5(*a*). Thus, for a circular arch of $L/h = 3.5$, the basic influence ordinates are obtained by multiplying the basic influence ordinates of the parabola (Fig. 5(*a*)) by $3.5 \times 0.950 = 3.325$, the factor 0.950 being taken from the basic reference curve in Fig. 12. Two similar average reference curves for the average curves A and B are plotted in Fig. 12 as dotted lines. For other arch forms, such as a multi-centered arch, segment of an ellipse, and others, the comparative curve form can be plotted in Fig. 10, and influence values determined from that curve which most nearly approximates it.

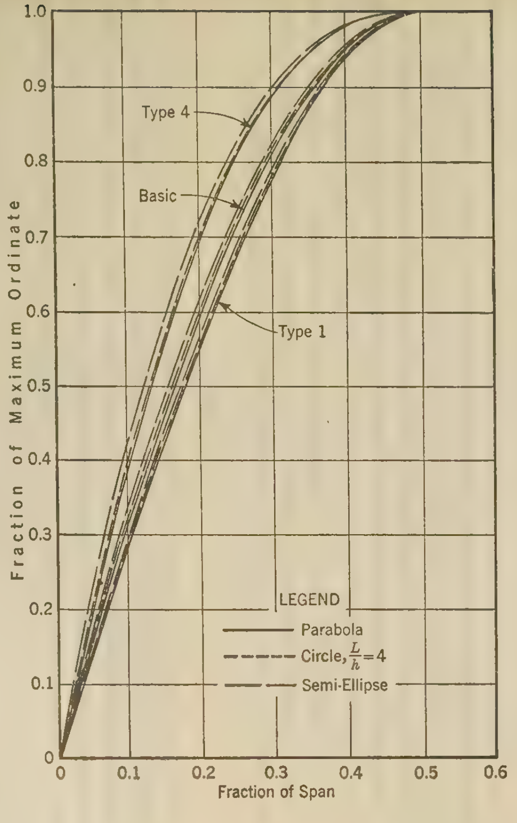


FIG. 11.—COMPARISON OF INFLUENCE LINE SHAPES

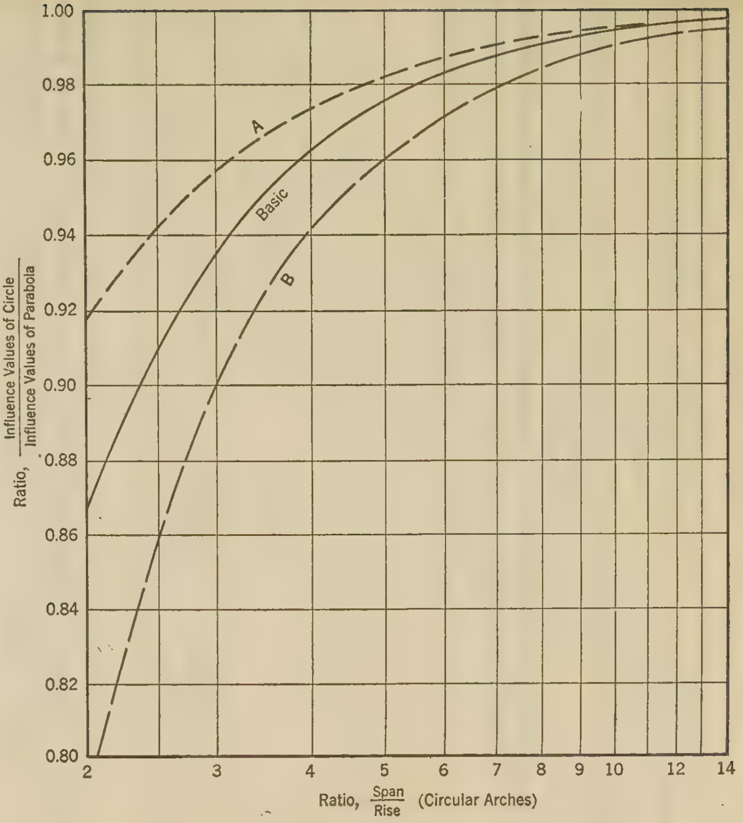


FIG. 12.—REFERENCE CURVES FOR TWO-HINGED ARCHES

ARCH TRUSSES

The reference curves in this paper apply equally well to two-hinged arch trusses. Computations show that the relation between the reduced equation $\frac{\sum \sigma u}{\sum u^2}$ and the classic equation $\frac{\sum \frac{\sigma u L}{A E}}{\sum \frac{u^2 L}{A E}}$ for truss structures is substantially the same as for solid structures.[2] Illustrative data are presented for three framed arches in Table 6. The Tyngsborough arch is parabolic in shape and of constant depth, corresponding to constant $\left(\frac{1}{E I} \frac{ds}{dx}\right)$-values. The Hell Gate and the Cottage Farm arches both have parabolic lower chords and are haunched in shape, the minimum depth being at the crown. The rise in each case is from the hinges to the midpoint between top and bottom chords at the crown. The Hell Gate and Cottage Farm arches are taken as parabolic in Table 6.

TABLE 6.—COMPARATIVE DATA FOR THREE TWO-HINGED ARCHES

No.	Arch	Location	Type	Ratio, span to rise	MAXIMUM INFLUENCE ORDINATE	
					"Reduced"	Actual
1	Tyngsborough	Lowell, Mass.	Basic	5.71	1.113	1.104
2	Cottage Farm	Boston, Mass.	A	4.165	0.852	0.838
3	Hell Gate	New York, N. Y.	A	4.16	0.849	0.841

LIMITATIONS

The accuracy of the reference curves decreases as the arch form becomes more curved, the errors varying from approximately 1% to 3% for the parabola to approximately 5% to 8% for the semi-ellipse, with usual arch shapes that do not have excessive differences in thickness between crown and haunch.

These reference curves will not apply to rigid-framed bents, and other similar structures, solved as two-hinged arches, because of their near-rectangular shape and consequent lack of "correspondence of ordinates." A special study of these shapes is necessary to establish the relationship between the geometry of the bents and the large errors resulting from reducing the equations.

CONCLUSION

The purpose of this paper has been to state a principle and to illustrate its application, but not to present an exhaustive treatise. Consequently the following considerations and many more have been omitted intentionally: Initial selection of the curve and shape of the arch rib or arch truss; rare arch forms of curves falling between a parabola and a straight line; rigid-frame bents and rectangular two-hinged structures; rib shortening; temperature effects; the effect of dissymmetry; and structures statically indeterminate to the second degree and higher. Furthermore, the study of the effect of variations in the thickness of the arch rib has been restricted to a small group of representative sections which is deemed sufficient for the practical application of the results. There would be no end to a "complete" investigation of this effect since there is an infinite number of possible variations.

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[2] "A Simplified Method for Solving Statically Indeterminate Framed Structures," by Lee H. Johnson, Jr., presented to Harvard Univ., Cambridge, Mass., in May, 1935, in partial fulfilment of the requirements for the degree of Doctor of Science in Civil Engineering.

The basic principle is applicable, however, to the analysis of certain highly indeterminate structures. A simple example is a three-span continuous truss, statically indeterminate to the second degree. The classic equations for the indeterminate reactions, R_1 and R_2, can be written:

$$R_1 \sum \frac{u^2_1\,L}{A\,E} + R_2 \sum \frac{u_1\,u_2\,L}{A\,E} + \sum \frac{\sigma\,u_1\,L}{A\,E} = 0 \quad \text{.........(30}a\text{)}$$

$$R_1 \sum \frac{u_1\,u_2\,L}{A\,E} + R_2 \sum \frac{u^2_2\,L}{A\,E} + \sum \frac{\sigma\,u_2\,L}{A\,E} = 0 \quad \text{.........(30}b\text{)}$$

The reduced equations would be:

$$R_1 \sum u^2_1 + R_2 \sum u_1\,u_2 + \sum \sigma\,u_1 = 0 \quad \text{..............(31}a\text{)}$$

$$R_1 \sum u_1\,u_2 + R_2 \sum u^2_2 + \sum \sigma\,u_2 = 0 \quad \text{..............(31}b\text{)}$$

Computations of continuous trusses in a paper by Albert Haertlein,[3] M. Am. Soc. C. E., show good agreement between the results obtained from the two sets of equations.

The "method of reduced equations" can be used to analyze structures statically indeterminate to the first degree where there is "correspondence of ordinates." It reduces the labor required of the designer and makes possible quicker solutions.

The factors governing the application of the method to the analysis of more highly indeterminate structures are not known to the writer. The development of such applications would be of considerable value to the designing engineer. The few computations the writer has made indicate that, if small deformations or movements of a structure produce large changes in any of its reactions, the method of reduced equations will not necessarily give close approximations to those reactions. The moments at the ends of a stiff, fixed-end arch are such reactions.

APPENDIX

NOTATION

The following letter symbols, used in this paper, conform essentially to American Standard Symbols for Mechanics, Structural Engineering and Testing Materials[4] prepared by a Committee of the American Standards Association and approved by the Association in 1932:

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[3] "The Design of Statically Indeterminate Trusses," by Albert Haertlein, *Journal*, Boston Soc. of Civ. Engrs., April, 1936, p. 57.

[4] ASA—Z10a—1932.

A = area of the cross section of the arch rib;
a = coefficient; the letters a, b, B, c, and d are used as mathematical coefficients, or arbitrary constants, in the introduction part of this paper (see "Basic Principle"); also a = distance along the span from the nearest support (see Fig. 4);
B = an arbitrary constant (see a);
c = an arbitrary constant (see a);
d = an arbitrary constant (see a);
E = modulus of elasticity;
f = "function of," distinguished by numerical subscripts as in Eq. 4; see also ϵ and η;
G = difference term as defined by Eqs. 2 and 6;
g = substitution factors g_1, g_2, g_3, and g_4, defined by Eqs. 25;
H = horizontal reaction;
h = rise of the arch;
I = moment of inertia of the cross section of the arch rib;
j = a fraction of the half span $l = a/l$ (see Eq. 16a);
k = mathematical multipliers k_1, k_2, $\cdots$, k_n, used to demonstrate the basic principle (see Eq. 1);
L = span length of arch axis = $2\,l$;
l = half the span length = $L/2$;
M = bending moment due to given loads and vertical reactions only;
m = arbitrary constant in the difference terms (Table 2, Col. 3) for arches that taper downward from the support to the center; m' = an arbitrary constant for arches that taper;
n = number of terms, as in Eq. 3;
R = reactions (see Eqs. 30);
s = length of an arch measured along its center line axis; Δs = length of a small segment of arc;
u = total stress in an arch member due to a horizontal force of unity applied at the reaction;
x = abscissa distance to any point x,y;
y = ordinate distance to any point x,y; the rise of the arch at a distance x from the left support;
ϵ = "function of"; $\epsilon(x)$ = a function whose absolute value can be made arbitrarily small; see also f and η;
η = "function of"; see f and ϵ;
σ = total stress in an arch member due to given loads and vertical reactions only.

DISCUSSION

Josef Sorkin,[5] M. Am. Soc. C. E.—The thesis presented in this paper is offered (see "Synopsis") as one that "* * * is useful in simplifying the analysis of certain types of statically indeterminate structures * * *." The paper belongs to the category of a number of others published since 1930, all attempting to simplify the application of the classical elastic theory and nearly all, unfortunately, ending in utter failure. The only notable exception is the "Analysis of Continuous Frames by Distributing Fixed-End Moments,"[6] by Hardy Cross, M. Am. Soc. C. E., in which a radically new approach substitutes simple arithmetic for the solution of simultaneous equations. This statement implies no reflection upon the sincerity of purpose of the other writers.

The simple fact is that most statically indeterminate structures, as encountered in actual practice, do not lend themselves to solution by specific simple mathematical expressions. Various solutions may be devised for any assumed academic cases, but it is impossible to assume enough cases to meet all, or even most, of the conditions encountered in actual design problems. That is primarily the reason why efforts to develop a "cure-all formula" are futile.

The author states clearly that the method proposed does not replace other methods and that it does not apply to all structures. Its primary purpose appears to be simplification of approximations of members preliminary to final design. The design of a statically indeterminate structure by any method requires that the size and the properties of the members must be assumed before proceeding with the stress analysis. It is not clear why the same principle does not apply to the method proposed by the author, regardless of his statement to the contrary. Irrespective of the method used for determining the reactions, the shape of the member originally assumed must finally be checked for stresses. Any part of the structure found to be either deficient or extravagant must be modified correspondingly with suitable readjustment of the reactions and the stresses. In prognosticating the proportions of such a structure, nothing can ever take the place of a designer's judgment based on past experience with similar structures. The question appears to be whether the methods proposed are in fact simpler than other methods available.

In so far as two-hinged arches are concerned, certain rudimentary rules have been in existence for many years. Thus, for the preliminary design of a parabolic arch, it is customary to assume that[7]

$$H = \frac{5}{8}\frac{P\,L}{h}\,(k - 2\,k^3 + k^4) \ldots\ldots\ldots\ldots\ldots\ldots (32)$$

in which $k\,L$ = distance from support. Incidentally, Eq. 32 is the same as Eq. 16*a*, since $j = 2\,k$. This formula can be employed with only a slight error for the preliminary proportioning of a circular arch of usual proportions.

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[5] Engr. of Design, Howard, Needles, Tammen & Bergendoff, Kansas City, Mo.

[6] "Analysis of Continuous Frames by Distributing Fixed-End Moments," by Hardy Cross, *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 1.

[7] "The Theory and Practice of Modern Framed Structures," by J. B. Johnson, C. W. Bryan, and F. E. Turneaure, 9th Ed., John Wiley & Sons, Inc., New York, N. Y., 1910–1916.

That no greater precision is necessary is indicated by the use of Table 6 as illustrated in Table 7. Values for the maximum influence ordinate, as obtained by the use of Eq. 32, are well within the required accuracy for preliminary design, compare favorably with the values obtained by the use of the author's reduced equations, and are more simple to compute. Preliminary determinations within 3% are fairly close. Variation of details may account for greater differences.

TABLE 7.—COMPARISON OF INFLUENCE ORDINATES IN TABLE 6 WITH COMPUTATIONS BY EQ. 32

No.	Arch (see Table 6)	MAXIMUM INFLUENCE ORDINATE			PERCENTAGE ERROR	
		"Reduced"	Actual	Eq. 32	"Reduced"	Eq. 32
1	Tyngsborough	1.113	1.104	1.114	+1.0	+1.0
2	Cottage Farm	0.852	0.838	0.813	+1.7	−3.0
3	Hell Gate	0.849	0.841	0.813	+1.0	−3.4

At its best, the method appears to be directly applicable only to solid concrete ribs of such shapes as the author (see paragraph following Eq. 16*b*) has selected "primarily because of their mathematical simplicity." It is clear that for steel ribs composed of shapes and plates, where changes in sections are abrupt, the mathematics would become so involved as to make the method too cumbersome for practical use. The author claims that the method is also applicable to arch trusses and to three-span continuous trusses; it is yet to be demonstrated, however, just how such application would be useful in any but very simple cases, where any relation to a practical design problem would be purely coincidental.

It may be pertinent to state that the actual time required for the analysis of any usual structure is a very small part of the total time required to prepare designs, plans, and specifications. Hence, the difference between one or two, or even three, trial designs reflects the total effort only in a minor way. Each designer as a rule develops his own peculiar approach to a problem and recognizes "short cuts" as his experience broadens. The quickest design method for any structure is the method that the designer knows and understands best.

Of late entirely too much emphasis seems to have been placed on methods and not enough on the physical meaning of the mechanics of calculations. Thus, as an example, the significance of Eq. 8*a*, for a horizontal reaction of a two-hinged arch, is simply that the horizontal reaction is equal to the horizontal movement of an arch end due to superimposed loads (if the end were free to move) divided by the similar movement caused by a unit load applied at that end. Such a definition is easily understood and the problem then is one of determining these movements of the arch end. Unfortunately, the true meaning of Eq. 8*a* can easily be befogged by bending moments, ordinates, and differential elements—the actual significance of the meaning being lost in the shuffle.

I. Oesterblom,[8] M. Am. Soc. C. E.—It is interesting to note that even the ancient Greek mathematicians have something to add to modern calculus, since the basic formula, Eq. 1, of the paper was set up and used by them in discussions of "proportionality." The development of the idea to assist modern engineers, when infinitesimal differences have to be computed, no doubt will have a much greater scope than Dean Johnson has so modestly shown.

Influence lines for horizontal reactions in arch bridges have been developed before, but never so easily and gracefully. On this account, one may say that the procedure presented in the paper is too late and no longer needed. This is not so, however; there is always room for a more graceful method. Then, there are also the broader needs—better understanding of a type, clearer comparisons of different arches or of loadings (especially unusual loadings) for various designs, comprehension of the stress behavior near the danger limits for unusual loads and for comparative alternates, and knowledge of how to use materials to the best advantage. Engineers can afford to spend more time on all these questions, and they will build better balanced as well as safer structures by doing so.

For the writer, Dean Johnson's method is so far only a dream—but a fine and fair one. He hopes, however, that opportunities will come soon to turn the vision into structural beauty. For the young engineer the paper presents more than a vision; it is a challenge to build always with grace—not only bridges but all kinds of indeterminate structures.

Indeterminate structures often involve small increments, and the method should be applicable to such problems. The writer expressed similar thoughts when Hardy Cross, M. Am. Soc. C. E., first presented his great vision of frame relaxation.[6] How well that idea was justified is now clear. The writer hopes sincerely that his present forecast will be equally justified.

Leon Beskin,[9] Assoc. M. Am. Soc. C. E.—The transformation of crude and well-known rules of the thumb used in preliminary design into a method of analysis for indeterminate structures has been attempted in this paper. It appears to the writer that this effort is misleading, because the examples selected and the internal forces computed are so chosen that they lead to erroneous general conclusions. For the special problem (hinged arches) for which the method has been developed, the charts and tables already available give enough information for preliminary design purposes without resorting to the author's approximations. Among such tables are those prepared by Eduard Winkler and presented in a French manufacturer's catalog and handbook.[10] These tables are computed for two-hinged arches, for built-in arches, and for various ratios of rise to span as well as for differences in moment of inertia along the span. Maximum positive and negative moments are given, as are the corresponding vertical and horizontal components of the force applied at the centroid, under the action of a uniform live load. The fraction of the span along which the live load is distributed is chosen to obtain the maximum moment. Tables of this type have considerable interest for a designer, and the author could have served his stated purpose better by completing existing tables of influence lines for arches of various proportions.

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[8] Engr., Carbide & Carbon Chemical Corp., South Charleston, W. Va.

[9] Design Specialist, Consolidated Vultee Aircraft Corp., San Diego, Calif.

[10] *Acitries de Longwy,* Longwy Steel Co., Paris, France, 1928 Ed.

Theoretical Considerations.—The fact that "the basic principle is one of simple arithmetic" (see heading, "Basic Principle") reveals one limitation of the author's development: The designer must assume that factors of the type of $M\,y$ are invariably plus or minus—never both. Otherwise, it is obvious that the value of a fraction such as that on the left-hand side of Eq. 1 (in which a_1 and a_2 have opposite signs) is not represented, to any degree of accuracy, by the fraction on the right-hand side. The two fractions may have opposite signs. Thus, the value of the redundant defined by the first fraction cannot be found by application of the author's method.

It may appear that the method is acceptable when ratios such as a_n/b_n (see Eq. 3) are approximately equal, so that the error G—which the author designates as "difference term"—is small. In this case, however, the method may lead to an even more serious failure than when the error G is great, as may be observed by comparing concentrated loads (large values of G) and distributed loads (small values of G). This fact can be demonstrated theoretically as follows: The values of the thrust computed by the correct formula (H) and by the approximate formula (H'), respectively, are

$$H = \frac{\Sigma M\,y\,\Delta s/I}{\Sigma y^2\,\Delta s/I} \quad\quad (33a)$$

and

$$H' = \frac{\Sigma M\,y\,\Delta s}{\Sigma y^2\,\Delta s} \quad\quad (33b)$$

Introduce the moments M' corresponding to H':

$$M' = M - H'\,y \quad\quad (34)$$

which can be considered as statical moments. Introduced into Eqs. 33, the thrust correction is found to be equal to zero if the approximate formula (Eq. 33b) is used, or to G if the correct formula (Eq. 33a) is used; thus:

$$G = \frac{\Sigma M'\,y\,\Delta s/I}{\Sigma y^2\,\Delta s/I} = \frac{\Sigma M\,y\,\Delta s/I}{\Sigma y^2\,\Delta s/I} - \frac{\Sigma M\,y\,\Delta s}{\Sigma y^2\,\Delta s} \times \frac{\Sigma y^2\,\Delta s/I}{\Sigma y^2\,\Delta s/I} = H - H' \quad (35)$$

The final moment M_0 can be expressed in terms of M' by the relation:

$$M_0 = M - H\,y = M' - G\,y \quad\quad (36)$$

Since M and y are assumed approximately proportional to each other, the quantities M' are very small in comparison with the quantities M. Thus, $G\,y$ and M' are of the same order or magnitude, and the error resulting from the use of H' (leading to M') instead of H (leading to M_0) may yield considerable differences between the moments, although the thrusts are practically equal.

Application to a Two-Hinged Arch.—The significance of the foregoing comments can be applied to the paper under discussion by the analysis of a two-hinged parabolic arch, under two conditions of loading. The following comparison is made between an arch that has a constant value of $E\,I\,dx/ds$ and an arch defined by the formula,

$$\frac{1}{E\,I} = \frac{1}{E\,I_0}\frac{ds}{dx}(1 + \beta\,\xi^2)\ldots\ldots\ldots\ldots\ldots\ldots(37)$$

in which $\xi = \frac{x}{l}$, the distance x being measured from the center line. The constant β may be positive or negative. Eq. 37 defines the arch ribs of types 3 and 6 (Fig. 5(*b*)). The extreme values of m and m' are $+40$ and -0.975 (see Tables 3 and 4), which correspond approximately to $\beta = -0.975$ and $\beta = 40$, since the relation between m (or m') and β is

$$(1 + m)(1 + \beta) = 1\ldots\ldots\ldots\ldots\ldots\ldots\ldots(38)$$

Case (*a*). Two Symmetrically Placed Concentrated Vertical Loads (*P*).—The expression for the thrust is (see Fig. 4, with $\alpha = 1 - a/l$):

$$\frac{H}{P} = \frac{l\int_0^{\alpha}(1-\alpha)(1-\xi^2)\,ds/I + l\int_{\alpha}^{1}(1-\xi)(1-\xi^2)\,ds/I}{h\int_0^{1}(1-\xi^2)^2\,ds/I}\ldots(39a)$$

or

$$\frac{H\,h}{l\,P} = \frac{\int_0^{\alpha}(1-\alpha)(1-\xi^2)(1+\beta\,\xi^2)\,d\xi + \int_{\alpha}^{1}(1-\xi)(1-\xi^2)(1+\beta\,\xi^2)\,d\xi}{\int_{\alpha}^{1}(1-\xi^2)^2(1+\beta\,\xi^2)\,d\xi}\ldots(39b)$$

The final value is:

$$\frac{H\,h}{l\,P} = \frac{5\,(5 - 6\,\alpha^2 + \alpha^4) + \beta\,(3 - 5\,\alpha^4 + 2\,\alpha^6)}{32\,(1 + \beta/7)}\ldots\ldots\ldots\ldots(40)$$

Table 8 contains the coefficients for thrust and moment, resulting from the application of Eq. 40, for various values of α and β. Moments are determined at the points of application of the concentrated loads. Percentage errors in thrust and moment, as related to the "basic arch" ($\beta = 0$), are included for comparison.

TABLE 8.—INFLUENCE VALUES RESULTING FROM EQS. 40 AND 43

Constant β	CASE (*a*)								CASE (*b*)	
	Thrust, $\frac{h\,H}{l\,P}$				Moment, $\frac{M}{l\,P}$				Moment, $\frac{M}{l^2\,w_2}$	
	Computation		Error (%)		Computation		Error (%)		Computation	Error (%)
	$\alpha = 0$	$\alpha = 0.5$	$\alpha = 0$	$\alpha = 0.5$	$\alpha = 0$	$\alpha = 0.5$	$\alpha = 0$	$\alpha = 0.5$		
− 0.975	0.802	0.551	− 3	−1	0.196	0.101	+12	+12	−0.00934	+27
− 0.900	0.800	0.550	− 3	−1	0.200	0.102	+10	+11	−0.00956	+24
0	0.781	0.556	0	0	0.219	0.113	0	0	−0.0119	0
+10	0.708	0.580	+10	+4	0.292	0.160	−25	−29	−0.0212	−44
+40	0.676	0.590	+15	+6	0.325	0.179	−32	−37	−0.0254	−53

In his presentation, Dean Johnson has not shown the errors in moment values, which reach 37%. He has reduced the error in thrust (which reaches

15%) to a fictitious value of 6% (see line 14 preceding Eq. 26) by the use of average arches with varying moments of inertia that have no relation to the basic arches introduced at the beginning of the paper. Since the object of the paper is to establish a method based on the properties of the basic geometric arches, it seems inconsistent to use arches with varying moments of inertia for the evaluation of errors due to the use of the basic arch. The result is an apparent compromise between the actual arch and the basic arch, which the writer cannot accept as tenable.

Case (*b*). A Variable Distributed Load (*w*).—Consider a variable load defined by

$$w = w_0 + w_2\,\xi^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (41)$$

When the ratio w_2/w_0 is small, the load is approximately uniform; and, when the ratio is large, the variation in load is relatively great. Then the formula for thrust is

$$\frac{h\,H}{l^2} = \frac{1}{2}\,w_0 + w_2\,\frac{\int_0^1 (1 - \xi^2)\,(1 = \beta\,\xi^2)\,(1 - \xi^4)\,d\xi}{12 \int^1 (1 - \xi^2)^2\,(1 + \beta\,\xi^2)\,d\xi}$$

$$= \frac{1}{2}\,w_0 + w_2\,\frac{6 + \beta}{9\,(7 + \beta)} \ldots\ldots\ldots\ldots\ldots\ldots (42)$$

and the moment at the crown is:

$$\frac{M}{l^2} = \frac{w_2}{12}\left[1 - \frac{4\,(6 + \beta)}{3\,(7 + \beta)}\right] = -\frac{(3 + \beta)\,w_2}{36\,(7 + \beta)} \ldots\ldots\ldots (43)$$

Values of the moment and of the error resulting from the use of the basic arch for this condition are included in Table 8. They show that errors are greater in the case of distributed loads than in the case of concentrated loads. The relative error in the value of the thrust is a function of the ratio $\rho = w_0/w_2$; thus:

$$\epsilon = \frac{\beta}{7\,[6 + \beta + 9\,\rho\,(7 + \beta)/2]} \ldots\ldots\ldots\ldots\ldots (44)$$

The error is infinite if ρ has the value,

$$\rho = -\frac{2\,(6 + \beta)}{9\,(7 + \beta)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (45)$$

Eq. 45 shows that, with a proper choice of ρ, any arbitrary value of ϵ can be selected. For instance, if $\epsilon = -2$, the approximate value of the thrust is equal and opposite to the actual value.

Loading conditions that lead to such results are not theoretical. For instance, in frames, used to reinforce a shell structure such as an airplane fuselage, it is possible to find cases for which the frame tends to open or close under a given applied load, depending on the distribution of the moments of inertia. Thus, depending on the structure, there may be thrusts of opposite signs—a fact that cannot be shown if basic arches are used.

Incidental Comment.—The author introduces expressions such as "difference term" and "reduced equation" when the expressions, "error" and "approximate equation," are well known and fulfil the same requirements, without obscuring the approximate character of the method. The expression, "correspondence of ordinates," is loosely defined. For instance, in Fig. 1(*c*), there is no reason to assume that the correspondence is stopped precisely at the center line. The concept of "correlation," which has a definite significance in statistics, could be usefully applied, because it would supply a definite quantity to express the "degree of correspondence" between various curves.

The author has devoted too much of his paper to a description of elementary methods of integration and to well-known tricks, such as the use of two symmetrical loads and the expedient of integrating along the half span when determining the thrust due to one load. The distinction between m and m', to separate positive and negative values, leads to the writing of two sets of formulas instead of one.

A difference is claimed between parabolic and circular arches, which actually does not exist. For example, in the sentence following Eq. 27, the author states: "The influence values [for circular arches] * * * are not a linear function of L/h, as in the case of the parabola." This is correct for a true "basic arch"; but, because the author uses a substitute basic arch in the case of a parabolic profile for expediency (in which $I\,dx/ds$ is constant, instead of I), he finds that the thrust is a linear function of L/h. If a similar adjustment is made for circular arches, the property of linearity may be shown to exist for this second case as well as for parabolic arches.

Conclusions.—In recent years, many writers have tried to simplify the analysis of redundant structures. With a few exceptions (which all concerned methods of successive approximations), these attempts have proved failures. This paper, likewise, has failed in its purpose.

JAROSLAV POLIVKA,[11] M. AM. SOC. C. E.—The results of analyses of a two-hinged arch of parabolic, circular, and elliptical shape, with a variable moment of inertia, I, are presented in this excellent paper. Tables and graphs are included showing clearly the effect of the variation of I, and the effect of the shape upon the horizontal reaction H. With this evidence, Dean Johnson arrives at the conclusion that H can be found with sufficient accuracy from three types of average influence lines, referring to three general types of two-hinged arches. The author discusses the correlation of error resulting from average values and the "correspondence of ordinates"; and he expresses this error in a novel manner by means of "difference terms."

Furthermore, the author defines the limitations of the method of using typical average values or reference curves. He states that such reference curves will not apply to rigid-framed bents and other similar structures, solved as two-hinged arches, because of the lack of "correspondence of ordinates." Finally, he concludes that such average reference curves will not necessarily give close approximations to redundant reactions for structures in which small deformations produce large changes in reactions—as, for example, those produced by the moments at the abutments of a restrained arch.

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The author derives the "reduced equation" (Eq. 10) from the "classic equation" (Eq. 9) by eliminating the variable term $E\,I$. This is possible[12,13] by making $E\,I$ proportional to $\frac{1}{\cos\phi} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$, in which $\frac{dy}{dx} = \tan\phi$. Eqs. 16 used to determine the influence line for H in the case of a parabolic arch has been derived by W. Ritter as follows:

$$H = \frac{x\,(L - x)\,(L^2 + L\,x - x^2)}{2\,L^3\,m} \qquad (46)$$

in which

$$m = \frac{4}{5}h\left(+ \frac{3\,I}{2\,A\,h}\right) \qquad (47)$$

The second term in Eq. 47 is an expression of rib shortening; and, for $x = \frac{L}{2}$ (or for $j = 1$ in the paper), Eq. 46 yields:

$$H_{\max} = \frac{25\,L}{128\,h} \qquad (48)$$

The effect of a variation in the thickness of the ribs is obtained by expressing the variation in simple terms which allow easy integration—a procedure which has been proposed by several writers.[14,15] The new version of the reduced equation should be a great help in the design of two-hinged arches with variable moments of inertia because the conventional methods involve a great many cumbersome computations.

LEE H. JOHNSON, JR.,[16] M. AM. SOC. C. E.—The writer is indebted to those who have prepared discussions of this paper. He has been aware of the fact that it presented only an idea, and was prompted to submit it in the hope that others would be stimulated to investigate the possibilities of the idea. In statically indeterminate structures both the geometry and the properties of the cross sections and materials have an influence upon the reactions and the internal forces. It is conceivable that the relation between these two influences may be determined sufficiently to be of value to the designer, at least for some types of structures. The use of reduced equations may provide a method of limited applicability for determining this relation. The possibilities of the method for this purpose were illustrated in the application to two-hinged arches, where it was shown that the influence of cross sections and materials upon reactions was relatively small and could be correlated to the influence of geometry.

Mr. Oesterblom's discussion is a great encouragement to pursue the idea still further. The writer agrees with him that at present the possibilities of

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[12] "Der elastische Bogen berechnet mit Hilfe der graphischen Statik," by W. Ritter, Albert Raustein, Zurich, Switzerland, 1886.

[13] "Anwendungen der graphischen Statik," by W. Ritter and C. Culmann, Albert Raustein, Zurich, Switzerland, 1906, Vol. 4, pp. 124 and 245.

[14] "Beitraege zur Theorie der vollwandigen Bogentraeger ohne Scheitelgelenk," by Max Ritter, W. Ernst & Son, Berlin, Germany, 1909.

[15] "Neuere Methoden zur Statik der Rahmentragwerke und der elastische Bogentraeger," by A. Strassner, Vol. 2, W. Ernst & Son, Berlin, Germany, 1921.

[16] Dean and Prof. of Civ. Eng., School of Eng., Univ. of Mississippi, University, Miss.

the method largely remain to be determined. The writer is indebted to Mr. Polivka for emphasizing the usefulness of the two-hinged arch data in eliminating certain cumbersome computations.

Although Mr. Beskin's mathematical analyses appear to be correct, his criticism comes from misleading inferences which he draws from them. He points out several apparent limitations to the writer's method—one to the basic principle as expressed in Eq. 1, another to the computation of moment values, M_0, in Eq. 36, and a third in the computation of thrust for a variable load in Eq. 45. None of these are actual limitations since they are valid only for values in the neighborhood of zero.

Consider two influence lines whose corresponding ordinates differ by relatively small amounts and which do not cross the horizontal or zero axis at the same point. It is obvious that the relative error becomes infinite at one of the zero points and is very large in the vicinity of that point. The small ordinates to influence lines, moment diagrams, and shear diagrams near the zero points are seldom significant in a design. Mr. Beskin has attempted to make them so, in discussing the writer's two-hinged arch data, by using a combination loading consisting of a uniform load and a load that increases parabolically from the center to the ends of the arch. He finds under this loading that the error in thrust, using the reduced equations, is infinite for a certain ratio of the two load constants, which ratio, it should be noted, is negative. Again it is obvious that if an arch is subjected to two loads, one downward and one upward as is indicated by the negative sign, the thrust on the arch can be made zero by properly balancing the loads, in which case the error in making any approximation to the thrust must be infinite. The stresses computed with a thrust of zero and the stresses computed with a very small thrust would be approximately the same in spite of the fact that the error in thrust is infinite. In a problem of this type where the thrust is zero, or is so close to zero that the approximation may be of the opposite sign, the thrust is not significant in determining the stresses and the arch behaves more as a curved beam with unusual support and loading than as an arch. Contrary to Mr. Beskin's statement, the method of reduced equations can be used to examine loadings of this type, and he, himself, has made a start in this direction.

The comparative values which Mr. Beskin shows in Table 8 are with respect to the basic influence values rather than with respect to curve A or B, Fig. 12; hence the large relative errors. He contends that there is no justification for curves A and B; but the writer is of the opinion that much more justification for these curves can be found in mathematical analyses than can be found for the use of any "allowable stress" in the design of a structure.

The writer made no claim to a difference between parabolic and circular arches such as Mr. Beskin states. He did claim, and still does, that the "basic influence values" for circular arches are not a linear function of the (L/h)-ratio as are those of the parabolic and semi-elliptic arches. The term "basic arch" was not used in the paper.

Mr. Beskin's suggestions about the terminology are noteworthy. The writer agrees that the term "correlation" would have been better than "correspondence" and that "error" would have done as well as "difference term."

He prefers the term "reduced equation," however, as it implies a particular type of approximation. There is a reason to assume that the correspondence in Fig. 1(*c*) is stopped precisely at the center line, for it is precisely there that the slope of curve F changes from positive to negative.

Mr. Sorkin's criticism arises from a misinterpretation of the details of the paper, the significance of the fundamental idea, and of its application to two-hinged arches. Apparently, he denies the possibility of beginning the design of a statically indeterminate structure without first assuming the size and properties of the members. The writer has shown mathematically under what conditions these quantities may be omitted from the first computations.

Mr. Sorkin states that the method is too cumbersome for practical use in the case of steel ribs composed of plates and shapes because of the involved mathematics. The reader should understand that the result of the application of the method to two-hinged arches is a set of three charts, Figs. 5, 10, and 12, which apply to steel girder ribs and to truss ribs as well as to solid concrete ribs. In the paper the writer did not give the tabular data for plotting the basic influence line, curve A, and curve B in Fig. 5. These data are given in Table 9.

TABLE 9.—ORDINATES FOR INFLUENCE LINES FOR PARABOLA (SEE FIG. 5)

(Values Are in Terms of L/h)

Fraction of span	Curve A	Basic influence line	Curve B
0.05	0.028	0.031	0.036
0.10	0.058	0.061	0.069
0.15	0.085	0.090	0.100
0.20	0.112	0.116	0.124
0.25	0.136	0.139	0.144
0.30	0.159	0.159	0.159
0.35	0.177	0.175	0.169
0.40	0.192	0.186	0.176
0.45	0.201	0.193	0.179
0.50	0.204	0.195	0.180

The parabolic formula, Eq. 32, which Mr. Sorkin mentions, can be used with small errors only for parabolic arches, and for circular arches of the higher (L/h)-ratios—say greater than 6. It will not give accurate results for these arches in cases where the crown thickness is much greater than the thickness at the haunch. The writer's data give closer approximations in all cases than the parabolic formula and also apply to any other arch form within the limits of the data in Figs. 10 and 12. The writer does not agree with Mr. Sorkin's implication that since "the difference between one or two, or even three, trial designs reflects the total effort only in a minor way," no attempt should be made to improve this phase of the work in the creation of a structure.

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COLUMN FORMULAS

By William R. Osgood,[1] M. Am. Soc. C. E.

With Discussion by Messrs. L. E. Grinter, George Winter, Jonathan Jones, Edward Godfrey, Ralph E. Spaulding, James H. Cissel, Leon Beskin, Louis Balog, and William R. Osgood.

Synopsis

An empirical column formula should be one that expresses theoretically possible relations. The prime requisites of any column formula or formulas are that the average stress be a continuous, monotonically decreasing function of the ratio of slenderness and that it approach the Euler value as the ratio of slenderness becomes infinite. Another desirable requisite is that the first derivative of the average stress with respect to the ratio of slenderness be continuous. These requirements impose certain restrictions on the constants in empirical formulas, and these restrictions are considered for a number of well-known and less well-known column formulas.

Introduction

Column formulas are a convenience, not a necessity, for relating the strength of a column to its dimensions. It would be quite possible to plot the results of tests, determine a curve from the plot, and select design values from this curve or from a table of values read from the curve. When a formula is used, however, as it usually is, there is some virtue in having it of correct form, if only because the designer may have to extrapolate beyond the values on which the formula is based. If, for example, at large ratios of slenderness a formula covering all ratios of slenderness leads to values of average stress that are known to be impossible, the formula will lead at less large ratios of slenderness to values that are likely to be impossible. Therefore it is desirable not to preclude extrapolation. Is it not at least slightly better to know that extrapolated values may be right than to know that they probably are wrong?

There have been numerous and elaborate attempts to devise formulas based on elastic theory for the design of centrally loaded columns in the plastic range.

Note.—Published in December, 1944, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

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Before the development of the double-modulus theory toward the end of the past century these attempts were consonant with good engineering practice. Today, however, it should be realized how futile such attempts must be. It would be possible to design by means of the double-modulus theory, given the compressive stress-strain curve of the material; but the best testing technique results in wider discrepancies between theoretical and experimental values in the plastic range than in the elastic range, and most engineers would undoubtedly prefer to use an empirical formula based on tests. An empirical formula, however, should be one that expresses theoretically possible relations. Besides being of correct form, the formula should contain separately one constant of the structure, implicitly or explicitly, and one or more constants related to the material. It would be very hard to justify more than two constants of the material in any practical column formula.

As to the form, the prime requisites of any column formula or formulas are that the average stress be a continuous, monotonically decreasing function of the ratio of slenderness and that it approach the Euler value as the ratio of slenderness becomes infinite. Another desirable requisite is that the first derivative of the average stress with respect to the ratio of slenderness be continuous.

It is proposed to present a number of formulas in their most general forms to satisfy the requirements of the preceding paragraph. For comparative purposes the formulas will all be written in terms of the two non-dimensional variables, λ_o (the reduced ratio of slenderness) and σ (the reduced average stress):

$$\lambda_o = \frac{1}{\pi}\frac{l_o}{i}\sqrt{\frac{S}{E}} \quad \text{(1a)}$$

and

$$\sigma = \frac{P}{A\,S} \quad \text{(1b)}$$

as well as in terms of the customary variables—the average stress P/A, and the ratio of slenderness l_o/i. The free length is:

$$l_o = k\,l \quad \text{(2)}$$

l being the actual length (panel-point length) of the column and k being the constant of the structure—namely, the factor depending on end conditions by which l must be reduced or increased to obtain the distance, l_o, between successive points of inflection. For simply supported ends, $k = 1$; for fixed ends, $k = \frac{1}{2}$; for one end free and the other end fixed, $k = 2$; etc. In Eqs. 1, E is the modulus of elasticity of the material, and S is an arbitrary constant of the dimensions of stress. In analyzing column-test data, S should be taken as a compressive secant yield strength. (The secant yield strength is the stress at the intersection with the stress-strain curve of a line through the origin having a slope $m\,E$, where $0 < m < 1$.)

The most general forms of some well-known and other, less well-known formulas will now be given. The dimensionless constants β, γ, and n, and the constants B and C that appear are arbitrary, subject to the conditions stated in each case. They are constants related to the material. The load, P, is to be considered the maximum load, and after assigning values to the constants, P is to be taken as the working load multiplied by a factor of safety.

The only formula containing but one arbitrary constant that will be considered is the Rankine formula. This formula is used extensively. The justification for it is that at low and at high ratios of slenderness it fits test results well, and at intermediate ratios of slenderness, where the scatter in test results is relatively large, the formula gives lower values of the average stress than the mean of the average stresses found in tests of most materials. The formula applies for all values of ratio of slenderness.

I. The Rankine Formula

The Rankine formula is expressed by

$$\sigma = \frac{1}{\beta + \lambda^2_o}; \qquad \beta \geqq 0 \ldots\ldots (3a)$$

or

$$\frac{P}{A} = \frac{\pi^2 E}{B + \left(\frac{l_o}{i}\right)^2}; \qquad B \geqq 0 \ldots\ldots (3b)$$

When $\beta = B = 0$, Eqs. 3 reduce to the Euler formula,

$$\sigma = \frac{1}{\lambda^2_o} \ldots\ldots (4a)$$

or

$$\frac{P}{A} = \frac{\pi^2 E}{\left(\frac{l_o}{i}\right)^2} \ldots\ldots (4b)$$

It is easy to determine β from a series of observed values of σ and λ_o by plotting $\sigma \lambda^2_o$ against σ, for

$$\sigma \lambda^2_o = -\beta \sigma + 1 \ldots\ldots (5)$$

and $-\beta$ is the slope of the resulting straight line (if one results!) through the point $\sigma = 0$, $\sigma \lambda^2_o = 1$. Alternatively, since

$$\beta = \frac{1}{\sigma} - \lambda^2_o \ldots\ldots (6)$$

β may be taken as the average of the values of $\frac{1}{\sigma} - \lambda^2_o$.

There are several formulas containing two arbitrary constants, and one of them, and often more than one, can be made to fit almost any column-test data that it may be desired to fit. First will be considered formulas covering the entire range of ratios of slenderness.

II. The Secant Formula

The secant formula is expressed by:

$$\lambda_o = \frac{2}{\pi \sqrt{\sigma}} \sec^{-1}\left(\frac{1}{\beta \sigma} + C\right); \qquad \beta \geqq 0, \qquad C \leqq 1 \ldots\ldots (7a)$$

or

$$\frac{l_o}{i} = \frac{2\sqrt{E}}{\sqrt{\frac{P}{A}}}\sec^{-1}\left(\frac{1}{B\frac{P}{A}} + C\right); \qquad B \geqq 0, \qquad C \leqq 1 \ldots\ldots\ldots (7b)$$

When $\beta = B = 0$, Eqs. 7 reduce to Euler's formula.

The constants β and C may be determined by plotting $\sec\frac{\pi}{2}\lambda_o\sqrt{\sigma}$ against $1/\sigma$, for

$$\sec\frac{\pi}{2}\lambda_o\sqrt{\sigma} = \frac{1}{\beta\,\sigma} + C \ldots\ldots\ldots\ldots\ldots\ldots (8)$$

and $\frac{1}{\beta}$ and C are the slope and intercept, respectively, of what may turn out to be a straight line.

III. AARFLOT'S FORMULA

A column formula due to M. G. Aarflot[2] is:

$$\sigma = \frac{1}{C\lambda^2_o + (1 - C)\sqrt{\lambda^4_o + \beta}}; \qquad \beta \geqq 0, \qquad 0 \leqq C \leqq 1 \ldots\ldots (9a)$$

or

$$\frac{P}{A} = \frac{\pi^2 E}{C\left(\frac{l_o}{i}\right)^2 + (1 - C)\sqrt{\left(\frac{l_o}{i}\right)^4 + B}}; \qquad B \geqq 0, \qquad 0 \leqq C \leqq 1 \ldots (9b)$$

When $\beta = B = 0$, or $C = 1$, Eqs. 9 reduce to Euler's formula.

The constants β and C may be determined from a series of observed values of σ and λ_o by plotting $\frac{1}{\sigma^2} - \lambda^4_o$ against $\lambda^2_o\left(\frac{1}{\sigma} - \lambda^2_o\right)$, for

$$\frac{1}{\sigma^2} - \lambda^4_o = 2\,C\lambda^2_o\left(\frac{1}{\sigma} - \lambda^2_o\right) + \beta\,(1 - C)^2 \ldots\ldots\ldots\ldots (10)$$

and $2\,C$ and $\beta\,(1 - C)^2$ are the slope and intercept, respectively, of the straight line represented by Eq. 10.

IV. YLINEN'S FORMULA

A column formula due to A. A. J. Ylinen[3] is:

$$\lambda_o = \sqrt{\frac{1 - \beta\sigma^n}{\sigma}}; \qquad \beta \geqq 0, \qquad n > 0 \ldots\ldots\ldots\ldots (11a)$$

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[2] "A New Column Formula," by Martin G. Aarflot, *Bulletin*, Am. Soc. of Swedish Engrs., Vol. 26, February, 1931, p. 7.

[3] "Keskeisesti Puristetun Sauvan Nurjahtamisesta Suhteellisuusrajan Yläpuolella," by Arvo A. J. Ylinen, *Teknillinen Aikakauslehti*, Vol. 21, 1931, p. 95.

or

$$\frac{l_o}{i} = \pi\sqrt{E}\sqrt{\frac{1 - B\left(\frac{P}{A}\right)^n}{\frac{P}{A}}}; \qquad B \geqq 0, \qquad n > 0 \ldots\ldots (11b)$$

When $\beta = B = 0$, Eqs. 11 reduce to Euler's formula, and when $\beta > 0$, $B > 0$, and $n = 1$, they reduce to Rankine's formula.

It is easy to determine whether the formula is a good fit to test data and to determine the constants if it is, for

$$\log (1 - \lambda^2_o \sigma) = n \log \sigma + \log \beta \ldots\ldots\ldots\ldots\ldots (12)$$

and it is only necessary to plot observed values of $1 - \lambda^2_o \sigma$ against σ on logarithmic paper. If the plotted points fall on a straight line, the constants may be obtained from the slope and intercepts of this line.

V. FORMULAS PROPOSED BY THE WRITER

A formula proposed by the writer is:

$$\sigma = \frac{1}{\beta + \gamma\lambda_o + \lambda^2_o}; \qquad \beta \geqq 0, \qquad \gamma \geqq 0 \ldots\ldots\ldots\ldots (13a)$$

or

$$\frac{P}{A} = \frac{\pi^2 E}{B + C\frac{l_o}{i} + \left(\frac{l_o}{i}\right)^2}; \qquad B \geqq 0, \qquad C \geqq 0 \ldots\ldots\ldots (13b)$$

When $\beta = \gamma = B = C = 0$, Eqs. 13 reduce to Euler's formula, and, when $\beta > 0$, $B > 0$, and $\gamma = C = 0$, they reduce to Rankine's formula.

The constants β and γ may be determined by plotting $\frac{1}{\sigma} - \lambda^2_o$ against λ_o, for

$$\frac{1}{\sigma} - \lambda^2_o = \gamma\lambda_o + \beta \ldots\ldots\ldots\ldots\ldots\ldots\ldots (14)$$

and, if the resulting plot is approximately a straight line, γ and β are its slope and intercept, respectively. If a straight line does not result, Eqs. 13 are inapplicable. (Eqs. 13 may not be new, and the writer would appreciate having a reference to them if any reader knows of one.)

Another suggestion by the writer is:

$$\lambda_o = \frac{1}{\sqrt{\sigma + \beta\sigma^n}}; \qquad \beta \geqq 0, \qquad n > 1 \ldots\ldots\ldots\ldots (15a)$$

or

$$\frac{l_o}{i} = \frac{\pi\sqrt{E}}{\sqrt{\frac{P}{A} + B\left(\frac{P}{A}\right)^n}}; \qquad B \geqq 0, \qquad n > 1 \ldots\ldots\ldots (15b)$$

When $\beta = B = 0$, Eqs. 15 reduce to Euler's formula. As $n \to \infty$, Eq. 15*a* approaches a broken curve consisting of the axis of σ down to $\sigma = 1$, the line $\sigma = 1$ between $\lambda_o = 0$ and $\lambda_o = 1$, and the reduced Euler curve beyond $\lambda_o = 1$.

As in the case of Eqs. 11 it is easy to determine whether Eq. 15*a* represents a good fit to test data, and to determine the constants if it does, for

$$\log\left(\frac{1}{\lambda^2_o} - \sigma\right) = n \log \sigma + \log \beta \text{................} (16)$$

Eqs. 15 are derived by the double-modulus theory from a stress-strain curve having the equation,

$$e = \frac{P}{A\,E} + K\left(\frac{P}{A\,E}\right)^n \text{....................} (17)$$

in which K and n are empirical constants.

A third suggestion by the writer is:

$$\lambda_o = \frac{2}{\sqrt{\sigma} + \sqrt{\sigma + \beta\,\sigma^n}}; \qquad \beta \geqq 0, \qquad n > 1 \text{.........} (18a)$$

or

$$\frac{l_o}{i} = \frac{2\,\pi\,\sqrt{E}}{\sqrt{\frac{P}{A}} + \sqrt{\frac{P}{A} + B\left(\frac{P}{A}\right)^n}}; \qquad B \geqq 0, \qquad n > 1 \text{......} (18b)$$

When $\beta = B = 0$, Eqs. 18 reduce to Euler's formula. Like Eq. 15*a*, as $n \to \infty$, Eq. 18*a* approaches a broken curve consisting of the axis of σ down to $\sigma = 1$, the line $\sigma = 1$ between $\lambda_o = 0$ and $\lambda_o = 1$, and the reduced Euler curve beyond $\lambda_o = 1$. Also, like Eqs. 15, Eqs. 18 are derived by the double-modulus theory from a stress-strain curve of the form of Eq. 17.

As in the case of Eqs. 11 the constants in Eqs. 18 may be determined easily and accurately, for

$$\log \frac{4}{\lambda_o}\left(\frac{1}{\lambda_o} - \sqrt{\sigma}\right) = n \log \sigma + \log \beta \text{..............} (19)$$

The next two formulas (Eqs. 20 and 21) contain two arbitrary constants each, but the formulas are intended for use only above the limiting average stresses indicated. The curves they represent are tangent to the Euler curve, and for stresses below the point of tangency the Euler formula, Eqs. 4, should be used.

VI. The Parabolic-Type or Hyperbolic-Type Formula

The following formula is of a parabolic or a hyperbolic type:[4]

$$\sigma = \beta\left[1 - \frac{2}{n+2}\left(\frac{n\,\beta}{n+2}\right)^{n/2} \lambda^n_o\right];$$

$$\beta > 0, \qquad n > 0, \qquad \sigma \geqq \frac{n\,\beta}{n+2} \text{..............} (20a)$$

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[4] "Column Curves and Stress-Strain Diagrams," by William R. Osgood, *Journal of Research*, National Bureau of Standards, Vol. 9, October, 1932, p. 571.

or

$$\frac{P}{A} = B\left\{1 - \frac{2}{n+2}\left[\frac{n\,B}{(n+2)\,E}\right]^{n/2}\frac{1}{\pi^n}\left(\frac{l_o}{i}\right)^n\right\};$$

$$B > 0, \qquad n > 0, \qquad \frac{P}{A} \geqq \frac{n\,B}{n+2} \ldots\ldots\ldots\ldots\ldots (20b)$$

When $n = 1$, Eqs. 20 become the straight-line formula; and when $n = 2$, they become the Johnson parabolic formula.

VII. KREÜGER'S FORMULA

A formula due to H. Kreüger[5] is

$$\sigma = \frac{\beta - \lambda^2_o}{\frac{1}{4}(\beta - \gamma)^2 + \gamma\,\lambda^2_o}; \quad \beta > 0, \qquad -\beta \leqq \gamma \leqq \beta, \qquad \sigma \geqq \frac{2}{\beta - \gamma} \ldots (21a)$$

or

$$\frac{P}{A} = \frac{\pi^2\,E\,B - \left(\frac{l_o}{i}\right)^2}{\frac{1}{4}\,\pi^2\,E\,(B - C)^2 + C\left(\frac{l_o}{i}\right)^2};$$

$$B > 0, \qquad -B \leqq C \leqq B, \qquad \frac{P}{A} \geqq \frac{2}{B - C} \ldots\ldots\ldots\ldots (21b)$$

When $\gamma = C = 0$, Eqs. 21 reduce to the Johnson parabola.

SUMMARY

There is little to be said for a formula on the grounds of its simplicity, for any formula can be plotted to as large a scale as desired and values read from the graph, or it may be tabulated with increments in the argument as small as desired. On the other hand, there is no point in using a formula that is applicable only down to a given value of the average stress, or up to a given value of the ratio of slenderness, beyond which the Euler formula must be used. The column formulas in Sections II to V, inclusive, apply for all values of the ratio of slenderness, and one or all of them can almost be guaranteed to fit adequately any reasonable column-test data it may be desired to fit.

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[5] "Bidrag till frågån om sträfvors knäckning," by H. Kreüger, *Teknisk Tidskrift*, Väg- och Vattenbyggnadskonst, Vol. 45, September 15, 1915, p. 101.

DISCUSSION

L. E. GRINTER,[6] M. AM. SOC. C. E.—Since the column is probably the most critical structural member, it is well for the structural engineer to review, at intervals, the background of procedures for column design. Design of columns has long been associated with the use of a "column formula" and, therefore, the author's excellent paper again stimulates the profession to re-examine its methods.

Test results for columns of varying slenderness, when plotted on a graph of failure stress versus slenderness ratio, do not fall along a single line but represent a broad band even when the tests were all performed in one laboratory. This is true because of the virtual impossibility of producing a centered load or a perfectly homogeneous column, particularly if it must be fabricated from parts. No doubt the spread of failure loads would be much greater for columns in actual structures where soil movements, temperature movements, and strains of fabrication have an influence upon resistance to failure.

The choice of a column formula, therefore, is something of a psychological or philosophical matter rather than a strictly scientific decision. A straight line, a broken line, a parabola, or a reversed curve can each be chosen to represent test results, probably with equal satisfaction and accuracy, within the common working range of slenderness ratios. If this is true, then the choice of a formula should be influenced by another consideration: What column formula will produce the best results in terms of design when placed in the hands of the ordinary designer who is not always a highly competent structural engineer?

There are two mistakes that can be made in the use of a column formula. The formula itself may be misinterpreted; and this mistake will become more probable as the formula becomes more complex. For this reason either the Kreüger formula or the Aarflot formula mentioned by the author could become a source of errors in design. The second mistake is in extending the use of the formula beyond its specified range. This can be counteracted by emphasizing the necessary limitations on the formula or by selecting a formula of the kind recommended by the author for which extrapolation beyond test results will give reasonably safe results.

Perhaps it is unfortunate, but, nevertheless, much design work is actually performed by persons of limited competency in structural design. For them the straight-line formula or the Rankine formula is the reasonable limit of useful complexity. Since no greater accuracy can be obtained by use of a more complex formula, the advantage of adding terms is not very clear. The best form of printed specification and one which would meet all objections would be a complete statement of the design formula to cover each range of slenderness ratio. The only mistake that has been common in specification writing has been to cover only the normal range and simply to neglect entirely the upper and lower ranges with resulting confusion in the minds of many persons who must, and do, perform design work.

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[6] Vice-Pres. and Dean, Graduate School, Illinois Inst. of Technology, Chicago, Ill.

George Winter,[7] M. Am. Soc. C. E.—Until a decade or two ago practical column formulas were exclusively of the empirical type. Mathematical expressions were simply chosen to approximate, as closely as possible, the results of a greater or smaller number of column tests. Little attention was paid to the physical significance of these formulas, to correct representation of end fixities, manufacturing peculiarities, and sometimes even to physical characteristics of the material. Since then the development has been along more rational lines. Methods were developed to account for column strength by considering, in a rigorous analytical way, properties of material, end fixity, initial shape of the member, manner of load application, etc. Two types of such methods have evolved:

(*a*) The one designated by the author as double-modulus theory which takes account of the deviation from the straight line of the stress-strain curves, particularly at higher stresses. This method proved rather cumbersome in its application and, as the author states, is of only limited practical consequence.

(*b*) The other takes account of the fact that the strength of a column depends to a great extent on its initial shape and the manner of load application. Initial deviations from straightness, eccentricities of loading, and other factors which might be termed imperfections were found to decrease the strength of columns considerably, particularly in the range of low and medium slenderness ratios. The secant formula is of this latter type as is the method proposed by D. H. Young[8] in 1934.

The great advantage of methods (*b*) consists in the fact that any term entering such a column formula has a definite physical significance and the entire formula is based on rigorous elastic theory rather than merely representing a necessarily limited number of test results. In the hands of a discriminating designer such a formula can be adapted to almost any set of particular circumstances, such as eccentricities of loading, imperfections of shaping, etc., which is entirely impossible with the purely empirical type of formula. Too much reliance on the latter actually may lead to unsafe design, inasmuch as a column, with larger amounts of eccentricity or crookedness than were inherent in the test specimens, actually fails at loads often considerably below those given by the empirical formula. After many years of extensive experimental research the Society's Special Committee on Steel Column Research[9] recommended a formula of this rational type—the secant formula—as the basis for design specifications. It found this formula not only to be rational and definite in the physical significance of its mathematical terms, but also established its excellent agreement with the results of an unusually large number of tests. The formula is directly adaptable to any given properties of material, eccentricity of loading, imperfection of initial shape, factor of safety, and other special conditions. The design specifications for railway bridges of the American Railway Engineering Association (A.R.E.A.) are presently based on this formula and, taking advantage of its versatility, are adapted not only to standard structural steel but also to a number of special steels used in bridges.

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[7] Asst. Prof., Civ. Eng., Cornell Univ., Ithaca, N. Y.

[8] "Rational Design of Steel Columns," by D. H. Young, *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 422.

[9] *Ibid.*, Vol. 98 (1933), p. 1376.

The author's proposed formulas are exclusively of the first, purely empirical type and, therefore, in the writer's opinion, represent a definite step backward in the direction of formal expressions merely adapted to a given set of tests but devoid of fundamental physical meaning. Therefore they are not applicable to any structures except those which duplicate the test setup in all significant details. They are purely mathematical expressions representing S-shaped curves with zero slope at the lower end and zero asymptote for large values of l/i; that is, they merely have the formal mathematical characteristics of all column curves without being capable of physical interpretation.

In the writer's opinion the desirable development in this field lies in the direction of rational column formulas of the type mentioned. For standard design conditions the way a column curve was originally determined may be of little consequence. Under nonstandard circumstances—such as columns of unusual alloys, special loading conditions (eccentricities), and particular fabricating methods—which may involve either larger or smaller imperfections of shape than are commonly met with, formulas of the rational type give considerable guidance to the designer even without, or with very limited, test evidence. The author's formulas, to be applicable, would require elaborate tests in each of these special cases. It is precisely for this reason (to take care of unusual situations) that the A.R.E.A., in an appendix to its specifications, included the complete secant formula with the physical significance of all its terms explicitly stated. From his own design experience with structures of nonstandard material and nonstandard but at least approximately known imperfections, the writer can testify to the reliability of this method which, in some cases he dealt with, was very succesfully verified by a small number of full-scale confirmatory tests.

The secant formula, and others of the same type, permit the engineer to predict the maximum fiber stress under a given load for a column of any material provided the following characteristics are known with reasonable accuracy: Properties of material, end conditions, eccentricities of loading, and initial deviations from straightness (crookedness). By equating this maximum fiber stress to some stress considered as dangerous (such as the yield point for steel) and by applying a suitable factor of safety, a practical column formula is immediately derived which covers the entire range of slenderness ratios. The reason that, despite these advantages, the explicit secant formula has not become too popular with designers lies in the fact that it is rather unwieldly mathematically. It requires the use of trigonometric tables and cannot be solved explicitly for the working stress for a given slenderness ratio; that is, it necessitates in each case the drawing of a complete column curve. This practical difficulty is easily overcome by developing general column formulas of the standard familiar type (Rankine and Johnson formulas), which are made to fit very closely the values of the exact secant formula.

The general form of the secant formula is

$$\left(\frac{P}{A}\right)_w = \frac{\dfrac{\sigma_y}{\eta}}{1 + a \sec\left[\dfrac{k\,l}{2\,i}\sqrt{\dfrac{\eta}{E}\left(\dfrac{P}{A}\right)_w}\,\right]} \quad\ldots\ldots\ldots\ldots(22)$$

in which (supplementing the notation of the paper): $\left(\frac{P}{A}\right)_w$ = working stress in compression; σ_y = limiting (that is, dangerous) value of extreme fiber stress, generally the yield point of the material; η = factor of safety; and a = "degree of imperfection"; that is—

$$a = \frac{(e + d)\,c}{i^2} \quad\ldots\ldots\ldots\ldots(23)$$

in which e = eccentricity of loading; d = maximum deviation from straightness (crookedness) of column; and c = distance from center of gravity to outer fiber of cross section.

Accepted values for these constants are: η = 1.7 to 1.8 for bridges and 1.5 to 1.6 for buildings; a = 0.25 for standard manufacturing conditions and centered connections; and k = 0.75 for riveted and welded connections and 0.875 for pin connections.

A very close approximation of the values of the secant formula in the low and medium range of l/i is obtained by the expression

$$\left(\frac{P}{A}\right)_w = \frac{\sigma_y}{n_s} - \frac{\sigma^2_y\,k^2}{4\,n_s\,\pi^2\,E}\left(\frac{l}{i}\right)^2 \quad\ldots\ldots\ldots\ldots(24)$$

in which

$$n_s = \eta\,(1 + a) \quad\ldots\ldots\ldots\ldots(25)$$

Eq. 24 is of the Johnson parabolic type familiar to designing engineers. Most column specifications, such as those of the American Institute of Steel Construction (A.I.S.C.) and the American Railway Engineering Association (A.R.E.A.), are written in terms of the Johnson formula for the low and medium range of l/i. The values obtained from Eq. 24 are extremely close to those of the secant formula, Eq. 22, for all values of the working stress down to

$$\left(\frac{P}{A}\right)_w = 0.5\left(\frac{P}{A}\right)'_w = 0.5\,\frac{\sigma_y}{n_s} \quad\ldots\ldots\ldots\ldots(26)$$

in which $\frac{\sigma_y}{n_s}$ is the working stress at $l/i = 0$. The limiting value of l/i at which $\left(\frac{P}{A}\right)_w = 0.5\left(\frac{P}{A}\right)'_w$ is

$$\left(\frac{l}{i}\right)_{\text{limit}} = \frac{\pi}{k}\sqrt{\frac{2\,E}{\sigma_y}} \quad\ldots\ldots\ldots\ldots(27)$$

For values of l/i larger than those given by Eq. 27, the Johnson curve deviates increasingly from the secant curve and, therefore, should no longer be used for design purposes. It will be found that for structurally useful alloys the value of $\left(\frac{l}{i}\right)_{\text{limit}}$ ranges from about 80 to 160, depending on mechanical properties. (The use of 0.5 $\left(\frac{P}{A}\right)'_w$ as the end point for the Johnson formulas was first suggested to the writer by Prof. J. H. Cissel, M. Am. Soc. C. E., of the University of Michigan, Ann Arbor, in a discussion concerning general questions of column design.)

For slenderness ratios larger than $\left(\frac{l}{i}\right)_{\text{limit}}$ a very close approximation of the secant formula is obtained by the expression,

$$\left(\frac{P}{A}\right)_w = \frac{B}{1 + \frac{1}{C}\left(\frac{l}{i}\right)^2} \quad \text{.....................(28)}$$

in which

$$C = \frac{500{,}000\,\pi^2\,E\,\eta\,a}{250{,}000\,k^2\,\eta\,\sigma_y - 2\,n_s\,\pi^2\,E} \quad \text{...............(29a)}$$

and

$$B = \frac{\sigma_y}{2\,n_s} + \frac{1}{C}\,\frac{\pi^2\,E}{k^2\,n_s} \quad \text{.....................(29b)}$$

Eq. 28 is adjusted to give working stresses extremely close to those of the secant curve up to values of $l/i = 400$ and even higher. This was achieved by making the value of $\left(\frac{P}{A}\right)_w$ from Eq. 28 equal to (Euler value)/η at $l/i = 500$. At such large slendernesses the values of the secant formula are almost identical with those of the Euler equation which results in the close approximation of Eq. 28 with the secant values.

It is seen that Eqs. 24 and 28 are given in the same general terms as is the secant formula (σ_y, E, η, k, a) and result in column formulas of the type familiar to the designer, that is, the Johnson formula for small and medium slenderness ratios and the Rankine formula for large slenderness ratios. Thus, for a given material and given conditions, design formulas, Eqs. 24 and 28, are computed once and for all by substituting appropriate values for the constants. The resulting formulas are then used in the same simple manner as any standard column formula in current codes. In fact, they are of the same form as those used in the A.I.S.C. specifications. In this manner the use of the arithmetically cumbersome secant formula is entirely eliminated, whereas all the advantages are retained. The formulas are adaptable to any type of material of known mechanical properties, which is likely to become advantageous with the increasing use of various ferrous and nonferrous alloys for structural purposes. The approximation is extremely close for all values of a from 0.1 to about 0.6; that is, in the practically important range.

The use of these formulas and the excellent approximation they furnish to the values of the secant formula are illustrated in the following two examples:

Example 1.—Consider a column of structural grade steel, under the following design conditions: $\sigma_y = 33{,}000$ lb per sq in.; $E = 29{,}500{,}000$ lb per sq in.; $\eta = 1.76$; $a = 0.25$; and $k = 0.875$. These data are precisely the same as those assumed for pin-ended members in the A.R.E.A. Specifications for Steel Railway Bridges, 1941 (see Appendix A). Substitution of the foregoing values in Eqs. 24 to 29 gives the following working formulas: For $l/i = 0$ to $l/i = 152$—

$$\left(\frac{P}{A}\right)_w = 15{,}000 - 0.323\left(\frac{l}{i}\right)^2 \qquad (30a)$$

and, for l/i larger than 152—

$$\left(\frac{P}{A}\right)_w = \frac{34{,}000}{1 + \frac{1}{6{,}510}\left(\frac{l}{i}\right)^2} \qquad (30b)$$

Eq. 30*a* is almost identical with that of the A.R.E.A. Bridge Specifications (Section III) for slenderness ratios as great as 140:

$$\left(\frac{P}{A}\right)_w = 15{,}000 - \frac{1}{3}\left(\frac{l}{i}\right)^2 \qquad (31)$$

For a greater slenderness ratio the A.R.E.A. specifications suggest the direct use of the secant formula. It is thought that the use of the more convenient Eq. 30*b*, instead of the secant formula, is preferable in a practical way.

The exact secant curve (solid curve), as well as the approximating working curves (dotted curves), is shown in Fig. 1(*a*). The close approximation obtained is immediately evident.

Example 2.—Consider a column of high-strength, corrosion-resisting steel under the following design conditions: $\sigma_y = 65{,}000$ lb per sq in.; $E = 29{,}500{,}000$ lb per sq in.; $\eta = 1.55$; $a = 0.35$; and $k = 0.75$. Substitution of these values in Eqs. 24 to 29 gives the following working formulas: For $l/i = 0$ to $l/i = 126$—

$$\left(\frac{P}{A}\right)_w = 31{,}000 - 0.971\left(\frac{l}{i}\right)^2 \qquad (32a)$$

and, for l/i larger than 126—

$$\left(\frac{P}{A}\right)_w = \frac{55{,}700}{1 + \frac{1}{6{,}140}\left(\frac{l}{i}\right)^2} \qquad (32b)$$

The curves for high-strength steel corresponding to Eqs. 32*a* and 32*b* and to the secant formula are given in Fig. 1(*b*).

Examples 1 and 2 demonstrate the adaptability of the writer's proposal, the simple form of the resulting working formulas, and the good degree of approximation as compared with the values of the secant formula.

For each of his nine formulas the author has indicated how they can be made to fit a particular series of tests. The same can be accomplished by the secant formula, as was actually done in the Society's column investigation.[9] Whereas fitting an empirical formula to a series of tests consists in the adjustment of arbitrary coefficients, the rational way to coordinate the secant formula with test results is to determine experimentally the degree of imperfection entering the formula. To establish the magnitude of this quantity it is only necessary to know $e + d$; that is, eccentricity plus crookedness (compare Eq. 23).

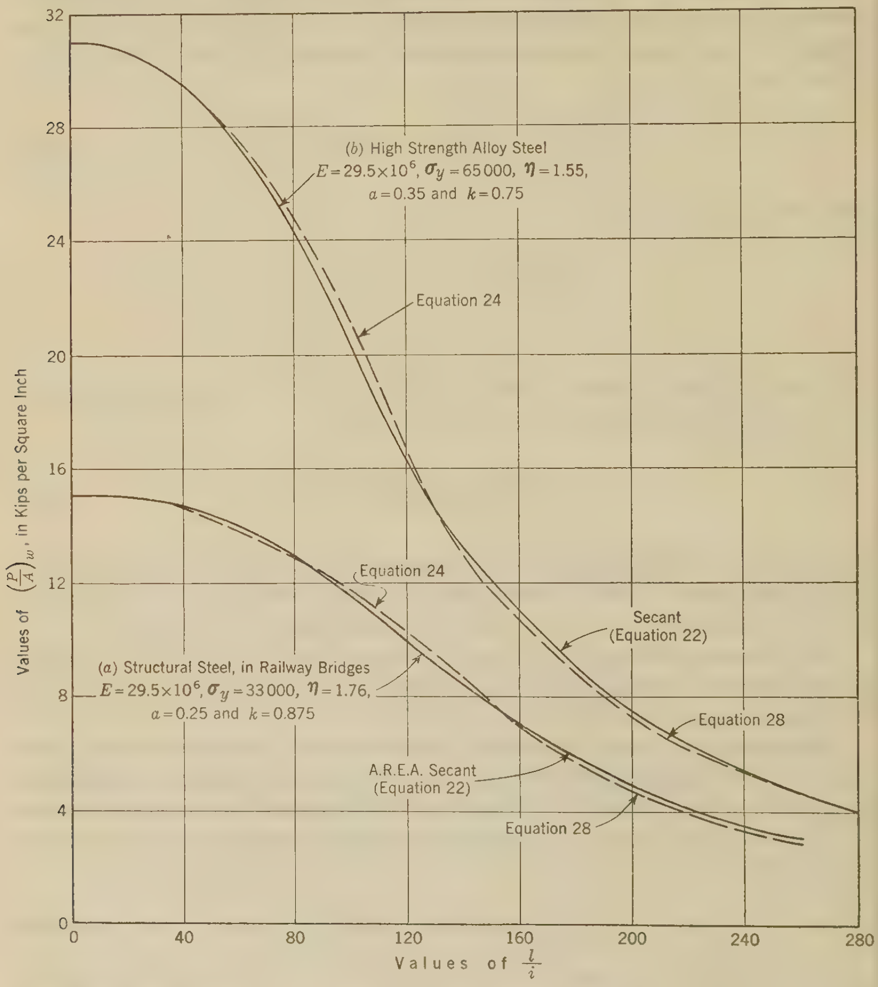

FIG. 1.—DESIGN CURVES FOR STEEL COLUMNS

The effect of these two factors is known to be almost identical; that is, an eccentricity of given amount has the same effect as a deviation from straightness of the same amount. For this reason it is superfluous to determine e and d separately, since only their sum enters the secant formula (and Eqs. 24 and 28). This quantity $(e + d)$ is easily determined in standard ways by the well-known Southwell method.[10] By this method it is even possible to determine the degree of imperfection without loading the element to destruction, which is of considerable advantage in the case of complicated special structures. Once this quantity and the physical characteristics of the material are known, the secant formula and the corresponding simplified expressions, Eqs. 24 and 28, can be applied directly.

It is quite possible that one of the author's several formulas can be made to fit the secant equation equally as well as Eqs. 24 and 28. The writer has chosen these particular formulas because they are of a form familiar to designers and widely used in design codes. Experience has shown that there is no particular disadvantage in using two different formulas for different ranges of l/i. The main point the writer wishes to convey is that, in his opinion, analytically rigorous formulas which lend themselves to complete physical interpretation are vastly preferable to empirical expressions of a purely formal, mathematical character.

A discussion by the author of the full significance of his arbitrary constant S, referred to as the secant yield strength, would contribute to a better understanding of this paper.

Jonathan Jones,[11] M. Am. Soc. C. E.—This paper is of a philosophical nature, comprising a discussion of how various curves, each defined by a formula, can be made to fit the data derived from a large number, and wide variety, of tests made on columns. It is of no assistance to a practicing engineer in what is considered "structural" work, because it proceeds, as expressed by the author in the "Synopsis" and in the "Summary," from postulates which, from the viewpoint of such an engineer, are untrue or unimportant.

The "Synopsis" states that a prime requisite of any column formula is that the average stress "approach the Euler value as the ratio of slenderness becomes infinite." As the Euler value at that ratio is zero, and as the column formulas in practical use do approach zero as the ratio of slenderness becomes infinite, this statement seems of little importance. However, it is here implied, and elsewhere stated, that a desirable property of a column formula is that it shall be susceptible to extrapolation. The practicing engineer is not concerned with extrapolation (as to values of slenderness ratio), as his entire work is within a range of slenderness ratios defined by specification.

The "Synopsis" further states that a desirable requisite is that "the first derivative of the average stress with respect to the ratio of slenderness be continuous." This condition certainly does not present itself as a necessity to the practicing engineer. Why is it stated as axiomatic?

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[10] "Theory of Elastic Stability," by S. Timoshenko, McGraw-Hill Book Co., Inc., New York and London, 1st Ed., 1936, p. 177.

[11] Chf. Engr., Fabricated Steel Constr., Bethlehem Steel Co., Bethlehem, Pa.

It is stated in the "Summary" that "there is no point in using a formula that is applicable only * * * up to a given value of the ratio of slenderness * * *." As a matter of fact, almost all of the critical work done by the structural designer is in a range up to a given value of the ratio of slenderness, and a formula that is applicable within that range is exactly what is needed.

It is stated in the "Summary" that

> "There is little to be said for a formula on the grounds of its simplicity, for any formula can be plotted to as large a scale as desired and values read from the graph, or it may be tabulated with increments in the argument as small as desired."

As a matter of fact, a practicing engineer will be confronted frequently with a need for an estimate of column capacity, at a time and place where neither graphs nor tables are available; the writer is certain that he has made such estimates, in the course of his experience, thousands of times, by the simple application of mental arithmetic, either at a saving of effort as compared with looking up a graph or a table, or when neither was at hand. It is very largely for this reason that the practicing engineer is so fond of "straight-line" or "Johnson's parabola" formulas, which lend themselves to quick, mental solution.

The attitude of the structural engineer toward column formulas has undoubtedly been oversimplified; but the correct approach for him to take is entirely different from the path marked out in this paper.

The theory leading to the "secant formula" is as valid a piece of mechanical reasoning as that leading to the customary beam theory, the "theorem of three moments," and other tools of the structural engineer. Furthermore, the secant formula has been shown to conform well to tests, where the tests have been controlled with respect to the variables entering into the formula. The point which the structural engineer must remember, however, is that, in order to use the secant formula, there must be established not only the properties of the material and the radius of gyration of the column but also an effective length (between points of contraflexure) and a constant representing the possible external moment applied through frame action, eccentricity of end loading, and crookedness resulting from fabrication, shipment, or erection. If practical column formulas need to be rendered more precise and, therefore, more complicated, the need is to recognize that these moments vary quite considerably over the entire column field. They may be safely estimated for one category of columns, and again safely estimated for some other category of columns; but it is probably useless to attempt what has been tried in the past—namely, to estimate them once and for all over the entire field.

It would appear to the writer that the most fruitful field of inquiry now open to the structural engineer is to determine, for each different type of column (compression members in framed trusses, columns in mill buildings, columns in story buildings, derrick masts, etc.), the most reasonable allowances for free length and for applied moments that should be inserted in the general secant formula, to safely and economically meet the probable loading conditions in that particular category of columns.

Having established these several allowances for moment, there can be written down, for steel of any one yield-point strength, a secant formula expressing the average unit stress at that load which causes a yield-point stress on the most severely stressed fiber. For a series of other steels, having other yield points, all these formulas can again be written, and the difference of yield point will make them all different, in detail only, from the corresponding members of the first series.

It is reasonable to believe, from the work already reported by a committee of the Society, that each of these formulas would quite reasonably represent the maximum loads that would be obtained from a series of tests, introducing in each particular case the material and the moments postulated in setting up the formulas.

The structural engineer would then affect each of these formulas by a factor of safety, representing the number of times (usually between 1.50 and 2), by which the anticipated total load could be assumed to be safely multiplied before the extreme fiber stress would reach the yield point in question. These then would be a set of safe-load formulas, all based upon the same reasonable premises, and each safe for practical use. However, one serious practical difficulty would remain—that the secant formula does not lend itself readily to arithmetical solution. It therefore would remain to be seen whether it would be possible to substitute, for the safe-load formulas thus in hand, simpler formulas, easier to handle, and not departing unduly from the secant series.

In the case of compression members in railroad bridge trusses, it is common knowledge that the American Railway Engineering Association (A.R.E.A.) has found such a substitution possible—

$$P = 15{,}000 - 0.25\left(\frac{l}{i}\right)^2 \ldots\ldots\ldots\ldots\ldots\ldots (33a)$$

—for values of slenderness ratio up to 140. The American Institute of Steel Construction (A.I.S.C.) has established a similar parabola—

$$P = 17{,}000 - 0.485\left(\frac{l}{i}\right)^2 \ldots\ldots\ldots\ldots\ldots\ldots (33b)$$

—for columns in buildings, up to a slenderness ratio of 120. Beyond these slenderness ratios it has been found, by these specification-writing bodies, that the secant curve and the second-degree parabola depart from each other rapidly. They have accordingly adopted other curves for slenderness ratios greater than those just stated—A.R.E.A. choosing the secant formula itself, and A.I.S.C. choosing the Rankine-Gordon type, as a reasonable equivalent.

In actual bridge and building practice, inasmuch as columns of greater slenderness ratios than those to which the second-degree parabola can be applied either are extremely rare, or are relegated by the specifications to non-critical elements of the structure, it would seem to the writer to be fully justifiable to adopt, for simplicity, for the second portion or end of each curve, a curve of the Euler form—namely, some easily remembered constant number in the numerator and the familiar $\left(\frac{l}{r}\right)^2$ in the denominator (the author's

$\left(\frac{l}{i}\right)^2\bigg)$; or an inverted parabola, which also can be kept simple in form and easy to solve.

The writer would not wish it to be inferred that he considers any of the foregoing remarks to be applicable outside the field of usual columns of structural steel, in bridges, buildings, and like structures; in that field, however, he feels that the discussion of column formulas should be entirely uninfluenced by the hypotheses and developments of the present paper.

Edward Godfrey,[12] M. Am. Soc. C. E.—The subject of this paper is of great importance to designers. With the exception of the Euler formula, rational column formulas are conspicuously absent from engineering works. The Euler formula is the one rational formula in common use, but it is applicable to slender columns only and has no meaning when applied to columns with a slenderness ratio usually specified as proper in ordinary structural work. Furthermore, column formulas in common use give totally false unit values when used for columns with slenderness ratios to which the Euler formula is applicable.

In spite of statements in column-design literature, the Euler load, instead of being the load a column can sustain in equilibrium, is the absolute maximum load—the load at which the column is certain to fail. The proof of this statement is quite simple. Assume that a column is in the shape of a bow with a finite versed sine or offset. The Euler load, by its derivation, will double that offset. It will not stop there, however. The column is still a bow with an offset and that offset will be increased again for the same reason that the first increase occurred. This process will continue indefinitely, or until the column has failed completely, since the increments are not decreasing. If the load adds one half to the original bow, the final bow will be only twice the original bow or offset. When $x = \frac{1}{2}$, the infinite series $1 + x + x^2 + x^3 + \cdots$, etc., is 2.

In the "Summary," Mr. Osgood disparages the use of one formula for short columns and another for slender columns. The writer does not agree with this opinion since it is on this very point that he has always differed with standard theories on the subject.

The criterion for shorter columns—those in structures such as bridges and buildings where the ratio of slenderness is properly restricted—is stress per square inch on the extreme fibers of the section. Based on the strength of the material of the column, the stress must not exceed certain values which are limited by a factor of safety. The criterion for slender columns—those outside the limit of the slenderness ratio permitted by specifications—is not in any sense the compressive strength of the material of the column, since only the modulus of elasticity (a property nearly constant for the various grades of steel) is needed for the derivation of the formula.

There are two distinct phases to be considered in determining the strength of a compression member. Failure to appreciate this fact has caused much misapprehension. Formulas have been derived, and accepted, that attempt to combine the two phases of the strength of columns in utter disregard of the distinction between the two.

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Every column of elastic material is a spring or bow as well as a shaft subject to endwise compression, and the strength of a column considered as a spring is independent of the ultimate or elastic strength of the material. In addition, the formula showing this strength is quite distinct from any formula based on ultimate strength of the material. The Gordon-Rankine formula has limited application. This formula is fraught with danger, if it is used in the range of slender columns.

That the straight-line formula for the strength of a column can be substantiated, rationally and theoretically, has been demonstrated elsewhere by the writer.[13]

In former years at least, European engineers used the Euler formula with a factor of safety of five. There is no reason for so high a factor of safety in the region where the Euler formula is applicable. The Euler formula is never applicable to short columns. A factor of safety of two, or even less, is ample for use in the Euler load, if the Euler formula is applicable, because the condition that causes failure is not reached until the load approaches the Euler value. Also, columns not in the slender bracket are likely to fail at the first observable sidewise deflection.

In tests described by C. P. Buchanan,[14] the deflection due to the load did not exceed 0.2 in. within the elastic value of the column for any column tested, and at the ultimate load, in many cases, this deflection did not increase.

In his previously cited work,[13] the writer shows the locus of an exceedingly complex formula and comparison with the Euler load in the slender range (with a factor of safety of two). The complex formula referred to is completely rational. It is based on the simple assumption that a column is bowed with an offset 1/300 of its length. The compressive load increases the bow by an infinite series of decreasing terms. The bow is not "frozen," as in the derivation of the Gordon-Rankine formula.

The formula is too cumbersome to use in practice, but the locus, up to 90 radii, is a straight line, and beyond this it deviates but little from a straight line in the range of structurally permitted columns. In the "slender" range it almost coincides with the Euler value with a factor of safety of two, which is ample for slender columns.

However, instead of being a reason for the use of one formula for both slender and short columns, it is a sound reason for the use of the ordinary straight-line formula for bridge and building columns and the Euler load for light struts, such as are used in transmission and radio towers.

Ralph E. Spaulding,[15] M. Am. Soc. C. E.—In the study of column formulas it is not generally recognized that the Euler formula,

$$\frac{P}{A} = \frac{K\,\pi^2\,E}{(L/i)^2} \quad \text{.......................} (34)$$

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[13] "Steel Designing" (Book Three of a Series on Structural Engineering), by Edward Godfrey, 1913, Chapter X, pp. 152–162.

[14] "A Series of Failure Tests of Full-Size Compression Members, Made for the Pennsylvania Lines West of Pittsburg," by C. P. Buchanan, *Engineering News*, December 26, 1907, p. 685.

[15] Cons. Engr., Jacksonville, Fla.

gives true theoretical values of P/A for all values of L/i from zero to infinity when, and only when, the yield strength is equal to the modulus of elasticity ($\sigma_y = E$). Since this relationship does not exist in metals, it is necessary to modify, or complete, the Euler formula in order to adapt it to any required use. This may be done by introducing the yield-strength factor into the Euler formula; thus:

$$\frac{P}{A} = \frac{K\,\pi^2\,E}{(L/i)^2} \times \frac{\dfrac{K\,\pi^2\,E}{(L/i)^2} + \dfrac{\sigma_y\,(L/i)^2}{K\,\pi^2} + (E + \sigma_y)}{\left[\dfrac{K\,\pi^2\,E}{(L/i)^2} \times \dfrac{E}{\sigma_y}\right] + \dfrac{\sigma_y\,(L/i)^2}{K\,\pi^2} + (E + \sigma_y)} \ldots\ldots (35)$$

Eq. 35 can be applied to any metal that follows Hooke's law by simply inserting the respective values of σ_y, E, and the end condition factor K. When σ_y becomes equal to E, Eq. 35 reverts to the Euler formula.

It is interesting to test this formula in connection with available data concerning the values of σ_y and E. In Fig. 2(a), the writer has plotted Eq. 35 for values of L/i as high as 400, for three metals with different characteristics. Apparently, this formula fulfils all the desirable requisites for a column formula as enumerated in this paper.

If the yield strength and the modulus of elasticity are known, the values may be inserted in Eq. 35 and the resultant values of P/A may be determined with reasonably dependable accuracy for all values of the ratio of slenderness.

There is a distinction between the column formulas proposed for the determination of allowable stresses in columns as built in engineering structures and formulas that reflect the probable ultimate load on columns at failure under laboratory conditions. Probably the author did not intend to open for discussion the relative merits of these many formulas. The column behavior under laboratory conditions can be predicted with reasonable accuracy; but, because of unexpected eccentricities and undetected defects in manufacture and erection, the design of columns for practical use becomes more a matter of judgment and experience aided by theory based on observed laboratory tests. The laboratory tests indicate the maximum limits beyond which the working formula cannot go.

Hundreds of formulas have been devised, and used, for the determination of the working stresses in columns. The writer takes the liberty of suggesting one more.

In columns, as built into structures (in contrast with columns tested under perfect laboratory conditions), the end condition factor, K, is a variable rather than a constant. For example, in a flat end column this factor would be 4 when the column was very short; but, as the column became relatively longer and the effects of manufacturing defects, residual stresses, etc., began to have more influence on the ultimate strength, the factor should be reduced. Reduction in accordance with the following formula is suggested—

$$K \text{ (effective)} = K \text{ (initial)} \times \frac{\sigma_y}{\sigma_y + (L/i)^2} \ldots\ldots\ldots\ldots (36)$$

A somewhat simplified and closely approximate form of Eq. 35 with the factor

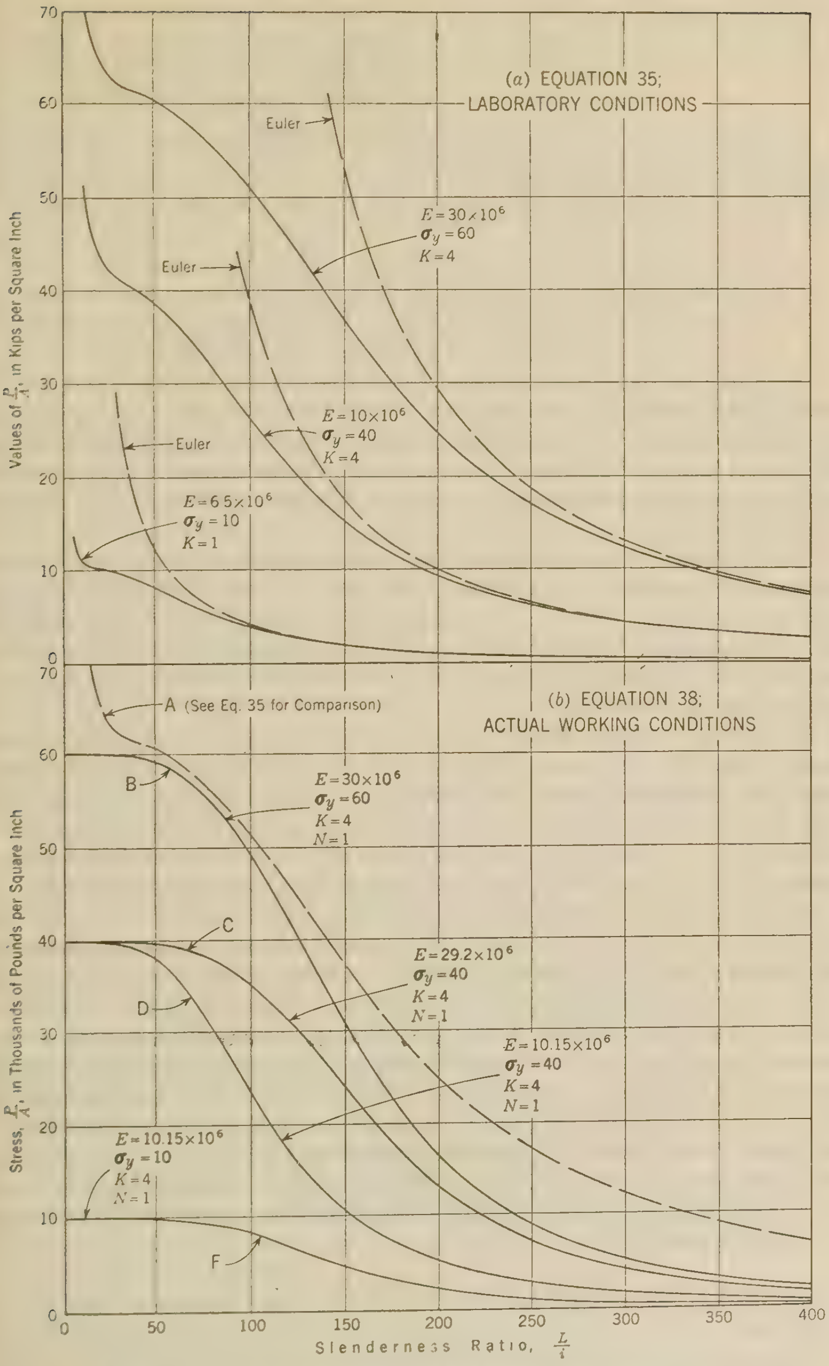

FIG. 2.—COMPARISON OF COLUMN FORMULAS FOR STEELS WITH VARIOUS PROPERTIES

of safety, N, added is—

$$\frac{P}{A} = \frac{\sigma_y}{N} \times \frac{1 + \dfrac{K\pi^2 E}{\sigma_y (L/i)^2}}{1 + \dfrac{K \pi^2 E}{\sigma_y (L/i)^2} + \dfrac{\sigma_y (L/i)^2}{K \pi^2 E}} \dots\dots\dots\dots (37)$$

and, by inserting the value of K proposed by Eq. 36 for the K-value in Eq. 37—

$$\frac{P}{A} = \frac{\sigma_y}{N} \times \frac{1 + \dfrac{K \pi^2 E}{\sigma_y (L/i)^2 + (L/i)^4}}{1 + \dfrac{K \pi^2 E}{\sigma_y (L/i)^2 + (L/i)^4} + \dfrac{\sigma_y (L/i)^2 + (L/i)^4}{K \pi^2 E}} \dots\dots (38)$$

which is the writer's suggested column formula for allowable working stresses in columns as built. The value of the end condition factor, K, still remains to be inserted in accordance with the actual initial end condition. The factor of safety can best be determined by judgment of existing conditions and requirements.

Eq. 38, like Eq. 35, is applicable to any metal or alloy that follows Hooke's law and for any value of the ratio of slenderness—in contrast to many formulas that are applicable only to one metal and often only within a narrow range of L/i. Although Eq. 38 is not quite as simple as some others that do not have such universal applicability, it is a reasonably simple formula, having only one variable together with its reciprocal.

In Fig. 2(*b*), curves are plotted from Eq. 38. Curve A (the only curve plotted from Eq. 35) shows the probable ultimate column strength under laboratory conditions for an alloy with the properties indicated in Fig. 2(*b*), whereas curve B for the same metal, and for actual working conditions, gives the suggested maximum design stress before application of a desired factor of safety. Curves C, D, and F (Fig. 2(*b*)) are of the same type as curve B.

Curves C and D illustrate the comparison in column strength between two metals having the same yield strength but very different moduli of elasticity; and curves D and F illustrate the difference between two metals having the same modulus of elasticity but quite different yield strengths. When the ratio of slenderness is from about 120 to 140, column C has an allowable working stress equal to twice that of column D although the respective yield strengths of the columns are the same. Column D has an allowable working stress only twice that of column F, whereas the yield strength of metal D is four times that of metal F. This relationship may be verified by reference to actual test data. In other words, as has always been known, except for very short columns, a high elastic modulus is of greater economic importance than a high yield strength.

JAMES H. CISSEL,[16] M. AM. SOC. C. E.—In the "Synopsis," the author states that

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> "The * * * requisites of any column formula * * * are that the average stress be a continuous, monotonically decreasing function of the ratio of slenderness and that it approach the Euler value as the ratio of slenderness becomes infinite."

It may be questioned whether this requirement in itself is an objective of formulas for compression members. The main purpose of any column formula should be to express, in convenient form, the behavior of a column in terms of dimensions and critical load.

The "Summary" states that "There is little to be said for a formula on the grounds of its simplicity * * *." This viewpoint completely ignores the need of the practicing engineer for a procedure of estimating the capacity of a given member quickly and accurately.

Any acceptable column formula must correctly appraise the probable ultimate axial load on a column by proper consideration of any combination of the following four possible modes of failure: (1) Compressive failure by crushing the material; (2) column failure or bending over the unsupported length of the member (in this type of action, the cross sections are translated laterally in the direction of least resistance of the member); (3) torsional failure in which the cross sections are rotated with respect to the longitudinal axis of the member; and (4) plate buckling or local failure of elements of the section by wrinkling over a limited length of member.

Failure (1) is the type that would occur in a very short member (l/i less than, say, 20) with elements or walls of such ratios of width to thickness as are necessary to preclude local wrinkles or buckles. Current specifications such as those of the American Railway Engineering Association (A.R.E.A.) or the American Institute of Steel Construction prescribe minimum thickness and maximum values of width to thickness for plates and elements; and, hence, short members complying with such specifications will not fail until the yield stress in the material has been developed by an axial load.

Failure (2) implies bending over the length of the column induced by eccentricity of loading and crookedness of member. If the thickness of elements is sufficient to prevent local wrinkling, failure will not occur until the stress on the extreme fiber due to direct stress and bending has reached the yield stress of the material. The appraisal of this action is the general purpose of column formulas, and the unstated assumption incorporated in the commonly used equation is that the elements of the section are elastically stable for unit stresses up to the yield stress of the material.

Failure (3) represents a twisting of the section with respect to the longitudinal axis, and is particularly likely to occur in members lacking torsional resistance, or subject to eccentric loads. It does not appear to have great practical importance for members with the proportions usually found in average structures, but may become significant for unsymmetrical sections or for long, slender members.

Failure (4) is sometimes called local failure by buckling of elements comprising the section. Current specifications attempt to eliminate this type of failure from consideration by specifying minimum thicknesses and proportions of parts, such as outstanding legs of flange angles. Such limitations merely prevent the use of thin-walled members, which may, in fact, be perfectly satisfactory and economical for a given purpose.

As a result of its extensive study of steel columns, the Society's Special Committee on Steel Column Research[9] recommends the secant formula as best interpreting the test results. Allowing for adaptation to the author's notation, this formula is commonly expressed in the form:

$$\sigma_w = \frac{P}{A\,\eta} = \frac{\dfrac{\sigma_y}{\eta}}{1 + a \sec \dfrac{l'}{2\,i}\sqrt{\dfrac{P\,\eta}{A\,E}}} \qquad (39)$$

in which σ_w is the allowable working stress; η is the safety factor equal to unity for ultimate load; P is the ultimate axial load; σ_y is the yield strength of material; A is the cross-sectional area; a is equal to $e\,c/i^2$ equal to the eccentric ratio; i is the radius of gyration of section; l' is the free length between inflection points; and E is the modulus of elasticity.

The validity of the secant formula has been proved repeatedly by test, and probably is as rational an equation as can be developed for expressing the ultimate axial load on columns that fail in accordance with failures (1) and (2). It cannot be safely applied in the form expressed in Eq. 39 to thin-walled sections or to members that are critical as regards torsional resistance. Moreover, since it is difficult in practical cases to determine precisely the value of the eccentric ratio expressed by the term a in Eq. 39, the use of a is inconvenient for practical design purposes. In recognizing the latter objection, the Society's committee recommended[9] the use of an average value of 0.25 for a, and a parabolic equation for values of l'/i to 160. In following this recommendation, the A.R.E.A. permits the use of the following formula for allowable working stress on structural grade steel columns:

$$\sigma_w = 15{,}000 - 0.33\left(\frac{l}{i}\right)^2 \qquad (40)$$

Eq. 40 gives a close approximation of values obtained from Eq. 39, when $\sigma_y = 33{,}000$, $a = 0.25$, $l' = 7/8\,l$, and $\eta = 1.76$. It should be observed that Eq. 40 will produce results consistent with those obtained by the secant formula, only when $a = 0.25$. This value, set by the Society's committee after a comprehensive study of the tests, provides an allowance for eccentricity of loading and eccentricity due to crookedness of member, as follows:

Eccentricity of applied loading $= 0.15\,i^2/c$
Eccentricity due to crookedness $= 0.10\,i^2/c$

Thus, for a cross section whose radius of gyration is 0.8 c, the load is assumed to be applied at a distance of 0.096 c from the axis, and the deviation of the axis from an absolute straight line is assumed to be 0.064 c, regardless of the length of the member. Also, the allowance for crookedness was based on a tolerance of $\frac{1}{1{,}200}\,l$ for a member with length $l = 100\,i$, and with a radius of gyration $i = 0.8\,c$. The effect of the eccentric ratio in producing variation in values of P/A is shown diagrammatically in Fig. 3.

Considering the fact that, in nearly all actual cases, the allowances for eccentricity in the secant formula are mere guesses, the writer prefers the Johnson formula as a means of estimating column capacity. This formula expresses the behavior as a parabolic curve tangent to the Euler curve when $P/A = 1/2\,\sigma_y$. Thus:

$$\sigma_w = \frac{P}{A\,\eta} = \frac{\sigma_y}{\eta} - \frac{\frac{\sigma^2{}_y}{\eta}}{4\,C\,\pi^2\,E}\left(\frac{l}{i}\right)^2 \dots\dots\dots\dots\dots\dots (41)$$

in which l is the unbraced length of member and C is a factor determined by the conditions of end restraint affecting the member (1.0 for hinged ends and 4.0 for fixed ends).

When $\frac{P}{A} = \frac{1}{2}\,\sigma_y$, $\frac{l}{i} = 4.44\sqrt{\frac{C\,E}{\sigma_y}}$; and, for greater values of $\frac{l}{i}$, the Euler formula is used, in which

$$\frac{P}{A} = \frac{C\,\pi^2\,E}{(l/i)^2} \dots\dots\dots\dots\dots\dots\dots\dots (42)$$

A comparison of the Johnson-Euler curves with those obtained from the secant formula is shown in Fig. 4.

Since Eqs. 39 to 42 apply only to cases where failure is of the types described herein as failures (1) and (2), they cannot be used without modification for thin-walled sections where width:thickness ratios of elements are outside the limits set by such specifications as those of the A.R.E.A. For sections outside such width:thickness limitations, the ultimate resistance is less than the yield strength of the material, and is determined by the elastic stability of the elements composing the section. The maximum critical load on such sections is that load which will induce the average critical buckling stress in the elements comprising the section, and Eqs. 39 to 42 will not be effective for loads greater than this value. The approximate value of the critical buckling stress for a section composed of rectangular-shaped elements of area can be determined by application of the Bryan-Timoshenko formula for critical stress in the following manner:

(*a*) Assume the sections to be divided into rectangular elements, or plates, attached to other similar elements at one or both edges.

(*b*) Compute the value of the critical stress for each such element from the Bryan-Timoshenko formula, and designate the ratio of this value to the yield stress of the material as the *z*-factor for the element. The Bryan-Timoshenko formula for the local crippling stress in rectangular plates may be written in the form:

$$z = \frac{\sigma_{cr}}{\sigma_y} = \frac{K\,\pi^2\,E}{12\,\sigma_y\,(1 - \mu^2)}\left(\frac{t}{b}\right)^2 \dots\dots\dots\dots\dots\dots (43)$$

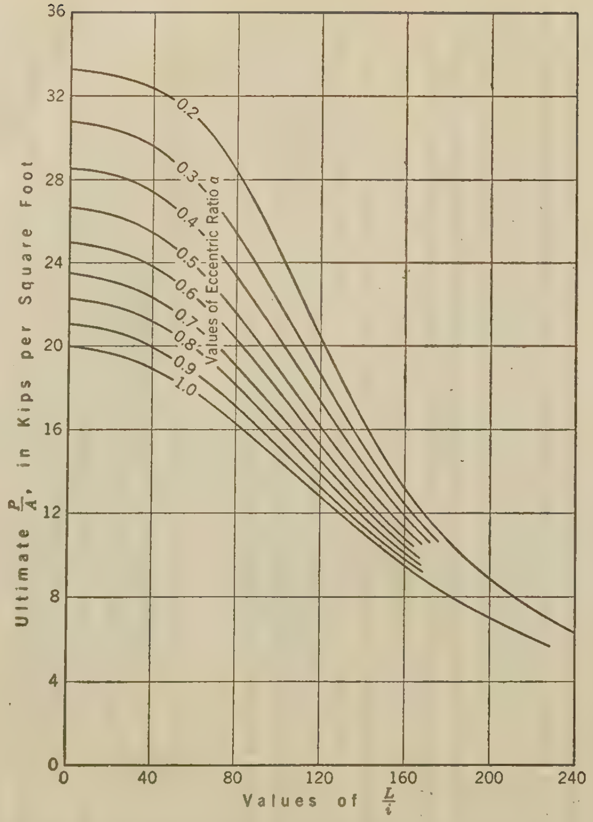

FIG. 3.—ULTIMATE AXIAL LOADS BY THE SECANT FORMULA, EQ. 39 (σ_y = 40 KIPS PER SQ IN., $\eta = 1$, $l' = \frac{7}{8} l$ FOR PIN ENDS, AND $E = 29.4 \times 10^6$)

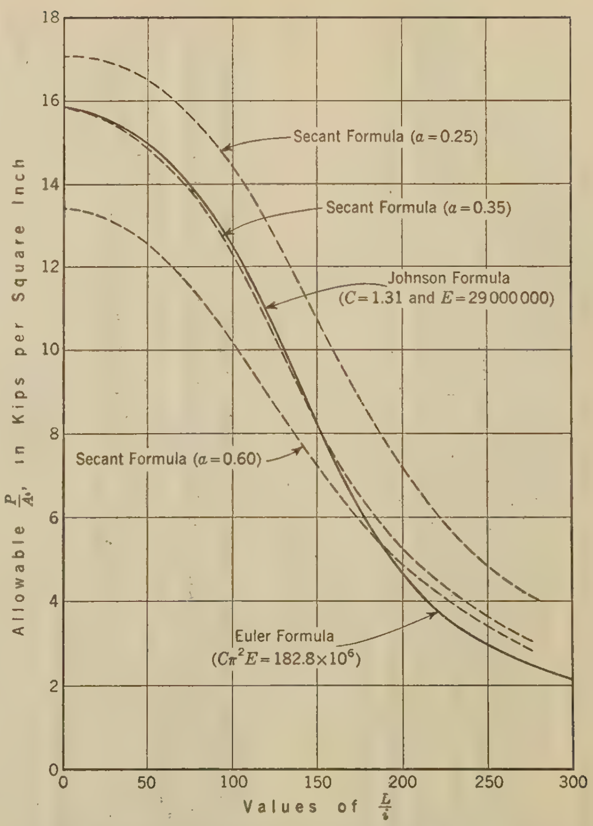

FIG. 4.—COMPARISON OF SECANT FORMULA, EQ. 39 ($\eta = 1.55$ AND σ_y = 33 KIPS PER SQ IN.), WITH JOHNSON FORMULA, EQ. 41 ($\eta = 2.09$ AND σ_y = 33 KIPS PER SQ. IN.)

in which σ_{cr} is the critical unit stress; t is the thickness of plate; b is the width of plate; μ is Poisson's ratio (0.3 for steel); and K is a coefficient determined primarily by the condition of edge support and restraint. It should be noted that, in Eq. 43, z must not be taken greater than unity.

(c) Compute the value of the form factor F for the entire section as the ratio:

$$F = \frac{\Sigma k\, b'\, t'}{\Sigma b'\, t'} \qquad (44)$$

in which b' is the width of any element and t' is the thickness of any element. The form factor, F, represents the approximate ratio of the average critical stress on the section to the yield stress of the material, and the limiting value of P/A is therefore $F\,\sigma_y$.

Since the formulas presented in this paper provide no limitation based on crippling stress, they should not be used without proper modification for sections whose form factor is less than unity.

Leon Beskin,[17] Assoc. M. Am. Soc. C. E.—That a column formula may be used as a curve on a graph, a set of precomputed values in a table, or as an algebraic expression, solved mathematically for each case that occurs in practice, is obvious. Nearly all structural designing departments plot column formulas instead of using them in their mathematical form, even if specifications give column strengths in terms of a mathematical expression. The author makes the point that, if this is so, an otherwise acceptable formula should not be rejected merely on the ground that it is complicated, since a graph of the formula is as easy to apply as the most elementary empirical expression. This seems to imply that the author proposes first to adjust a formula to fit experimental results and then to plot the resulting expression. This proposal seems to eliminate all interest in the formula since it is more straightforward to use an averaged curve of experimental results, directly without bothering with its mathematical expression.

Mathematical column formulas may be useful, however, when materials with different characteristics (such as aluminum alloys or concrete) are being considered. Graphical representation of a formula then involves a set of curves that can be applied to any material of the group. In this case it may be convenient to derive a formula in which the strength or the modulus of elasticity, or both, appear explicitly. Given the material to be used, it is then a simple matter to plot an acceptable column curve. In about 1934 the French Air Department introduced the following expression:

$$\sigma = \frac{1 + \frac{3\sqrt{3}}{2}\lambda^3_o}{1 + \lambda^2_o\left(1 + \frac{3\sqrt{3}}{2}\lambda^3_o\right)} \qquad (45)$$

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[17] Structures Engr., Consolidated Vultee Aircraft Corp., San Diego, Calif.

Eq. 45 is valid for $\lambda^2_o > \frac{1}{3}$, whereas, for $\lambda^2_o < \frac{1}{3}$, σ is assumed equal to unity. The value of σ is asymptotic to the Euler value $\sigma = \frac{1}{\lambda^2_o}$ for infinite values of λ_o, is equal to 1, and has a derivative equal to zero, for $\lambda^2_o = \frac{1}{3}$, so that the conditions of continuity, monotony, and continuity of the derivative are satisfied. The same result is obtained when, for a wide class of aluminum alloys, a straight line or a Johnson parabola, tangent to the Euler curve and corresponding to the block strength for $\lambda_o = 0$, is used.

The curves resulting from Eq. 45 do not satisfy the author's condition of extrapolation, and it is not necessary that they do so since the Euler curve in one case and the limiting value $\sigma = 1$ in the other are so defined that errors in extrapolation are normally avoided. There seems no doubt that the condition of possible extrapolation is unnecessary if the entire range of possible slenderness ratios is covered by a formula (or by several formulas). One such limitation may be defined as the Euler formula, whereas, for small slenderness ratios, a "cutoff" constant value may be used.

Furthermore, a system of several formulas satisfying the conditions of continuity, monotony, and continuity of the derivative as a whole can always be replaced, either exactly, or within the desired degree of accuracy, by a unique formula satisfying the same conditions so that the condition of a unique formula has no mathematical significance. Consequently, there is no practical, mathematical, or logical objection to the use of a limited number of simple formulas, instead of one complicated formula covering the entire range of possible slenderness ratios. An engineer can memorize simple formulas and compute allowable stresses on the slide rule, or by any other method best suited to his immediate needs, whereas a complicated formula leaves the practical engineer no choice but to plot a graph. Since that is his only choice, the best possible graph is that plotted directly from test data.

The writer cannot sense the importance of the condition stated in the "Introduction" that "An empirical [column] formula * * * should be one that expresses theoretically possible relations." The "theoretically possible relations" referred to by the author seem limited exclusively to the condition defined by the Euler formula. If a given empirical formula is claimed to be valid only for stresses less than the Euler values, that formula is absolutely legitimate as soon as it has been shown to fit the experimental results. In addition, if the formula is represented by a curve tangent to the Euler curve, it satisfies the condition of continuity of the derivative; but this condition is not important, since the stress-strain curve is never considered necessary to satisfy it. Since the column curve is a function of the stress-strain curve, it is possible that none of these curves satisfies the condition of continuity.

Thus, of all the conditions expressed by the author, the condition of monotony appears to be the only justifiable one. Incidentally, a decreasing function is always a monotonically decreasing function, because the concept of monotony is broader than the concept of the decreasing function. Mathematically, the word "monotonic" means "not decreasing or not increasing as the independent variable increases."[18] When Mr. Osgood refers to a "monotonically decreasing function" in the "Synopsis," it is understood to mean that the dependent variable "stress" does not increase as the independent variable "slenderness ratio" increases.

As stated by the author, the derivation of a column formula from a stress-strain curve is a difficult task. The double-modulus theory, which has been used for a long time, involves a basic error in theory, in that it presupposes a theoretically straight column. When a column with some initial curvature or eccentricity is considered, the double-modulus theory cannot be applied, and then the single-modulus theory appears to be more representative of both the theoretical picture and the actual facts.

A so-called "theoretical" formula, derived from an empirical stress-strain curve, cannot be considered theoretical, according to the precise meaning of the word as used by physicists. A formula of "correct form" (see "Introduction") cannot be devised without introducing at least one observed constant of the structure, and one observed constant related to the material. A formula of correct form should contain at least the slenderness ratio and one constant related to the strength of the material in compression (for a slenderness ratio of zero). In addition, since the author's opinion is that the "correct form" implies that the curve must be asymptotic to that of the Euler formula, the modulus of elasticity of the material must appear explicitly in the formula.

In the "Introduction," the author declares that "It would be very hard to justify more than two constants of the material in any practical column formula." All the formulas that he indicates, with the sole exception of the Rankine formula, contain two dimensionless constants. If only two constants of the material, one of them being the modulus of elasticity, were to be used, a single dimensionless constant, related to the ratio between the yield point (or the ultimate strength) and the modulus of elasticity, would be sufficient to define a column formula. The occurrence of two dimensionless constants in nearly all formulas shows that at least three constants of the material must be considered. Possibly, these constants could be taken as the average modulus of elasticity before the yield point, the average modulus of elasticity after the yield point, and the yield strength of the material. In the relation between free length and actual length (Eq. 2), the constant k may be different from the theoretical values such as $k = \frac{1}{2}$ for fixed ends, when the material is not in the elastic range, because of the important effect of initial eccentricities. In addition, when the ends are restrained elastically, the constant k is a function of the moduli of elasticity of both the support and the beam, and varies with the load when the column is not in the Euler range.

In the text following Eq. 2, the author states that the arbitrary constant S "* * * should be taken as a compressive secant yield strength." A yield strength defined by an arbitrary value of the permanent strain could be used as well as the secant yield strength. It would be interesting to learn if the author has a definite reason for using the secant yield strength rather than some other constant of the same nature.

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[18] "Webster's New International Dictionary of the English Language," G. & C. Merriam Co., Springfield, Mass., 2d Ed. (unabridged), 1939, Vol. I, p. 1588.

A better choice for the constant S seems to be the intercept, S_o, of the column curve with the axis, σ. This choice eliminates one constant from all the dimensionless equations summarized by the author, with the exception of the two formulas (Eqs. 15 and 18) derived by means of the double-modulus theory. Eqs. 15 and 18 are not practical since they give an infinite column load for a slenderness ratio of zero. When the value of S is assumed equal to that of S_o, the various nondimensionless equations are transformed as follows:
The Rankine formula (Eq. 3*a*)—

$$\sigma = \frac{1}{1 + \lambda^2_o} \quad \text{(46)}$$

the secant formula (Eq. 7*a*)—

$$\cos \frac{\lambda_o \pi \sigma}{2} = \frac{1}{\frac{1 - C}{\sigma} + C} \quad \text{(47)}$$

the Aarflot formula (Eq. 9*a*)—

$$\sigma = \frac{1}{C \lambda^2_o + \sqrt{1 + (1 - C)^2 \lambda^4_o}} \quad \text{(48)}$$

the Ylinen formula (Eq. 11*a*)—

$$\lambda^2_o = \frac{1 - \sigma^n}{\sigma} \quad \text{(49)}$$

the author's formula (Eq. 13*a*)—

$$\sigma = \frac{1}{1 + C \lambda_o + \lambda^2_o} \quad \text{(50)}$$

the parabolic formula (Eq. 20*a*)—

$$\sigma = 1 - \frac{1}{n + 1} \left(\frac{n}{n + 1} \right)^n \lambda^{2n}_o \quad \text{(51)}$$

and the Kreüger formula (Eq. 21*a*)—

$$\sigma = \frac{1 - \lambda^2_o C^2}{1 + (1 - 2 C) \lambda^2_o} \quad \text{(52)}$$

Experiments show that, when the shape of the cross section of a column is modified for a constant value of the slenderness ratio, the ultimate load is also changed. The constants which appear in the formulas are thus shown to be related to the shape of the cross section, as well as to the material—a fact that is easily explained when the influence of an initial curvature or eccentricity is considered.

Regrettably, the dimensions of the constants in Eqs. 3 to 21 are not the same throughout all the formulas. In the following tabulation, dimensionless constants are denoted by zero, and S represents a constant of the dimension of a stress:

Constant	B	C
The Rankine formula, Eq. 3	0	....
The secant formula, Eq. 7	$1/S$	0
The Aarflot formula, Eq. 9	0	0
The Ylinen formula, Eq. 11	$(1/S)^n$	....
The author's formula, Eq. 13	0	0
The author's formula, Eq. 15	$(1/S)^{n-1}$	....
The author's formula, Eq. 18	$(1/S)^{n-1}$	....
The hyperbolic formula, Eq. 20	S	....
The Kreüger formula, Eq. 21	$1/S$	$1/S$

In the "Summary," the author indicates that "The column formulas in Sections II to V, inclusive, apply for all values of the ratio of slenderness * * *." For very small slenderness ratios, Eqs. 15 and 18 are exceptions to this statement. When these formulas are used, a "cutoff" must be made for the strength corresponding to very short columns, which violates the author's principle that column formulas must be valid for all slenderness ratios. Incidentally, it would be interesting to know how two column equations have been derived from the same basic assumptions; that is (see paragraph containing Eqs. 18), "* * * derived by the double-modulus theory from a stress-strain curve of the form of Eq. 17."

Both the secant formula, when $C > 0$, and Eq. 13 give values less than those of the Rankine formula. This indicates that these formulas have only a limited usefulness, since, as noted by the author in the "Introduction," "* * * the [Rankine] formula gives lower values of the average stress than the mean of the average stresses found in tests of most materials." This fact disqualifies all column curves below the Rankine curve.

Both the secant formula and the Ylinen formula are extremely difficult to apply since the stresses are not given explicitly in terms of the slenderness ratio. Consequently, since the formulas cannot be used directly, tables must be compiled for the computation, which makes these formulas impractical.

From the foregoing, it can be stated that only the Rankine, Aarflot, parabolic (including the Johnson parabolas and the straight line), and Kreüger formulas are of practical value in the design of columns. The remaining formulas cannot be recommended, because they do not fit experimental results, and because they do not define the strength as an explicit function of the slenderness ratio.

LOUIS BALOG,[19] ESQ.—Imperfections in technical processes, and economic considerations, limit the precision attained in making and shaping of metals and in fabricating and erecting structural members. Homogeneity and physical properties of materials, the geometric accuracy of members and structures, the conformity to the contemplated points of application, and the character of the calculated forces depend on practices that must be justified economically for various types of structures. Since all these circumstances influence the carrying capacity of a compression member, the derivation of empirical column formulas, of universal validity, is impossible.

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[19] Cons. Engr., New York, N. Y.

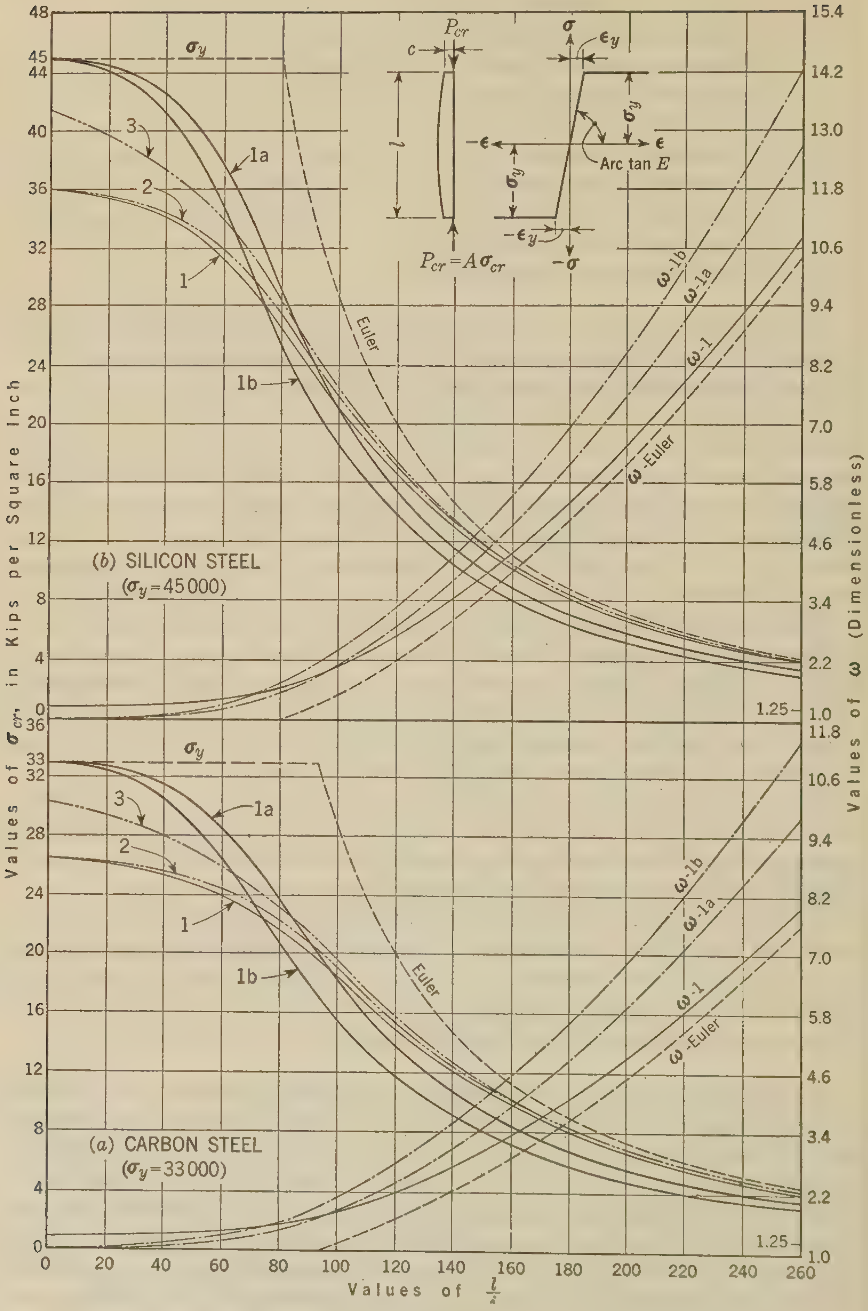

FIG. 5.—COLUMN CURVES FOR CARBON-STEEL AND SILICON-STEEL STRUCTURES (E = 29,400,000 Lb per Sq In.)

For absolutely ideal conditions the $\left(\sigma_{cr} - \frac{l}{i}\right)$-curve is composed of Euler's hyperbola and of a horizontal line $\sigma_{cr} = \sigma_y$, as shown in Fig. 5. The most exacting laboratory tests have failed to conform to this curve, due to the sensitivity of columns to deviations from ideal conditions. Study of the effects of tolerances in metal and of rolled shape characteristics, and experience with the accuracy of statical calculations and with that of shop and erection work, lead to the reduction of the theoretically ideal curve to curves shown below it in Fig. 5, which curves indicate the practical carrying capacity of carbon-steel and silicon-steel columns in bridge and structural work. The deviations between the ideal and actual conditions are so great in this case that the mathematical criteria, set up by the author for establishing the shape of column curves, seem to be of little practical consequence.

The practical validity of the formulas in the paper depends not so much on the primary requisites defined by Mr. Osgood as on the selection of the arbitrary constants they contain. These constants can indeed be so selected that the formulas will approximate special conditions more or less satisfactorily; but the restrictions on the constants, to obtain curves representing monotonically decreasing functions (the first derivatives of which are continuous) are irrelevant requirements from point of view of practical design.

Although columns have been designed by the use of empirical formulas, the general requirements of engineering practice could never be met with such formulas. This alone would be a sufficient reason for their rejection; but empirical formulas also hinder progress in design. The problem, therefore, is not to fit a formula adequately to a series of column tests (which seldom represent actual structural conditions), but to fit a theory to reality. Rational theories, by which the effects of varying all characteristics that enter into the problem can be measured, should be the basis of design.

In the "Introduction" the author states that

> "It would be possible to design by means of the double-modulus theory * * * but * * * most engineers would undoubtedly prefer to use an empirical formula based on tests."

Bridge and structural design practice does not support this opinion. Empirical formulas, based on tests, have been abandoned and theoretical formulas, in the derivation of which the modulus variation is neglected, are in general use. No other conclusion can be drawn from the widely scattering results of column tests than that only theoretical formulas are rational. Tests have failed to furnish data for the basic constants of the theoretical formulas—the eccentric ratio a and the reduced length l_o (Eq. 2)—on which two factors the validity of any practical column formula depends. Values for both of them can be determined, however, from the characteristics of actual structures and from theoretical considerations. It is quite certain that imperfections in testing procedure render such experiments useless. Extremely careful tests of small specimens, to determine the limits of validity of some basic theoretical assumption, constitute the reasonable experimental investigation.

Inasmuch as axially loaded columns do not exist in an ideal sense, only eccentric loading needs to be considered. A distinction must be made between small eccentricities ($a < 0.01$) and large eccentricities ($a > 0.1$); and actual structural columns belong in the latter category.

Numerous difficulties hinder the mathematical expression of the yielding process; and therefore the actual stress-strain relation and the actual shape of the deflection line can be considered only in graphic-analytic solutions of the problem.[20,21,22,23,24,25] The labor involved in these methods restricts their practical use to the most unusual investigations. The same is true of the approximate methods in which some assumption is made for the deflection line and in which the stress-strain relation is replaced by a combination of straight and parabolic lines.[26,27,28,29,30,31,32,33] In spite of their limited practicability of direct application in design these theoretical investigations are of much greater practical value than the tests made to determine empirical column formulas.

Diagrams containing the curves representing the functional relations,

$$\frac{l}{i} = f(\sigma_{cr}, a) \qquad \text{(53)}$$

arranged according to parameter a (see Fig. 6), facilitate the clear visualization of the behavior of compression members as influenced by the characteristics of the metal and by geometric relations. The upper limit of such a group of curves is $a = 0$ in Fig. 6, or the case of ideal axial load. This curve consists of Euler's hyperbola for slenderness ratios larger than that corresponding to the proportional limit; it consists of the Engesser-Kármán (double-modulus) line for slenderness ratios that correspond to the range between the point of strain hardening and proportional limit, and, for smaller slenderness ratios, it consists of a rapidly ascending line. Since structural steels have a long yield range, the limit of hardening will be smaller than 15; or it will be always smaller than the practical minimum slenderness ratio, which is about 25. Therefore, actual compression members would reach their ultimate carrying capacity

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[20] "Untersuchungen über Knickfestigkeit," by Theodor von Kármán, *Mitteilungen über Forschungsarbeiten*, V.D.I., Heft 81, 1910.

[21] "Die Stabilität zentrisch und exzentrisch gedrückter Stäbe aus Bausthal," by E. Chwalla, *Sitzungsbericht der Akademie der Wissenschaften in Wien*, Abteilung IIa, July, 1928, p. 469.

[22] Rept., 2d International Cong. for Bridge and Structural Eng., Vienna, September 24–28, 1928, p. 608.

[23] Final Rept., 1st Cong., International Assn. for Bridge and Structural Eng., Paris, May 19–25, 1932, p. 53.

[24] "Über die experimentelle Untersuchung des Tragverhaltens gedrückter Stäbe aus Bausthal," by E. Chwalla, *Der Stahlbau*, Vol. 7, 1934, p. 17.

[25] "Theorie des aussermittig gedrückten Stäbes aus Bausthal," by E. Chwalla, *ibid.*, pp. 161–165, 173–176 and 180–184.

[26] "Die Knicksicherheit von an beiden Enden gelenkig gelagerten Stäben aus Konstruktionsthal," by M. Roš and J. Brunner, *Bericht No. 13* der EMPA und der Gruppe VI der TKVSB, Zürich, August, 1926.

[27] *Proceedings*, 2d International Cong. for Applied Mechanics, Zürich, September 12–17, 1926, pp. 368–378.

[28] "Die Bemessung zentrisch und exzentrisch gedrückter Stäbe auf Knickung," Rept., 2d International Cong. for Bridge and Structural Eng., Vienna, September 24–28, 1928, p. 282.

[29] "La stabilité des barres comprimées par des forces excentrées," by M. Roš, Preliminary Rept., 1st Cong., International Assn. for Bridge and Structural Eng., Paris, May 19–25, 1932, p. 57.

[30] "The Strength of Steel Columns," by H. M. Westergaard and William R. Osgood, *Transactions* A.S.M.E., Vol. 49–50, Pt. I (1928), APM-50-9, pp. 65–80.

[31] "Die einseitige Druck bei Stäben aus Bausthal," by Friedrich Hartmann, Final Rept., 1st Cong. International Assn. for Bridge and Structural Eng., Paris, May 19–25, 1932, p. 40.

[32] Zeitschr. d. ÖIAV, 1933, H.11–12.

[33] Knickung Kippung Beulung, Franz Deuticke, Wien, 1937, p. 43.

before strain hardening can set in, even if they could be loaded axially. The region of hardening, therefore, has no practical significance.

The shape of the Engesser-Kármán (double-modulus) line depends on the shape of the stress-strain diagram between the proportional limit and yield stress. An infinite variety of double-modulus curves is possible between these two values for the same steel. This indicates that a reasonably assumed shape for this curve is just as close to reality as any of those which correspond to a certain actual stress-strain diagram. Comparative examinations of the effect of the variations of the double-modulus curve on the values of the carrying capacity of columns, of the effects of the practically unavoidable eccentricities of the load ($a > 0.1$), and of the fluctuation in the values of σ_p and σ_y, indicate that the double-modulus theory has no practical significance in bridge design or in structural design generally. Therefore, it is not only the simplest procedure, but also the most reasonable, to base structural column formulas on the ideal stress-strain relation.

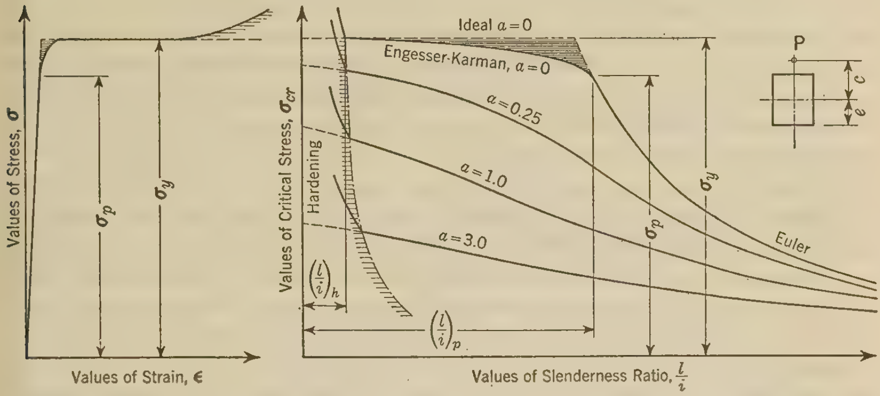

FIG. 6.—RELATION BETWEEN STRESS-STRAIN CURVES AND $\left(\sigma_{cr} - \frac{l}{i}\right)$-DIAGRAMS

The ideal stress-strain variation (see stress-strain diagram in Fig. 5), and the validity of the classical theory of the yielding process, are the fundamental assumptions of practical column investigations. It is assumed that yielding begins when the stress in the extreme fiber of the section reaches the yield stress of the tension-test specimen subjected to the standard tension test, and extends continuously with continuously increasing load.

Actually the stress-strain diagram is different for tension and compression, for increasing and decreasing load. It is also curved beyond the proportional limit, and the yielding process is characterized by the formation of flow layers caused by the superimposed bending stresses resulting from the unavoidable eccentricity of the loading.[34] Yielding, therefore, does not progress continuously with an increasing load but in layers, and this fact influences the stress distribution in the section and brings the shape of the section into play. The consideration of flow layers is generally neglected in present-day structural design. The curved portion of the stress-strain diagram between the proportional limit and yield stress is short for structural steels. The fluctuation of σ_y for specimens of the same steel is often larger than the length of this region. The consideration of these conditions indicates that, for structural steel columns, the foregoing idealizing simplifications are justified. Based on them, and on Bernoulli's hypothesis that originally plain sections remain plain and normal to the longitudinal column fibers after bending, the carrying capacity of compression members is adaptable to a theoretic evaluation that is entirely satisfactory for structural design.

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[34] "Plasticity," by A. Nadai, McGraw-Hill Book Co., Inc., New York, N. Y., 1931, p. 104.

In view of the foregoing simplifying assumptions, three different stress conditions are possible in a rectangular column section: (*a*) The condition corresponding to purely elastic deformation; (*b*) the stress pattern when there is a plastic area at the compression side; and (*c*) the stress pattern when there is a plastic area at both compression and tension sides. ("Compression side" or "tension side" denotes that side of the section in which the stress due to bending alone is compression or tension—quantities referring to these sides will be designated by subscripts *c* and *t* herein.) For the three different stress conditions of the center section (*a*), (*b*), and (*c*), the following formulas give the carrying capacity:[35,36]

Case (*a*)—

$$\left(\frac{l}{i}\right)^2 = \frac{\pi^2 E}{\sigma_{cr}}\left(1 - \frac{a\,\sigma_{cr}}{\sigma_y - \sigma_{cr}}\right) \qquad (54a)$$

Case (*b*)—

$$\left(\frac{l}{i}\right)^2 = \frac{\pi^2 E}{\sigma_{cr}}\left[1 - \frac{a\,\sigma_{cr}}{3\,(\sigma_y - \sigma_{cr})}\right]^3 \qquad (54b)$$

Case (*c*)—

$$\left(\frac{l}{i}\right)^2 = \frac{\pi^2 E}{\sigma_{cr}}\sqrt{\frac{\sigma_{cr}}{\sigma_y}\left(\frac{\sigma_y}{\sigma_{cr}} - \frac{\sigma_{cr}}{\sigma_y} - \frac{2\,a}{3}\right)^3} \qquad (54c)$$

For the solution of Eqs. 54 only the values of E and σ_y are necessary. These formulas apply to any kind of structural steel. Eq. 54*a* is valid until the yield stress in the extreme fiber is reached; Eq. 54*b* is valid until $a \leqq 3\left(1 - \frac{\sigma_{cr}}{\sigma_y}\right)$; and Eq. 54*c* is valid until $a \geqq 3\left(1 - \frac{\sigma_{cr}}{\sigma_y}\right)$.

The practical carrying capacity, or the lower limit of the carrying capacity of columns, is defined by the load at which the yield stress is reached in the extreme fiber. Eq. 54*a* gives this lower limit on which design is usually based. It is valid for any shape of section and gives practically identical results with the "secant formula":

$$\sigma_{cr} = \frac{\sigma_y}{1 + a \sec \frac{l}{2\,i}\sqrt{\frac{\sigma_{cr}}{E}}} \qquad (55a)$$

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[35] "Näherungsberechrung der Tragkraft exzentrisch gedrückter Stahlstäbe," by Karl Ježek, *Der Stahlbau*, Vol. 8, 1935, p. 89.

[36] "Die Festigheit von Druckstäben aus Stahl," by Karl Ježek, Julius Springer, Wien, 1937, p. 79.

Eq. 55*a* must be solved by the cumbersome trial-and-error method; but, transformed into the form,

$$\frac{l}{i} = 2\sqrt{\frac{E}{\sigma_{cr}}}\,\text{arc cos}\,\frac{a\,\sigma_{cr}}{\sigma_y - \sigma_{cr}} \quad \ldots\ldots\ldots\ldots\ldots (55b)$$

it is convenient for calculations because tables for circular cosines for radian arguments are readily available; the arc of the cosine can be read from such tables, and Eq. 55*b* can be solved directly.

The upper limit of the carrying capacity is defined by the load that produces the critical equilibrium condition. Eqs. 54*b* and 54*c* give these values for rectangular section.

Rigorous investigations[37] have shown that the upper limit of the carrying capacity is appreciably influenced by certain cross-sectional shapes. A formula by which the sectional form can be considered is:[38,39]

$$\left(\frac{l}{i}\right)^2 = \frac{\pi^2 E}{\sigma_{cr}}\left(1 - \mu_1\frac{a\,\sigma_{cr}}{\sigma_y - \sigma_{cr}}\right)\left(1 - \mu_2\frac{a\,\sigma_{cr}}{\sigma_y - \sigma_{cr}}\right) \ldots\ldots (56)$$

Values of μ_1 and μ_2 for use in Eq. 56 are given in Table 1. Eq. 56 has unlimited validity for each of these sections except sections (d) and (e), for which it is valid, provided

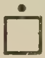
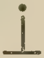
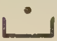
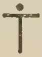
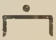
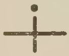
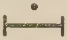
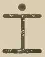
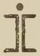
$$\frac{\sigma_{cr}}{\sigma_y} \geqq \frac{S_c - S_t}{S_c + S_t} \ldots\ldots\ldots\ldots\ldots (57)$$

in which S_c and S_t are the section moduli at the compression and tension sides, respectively. If the relation defined by Eq. 57 does not exist, failure occurs at the tension side and the ultimate carrying capacity becomes

$$\left(\frac{l}{i}\right)^2 = \frac{\pi^2 E}{\sigma_{cr}}\left[1 - \mu_1\frac{S_c\,a\,\sigma_{cr}}{S_t\,(\sigma_y + \sigma_{cr})}\right]\left[1 - \mu_2\frac{S_c\,a\,\sigma_{cr}}{S_t\,(\sigma_y + \sigma_{cr})}\right] \ldots (58)$$

As the eccentricity ratio decreases (that is, as *a* approaches zero), the influence of the shape of the section diminishes.

TABLE 1.—VALUES OF μ FOR THE SOLUTION OF EQ. 56

μ	 (*a*)	 (*b*)	 (*c*)	 (*d*)	 (*e*)	 (*f*)	 (*g*)	 (*h*)	 (*i*)
μ_1	0.5	0.5	0.5	0.8	0.8	0.4	0.4	0.9	0.9
μ_2	0.5	0.5	0.5	0.2	0.2	0.4	0.4	0.1	0.1

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[37] "Der Einfluss der Querschnittsform auf das Tragvermögen aussermittig gedrückter Bausthalstäbe," by Ernst Chwalla, *Der Stahlbau*, Vol. 8, 1935, p. 193.

[38] "Die Festigkeit aussermittig gedrückter Stahlstäbe beliebiger Querschnittsform," by Karl Ježek, *Der Bauingenieur*, Vol. 17, 1936, p. 306.

[39] "Die Festigheit von Druckstäben aus Stahl," by Karl Ježek, Julius Springer, Wien, 1937, p. 213.

It is possible to base column formulas on the ratio of the yield and critical stresses, or, for design purposes, on the ratio of a constant allowed stress and the critical stresses which correspond to the various $\frac{l}{i}$-ratios. In this case the force to which the column is subjected will be multiplied by this ratio (the dimensionless parameter $\omega \geqq 1$) and it is required that the resulting axial unit stress shall not exceed the constant allowed stress value. An exceedingly practical column formula of this type,[40] in the derivation of which the center deflection was not approximated by $y_m = \frac{c\,\sigma_E}{\sigma_E - \sigma_a}$ (which corresponds to the sine curve assumption of Eq. 54*a*) but by the more exact expression,[41]

$$y_m = \frac{c}{\sigma_E - \sigma_a}\left(\sigma_E - 0.25\,\sigma_a\right) \qquad (59)$$

is as follows:

$$\omega = \tfrac{1}{2}\left[\varphi\left(\frac{l}{i}\right)^2 + a + 1\right]\left\{1 + \sqrt{1 - \frac{4\,\varphi\left(\frac{l}{i}\right)^2 (1 - 0.25\,a)}{\left[\varphi\left(\frac{l}{i}\right)^2 + a + 1\right]^2}}\right\} \qquad (60)$$

in which: $\sigma_E = \frac{\pi^2 E}{\left(\frac{l}{i}\right)^2}$; $\sigma_a = \frac{P}{A}$; and $\varphi = \frac{\sigma_y}{\pi^2 E}$. The critical stress becomes

$$\sigma_{cr} = \frac{\sigma_y}{\omega} \qquad (61)$$

Eq. 60 can be solved directly for any $\frac{l}{i}$-ratio; and for any variation of *a* it gives practically identical results with Eqs. 55.

All imperfections of the column in the structure are translated in column formulas to a single factor representing a certain equivalent eccentricity of the load. The functional relation between the eccentricity and the stress condition of the center section makes such a summary consideration of the imperfections possible. The practical validity of a column formula depends greatly on the assumption of the magnitude and variation of this measure of the degree of imperfection, expressed by the eccentricity ratio *a*.

The column formulas specified by the American Railway Engineering Association (A.R.E.A.; see Eq. 33*a*) and the American Association of State Highway Officials (A.A.S.H.O.) approximate the "secant formula," Eqs. 55 and 61 in which $a = 0.25$ $\left(\text{that is, } c = 0.25\,\frac{i^2}{e} \text{ has been assumed}\right)$. For the same symmetrical column section $\frac{i^2}{e}$ is constant and therefore the value of c remains constant for any l/i-ratio. This results in an increasing reduction of the calculated carrying capacity, or allowed unit stress, as the slenderness decreases. In actual structures $\frac{i^2}{e}$ is not constant; it increases with a decreasing l/i-ratio, and therefore the assumption of $a = 0.25 =$ constant results in increasingly more severe sectional requirements for compression members as the slenderness ratio decreases. The correctness of this statement will be demonstrated by an investigation of actual structures.

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[40] "Vorschlage zur wirtschaftlichen Bemessung gedrückter Baustahlstäbe," by Ernst Chwalla, *Deutscher Stahlbau-Verband*, Berlin, Ernst, 1939, p. 18.

[41] "The Imperfect Strut and the Arch," by R. H. W. Hawken, *Proceedings*, World Eng. Cong., Tokyo, 1929, Vol. III, Pt. 1, pp. 73–114.

In Fig. 7 the $\frac{i^2}{e}$-values (or, for unsymmetrical sections, the average value) of the core radii $\frac{1}{2}\left(\frac{i^2}{e_t} + \frac{i^2}{e_c}\right)$ of main and bracing sections of several highway and railway bridges are plotted against the $\frac{l}{i}$-value of the members. These bridges have been built since 1920 and include long-span truss bridges, stiffening trusses of large suspension bridges, and other smaller structures. The indicated curve represents the average of scattering points. It appears from this curve that the variation of the actual $\frac{i^2}{e}$-values is hyperbolic in bridge structures, the product $\frac{i^2}{e} \times \frac{l}{i}$ being nearly constant. In addition to the ordinates $\frac{i^2}{e}$, Fig. 7 gives the values of c, the unavoidable eccentricities, on which the specification column formulas are based. The sensitivity of compression members to eccentric loads in the small $\frac{l}{i}$-region, in which rapidly increasing eccentricities are specified, indicates that, in bridge design, it is not reasonable to base a column formula on the assumption that $a = 0.25$ is a constant eccentricity ratio, because it results, unjustifiably, in an increasing factor of safety as $\frac{l}{i}$ decreases. A more uniform factor of safety will be obtained by assuming a linear increase of the unavoidable eccentricities c, with increasing $\frac{l}{i}$. In view of the hyperbolic variation of the actual $\frac{i^2}{e}$-values, this assumption results in an eccentricity,

$$c = \kappa \frac{1}{2}\left(\frac{i^2}{e_t} + \frac{i^2}{e_c}\right)\left(\frac{l}{100\,i}\right)^2 \qquad \text{.................. (62)}$$

By Eq. 62 the design eccentricity becomes a function of the structural, sectional and dimensional characteristics of the compression member, and the expression of the eccentric ratio becomes:

$$a = \frac{\kappa}{2}\left(1 + \frac{e_c}{e_t}\right)\left(\frac{l}{100\,i}\right)^2 \dots\dots\dots\dots\dots\dots\dots (63)$$

With present-day structural imperfections the value of the constant κ, for bridges and structures, is 0.25. The most unfavorable section actually in use is that composed of two angles, back to back. If the load produces compression in the extreme fibers of the legs in contact, the ratio of the distances from the center of gravity to the extreme compression and tension fibers is $e_c/e_t = 2.5$, for such sections. If the load is at the other side of the section e_t/e_c must be introduced into Eq. 63 and the yield stress in this case will occur at the tension side of the section. For symmetrical sections these ratios become unity, and for $\frac{l}{i} = 100$, Eq. 63 reduces to the present specification value of $a = 0.25$.

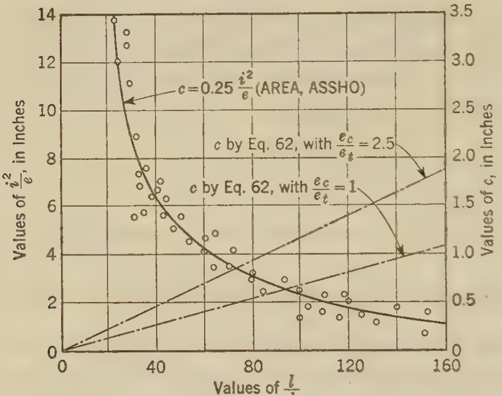

FIG. 7.—VARIATION OF THE RATIO $\frac{i^2}{e}$ WITH THE SLENDERNESS RATIO $\frac{l}{i}$ IN ACTUAL BRIDGES

Fig. 5 is an evaluation of Eqs. 54*a*, 54*b*, 55, 56, 60, and 61 for carbon steel and silicon steel. Curves 1 correspond to Eqs. 55 and 61 with $a = 0.25$; and curves 1a and 1b correspond to the same formulas with $a = \frac{0.25}{2}(1 + 1)\left(\frac{l}{100\,i}\right)^2$ and $a = \frac{0.25}{2}(1 + 2.5)\left(\frac{l}{100\,i}\right)^2$, respectively. Curves 2 correspond to Eq. 54*a* with $a = 0.25$. The critical stress, σ_{cr}, given by curves 1, 1a, 1b, and 2, is the axial, or average, stress at which the yield stress is reached in the extreme compression fiber, and $P_{cr} = A\,\sigma_{cr}$ is the lower limit, or design, carrying capacity of the column. Curves 3, Fig. 5, correspond to Eq. 54*b* and give the upper limit, or ultimate carrying capacity $P_{cr} = A\,\sigma_{cr}$ of a column with a rectangular section based on the classical theory of the yielding process and on the assumption that $a = 0.25$ is a constant eccentricity ratio.

The $\left(\omega = \frac{\sigma_y}{\sigma_{cr}}\right)$-curves are also given in Fig. 5. Examination of these curves reveals that they can be approximated by simple parabolic power relations, or, by means of these ω-curves, doubly curved cubic-hyperbolic $\left(\sigma_{cr} - \frac{l}{i}\right)$-curves, corresponding to eccentricity ratios which vary with $\frac{l}{i}$, can easily be represented.

Comparison of curves 1 with 1a and 1b in Fig. 5 indicates that, for a given value of E and σ_y, the magnitude and variation of the eccentric ratio a has a much greater effect on the values of the critical stresses and on the shape of the column curves than the other variables entering into the design of compression members.

In 1932 the Society's Special Committee on Steel Column Research concluded its 10-yr investigations with the statement that:[42] "A rational column formula can best be constructed on the basis of the theoretical formula for eccentrically loaded columns * * *." This statement is in agreement with the findings of numerous valuable investigations presented since 1926. Progress in the design of compression members, during the intervening years, is characterized by the adoption of the theoretical formula as evidenced also by both the A.R.E.A. and A.A.S.H.O. specifications which have been in force since before 1935. Improvements on the A.R.E.A. formulas and the A.A.S.H.O. formulas so that they will conform to actual structural conditions of eccentricity and the use of proper reduced lengths, constitute the criteria for their usefulness, rather than the formal requirements suggested by Mr. Osgood.

The determination of the reduced length should be based on consistent assumptions. Inasmuch as the design of the members is based on the condition of yield stress in the extreme fiber it is reasonable to consider the reduced lengths which correspond to this condition of the structure. This indicates that such summary assumptions for the buckling lengths as those made by Mr. Osgood and those involved in the A.R.E.A. and A.A.S.H.O. column formulas are neither reasonable nor tenable. The variation of the stress and rigidity of members adjacent to the same panel point, the character of the intermediate supports of members in various truss systems, and other factors influence the buckling length. For all members and conditions the specification formulas assume 0.75 l if the connections are riveted. This fact, combined with the assumed constant eccentricity ratio, makes the design value of these fundamentally correct formulas no greater than rules of thumb.

The critical stresses represented by curves 1a and 1b in Fig. 5—in combination with reduced lengths obtained from the yield stress in the extreme-fiber condition of the members in the structure and correspondence between the allowed stress in tension and compression—are more likely to result in a homogeneous factor of safety at economic advantage than the prevailing specification column formulas which are not fitted to the characteristics of actual structures.

The writer has attempted to explain in the foregoing why there is no need for the application of the double-modulus theory in structural design. The difference between the Engesser-Kármán (double-modulus) line and the line of the ideal buckling stresses is greatest where the Euler hyperbola intersects the horizontal of σ_y (Fig. 6). Assuming this difference to be as great as 10% of σ_y, introducing this value into Eq. 54a, and solving, $a = 0.01$. Inasmuch as the design value of a for actual members in this region is between 0.25 and 0.6, the carrying capacity resulting from the assumption of the ideal stress-strain diagram is only from 1.5% to 4% different from that obtained by the use of the double-modulus line in the extreme case.

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[42] "Final Report of the Special Committee on Steel Column Research," *Transactions*, Am. Soc. C. E., Vol. 98 (1933), Conclusion 5, p. 1460.

The problem in design is to ascertain a desired structural performance with the greatest economy. The comparison of various structural schemes is equitable only if their factors of safety are the same. Empirical equations, and the prevailing specification formulas, do not result in useful comparisons because their deviations from structural reality affect members of different proportions in different ways. It can generally be stated that the most frequent stress condition—compression combined with bending—is not being given proper attention in design. Laborious calculations, by which internal forces and moments are often determined, usually end in taking the summation of the divisions of force by area and moment by section modulus. The result is not indicative for the factor of safety of the members unless the critical stress that corresponds to the loading condition is known.

The writer agrees with the statements made by Mr. Osgood in the "Summary." There is no point to approximating a correct formula by substituting another of limited validity. It is practical to prepare tables computed from a formula which will apply correctly to the case under consideration in design specifications. Such a method, however, is not applicable directly to many of the infinitely varying complex problems of engineering practice. The simplification of such problems to manageable form requires judgment ripened by thorough structural knowledge, and it requires the study of analytic and experimental investigations in the entire field of stability phenomena.

William R. Osgood,[43] M. Am. Soc. C. E.—Although the paper seems to have been written so that it could be understood, it evidently was not written so that it could not be misunderstood.

The paper contemplates only column failure as distinct from local failure and from crushing of the material, and it contemplates only central loading. Before account can be taken of accidental eccentricities, initial curvatures, and that sort of thing, it seems desirable to eliminate the effects of these disturbing factors so far as possible. When a resulting basic column curve has been obtained, the effects of accidental imperfections in modifying the curve may be considered.

Some of the discussers consider the secant formula to be a rational one. The secant formula is simply the familiar formula,

$$f = \frac{P}{A} + \frac{M c}{I} \qquad \text{(64}a\text{)}$$

written for columns,

$$f = \frac{P}{A} + \frac{P\, e \sec \dfrac{l}{2\,i}\sqrt{\dfrac{P}{E\,A}}}{I}\, c \qquad \text{(64}b\text{)}$$

Eqs. 64 express accurately the relations between the variables that enter so long as Hooke's law applies. For slight departures from Hooke's law, there will be slight inaccuracies. At failure, of either a beam or a column of ordinary structural material, Hooke's law no longer applies even approximately; the inaccuracies are great, and a rational basis can no longer be claimed for the formulas. The computed value of f at failure has no physical meaning whatsoever. In the case of a simple beam ($P = 0$ in Eq. 64a), the computed value of f will vary widely with the shape of the cross section, for the same material. In the case of a column, the computed value of f will vary widely with the shape of the cross section, with the eccentricity, and with the length. Likewise, with a given value of f, the computed values of $\frac{P}{A}$ will bear little relation to the actual values of $\frac{P}{A}$ at failure of columns of different lengths, eccentricities, and shapes of cross section, but of the same material. An unknown, variable factor of safety results when design is based on Eq. 64b with a constant f.

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[43] Materials Engr., National Bureau of Standards, Washington, D. C.

The view that the secant formula is rational seems similar to maintaining that Rankine's formula is rational, as was often done in the past. Frequently, the assumptions of a theory are not ideally satisfied; but, for the theory to be rational, the assumptions must be reasonably well satisfied. The assumptions underlying the secant formula, considered as a rational one, are not reasonable, and the formula does not and cannot in general produce results in agreement with tests. For example, a series of tests made recently has shown that, for eccentricity ratios of $\rho = \frac{e\,c}{i^2}$ up to 1, the formula,

$$\lambda = \frac{\lambda_o}{1 + 4\,\rho\,\sigma^2} \qquad \text{(65}a\text{)}$$

fits the test results well. In Eq. 65a,

$$\lambda = \frac{1}{\pi}\frac{l}{i}\sqrt{\frac{S}{E}} \qquad \text{(65}b\text{)}$$

in which l is the length of the eccentrically loaded column. In the secant formula,

$$\lambda = \frac{2}{\pi\sqrt{\sigma}}\sec^{-1}\left(\frac{1}{\beta\,\sigma} + C\right) \qquad \text{(65}c\text{)}$$

β and C cannot be determined to fit these test results at all accurately, except possibly for a narrow range. Much less are β and C related to properties of the material. The only rational consideration of the strength of eccentrically loaded columns is along the lines pursued by some of the authors cited by Mr. Balog, notably Karl Ježek and E. Chwalla.

All this does not infer that a formula of the secant type may not be useful, considered as an empirical formula.

A quick and easy formula is demanded by some of the discussers. In comment the writer would quote Mr. Beskin:

> "* * * there is no practical * * * objection to the use of a limited number of simple formulas, instead of one complicated formula covering the entire range of possible slenderness ratios."

To cover the entire range accurately with one or even two simple formulas is asking a lot. It will rarely be possible, depending, of course, on one's ideas of simplicity.

Some object to a formula, like the secant formula, that expresses the ratio of slenderness explicitly as a function of the stress, rather than the stress as a function of the ratio of slenderness. In practice both the load and the length are given. In a formula is it important which one is expressed in terms of the other?

Dean Grinter's criticism indicates the real problem—on the one hand, the need for re-examining methods from time to time, and, on the other hand, the need for keeping them sufficiently simple for use by the ordinary designer. Dean Grinter raises a practical question that must be faced when he says that "* * * much design work is actually performed by persons of limited competency in structural design." The final answer might be to increase the competence of designers! Whether the immediate answer is to confine the use of column formulas to the Rankine formula and the straight-line formula the writer does not know, especially in view of some of the complicated formulas in use by structural designers in the field of aircraft.

Professor Winter and Mr. Beskin suggest that a discussion of the secant yield strength, S, would be of interest. This question has been raised before. It has been answered partially, in reference to beams, in another paper.[44] To answer it for columns, one may write the double-modulus column formula,

$$\sigma = \frac{1}{\lambda^2_o} \frac{\bar{E}}{E} \qquad (66)$$

and ask under what conditions Eq. 66 represents the same reduced column curve for different materials. In Eq. 66,

$$\bar{E} = \frac{E' I' + E I}{I_g} \qquad (67)$$

in which:

E' is the tangent modulus of elasticity of the material at the stress $s = P/A$;

I' is the moment of inertia about the axis of average stress of the part of the cross-sectional area that is subjected to an increase of stress at the instant of failure of the column;

I is the moment of inertia about the axis of average stress of the part of the cross-sectional area that is subjected to a decrease of stress at the instant of failure; and

I_g is the moment of inertia of the cross-sectional area about a gravity axis parallel to the axis of average stress.

The position of the axis of average stress is defined by the relation $E' Q' = E Q$, in which Q' and Q are the statical moments about the axis of average stress of the two parts of the cross-sectional area just mentioned.

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[44] "A Rational Definition of Yield Strength," by William R. Osgood, *Journal of Applied Mechanics*, Vol. 7, June, 1940, p. A-61.

If, now, for two materials, Nos. 1 and 2, and for geometrically similar cross sections, the column curves are defined by:

$$\sigma_1 = \frac{1}{\lambda^2_1}\frac{\bar{E}_1}{E_1} \quad \text{(68a)}$$

and

$$\sigma_2 = \frac{1}{\lambda^2_2}\frac{\bar{E}_2}{E_2} \quad \text{(68b)}$$

—what conditions are required to make these two curves the same? For any value of $\sigma_1 = \sigma_2$, the corresponding value of λ_1 is equal to λ_2 if $\bar{E}_1/E_1 = \bar{E}_2/E_2$. From the definition of $\bar{E}$, and the axis of average stress, this is the same as requiring that $E'_1/E_1 = E'_2/E_2$; or

$$\frac{1}{E_1}\frac{ds_1}{de_1} = \frac{1}{E_2}\frac{ds_2}{de_2} \quad \text{(69)}$$

in which e is the compressive strain at the stress s. Since $\sigma_1 = \sigma_2$,

$$\frac{s_1}{S_1} = \frac{s_2}{S_2} \quad \text{(70)}$$

and

$$\frac{1}{E_1}\frac{ds_1}{de_1} = \frac{1}{E_2}\frac{S_2}{S_1}\frac{ds_1}{de_2} \quad \text{(71)}$$

or, upon integration,

$$\frac{E_1\,e_1}{S_1} = \frac{E_2\,e_2}{S_2} \quad \text{(72)}$$

The stresses and strains of the two materials must be related as expressed by Eqs. 70 and 72.

Eqs. 70 and 72 must apply in particular when $\sigma_1 = \sigma_2 = 1$—that is, when s_1 and s_2 are equal to the yield strengths of the two materials. If e_{11} and e_{22} denote the strains corresponding to the yield strengths, Eq. 72 becomes, for this case:

$$\frac{S_1/e_{11}}{S_2/e_{22}} = \frac{E_1}{E_2} \quad \text{(73)}$$

which states that the slopes of the lines through both the origins and the yield strengths of the stress-strain curves must be proportional to the moduli of elasticity of the two materials. This requires that the yield strengths of the two materials be determined as the stresses at the intersections with the stress-strain curves of secants through the origin having slopes equal to $m\,E_1$ and $m\,E_2$, in which m is an arbitrary constant $0 < m < 1$—hence, the name "secant yield strength" for yield strengths so determined.

(In a conversation with the writer, Nathan M. Newmark, M. Am. Soc. C. E., has pointed out that, since $E'_1/E_1 = E'_2/E_2$, at the yield strength, $E'_{11}/E_1 = E'_{22}/E_2$ and therefore the yield strengths may be determined as the stresses at which the slopes of the stress-strain curve are $n E_1$ and $n E_2$, with $0 < n < 1$. This is probably the first time that "Johnson's elastic limit" has been justified on any rational basis. Since it is the slope of the stress-strain curve that is particularly significant in compression members, it may be that Johnson's elastic limit is a better measure of the yield strength than the secant yield strength. However, Johnson's elastic limit does require greater care in its determination than does the secant yield strength.)

The necessary and sufficient conditions, therefore, that Eqs. 68 represent the same curve are that Eqs. 70 and 72 be satisfied at every point and that S_1 and S_2 be secant yield strengths determined with the same value of m. It will always be possible to determine S_1 and S_2 as required. Whether Eqs. 70 and 72 are satisfied at every point depends on the materials considered. Many tests have shown, however, that, for nominally similar materials, such as materials complying with a given specification, Eqs. 70 and 72 are in fact satisfied to a high degree of approximation, even when the yield strengths differ widely.

Finally, the generalization from two materials to any number of materials is obvious, the practical point being that the yield strengths must be determined as secant yield strengths with slopes of the secants equal to $m E$, in which m is the same constant for all the materials being considered and E is the modulus of elasticity of each individual material.

Incidentally, whether Eqs. 70 and 72 are satisfied or not, the scatter in the column data can be reduced by plotting σ against λ_o rather than s against l_o/i, provided S is the secant yield strength. Moreover, no other method is likely to give as good results consistently; and no other method can give as good results if Eqs. 70 and 72 are satisfied at every point and if S is the secant yield strength.

In Mr. Godfrey's opinion, "The Gordon-Rankine formula has limited application. This formula is fraught with danger, if it is used in the range of slender columns." One of the objectives of the paper was to call attention to this kind of danger. If the Rankine formula is written in the form of Eq. 3*b*, in which $B \geqq 0$, the average stress can never exceed the Euler value. If B is chosen to give safe values in the range of "medium" ratios of slenderness, safe values will be obtained for slenderer columns also. The writer agrees with Mr. Godfrey that a factor of safety of 5 in the Euler range is excessive. If a variable factor of safety is used (and it probably should be used in some form), it is the opinion of the writer that the factor should be less for short and for long columns than for columns of medium length, for which λ_o is usually about 1, depending on how S is measured.

In its most general, dimensionless form, Mr. Spaulding's first formula, Eq. 35, may be written

$$\sigma = \frac{1}{\lambda^2_o} \frac{\lambda^4_o + \beta \lambda^2_o + \gamma}{\lambda^4_o + \delta \lambda^2_o + \epsilon} \qquad (74a)$$

and his second one, Eq. 37,

$$\sigma = \frac{\beta + \lambda^2_o}{\gamma + \delta \lambda^2_o + \lambda^4_o} \qquad (74b)$$

(Mr. Spaulding's σ_y—a yield strength—is not to be confused with the writer's σ, which is a dimensionless "stress".) Eq. 74*a* contains four arbitrary constants; and Eq. 74*b*, three. These are too many to determine conveniently and more than are necessary. Mr. Spaulding arbitrarily takes $\beta = \gamma + 1 = \delta = \epsilon + 1$ in Eq. 35 and $\beta = \gamma = \delta$ in Eq. 37, thus reducing the number of constants to one in each formula. In terms of λ_o and σ, the solid curves of Fig. 2(*a*) reduce to one curve.

Laboratory conditions differ from conditions in engineering structures primarily in that the former conditions are usually better known than the latter. Undoubtedly, the most troublesome unknown is k. Mr. Spaulding and Mr. Beskin consider this factor a variable rather than a constant. Perhaps it is better called a parameter. It is usually exceedingly difficult (that is, particularly time consuming) to determine k rationally for a given structure. It seems that k must be determined largely by judgment, although the excellent studies of F. Bleich, M. Am. Soc. C. E., may serve as a helpful guide. The writer is of the opinion that conditions such as unintentional eccentricities and initial curvature are best taken care of by the factor of safety.

Professor Cissel lists four possible modes of failure. The first, compressive failure by crushing the material, is important for "brittle" materials—in structural work, primarily for concrete. It was important for cast iron and may become so again for metals, with the development and use of alloy steels, alloys of aluminum, and alloys of magnesium, for example. The formulas of the paper were concerned implicitly with column failure, Mr. Cissel's second mode of failure. His third mode of failure, twisting, is sometimes lost sight of but must definitely be considered; and perhaps the fourth mode, local failure, is the type most likely to occur, certainly in short columns.

The objection to using, instead of a formula, an averaged curve of experimental results (which Mr. Beskin suggests as a possibility) is that such a curve cannot be transferred readily. No two users would be dealing with the same curve, although a sufficient number of corresponding abscissas and ordinates might be read from a master curve and a list of these points published so that practically the same curve would be drawn by everyone.

It is true, as Mr. Beskin states, that a decreasing function is always a monotonically decreasing function; but a monotonically decreasing function is not always everywhere a decreasing function.

The writer disagrees with Mr. Beskin that the double-modulus theory involves a basic error in theory, in that it presupposes a theoretically straight column. In carefully made and carefully centered columns, if the departures from the ideally assumed conditions were small but sufficiently great to give results differing significantly from those predicted by the theory, then the theory would be in error. Actually, no such significant differences in results are obtained. It is presumably the column that is in error, not the theory; that is, the imperfections in the column lead to results not in complete accord with those predicted by the theory.

When the writer stated "It would be very hard to justify more than two constants of the material in any practical column formula," he had in mind the dimensionless constants of the formulas written in terms of λ_o and σ.

As Mr. Beskin states, the formulas in terms of l_o/i and P/A all contain one more constant—namely, E, the modulus of elasticity of the material.

To choose for the constant S the intercept S_o of the column curve with the axis of P/A, as Mr. Beskin would prefer, sounds easier than it is. It is not so easy to determine this intercept. A column of zero length has practically infinite strength; and, if S_o is to be determined by extrapolation, extrapolation of what? If the material tested is all uniform and has the same mechanical properties, and if the cross sections of all specimens are geometrically similar, it may be possible by extrapolation from low values of l_o/i to arrive at a fairly definite value of S_o for that material and that cross section. This value of S_o has no significance for a different, but nominally the same, material and a different shape of cross section.

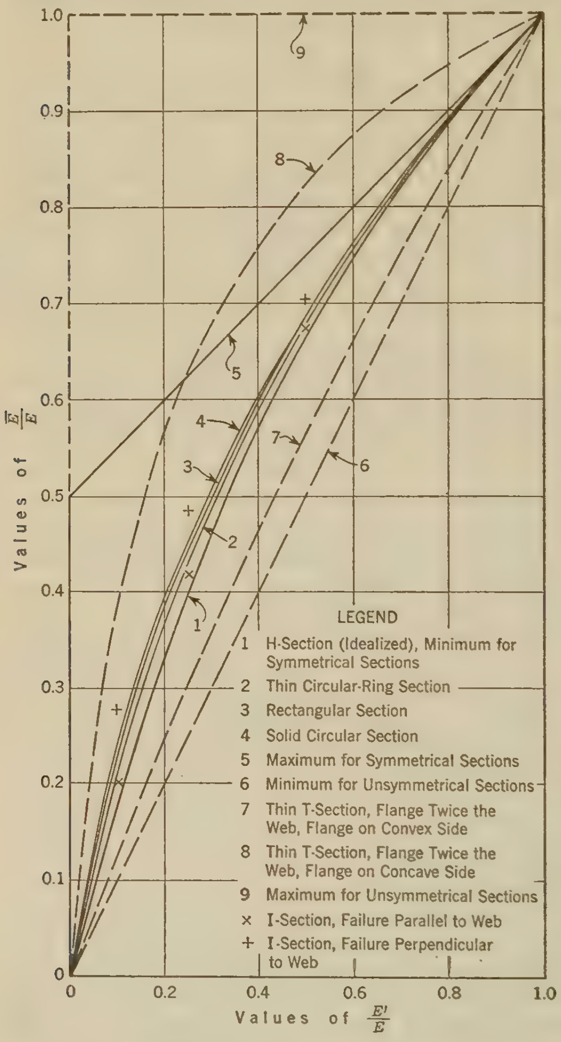

FIG. 8.—THE EFFECT OF THE SHAPE OF THE CROSS SECTION ON THE STRENGTH OF COLUMNS

Mr. Beskin calls attention to the effect of the shape of cross section on column strength. The effect of shape is illustrated in Fig. 8, in which $\bar{E}/E$ is shown as a function of E'/E for a number of cross sections.[4] Since E' is the slope of the stress-strain curve—that is, the tangent modulus at a particular stress—the abscissas also represent stresses to some scale. For a given value of E'/E that is not too small, the spread in $\bar{E}/E$ for practical symmetrical sections is seen not to be excessive. The same is not true of unsymmetrical sections, where large variations in $\bar{E}/E$, and thus in the strength (Eq. 66), may be obtained.

Mr. Beskin regrets that the dimensions of the constants in Eqs. 3 to 21 are not the same throughout all the formulas, but he does not say why. Perhaps he would have liked the dimensionless constants of the first forms, Eqs. 3*a* to 21*a*, to have been retained in the second forms, Eqs. 3*b* to 21*b*; but it seemed to the writer that this would merely clutter up these formulas unnecessarily. The constants are not intended to have the same values in each formula. Thus, whether Eq. 13 gives values less than the Rankine formula depends on the values of B and C in Eq. 13, on the one hand, and on the value of B in Eq. 3 on the other hand. There is no connection between the value of B in Eq. 13 and the value of B in Eq. 3. The writer regrets the confusion that might have been avoided by using different letters for the constants in all the formulas.

As for the necessity for a cutoff when Eqs. 15 and 18 are used, it may be stated in general that any formula may need a cutoff if the material fails at a lower stress than the column stress. Cutoffs may also be needed to take account of crinkling failure without bending of the column as a whole.

Eqs. 15 and 18 were derived by methods published elsewhere.[4] When the column strength is represented by

$$\sigma = \frac{1}{\lambda^2_o} \frac{E'}{E} \qquad \text{(75)}$$

and, when

$$K\,n \left(\frac{S}{E}\right)^{n-1} = \beta \qquad \text{(76a)}$$

Eq. 17 leads to Eq. 15*a*. When the column strength is represented by Eq. 66 with $\bar{E}$ the double modulus for an idealized H-section (negligible web); and, when

$$\frac{1}{2} K\,n \left(\frac{S}{E}\right)^{n-1} = \beta \qquad \text{(76b)}$$

Eq. 17 also leads to Eq. 15*a*. When, in Eq. 66, $\bar{E}$ is the double modulus for a rectangular cross section, and β is defined by Eq. 76*a*, Eq. 17 leads to Eq. 18*a*.

Both Eqs. 15 and 18 were intended to be purely empirical column formulas, with the β's and n's to be determined as the results of tests. On the other hand, Eq. 75 and the resulting formula, Eq. 15*a*, with β in effect given by Eq. 76*a*, have received wide experimental confirmation in ordinary, carefully made laboratory tests for a wide variety of structural materials. The writer would feel much more confident in predicting column strength of a new material from Eq. 75, Eq. 15*a* and Eq. 76*a*, or Eq. 65*a*, than from the secant formula.

Mr. Balog's "ideal conditions" (second paragraph of his discussion), leading to a column curve composed of Euler's curve and a horizontal line (Fig. 5), include of necessity an "ideal" compressive stress-strain curve composed of a modulus line and a horizontal yield line. Most structural materials do not have so simple a stress-strain curve; in particular, they do not have the sharp knee of Mr. Balog's ideal material. The blunter the knee, the wider the gap will be between the "ideal" column curve and the actual column curve.

Mr. Balog writes that an infinite variety of double-modulus curves is possible between the proportional limit and yield stress for the same steel. If the steel is really the same, only one double-modulus curve is possible, and the writer believes Mr. Balog is mistaken, unless he means that an infinite variety is possible depending on the shape of the cross section; but, as noted previously, the column strength does not vary very much with different symmetrical shapes in common use.

The writer was not aware that he made any "summary assumptions for the buckling lengths." The free length, or the buckling length, is one of the most difficult variables, if not the most difficult, to evaluate in the design of columns; and the writer is just as much against summary assumptions in this case as is Mr. Balog.

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THE EFFECT OF BLANKETS ON SEEPAGE THROUGH PERVIOUS FOUNDATIONS

BY PRESTON T. BENNETT,[1] ESQ.

WITH DISCUSSION BY MESSRS. REGINALD A. BARRON, V. A. ENDERSBY, HARRY R. CEDERGREN, W. J. TURNBULL, KENNETH S. LANE, AND PRESTON T. BENNETT.

SYNOPSIS

A mathematical analysis of seepage through natural or artificial blankets of relatively impervious soil overlying a pervious foundation is presented in this paper, indicating methods for the application of formulas to the design of earth dams. Since natural foundations rarely if ever approach the simplicity of structure and the uniformity assumed in this analysis, the blanket formulas should not be considered as an accurate means of computing flow conditions, but as an aid to the judgment of the designer.

INTRODUCTION

In the design of earth dams built on pervious foundations, there appears to be a trend toward the control of seepage through the foundation by construction of an upstream blanket connected to the impervious section of the dam, and providing drainage for the downstream section of the dam by horizontal pipe drains or by a system of vertical drainage wells. In dams where the control of seepage is undertaken primarily to increase the stability of the structure, rather than to reduce seepage losses to a minimum, this design offers several advantages over a central impervious section in the dam, and a cutoff through the foundation.

In the latter design, practically all the hydrostatic pressure of the reservoir is exerted on the central core and cutoff, and the downstream half of the dam must have slopes sufficiently flat to withstand the full pressure of the reservoir. If the reservoir head is dissipated through a blanket and upstream impervious section, a much larger portion of the dam is effective in offering resistance to the hydrostatic pressure. Furthermore, the flow of water through the upstream impervious section is almost vertically downward toward the pervious founda-

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NOTE.—Published in January, 1945, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

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tion, which is maintained at a relatively low hydrostatic potential by the combined action of the upstream blanket and the downstream drainage system. There is little tendency, upon sudden drawdown of the reservoir, for the water saturating the upstream section to flow back into the lowered reservoir, and hence the adverse conditions resulting from drawdown are reduced to a minimum. If an excess of impervious material unsuitable for embankment construction is available, the most important advantage of the blanket lies in the economic use of waste material, rather than in the use of a costly cutoff wall.

Quite commonly the alluvial deposit in a river valley consists of a stratum of sand and gravel overlain by a surface layer of relatively impervious finer soils. It is advantageous to leave such a blanket in place for seepage control, but it may be desirable, on the other hand, to remove a part of it to reduce the haul from upstream borrow pits. If the natural blanket is sufficiently uniform in thickness and permeability to justify the application of blanket design theory, the formulas developed herein may be of assistance to the designer in establishing limits for borrow-pit excavation, which will not reduce the blanketing effect materially.

The mathematical analysis of flow through blankets presented herein is of necessity based on idealized assumptions as to the uniformity of permeability and thickness of the pervious stratum and blanket. As in any other phase of soil mechanics, the validity of any conclusions based on this analysis is limited by the degree to which these idealized conditions are approached in nature, and likewise by the degree of accuracy obtainable in the determination of the permeability of the soils involved.

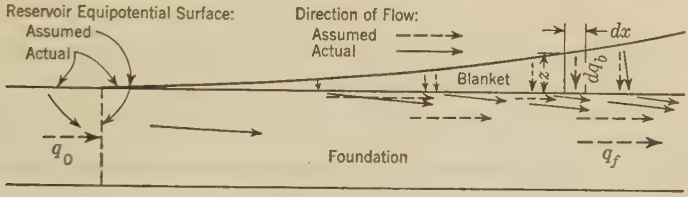
Since natural deposits of soils cannot be expected to be uniform in thickness or permeability, and since the determination of the permeability of such deposits can result at best only in a good approximation, the formulas in the paper are not intended to represent, accurately, the actual conditions of flow and potential distribution under a dam. Nevertheless, if several values for the permeability and thickness of the blanket and pervious stratum are used, covering the estimated range of variation of these factors, the resulting analytical evaluation of a blanket should be of value as an indication of the probable limits of its effectiveness.

NOTATION

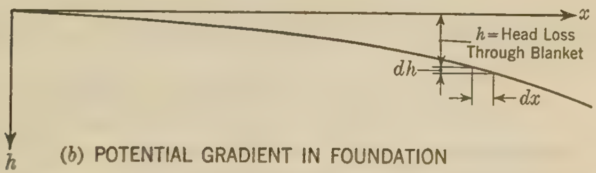
The letter symbols in this paper are defined where they first appear, in the text or by illustration, and are assembled for convenience of reference in Appendix I.

ASSUMPTIONS

The following derivation is applicable only when the ratio of foundation permeability to blanket permeability is large—say 10 to 1 or greater. If the contrast in permeability is small, the assumption that flow is vertical through the blanket and horizontal through the foundation may not be warranted. Furthermore, it has been assumed that the vertical plane through the upstream end of the blanket is an equipotential surface at full reservoir head. Actually, the equipotential plane coincides with the bottom of the reservoir, and in the cases represented subsequently by Eqs. 9*a*, 12*a*, and 18*a*, a small loss of head occurs between the true and assumed equipotential surfaces. However, if the length of the path of flow through the foundation is great as compared to the depth of the foundation, this head loss is negligible as compared to the errors due to nonuniformity of natural soils. The assumptions relative to direction of flow and potential distribution at the boundaries are shown in Fig. 1.


Reservoir Equipotential Surface:
Assumed
Actual
Direction of Flow:
Assumed
Actual
dx
Blanket
z
dq_b
q_0
Foundation
q_f

(a) SECTION THROUGH BLANKET


x
h = Head Loss Through Blanket
dh
dx
h

(b) POTENTIAL GRADIENT IN FOUNDATION

Fig. 1.—Assumed Direction of Flow, and Potential Distribution at the Boundaries

General Case

At any point x, under the blanket (Fig. 2), the horizontal flow through the foundation is equal to the flow through the blanket upstream from that point, plus the inflow under the upstream end of the blanket. That is:

$$q_f = \int_0^x dq_b + q_{f0} \qquad (1)$$

The vertical flow dq_b, through a small element of the blanket, of width dx and thickness z_b, varies directly with the head loss h, and inversely with blanket thickness z_b, so that

$$dq_b = \frac{k_b\, h\, dx}{z_b} \qquad (2)$$

In the interval dx, the horizontal flow in the pervious stratum is increased by an amount equal to the vertical inflow dq_b, so that

$$\frac{dq_f}{dx} = \frac{dq_b}{dx} = \frac{k_b\, h}{z_b} \qquad (3)$$

Since

$$q_f = k_f\, z_f \frac{dh}{dx} \qquad (4a)$$

it follows that

$$\frac{dq_f}{dx} = k_f\, z_f \frac{d^2h}{dx^2} \qquad (4b)$$

and, from Eq. 3,

$$\frac{d^2h}{dx^2} = \frac{k_b\, h}{z\, k_f\, z_f} \qquad (4c)$$

Using the notation $a = \sqrt{\dfrac{k_b}{z_b\, k_f\, z_f}}$ for blankets of uniform thickness, and $\alpha = \dfrac{k_b}{k_f\, z_f}$ for blankets of variable thickness, Eq. 4c may be written:

$$\frac{d^2h}{dx^2} = a^2\, h \qquad (5a)$$

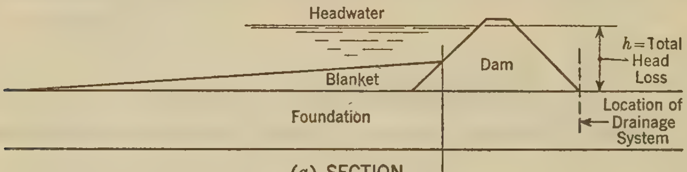
or

$$\frac{d^2h}{dx^2} = \frac{\alpha\, h}{z_b} \qquad (5b)$$

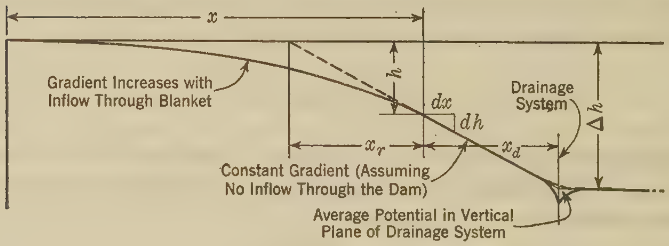
For a blanket of constant thickness and infinite horizontal extent, the solution to Eq. 5a is

$$h = h_0\, e^{ax} \qquad (6)$$

in which h_0 is the head loss through the blanket at point $x = 0$, which is taken for convenience in this case at the downstream end of the blanket. This solution is verified by successive differentiation:


Headwater
Dam
Blanket
Foundation
h=Total Head Loss
Location of Drainage System

(a) SECTION


x
Gradient Increases with Inflow Through Blanket
h
dx
dh
Drainage System
Δh
x_r
x_d
Constant Gradient (Assuming No Inflow Through the Dam)
Average Potential in Vertical Plane of Drainage System

(b) POTENTIAL GRADIENT

Fig. 2.—Relation Between Resistance, Head Loss, and Gradient

$$\frac{dh}{dx} = a\, h_0\, e^{ax} = a\, h \qquad (7a)$$

and

$$\frac{d^2h}{dx^2} = a^2\, h_0\, e^{ax} = a^2\, h \qquad (7b)$$

The resistance x_r is a measure of the efficiency of a blanket. It may be defined as the length of a prism of the foundation material, of thickness z_f and permeability k_f, which, under head loss h, would carry a flow equivalent to the flow which passes the blanket system under the same head loss. The relation between resistance, head loss, and gradient is shown graphically in Fig. 2.

Defined algebraically, $x_r = \frac{h}{\frac{dh}{dx}}$; or, for this case,

$$x_r = \frac{h}{a\,h} = \frac{1}{a} = \sqrt{\frac{z_b\,k_f\,z_f}{k_b}} \qquad (8)$$

Eq. 8 may be used to estimate the effectiveness of a natural surface layer as a blanket when none of the fine material is removed for borrow, and all the seepage enters the substratum by percolation through the blanket.

If part of a natural blanket is removed, the substratum is exposed directly to full reservoir head, and a part of the flow enters the foundation sand at the upper limit of the blanket. For this case, the solution of Eq. 5*a* is

$$h = h_n\,(e^{ax} - e^{-ax}) \qquad (9a)$$

$$\frac{dh}{dx} = a\,h_n\,(e^{ax} + e^{-ax}) \qquad (9b)$$

$$\frac{d^2h}{dx^2} = a^2\,h_n\,(e^{ax} - e^{-ax}) = a^2\,h \qquad (9c)$$

and

$$x_r = \frac{e^{ax} - e^{-ax}}{a\,(e^{ax} + e^{-ax})} = \frac{e^{2ax} - 1}{a\,(e^{2ax} + 1)} \qquad (9d)$$

In Eqs. 9, $x = 0$ at the upstream edge of the blanket, and h_n is a constant depending on the total head loss of the system of which the blanket is a part, and on the ratio of the blanket resistance to the remainder of the system. The value of h_0 in Eq. 7*a* is determined in the same manner. For example, assume a top stratum varying in thickness from 10 ft to 15 ft, overlying a sand foundation 50 ft to 60 ft thick, and assume that the ratio of blanket permeability to foundation permeability varies between 0.005 and 0.01. The extreme values of a, as computed from these dimensions, are 0.00246 and 0.0045, corresponding to resistances of 425 ft and 225 ft. If Δh and x_d (see Fig. 2) are, respectively, 80 ft and 750 ft for the dam in question, and, if 300 ft is adopted as the average resistance of the blanket, the effective total length of the percolation path is 1,050 ft, and the head loss through the blanket is 300/1,050 × 80, or 23 ft. Using $a = 0.0033$, corresponding to $x_r = 300$, and $h_0 = 23$, Eq. 7*a* becomes, in this case,

$$h = 23\,e^{0.0033x} \qquad (10)$$

The value of h_n, in Eq. 9*a*, is determined after computation of a definite value of h at the lower end of the blanket, using the relation between Δh, x_d, and x_r indicated graphically in Fig. 2 and as illustrated in the preceding example.

Ordinarily, for upstream blankets, the pressure distribution under the blanket is not of great interest, and the computation of constants for Eq. 6 or 9*a* need not be made, since the determination of x_r is all that is necessary for the estimation of pressures under the dam, or for computation of the approximate horizontal flow or discharge per linear foot of dam, which is

$$q_f = \frac{k_f\,z_f\,\Delta h}{x_r + x_d} \qquad (11)$$

Under downstream blankets, however, the substratum pressures are of considerable importance, and Eqs. 6 and 9*a* may be used in the design of the downstream drainage system to estimate the pressures that would exist if no drainage were provided, and if the underseepage were forced to find its way to the surface against the resistance of an impervious top stratum. When used for downstream blankets, the head loss h is referred to the ground surface (assumed level) downstream from the dam, or to tailwater elevation if the downstream toe is submerged.

The curves in Fig. 3 give values of x_r computed from x and a by the use of Eq. 9*d*. The curves for x_r and $\frac{z_b\,x}{\alpha}$ are plotted on the a-scale and the x-scale; and the curves for z_b are plotted on the a-scale and the α-scale. Suppose, in the foregoing example, that it is desired to excavate the natural blanket for use as fill material, leaving in place enough to provide a resistance of 200 ft. Using the higher of the limiting values of a (Fig. 3), the curve $x_r = 200$ intersects $a = 0.0045$ at $x = 330$ ft, the length of blanket to be left in place.

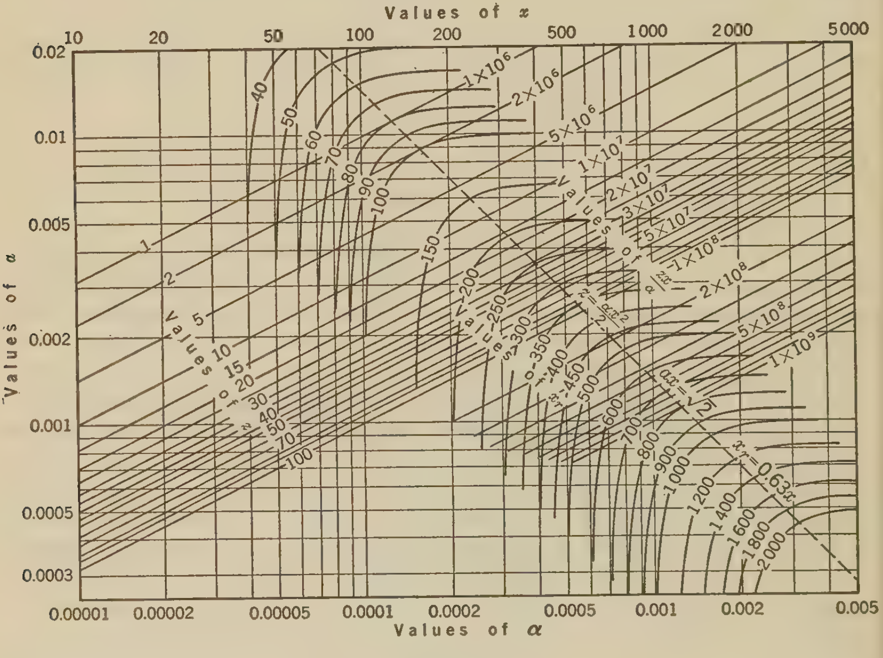


Fig. 3.—Resistance Curves

It is apparent from the curves of Fig. 3 that the ratio of resistance to blanket length, for any value of a, decreases rapidly as the length is extended beyond that indicated by the diagonal line $a\,x = \sqrt{2}$ in Fig. 3, and that very little benefit is to be gained either by leaving a natural blanket in place to great distances above a dam, or by the construction of an artificial blanket of uniform thickness and excessive length. These considerations lead to the conclusion that, if a blanket is to be built, its thickness probably should be reduced at the upstream where the blanket is relatively ineffective, and increased near the dam, interposing an additional thickness of impervious material where the head loss through the blanket is greatest.

This procedure would result in a blanket of triangular section, for which the resistance may be determined by substituting $s\,x$ for z_b in Eq. 5*b*, s being the slope of the blanket surface, and x the blanket length measured from its upstream edge. The solution for this case, as developed by Stanley U. Benscoter, Jun. Am. Soc. C. E., is as follows:

$$h = h_n \left\{ x + \frac{\alpha\,x^2}{2\,s} + \frac{\alpha^2\,x^3}{2!\,3!\,s^2} + \cdots + \frac{\alpha^{(n-1)}\,x^n}{(n-1)!\,n!\,s^{(n-1)}} \right\} \ldots\ldots (12a)$$

$$\frac{dh}{dx} = h_n \left\{ 1 + \frac{\alpha\,x}{s} + \frac{\alpha^2\,x^2}{[2!]^2\,s^2} + \cdots + \frac{\alpha^{(n-1)}\,x^{(n-1)}}{[(n-1)!]^2\,s^{(n-1)}} \right\} \ldots\ldots (12b)$$

and

$$\frac{d^2h}{dx^2} = h_n \left\{ \frac{\alpha}{s} + \frac{\alpha^2\,x}{2\,s^2} + \cdots + \frac{\alpha^{(n-1)}\,x^{(n-2)}}{(n-2)!\,(n-1)!\,s^{(n-1)}} \right\} \ldots\ldots (12c)$$

If Eq. 12*c* is multiplied by $\frac{s\,x}{\alpha}\left(= \frac{z_b}{\alpha}\right)$, the right-hand side becomes identical with the value of h in Eq. 12*a*, and the resulting equation

$$\frac{d^2h}{dx^2}\,\frac{z_b}{\alpha} = h \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (13)$$

verifies Eq. 12*a* as a solution of Eq. 5*b* for the triangular blanket.

The value of x_r, obtained by dividing Eq. 12*a* by Eq. 12*b*, is

$$x_r = x - \frac{\alpha\,x^2}{2\,s} + \frac{\alpha^2\,x^3}{3\,s^2} - \frac{11\,\alpha^3\,x^4}{48\,s^3} + \frac{19\,\alpha^4\,x^5}{120\,s^4} \ldots\ldots\ldots\ldots (14)$$

This series converges very slowly, and is not adapted to numerical computations. Eqs. 12*a* and 12*b* are somewhat better suited to numerical work, and x_r can be obtained from h and $\frac{dh}{dx}$ as computed from these series. Using this method, values of x_r for several triangular sections have been computed, and compared with the resistances of uniform sections of equal end area. From these comparisons, it appears that the resistance of a triangular section is approximately equal to that of a rectangular section of the same end area, if the length of the latter is one half the length of the triangular section.

The numerical difficulties involved in computing the resistance of a triangular section can be avoided by substituting a vertically curved surface profile,

$$z_b = \frac{\alpha\,x^2}{n\,(n-1)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (15)$$

for the tangent profile $z_b = s\,x$. This substitution is made only to facilitate computations, and it is not recommended that blankets be constructed to fit the curves used for purposes of analysis. For the curved surface indicated by

Eq. 15 the solution to the general formula (Eq. 5*b*) is

$$h = h_n x^n \qquad (16a)$$

$$\frac{dh}{dx} = n\, h_n\, x^{(n-1)} \qquad (16b)$$

and

$$\frac{d^2h}{dx^2} = n\,(n-1)\, h_n\, x^{(n-2)} \qquad (16c)$$

The solution is verified by substituting values of z_b and h from Eqs. 15 and 16*a* in Eq. 16*c*. From Eqs. 16*a* and 16*b* the resistance is

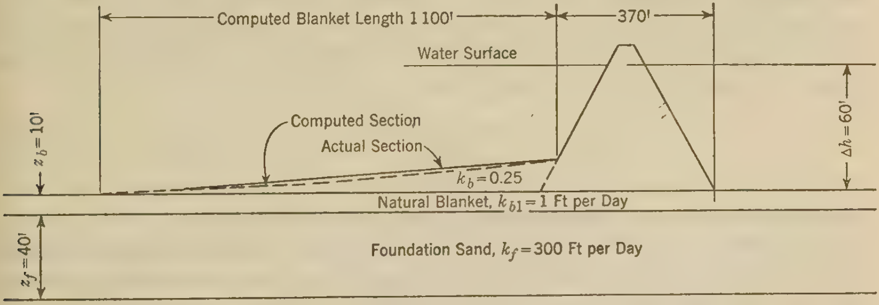
$$x_r = \frac{x}{n} \qquad (17)$$

The use of Eqs. 15 and 17 to approximate the resistance of a triangular section is shown by the following examples, computed for a blanket 500 ft long, which varies uniformly from 0 to 10 ft in thickness, and is built on a pervious stratum 40 ft thick, of such permeability that $\frac{k_b}{k_f} = 0.005$. For these conditions, $\alpha = \frac{k_b}{k_f\, z_f} = 0.000125$ and a curved section conforming to Eq. 15 will have a thickness of 10 ft at $x = 500$ ft when $n\,(n-1) = 3.12$, or $n = 2.34$. The resistance of this section is $500/2.34 = 214$ ft. Another approximation of the triangular section may be made by selecting n (Eq. 15) so that the end area of the parabolic section equals that of the triangular shape. For the preceding example, this gives $n = 2.04$ and $x_r = 245$ ft. Using the assumption that the triangular section 500 ft by 10 ft is equivalent to a uniform section 250 ft by 10 ft, Fig. 3 gives $a = 0.0035$ (for $z_b = 10$ and $\alpha = 0.000125$) and $x_r = 200$ when $x = 250$.

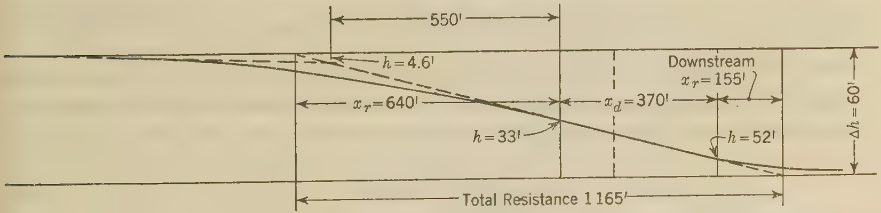
Considering the fact that the basic data used in the computations will be only approximately correct, the foregoing approximate values of the resistance of a triangular section are in sufficiently close agreement to justify the use of either Eq. 9*d* or 17 in estimating the efficiency of this type of blanket. The approximate relationship between rectangular and triangular sections, thus far developed, however, does not furnish any answer to the question raised as to the relative merits of the two sections. Referring again to Fig. 3, it will be seen that the diagonal lines that represent constant blanket thickness, when referred to the α-scale and the a-scale, also represent constant values of the sectional area $z_b\, x$ when referred to the x-scale and the a-scale on which the x_r-curves are plotted. These constant-area curves may be used to estimate the optimum dimensions of a rectangular section if the quantity of blanket material is fixed. For example, suppose that 20,000 cu yd of blanket material per station is available, and that its permeability is such that $\alpha = 0.0001$. The sectional area is then 5,400 sq ft, and $\frac{z_b\, x}{\alpha} = 5.4 \times 10^7$. The maximum resistance (300 ft) is found at $x = 475$, and the corresponding thickness is 11.4 ft. If the blanket material were much more pervious, so that $\alpha = 0.0027$ and $\frac{z_b\, x}{\alpha} = 2 \times 10^6$, the optimum length would be 160 ft, the thickness 34 ft, with a resistance of 100 ft. However, by following the line $\frac{z_b x}{\alpha} = 2 \times 10^6$ to intersect $x_r = 90$ at $x = 105$ and at $x = 250$, it is seen that the optimum length may be reduced by about 34% or increased by about 56% with a loss of only 10% in resistance, if the thickness is correspondingly changed to retain the same sectional area. Since the x_r-curves are spaced to conform to the logarithmic x-scale, the same ratio of loss of resistance to increase or decrease from the optimum value of x holds true for any value of x on the broken line $a x = \sqrt{2}$.

Since Fig. 3 is approximately applicable to triangular sections if the base length is considered to be $2x$, it appears that either the triangular or rectangular shape may be used with equal effectiveness, and that for either section a considerable departure may be made from the theoretical optimum dimensions without great loss of efficiency. It is concluded, therefore, that feasibility of construction should govern the dimensions of a blanket, as long as the extremes of inefficiency represented by the ends of the x_r-curves in Fig. 3 are avoided.

For a combination of natural and artificial blankets, such as is shown in Fig. 4(a), it is again convenient to represent the triangular section by an approximately equivalent parabolic section to simplify the computations. It can be shown analytically that, if $n = 2$ in Eq. 15, the maximum resistance can be obtained from a fixed sectional area. (It is of interest to note that the relationship, between z_b and x for the most favorable ratio of $\frac{x_r}{z_b x}$, is identical


Computed Blanket Length 1 100'
370'
Water Surface
Computed Section
Actual Section
$k_b = 0.25$
$\Delta h = 60'$
$z_b = 10'$
Natural Blanket, $k_{b1} = 1$ Ft per Day
Foundation Sand, $k_f = 300$ Ft per Day
$z_f = 40'$

(a) SECTION THROUGH DAM


550'
$h = 4.6'$
Downstream
$x_r = 155'$
$x_r = 640'$
$x_d = 370'$
$h = 33'$
$h = 52'$
$\Delta h = 60'$
Total Resistance 1 165'

(b) COMPUTATION OF HEAD IN SUBSTRATUM

FIG. 4.—ILLUSTRATIVE EXAMPLE WITH IMPERVIOUS DAM

for the curved section as obtained analytically, and for the rectangular section as obtained graphically by the curves in Fig. 3.) The arbitrary use of $n = 2$ simplifies computation, and at the same time permits the development of a formula adapted to use in designing combination blankets. Let the vertical gradient $\frac{h}{z_b}$ be a constant, i. Then

$$z_b = \frac{h}{i} = \frac{\alpha x^2}{2} + \beta x + z_{b0} \qquad (18a)$$

$$\frac{dh}{dx} = i(\alpha x + \beta) \qquad (18b)$$

$$\frac{d^2h}{dx^2} = i\alpha = \frac{\alpha h}{z_b} \qquad (18c)$$

and

$$x_r = \frac{z_b}{\alpha x + \beta} \qquad (18d)$$

At the upstream end of this section $x = 0$, $h = i z_{b0}$, and $\frac{dh}{dx} = i\beta$. If the upstream end adjoins the lower end of another blanket for which the head loss and foundation gradient are numerically equal to $i z_{b0}$ and $i\beta$, there will be no discontinuity in the foundation flow. This condition may be met by choosing values of β and z_{b0} in Eq. 18a such that

$$\frac{z_{b0}}{\beta} = \frac{h}{\frac{dh}{dx}} = x'_r \qquad (19)$$

in which x'_r is the resistance of the upstream blanket.

An application of Eqs. 18 and 19 is shown in Fig. 4, representing an artificial blanket, whose permeability k_{b1} is 0.25 ft per day, constructed on a natural blanket 10 ft thick with a permeability $k_{b2} = 5$ ft per day. The natural blanket may be replaced, for purposes of computation, by one having a thickness of 0.5 ft and a permeability of 0.25 ft per day, equal to that of the artificial section, without changing its resistance, which for infinite length is $\sqrt{\frac{10 \times 300 \times 40}{5}}$ or 155 ft. From the dimensions shown in Fig. 4, the value of α for the artificial blanket is 1/48,000 and z_{b0} is 0.5 ft, the reduced thickness of the natural blanket. Using these constants in Eq. 18a, and $\beta = 0.0032$, computed from Eq. 19,

$$z_b = \frac{x^2}{96{,}000} + 0.0032\,x + 0.5 \qquad (20)$$

in which z_b includes the reduced thickness of the blanket already in place. If the size of the blanket is limited by availability of material to a sectional area of 6,000 sq ft for the case under consideration, the length is found to be about 1,100 ft from Eq. 20, and the corresponding thickness at the dam is 16.2 ft, or 16.7 ft, including the reduced thickness of the original surface layer. From Eq. 18d, the resistance is $\frac{16.7}{0.023 + 0.003}$ or 640 ft, which includes the effect of the natural blanket upstream from the point where no additional material is placed.

From Fig. 4(*a*), the length of the path of flow under the dam is 370 ft, and the resistance of the system 1,165 ft if it is assumed that no seepage takes place through the dam proper. The pressure gradient in the foundation, as determined by these computed resistances, is shown in Fig. 4(*b*). In this example it has been assumed for the sake of simplicity that the entire dam is impervious, and that all the seepage must escape through the downstream blanket. In actual design, foundation drainage would ordinarily be provided in the form of a pervious drainage blanket within the dam, a system of pressure relief wells, or a horizontal pipe drain, and the path of flow or resistance downstream from the blanket would be less than in the example given. A discussion of downstream drainage facilities is outside the scope of this paper, but, since a knowledge of the downstream resistance is necessary in evaluating the effectiveness of a blanket, approximate methods for estimating the path of flow under the dam are included in Appendix II.

As previously stated, the curved section, shown by the dotted line in Fig. 4(*a*), was adopted for purposes of computation only, and would be replaced in actual construction by an equivalent section of similar shape, as indicated. Theoretically, the vertical gradient is constant over the entire length of the curved section, and therefore the curve represents an efficient and well-balanced section which should serve as a good guide in the selection of the actual blanket shape adopted for construction.

Subject to the limitations previously stated as to accuracy of the basic data used in the computations, the formulas for uniformly thick blankets may be used to estimate the substratum pressures downstream from a dam when no drainage is provided, as in the section shown in Fig. 4(*a*). In this example, although the natural blanket has a relatively low resistance, the computations indicate the desirability of a downstream drainage system, and at the same time provide a method whereby the discharge from the system may be estimated.

SUMMARY

The foregoing analysis of flow through blankets is based on assumed conditions which are approached very closely when the ratio of foundation to blanket permeability is large, and when the length of the horizontal path of flow is large as compared to the thickness of the pervious foundation. When these conditions are met, the formulas are mathematically exact for practical purposes.

The difficulty of accurate determination of the permeability of the soils in place, and the nonuniformity of thickness and permeability of natural deposits, constitute the greatest source of error in use of this analysis in design. However, since the same sources of error are present in analysis of flow by models, the analytical method is to be preferred for blanket studies when the foundation may be assumed to consist of a single pervious layer. If the foundation is separated into two or more distinct strata by impervious lenses, a model study would be the better method to investigate the resulting complex flow pattern.

In view of the uncertainties as to the thickness and permeability of natural strata involved, it might first appear that the methods of analysis presented herein are unnecessarily elaborate. The methods have been developed in considerable detail, however, in the belief that the general principles involved, in the design of blankets, could best be presented by a mathematical analysis, and that the application of these principles to blanket problems, although not leading to exact results, will provide, in many cases, an improved method of estimating blanket performance.

APPENDIX I

NOTATION

The following letter symbols, used in this paper, conform essentially with "Soil Mechanics Nomenclature"[2] adopted by the Society's Board of Direction in 1941:

a = an abbreviated notation for $\sqrt{\frac{k_b}{z_b\, k_f\, z_f}}$;

α = an abbreviated notation for $\frac{k_b}{k_f\, z_f}$;

β = a constant, equal to the slope of a curved blanket at $x = 0$;

e = base of natural logarithms;

h = head or potential at any point, x, in the foundation (referred to reservoir potential) and equal to the head loss through the blanket at point x; h_1, h_2, etc. = head at specified points in the foundation:

$\frac{dh}{dx}$ = horizontal potential gradient in the foundation;

$\frac{d^2h}{dx^2}$ = rate of change of $\frac{dh}{dx}$, proportional to inflow through the blanket;

h_n = a constant factor for computing h, determined by the relationship between the blanket and the flow system as a whole;

h_0 = head loss through the blanket at point $x = 0$;

i = vertical potential gradient through blanket, for special case of constant gradient;

k = permeability; k_b = permeability of the blanket material, which influences the vertical seepage path; and k_f = permeability of the foundation material, which influences the horizontal seepage path.

n = an arbitrary constant defining the profile of curved blanket surfaces;

q = flow per unit width; q_1, q_2, q_3, etc. = flow through designated sections of the blanket or foundation;

q_b = vertical flow per unit width, through the blanket; d_{qb} = element of vertical flow through an element dx of the blanket;

q_f = horizontal flow, per unit width, through the foundation;

q_0 = inflow to the foundation under the upstream end of the blanket;

s = z/x, the slope of the surface of a triangular blanket;

x = horizontal distance measured positive in the direction of flow; dx = an element of the distance x;

x_r = resistance of a blanket, being the effective length of the path of flow under the blanket;

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[2] *Manual of Engineering Practice No. 22*, Am. Soc. C. E., 1941.

z = thickness of a blanket, when the thickness is variable;
z_0 = thickness of blanket at $x = 0$;
z_b = thickness of a blanket when the thickness is constant; and
z_f = thickness of the foundation stratum.

APPENDIX II

Flow Under the Dam Proper

A principle which may be used, in some cases, to estimate the characteristics of flow under the dam itself is illustrated by analogy with the flow under the composite blanket shown in Fig. 4(*a*). Controlling points and slopes on the pressure gradient curve of Fig. 4(*b*) may be obtained without recourse to the equations defining h in terms of x, as follows: Assign any convenient value, say, $h_1 = 1.55$ ft to the head loss through the natural blanket at its junction with the artificial blanket. Under this head, the foundation gradient at this point is $1.55/155 = 0.01$, since the natural blanket resistance is 155 ft, and $q_1 = 0.01\ k_f\ z = 120$ cu ft per day. For this particular case, the vertical gradient through the artificial blanket is constant throughout its length, and is equal to the maximum gradient through the natural blanket. In terms of the reduced thickness of 0.5 ft, the constant vertical gradient is $1.55/0.5 = 3.1$, and the flow through the artificial blanket is $q_2 = 3.1 \times 0.25 \times 1{,}100 = 853$ cu ft per day, making the

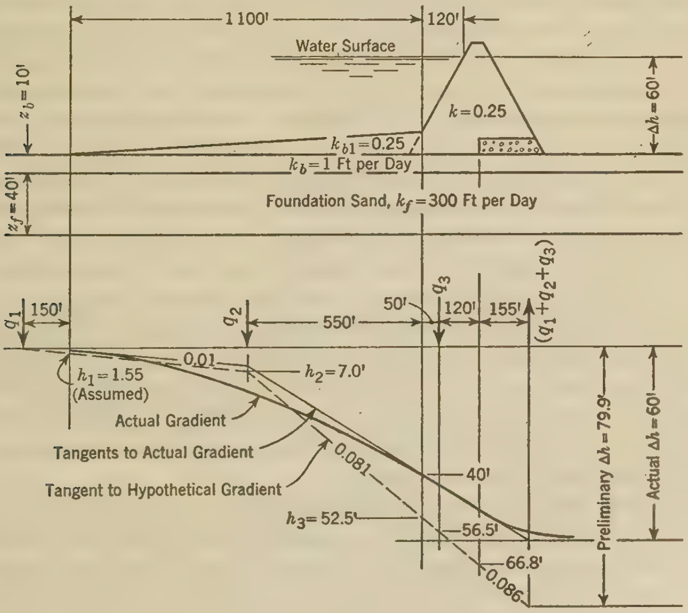
1 100'
120'
Water Surface
k=0.25
Δh=60'
z_b=10'
k_b1=0.25
k_b=1 Ft per Day
Foundation Sand, k_f=300 Ft per Day
z_f=40'
q_1
150'
q_2
550'
50'
q_3
120'
155'
(q_1+q_2+q_3)
0.01
h_1=1.55 (Assumed)
h_2=7.0'
Actual Gradient
Tangents to Actual Gradient
Tangent to Hypothetical Gradient
0.081
40'
h_3=52.5'
56.5'
66.8'
0.086
Preliminary Δh=79.9'
Actual Δh=60'

Fig. 5.—Illustrative Example with Flow Through Dam

total foundation flow at the lower end of the blanket system 973 cu ft per day, with a corresponding horizontal gradient of $\frac{q_1 + q_2}{k_f z_f} = 0.081$. For purposes of computing the head loss at the lower end of the blanket system, the uniform inflow through the artificial blanket may be considered concentrated at its midpoint, and the inflow through the natural blanket as concentrated at a point 155 ft upstream from its lower end. For this assumed condition, the foundation gradient would be two tangents to the actual gradient curve, intersecting at the midpoint of the artificial blanket as shown in Fig. 4(*b*), and the numerical value of $h_2 = 52.5$, corresponding to the assumed value of $h_1 = 1.55$, may be computed from the tangent slopes as indicated in Fig. 5.

In Fig. 5, the blanket is identical with that in Fig. 4, but the dam section consists of an upstream "impervious" section having the same permeability as the blanket, and has a downstream drainage blanket of very high permeability, such that seepage entering it from the foundation will be carried off without appreciable resistance, and the drainage blanket may be assumed to be at tailwater potential. The flow through the impervious upstream section will be principally downward, and the vertical gradient may be roughly estimated as a mean between the value at the lower end of the blanket and the value at the line of saturation, where the vertical gradient for gravity flow may be taken as 1.0. On the basis of these assumptions, the average downward gradient through the dam is 2.0, and over a horizontal distance of 120 ft as shown in Fig. 5, the inflow q_3 is $2 \times 0.25 \times 120 = 60$ cu ft per day. The "center of gravity" of the inflow is somewhat upstream from the center of the 120-ft interval—say 50 ft downstream from the end of the blanket. At this point the hypothetical gradient shown by the broken line in Fig. 5 changes from 0.081 to 0.086 as a result of the increased flow, and the foundation head loss at the upstream end of the drainage blanket is accordingly 66.8 ft. Assuming no back pressure in the drainage blanket of the dam, the resistance of the natural blanket remains 155 ft and in this distance downstream the additional head loss, computed from the gradient of 0.086, is 13.5 ft making a total hypothetical head loss of 79.9 ft resulting from the initial arbitrary assumption of 1.55-ft loss through the upstream natural blanket. Since the actual head on the system is 60 ft, the preliminary head losses and gradient are corrected by the factor 60/79.9 to give the controlling points on the actual head loss profile, as shown by the solid line in Fig. 5. The computed head loss through the blanket system is 39 ft, or two thirds the total loss of the system.

The foregoing example illustrates, in a general way, the methods which may be used for estimating the over-all flow characteristics of a pervious foundation, and which may be modified to take into account toe drainage by longitudinal drains or pressure relief wells. It is to be noted that the effect of horizontal flow in the dam has been neglected entirely in this approximation, and that this method is not applicable where the flow is substantial as compared to the seepage through the foundation.

DISCUSSION

REGINALD A. BARRON,[3] JUN. AM. SOC. C. E.—The efficiency of upstream blankets for dams and dikes has been presented in an interesting manner by Mr. Bennett. The writer agrees with the author as to the effects of variations in extent and physical properties of natural soil deposits, and believes that the simplifying assumptions made in the paper are quite acceptable. The difference in permeability between natural blanket and underlying pervious layer quite easily may be greater than 1 to 10 when the effect of stratification is considered.

The writer has developed and used comparable equations particularly for the study of hydraulic pressures on the lower side of a natural blanket for conditions shown in Fig. 6. In this case an earth embankment with a central impervious core and flanking pervious shells rests upon a natural blanket which in turn rests upon a more pervious deposit in direct contact with the upstream pool. The assumptions are quite similar to those of the author. In the length L_1, it is assumed that the flow is vertically downward in the blanket. In the length $L_2 - L_1$, seepage through the embankment and the underlying blanket is neglected. In the remaining length $L_3 - L_2$, the seepage through the blanket is vertically upward. In the pervious deposit the seepage is considered to occur in a horizontal direction only.

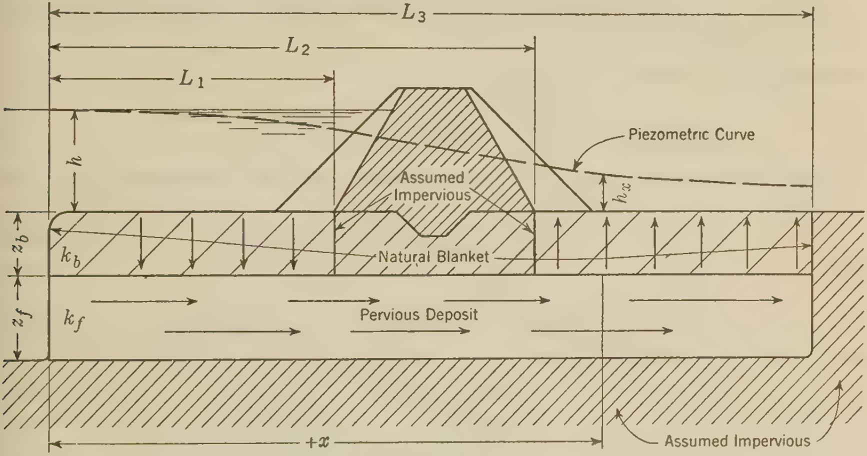

FIG. 6.

The differential equations based upon continuity of flow are: For $0 < x < L_1$,

$$-k_f \frac{d^2h_x}{dx^2} \cdot z_f = +k_b \frac{h - h_x}{z_b} \qquad \text{(21a)}$$

for $L_1 < x < L_2$,

$$-k_f \frac{d^2h_x}{dx^2} \cdot z_f = 0 \qquad \text{(21b)}$$

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and, for $L_2 < x < L_3$,

$$+ k_f \frac{d^2 h_x}{dx^2} \cdot z_f = + k_b \frac{h_x}{z_b} \qquad \text{(21}c\text{)}$$

in which h_x is the elevation of the piezometric height above land-side tailwater.

The boundary conditions are:

1. At $x = 0$, $h_x = h$;
2. At $x = L_1$, h_x obtained from Eq. 21*a* equals h_x obtained from Eq. 21*b*;
3. At $x = L_1$, the gradient, $\frac{dh_x}{dx}$, obtained from Eq. 21*a*, is equal to the gradient, $\frac{dh_x}{dx}$, obtained from Eq. 21*b*;
4. At $x = L_2$, h_x obtained from Eq. 21*b* equals h_x obtained from Eq. 21*c*;
5. At $x = L_2$, the gradient, $\frac{dh_x}{dx}$, obtained from Eq. 21*b*, is equal to the gradient, $\frac{dh_x}{dx}$, obtained from Eq. 21*c*; and
6. At $x = L_3$, horizontal flow ceases—that is, $\frac{dh_x}{dx} = 0$.

The solutions of Eqs. 21, subject to the boundary conditions, are: For $0 < x < L_1$—

$$h_x = h + A \sinh (a\,x) \qquad \text{(22}a\text{)}$$

for $L_1 < x < L_2$—

$$h_x = h + A\,[\sinh (a\,L_1) + a\,(x - L_1) \cosh (a\,L_1)] \qquad \text{(22}b\text{)}$$

and, for $L_2 < x < L_3$—

$$h_x = B\,[e^{ax} + e^{a(2L_3 - x)}] \qquad \text{(22}c\text{)}$$

in which

$$A = B \left\{ \frac{[e^{aL_2} - e^{a(2L_3 - L_2)}]}{\cosh (a\,L_1)} \right\} \qquad \text{(23}a\text{)}$$

$$B = h \div \{[1 - \tanh (a\,L_1) - a\,(L_2 - L_1)]\,e^{aL_2} + [1 + \tanh (a\,L_1) + a\,(L_2 - L_1)]\,e^{a(2L_3 - L_2)}\} \qquad \text{(23}b\text{)}$$

and

$$a = \sqrt{\frac{k_b}{k_f\,z_b\,z_f}} \qquad \text{(23}c\text{)}$$

The seepage rate per unit width at any section in the pervious deposit may then be obtained from:

$$Q_x = -\,k_f \frac{dh_x}{dx} z_f \qquad \text{(24}a\text{)}$$

and the seepage rate per unit width at any point x through the downstream natural blanket may be obtained from

$$Q'_x = k_b \frac{h_x}{z_b} \qquad \text{(24}b\text{)}$$

If the dimension L_3 approaches infinity, Eqs. 22 become: For $0 < x < L_1$—

$$h_x = h \left\{ 1 - \frac{\sinh (a\,x)}{[1 + \tanh (a\,L_1) + a\,(L_2 - L_1)] \cosh (a\,L_1)} \right\} \ldots (25a)$$

for $L_1 < x < L_2$—

$$h_x = h \left[\frac{1 + a\,(L_2 - x)}{1 + \tanh (a\,L_1) + a\,(L_2 - L_1)} \right] \ldots\ldots\ldots\ldots (25b)$$

and, for $L_2 < x$—

$$h_x = \frac{h\, e^{a(L_2 - x)}}{1 + \tanh (a\,L_1) + a\,(L_2 - L_1)} \ldots\ldots\ldots\ldots (25c)$$

If the distance L_1 approaches minus infinity and L_3 approaches plus infinity, then, with the origin at the upstream toe of the impervious core of the earth embankment and with $(L_2 - L_1)$ equal to L, Eqs. 25 further simplify as follows: For $x < 0$—

$$h_x = h \left[1 - \frac{e^{ax}}{2 + a\,L} \right] \ldots\ldots\ldots\ldots (26a)$$

for $0 < x < L$—

$$h_x = h \left[\frac{1 + a\,(L - x)}{2 + a\,L} \right] \ldots\ldots\ldots\ldots (26b)$$

and, for $L < x$—

$$h_x = \frac{h\, e^{a(L - x)}}{2 + a\,L} \ldots\ldots\ldots\ldots (26c)$$

Eqs. 25a and 26a are in agreement with Eqs. 6 and 9a, respectively, when proper consideration is given to the physical constants and boundaries.

The foregoing development neglects the fact that the seepage paths are curved lines in the pervious deposit. Because of this phenomenon, the piezometric heads along the upper part of the pervious deposit will be somewhat less at the river-side toe of the embankment and somewhat greater at the land-side toe, as compared with the piezometric heads obtained from flow-net studies. Since it is possible that the land-side natural blanket may be disrupted by excess water pressure from pervious deposit, this difference in calculated heads is conservative. The neglect of seepage through the embankment and through the natural blanket immediately below reduces the seepage calculated by Eq. 24a as compared with that calculated by flow-net studies. The reduction is not large and is well within the permeability range of natural deposits.

Although the author indicates that the law of diminishing returns applies to an upstream blanket extended beyond certain limiting lengths, the writer would be somewhat reluctant to destroy or weaken any such natural blanket because of the increased water pressures that would occur under the blanket in the downstream area. If upstream borrow pits are used, adequate drainage facilities should be provided at the downstream toe or at adjacent areas to minimize the danger from increased water pressures. Downstream borrow pits, if available, would appear to be more desirable, and might aid in partly controlling any excessive pressures under the blanket between the downstream embankment toe and the borrow pits. However, borrow pits should be excavated so as to minimize the danger from piping or underground erosion which might work back under the natural blanket to the upstream area.

An upstream blanket with variable section is not a new idea but the author has done well to restate the case and to indicate the proper and most beneficial use of available material. A review of a number of dams indicates that blankets with triangular sections have been utilized infrequently. However, triangular-shaped blankets have certain practical limits that should be considered. For instance, it is doubtful that a blanket shaped to a featheredge would be either desirable or practical to construct. For flood control structures that do not have permanent storage, partial loss of imperviousness may occur in an exposed upstream blanket near the surface due to loosening of the blanket by frost action or due to other causes which promote soil expansion.

V. A. Endersby,[4] M. Am. Soc. C. E.—The principle of applying a blanket of low permeability to the dam face and part of the adjacent reservoir bottom is advocated by the author as being preferable to the core and cutoff wall method because more of the dam remains to resist hydrostatic pressure. This procedure has the additional advantage of usually being cheaper; however, it has the disadvantage of not creating as positive a cutoff.

An optimum combination of economy and successful operation should result from blanketing the face of the dam and connecting the blanket with a cutoff at the upstream toe, provided that this location is suitable for a cutoff. The use of an upstream blanket would also permit the cutoff to be placed at some distance from the dam, if such a location were feasible or economical—as may often be the case because of the natural slope of the bedrock. The writer is under the impression that such combinations have not been considered very often.

Of course, the use of such combinations, as well as the use of blankets of considerable area, in general, depends on the cost of blanket construction. A blanket used with a cutoff wall usually would have to be impermeable or almost so. In many cases conservation of water is important.

Mr. Bennett's analysis, favoring a blanket section of variable thickness, also leads to economy of blanket construction where natural, partly pervious materials are being considered. Such materials at the dam usually would be quite thick. If the materials were not already in place, the cost of placing them would justify a comparison with the cost of placing positively impermeable, thin, artificial membranes.

Since 1935 or thereabouts, asphaltic concrete membranes have been used increasingly for canal, flood channel, and reservoir linings. Asphaltic concrete has the advantages of being flexible and plastic, and thus not subject to the cracking due to the ground settlements which usually occur under an impermeable blanket. For this reason, some large dams in Africa were faced several years ago with asphaltic layers under concrete facing.

If an asphalt plant is near by, or if the work is important enough to justify a large portable plant, asphaltic concrete is quite low in cost. Since a 2-in.

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blanket, properly sealed, is sufficient for high water heads, asphaltic concrete is in close competition with natural materials.

However, if complete impermeability is required, special sealing treatment for the surface is necessary, unless the mix itself is made impermeable. The latter method requires special proportioning and close control in construction. If sealing treatment is used, requirements for aggregates are less rigid than those for road work, the chief difference being the need for considerably more asphalt in the hydraulic construction.

Asphaltic concrete is suitable for a wide range of gradings and compositions of aggregates, and is free from chemical interaction between binder and aggregate.

The construction of a durable and impermeable asphaltic membrane is highly technical, requiring as much expert supervision as the mixing of Portland cement concrete.

HARRY R. CEDERGREN,[5] JUN. AM. SOC. C. E.—In presenting a mathematical analysis of seepage through impervious blankets, Mr. Bennett has defined one of the major problems encountered in the design of earth dams—the control of underseepage. The author has indicated the trend toward the use of graded filters and vertical drainage wells for the control of seepage and has discussed the value of impervious upstream blankets for reducing seepage quantities through pervious foundations. He has presented a mathematical analysis for determining the head loss through impervious blankets—evaluating the efficiency of the blankets in terms of head loss.

Undoubtedly, the formulas in the paper will find favor among engineers interested in obtaining quick evaluations of the general efficiency of proposed blanket designs. Such analyses should be particularly helpful for preliminary studies in which basic types of treatment and approximate costs of projects are being studied. In some cases, no further analysis may be required; however, in the preparation of final designs, more detailed analysis of the probable development of seepage pressures and gradients is often feasible. For this purpose many engineers make extensive use of the graphical flow-net[6,7] method. Although a moderate amount of study and practice is required for the development of skill in the construction of flow nets, the required effort is not out of proportion to the value. Frequent use of the flow net develops an understanding of seepage that often permits an engineer to detect inadequate designs quickly without detailed analysis, thus saving considerable time and expense. Furthermore, the flow net gives a graphical picture of the probable distribution of seepage pressures and gradients that is an important aid to the engineer in the design of water impounding structures.

As the author has stated, soil deposits are never uniform, and no seepage analysis is more dependable than the degree of accuracy of the conditions assumed in the analysis. Karl Terzaghi, M. Am. Soc. C. E., has stressed this limitation of soil mechanics analyses repeatedly and recommends[8] field observations during construction to determine the degree of safety of structures. Observation wells or piezometers at strategic points in earth dams and in dam foundations will permit the determination of actual pore pressures and thus will reflect the degree of safety of structures, both during and after construction. An adequate system of pore pressure measuring devices should be installed in all important earth dams, and reliable records of pore pressure data, together with pertinent information regarding the installations, should be made available to the engineering profession.

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[6] "Seepage Through Dams," by Arthur Casagrande, *Journal*, New England Water Works Assn., June, 1937, pp. 131–172 (also reprinted in *Publication No. 209*, Graduate School of Eng., Harvard Univ., Cambridge, Mass., June, 1937; and in "Contributions to Soil Mechanics, 1925–1940," Boston Soc. of Civ. Engrs., Boston, Mass., 1940).

[7] "Theoretical Soil Mechanics," by Karl Terzaghi, John Wiley & Sons, Inc., New York, N. Y., 1943, pp. 235–247.

In evaluating the performance of an impervious blanket, or any other design feature relating to seepage control, the first consideration should be the safety of the dam. The safety of earth dams on pervious foundations depends in a large measure on uplift pressures in the vicinity of the downstream toe. The author has emphasized the value of downstream drainage systems for the control of uplift pressures downstream from dams, and his two examples (Figs. 4 and 5) show the beneficial influence of a pervious blanket installed beneath the downstream part of the dam. This type of drain increases the degree of safety of the structure but it may not be entirely adequate. Under the assumed condition that the soils are isotropic (horizontal permeabilities (k_{I}) equal to vertical permeabilities (k_{II}), Mr. Bennett's analysis indicates that the escape gradient at the downstream toe is 0.8 for the section in Fig. 4 without drainage and 0.20 for the section in Fig. 5, in which a gravel blanket has been provided beneath the downstream part of the dam. The former condition (Fig. 4) is decidedly unsafe and the latter may be considered barely adequate provided the soils are relatively free of stratification.

Before accepting a blanket design as adequate (for example, the section in Fig. 5), an investigation should be made to determine the probable degree of stratification of the foundation, since natural soil deposits are always more or less stratified and even moderate stratification can have an important influence upon uplift pressures. The effect of stratification can be demonstrated clearly with the aid of the flow net. For example, let it be assumed that all soils in the basic section given in Fig. 5 are moderately stratified and that all the horizontal permeabilities (k_{I}) are nine times the vertical permeabilities (k_{II}). The principal question to be answered is "What effect does this degree of stratification have upon uplift pressures at the downstream toe?" The results of a flow-net analysis of the basic section in Fig. 5, together with a number of modifications, are summarized in Table 1. The section given in Fig. 5 is designated "case 1," and modified sections are designated cases 2, 3, and 4, respectively. All flow nets were constructed by the Forchheimer graphical method.[6,7] To show the influence of stratification, the original section with a 1,100-ft blanket upstream and a drainage blanket under the downstream half of the dam was analyzed for two conditions—namely, all soils homogeneous and isotropic ($k_{\mathrm{I}} = k_{\mathrm{II}}$) and all soils moderately stratified ($k_{\mathrm{I}} = 9\, k_{\mathrm{II}}$). To determine the benefit derived from the blanket, a similar analysis was made with the blanket omitted. This analysis indicated that the blanket reduced the seepage quantity and discharge gradients at the toe approximately 50%. The values obtained from the flow-net analysis of Mr. Bennett's section with homogeneous soils agree substantially with the results obtained from his formulas but indicate a slightly higher discharge gradient of 0.25 at the downstream toe. For the stratified condition, the discharge gradient is increased to 1.0 which represents a critical condition. The magnitude of the discharge gradient in this analysis varies approximately in proportion to the square root of the ratio of k_I to k_{II}; hence, the approximate gradient can be estimated for other degrees of stratification. The original design incorporating the 1,100-ft blanket (case 1, Table 1) may be considered barely adequate, provided the soil is essentially nonstratified, but it is not adequate if any appreciable degree of stratification exists. Omitting the blanket (case 2, Table 1) reduces the degree of safety by approximately 50%; hence, the blanket does definitely contribute to the safety of the structure, even though it may not be entirely adequate.

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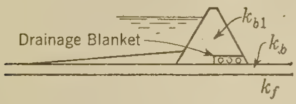
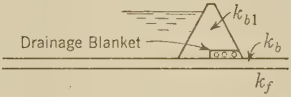
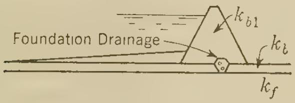
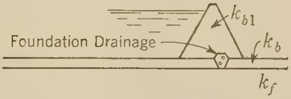
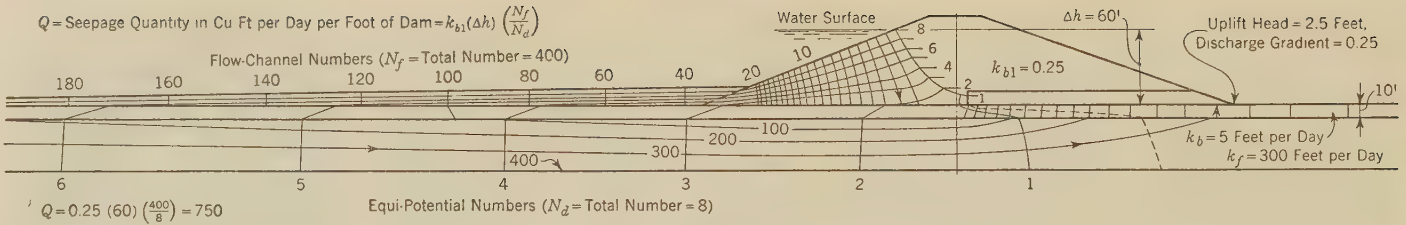
[8] "Theoretical Soil Mechanics," by Karl Terzaghi, John Wiley & Sons, Inc., New York, N. Y., 1943, pp. 1–3.

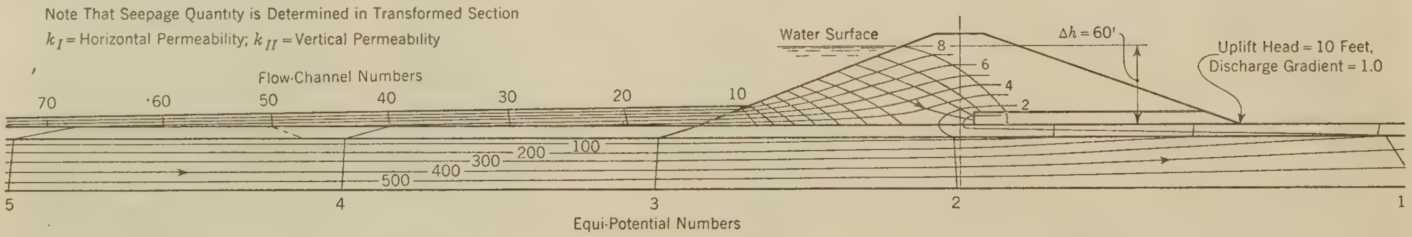
TABLE 1.—EVALUATION OF THE BLANKET EFFICIENCY BY THE FLOW-NET ANALYSIS (BASIC SECTION OF FIG. 5, WITH MODIFICATIONS[a])

<table>
<tr><th colspan="6">WITH UPSTREAM BLANKET</th><th colspan="6">WITHOUT UPSTREAM BLANKET</th></tr>
<tr><th colspan="3">All Soils Isotropic ($k_I = k_{II}$)</th><th colspan="3">All Soils Anisotropic ($k_I = 9\,k_{II}$)</th><th colspan="3">All Soils Isotropic ($k_I = k_{II}$)</th><th colspan="3">All Soils Anisotropic ($k_I = 9\,k_{II}$)</th></tr>
<tr><th>Seepage[b]</th><th>Gradient[c]</th><th>Stability[d]</th><th>Seepage[b]</th><th>Gradient[c]</th><th>Stability[d]</th><th>Seepage[b]</th><th>Gradient[c]</th><th>Stability[d]</th><th>Seepage[b]</th><th>Gradient[c]</th><th>Stability[d]</th></tr>
<tr><td colspan="12">(a) BLANKET DRAINAGE UNDER DOWNSTREAM HALF OF DAM</td></tr>
<tr><td colspan="6">
Case 1</td><td colspan="6">
Case 2</td></tr>
<tr><td>750</td><td>0.25</td><td>Barely adequate</td><td>1,100</td><td>1.0</td><td>Critical</td><td>1,400</td><td>0.41</td><td>Critical</td><td>1,900</td><td>1.73</td><td>Extremely critical</td></tr>
<tr><td colspan="12">(b) FOUNDATION DRAINAGE</td></tr>
<tr><td colspan="6">
Case 3</td><td colspan="6">
Case 4</td></tr>
<tr><td>850</td><td>→0</td><td>Very safe</td><td>1,400</td><td>→0</td><td>Very safe</td><td>1,700</td><td>→0</td><td>Very safe</td><td>2,800</td><td>→0</td><td>Very safe</td></tr>
</table>

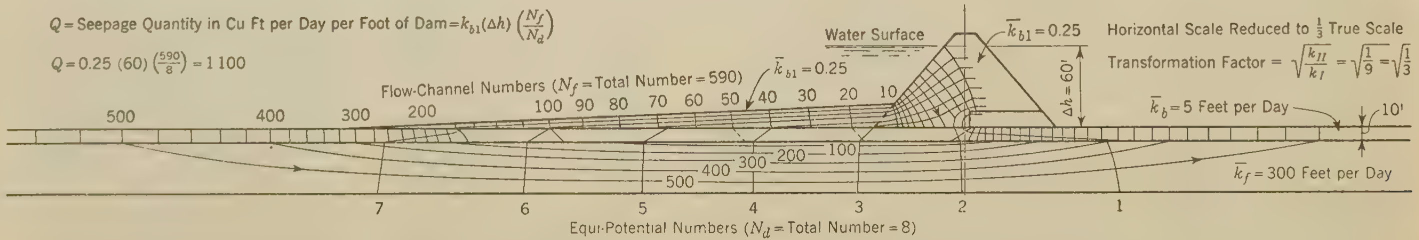
[a] In the diagrams: $k_{b1} = 0.25$ ft per day; $k_b = 5.0$ ft per day; and $k_f = 300.0$ ft per day. [b] Seepage quantity under the given permeability assumption, in cubic feet per day per foot of dam. [c] The discharge gradient at the downstream toe. [d] Comment on the degree of stability of the dam.


Q = Seepage Quantity in Cu Ft per Day per Foot of Dam $= k_{b1}(\Delta h)\left(\frac{N_f}{N_d}\right)$
Flow-Channel Numbers (N_f = Total Number = 400)
180
160
140
120
100
80
60
40
20
10
Water Surface
8
6
4
2
1
$\Delta h = 60'$
$k_{b1} = 0.25$
Uplift Head = 2.5 Feet, Discharge Gradient = 0.25
10'
k_b = 5 Feet per Day
k_f = 300 Feet per Day
100
200
300
400
6
5
4
3
2
1
$Q = 0.25\ (60)\left(\frac{400}{8}\right) = 750$
Equi-Potential Numbers (N_d = Total Number = 8)

(*a*) ALL SOILS ISOTROPIC ($k_I = k_{II}$)


Note That Seepage Quantity is Determined in Transformed Section
k_I = Horizontal Permeability; k_{II} = Vertical Permeability
Flow-Channel Numbers
70
60
50
40
30
20
10
Water Surface
8
6
4
2
1
$\Delta h = 60'$
Uplift Head = 10 Feet, Discharge Gradient = 1.0
100
200
300
400
500
5
4
3
2
1
Equi-Potential Numbers

(*b*) ALL SOILS ANISOTROPIC ($k_I = 9k_{II}$); TRUE SECTION


Q = Seepage Quantity in Cu Ft per Day per Foot of Dam $= k_{b1}(\Delta h)\left(\frac{N_f}{N_d}\right)$
$Q = 0.25\ (60)\left(\frac{590}{8}\right) = 1\,100$
Flow-Channel Numbers (N_f = Total Number = 590)
500
400
300
200
100
90
80
70
60
50
40
30
20
10
$\bar{k}_{b1} = 0.25$
Water Surface
$\bar{k}_{b1} = 0.25$
$\Delta h = 60'$
Horizontal Scale Reduced to $\frac{1}{3}$ True Scale
Transformation Factor $= \sqrt{\frac{k_{II}}{k_I}} = \sqrt{\frac{1}{9}} = \sqrt{\frac{1}{3}}$
$\bar{k}_b$ = 5 Feet per Day
10'
$\bar{k}_f$ = 300 Feet per Day
100
200
300
400
500
7
6
5
4
3
2
1
Equi-Potential Numbers (N_d = Total Number = 8)

(*c*) ALL SOILS ANISOTROPIC ($k_I = 9k_{II}$); TRANSFORMED SECTION

FIG. 7.—FLOW-NET ANALYSIS; SECTION FROM FIG. 5

Examination of the flow nets for the original section with the 1,100-ft blanket (see Fig. 7) indicates that uplift pressures in the pervious substratum should be relieved by a suitable drainage system such as vertical drainage wells drilled to a substantial depth or a drainage trench excavated to the pervious substratum. The results obtained from flow-net analyses of the latter design are summarized in Table 1 under cases 3 and 4. Case 3 designates the section with a 1,100-ft blanket and with a drain excavated into the pervious substratum; case 4 is identical, except that the upstream blanket has been omitted. Flow nets drawn for the assumed conditions indicate that uplift pressures at the toe have been eliminated by the foundation drain and that the embankment is absolutely safe against blowouts, even with the blanket omitted. The data in Table 1 illustrate that the foundation drain increases the seepage quantity in varying amounts to a maximum of 47%; hence, the value of the lost water (if the structure is to provide storage) and the cost of removing seepage from developed lands on the back side of the structure must be considered in establishing the design.

Where conditions are favorable to the use of a positive cutoff, this alternative may be the most economical long-time solution. In many locations, however, pervious strata are so deep that a cutoff is impracticable; and, in such instances, some other method must be adopted for the control of underseepage. In preliminary studies of embankments incorporating upstream blankets, the method outlined by the author should be a valuable asset. In detailed design studies, many engineers will prefer a more general method that can be adapted to the analysis of a variety of methods of seepage control. For this purpose the graphical flow-net method offers advantages that should not be overlooked by the designing engineer.

W. J. TURNBULL,[9] M. AM. SOC. C. E.—Mr. Bennett should be complimented on his modesty in presenting the mathematical analysis of seepage through natural or artificial blankets as not representing an absolutely concise or correct analysis, but rather furnishing a good estimate on which to base computed seepage flows and to determine required length of blankets.

Of all the assumptions which Mr. Bennett makes in his mathematical treatise, there is no question but that the correct determination of actual soil conditions is the critical one and as a result would be the greatest source contributing to errors. Since the accurate determination of the true soil conditions is very necessary for a reasonably good estimate of blanket length, then it becomes all important that every effort be made to obtain reliable and accurate data as to the soil profile and the soils test applicable for each type of soil in the profile. This can be done only by very careful borings, both disturbed and undisturbed. The undisturbed borings are particularly necessary for the proper determination of density and permeability. In the case of cohesionless materials, such as very coarse silts, sands, and sand gravels, it is impossible, of course, to obtain undisturbed samples by boring, and therefore it is necessary to obtain the best disturbed samples possible, particularly with reference to the finer fractions so that none of them are lost, and a true gradation of the material is determined. It is possible to secure relatively undisturbed samples by means of test pits. However, to collect a sufficient number to determine the true average permeability of the pervious stratum is a costly process. In many cases it may be very desirable to conduct large-scale field pumping tests from which the permeability of the pervious stratum may be obtained. The field permeability test has been used very little by engineers in the past. It is believed, however, that this test will become more and more important and that continued improvement in the technique of the test will result in greater reliability and confidence in the results. There is no question but that such results are much better than laboratory tests on the disturbed material or estimates of permeability made from the grain sizing of the material.

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[9] Prin. Engr. Executive Asst., U. S. Waterways Experiment Station, Vicksburg, Miss.

Mr. Bennett presents four conditions to be satisfied before his analysis can be applied with a practical degree of accuracy:

(1) The ratio of the permeability of the pervious stratum to the natural or artificial top stratum is at least ten or greater;

(2) The ratio of the length of the path of travel of the seepage water to the thickness of the pervious stratum is relatively great;

(3) There is only a single pervious stratum which is largely responsible for the underseepage flow; and

(4) The horizontal flow through the embankment structure is relatively small as compared with that through the pervious stratum.

All the foregoing criteria will be met by the greater proportion of embankments, thus making them fall within a category to which Mr. Bennett's analysis can be applied.

If the information on the artificial or natural top stratum, and on the semipervious or pervious layers beneath them, is correct, the results of the analysis demonstrated by Mr. Bennett should be regarded with as much confidence as many other soil tests or design features that comprise the over-all design of an embankment and its drainage facilities. A study of existing embankments and their blankets, assuming that accurate original data were available, would be a valuable contribution to the practicability of Mr. Bennett's analysis. With such a study, one could compare the actual performance record with the estimates derived by the methods of the paper.

The analysis presented in this paper is especially important in that it presents a rational means of estimating blanket length and thickness requirements. Such a procedure may be a means of ending arguments that always arise in the design of an embankment as to the proper length of a natural or artificial blanket of a given thickness and permeability. Many engineers are attaching more and more importance to the design of blankets, because it is believed that properly designed blankets may be used to take the place of the conventional sheet-pile cutoff. Many dams now being built include both the sheet-pile cutoff and a blanket, and once engineers learn to design blankets properly and with confidence, proved by satisfactory performance, the older and more costly, and in many cases ineffective (as far as underseepage is concerned), sheet-pile cutoff may be discarded.

KENNETH S. LANE,[10] ASSOC. M. AM. SOC. C. E.—The mathematical analysis presented by Mr. Bennett furnishes a relatively rapid method of studying certain seepage problems—particularly those concerning levees and dams across valley flood plains where the soil profile frequently consists of a relatively impervious top stratum or natural blanket overlying pervious sands and gravels. The equations can be solved for uplift pressure or piezometric head acting on the bottom of the top stratum and caused by seepage through the foundation beneath the structure. This uplift pressure on bottom of the top stratum is of particular interest since it is the pressure that causes flotation in the zone of seepage emergence—whenever the upward pressure equals or exceeds the weight of the overlying soil. In the state of flotation either the soil of the top stratum suffers a great loss of shear strength (acting as a heavy liquid with the soil grains held in suspension by the upward seepage forces which equal or exceed the buoyant weight of the grains) or the top stratum may be slightly lifted bodily, followed by some relief of uplift pressure through cracks in the top stratum. In either case, boils frequently ensue. If these boils are not arrested, eventually they may develop into piping that removes the soil from beneath the structure and causes failure. It has been well stated that flotation is the cause and piping the effect; and, if flotation can be prevented, piping cannot occur.[11] Therefore, these mathematical solutions for investigating the uplift pressure causing flotation are felt to be valuable contributions to the methods of seepage analysis.

In developing a theory for relief wells as a treatment for flotation, W. H. Jervis, Assoc. M. Am. Soc. C. E., has obtained a solution for flow through a blanket of finite length which was published in 1939 for limited distribution in a report of the U. S. Engineer Office, Vicksburg (Miss.) District.[12] Mr. Jervis considered the case of flow to a slot of infinite permeability with the boundary conditions shown in Fig. 8. As converted to the notation of the paper, Mr. Jervis' solution is as follows:

For $0 < x < L_1$

$$h_x = h \left[1 - \frac{\sinh (a\, x)}{\sinh (a\, L_1) + a\, (L_2 - L_1) \cosh (a\, L_1)} \right] \ldots\ldots (27)$$

in which

$$a = \sqrt{\frac{k_b}{k_f\, z_b\, z_f}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (28)$$

For $L_1 < x < L_2$, the curve of h_x is a straight line falling to zero at $x = L_2$. The author's solution for these boundary conditions is presented in Eq. 9*a*; but, for reasons given subsequently, it is felt that a denominator term in Eq. 9*a* should read as follows, provided h_n has been correctly interpreted to be defined as shown in Fig. 8:

$$h = h_n \frac{e^{ax} - e^{-ax}}{e^{aL_1} - e^{-aL_1}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (29)$$

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[10] Head, Soils Laboratory, U. S. Engr. Office, Providence, R. I.

[11] "Uplift and Seepage Under Dams on Sand," by L. F. Harza, *Transactions*, Am. Soc. C. E., Vol. 100 (1935), p. 1352.

[12] "Underseepage Studies, Black Bayou Levee," by W. H. Jervis, Rept. to U. S. Engr. Office, Vicksburg Dist., Vicksburg, Miss., June, 1939.

Writing Mr. Jervis' solution in terms of head lost (Δh), as considered by the author:

$$\Delta h = h - h_x = \frac{h \sinh (a\,x)}{\sinh (a\,L_1) + a\,(L_2 - L_1) \cosh (a\,L_1)} \qquad (30)$$

at $x = L_1$—

$$\Delta h_{(x=L_1)} = h_n = \frac{h \sinh (a\,L_1)}{\sinh (a\,L_1) + a\,(L_2 - L_1) \cosh (a\,L_1)} \qquad (31)$$

or

$$h = h_n \frac{\sinh (a\,L_1) + a\,(L_2 - L_1) \cosh (a\,L_1)}{\sinh (a\,L_1)} \qquad (32)$$

Substituting Eq. 32 in Eq. 30:

$$\Delta h = h_n \frac{\sinh (a\,x)}{\sinh (a\,L_1)} \qquad (33a)$$

or

$$\Delta h = h_n \frac{e^{ax} - e^{-ax}}{e^{aL_1} - e^{-aL_1}} \qquad (33b)$$

which is the same case as that covered in Eq. 9a, using $h = \Delta h$ for lost head.

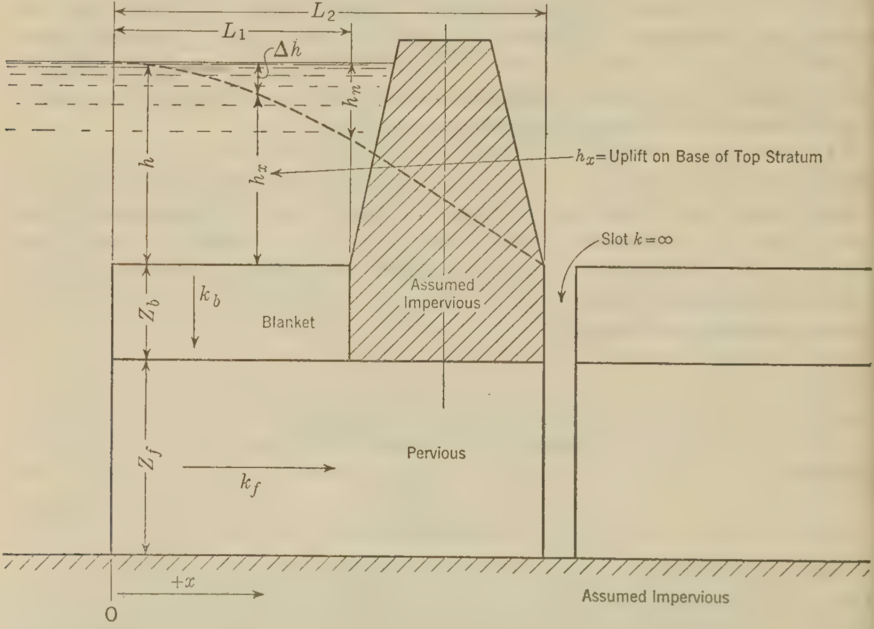

L_2
L_1
Δh
h_n
h
h_x
h_x= Uplift on Base of Top Stratum
Slot $k=\infty$
Assumed Impervious
k_b
Z_b
Blanket
Z_f
k_f
Pervious
$+x$
O
Assumed Impervious

FIG. 8

For h_n defined as in Fig. 8, with this transformation of Mr. Jervis' solution, Eq. 9a would read as in Eq. 29 or 33b. This is also confirmed by comparison with uplift pressures determined from flow nets and by computations from Eqs. 25.

It is of interest to compare computations from formulas with uplift curves determined by flow nets. (There are several good references to the utility of

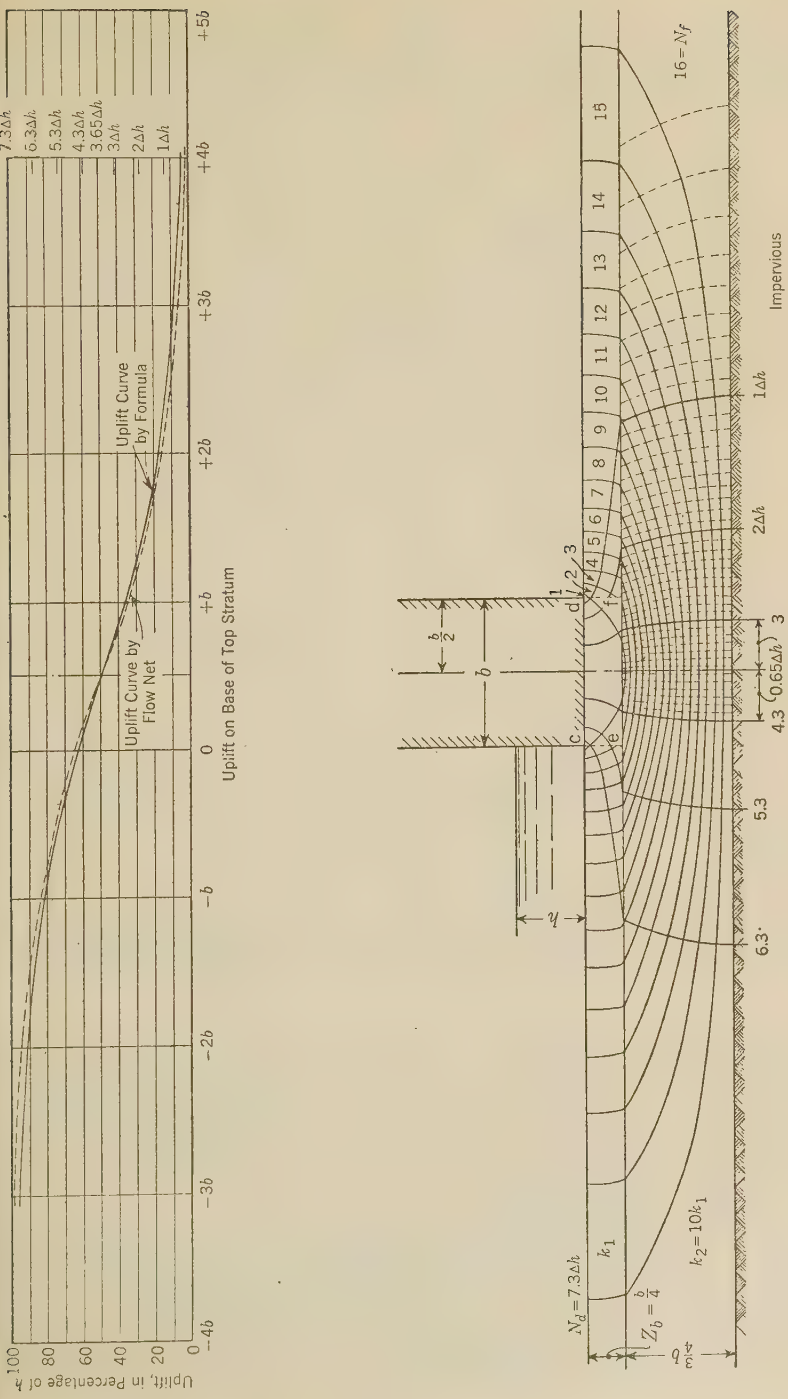
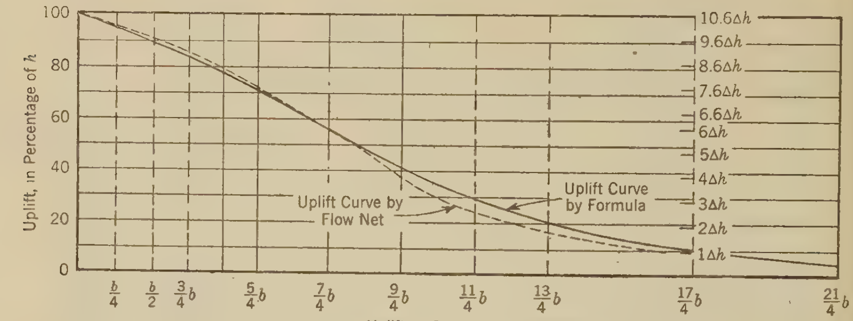

Uplift, in Percentage of h
100
80
60
40
20
0
−4b
−3b
−2b
−b
0
+b
+2b
+3b
+4b
+5b
Uplift on Base of Top Stratum
Uplift Curve by Flow Net
Uplift Curve by Formula
7.3Δh
6.3Δh
5.3Δh
4.3Δh
3.65Δh
3Δh
2Δh
1Δh
y
b
b/2
c
d
e
f
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16 = N_f
N_d = 7.3Δh
Z_b = b/4
b/4
k_1
k_2 = 10k_1
Impervious
1Δh
2Δh
3
0.65Δh
4.3
5.3
6.3

FIG. 9

the flow net.[13,14]) This has been done by using the applicable equations in the paper for four cases of a blanket over a buried pervious deposit. The results are shown in Figs. 9 to 12, inclusive. The two methods of solution give quite close agreement and indicate the validity of the formulas.



Uplift on Base of Top Stratum

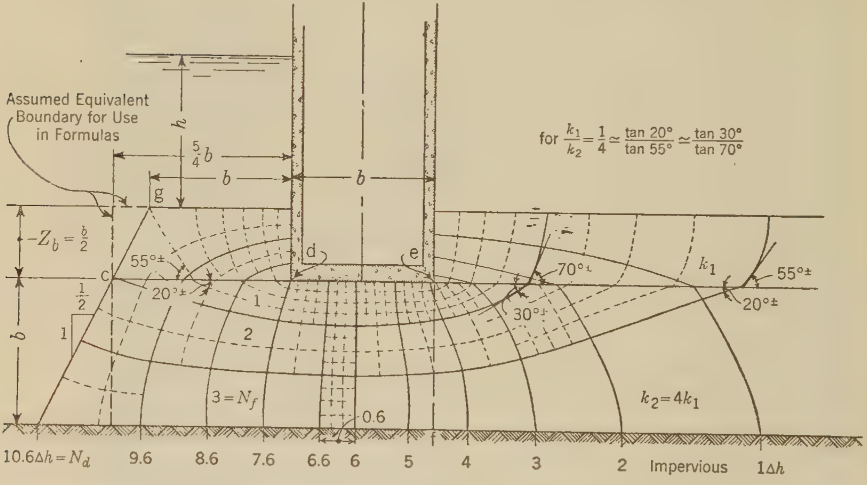

Assumed Equivalent Boundary for Use in Formulas
h
5/4 b
b
b
g
d
e
c
$-Z_b = \frac{b}{2}$
1/2
b
1
55°±
20°±
1
2
3 = N_f
0.6
for $\frac{k_1}{k_2} = \frac{1}{4} \simeq \frac{\tan 20°}{\tan 55°} \simeq \frac{\tan 30°}{\tan 70°}$
70°±
30°±
k_1
55°±
20°±
$k_2 = 4k_1$
10.6Δh = N_d
9.6
8.6
7.6
6.6
6
5
4
3
2
Impervious
1Δh

FIG. 10

All the foregoing formulas were derived for flow through a two-layer case (impervious top stratum above buried pervious stratum) and are based on the same differential equation to set up which it was necessary to assume that:

1. The flow is vertical through the top stratum and horizontal through the buried pervious stratum; and

2. No flow occurs in the top stratum under the impervious section of the structure; or, expressed otherwise, the impervious section of the structure extends down to the bottom of the top stratum as shown in Fig. 8.

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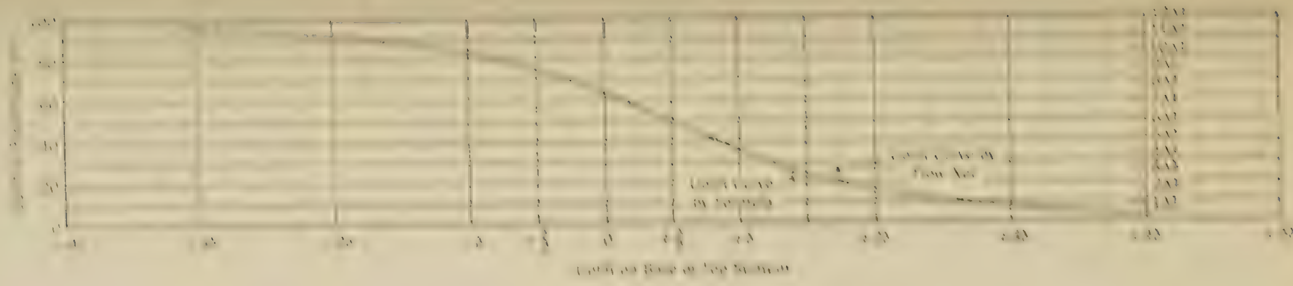
[13] "Seepage Through Dams," by A. Casagrande, *Journal*, New England Water Works Assn., June, 1937, p. 131.

[14] "Engineering for Dams," by William P. Creager, Joel D. Justin, and Julian Hinds, John Wiley & Sons, Inc., New York, N. Y., Vols. 1 and 3, 1945.



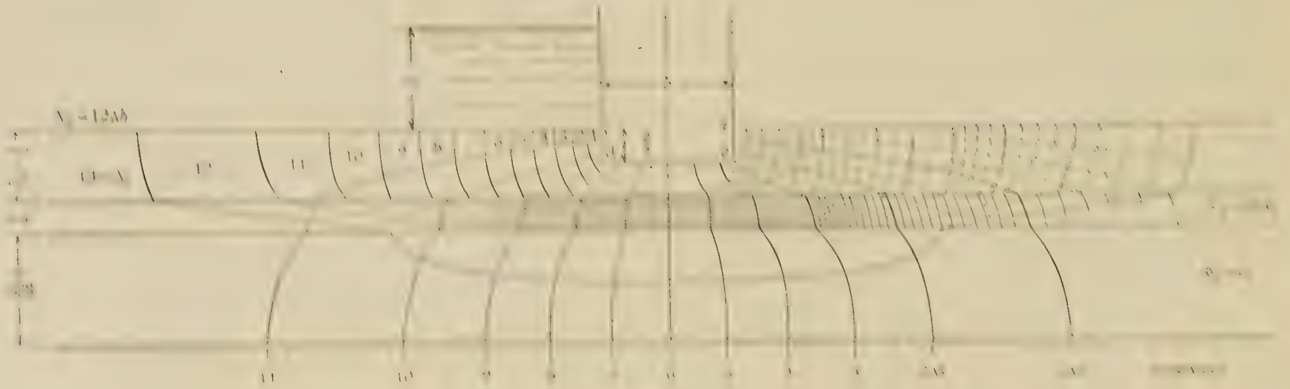


Fig. 11

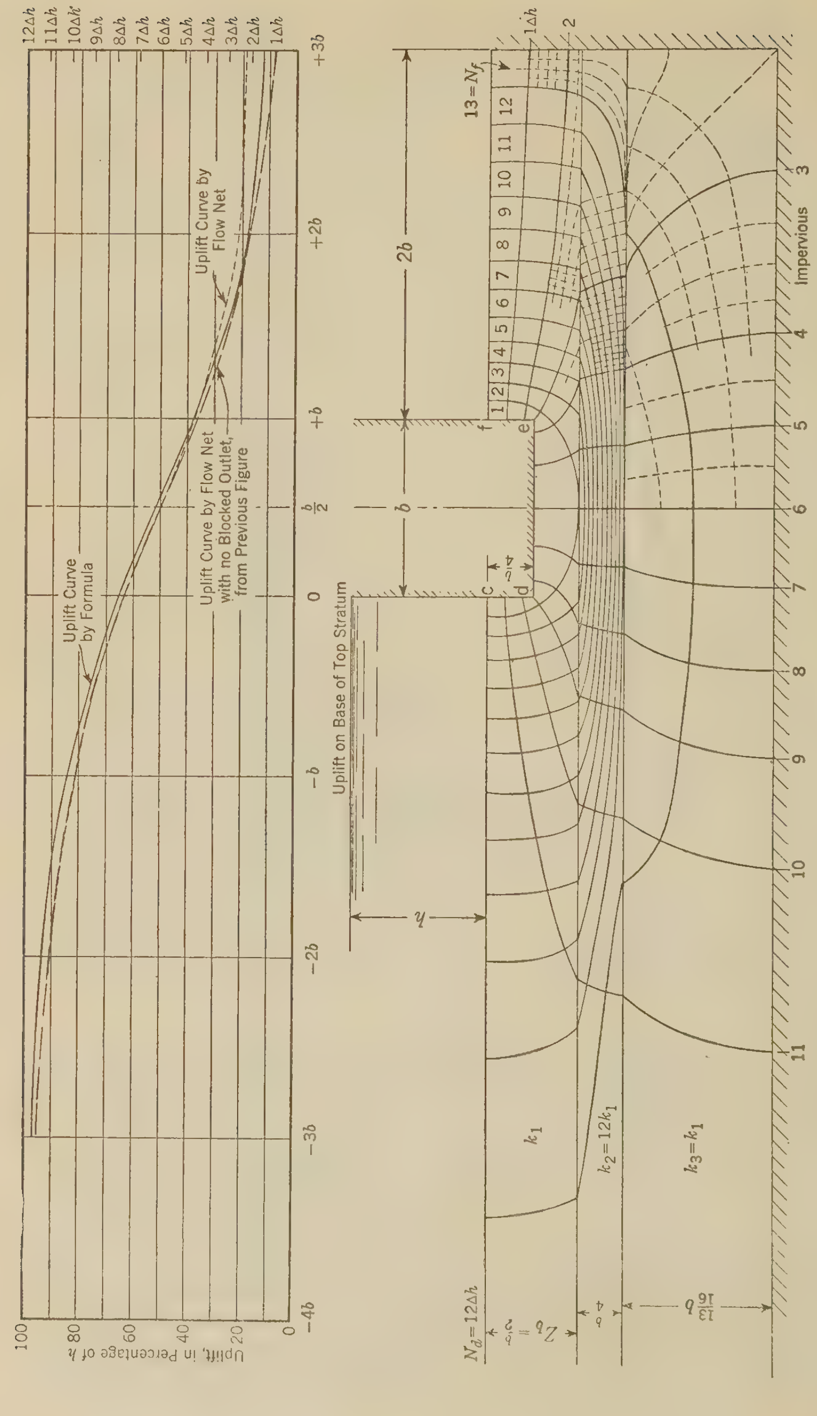

12Δh
11Δh
10Δh
9Δh
8Δh
7Δh
6Δh
5Δh
4Δh
3Δh
2Δh
1Δh
100
80
60
40
20
0
Uplift, in Percentage of h
−4b
−3b
−2b
−b
0
b/2
+b
+2b
+3b
Uplift Curve by Formula
Uplift Curve by Flow Net with no Blocked Outlet, from Previous Figure
Uplift Curve by Flow Net
Uplift on Base of Top Stratum
y
b
2b
13 = N_f
1
2
3
4
5
6
7
8
9
10
11
12
f
e
c
d
b/4
1Δh
2
3
4
5
6
7
8
9
10
11
Impervious
k_1
k_2 = 12k_1
k_3 = k_1
N_d = 12Δh
Z_b = b/2
b/4
13/16 b

FIG. 12

Actually, the flow paths are somewhat curved in both the top stratum and the underlying pervious layer, and some flow always occurs in the section of the top stratum beneath the structure; but for practical purposes the foregoing assumptions give results sufficiently accurate for analysis considering the variations in soils and their permeability which are encountered in nature and must be handled by replacing nature's conditions with a simplified or generalized section for the purposes of analysis—a condition which the author has well emphasized. Since they are graphically sketched solutions, flow nets are not precise determinations but rather are good approximations for the chosen boundary conditions. Therefore, although the flow nets of Figs. 9 to 12 are not exact, nevertheless, they give good illustrations of the effect of the foregoing two assumptions which were necessary in deriving the formulas.

Fig. 9 exhibits a flow net solved by K. A. Linell, Jun. Am. Soc. C. E. Here the formula gives a value for uplift slightly too low on the upstream side at point e and slightly too high on the downstream side of the structure at point f—largely because of neglecting the flow through the top stratum in the zone cdef, which in this case is less than one twelfth of the total underseepage. However, the formula is conservative in showing slightly too high a value at point f, as this is the point of most interest in computing the uplift causing flotation in the zone of seepage emergence. Divergence of the formula in the zones from $-b$ to $-3b$ and from $+2b$ to $+4b$ is probably caused by neglecting the curvature of the flow lines in the pervious stratum.

The flow net of Fig. 10 has been drawn for a relatively low ratio of foundation permeability to blanket permeability of 4 to 1, which is likely to be much lower than usual cases in nature. Although the author has warned that the formulas may not be applicable for permeability ratios of less than 10 to 1, the agreement in the uplift curves shown by Fig. 10 is still quite good, and again the formula gives slightly high or conservative results at the point of most interest in the emergence zone, point e (Fig. 10). The formula would agree much more closely with the excess uplift pressure measured in the flow net at point f on the bottom of the pervious stratum as here the flow is horizontal according to the first assumption. The geometry of Fig. 10 exerts a major influence on the flow pattern; and, if the blanket of Fig. 10 had been longer than the relatively short distance of $\frac{5}{4}b$, the agreement between formula and flow net might have been much closer.

The flow net of Fig. 11 covers a three-layer case with a relatively thin pervious layer placed within a deposit which is one twelfth as pervious. Although the formulas are not intended to apply to this case, it appears that they could be used since the check with the flow net is very close. Actually, the influence of the third or lowest stratum is small as it carries only two thirteenths of the total underseepage and the great bulk, or ten thirteenths, of the flow passes through the pervious deposit, leaving only one thirteenth of the flow in the top stratum from points d to e.

In Fig. 12, the upstream half of the flow net is the same as that in Fig. 11, but on the right or downstream half an impervious barrier has been considered at $+3b$ to give a partly blocked outlet. This raises the uplift over the entire emergence zone as shown by the comparison of the two uplift curves from the flow nets drawn in the upper diagram of Fig. 12. The author's solutions do not cover this case and, therefore, the computations were made from Eqs. 22 and 23, which are considered solutions for a more general case. At the point of most interest on line ef, the agreement is good; but, in the zones from + 2 *b* to + 3 *b*, the influence of the third stratum becomes important and the use of the formula is not warranted for obtaining the complete uplift curve of this particular three-stratum case.

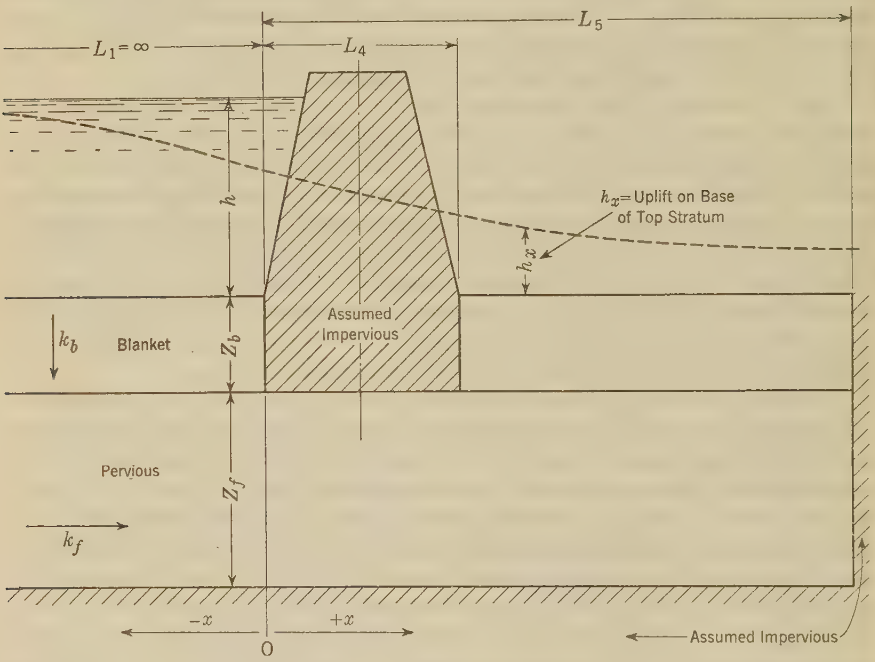

L_5
$L_1 = \infty$
L_4
h
h_x= Uplift on Base of Top Stratum
h_x
k_b
Blanket
Z_b
Assumed Impervious
Pervious
Z_f
k_f
$-x$
$+x$
0
Assumed Impervious

FIG. 13

In using Eqs. 22 and 23 for the solution of Fig. 12, it was necessary to select finite and large values of L_1, making the computation rather cumbersome. A further simplification, for the boundary conditions illustrated in Fig. 13, has been derived by R. A. Barron, Jun. Am. Soc. C. E., and is as follows: For $x < 0$,

$$h_x = C\,e^{ax} + h \quad \text{(34a)}$$

for $0 < x < L_4$,

$$h_x = C + h + C\,a\,x \quad \text{(34b)}$$

and, for $L_4 < x < L_5$,

$$h_x = D\left[e^{ax} + e^{a(2L_5 - x)}\right] \quad \text{(34c)}$$

in which the constants,

$$C = \frac{h}{\dfrac{1 + e^{2a(L_5 - L_4)}}{1 - e^{2a(L_5 - L_4)}} - (1 + a\,L_4)} \quad \text{(35a)}$$

and

$$D = \frac{C}{e^{aL_4}\left[1 - e^{2a(L_5-L_4)}\right]} \qquad \text{(35}b\text{)}$$

Two advantages of upstream blankets are reduction in uplift pressures in the seepage emergence zone and reduction in quantity of seepage. These apply particularly to dams holding a permanent pool where the blanket is protected by the water above and where the head lost through the blanket exerts a force compressing the blanket and thus increasing its impermeability. The author's use of the expression for x_r as a measure of blanket efficiency offers a quick method of comparing the value of different blanket dimensions.

A triangular blanket is theoretically sound and could be applied where the impervious material is very scarce; but the usual case is likely to be that of a more uniform blanket thickness. In this connection, it is difficult to visualize the author's findings (see paragraph containing Eq. 14) from comparative computations that, for the same area or volume of impervious material, a triangular blanket should be twice as long as one of uniform thickness to provide the same resistance to flow. Any further elaboration the author could inject on this point would be helpful.

In contrast to dams holding a permanent pool, an upstream blanket is not quite as advantageous for flood control dams where water is stored only intermittently with frequent rapid drawdowns of the pool and where the volume of loss from seepage is seldom important. For this use, thicker blankets are apt to be required as the frequent exposures of the blanket during nonstorage periods offer opportunities for erosion and, in the colder climates, for loosening the upper part of the blanket by frost, substantially reducing its impermeability. The latter would apply if the impervious soil of the blanket is subject to frost action as is likely to be the case.

TABLE 2.—PERMEABILITY OF NATURAL BLANKET OVERLYING A GLACIAL DEPOSIT OF PERVIOUS SAND

(All Values Are the Averages of Several Tests)

Description	PEAT		TOPSOIL		SUBSOIL[a]
	Undisturbed	Remolded	Undisturbed	Remolded[b]	Remolded
Organic content (percentages)	23.8		6.1		1[c]
Specific gravity	2.25		2.45		2.62
Permeability (cm per sec × 10^{-4})	51	49	87	28	13
Void ratio	3.66	1.51	1.21	0.98	0.73
Dry density (lb per cu ft)	30.1	56.0	69.1	77.1	94.5

[a] Medium sand. [b] Sod removed. [c] Estimated.

The test results in Table 2 were obtained from sampling the surface soils found over a large glacial deposit in northern Connecticut of highly pervious sand (the parent soil), with a permeability of from 100 × 10^{-4} cm per sec to 400 × 10^{-4} cm per sec. The peat represented low-lying swamp deposits, the topsoil represented the top horizon on higher ground, and the subsoil was the horizon of weathering immediately beneath the topsoil. The high values of permeability may seem surprising at first but they are considered as being caused by the very loose state found in nature and attributed to the annual loosening by frost action. In an impervious blanket for flood control works, it is judged that the upper part of an exposed blanket should be expected to be loosened similarly where frost action is a factor.

A further precaution in using blankets on flood control structures is to avoid placing the blanket in such a position that the pressure of water trapped behind it on drawdown might be sufficient to cause flotation from seepage up through the blanket—often termed a blowout in the blanket. This situation is likely to be encountered more frequently in the case of levees than in the case of dams; and one solution, effective but expensive, is, of course, to weight the blanket down by a cover of pervious material. Another solution, occasionally possible, might be to provide a measure for relieving this back pressure as by avoiding a complete cutoff or by shortening the blanket. Blanket failures from high back pressure of trapped water have been fairly frequent when blankets were extended down over existing slopes, when inclined blankets without sufficient covering weight were placed on dams of large width and poor internal drainage, and when blankets formed canal linings. An outstanding example of piping following a blowout in a blanket is the Memphis (Tenn.) subsidence described by Karl Terzaghi,[15] M. Am. Soc. C. E., although in that case the placing of the blanket down over a river slope was due to nature.

Acknowledgment is made of assistance given by Mr. Barron in checking the computations for the uplift diagrams and by Mrs. R. D. Lacey in preparing the illustrations and making some improvements to the flow nets during tracing.

Corrections for *Transactions:* In January, 1945, *Proceedings*, change "k_z" to "k_b" in Eq. 4c (page 21), Eq. 8 (page 23), and in lines 1 and 2, page 22. Also change "m" to "n" in Eq. 15 (page 25) and in line 26, page 30.

PRESTON T. BENNETT,[16] ESQ.—The importance of anisotropic permeability, resulting from stratification of natural deposits is emphasized clearly in Mr. Cedergren's discussion. If the symbols k_b and k_f are interpreted properly, as vertical and horizontal permeabilities, no further correction for anisotropy needs to be made to bring the formulas to a degree of accuracy greater than that obtainable by graphic methods.

As Mr. Cedergren has stated, considerable skill and experience in the construction of flow nets are necessary before such graphic solutions can be relied on. The writer's dissatisfaction with his own efforts at graphic solution caused him to use electric conduction models, and his observation of the nearly straight-line equipotential lines obtained from models of blanket systems led, in turn, to the opinion that an analytical solution for simple blanket problems was entirely feasible and less time consuming than either of the other methods.

A blanket over a more pervious stratum constitutes what might be termed a foundation with "coarse-grained" anisotropy. The horizontal coefficient of transmissibility for such a foundation is the ratio, $k_f\,z_f$; and, by analogy, the term "vertical coefficient of transmissibility" may be coined for k_b/z_b. Likening these coefficients to horizontal and vertical permeabilities k_I and k_{II}, whose transformation factor is $\sqrt{\frac{k_{II}}{k_I}}$ the comparable ratio $\sqrt{\frac{k_b}{z_b\, k_f\, z_f}}$ is obtained, which is the factor common to the solutions of blanket problems by W. H. Jervis, Assoc. M. Am. Soc. C. E., by Messrs. Barron and Lane, and by the writer.

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[15] "Earthslips and Subsidences from Underground Erosion," by Karl Terzaghi, *Engineering News-Record*, July 16, 1931, p. 91.

[16] Engr. (Soil Mechanics), U. S. Engr. Office, Omaha District, Omaha, Nebr.

The similarity of the factor a to an anisotropic transformation factor is mentioned to emphasize again the writer's opinion that no second transformation of dimensions is necessary if k_b and k_f in his notation are interpreted properly. This similarity also suggests the definition "coefficient of blanketed transmissibility" for the hitherto undesignated quantity a.

Messrs. Barron and Lane have both presented solutions for the uniform blanket which contain the term $(L_2 - L_1)$. This is equivalent to x_d of the paper. Inclusion of the term x_d eliminates the necessity for a separate equation and computation to determine values such as h_n in Eq. 9a.

Mr. Lane was not entirely correct in assuming that the writer's h_n was as defined in Fig. 8, but the modification of Eq. 9a as indicated in Eq. 29 is necessary if it is desired to generalize the blanket formula by adding factors to include flow under the impervious dam

From Fig. 2 it is evident that

$$\frac{h}{\Delta h} = \frac{x_r}{x_r + x_d} \qquad (36)$$

Rewriting Eq. 9d in hyperbolic functions, the resistance for a blanket of length L_1 is

$$x_r = \frac{\sinh (a\, L_1)}{a \cosh (a\, L_1)} \qquad (37)$$

Referring to Fig. 8, the h and Δh in the paper correspond, respectively, to h_n and h in Mr. Lane's discussion. In Mr. Lane's notation, using $(L_2 - L_1)$ for x_d, Eq. 36 becomes

$$\frac{h_n}{h} = \frac{\dfrac{\sinh (a\, L_1)}{a \cosh (a\, L_1)}}{\dfrac{\sinh (a\, L_1)}{a \cosh (a\, L_1)} + (L_1 - L_2)} \qquad (38)$$

which reduces to identity with Eq. 31, and leads to the conclusion that the writer's use of two formulas, Eq. 9a and Eq. 36, is exactly equivalent to Mr. Jervis' solution.

Although the writer's use of the concept of blanket resistance introduces extra equations into the solutions, it actually simplifies the numerical work to be done. Consider, for example, the system shown in Fig. 14, which is similar to Fig. 8 except that a line of relief wells is used instead of the slot of infinite permeability. The uplift pressure at the toe will vary from a minimum at the wells to a maximum between wells, but it may be assigned an average value, h_{x2}, dependent on characteristics of the well system and the land-side flow pattern. If the average value h_{x2} is above the land-side surface area, the line of wells is equivalent in effect, at a distance beyond the radius of influence of the wells, to a slot of infinite permeability, as in Fig. 8, except that it acts as a source of flow from potential h_{x2} to the land-side surface at potential zero.[17]

The solution of such a well and blanket system, involving a discontinuity of discharge at the line of wells, is considerably simplified by the use of blanket resistances as follows: Assume that a maximum permissible value for h_{x2} has been established. The curvilinear gradients A′B′ of Fig. 14 can be replaced, for the computation of discharge, by the tangent gradients AW and WB if the resistances x_{r1} and x_{r3} are known. The slopes of these tangents, evident from Fig. 14, determine the foundation flows Q_1 ($= s_1\, k_f\, z_f$) and Q_2 ($= s_2\, k_f\, z_f$) on either side of the wells. The discharge of the well system required to reduce the land-side toe uplift pressure is $Q_1 - Q_2$. The size, spacing, and outlet elevation for the well system can readily be computed when the required well discharge is known.[17]

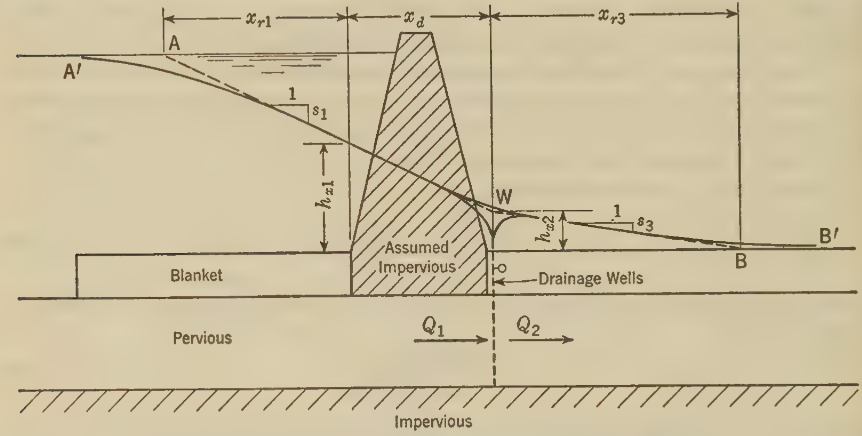

FIG. 14.—COMBINATION OF BLANKET AND DRAINAGE WELLS

If it were desired to design a well system to reduce the land-side uplift pressures in the system shown by Mr. Barron in Fig. 6, Eqs. 22 and 23 would probably be less easily applied than a solution involving the resistance values of blankets. In fact, Mr. Lane has noted that these equations are rather cumbersome for numerical work, even without the introduction of a drainage system.

Rewriting Eq. 22c with the origin at $x = L_3$, $x' = 0$:

$$h_{x'} = B' \left(e^{ax'} + e^{-ax'}\right) \qquad (39)$$

in which B' is a factor similar to B in Eq. 22c. Thus, the resistance of the interval between L_2 and L_3 in Fig. 6 is

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[17] "Conference on Control of Underseepage," published by U. S. Waterways Experiment Station, Vicksburg, Miss. (see "Design of Relief Well Systems," by W. H. Jervis, p. 81; and "Comments on Design of Relief Wells," by P. T. Bennett, p. 87), April 1, 1945.

$$x_{r3} = \frac{h_{x'}}{\dfrac{d(h_{x'})}{dx'}} = \frac{e^{ax'} + e^{-ax'}}{a\,(e^{ax'} - e^{-ax'})} \qquad \text{.................(40)}$$

Fig. 15 is Mr. Barron's Fig. 6 redrawn with self-explanatory changes in notation which are made for easier use in resistance formulas. In the notation of Fig. 15, from Eqs. 9*d* and 40,

$$x_{r1} = \frac{\sinh\,(a\,x_1)}{a\,\cosh\,(a\,x_1)} \qquad \text{.....................(41}a\text{)}$$

and

$$x_{r3} = \frac{\cosh\,(a\,x_3)}{a\,\cosh\,(a\,x_3)} \qquad \text{.....................(41}b\text{)}$$

From Fig. 15,

$$h_{x1} = \frac{h\,(x_{d1} + x_{r3})}{x_R} \qquad \text{.....................(42}a\text{)}$$

and,

$$h_{x2} = \frac{h\,x_3}{x_R} \qquad \text{.........................(42}b\text{)}$$

and, from Eq. 39,

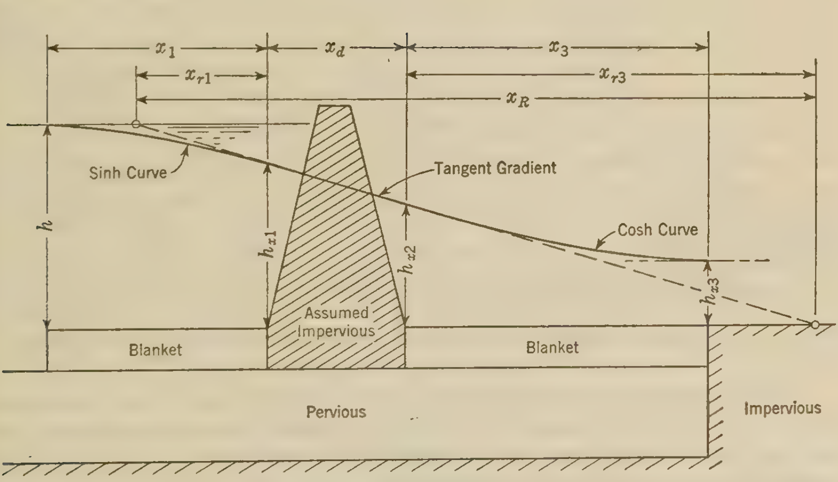

x_1
x_d
x_3
x_{r1}
x_{r3}
x_R
Sinh Curve
Tangent Gradient
Cosh Curve
h
h_{x1}
h_{x2}
h_{x3}
Assumed Impervious
Blanket
Blanket
Pervious
Impervious

FIG. 15.—APPLICATION OF "RESISTANCES" TO PERVIOUS FOUNDATION TO FINITE LENGTH

$$\frac{h\,x_3}{h\,x_2} = \frac{h\,(x' = 0)}{h\,(x' = -\,x^3)} = \frac{e^0 + e^{-0}}{e^{-ax_3} + e^{ax_3}} = \frac{1}{\cosh\,(a\,x_3)} \qquad \text{........(43}a\text{)}$$

or

$$h\,x_3 = \frac{h\,x_2}{\cosh\,(a\,x_3)} \qquad \text{......................(43}b\text{)}$$

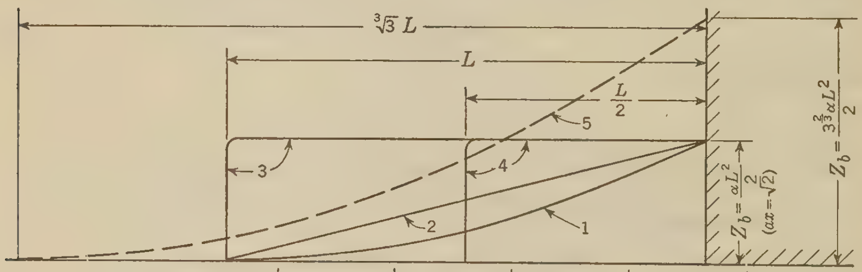
The transformation of Eqs. 22 and 23 into the forms given in Eqs. 41, 42, and 43 seems to render them more useful in numerical applications.

Quite properly, Mr. Lane has questioned the writer's comparison of uniform and triangular blankets. The conclusion (paragraph containing Eqs. 12, 13, and 14, last sentence) that a triangular blanket of maximum thickness z_b and length x is equivalent to a uniform blanket of the same thickness and length $x/2$ is obviously untrue if k_b were zero. In the latter case, the resistances would be x and $x/2$, respectively. The writer's comparison was made on blankets of optimum dimensions, such that $a\,x = \sqrt{2}$. These conclusions may be illustrated by Table 3, in which five blanket sections are superimposed for comparison of end areas and resistances. Blankets No. 1 and No. 5 are parabolic sections (see diagram in Table 3) described by

$$z = \frac{a\,x^2}{2} \qquad \text{(44)}$$

Eq. 44 can be shown analytically to represent the curve section that provides maximum resistance for a fixed-end area. The uniform blanket of length L (Section 3, Table 3) is also of optimum proportions, as can be noted from Fig. 3.

TABLE 3.—COMPARISON OF BLANKET SECTIONS


$\sqrt[3]{3}\,L$
L
$\frac{L}{2}$
5
3
4
2
1
$Z_b = \frac{\alpha L^2}{2}$
$(ax = \sqrt{2})$
$Z_b = \frac{3^{\frac{2}{3}}\alpha L^2}{2}$

Description	Section 1	Section 2	Section 3	Section 4	Section 5
Comparative end area	1/3	1/2	1	1/2	1
Resistance in terms of L	0.5	0.5 to 0.63	0.63	0.44	0.72
Ratio, resistance to area	1.5	1.0 to 1.26	0.63	0.88	0.72

Table 3 indicates some advantages of the vertically curved and triangular sections over the uniform section, and is the basis for the writer's assumption that sections of equal area, shaped similarly to Sections 2 and 4, are about equally effective. Even with careful determination of permeabilities as advocated by Mr. Turnbull, the best possible estimate of the factor a will not represent field conditions closely enough to justify hairsplitting in the selection of blanket shapes and dimensions.

This opinion is not to be interpreted as nonconcurrence with Mr. Turnbull's statements that field conditions should be determined with the greatest care, for only when this is done can the formulas developed in this paper and in the discussions be used to best advantage.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2271

A PLAN FOR A MOVABLE DAM

By Isaac DeYoung,[1] M. Am. Soc. C. E.

With Discussion by Messrs. C. L. Hall, C. E. Meyerdick, Ralph R. Leffler, Jay L. Southworth, L. C. Sabin, A. W. Sargent, and Isaac DeYoung.

Synopsis

There are two general conditions under which movable dams are operated: One in which it is required to close a channel in an orderly predetermined plan and without immediate urgency; and the other, in which a sudden release of the water in a canal caused by an accident due to the wrecking of a lock gate by collision or otherwise, requires a structure that can be operated in the rushing waters, checking the flow.

The plan for a movable dam presented herewith is of the latter type. It is an emergency dam, and is designed so that it can be put in place during the rush of waters thus stopping the uncontrolled stream. It consists of plate girders (which form the dam), connected by hinges fastened together at their flanges and folded together in small compass in the bottom of the lock. The dam is held in place by rigid anchors, hinged to the girders, and embedded anchorages under the floor of the lock. The dam is raised through multiple reaved cables by two hoisting engines. The raising operation required only a short time. A single operator in a central station manipulates the controls in an orderly way as soon as the upper canal is free from all floating craft.

Introduction

The movable dam proposed in this paper is a development from the studies made in the early 1900's in connection with the design and construction of two movable dams for St. Marys Falls Canal, Michigan, in which the writer had a part. The writer's ideas leading to the proposed dam have been developed through the years intervening since that first experience.

Forms of Movable Dams

There are four general types of movable dams: (1) Overhead bridges built in conjunction with wickets, needles, and other devices; (2) submerged structures;

Note.—Published in February, 1945, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

[1] Consultant to City Mgr., Sault Ste. Marie, Mich.; formerly Supt. of St. Marys Falls Canal, Michigan (retired).

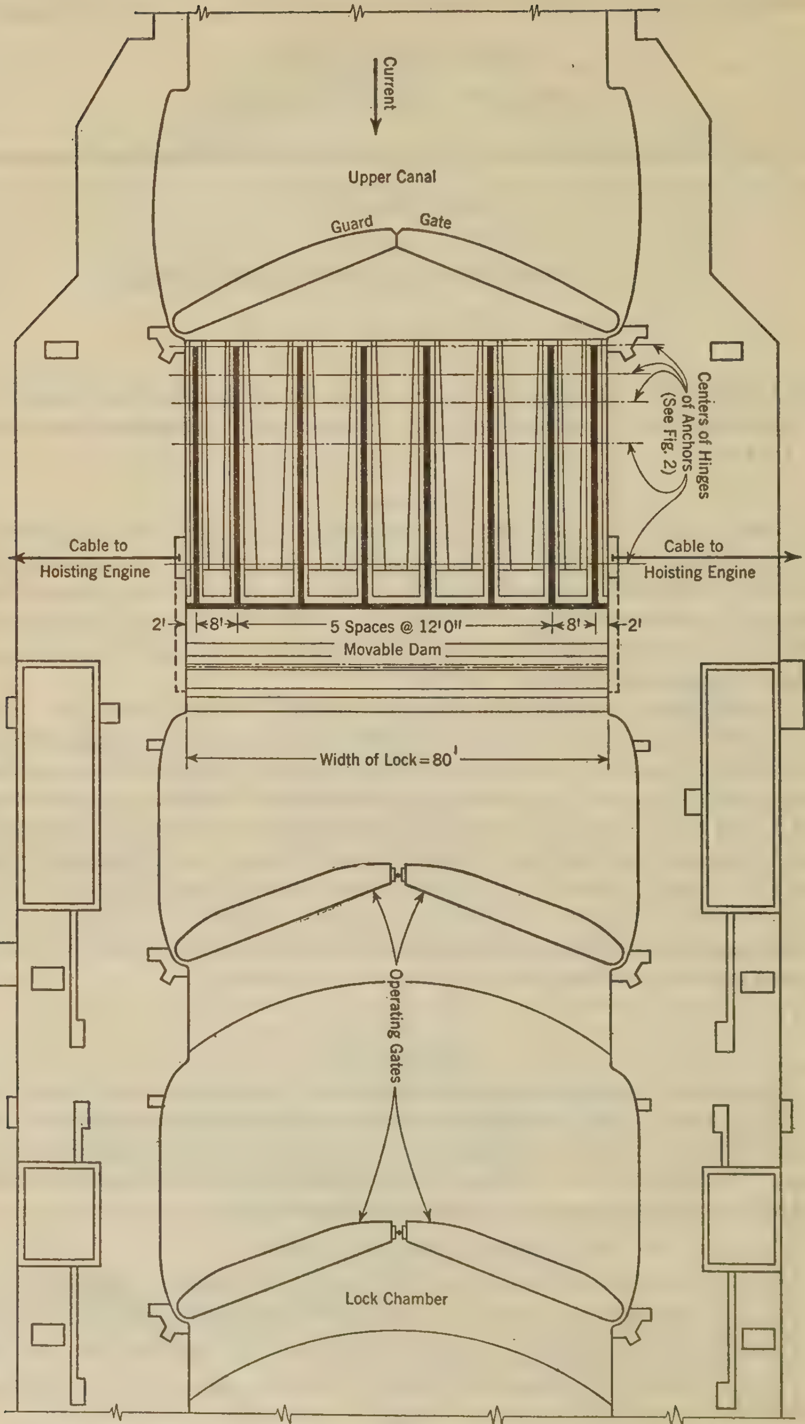

Current
Upper Canal
Guard Gate
Centers of Hinges of Anchors (See Fig. 2)
Cable to Hoisting Engine
Cable to Hoisting Engine
2'
8'
5 Spaces @ 12' 0"
8'
2'
Movable Dam
Width of Lock = 80'
Operating Gates
Lock Chamber

Fig. 1.—Plan of Hypothetical, Flexible Shutter Dam, Designed for the Upstream End of Lock St. Marys Falls, Michigan

(3) floats; and (4) lock gates (sheathed or skeleton), horizontal sliding gates, rolling segmental gates, and lift gates.

(1) Overhead bridges of the swing type, from which either horizontal or vertical stop planks, wickets, or needles are manipulated, are fairly sure to be operated so that the flow of water is checked gradually, thus minimizing the shock on all parts of the structure. The operating parts are readily accessible and easily repaired. Some of the disadvantages are that they are limited as to length of span, possibility that a sunken vessel in the locality may prevent the bridge from closing, and the lodgment of stone or other material on the sill.

(2) Among dams of the submerged type may be mentioned trestle dams, bear traps, Taintor gates, etc. The flexible shutter type of dam described in this paper belongs in this class (see Figs. 1 and 2). Some inherent objections to the typical submerged dam are that a dragging anchor may injure the structure and that, when damaged, the structure is relatively inaccessible for repairs. These objections, as applied to the flexible shutter dam, are discussed subsequently.

(3) Under the subject of floats, there is the barrier created by sinking a vessel of any shape. A circular caisson built of timber or steel can be used and floated into place at the jaws of the narrowest part of the canal or lock.

(4) Having two sets of operating gates at each end of the lock offers some safety because of the double barrier in front of a moving vessel. A gate located in the canal above the lock would accomplish a similar result, the gate being operated in such a manner as to be always closed during a danger period when the lock gates are open; but the operations under such conditions would be subject to excessive delays to navigation.

Description of the Flexible Shutter

The structure described in this paper is composed mostly of plate girders, fastened together on the downstream side by plate hinges extending the entire length of the girders (Figs. 2(*a*), 3, 4, 5, and 6). On the upstream side the dam is held by anchors connected to embedded anchorages. It is raised by two hoisting engines, through six-part cable, one on each side of the lock. In the lowered position, the dam is folded in small compass below the lock floor level (see Figs. 3 and 7). It is designed for a lock 80 ft wide, having 30 ft of water on the sills.

The selection of a type of movable dam should be based on the following requirements: (*a*) Certainty of operation when required; (*b*) reasonable facility of operation; (*c*) no obstruction to navigation; (*d*) ability to withstand shock; (*e*) simplicity of construction; (*f*) accessibility for inspection and repairs; (*g*) appearance; (*h*) freedom from enemy attack during stress of war; and (*i*) cost of construction and maintenance.

(*a*) A movable dam should be free from the possibility of being clogged or broken by the ordinary use of the canal or by the exigencies arising at the time of the accident.

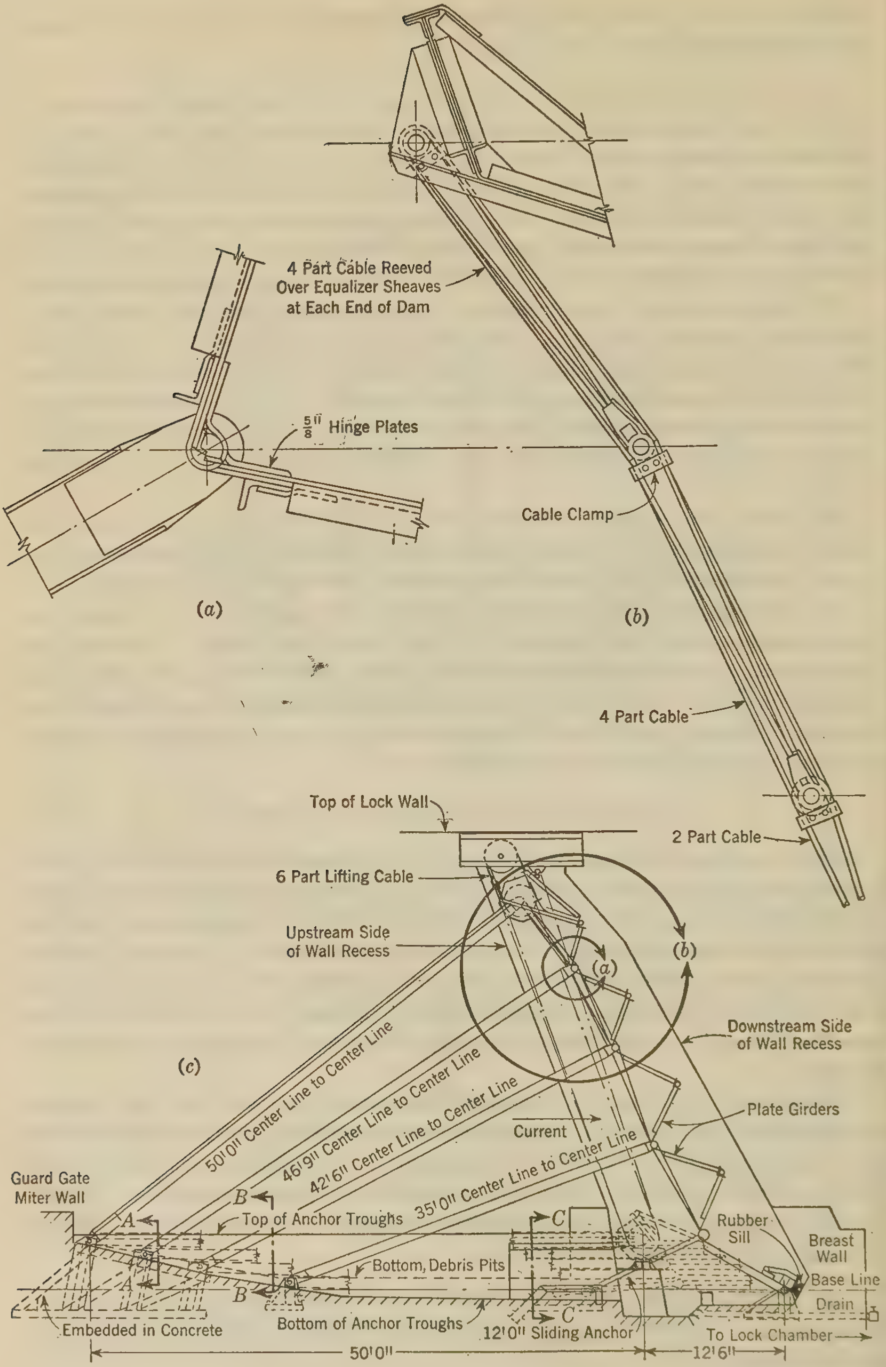

4 Part Cable Reeved Over Equalizer Sheaves at Each End of Dam
$\frac{5}{8}''$ Hinge Plates
(a)
Cable Clamp
(b)
4 Part Cable
2 Part Cable
Top of Lock Wall
6 Part Lifting Cable
Upstream Side of Wall Recess
(a)
(b)
Downstream Side of Wall Recess
(c)
50'0'' Center Line to Center Line
46'9'' Center Line to Center Line
42'6'' Center Line to Center Line
35'0'' Center Line to Center Line
Current
Plate Girders
Guard Gate Miter Wall
A
A
B
B
C
C
Top of Anchor Troughs
Bottom, Debris Pits
Rubber Sill
Breast Wall
Base Line
Drain
Embedded in Concrete
Bottom of Anchor Troughs
12'0'' Sliding Anchor
To Lock Chamber
50'0''
12'6''

FIG. 2.—LONGITUDINAL SECTION OF DAM, SHOWING GENERAL OUTLINES AND LOCATION OF SECTIONS (FOR SECTIONS, SEE FIG. 7)

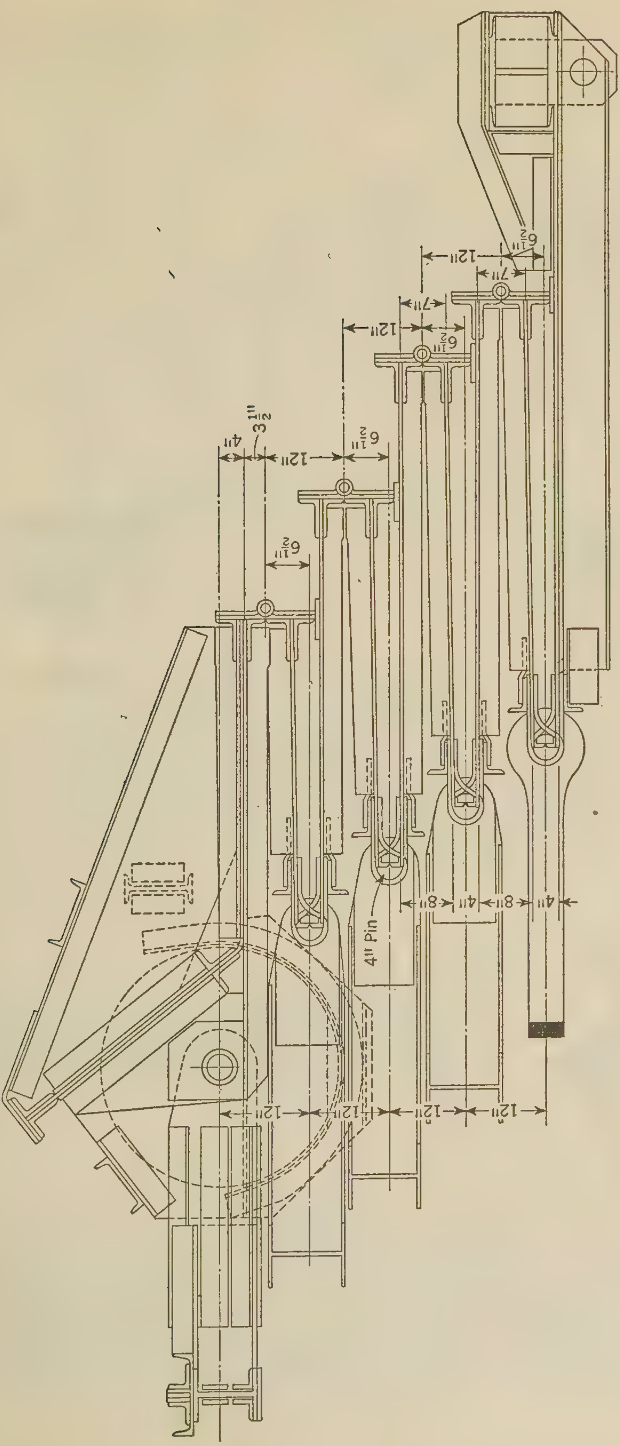
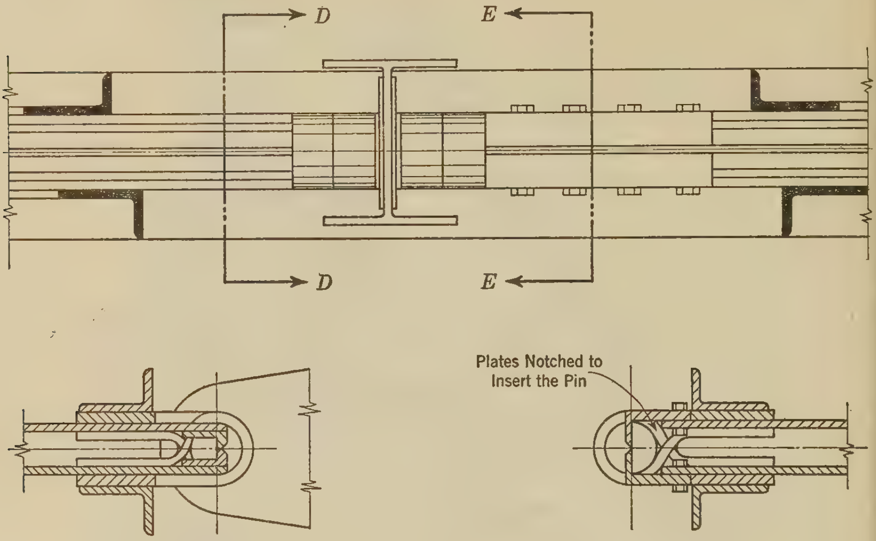

4"
3½"
12"
6½"
6½"
12"
6½"
7"
12"
7"
6½"
4" Pin
8"
4"
8"
4"
12"
12"
12"
12"

FIG. 3.—DETAILED VIEWS OF DAM IN A LOWERED POSITION

In the proposed plan the dam is folded in small compass in the bottom of the lock and protected by transverse breast walls, preventing damage from dragging equipment, such as anchors.



SECTION *D-D* SECTION *E-E*

Fig. 4.—Sections Through Upstream Hinges

As debris may be moved by the current created in the canal when a lock gate is wrecked, some provision should be made to prevent the collection of such debris on parts of the dam, so as to interfere with its successful operation. In the plan, the anchor slots, eight in number, are covered by the top anchor having angle iron attached, whose outstanding legs cover the slots completely from end to end of anchors, preventing the debris from entering these slots (see Fig. 7). Also, funnel-shaped pits between the anchor slot walls catch this debris.

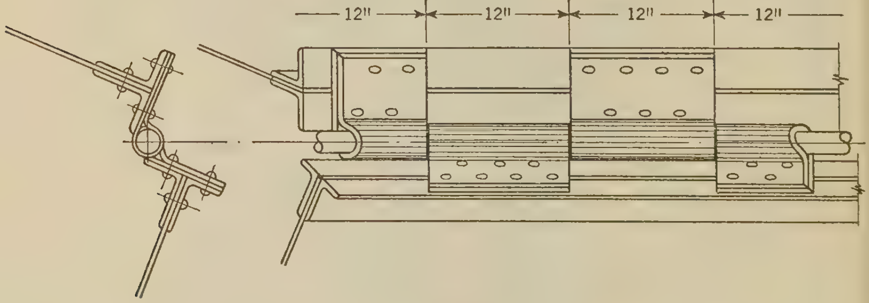


Fig. 5.—Downstream Hinges, Continuous End to End of Dam

It is not considered likely that a vessel would become stuck, because then it would itself become a barrier, building up pressure sufficient to move it through the lock.

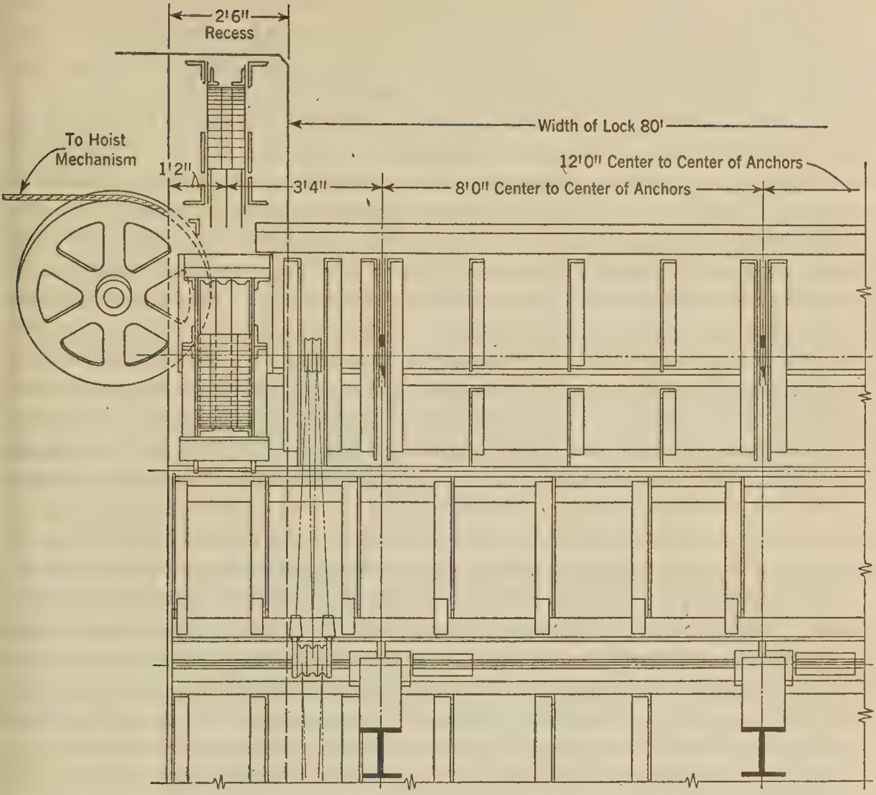
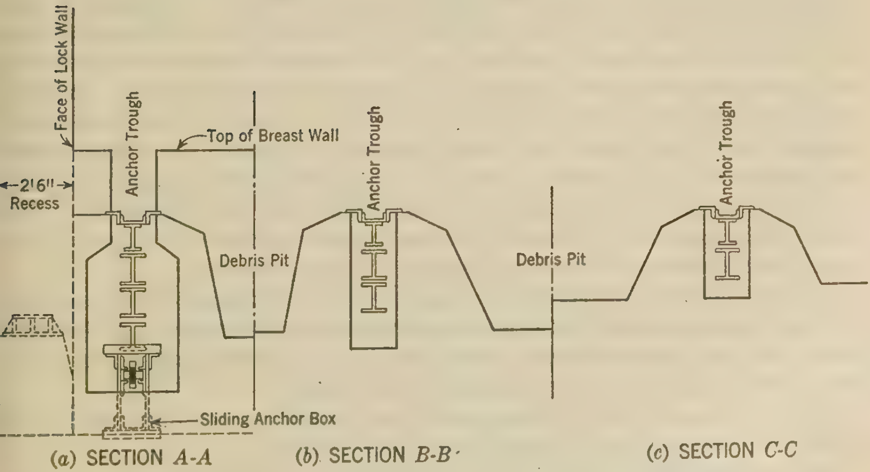

2'6"
Recess
Width of Lock 80'
To Hoist Mechanism
12'0" Center to Center of Anchors
1'2"
3'4"
8'0" Center to Center of Anchors

Fig. 6.—Rear Elevation, Facing Downstream


Face of Lock Wall
Anchor Trough
Top of Breast Wall
Anchor Trough
Anchor Trough
2'6"
Recess
Debris Pit
Debris Pit
Sliding Anchor Box

(*a*) SECTION *A-A* (*b*) SECTION *B-B* (*c*) SECTION *C-C*

Fig. 7.—Sections A–A, B–B, and C–C, Fig. 2, Showing Anchors in the Lowered Position

It would be desirable if the dam could be closed before the current in the canal has reached a dangerous velocity, but this ideal is impossible in any form of dam. The flexible shutter dam can be raised in a few minutes, and as soon as all floating craft in the upper canal has passed through.

(*b*) The operating mechanism should be simple, not likely to get out of order, and it should be easily accessible for quick operation. The flexible shutter dam is operated by means of two hoisting engines, one on each side of the lock. The operation may be performed from one station through appropriate electrical connections similar to the Selsyn method of operation. (The Selsyn system of signaling and control consists of two small specially wound induction motors electrically interconnected in such a way that the movement of one armature causes the other armature to move by a corresponding amount and in the same direction. One motor is operated at the sending point as a generator and is called the transmitter and the other is operated as a motor and is known as the receiver.) The dam is operated as a single unit. The plan provides for a dam in a lock 30 ft deep and 80 ft wide. In a lock of less depth and width, the operation may be performed by hand through a train of gears.

(*c*) The structure should be of such a form as to offer no obstruction to navigation. These requirements have been met in the flexible shutter dam in that all parts are outside of the limits of the navigable cross section of the lock.

(*d*) Since the proposed structure is held rigidly in place by substantial anchor beams, which revolve slowly about their centers, there will be no sudden change of position of any part of the dam to receive any shock. The flow will be stopped gradually and there will be a smoothness of operation. It is expected that the dam will not be raised until the upper part of the canal has been entirely clear of any floating craft that may have been moored in the canal.

(*e*) This structure can be built largely of standard structural sections. The dam proper is composed of plate girders. The anchors for holding the dam are of H-sections, channels, angles, and eyebars. The hinges are of steel plates, continuous on the downstream side the full width of the lock. These hinges are riveted to the flanges of the plate girders with the hinge pins in place; the pins may be made in sections of any length. The anchor hinges on the upstream side, holding the dam, are riveted to the webs of the plate girders. All the hinge connections have rivets in transverse or direct shear. It is not considered necessary that these hinges be bushed because movement of the parts is slow and infrequent. The upstream flanges of the plate girders consist of angles having their meeting legs machined, forming a close and watertight joint. The lower girder is provided with a box section, one side of which, when the dam is raised, abuts against a rubber sill, making a watertight joint.

(*f*) The flexible shutter dam is placed within the lock enclosure, between the upper guard gate and the upper operating gate at the elevation of the canal bottom of the upper approach. When the lock is unwatered it can be completely inspected, painted, and repaired, if required.

(*g*) As all parts will be placed out of view, the proposed dam has no objectionable features from an appearance point of view.

(*h*) Because the structure is submerged, it is not a likely subject for enemy attack. The power plant can also be placed out of view, if desired.

(*i*) As the flexible shutter dam is light in weight, compared with other types of movable dams, the cost will be considerably less. Cost of maintenance is small because of the smaller number of parts and the ease of access and inspection when the lock is unwatered. The weights (in pounds) of parts of the proposed dam (lock 80 ft wide and 30 ft deep) are estimated approximately as follows:

Dam proper, including girders, connections, and half of anchors	310,000
Other half of anchors (supported on lock floor)	53,000
Wire rope and sockets	7,000
Embedded anchorages	40,000
Contingencies	15,000
Total	425,000

The weight of two hoisting engines has not been included in the foregoing estimate. Because of abnormal wartime conditions, no estimate of cost is presented. Under normal conditions the average cost of steel and iron in the structure should not exceed 15¢ per lb.

SUMMARY

Probably no engineering structure has been given more study in connection with the operation of locks than a movable dam. Should a lock gate be wrecked by collision or otherwise while restraining the waters in a canal, sudden release of the water would cause a wave that would probably result in carrying away all vessels moored at the canal walls, and such vessels would be subject to collision below the locks and be wrecked. The magnitude of the catastrophe would depend on the head depth and size of the lock and canal. Having a movable dam will not avert the wrecking of a lock gate, but such dam will serve to stop the flow of water, permitting repairs to be made and restoring the lock for service.

The importance and necessity of having a movable dam is evident. The task of stopping a flow of water in a ship canal without such a structure would be tremendous.

DISCUSSION

C. L. HALL,[2] M. AM. SOC. C. E.—In his classification of movable dams, Mr. DeYoung includes submerged structures as type (2). Among the submerged structures mentioned are trestle dams, bear traps, Tainter gates, etc., but not wicket dams of the Ohio River type. Wicket dams are found on a great many canalized rivers. In particular there are forty-three such structures on the Ohio River and through them pass the heaviest river-borne commerce in the western hemisphere. Any general discussion of movable dams which does not mention dams of the Ohio River type is likely to give a false impression. Moreover, the classification of a lock gate as a form of movable dam is still more likely to confuse the reader.

The structure described by Mr. DeYoung is a substitute for the Poiree dam[3] which has been installed in several places to stop downstream flow through a damaged lock, thus permitting repairs to be made. The main difference between the two structures is that the Poiree dam merely provides the horses and props against which wickets may be placed to establish a closure, whereas the flexible shutter type of dam includes substitutes for wickets as integral parts of the dam, thus establishing closure automatically as soon as the dam is raised. Clearly this feature is desirable; but also it is much more expensive. There are comparatively few places where such complex dams are justified. Also, there are only a few locks where a gate failure would be so disastrous that both a submerged movable dam and a set of guard gates would be economically justifiable. For most waterways the extra insurance is not worth the cost.

A study of the drawings indicates that natural sedimentation into the recesses is not feared. In a clear stream like St. Marys River there may be no sedimentation during the long periods when the dam lies on the bottom. In streams that are closed every winter, it may be possible to raise and lower the dam every spring before navigation begins; but on the Ohio River this form of dam would probably not operate. In the upper river, acid would probably corrode the steel, unless it were made of a very expensive alloy. Throughout the entire river, sediment would be deposited in any recess, in such a way that it would probably be extremely difficult to overcome initial inertia when dam raising is undertaken. With the river open to navigation all year, it would never be easy to find a convenient time to raise and inspect a flexible shutter type of dam.

The writer expresses no opinion on the proposed dam, considered from the standpoint of the mechanical engineer. He has great doubt as to the practicability of using such a dam except in a canal or canalized river with far from normal characteristics.

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[2] Col., Corps of Engrs., U. S. Army, Columbus, Ohio.

[3] "The Design and Construction of Dams," by Edward Wegmann, John Wiley & Sons, Inc., New York, N. Y., 8th Ed., p. 309.

C. E. MEYERDICK,[4] ESQ.—There are various methods of operating locks with guard gates and supplemental gates. The contribution by Mr. DeYoung should be greatly appreciated because it is reasonable that few locks should be built without provision for stopping the flow through the lock chamber in case of an accident. There are, of course, many ideas on how to put a set of lock gates "into the dry"—by constructing bulkheads or by making use of the various forms of movable dams enumerated by Mr. DeYoung. A lock is not necessarily guarded by the use of a set of guard gates as illustrated in Fig. 1.

A simple plan for operating a lock is the plan used on most of the Mississippi River locks from St. Louis, Mo., to St. Paul, Minn. A broken set of gates would tie up traffic through the lock, in which case an auxiliary lock is usually available. Should the lock be left entirely open, the quantity of flow through the lock would not affect other traffic activities on the river since this flow would unquestionably be less than normal flow in the river. If a navigation canal is small in size, its capability to absorb a high flow such as might surge through an open lock chamber is limited. Lock chambers on the Mississippi River are equipped with only a single set of gates at each end of the lock. Normally, one set of gates is always closed. For any other arrangement of gates, if more are considered essential, it does not necessarily follow that expenses should be increased by operating all the gates, as some of them may be left recessed to be used as stand-by units in cases of emergency. The fact that a lock has two sets of gates at each end does not necessitate utilizing both sets to lock a vessel through the river. The cost of these extra gates, including that of the extra set of gates to be used as guard gates, and the cost of movable dams (bulkheads) recessed in the bottom of the lock or otherwise installed add up to a tidy sum for construction—not to mention operation and maintenance.

There are two types of lock gates commonly considered for navigation projects. They are the miter gate and the sector gate. Both types are usually installed in pairs with a sealing arrangement on the gates to effect a seal at the center of the lock where the gates meet, and with other seals at each wall from which each gate swings. The operating procedure for the miter gate is similar to that for the garden gate since the force usually is applied at the top of the gate. In the case of the miter lock gate, considerably more force is required to swing the gate through the water than is required to swing it free in air.

In sector gate operation, the face plate, being an arc, slices through the water as the gate is swung. The resistance against the sector gate is much less than that against a miter gate unless the gate has a plate on one side in addition to the radial face plate. The operating force on the miter gate commonly is applied through the medium of an operating arm attached to the top of the gate, whereas on the sector gate the force might be applied through a pinion mounted on the lock walk and meshed with a curved rack attached to the top of the face plate of the gate.

The sector gate recess requires more room than does the miter gate recess and both gate types have other advantages and disadvantages. One of the chief advantages of the sector gate is that, because of its slicing action, it can operate against a head differential and in high velocities of flow, whereas operation of the miter gate must never be risked in high velocities. Sector gates in any combination of sets in a lock, by using any one set of the gates, provide a movable dam. The lock chambers can be "valved" by the operation of the sector gates, and there is an indication of the economy of their uses—namely, for "valving," "locking," and emergency operation against high velocities.

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[4] Mech. Engr., U. S. Engr. Office, Sacramento, Calif.

The writer favors sector gates because of their sturdiness and minimum deflection in the gate structure during operation. The fact that the resultant of all hydrostatic load lies through the center of the hinge (trunnion) indicates the reason for mechanical economies and utility of operation. A lock can be built with only two sets of sector gates that will meet the necessary requirements of many of the most important navigation structures.

Obviously, as proposed by Mr. DeYoung, the safety of lock operation should extend to closing off the lock in case of accidental damage to any of the locking or valving mechanisms. Mr. DeYoung is to be commended for emphasizing the necessity for safety requirements for navigation structures. The writer believes, however, that by using sector lock gates the margin of safety that can be attained will be comparable to what might be expected by installing a movable dam mechanism as proposed by Mr. DeYoung.

RALPH R. LEFFLER,[5] ESQ.—The plan outlined in this paper deserves full consideration wherever the problem of designing a movable dam arises. On certain waterways that are important in wartime, such a movable dam should be installed in preparation for World War III. The cost is relatively so small that many major peacetime locks should be equipped with it.

The prospects that Mr. DeYoung's plan will receive merited consideration are not good. The proposal is new; it is novel; it is unprecedented; it is irritating to the "rut-worn" mind; and it has not been done before. It must run the gauntlet of the heavy hands of the finical designers, the high-powered executives, the "solid-minded" citizens, the "buttocky" bureaucrats, the "practical" men, and a whole host of others, adept and zealous at dodging responsibility to safeguard themselves. The design has so many merits, nevertheless, that it will overcome these obstacles.

JAY L. SOUTHWORTH,[6] ASSOC. M. AM. SOC. C. E.—The dam described in this paper is worthy of consideration both as a movable and an emergency dam. As an emergency dam, it would afford a fast remedy in case of accident to the miter gates. As a movable dam, it would greatly reduce the cost of maintenance of the gates and lock chamber. It is particularly adapted to a lock which has a shallow breast wall as illustrated. This construction should prevent silt from depositing to a harmful depth on the structure in its supine position. For a lock with a breast wall of sufficient height, it would be more economical to use the Lockport (Ill.) type of emergency dam[7] which is similarly placed between the two upper breast walls.

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[5] Engr. of Structural Design, The San. Dist. of Chicago, Chicago, Ill.

[6] Civ. Engr., Federal Power Comm., Washington, D. C.

[7] "Engineering Features of the Illinois Waterway," by Walter M. Smith, *Transactions*, Am. Soc. C. E., Vol. 98 (1933), Fig. 11, p. 328.

The dam should be designed to withstand the warping stresses resulting from the dynamic effect of the swirl of water under high velocity. In actual experience, this swirl of water has been caused by one upper operating gate leaf hanging by its anchorage in a diagonal obstructive position after the other gate leaves have been swept clear away.

The writer is not convinced of the necessity for an emergency dam in any lock. Since 1925, the hydraulic system for emptying and filling lock chambers has been so improved that the surge—the principal cause of lock gate damage in the past—has been sufficiently reduced so that the danger is practically removed. In thirty years of operation, the emergency gates of the Panama Canal have not been used as such. There has been no occasion to test them. Since the lock gates in the old Welland Canal between Lake Erie and Lake Ontario in Canada were damaged at the rate of two or three accidents per year for a long period, metal horns were provided at great expense at the miter ends of the new Welland Canal lock gates to prevent damage by absorbing the shock of a blow from a moving vessel. Today operators have been trained to use extreme care in moving vessels through a lock.

Inasmuch as the structure described can be used at great saving in ordinary gate and lock chamber maintenance, necessity seems to have been the "prime mover" in this unique conception. This type of dam would supplant the elaborate and unwieldy emergency structures that have been in vogue at Sault Sainte Marie, Mich., and other places in the past.

L. C. Sabin,[8] Hon. M. Am. Soc. C. E.—A plan for a movable dam based on experience in the construction and operation of canals and locks at Sault Ste. Marie, Mich., is presented in this paper. Although the dimensions assumed and the observations made as to the general problem relate particularly to the conditions there found, the application of the principle is not confined to those conditions. Any criticism that may apply to these specific requirements may not be applicable to less severe situations.

Need of safety appliances to prevent or stop the free flow of water through the canal serving the navigation locks has been in the minds of all engineers who have had the responsibility for the successful and continuous operation of the waterway through the many years of development at Sault Ste. Marie. Conditions have changed as each new unit of facility has been added and the problem thus modified. The variety of devices adopted was not only the result of such changes and of the experience gained in operation of the earlier types, but perhaps these changes have been as much the result of the ideas of the responsible executives in charge at the time.

The situations involved may be considered as presenting two problems, and the first decision to be made is whether both or only one of the requirements is to be met. First, shall dependence be placed on prevention, or second, shall provision be made for stopping the flow once established? If the arrangement is such that the separation of water levels is always made to depend on at least two gates the chances of free communication by reason of a collision with a gate become very small. Such a device has been depended upon in certain situations but it has not for long been the sole dependence at the Sault Canals. Either the inconvenience of operation, or the realization of the possibility and serious result of a failure to properly operate, has resulted in the second requirement; that is, to stop the free flow once established as early as practicable. This is illustrated by a history of the developments at the Sault.

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[8] Vice-Pres., Lake Carriers Assn., Cleveland, Ohio.

The difference in level overcome by the locks at Sault Ste. Marie is normally about 21 ft but varies from 18 ft to 22 ft according to the stages of water in Lake Superior and the lower St. Marys River. It has been estimated that, under conditions existing in some of the canals, velocity may reach from 16 ft per sec to 21 ft per sec in case of communication of levels. The upper level is a canal a mile or more in length, whereas the lower approach is quite short.

The first attempt to provide for the passage of the St. Marys rapids by other than canoes was a lock built (1853–1855) by the State of Michigan having two lifts of about 9 ft each. In connection with this construction a caisson gate was built near the head of the canal. The difficulty of operating this gate was such that about 1859 a set of guard gates, similar to the lock gates then in use, was built just below the caisson gate. For a time, beginning about 1865, these gates were closed whenever a lockage was in progress—that is, when only one pair of lock gates was restraining the upper level. These gates thus became a preventive measure. They probably could not have been closed had a free flow through the canal been established by an accident.

South or landward of the State Locks the single-lift Weitzel Lock was built (1873–1881) with a length of 515 ft, and a width of 80 ft in the chamber and 60 ft at the gates. The Weitzel Lock and the State Locks were served by the same canal, deepened to 16 ft. The State Locks were operated until 1888 when they were destroyed to make way for the next improvement.

The work done in connection with building the Weitzel Lock included a movable dam designed by the late Alfred Noble, Past-President, Am. Soc. C. E. This structure introduced the use of a swing bridge carrying girders and wickets with hand-power machinery for operation. The method of operation was first to close the bridge, the bottom chord resting against the abutments on either side. The iron girders carrying the wooden wickets were then lowered successively against the horizontal sill, the girders turning on the horizontal axis along the lower chord of the bridge. When the girders were in place the wickets, hinged within the girders, were revolved into an upright position closing the canal. This dam was in place until 1896 when the canal was deepened to 25 ft, making the dam no longer operative.

The next change was brought about by the construction (1887–1896) of the Poe Lock which replaced the old State Locks. The Poe Lock is 800 ft long between operating gates. Its original chamber width of 100 ft was reduced to 95 ft in 1942 by placing a 5-ft concrete curtain on the south wall to repair damage caused by blasting for the MacArthur Lock adjacent to it. At first the Poe Lock was served by the old canal deepened to 25 ft and having a width of 108 ft at its narrowest place where it is crossed by the International Railroad Bridge with swing span. To fit this situation a canal gate was first built of the mitering type similar to the lock gates, but constructed of timber and iron.

It was proposed at first to use this as a safety gate, having it closed during any danger period—that is, when the levels were separated by a single lock gate. To serve this purpose only the lower 18 ft was sheathed, the upper part being left open for the passage of water to fill the locks. It soon was realized that delays resulting from such operation would be intolerable in view of the heavy traffic. Accordingly, it was utilized only as a head gate and fortunately no emergency occurred to call for its closure in a current.

After the completion of the Poe Lock in 1896 the upper canal was widened (1908–1911) giving two channels at the railroad bridge location. These channels, each 108 ft wide, were separated by an island about 80 ft wide carrying the center pier of the bridge which was used also for the center pier of a new movable dam. The closing of the widened and deepened canal, in which the flow in two channels would have to be stopped in case of an emergency, presented a more serious problem and was given much study, including rough plans of many types of structure. The conclusion reached was to build a dam of the swing-bridge type provided with wickets on both leaves.

During the final planning of this dam there occurred, in the canal on the Canadian side, the only accident resulting in the actual communication of water levels. A dam of similar form had been built for this emergency and its use in stopping the flow developed some weaknesses. Advantage was taken of this fact and the plans for the new dam were modified to strengthen certain members.

Each leaf of the bridge carried sixteen steel wicket girders of box cross section about 6 ft 9 in. wide, hung on a horizontal axis at the downstream chord of the bridge and stored under the floor system of the bridge when not in use. When the girders were swung to position, the lower ends resting against the sill, the wickets were released to slide into place. In general form it thus resembled the Alfred Noble dam of 1880. This dam[9] remained in place for about thirty-four years until, in 1944, the deepening of the South Canal in connection with the construction of the MacArthur Lock made it inoperative. Although its use never had been required to stop free flow, it provided insurance and was of some service as a cofferdam in unwatering the canal. Between 1908 and 1919 two new locks were built with a separate canal north of the one to which the preceding description has referred. This north canal has a least width of 280 ft with a depth of 24 ft; the locks and lower approach were built to 24.5 ft below low water datum.

The types of dam considered advantageous were not thought suited to a span of 280 ft and, moreover, it was evidently not desirable to close the entire canal should only one lock be disabled. In the construction of these locks an important innovation was made in providing double service gates at each end. The proposed method of lock operation was such that at no time would the separation of levels depend on one set of gates. Although this is not a certain preventive of level communication it is extremely unlikely that two pairs of gates would be carried away by the collision of a ship. This being still a possibility, however, a movable dam was provided to operate at the upper end of either lock.

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[9] *Engineering Record*, August 5, 1911, p. 161.

This dam consists of an 85-ton bridge capable of serving either the Davis Lock or Sabin Lock and when not in use is stored on the canal wall just above the upper guard gates of the lock. Two large derricks, one on either side, swing this wicket bridge into position in recesses provided in the canal walls. An operating bridge of lighter design is also provided to be placed over the canal about 40 ft upstream from the main bridge to carry a winch truck. Thirty wickets to close the opening, also stored on the canal wall, are placed consecutively by the derrick. A wicket picked up by the derrick is first hung on hinge pins on the wicket bridge; the winch truck moving on a track on the operating bridge then serves to lower the wicket into place against a timber sill at the bottom of the canal. The width of the canal section at the dam is about 90 ft, each wicket closing about 3 ft.

The principal objection to this type is the time consumed in operation. As many as two days have been required for complete closing, although it is thought that this time could be greatly reduced if the operating crew were kept in training. This objection is not as important as might appear, since the principal damage that could occur probably would take place within a few minutes, or at least within an hour after the accident; and no type has been tried which could be operated within that time. However, no traffic in the canal could be resumed until the flow in the damaged lock could be stopped.

The construction of the MacArthur Lock in 1942–1943 presented again the problem of providing means to stop the flow if the wrecking of a lock gate should permit communication of the water levels. This lock replaced the old Weitzel Lock and thus utilizes the south canal serving the Poe Lock.

The plans included the removal of "Bridge Island," thus destroying the movable dam at that location. The resulting width of the canal, with Bridge Island removed, being too great to permit the adoption of any of the types in use, it was decided to place a dam immediately above the new lock and a similar dam at the head of the Poe Lock.

The structure adopted is simple in general plan as it consists of six girders spanning the canal and sliding in vertical grooves in the canal wall. For the MacArthur Lock these girders are about 87 ft long and 6 ft high. The depth of the girders at the center is 6 ft. At the ends they are 3 ft deep and are provided with both reaction and end guide rollers to facilitate sliding to place and removal. The girders are stored on the canal wall and placed by a stationary derrick.

The dam was used as a cofferdam during one winter and considerable difficulty was experienced in removing the girders in the spring because of the formation of ice. Similar trouble might attach to other types if used in winter, but prior steaming should relieve this difficulty. As in the case of other dams its placing in a current has not been tested at the Sault but when not under pressure the time of operation is estimated at two or three hours. A similar structure was later provided at the head of the Poe Lock where the span is about 106 ft, thus requiring much heavier construction.

It may be noted that this stop-log type of dam, properly designed, has very little leakage and thus may serve as a guard gate permitting full access to the regular guard gate of the mitering type.

It is realized that some question may arise as to the justification for the expense incurred in providing the rather elaborate structures described when the probability of an accident that might call for their use has proved so remote; but the volume of the traffic and the effect of any prolonged delay must be considered.

On the completion of the Weitzel Lock in 1881, the first lock built by the United States, the traffic passing the canal the following year was 2 million tons. After the completion of the Poe Lock in 1896 the commerce passing this lock alone in its first full year of operation reached 13 million. In the first full year of operation the Davis Lock in 1915 passed 57 million; the corresponding figure for the Sabin Lock was 51 million in 1920.

A rough value for the cost of the transport concerned may be taken as 10¢ per ton per day. For an annual traffic of 100 million tons, which has been exceeded, this represents a loss of some $40,000 per day and for a delay to one lock a loss proportional to its traffic, except for the amount which may be passed through other locks. Being thus concerned with a commerce volume in excess of the total import and export of all United States ocean and gulf ports, assurance of uninterrupted passage is of such importance as to warrant considerable expenditure.

The author has indicated the relative importance of the desirable features of a movable dam, certainty and reasonable facility of operation being prime requisites, and with this there must be general acquiescence. The dam described has many excellent and novel features and appears to fulfil these prime requirements. The criticism which might be suggested by this discussion relates to the large number of movable parts and the possibility of collecting mud on any submerged structure in a silt-bearing stream. The latter may not be serious in waters of the Great Lakes.

The conservative engineer may wish to see this novel form tested under conditions somewhat less rigorous than those prevailing at Sault Ste. Marie before adopting it for that particular location, where frequent operation is not needed and where a failure to operate when required might prove disastrous. However, for these less rigid conditions it would seem to be well adapted and the author deserves much credit for his contribution in design and description to the general subject of movable dams.

A. W. SARGENT,[10] M. AM. SOC. C. E.—Some means should be provided at ship canal locks to stop the rush of water through the chamber, in case a lock gate is rammed by a ship or otherwise damaged. The method usually adopted is a dam—an emergency dam—that can be installed in swift water. The type would depend somewhat on the time that should be allowed for placing the dam, and this would vary in different localities. In some canals, the excessive loss of water from the upper pool would seriously affect shipping and the dam should be designed to be placed with the least practical delay. In other localities excessive discharge from the pool would not be of great importance and a less expensive dam, which would require several hours to install, could be used.

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[10] Stanwood, Wash.

An emergency dam must be reliable; it must be ready for use at all times and therefore should be stored where it can be inspected and kept in perfect repair.

The author proposes an emergency dam located in a sump in the bottom of the lock where inspections and repairs can be made only when the lock is unwatered. Proper maintenance would be very difficult and wire cables could be weakened by corrosion in a short time, especially in locks bordering on sea water. Sunken timbers or other débris could lodge on the dam, preventing its successful operation and, after practice operations of the dam when the lock is not unwatered, there is no visible means of determining whether the dam has settled completely back into place or whether it is partly raised where a vessel could wreck it.

Some of the objections to this type of emergency dam are mentioned in the preceding paragraph. However, at the St. Marys Falls Canal, where the locks are not operated during winter months and there is ample time for repairs, this type of dam may prove satisfactory. It would not be a logical type to use in a lock bordering on sea water, such as Lake Washington Ship Canal, Seattle, Wash., where there is a single large lock for freight traffic which is operated throughout the year except for a very short period for general repairs.

The stop-log type of emergency dam, composed of steel girders placed horizontally across the lock, one on top of another, by a derrick and stored adjacent to the lock when not in use, is a very reliable type and perhaps one of the most costly types for large locks. The type used at St. Marys Falls Canal locks, with all parts stored on the lock walls, and where three to four hours are required to install it, is a satisfactory type at a reasonable cost.

The cost of the structure, or time required for installing, should not be the deciding factor in determining the type of emergency dams for locks. Reliability should be the first consideration.

A skeleton miter lock gate, powered to be safely closed in swift water, after which the complete closure would be made by needles or wickets, might prove a satisfactory type of emergency dam in some locks.

Isaac DeYoung,[11] M. Am. Soc. C. E.—In Mr. Sabin's discussion, a historical review is given of the developments of the various forms of safety appliances adopted at the Sault canals to stop the free flow of water, in the event a lock gate is rammed by a vessel or otherwise damaged. Seven different devices are enumerated beginning with the caisson gate for the lock of 1855 to the present form of dam of the box girder type for the MacArthur Lock, completed in 1943.

The plan of the proposed dam is based on the latest conditions existing, in that the lock has a depth of 30 ft. The application of the form of dam will be more favorable to less severe conditions.

Since the first canal and lock were opened to navigation, at St. Marys Falls Canal, Michigan, in 1855, no accident has occurred that resulted in the urgent need for a movable dam. At various times, however, the gates of the locks at St. Marys Falls Canal have been struck by vessels, with potentially serious consequences, while maneuvering through the locks (see Table 1). An accident occurred at the Sault Ste. Marie Canal, Ontario, on June 9, 1909, in which all the gates, except the intermediate and guard gates, were wrecked, making the canal and lock an open raceway. A movable dam, of the swing-bridge type, was available to stop the free flow.

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[11] Consultant to City Mgr., Sault Ste. Marie, Mich.; formerly Supt. of St. Marys Falls Canal, Michigan (retired).

There have been no collisions with the gates of the Davis and Sabin locks since they were placed in commission in 1914 and 1919, respectively. The wooden gates of the Weitzel Lock, built in 1881, have been struck at various times by sailing vessels, but not to the extent that would require an emergency dam. More detailed information of the accidents may be obtained by reference to the Reports of the Chief of Engineers, U. S. Army.

All the gates of the locks at St. Marys Falls Canal are mitering gates. The Poe Lock, in addition to the upper and lower guard gates (which are used only for unwatering the lock), has one pair of operating gates at the upper end of the lock, and two pairs at the lower end. All the other locks have two pairs of operating gates at both ends of the lock. In the lock of the Sault Ste. Marie Canal, Ontario (item 8, Table 1), there is one pair of operating gates at the upper end of the lock and two pairs of operating gates at the lower end.

TABLE 1.—REPORTS OF VESSELS COLLIDING WITH LOCK GATES

Accident No.[a]	Date	Name of steamer	Bound	Report, Chief of Engineers	
				Year	Page
1	August 4, 1899	*Sir William Siemens*	Up	1900	3961
2	June 3, 1901	*William R. Linn*	Up	1901	3173 3182
3	October 7, 1901	*Robert Fulton*	Down	1902	2225
4	November 10, 1909	*Isaac L. Elwood*	Down	1910	2231
5	August 3, 1910	*Lake Shore*	Up	1911	2424
6	September 13, 1912	*U. S. Steamer Essex*	Up	1913	2877
7	October 22, 1928	*Cambria*	Up	....	....
8	June 9, 1909	*Perry G. Walker*	Up	1909	2069

[a] Each collision occurred in the Poe Lock (St. Marys Falls Canal) except No. 8 which occurred at the Sault Ste. Marie Canal in Ontario, Canada.

In the accidents listed in Table 1, items 2, 4, 5, and 8 were the most serious. In accident 1, the upbound vessel struck the south leaf of the upper operating gate. Upon examination, after pumping out the lock, it was found that the gate was deformed only enough to prevent the use of the gate during the remainder of the season. Repairs were made during the following winter.

In accident 2, the vessel was approaching the lock upbound; it struck the downstream side of the lower south leaf, while the water in the lock was at the upper level and while upper gates were being opened for a downbound lockage. The leaf was driven upstream some distance against the high level of water; but, fortunately, the gate fell back, mitered partly with the opposite leaf, and, withstood the shock. Considerable damage was done, causing the lock to be out of commission for twenty-two days. Permanent repairs were made during the closed season.

In accident 3, the vessel struck the north leaf of the lower operating gate 3 ft from the miter post, the stem making a vertical dent 10 in. deep for 20 ft in the height of the gate. The north leaf passed the south one at the top of miter posts 3 ft to 4 ft, the water pouring in a large stream between the gates. These gates were out of commission the remainder of the season. Only the intermediate gates were used during this time, with the effect of shortening the lock by 93 ft. The repairs were made by the owners of the vessels during the closed season.

In accident 4, the steamer struck the south leaf of the upper operating gate, twisting the gate so badly that it could not be used the remainder of the season. The gate was temporarily replaced by a leaf of the upper guard gate. The lock was out of commission for seven days. The gate was completely wrecked; the work of repairs involved the dismemberment of practically all parts and the rerolling of the plates at the mill. Repairs were made by the vessel owners. The cost of repairs was extensive.

In accident 5, the vessel struck the south leaf of the upper operating gate, doing considerable damage. The gate was forced open against a high head, breaking the miter so that the distance between the toe cushions was estimated by the lockmen to be about 2 ft. Fortunately, the gates mitered perfectly when they were closed. The gate was damaged considerably.

In accident 6, the vessel struck the north wall with her anchor about 200 ft east of the upper operating gates, and scraped along the wall until the bowsprit rode up on the footbridge. The wooden bow of the steamer was wrecked for about 10 ft. The lower part of the wooden vessel was sufficiently decayed to make it a fairly effective fender or cushion.

In accident 7, the vessel backed into the intermediate gate which had a head of 3 ft on it. The pressure of the vessel's rudder broke the miter, moving the gate on its pintle support. After breaking the miter, the wheel of the steamer struck the toe of the gates, breaking off the four propeller blades The damage to the gate was not extensive.

In accident 8, in the Canadian Sault Canal, the steamer *Perry G. Walker* in approaching the lock upbound, rammed against the lower lock gate under a head of about 20 ft. The impact broke the miter, all the gates were carried away except the intermediate and guard gates, making the canal and lock an open raceway. The Canadian passenger steamer, *Assimmboria,* with one hundred passengers aboard was tied up in the lock waiting for the steamer *Crescent City*, an ore laden steamer, to enter the lock behind her. The *Assimmboria* was swept out of the lock, followed by the *Crescent City*. All three boats were damaged more or less by glancing blows when passing each other The movable dam was swung across the canal and the flow through the cana was sufficiently checked to place a spare set of gates, which had been built a short time before the accident. These gates were stepped in place on June 1 and the lock was again put in commission on June 21, 1909.

All the wickets of the dam were lowered into place successfully, except one which rested against an obstruction at the sills. That wicket became badl twisted and could not be operated. This gap was closed by stop planks This movable dam is similar to that of the South Canal of St. Marys Fall

Canal, Michigan, which was put out of commission when the canal was deepened in 1943. The Canadian dam (item 8, Table 1) is still ready for use.

Each of the accidents cited in Table 1 was the result of a misunderstanding of signals between the vessel masters and the engineers.

Colonel Hall criticizes the proposed movable dam on the premise that it will be expensive. The cost of such a structure should not be a deciding factor in determining the type of an emergency dam. In the paper it is stated that, because of the small quantity of steel and other metal required in its construction as compared to other forms of dams, the cost would be relatively less. The required weight of steel, and other metals, in the proposed dam is indicated as being about 215 tons, consisting largely of standard structural sections. In the construction of the swing-bridge, movable dam in the South Canal of the St. Marys Falls Canal, more than 2,100 tons of metal were used.

Colonel Hall questions the need for a movable dam, stating that "for most waterways the extra insurance is not worth the cost." If provisions are made by installing two sets of operating gates at each end of the lock, the chances of wrecking both sets of gates are very slight. Such an arrangement has been depended upon in certain localities, but it has not been the sole dependence at the Sault canals. As stated, all the accidents enumerated in Table 1 were the result of a misunderstanding of signals of the vessel masters and the engineers at the throttle, over which the lock personnel had no control.

The recess that houses the folded dam is protected by cross walls which will prevent the passage of boulders and other large objects from entering this recess. Large debris pits are provided upstream from these cross-walls. The anchor troughs are fully covered by angle flanges attached to the top anchors, thus preventing any debris from entering these troughs when the dam is in the lowered position. If sediment should be deposited on any or all parts of the dam, the structure can still be raised through a considerable load of sediment because, in the raising, the upper parts which constitute only a small part of the total weight of the dam are moved first.

Sedimentation will affect any form of movable dam, whether it has only a sill or any other underwater connection.

Mr. Southworth refers to the dynamic affect of a swirl of water against the dam caused by a wrecked gate leaf hanging by its anchorage in a diagonal obstructive position. The proposed dam will be placed upstream of all operating lock gates, and a swirl produced would be downstream from the dam.

Regarding Mr. Sabin's criticism that the proposed dam has a large number of parts—there are nine plate girders and forty anchors, which form practically the entire structure, and the entire assembly is operated as one unit. In comparison with the swing-bridge movable dam of the South Canal of the Sault canals it has, besides the bridge, thirty-two wickets and thirty-two wicket leaves. Each wicket and leaf is operated singly by cables through individual hoist mechanisms which are connected to a line shaft operated through motors located at the center of the bridge.

In the dam for the Davis and Sabin locks, there are two bridges to be lifted in place—the wicket bridge and the operating bridge. The dam is provided with thirty-two wickets, and into each wicket are to be inserted two 15-in.

steel channels. Each of these units is lifted singly into place by the 85-ton derrick.

A model of the proposed dam has been made to determine its workability. It was made from plans in the early stages of its development and is not exactly in accordance with the latest plan, but the general characteristics and method of operation are the same. The operation of the model is very satisfactory.

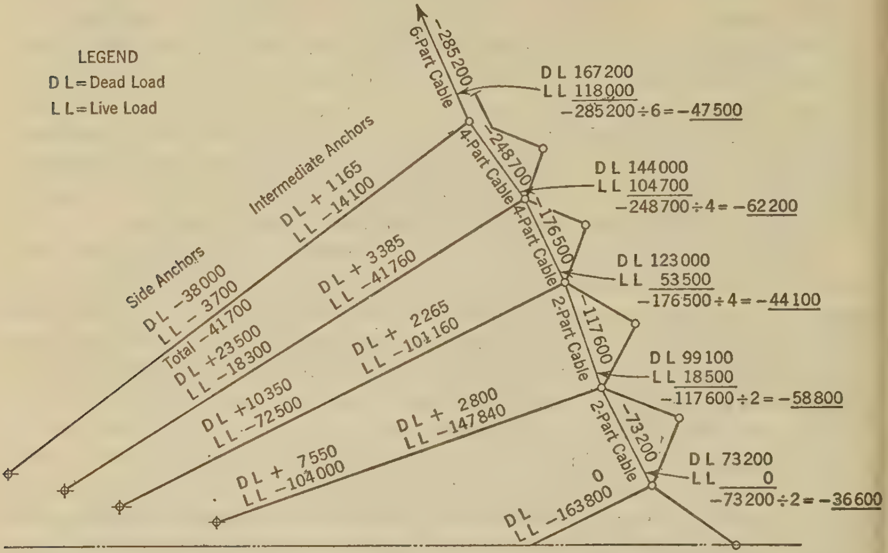
LEGEND
D L = Dead Load
L L = Live Load
-285200
6-Part Cable
D L 167200
L L 118000
-285200÷6=-47500
-248700
4-Part Cable
D L 144000
L L 104700
-248700÷4=-62200
-176500
4-Part Cable
D L 123000
L L 53500
-176500÷4=-44100
-117600
2-Part Cable
D L 99100
L L 18500
-117600÷2=-58800
-73200
2-Part Cable
D L 73200
L L 0
-73200÷2=-36600
Intermediate Anchors
D L + 1165
L L -14100
Side Anchors
D L -38000
L L - 3700
Total -41700
D L +23500
L L -18300
D L + 3385
L L -41760
D L + 2265
L L -101160
D L +10350
L L -72500
D L + 2800
L L -147840
D L + 7550
L L -104000
D L 0
L L -163800

FIG. 8.—STRESSES (POUNDS PER SQUARE INCH) IN PRINCIPAL MEMBERS OF MOVABLE DAM

Stress sheets have also been prepared for both the dead and live loads, which are based on a lock's having a depth of water on the sills of 30 ft, a width of 80 ft, and a head of water of 22 ft. Fig. 8 shows the stresses in the anchors and the cables. The stresses in the plate girders may be obtained readily from those indicated.

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Paper No. 2272

UTILIZATION OF GROUND-WATER STORAGE IN STREAM SYSTEM DEVELOPMENT

By Harold Conkling,[1] M. Am. Soc. C. E.

With Discussion by Messrs. Raymond A. Hill, Clarence S. Jarvis, John R. Charles, C. W. Sopp, J. A. Bradley, Irving B. Crosby, G. B. Drummond, Raphael G. Kazmann, Donald M. Baker, Hyde Forbes, and Harold Conkling.

Synopsis

The scope of this paper is limited to consideration of those formations which may be called underground reservoirs, into which water is charged naturally or can be charged artificially when there is a surplus, and from which water is available when needed either as gravity escape from the ground-water reservoir into a surface stream (rising water or return flow) or as controlled extractions by wells and pumps.

Introduction

The first developers of irrigation on the stream systems in western United States utilized such surface discharges as were available. In streams fed by melting snow these discharges might be fairly large until late in the growing season, but in streams where snowfall at the headwaters was small, or where the runoff season began early, the dependable surface flow was small in the latter part of the growing season. In this circumstance and where climate and geology were favorable, recourse was had at about the turn of the century to wells and pumps to utilize ground water for supplementary irrigation; but in locations where runoff was larger and in more favorable regimen, utilization of ground water was long delayed unless artesian flows could be developed. However (especially in the 1930's which in western United States was a decade of drought generally), pressure of need forced the use of ground water by individuals even in those areas favored by fairly stable surface flow in the irrigation season and by large surface reservoir capacity. Use of pumped water for irrigation is expanding rapidly in some of these areas and in others that formerly had available only such natural discharge as occurred in the flood season of early summer.

Note.—Published in January, 1945, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

[1] Deputy State Engr., Water Rights, Sacramento, Calif.

Complete development of water supplies for irrigation in the United States will require the greatest possible use of the underground portion of the streams and of the storage capacity resident in the unconsolidated alluviums of the stream valleys. The plans for large development by federal government agencies in cooperation with state and local agencies contemplate this use of underground sources. One of the principal features of the Central Valley Project in California is the use of underground reservoirs (17)(21).[2]

Such utilization is discussed in this paper from an engineering viewpoint and a brief review is presented of the somewhat inchoate body of knowledge of the subject. This paper is a preliminary attempt to correlate the subject matter, with emphasis upon those phases most useful to an engineering approach. Omission of other phases of the hydrology and geology of underground water does not deny their importance for other than the matters discussed herein nor their possible importance even in these matters.

The object of the paper places limitations on its scope:

1. Only that part of the United States west of the 97th to 100th meridian is included. This is the arid or semiarid portion, in practically all of which irrigation is needed for successful intensive agriculture.
2. Only those situations in which the following conditions exist are considered:
   - (*a*) Ground water in the alluvium is readily accessible to extraction by pumps;
   - (*b*) The alluvium is sufficiently permeable to yield water in commercial quantities;
   - (*c*) The alluvium is naturally charged with water or is susceptible of being charged artificially to such an extent that heavy commercial drafts can be sustained; and
   - (*d*) If artificial charging is necessary it can be done at feasible cost.

The foregoing conditions are necessary for the successful use of ground water in irrigation.

The scope of the paper excludes the lava beds of Idaho, Oregon, and Washington and the limestone formations of New Mexico. Immense quantities of water might be secured from the lavas judging from the large springs that discharge from the formation, and considerable quantities are being secured from limited areas of the limestone; yet experience is small concerning the exploitation of the ground water of the lavas by wells, and such exploitation would be generally hazardous and costly. The limestone formations are limited in extent, and utilization of their waters is still more limited.

The criteria eliminate from consideration almost all of the water-bearing strata of Tertiary and early Quaternary age which underlie so great a part of the Great Plains east of the Rocky Mountains. In parts of this area these strata are close enough to the surface for water to be pumped from them and often they yield water freely in commercial quantities. However, except in the bottoms where the streams have trenched themselves through the finer surface materials down to the water-bearing strata, or in locations adjacent to the streams where water from the streams is being used or can be used for irrigation of the uplands, there is no opportunity for artificial recharge. Except for the negligible percolation from streams into these strata, the natural supply is the very small portion of the insufficient precipitation which is not used by vegetation, or which does not run off and thus must percolate to the ground-water body. The area irrigated in these regions outside of the stream bottoms is not large.

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[2] Numerals in parentheses, thus: (17), refer to corresponding items in the Bibliography (see Appendix).

ALLUVIUM PROVINCES IMPORTANT FOR GROUND-WATER SUPPLIES

After the foregoing exclusions, only certain unconsolidated alluviums remain for consideration in this paper. These are (*a*) the Quaternary (with some Tertiary) alluviums of the "mountain and valley" ground-water province (8)(11); (*b*) the Tertiary and Quaternary alluviums in the stream bottoms of the Great Plains, and the same alluviums beneath irrigated areas near the major streams of the region. Of these the alluviums of the mountain and valley province are by far the most important for the purpose of this paper. Irrigation is practiced throughout this region and, as ground-water developments are extensive, it is believed that a greater volume of water is extracted from this

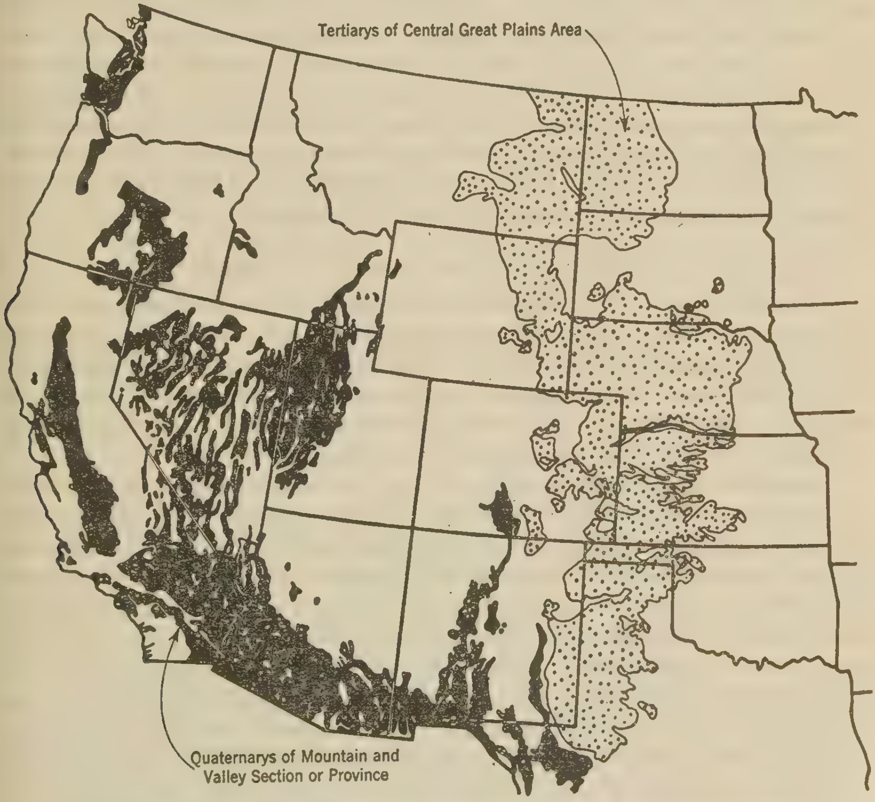

Tertiarys of Central Great Plains Area
Quaternarys of Mountain and Valley Section or Province

FIG. 1.—LOCATION OF ALLUVIUMS THAT YIELD IMPORTANT GROUND-WATER SUPPLIES

formation than from any other formation in the world. However, it is not implied that ground water can be produced in commercial quantities from all of the valleys of the province, nor even from most of them.

The mountain and valley province includes the Great Basin which embraces almost all of Nevada, western Utah, a part of southern Idaho, southeastern Oregon, and most of the southeastern desert region of California. The province also includes the coastal and central valleys of California, southern Arizona, and the Rio Grande Valley above El Paso, Tex. The portion of the Great Plains province considered herein borders the Rocky Mountains to the east and forms a wide band through North Dakota and southward almost through Texas. Fig. 1 which indicates the general locations of these sources is an adaptation of two illustrations in U. S. Geological Survey *Water-Supply Paper No. 489* (11).

The characteristic smaller valleys of the Great Basin are bolsons or semi-bolsons; that is, they either have no channel draining them and the water reaching them is disposed of by evaporation from central playas or they do have a longitudinal channel which partly drains them leaving the remainder of the water to be disposed of from playas. However, the larger streams of the basin discharge into perennial lakes and for purposes of this paper have characteristics similar to streams discharging into the ocean which drain the remainder of the mountain and valley region. In the Great Plains region all the streams under consideration are part of the Mississippi River system or of the smaller systems west of the Mississippi draining into the Gulf of Mexico.

Characteristics of Alluvial Valleys

A characteristic of the alluvial valleys of the mountain and valley province is the immense depth frequently attained by the unconsolidated alluvium, the large storage capacity of the alluvium below the water table, and its permeability. The water-yielding strata along the streams, and near-by uplands, of the Great Plains region are by contrast often of very limited thickness and thus of small water capacity although often highly permeable.

Beneath every surface stream in alluvium is an accompanying flow of water through the interstices of the alluvium. In exceptional circumstances this ground-water stream may be diverted to a direction different from that followed by the surface stream, but these circumstances do not invalidate the general rule. With the greater water supplies of a humid region this subsurface flow is almost always in contact with and supporting the surface stream. The low-water discharge of the stream is fed by the effluent seepage from the high water table. In the Great Plains area, at least in the western and drier parts, the discharge of the major streams except at times of local rain floods, in most cases, is supplied by runoff from the mountains. The natural water table of the major streams in this area is about at stream level. Generally (but not always) percolation is influent to the ground water; but only to the extent necessary to supply "evapo-transpiration" losses in the stream bottoms although there are reaches with a water table not in contact with the stream. (Effluence from the ground water to the stream when it does occur is also very small generally but a special case is the underflow—totaling annually several hundred thousand acre-feet—from the sand hills north of the Platte River in Nebraska. This flow augments the Platte from Bridgeport to Gothenburg, Nebr., a distance of 160 miles.)

In the mountain and valley province the streams discharge in general from steep mountains either in sudden rain floods or in more gentle snow-melt runoff. The flows are often sufficiently violent to cause the tributary streams to form steep cones of coarse debris along the sides of the valley and in almost all cases cones are evident. The slope along the trough of the valley is generally less steep. The water table is often far below the surface along the sides of the valley but may be near or at the surface along the trough. Obstructions beneath the surface caused by a narrowing of the valley or by a fault zone which has been active during the period of alluviation, or merely by a decrease in the permeability of the alluvium or by other reasons, will generally bring the underflow to the surface, thus joining the surface and underground streams. The seepage at this point becomes effluent; that is, it flows into the stream from the ground water, whereas in reaches where the water table is far below the ground surface it is influent.

The storage capacity in a reach of a surface stream is small or large with reference to the discharge in that reach depending on rate of flow—the slower the movement the greater will be the relative storage capacity. Water flowing through the interstices of alluvium moves very slowly; so the storage capacity in relation to discharge past any point is astronomically large. Thus any portion of the underground stream is a storage reservoir although not strictly so in the accepted sense of the term when a surface reservoir is under consideration. From this underground reservoir water can be drawn, and into it water may be charged if the water table is lower than the stream bed.

Rising water in these streams is the ground water moving downstream which has been brought to the surface by an obstruction. Even if the obstruction is not sufficient to so reduce the cross section that the underflow is brought to the surface, it will be raised to an elevation at which pumping extraction is easier. The term "basin" is often applied to the area above a subsurface or surface obstruction. Other criteria may also lead to the use of the word "basin" but there is no general definition. The term is used very loosely, but in spite of vagueness it is a convenient one. If the alluvium is deep enough, a basin in the water table could be created at almost any point by pumping and thus lowering the water table below the elevation that will be maintained some distance above or below or to the sides of the location of the pumping.

It is probable that the water supply of none of the stream systems of the west can be fully developed without utilizing some part of the storage possibilities of the unconsolidated alluvium; nor can it be fully developed without construction and utilization of surface reservoirs. The best development will balance the advantages and disadvantages of surface and underground storage, using both in the ratio found applicable to the particular stream system and the regimen of its flow.

A study of evaporation losses from hypothetical surface reservoirs on all streams in California (16) indicates that, no matter how large the reservoir capacity, streams of erratic annual and cyclic flow will yield for useful purposes no more than 50% or 60% of the annual average discharge because the remainder will be lost, over the years, by evaporation from the excessive water surface of the reservoir necessary to impound the water of the infrequent years of large discharge. Reservoirs of much smaller capacity would make almost as much water available. In streams of more regular regimen of flow the percentage of water that could be made available is larger; but it is evident from the aforementioned study, and it is common knowledge to those who have planned large developments, that the proper ratio of reservoir capacity to average annual stream discharge is definitely limited no matter what the extreme variations in annual discharge may be and that evaporation is one of the limiting factors.

On the other hand, if the elevation of the water table in a ground-water unit is below the root zone of overlying vegetation and below the limits of capillary action, loss by evaporation will be practically nil. Loss will occur, nevertheless, but it will be due to escape from the ground-water unit at the lower end of the unit whether as underflow or surface flow or transpiration from water-loving vegetation. Under the conditions presented in this paper, water underground is not static as it is in a surface reservoir when the water level is below the spillway lip and outlets are closed. On the contrary, it is moving as a stream or group of streams through the unit and toward the locus of escape.

Large and not directly controllable losses thus occur from both surface and underground reservoirs. For that reason water cannot be held in any reservoir for an indefinite period. Its supply must be replenished periodically and used before it is lost.

Factors that tend to make the use of ground water attractive are the gradually increasing cost of the gravity developments and the decreasing cost of pumping due to decreasing power cost and increasing efficiency of pumps. Furthermore, its use has two important advantages to the farmer—ground water does not carry weed seeds as does surface water, and it is available on demand for irrigation without waiting on rotation schedules. This immediate availability promotes over-all efficiency of farm operations although the cost of the pumped water may be greater than that of gravity water. Thus the relation between the desirability of construction and use of surface reservoirs as opposed to the use of underground reservoirs is altering progressively in favor of greater utilization of ground-water reservoirs in complete stream developments. The high prices paid for farm products is accelerating the construction of wells and the use of pumps. Once installed, the equipment will be used when desirable whether present prices for farm products continue or not.

In what follows frequent reference is made to the North Platte and Platte river basin of Colorado, Wyoming, and Nebraska, and to San Gabriel River Valley of Southern California. These two stream systems have been chosen because they involve extreme but quite typical conditions of the Great Plains province on the one hand and of the mountain and valley province on the other hand. In addition, an enormous amount of information is available about each.

In locations where utilization of the natural ground-water supply by pumping has been the paramount development (as has been particularly notable in California), it is being found that surface reservoirs must be built if much further conservation is to be accomplished even if the only function of such reservoirs is to equate the floods so that more of the stream flow will percolate to the water table in crossing the valley or so that it can be diverted feasibly in canals to off-stream spreading grounds.

A comparison of the situation on the coastal streams of California south from San Francisco with the situation in the Platte River basin is illustrative.

PLATTE RIVER BASIN

In the Platte River basin, surface reservoir development on both the North and South Platte rivers (which unite in Nebraska to form the Platte River) has been so large in comparison to annual stream discharge that it does not seem feasible to extend it farther although quantities of water are lost out of the irrigated area in both streams, particularly the North Platte, and severe shortages occur in parts of the irrigated areas. The irrigation projects that are diverted from the streams by gravity have created large return flows to these same streams which have been used by downstream irrigation developments. This return flow comes from the large amount of storage artificially induced beneath the surface of the lands irrigated by gravity along and near-by the streams and is fairly uniform as compared to the regimen of the natural stream flow. Individuals or groups can pump this water while it is still in storage, for use of immediately overlying lands, and thus in times of drought make it available to themselves instead of allowing it to proceed as return flow to others downstream from them. Such pumping was begun in earnest in 1934 (the worst of the drought years) and, since then, on the South Platte and on the main Platte rivers, its progress has been accelerating rapidly. It reduces the return flow and those dependent on such flow for gravity diversions are being forced to seek other methods of securing a supply. This need has been met in part by local surface reservoir construction and in part by pumping from the ground water beneath the local irrigated area which was deprived of gravity water by the new upstream draft on the ground water. Thus pressure of need is bringing about a development whereby the utilization of water is approaching the ultimate. This may be a logical development, but it is being made without long-range advance planning. The drainage area of the North Platte River

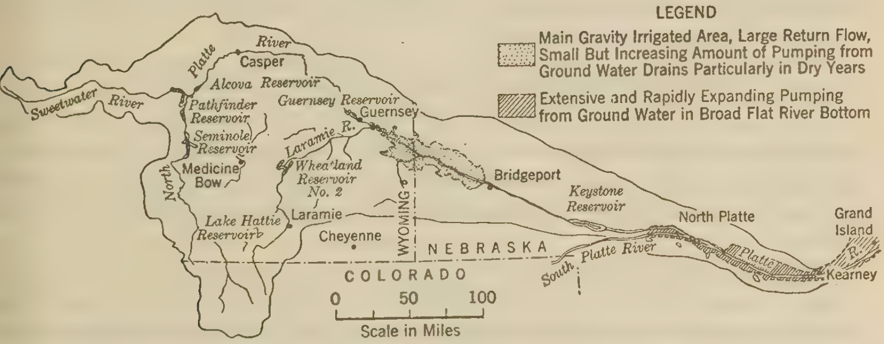

LEGEND
Main Gravity Irrigated Area, Large Return Flow, Small But Increasing Amount of Pumping from Ground Water Drains Particularly in Dry Years
Extensive and Rapidly Expanding Pumping from Ground Water in Broad Flat River Bottom
Platte
River
Casper
Alcova Reservoir
Sweetwater River
Pathfinder Reservoir
Guernsey Reservoir
Guernsey
Seminole Reservoir
Laramie R.
Medicine Bow
North
Wheatland Reservoir No. 2
Bridgeport
Keystone Reservoir
North Platte
Grand Island
Lake Hattie Reservoir
Laramie
Cheyenne
WYOMING
NEBRASKA
COLORADO
South Platte River
Platte R.
Kearney
0 50 100
Scale in Miles

FIG. 2.—NORTH PLATTE RIVER BASIN

basin is shown in Fig. 2. Since the water of the South Platte River is virtually all consumed in Colorado, the Platte River is treated as an extension of the North Platte River.

Development of ground-water use similar to that in the Platte River basin is under way in the lower Boise Valley, Idaho, and in the Rio Grande basin in Colorado. It is well advanced in Salt River Valley and Gila River basin, both in Arizona, to mention only a few. This extensive use of ground water was not planned although the early conception of Salt River Valley project did envision some use of pumped water. Such conception was lacking in other places. The developments apparently have just happened for the most part and this fact is no great compliment to engineers and hydrologists. However, the lessons learned will be of aid in the future.

In the coastal valleys of California south from San Francisco, development has been almost exclusively from ground water, resulting in some places either in cumulative overdraft or expensive pumping lifts. Reservoir development in the headwaters of Santa Clara County streams draining into southern San Francisco Bay and of Los Angeles County streams has been made or is in process, but these developments are not designed to provide gravity water to the irrigator, but rather to give greater recharge to the ground water from which the irrigator can pump it.

Thus, whether the first and the major development of any particular water supply is from surface or from underground supplies, utilization of all opportunities for water salvage within some practical limitation finally occurs, although it almost goes without saying that the percentage of salvage by surface reservoirs, in such situations as are found in the Platte River basin where large and feasible surface reservoir sites are plentiful, will always be far greater than the percentage of salvage by ground-water reservoirs. In such situations as exist in the coastal valleys of California where surface reservoir sites in the mountain drainage areas are few, of limited capacity, and costly to construct, and where ground-water reservoirs of such large capacity exist, the ground-water reservoir will always be paramount.

Characteristics of a Ground-Water Reservoir

The capacity of a surface reservoir is determined by the safe yield from the stream that supplies it, and by the cost of the reservoir per unit of such yield. The capacity and the desired useful outflow are designed to be in harmony with one another, the relation between the two depending on the inflow and its regimen.

The ground-water reservoir, on the contrary, exists in nature and its capacity is set by nature. When man appears on the scene and proceeds to upset the balance of nature the reservoir is empty or full depending on the relation of the surface stream level to the surrounding terrain. In such a situation as the North Platte Valley astride the Nebraska-Wyoming line (and of course many other valleys of the Great Plains area) the reservoir was almost empty when developments began because it was drained by the trough of the stream which is cut far below the bordering terrain and because precipitation is not sufficient to maintain a water table appreciably above stream level. In the case of San

Gabriel Valley in California (and many other valleys of the mountain and valley province), the reservoir was full because practically the only outlet is the Paso de Bartolo (Whittier Narrows) which forms a spillway lip at the southern side (see Fig. 3). The streams entering the valley do not drain the ground water but feed it, and precipitation on the valley floor is also a considerable source of supply. Development of irrigation in the North Platte Valley, which is practically all by gravity, has resulted in filling that ground-water reservoir. Development of irrigation in San Gabriel Valley, which is practically all by pumping, has resulted in partly emptying that ground-water reservoir.

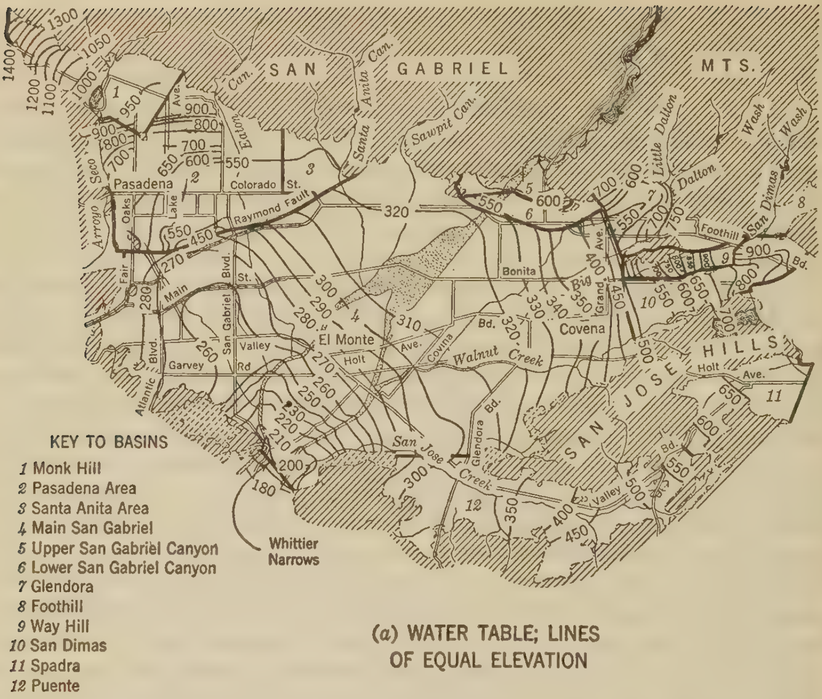
The ground-water reservoir exists in nature but its capacity is unknown and cannot be determined easily. Unlike a proposed surface reservoir its existence as such has rarely if ever been consciously envisaged until large use of water from it had developed. When pumping from it began the most important matter was whether good yields could be secured from wells at low cost. If artesian flows were discovered it was naturally so much the better.

As pumping increased and the underground reservoir was envisaged its safe yield became a most important consideration. One of the components that determines this is the capacity of the reservoir but, as before stated, its capacity cannot be measured directly as can that of a proposed surface reservoir. However, in the stage of development at which the ground-water reservoir is being consciously utilized, a mass of data will have accumulated concerning return flow, canal losses, precipitation, evaporation, consumptive use, stream-bed losses, change in water-table elevation, surface culture, stream discharge, annual pumping draft, etc. All of these items may not be applicable to any one ground-water basin but from those that are applicable, when properly assembled and interpreted, it may be possible to find the safe yield without determining capacity, as is necessary when a surface reservoir is contemplated. If such data are not available it may be necessary to estimate capacity through determination of specific yield of the alluvial material from which storage capacity may be calculated, approaching the problem much as an estimate of safe yield of a stream on which a surface reservoir is to be built is approached—that is, by determination of supply to, and demand on, the stream. This may be advisable sometimes with a ground-water unit even if all the applicable aforementioned data are available. Any such determination involves analysis of large amounts of data, but a direct determination based on recorded experience may involve less work than a determination based on capacity as an antecedent. Regardless of the work necessary the former method has merit because the result is found almost directly instead of as a residual quantity.

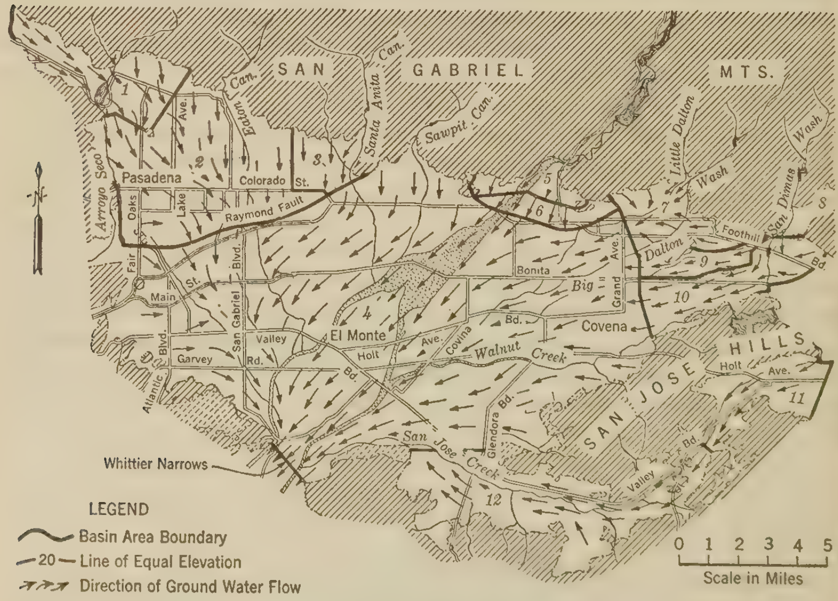
Safe Yield

Definitions.—"Safe yield" is the annual extraction from a ground-water unit which will not, or does not—

1. Exceed the average annual recharge;
2. So lower the water table that permissible cost of pumping is exceeded; or
3. So lower the water table as to permit intrusion of water of undesirable quality.


SAN GABRIEL MTS.
SAN JOSE HILLS
Pasadena
El Monte
Covena
Bonita
Whittier Narrows
Raymond Fault
Arroyo Seco
Eaton Can.
Santa Anita Can.
Sawpit Can.
Little Dalton
Dalton Wash
San Dimas Wash
Walnut Creek
Big Dalton
San Jose Creek
Colorado St.
Main St.
Valley Bd.
Holt Ave.
Garvey
Foothill Bd.
1400
1300
1200
1100
1050
1000
950
900
800
700
650
600
550
450
320
300
290
280
270
260
250
230
220
210
200
180
310
330
340
350
400
500
KEY TO BASINS
1 Monk Hill
2 Pasadena Area
3 Santa Anita Area
4 Main San Gabriel
5 Upper San Gabriel Canyon
6 Lower San Gabriel Canyon
7 Glendora
8 Foothill
9 Way Hill
10 San Dimas
11 Spadra
12 Puente

(*a*) WATER TABLE; LINES OF EQUAL ELEVATION


SAN GABRIEL MTS.
SAN JOSE HILLS
N
Pasadena
El Monte
Covena
Bonita
Whittier Narrows
Raymond Fault
Arroyo Seco
Walnut Creek
San Jose Creek
LEGEND
Basin Area Boundary
20 Line of Equal Elevation
Direction of Ground Water Flow
0 1 2 3 4 5
Scale in Miles

(*b*) GROUND WATER FLOW

Fig. 3.—Observations, San Gabriel Valley, California, in the Fall of 1941

"Specific yield" is the ratio (expressed as a percentage) of the volume of water which may be drained out by gravity from a given volume of soil. Possibly simpler terms would be "drainable voids" or "effective porosity."

"Specific retention" is the ratio (expressed as a percentage) of the volume of water which is held against the force of gravity in a given volume of soil. Possibly a simpler and sufficiently accurate term would be "field capacity."

Determination of Safe Yield.—Disposal must equal supply. This is fundamental to all physical processes. The supply of water to a unit or reach of a stream system is the surface flow and underflow into the unit and the precipitation falling on the unit. Final disposal is by surface flow or underflow or both, by evaporation, or by artificial withdrawal. If final disposal is in temporary imbalance with supply, the storage in the reach of the stream changes. The statement applies both to the part of the stream underground and to the portion on the surface.

Over a term of years the natural escape of ground water from a unit balances the natural supply to the unit. The amount escaping at any particular time is governed by the elevation of water table. In many cases the supply is also modified by this because the natural water table may be so high as to preclude percolation from parts of the streams. When pumping is begun the new and additional draft causes the water table to lower and natural outflow decreases. At the same time the lowered water table may induce a greater supply because of greater percolation opportunity. In other words, any reservoir, when half full, can and will impound more of the water that reaches it than it could if it were three fourths full or entirely full. Also, discharge through the uncontrolled outlets of a reservoir or over the spillway decreases as the water level in the reservoir decreases.

As a physical matter only, the pumping draft might become so large that outflow would be stopped. This possibility would depend on the slope of water table toward the outlet and on the depth of alluvium, but as a practical matter such lowering of the water table would probably bring about higher cost for the pumped water than would be feasible. Therefore, there is a financial as well as a physical limit to the safe yield of a ground-water unit. The essential problem of safe yield from a ground-water unit is to determine that part of the natural disposal that can be or has been diverted to beneficial use by pumping from the basin and to what extent the recharge will be or has been increased by development.

If the water is to be used on overlying land the salts in the return flow finally concentrated in the ground water would make it inexpedient to use it all. Some escape would be necessary to maintain the salt balance, for a ground-water unit needs "sewage" disposal as imperatively as does a municipality.

Thus there are two limitations on the safe yield as it might be calculated from consideration of only the supply to the unit and the void space in the alluvium. One of these is cost and the other is quality of water. However, a start must be made and the limitations due to cost and water quality may be considered after the physical possibilities have been explored.

It was stated previously that by the time the stage of development has been reached at which the ground-water basin is consciously treated as a ground-water reservoir, numerous data are available as to various phenomena that will aid in determination of safe yield. The question arises as to whether it would be possible to determine safe yield of a virgin ground-water basin. The quantities of water percolating to the ground-water basin or leaving it as transpiration, evaporation, and rising water would be the only criteria; but precipitation varies cyclically (to use a not wholly accurate but quite descriptive term) as well as annually and in the arid regions of the United States the cycle may be long and the percentage of variation in the cycle large. In Southern California the cycles during the period of record have been from 20 to 30 years long. If there had been no development in San Gabriel Valley and if an observer had measured the rising water at Paso de Bartolo for the period from 1905 to 1933 he would have been able to calculate the average surface outflow from the ground-water basin above. If he had been informed as to depth of the alluvium in the valley he would have known that sufficient capacity exists to give a safe yield equal to the average yield. However, he would not have known that the safe yield would be larger than the surface outflow because of greater recharge due to change from native to cultivated vegetation and to lowered water table which gives greater percolation opportunity. At any rate, only long-continued observation would have yielded satisfactory information. (The discharge of rising water from the valley varies with the elevation of the water table in the central part. Consistent measurements of rising water which began in 1923 and measurements of elevation of water table in the central part of the valley which go back to 1903 indicate that a fairly definite relation exists. The highest elevation was recorded in 1916 and the lowest in 1934. The highest average discharge of rising water taken from the relationship curve was 165 cu ft per sec in the water year 1915–1916 and the lowest (measured) was 38 cu ft per sec in 1933–1934.)

From some valleys, particularly those of the Great Basin, there is no surface drainage and the water supply is disposed of by evaporation from playas and transpiration from vegetation whose roots feed on the ground water near the low point of the valley. The area of high water table must vary with the climatic cycle and, although the draft on ground water per unit of area of such vegetation can be closely approximated by appropriate methods, it would seem that changes in the area of vegetation drawing on ground water would occur, thus making it necessary to wait years before this would be a definite measure of the safe yield.

It may be concluded that seldom, if ever, can safe yield be determined with any great accuracy until the data available from more or less haphazard development are at hand; but until such time comes probably approximations can be made by applying factors to a limited amount of data. However, only infrequently, if at all, would there be occasion to determine the yield of a virgin basin.

The Hydraulic Equation.—The various elements of disposal and supply for a stream in which a ground-water reservoir, basin, or unit is involved are the same as for a unit of a surface stream. The equation is as follows:

$$\left.\begin{array}{c}\text{Surface outflow}\\ \text{plus}\\ \text{Underflow out}\\ \text{plus}\\ \text{Consumptive use}\\ \text{plus}\\ \text{Change in storage}\\ \text{plus}\\ \text{Exportation}\end{array}\right\}\text{equals}\left\{\begin{array}{c}\text{Surface inflow}\\ \\ \text{plus}\\ \\ \text{Underflow in}\\ \\ \text{plus}\\ \\ \text{Precipitation on surface}\end{array}\right.$$

The equation is general but there are refinements which must be used under certain conditions.

Surface Outflow.—The lower end of a ground-water unit is generally at a geological formation either of relatively impervious alluvium or at a narrows which is a strategic location for a stream gaging station.

Surface Inflow.—The upper end of a ground-water unit is likewise generally at a strategic location for a stream gaging station. Strategic locations are generally available on tributaries also above the ground-water basin, but, if unimportant, their discharge can be estimated with sufficient accuracy.

Underflow.—Measurement of the outflow underground has been tried in various places with little success except in those cases in which its amount is negligible. As the underflow is generally small when compared to other quantities, because the area of permeable alluvium is constricted at points of outflow, a considerable error in the estimate of this item may not be important in the over-all result. Often the only inflow underground is the probably negligible amount from the rocks of the basement complex of the region.

Consumptive Use.—Much work has been done on consumptive use of individual crops and types of native vegetation by H. F. Blaney, M. Am. Soc. C. E., and associates (23)(26)(28)(29)(30)(31)(35)(39) and on total consumptive use of valleys as a whole. Therefore, it is possible to assign values to this item based on experience elsewhere, properly modified for variations in temperature and surface culture, or based on data from the unit under consideration itself (1)(4).

Change in Storage.—In all localities where ground water is important, interest has turned to measurement of depth to water from the surface at wells. As the years have passed it has come about that such measurements are generally made systematically and thoroughly. It is not always possible to determine whether a measurement discloses the distance from the surface to a column of water under pressure or whether it discloses the distance to a free water table. Consistency of measurements, study of results, and in some cases installation of recorders have helped in separating the two but difficulties will always be encountered. Changes in piezometric level, of course, do not indicate change in storage. For a pressure area the change in elevation of water table at the forebay (if such exists) is the indicator. These measurements give the change in volume of saturated alluvium which has taken place in a stated period of time and are thus indexes of change in amount of water stored.

Change in storage in an underground reservoir is the integration of the values of all the divergent forces acting on the ground-water basin. It includes the effect of the unknown underflows and, in fact, none of the other quantities noted in the equation except extractions need to be known for the determination of safe yield for any stated period. ("Safe yield" may be used as meaning the volume of water pumped or the net volume after consideration of return flow and other new recharge, which is lost to the ground-water unit due to pumpage. In this paragraph the former meaning is in mind. In the hydraulic equation the latter meaning is embraced in the terms "consumptive use" and "exportation." Consumptive use of pumped water is meant.) As compared to most of the other quantities and especially to the sum total, change in storage can be expected to be small. Even at the most it is unlikely that it would be more than 10% of the difference between total water entering (including precipitation) and leaving the unit and it will usually be far less. Error in this one quantity, even if in the same percentage order as the errors in other quantities involved, would therefore ordinarily be only a small part of the absolute errors involved in the other quantities.

Disregarding determination of underflow, determination of change of storage capacity via an antecedent determination of specific yield is the most difficult of all the items of the equation but is evidently the most important in those cases where such determination is desirable. It may be possible in many cases to estimate it from general physical conditions with sufficient accuracy for preliminary purposes and thus avoid elaborate determinations. Short cuts along this line have been evolved. The growing knowledge of values for consumptive use of vegetation is giving valuable aid.

Exportation.—Water used on overlying land gives return flow to the basin unless continuous impervious strata intervene between the surface and the aquifer. Exportations of pumped water do not give a return flow, and therefore the two must be kept separate in the hydraulic equation.

Precipitation.—In some regions fairly distant precipitation records are satisfactory. In others, notably the mountain and valley province, and particularly in the coastal valleys of California, variation is great in a few miles even on the valley floor and numerous records are necessary.

Observed General Values of Certain Phenomena.—It will be observed that the general hydraulic equation will give an accounting of the disposal of the supply for any stated period and involves both the surface flow and underflow of the reach of the stream in question. It does not necessarily provide a complete basis for extending experience into the future—that is, for the design of a project (if the term may be used), or for safeguarding prior rights dependent on the natural percolation. To do this it may be necessary to secure data on the variation in amount of percolation with variation in discharge of the streams, with variation in annual precipitation, with anticipated changes in surface culture, and with anticipated changes in elevation of water table.

In such case the hydraulic equation refers to supply to, and disposal from, the ground-water unit itself instead of from the entire portion of the stream system which contains the ground-water unit. The members of the left-hand side of the equation would be unaltered but "consumptive use," instead of being that of the entire basin, including that from precipitation direct, is only that which draws on the ground water; that is, the use by the area irrigated by pumping and by water-loving vegetation where the water table is high. "Surface inflow" on the right-hand side of the hydraulic equation becomes "percolation from stream flow," and "precipitation on surface" becomes "percolation of precipitation."

The determination of these quantities is much more difficult than the recording of precipitation and runoff since only the portion of these quantities which percolates is involved. These various items of recharge can be discussed only generally in this paper.

Percolation from Precipitation.—The portion of the rainfall on the alluvium of the valley floors which percolates past the root zone of vegetation is a supply to the underlying ground-water basin unless continuous impervious strata intervene. Average annual precipitation in the valleys of western United States varies from about 2 in. to about 30 in. East of the Rockies about 65% of the total falls in the late spring and summer, whereas on the Pacific Coast about 75% falls in the winter. In other areas the distribution through the year is more nearly uniform. Probably the runoff from any of these annual average rainfalls is not more than 5% in agricultural areas. The amount that gets through the root zone and thence to the ground water depends on the regimen of precipitation, the type of vegetation, the type of soil, and the temperature. The native vegetation tends to be of the highest type that the precipitation will support and hence consumes almost all the average rainfall even in a subhumid climate. Thus, percolation to the ground water in areas where the surface is covered with native vegetation probably occurs only in the years of considerably above-normal precipitation. A rough estimate would be that in such areas practically no precipitation escapes unless the annual average is above 15 in. and that with an annual average precipitation of 30 in. the average annual contribution to ground water is generally about 2 in., but that it varies from less than 1 in. to as much as 5 in. in exceptional circumstances.

In order to capture the rainfall the native vegetation often must develop tremendous root systems. These are lateral systems in extremely arid regions where depth of penetration of precipitation is necessarily very small and for this reason the individual plants are widely separated. On the other hand, where the soil is coarse and where precipitation is great enough to penetrate deeply, especially if it occurs in the non-growing season and thus must be stored in the soil because it is not consumed immediately, the root systems do not have disproportionately large lateral spread. Rather, they extend to great depths in order to use the greater part of that which has penetrated beneath the surface and been stored. As most cultivated vegetation except alfalfa, deciduous trees, and vines, has a comparatively shallow root system a larger amount of precipitation goes through to the water table after land in the less arid regions with the Pacific type regimen of precipitation has been placed in commercial crops.

Annual average precipitation on the well-developed Raymond basin area northeast of Los Angeles, Calif., is 22 in., of which 75% falls in the four winter months. In this area are the City of Pasadena, Calif., and several other cities, as well as irrigated groves (41). The average annual supply to ground water of this area from precipitation direct is about 5.3 in. in depth. When the area was in a virgin state, the deep percolation was about 2.0 in. Precipitation on the valley floor accounts for about 44% of total supply to the ground water. About 27% of the surface is impervious because of paving, sidewalks, and buildings. The culture on another 48% has been changed from the native state but change in root zone depth on this part has been immaterial. The major increase in penetration of rainfall to the water table has been from the remaining 25% of the area and the total amount of such increase is striking in the extreme when it is realized that it comes from so small a part of the area. In this part average annual penetration of precipitation is perhaps about 13 in., or six times as much as when it was in deep-rooted native vegetation. This increase, however, is partly due to runoff from impervious roofs of residential buildings to the pervious shallow-rooted areas of lawn and garden.

Such calculation of percolation from precipitation separate from irrigation water might not be possible where a large part of the precipitation occurs in the summer coincident with irrigation; nor in most cases would it be necessary. In such cases the precipitation is added to the irrigation water diverted to the area and the combined total is used in general calculations. Due account must be taken of increased over-all consumptive use due to almost total evaporation of small rainfalls.

After irrigation development takes place it would seem probable that in regions where precipitation occurs in the growing season contributions would occasionally occur to the ground water even with annual average precipitation of from 7 in. to 8 in. However, in a large part of the Great Plains region the soil is close textured and its field capacity is large. Although irrigation is practiced for general crops where average annual rainfall is as much as 24 in., precipitation in that region occurs mainly in the growing season when transpiration is heavy and it does not penetrate deeply; nor are deep-root systems necessary to take advantage of it inasmuch as it is used up very soon after falling and there is little accumulation. Hence contribution to ground water from precipitation on such soils is small even after the surface culture changes from native to cultivated types. Actually in this region (except in coarse soils such as the sand hills of western Nebraska, where the average annual rainfall is about 20 in.) the contribution to ground water from precipitation is almost negligible and exploitation may quickly cause overdraft.

Percolation from Stream Flow.—Under natural conditions and in regions of less than about 15 in. average annual precipitation, percolation from streams supplies practically all of the natural ground water of the valleys in the mountain and valley province. The water table is generally far below the surface and the material of the stream beds is generally very receptive of water. Percolation is greater in that portion of the stream beds consisting of coarse sands than it is in the portions wherein larger particles, termed gravel and boulders, are found because these particles occupy a considerable portion of the mass, whereas the percolate can travel only through the interstices of the sand which fills the space between the larger particles. In the San Gabriel River bed the material changes rapidly from coarse gravel, boulders, and sand near the mountains to rather coarse sand about six miles below. Farther downstream the sand is finer.

Percolation rates from this stream measured by the State Division of Water Resources varied from 0.90 cu ft per sec per acre near the mountains to 4.84 cu ft per sec per acre at about 6 miles below for the first few days of flood discharge (18)(20). From that point downstream, measured percolation rates decrease gradually to about 1 cu ft per sec per acre in the next two miles and presumably the rate is smaller farther downstream. A feature of percolation in this vicinity is its rapid decrease in the portion of the river bed near the mountains as the flow recedes and becomes more uniform after a flood. This is because of the deposit of calcareous material which forms on the stream bed near the mountains. In these reaches the gravel and boulders are of sufficient size to be undisturbed by the smaller flows. Downstream from this coarser material the lighter sands are moved continually by flows which do not disturb the heavier material and the calcareous coating is not formed until the flow gets smaller.

By a series of intensive measurements on San Gabriel River during flood times and subsequently until water ceased to flow into the ocean, a formula was established which has been expressed on a diagram (20) giving the volume of water which will percolate for different discharges at the mouth of the canyon from which the waters of the river debouch on to the valley. This diagram is somewhat similar to a rating curve for a stream measuring station but is more complicated.

A similar series of measurements and determination of percolation on Alameda Creek tributary to San Francisco Bay has also been made. The formulas derived apply only when the condition of the stream bottoms is as it was at the time of the investigations. The conditions in many of the coastal streams of California are similar to those of the San Gabriel River, and the formula applicable to the San Gabriel River has been used in calculations of loss from these other streams in the belief that substantial error would not result even if the ratio of percolation to discharge on these other streams was considerably different, which seems unlikely. This seems permissible and especially so because the number of days of flood flow in which the streams persist to the ocean is comparatively few. At other times the entire stream percolates and the cumulative total of percolation at such times is much more than the cumulative total during flood times.

Ordinarily, stream-flow measurements to determine percolation are too expensive to be practical except at low flows but, in these cases, were believed to be necessary for protection of prior rights, inasmuch as proposed reservoirs would disturb the flows and the percolation.

Percolation from Water Spread on Surface (*5*).—In Southern California at about the turn of the century, the practice was introduced of diverting the water of a stream over waste land below the canyon mouths where the water table was distant below the surface thus causing it to cover a greater area and giving it a greater opportunity to percolate. This is much the same as irrigating the land by wild flooding although for a different purpose than ordinary irrigation. Water is also caused to occupy a large area of the stream bed itself for the same purpose. In some cases the water diverted from the stream is ponded behind low dams or rock walls. The practice has since been adopted in other states.

Unless the stream is regulated by a reservoir which will also desilt the water, spreading cannot be done in times of violent rain floods because of the hazard or because the silty water will seal the surface of the gravel and decrease the percolation rate. Therefore, spreading on unregulated streams is usually done at other than flood times, and most of the discharge in many such times would percolate even if not spread (because of the small volume), although at some lower point on the stream.

The more uniform snow-melt floods carry less silt and fewer precautions are necessary. Spreading other than small quantities on unregulated streams may be quite a costly matter when net benefits are considered. Published data are misleading in that they give gross amounts diverted and spread without reference to the amount which would have percolated anyway. Often too, published data are not actual records of even gross amounts diverted.

Spreading from regulated streams is more successful and possibly will be ultimately developed in many streams where conditions are favorable. The surface reservoir regulates the flow so that it can be handled more easily. Spreading under such conditions is equivalent to a transfer of water from a surface on-stream reservoir to a surface off-stream reservoir, in this case represented by the underground reservoir. Even in such cases published records may give exaggerated values.

The initial rate of percolation from spreading grounds may gradually decrease through each spreading season and the average rate for each successive spreading season may decrease until spreading is no longer possible. This phenomenon may be due to changes in the soil caused by base exchanges. Other possible causes exist. This deterioration appears to be less evident, or entirely lacking, on debris cones of coarse material where native vegetation is undisturbed.

Water is sometimes diverted to wells which are not being used and the water is forced into the gravels by the head in the well thus created. The amounts thus caused to percolate have been comparatively insignificant and in some regions, due to deposition on the sides, the possible future utility of the well may have been impaired. However, in regions where deposition would not occur, it would appear possible to recharge ground water through wells in situations where a distribution system is installed having surplus capacity and surplus gravity water at a coincident period. Such recharge would seem to be of special importance in connection with demands on ground water for air conditioning now expanding so rapidly and also for general municipal drafts because such distribution systems lend themselves more readily to the practice and the draft is more localized than in the situations under discussion here. The matter is somewhat outside the scope of this paper and is here mentioned only in passing.

An important consideration often overlooked in evaluation of the benefit of spreading to induce additional underground supply is as follows: Unless the draft on the ground water of the basin into which the water is caused to percolate is greater than the recharge the water caused to percolate will be lost by increased transpiration from water-loving vegetation near the lower side of the basin or by escape as increased surface outflow or underflow. The sum of these increases would equal the additional water caused to percolate. The benefit from spreading in such basins is limited to the smaller pumping cost due to higher water table which exists primarily in the drought phase of the precipitation cycle inasmuch as, during the surplus phase, the water table would raise from the natural supply to about the same elevation as it would with the additional induced percolation. This benefit must be balanced against cost of spreading and loss of water to the stream due to increased transpiration in areas of high water table which reduces the supply to other users below.

Percolation from Gravity Irrigation.—Whenever water is used for irrigation the surplus beyond the amount consumed or returned directly to a water course percolates to the ground water. This deep percolation may be small or large depending on soil, crops, cost of water, etc., but it is inevitable. If the water is from a stream, thus constituting a new supply, the percolate finally fills the alluvium beneath to the level at which it will be disposed of in obedience with physical law. The disposal will be accomplished by evaporation if the topography is such that the water table approaches close to, or intercepts, the ground surface or by seepage into or by discharge of springs into drainage channels. Such discharge is called "return flow." It is the escape from the ground-water reservoir beneath the irrigated area which has been filled by percolation of water diverted from a stream and by any additional percolation of precipitation due to irrigation.

These irrigation enterprises accomplish much the same thing as "spreading" to induce percolation, but actually the amount of percolation is generally much larger although in most cases only an incident to direct beneficial use. Possibly the first conscious use of the alluvium as a water reservoir came about because of such deep percolation. This was so in San Joaquin Valley (California) where, in many parts, the excess water does not readily escape as return flow but swamps the land instead. Ground water was pumped through necessity near the beginning of the century, and, during subsequent dry periods which brought about recession of water table, water was diverted and placed upon the land during the non-growing season, or excess quantities were diverted during the growing season, in order to augment the ground water.

Specific Yield.—If the specific yield can be determined the water capacity of the underground reservoir can be calculated. Changes in elevation of water table registered in feet can be interpreted into changes in quantity of water—that is, into additions to and subtractions from the amount of water in storage. If extractions by pumping, specific yield, and change in water-table elevation are known, the safe yield for any stated period in which data are available can be determined. This information for a shorter period must be modified to find the long-time average safe yield.

In 1923 Mr. Meinzer listed seven methods for determining specific yield (11) and in 1932 he again listed the identical methods (13). In 1937 C. F. Tolman relisted them (8). It is safe to say that all direct determinations that have been made had one or more of the seven methods as a basis.

As given by Mr. Meinzer (headings are by the writer) the seven methods are:

A. Direct Determinations of Specific Yield:
  1. Laboratory saturation and drainage of columns of alluvium;
  2. Field saturation and drainage of alluvium as it lies in place;
  3. Determining from samples the specific retention and porosity after water has been drained in field;
  4. Pumping from ground water, exporting or using the water and observing the volume of alluvium unwatered; and
  5. Observing the volume of water entering alluvium and the corresponding volume of the alluvium in which a rise in water table occurs.

B. Indirect Laboratory Determinations of Specific Yield:
  6. By the centrifuge; and
  7. Mechanical analysis and determinations of porosity, estimating therefrom the specific retention and specific yield.

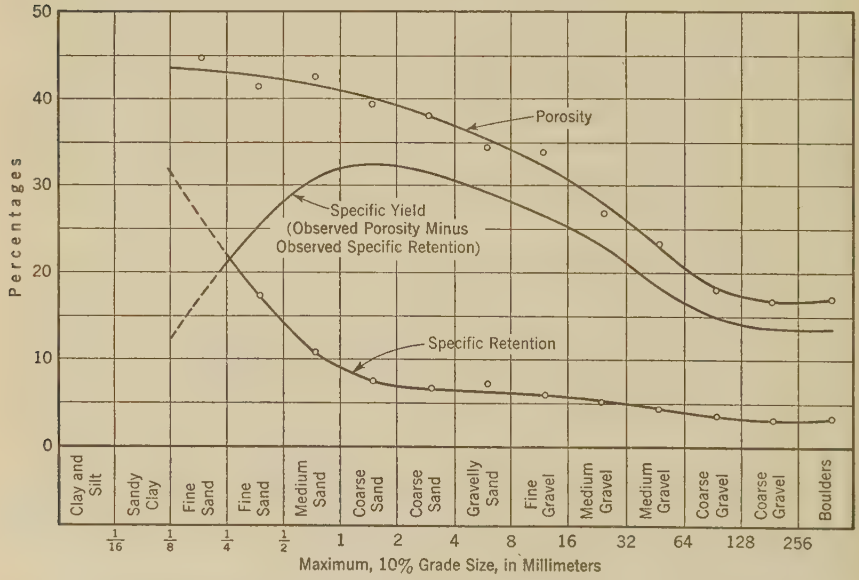


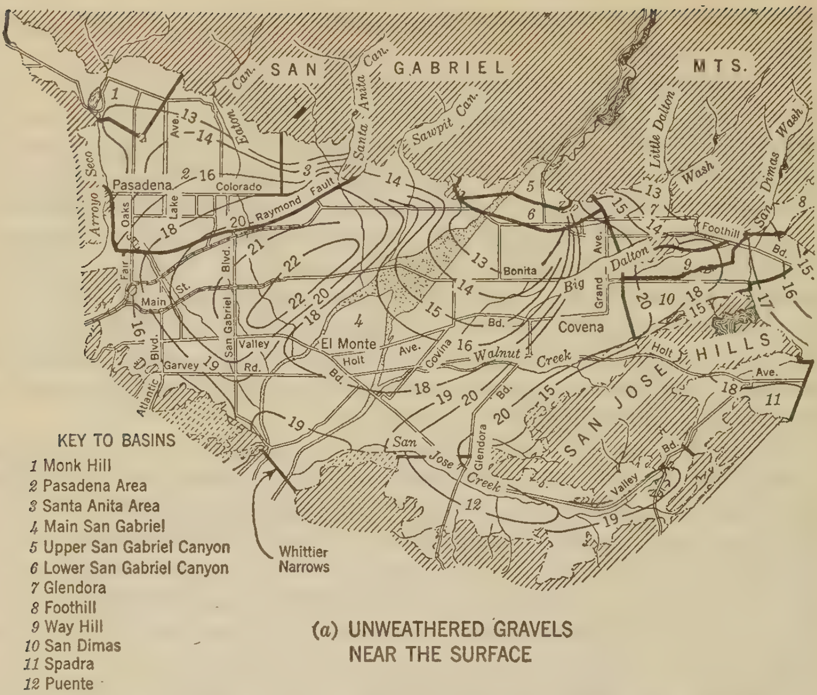
FIG. 4.—POROSITY, SPECIFIC RETENTION, AND SPECIFIC YIELD FOR SEDIMENTS OF THE SOUTH COASTAL BASIN

Adaptation in California.—Items 1 and 7, combined with much field work, are the basis of methods used for estimating safe yield in a large part of the coastal valleys of California south from San Francisco, by the State Division of Water Resources. In these valleys the water table is generally far below the surface. Supply to, and draft on, the ground water vary widely from year to year and data necessary to direct methods of determining safe yield are sometimes difficult to secure; or the methods may not be applicable in a large part of the area. In such cases specific yield must be known as an antecedent. In parts of the valleys, however, none of the aforementioned methods are applicable and other more roundabout procedures have been adopted although they are less satisfactory.

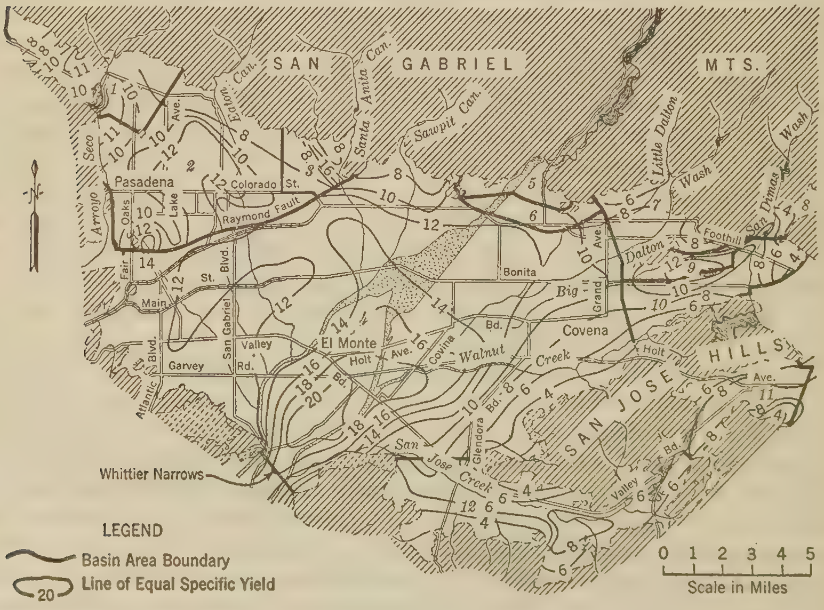
Laboratory methods naturally involve the application of the determination of specific yield at several points to the specific yield of a large mass and the procedure is open to objection from that standpoint; but this same sampling procedure is followed in determinations of many other physical properties. It happens that, in determining specific yield, it is possible to take advantage of other well-known physical phenomena for applying data secured at one point to the larger area. The result is to decrease materially the amount of work necessary, and to increase the accuracy. After several years of intensive work in the South Coastal Basin of California, notably in San Gabriel and Santa Ana river basins, procedures were evolved which may be applicable to the alluvium in other portions of the mountain and valley province where conditions are similar to those in coastal California (26).

A feature of this work was the objective of determining whether the slope of the debris cones formed by the streams could be used as a criterion for determining the variation in specific yield of unaltered surface material along radial lines from apex to distal portion, and whether the specific yield of unaltered surface material could be used as a criterion for the yield of altered similar material beneath the surface. It was thought that, if such relations did exist, a short cut would be available which could be used elsewhere in similar valleys under investigation with a minimum of field and laboratory work.

The study therefore involved laboratory determinations of porosity and specific retention of unaltered materials encountered in cut banks or well pits near the surface and at depth. It involved similar determinations of size of material at various distances from the mountains for streams of various sizes and various slopes of the surface. Fig. 4 shows the porosity and specific retention of a range of sands and gravels as determined by tests in the laboratory of more than 200 samples of unaltered material taken from near the surface (27). The curve labeled "specific yield" is the result of subtracting values in the "specific retention" curve from values in the "porosity" curve. It will be noted that even the finest of these materials is sufficiently permeable to yield water freely. As an easy method of determining the coarseness of the sample in the field the "maximum 10% grade size" is used. This is the grade size in which the cumulative total, beginning with the coarsest material, reaches 10% of the total sample and in these alluviums it is generally the next to largest grade size or sometimes the grade size next smaller. The maximum 10% grade size is the exact opposite of the "effective size" used by the late Allen Hazen, M. Am. Soc. C. E., in his investigations on sand filters which is the grade size in which the cumulative total beginning with the finest material reaches 10% of the total sample. For the entire South Coastal Basin a map showing lines of equal specific yield of surface materials was drawn and the part of it for San Gabriel Valley is shown in Fig. 5(*a*). Drillers' logs were studied and materials from wells in process of being drilled were tested so that it was found that certain expressions used by drillers such as "gravelly clay," for instance, referred to a material altered by decay from its original characteristics to a specific yield


SAN GABRIEL MTS.
SAN JOSE HILLS
Pasadena
El Monte
Covena
Whittier Narrows
Raymond Fault
Arroyo Seco
Eaton Can.
Santa Anita Can.
Sawpit Can.
Little Dalton Wash
San Dimas Wash
Big Dalton
Walnut Creek
San Jose Creek
Colorado
Foothill Bd.
Bonita
Holt Ave.
Valley Bd.
Garvey
KEY TO BASINS
1 Monk Hill
2 Pasadena Area
3 Santa Anita Area
4 Main San Gabriel
5 Upper San Gabriel Canyon
6 Lower San Gabriel Canyon
7 Glendora
8 Foothill
9 Way Hill
10 San Dimas
11 Spadra
12 Puente

(*a*) UNWEATHERED GRAVELS NEAR THE SURFACE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Raymond Fault
Arroyo Seco
Eaton Can.
Santa Anita Can.
Sawpit Can.
Little Dalton Wash
San Dimas Wash
Big Dalton
Walnut Creek
San Jose Creek
N
LEGEND
Basin Area Boundary
20 Line of Equal Specific Yield
0 1 2 3 4 5
Scale in Miles

(*b*) ALLUVIUM AT DEPTH

Fig. 5.—Lines of Equal Specific Yield, San Gabriel Valley, California

about one third that of the original material at or near the surface. Well logs were studied by groups and the specific yield for the group was placed at a point on the map at approximately the center of gravity of the well locations. Other criteria, such as decrease in porosity due to depth, and the steepness of cone, etc., were given weight. All this was used in drawing contours of equal specific yield for the material within the probable range of fluctuation of water table (see Fig. 5(*b*)).

It was found that distance from the mountains and the steepness of slope of the debris cone were useful as indicators of specific yield. The method resulting from this intensive study has been applied to some extent with a very small amount of laboratory work to other similar valleys in California. The results have not been tested by other means in those valleys. In the central San Gabriel Valley and in the Raymond basin area of San Gabriel Valley from which many of the samples analyzed were secured and in which much of the work was done, studies have indicated that the values for specific yield thus determined in those locations are sufficiently reliable. This method is well suited to a large part of the coastal valleys of the state and should be also suited to much of the entire mountain and valley province.

Direct Methods of Safe Yield Determination

The foregoing involves measurement of capacity for water storage between different elevations of the water table of the unit involved and this is analogous to a survey made to determine capacity of a surface reservoir. In some cases operation of a surface reservoir has shown that the surveys made to determine its capacity were in error and that actual capacity was considerably different. This has been indicated by the fact that, after allowing for losses and inflow, release of a certain quantity of water did not produce the change in elevation of the water table which, according to the surveys, it should have. In such a case safe yield can be determined from operational data and capacity can be ignored. In other words, safe yield may be determined directly without knowing capacity.

Similarly with underground reservoirs: Given a set of conditions which comply with the requirements, safe yield and other desired information can be determined. There are three variables concerned—supply, draft, and change in water-table elevation. The requirement is that one variable must be sufficiently uniform to be considered constant, thus leaving only two. The most convenient method is graphic by platting on rectangular coordinates.

Two cases will be examined by way of illustration:

Case I. Supply to the Ground-Water Unit Is Uniform Year by Year, or Period by Period.—The variables are (*a*) the annual change in water-table elevation, and (*b*) the annual draft.

Case II. Draft on the Supply of a Unit, Whether Surface or Underground, Is Uniform Year by Year, or Period by Period.—The variables in this case are (*a*) the annual changes in water-table elevation, and (*b*) the annual differences in the quantity of surface water retained in the unit, as found by measuring inflow and outflow.

In either case, even if the period of record includes only years in which the water table fell, or only years in which it rose, the points when platted will define the slope of a straight line which, prolonged, will intercept the line of zero change in water-table elevation. This value on the axis on which volume of water is platted is the safe yield in Case I. In Case II it is the quantity required to supply the demand, whereas the quantity retained as determined by the measurements of inflow and outflow is the safe yield.

For Case I the net supply to the ground water might be sufficiently uniform to be so assumed (*a*) under an irrigation project in a fairly arid country, (*b*) in an area where the supply is by lateral percolation from ground water at higher elevation, and (*c*) in an area where, even if the water reaching the overlying surface is variable from year to year, distance to the water table is so great and perviousness so small that annual variations in percolation of the surface supply are eliminated to a large extent. In general it could not be used in a region of fairly large precipitation during the non-growing season although special circumstances might make it applicable in limited areas in such regions.

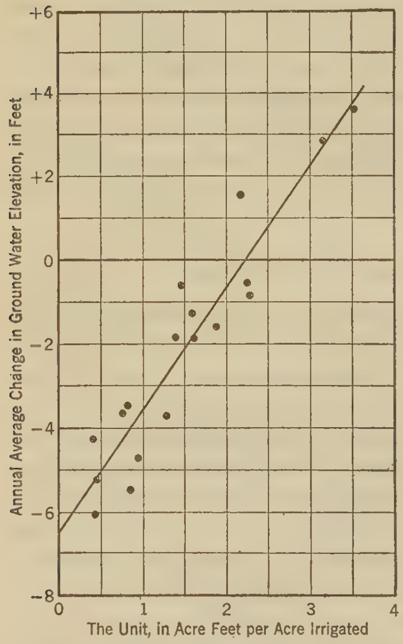

FIG. 6.—RELATION OF ANNUAL SURFACE INFLOW RETAINED TO CHANGE IN AVERAGE WATER-TABLE ELEVATION, TULE RIVER-DEER CREEK AREA

For Case II the net draft on the general water supply can be considered uniform in a unit where the irrigated area is fairly constant from year to year, and where deficiency in surface flow can be made up by pumping from the ground water. If precipitation on the surface of the unit were large and occurred in the non-growing season it might not be possible to use this procedure because of the great variability in apparent and actual consumptive use from year to year.

So far as is known to the writer, the Case I method was first used by Raymond A. Hill, M. Am. Soc. C. E., in work in Arizona. Similarly, the Case II method in this form was first used by S. T. Harding, M. Am. Soc. C. E. (17), in San Joaquin Valley and was used by the State Division of Water Resources in its studies for Central Valley Project. In both of these regions precipitation is small on the valley floors.

Fig. 6 illustrates Case II. It shows the surface inflow minus outflow per acre irrigated each year, platted against the average change in water-table elevation for that year, for the Tule River-Deer Creek area, which is one of the ground-water units of southern San Joaquin Valley (3)(8). The surface supply is from the two streams mentioned which head in the Sierra Nevada mountain range in eastern California. The flow is diverted by gravity and when deficient is augmented by pumping from ground water which is supplied by percolation from the streams from irrigated lands.

Underflow, either in or out, is unknown and is believed to be negligible; at least the difference is believed negligible. However, this is immaterial because the change in water table is the integration of all the forces acting on the ground-water supply. The net surface supply was sufficient in those years in which no change in water table occurred to supply the consumptive use plus or minus the net underflow.

The average area irrigated in the Tule River-Deer Creek area during the 18-yr period from 1921 to 1939, inclusive, was 67,800 acres and the maximum variation from this was about 10% plus or minus. The net average annual stream flow retained was 103,500 acre-ft or 1.53 acre-ft per acre irrigated. This is the safe yield for the 18-yr period. If 2.22 acre-ft per acre had been retained, the water-table elevation would have remained constant indicating that 2.22 acre-ft per acre (plus or minus any difference between underflow, in or out) is the net requirement or a total of 150,500 acre-ft for the 67,800 acres. Therefore, the average annual overdraft on the ground water of the unit for the 18-yr period is 47,000 acre-ft. The gross area of the unit is 239,000 acres and the water table lowered an annual average of 2.06 ft during the 18-yr period. Thus the soil volume drained was 492,000 acre-ft per yr and as the overdraft was 47,000 acre-ft the specific yield of the alluvium is 9.5%.

Whether the long-time average safe yield is 103,500 acre-ft is not disclosed by analysis of these data. To determine it would require a comparison of long-time average net surface inflow with that which occurred during the 18-yr period. A similar adjustment would be necessary with any other method. To make such adjustment would require a detailed analysis of the amount of percolation reaching the ground water from stream flow and precipitation during a long-time average period of precipitation to estimate the difference between that amount and the amount which did reach the ground water during the 18-yr period. Consideration would also be given to the changed amounts of underflow in and out which might occur with a changed elevation of water table.

In southern San Joaquin Valley the precipitation occurs in the winter and the annual average varies from 5 in. to 10 in. according to location. The direct contribution from precipitation to ground water may be regarded as negligible since the area with precipitation approaching 10 in. is small. Annual changes in water-table elevation at many wells in the various cones of the major streams or ground-water units have been recorded since the early 1920's and the average for the period of record can be calculated. Surface stream discharge into and out of each unit has been recorded. Underflow either in or out is probably negligible. The acreage irrigated each year is known approximately. In the particular unit under discussion, and in some other units, it has been fairly static for many years past.

In many situations large annual variations in supply or demand may be ironed out by using moving averages embracing sequences of several years. If this occurs it may be found that the methods of Case I and Case II will be applicable to the solution in particular areas, although not in regions of long swings in the precipitation cycle except perhaps in limited areas.

Dissolved Solids—Quality of Water

It is now known that not all the water of a stream system can be consumed in irrigation because the gradually increasing concentration of dissolved solids in it causes it finally to become so toxic to plant life that it cannot be used. A certain percentage of the water of a stream must be used to carry off the toxic material and to maintain a salt balance.

About 2500 years ago the great empires built upon the large irrigation developments of the Tigris and Euphrates river valleys deteriorated. It is surmised by some that these ancient empires retrograded because of lack of knowledge about the damage by dissolved salts to plant life; but, be that as it may, even as late as 1900 engineers in the United States knew so little about the matter that some irrigation developments (even in the part of the country where the geological formations are such that the moving water gathers large quantities of various salts) were designed without this difficulty in mind.

The problem is much less serious in some parts of the West than in others. Probably it is worst in west Texas, New Mexico, southwest Colorado, southwest Utah, Arizona, and the southeastern part of California. In some streams in other western regions there is little evidence of such toxicity, but the matter is one of degree. Water will dissolve some salts from all formations and, if the final residue of water is small enough, the solids will be sufficiently concentrated in it to be detrimental.

The same difficulty occurs with ground-water developments as with surface water but the likelihood exists, in the case of some ground-water developments, that the final disposal must be by pumping and exportation of sufficient water to carry off the dissolved solids.

Ground water suffers also in this respect from causes of concentration of solids which are not likely to exist in surface waters. Three examples may be cited:

1. Near the ocean beaches or other salt-water bodies, salinity may encroach into the pumping field if the water table is lowered sufficiently to reverse the normal gradient. This is not greatly different from the situation found in fresh-water streams for it is true also that salt water may encroach into such streams at their mouths and deteriorate the quality of the water in the lower reaches of the stream.

2. Well casings may be perforated incorrectly or they may be imperfectly set, allowing water with high concentrations of solids from a perched water table or other source to flow into the strata from which the wells of a field are drawing.

3. The ground-water basin was at one time a part of the ocean. Salt water was trapped in it and there remains below the fresh water because of its greater weight. As the weight of fresh water on it is lightened by pumping from it, and especially at the cones of depression and drawdown due to such

pumping, the salt and fresh waters tend to be mingled and difficulty occurs (2)(25).

Once the aquifers of a ground-water unit are impregnated with salt water it may be years, even with cessation of pumping, before the quality of water will be restored.

Movement of Ground Water

The rate of forward movement or flow of ground water would seldom be important to the purpose of this paper, because in most cases pumping is widely distributed over the surface of the ground-water unit and except in pressure areas the recharge is from the immediate surface. If the recharge is not predominantly vertical it may be possible to use the direct determination of safe yield by Case I without knowledge as to transmissibility. However if the recharge is largely lateral and if a part of the supply comes from a source in which concentration of dissolved solids is high (such as might be the case with a ground-water unit along an ocean beach) it would be found desirable to determine only the amount of the fresh-water supply. This could be done either by calculating the rate of movement of such supply into the ground-water unit (the cross section of the flow being known) or by calculating by other means the volume which must be entering the unit. Either method leaves much to be desired.

One important aid in determining direction of movement and influence of any one source of supply in any particular part of a ground-water unit is often the dissolved solids in the ground water and this matter should be given consideration in all ground-water studies (2)(25). However, the importance of this decreases as the slope of the water table steepens. If there are no discontinuities in the water table its slope should be a more definite indicator of direction of movement than the solid constituents of the water and their proportions. It would not, however, tell the amounts of water from each of two or more sources which reach the area of extraction. This would be best answered by determination of the dissolved solids.

Summary

This paper has sought to accentuate the fact that ground water is merely water beneath the earth's surface and that surface water is merely water on the earth's surface. The same drop of water moves from above the surface to below the surface and vice versa. Whether above or below, it follows the same physical laws at all times. Consideration of this fact should remove a part of the mystery which sometimes seems to surround the subject of ground water. The facts concerning movement of water underground are more difficult to obtain than those for water on the surface and this has caused less definite application of well-known principles than has been the case with water on the surface, but accumulating knowledge is giving methods for, and sureness in, dealing with ground water. This paper has presented, briefly, some of the values found and methods so far achieved, keeping in mind always that a solution of any water problem of the nature discussed in this paper is accomplished by merely substituting correct values in a simple equation. It is the values which are often difficult to determine correctly; but as data accumulate this difficulty decreases.

Ground water in alluvium is moving as a stream but at the same time the analytical approach to determination of yield is often, probably generally, best made if the ground-water body is treated as a reservoir. Regardless of this a clearer view will be had if the conception of it as a stream is not lost.

Each ground-water problem is unique in some of its elements. Even though the field has been very narrowly delimited, it is still so broad that it cannot be handled with anything approaching adequacy in a paper such as this; but, if the paper has served in any way to outline the possibilities in ground-water utilization and will promote discussion of the general engineering approach to such utilization, it has served its purpose.

APPENDIX

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DISCUSSION

Raymond A. Hill,[3] M. Am. Soc. C. E.—It might be inferred from the author's presentation of the subject that the safe yield of any ground-water unit is a definite quantity, as it is in the case of any normal surface reservoir formed by construction of a dam across a flowing stream. The safe yield of such a reservoir can be defined as the maximum rate at which water can be withdrawn continuously between times of filling, because all of the inflow is captured, until the reservoir fills and overflows, and losses are restricted to evaporation from the water surface and nominal leakage.

In the case of a subsurface reservoir, however, the proportion of the contributing supply which is captured and the magnitude of losses through evaporation and leakage are dependent in large degree upon the depth at which the ground water is held below the surface; this depth in turn is dependent upon the draft which is made on this reservoir. The safe yield—that is, the rate at which water may be withdrawn from any ground-water basin for use within or without the boundaries of the basin—is thus not a fixed quantity but a variable. Under any given set of conditions it is only necessary that the following general equation be satisfied:

Primary Equation of Hydrologic Equilibrium.—

$$\left.\begin{array}{c}\text{Surface inflow across the}\\ \text{boundaries of the basin}\\ \\ \text{plus}\\ \\ \text{Subsurface inflow across the}\\ \text{boundaries of the basin}\\ \\ \text{plus}\\ \\ \text{Precipitation within the}\\ \text{boundaries of the basin}\end{array}\right\}\text{ equals }\left\{\begin{array}{c}\text{Surface outflow across the}\\ \text{boundaries of the basin}\\ \\ \text{plus}\\ \\ \text{Subsurface outflow across the}\\ \text{boundaries of the basin}\\ \\ \text{plus}\\ \\ \text{Evapo-transpiration losses within}\\ \text{the boundaries of the basin}\end{array}\right.$$

The magnitudes of the terms on the left side of this equation are substantially independent of any condition or change in conditions within the boundaries of a ground-water basin; consequently, development of a supply of ground water generally requires a change in either the magnitude or the composition of one or more of the terms on the right side of the equation. If the unused surface outflow is decreased, then there can be a corresponding increase in the consumption of water within the basin, or an equal amount can be exported for use elsewhere. In like manner, exportations or greater use of water within the basin can be offset by restriction of nonbeneficial evapo-transpiration losses or by reduction of the subsurface outflow. The latter necessarily requires that the ground-water level be lowered within the basin except in the rare instance where subsurface outflow is cut off or reduced by construction of some barrier.

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[3] (Leeds, Hill, Barnard & Jewett), Los Angeles, Calif.

The term "safe yield," as applied to a ground-water basin, has frequently been defined as that quantity of water which can be extracted each year without progressive and permanent lowering of the ground-water level. Although this definition is applicable to alluvial valleys along a stream where the safe yield is limited only by the rate of channel absorption and by the time during which the flow of the surface stream is much greater than the rate of withdrawal from the ground-water supply, it is too restrictive when applied to closed basins and to those where the ground water is not normally near the surface.

In neither of these latter cases could any quantity of water be withdrawn continuously without causing permanent but not necessarily progressive lowering of the ground-water level. The following general definition of safe yield is therefore suggested:

> "The safe yield of a ground-water basin is that quantity of usable water which can be extracted therefrom each year without progressive lowering below the level at which the same quantity can be obtained at reasonable cost."

Determination of Safe Yield.—In general, the determination of the safe yield of a ground-water basin under any set of physical and economic limitations can be made by evaluation of all terms in two primary equations of hydrologic equilibrium: One for the conditions which prevailed in the past and another for the conditions assumed for the future. This method, which evidently is preferred by the author, is necessarily subject to considerable error, because second differences as well as first differences between quantities of relatively great magnitude must be computed.

The problem may frequently be simplified by eliminating all factors which do not affect directly the ground-water supply. The following general equation is then applicable:

Secondary Equation of Hydrologic Equilibrium.—

Absorption of surface inflow within the boundaries of the basin		Subsurface outflow across the boundaries of the basin
plus		plus
Subsurface inflow across the boundaries of the basin	equals	Exportations of water across the boundaries of the basin
plus		plus
Absorption of precipitation within the boundaries of the basin		Evapo-transpiration losses within the boundaries of the basin

Most of the terms on the left side of the secondary equation are subject to less variation than the corresponding terms in the primary equation. For example, although absorption of surface inflow within the boundaries of the basin must equal the difference between surface inflow and surface outflow across these boundaries, it can be computed from measurements of the wetted channel area of the streams and observed rates of percolation. Likewise, it is generally true that the rate of absorption of precipitation is limited and that, when the intensity of rainfall exceeds such observed rates, all the excess passes out of the area in the form of surface runoff.

TABLE 1.—Hypothetical Basin (Area 10,000 Acres)

Successive years	Acre-Feet		Feet
	Annual contribution less outflow[a]	Annual quantity pumped[b]	Change in water level during year
1	17,000	8,000	+ 9.0
2	15,000	20,000	− 5.0
3	13,000	6,000	+ 7.0
4	17,000	16,000	+ 1.0
5	15,000	22,000	− 7.0
6	13,000	10,000	+ 3.0
7	15,000	18,000	− 3.0
8	13,000	4,000	+ 9.0
9	17,000	12,000	+ 5.0
10	15,000	14,000	+ 1.0

[a] Total contributions to ground-water supply minus total natural outflow vary from 13,000 to 17,000 acre-ft per yr. [b] Pumping draft on ground-water supply varies from 4,000 to 20,000 acre-ft per yr.

Direct Methods.—In many cases there are more direct solutions of the problem, which take advantage of the fact that contributions to and losses from a ground-water basin are not characterized by the wide variations typical of other hydrologic phenomena. Such direct methods, described briefly by the author, involve correlation between changes in the ground-water level and observed variations in either the contribution to or the extraction of water from the basin. It is believed that they have much wider application than is assumed by him. The direct method developed by the writer (Case I of the author) may best be demonstrated by assuming, first, a hypothetical case and then showing the result of certain actual determinations of safe yield of ground-water basins. Such a hypothetical case is given in Table 1.

In Fig. 7(*a*) the observed change in the average depth to water throughout this hypothetical basin is plotted against the corresponding annual amount of water pumped. The curve which best fits the plotted data is evidently a straight line. The points that fall on the straight line correspond to years in which the annual net inflow equals the average net inflow. The points that fall to the right of the straight line correspond to years in which the net inflow exceeded the average, and the remaining points which fall to the left of the straight line are for those years in which the net inflow is smaller than the average.

The departure of such points from the straight line is equal in each case to the difference between the actual net inflow and the average net inflow. The intercept with the x-axis therefore indicates the safe yield, which in this case is 15,000 acre-ft per yr.

In Table 2 the relationships are given between actual fluctuations of the ground-water level in the Pasadena basin in Los Angeles County, California (Fig. 3), and the quantity of water pumped from that basin from 1922 to 1938. Each point in Fig. 7(*b*) represents the average annual amount of water pumped for five consecutive years in relation to the average annual change in water level during the same five years. Progressive 5-yr periods were taken covering the entire number of years of record to smooth out annual variations in supply.

The area of this basin is 11,900 acres, bounded on the north by a line extending generally east from Arroyo Seco below Devils Gate and Monk Hill.

The area immediately tributary to Santa Anita Wash is excluded, as is the southwest corner lying between Eagle Rock Fault and Raymond Fault.

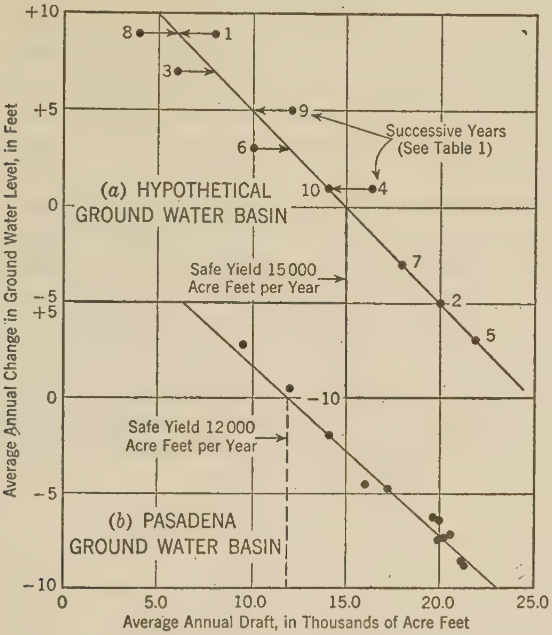

FIG. 7.—DIRECT METHOD FOR THE DETERMINATION OF SAFE YIELD

It is significant in this actual case that the plotted points lie substantially on a straight line and the greatest departure from that line does not exceed 1,000 acre-ft. It must be concluded, therefore, that the net inflow to the Pasadena basin is substantially constant. The straight line intersects the x-axis at 12,000 acre-ft per yr, as compared with the average pumping of about 18,500 acre-ft. Consequently, the safe yield—that is, the difference between the contribution to the ground-water supply and natural outflow and losses from that supply under the conditions which prevailed from 1922 to 1938—was a little less than two thirds of the draft.

TABLE 2.—HISTORICAL RECORD OF PASADENA (CALIFORNIA) BASIN (AVERAGE ANNUAL VALUES)

Successive periods[a]	Quantity pumped (acre-ft)	Change in water level (ft)
1922 to 1927	19,705	− 6.2
1923 to 1928	20,088	− 7.3
1924 to 1929	19,939	− 6.4
1925 to 1930	20,529	− 7.1
1926 to 1931	21,086	− 8.5
1927 to 1932	21,210	− 8.7
1928 to 1933	20,919	− 8.3
1929 to 1934	19,779	− 7.4
1930 to 1935	17,233	− 4.7
1931 to 1936	16,147	− 4.5
1932 to 1937	14,205	− 1.9
1933 to 1938	11,979	+ 0.4

[a] Five-year moving average.

The data pertaining to the Pasadena basin illustrate a single condition of equilibrium. In Fig. 8 the data are plotted corresponding to two different conditions for a ground-water basin located along a major fault in Southern

California. Prior to 1927 the safe yield of this small basin was approximately 3,100 acre-ft per yr, but since 1930 an average of only 1,700 acre-ft annually could be withdrawn without progressive lowering of the water level over a typical series of wet and dry years. The shift from one condition of equilibrium to another in the foregoing case was probably caused by some change in natural conditions, presumably by movement along a major fault in 1927.

Similar reductions in the safe yield of the Cortaro basin along the Santa Cruz River in Arizona were disclosed by correlating the amount of pumping from wells with changes in ground-water levels. The land overlying the ground-water basin was irrigated for a number of years, and the return flow from the application of water on the fields reduced the effective draft upon the ground-water supply. Subsequently, farming operations were shifted to another part of the project, and the water pumped from the wells was collected in canals and exported from the basin. The return flow was then limited to leakage from the canals, and lowering of the water level quickly took place. Later, to conserve the supply, the canals were lined with concrete, thereby increasing the net draft upon the ground-water basin. This again resulted in a reduction in the amount of water which could be pumped without progressive lowering of the ground-water level.

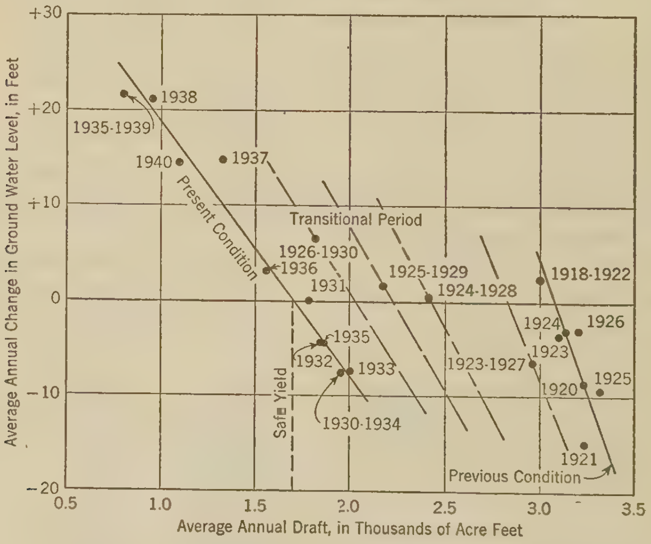

FIG. 8.—CHANGE IN THE SAFE YIELD OF A SMALL GROUND-WATER BASIN

Determination of Specific Yield or Area.—The specific yield or the area of a ground-water basin, when either is known, can be determined by the direct method from the slope of the straight line which best fits the plotted points. In the case of the hypothetical basin described in Table 1, a change of 1,000 acre-ft per yr in the rate of withdrawal produced a corresponding change of 1 ft in the average ground-water level. The area of this hypothetical basin is 10,000 acres, so that the gross displacement for a 1-ft change in level was 10,000 acre-ft; consequently, the specific yield (that is, the proportion of drainable voids) was 10%.

In the case of the Pasadena basin the area is 11,900 acres, and the average specific yield of the alluvium in this basin is 9.4%, as measured by the rate of change in level compared with the rate of change in draft.

In some cases the actual extent of the ground-water basin, or at least the zone of influence of pumping from the basin, is not known. The effective area can be approximated, however, by assuming a reasonable value for the average specific yield. An extreme case in point was a ground-water basin in Arizona, which was generally thought to extend over only about 1,000 acres. When the quantity of water pumped from this basin was plotted against the corresponding changes in ground-water level, the slope of the curve fitting the points was so flat that the specific yield would have had to be more than 50% if the area of the basin were only 1,000 acres. No natural alluvium has gross voids as large as 50%, and there are few instances in which the drainable voids exceed 15% of the total volume displaced. Obviously, in this case, the ground-water basin extended beyond what had been assumed to be its boundaries, and the effective area of the subsurface reservoir must have been almost 4,000 acres.

Summary.—The direct method of determining the safe yield of a ground-water basin necessarily depends upon the availability of records of performance for that basin under known conditions. However, most of the available supplies of ground water have already been tapped and used sufficiently to establish the relation between rates of withdrawal and changes in level. Even if it is found on trial that the variations in supply from year to year are such as to make indefinite the draft at which there would be no change in level under average conditions, these variations can be smoothed out by taking periods of time longer than one year. Essentially the method involves only recognition of the fact that continuous pumping in excess of the average net supply to a ground-water basin will produce progressive lowering, and that conversely any reduction in draft will cause the ground-water level to rise to some higher stage of equilibrium.

In the application of the indirect or difference method favored by the author, there is the same need for historical data. Furthermore, these data must be more accurate than those required for direct determination, because of the multiplication of errors resulting from the computation of small differences between large quantities, each subject to observational error.

Where historical records are not available, any determination of the safe yield of a ground-water basin can be only speculative between the reasonable upper and lower limits indicated by experience and sound judgment.

Clarence S. Jarvis,[4] M. Am. Soc. C. E.—A welcome and practical addition to the voluminous literature on the subject of ground-water storage is presented in this paper. The author's contribution in clarifying the basic concepts and translating them into familiar, readily understandable terms should benefit a wide circle of readers who must have awaited such a simple and logical approach.

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The comprehensive title would seem to invite consideration of the entire field of ground-water storage and utilization, without regard for any specified limitations, either geographical or functional. Viewed broadly, projects involving domestic stock watering, industrial water supply, municipal water supply, and irrigation are all dependent on ground-water storage. In addition, ground-water storage is important in a still broader field, governing integrated stream flow in the intervals between storms—namely, the maintenance of dependable navigation depths and power output. Essentially the same basic principles and physical laws govern the storage and withdrawal of ground water from any and all basins, whether the immediate purpose is to maintain irrigation in the semiarid regions of the Southwest, or to provide municipal supplies along the Atlantic seaboard. The wide range of behavior reflects the variety and integrated effect of modifying conditions.

The familiar example of depressed ground-water surface, due to overdraft by pumping for irrigation in the Santa Clara Valley of California, has much in common with the water supply situation of the Amboys in New Jersey. Rapid industrial expansion in Perth Amboy and South Amboy in 1917 and during succeeding years sorely overtaxed the supplies pumped from wells near Runyon, N. J., adjacent to the Raritan Bay and estuary. These wells were therefore subject to salinity intrusions. The installation of a pumping plant far enough up the estuary to avoid salt water insured the delivery of a bountiful water supply via wood-stave pipe to the Runyon area for the purpose of replenishing the depleted ground-water reservoir. Thus, the threatened contamination by sea water following excessive drawdown was thwarted.

Somewhat similar experiences attending overdraft of water supply wells have been noted from New England and Long Island to Florida and to the west. The remedy most often applied is a reduced pumping demand to permit natural replenishment. What part of the total volume supplied in this manner along the Atlantic and Gulf Coast regions has been diverted for such irrigation purposes as lawns, a large variety of domestic gardens, choice orchards (occasionally), market gardens, and seed farms?

Supplementary Irrigation in Humid Regions.—A legislative directive (42) required examination of the principal river systems of the United States with a view to formulating a coordinated plan for each system that would best serve the public interest with due consideration of the fourfold uses—navigation, power development, flood control, and irrigation. Some of the early reports disposed of the last-named with some such statement as the following: "Inasmuch as the normal precipitation is well distributed throughout the year and ample in amount for the maturing of standard crops, irrigation is not required." As a consequence, municipal and industrial water supplies were ordinarily substituted for irrigation in humid regions. Then came the years of drought in the early 1930's, causing widespread damage to crops, or their total loss. Wells and springs that had never failed became dry.

It was the writer's privilege to tour the outstanding agricultural regions from New York to the tidewater areas of Maryland and Virginia during the final days of the two successive years of pronounced rainfall deficiency, 1930 and 1931. The devastation in some of the most fertile and usually productive farming areas was quite reminiscent of scenes the writer had viewed in the semiarid and desert regions of the West when the water supplies, control structures, or distributing systems failed. One outstanding difference was noted: Here and there a field had grown and matured standard crops of at least medium quality and tonnage, in the midst of the widespread blight and ruin. These exceptional crops had been planted early enough to establish strong root systems before the period of drought and accompanying intensified summer heat had seared the more tender sprouts of later plantings. The declining water table and associated capillary fringe had evidently been followed to the increased depth by the strongly established root systems of the early plantings, whereas crops on adjoining fields gradually wilted beyond recovery.

During the week or more between preliminary wilting and utter ruin of the standard crops, many farmers missed golden opportunities to save their crops by hastily installed irrigation systems. With fresh-water supplies a fraction of a mile from the choicest fields, and with portable pumps to elevate the water from natural stream channels or from existing wells, lines of pipe or flume, or even backfurrowed ditches, would readily have conveyed a generous irrigation supply to the most accessible and the choicest areas to insure bounteous crops instead of total failures. The drenching rains that followed the months of drought, representing nearly the year's accumulated deficiency at some stations, were weeks too late to restore the wilting crops, some of which had been totally lost by neglect of irrigation resources within reach.

Opinions contributed by the writer and others familiar with the hydrology of the eastern states—which recognized the inviting field for supplemental irrigation not only during severe drought periods, but also during minor shortages such as may be expected within one month or more of the growing season, during years of normal annual precipitation—have gradually gained recognition and standing. The same inertia against the practical need for irrigation in humid regions as that quoted previously has operated against the incorporation of progressive views (42). Likewise, opinions in rural communities were so disinterested that the farmers were not alert, or prepared with the necessary equipment, when severe drought conditions demanded prompt and effective measures to save and insure growing crops. Many similar situations have been met, through ingenious adaptation and utilization of equipment already at hand, when the normal gravity supply in arid regions suffered interruption. For example, power from tractor, truck, or even automobile engines has operated centrifugal or other pumping machinery, thus supplying some of the most valuable growths (often including the orchards, vineyards, and selected field or garden crops) from ground-water reservoirs or sometimes from surface storage.

The several tens of thousands of acres along the Atlantic Coast devoted to market gardening, seed culture, and floriculture, together with the smaller, more numerous tracts that have been equipped by individual farmers to supply stand-by service during the inevitable months of subnormal rainfall, have apparently proved the benefits of irrigation in humid regions beyond challenge. The provision of sprinkling systems for lawns and shrubs in public park systems in all metropolitan districts of the United States, and the customary restrictions imposed on the use of city water for gardens and lawns in midsummer, in order to conserve pressure and volume for fire protection—both testify to the widespread need for supplemental irrigation throughout the United States. In Washington, D. C., the meticulous attention to the stately trees and to the park system, by copious watering to prevent midsummer defoliation, is both impressive and praiseworthy.

Considering specialized agricultural, horiticultural, and floricultural projects, which demand irrigation facilities, even in humid regions, to meet the inequalities of rainfall distribution and to maintain optimum soil-moisture conditions throughout the growing season, it must be admitted frankly that irrigation is almost as requisite for special crops in the eastern states as it is for general purposes in the southwestern region. Undoubtedly, with supplemental facilities for utilizing ground-water resources, both the quantity and the quality of orchards, gardens, and field products may be improved.

Coincidental with reliance on ground-water supplies to increase crop tonnage and marketability, normally each year interest in soil and water conservation would be awakened and stimulated. Practices and devices for decreasing surface runoff, or for practically eliminating surface runoff on both farm and woodland, have progressed to the extent that engineers recognize such procedures as both effective and profitable. Under the complete program as prescribed for retaining and utilizing the rain where it falls, rising water tables and augmented base flow in drainage channels have resulted in hundreds of localities. Whether the increased opportunity for infiltration is caused by terracing, contour plowing, check-basin development, grass sod or other vegetal cover, diversion into open cuts, sand or gravel pits, or abandoned quarry sites, volumes of water, many times greater than the capacities of the foregoing, may be successively detained and gradually conducted to the underground storage.

Man's progress in the solution of ground-water problems and in the thorough utilization of water resources has been evident mainly in the field of recharging or replenishment. The location, tapping, and conveyance of dependable streams from infiltration galleries seem to date back to the reigns of Darius and Cyrus of Persia in the sixth century, B. C., or to an earlier period (43).

The Quest for Ground Water.—Somewhere between a horizontal plane and the surface of talus or alluvial slopes flanking a river valley, the surface of saturation or the ground-water table is ordinarily located. In many instances, it approaches the position of the bisector of the angle of talus slope; or, in rare cases, it inclines in a reverse direction as if infiltration from the river channel were diverted outside the basin. Some of the great springs from limestone formations in Texas presumably are supplied in this manner, with no regard for surface topography. The remoteness of these springs from intake areas helps to insure against pollution; successive filtration and sedimentation along the underground courses seem to remove organic material, although hardness may increase progressively through contact with readily soluble minerals along the system of solution channels at varying depths underground. When may one hope to see patterns of underground channels delineated on maps that show the surface drainage channels superimposed, using distinctive symbols or colors for the separate systems? When such maps are available, hydrologists may have a logical explanation for the disparity in runoff yields of apparently similar tributaries, with nearly equal drainage areas and observed rainfall. Subdrainage systems may subtract the underflow from one basin and add it to the base discharge of a neighboring tributary.

In the nearly desert regions of the Southwest or any other semiarid region, what would an isolated traveler seek as an indicator of dependable ground-water supply? Some species of cactus might afford enough sap moisture to quench the first edge of thirst; but a zone of saturation would not likely be found within range of cactus roots. Better indicators would be rabbit brush, grease wood, or, best of all, tufts of willow growth. Likewise, some lush and prominent patches of bunch grass, reeds, rushes, or even salt grass might be within a few feet of the water table.

Structural and geologic, as well as geomorphic, features furnish just as positive and simple indicators of ground-water occurrence to specialists in that field. What thrilling chapters are yet to be written concerning the accomplishments of these indicators on every recent invasion and battle front in locating plentiful supplies of potable water! What enrichment both the graphical and the textual treatment of such practical problems, with approved solutions, will provide!

For ages, open wells and trench cuts leading to water tables even in the fine-grained alluvium of such valleys and deltas as those of the Nile River in Egypt and the Ganges River in India have served as dependable sources of irrigation supply. Whether the water is elevated to the field surface by human labor, animal power, windmill, or other mechanical devices, the expense must have been justified by resulting crop yields or the practice would not have spread and survived. Strange as it may appear at first sight, such irrigation may bring much richer returns in fairly humid regions than in distinctly arid ones. A single application of water to a depth of from 3 in. to 6 in. may suffice for the purpose of supplemental irrigation, whereas five times such depths ordinarily would be required for the growth and maturing of standard crops in desert surroundings. In the latter case, the power charge may be increased more than the volume of water required, as the water table recedes under such heavy demand and output throughout the long season of growth, or even throughout the year. Eventually there must be a reckoning, a balancing of accounts; either the pumping demand must be regulated or the intake increased by some positive and dependable means.

What portion of the cost entailed in steep mountain terracing, with successive units spilling into the next terrace down the slope, is represented by increased base flow from springs and seeps? Likewise, what portion of this cost is repaid by stabilized earth mantles, stream beds, and adjacent alluvial plains on which the increased base flow is utilized? Surely the parallel retaining walls on steep slopes represented an investment of more than the actual value of the narrow terraced strips; only an integrated program of soil and water conservation, stream regulation, flood protection, and promotion of the public welfare could have justified such projects.

Unmistakable evidences of rock wall barriers and earth terraces to insure the detention of rainfall for the immediate benefit of growing crops and accessions to ground-water storage have been identified both in northern Africa and in Palestine. The more ambitious and comprehensive terracing, still used in favored regions of oriental countries, the East Indies, and the Mediterranean areas, seems to have been equaled by similar works on the Andean slopes of Peru, Bolivia, and neighboring South American countries centuries ago.

For miles beyond some of the oases in desert regions the water table is within practicable limits for mechanical pumping, although it may be beyond the reach of native vegetation. This fact may account for the dependable water supply in the central area constituting the actual verdant oasis; the insulating blanket of desert earth conserves the underground storage for a series of years to provide a dependable supply for the limited and favored area in and about the oasis.

The monumental wells of Asia Minor, with helical strairways or inclined ramps providing access to the water table and supply pool, have been unfailing sources of water for the replenishment of rawhide bags and water jars for centuries; the variations between wet and dry cycles, if such have occurred, were registered volumetrically in the outlet springs and seeps wherever the descending water table outcropped. The failure of such springs is merely a signal that the time has come for replenishing the underground storage by conserving the usual wastage.

Within the Colorado River basin in the Southwest, the precipitous faces of the bold vermilion cliffs and other sedimentary formations abounding there are monuments to the action of ground water. Accelerated weathering and crushing, within the saturated strata outcropping at the base, account for the successive slicing and arching along the overhanging faces, from which falling masses supply shattered stone, placed within reach of the river. It is a fairly debatable question whether the steady pressure from within the sedimentary ledge masses, due to ground water, is more or less potent than the attrition and erosion caused by the turbulent river in carving the canyon sections (of course, reserving the actual transportation along the channel to stream flow—both water and wind).

Although the ordinary problems of ground-water occurrence may be essentially simple and understandable, those associated with faults and other deep-seated crustal disturbances may long await explanation. Note, for example: (*a*) The occurrence of distinctly ferrous, sulfur, and other mineral springs of varying temperatures within an area of one square mile on the slope of Mount Rainier, in the State of Washington, not far from Paradise Valley; and (*b*) the hot sulfur springs along the trace of the Wasatch Fault in Utah, emerging only a few miles from a cold spring in a lava bed, where ice forms and remains every day in the year until harvested by visitors in need of such a commodity. The escape of air under pressure from subterranean caverns, attended by rapid expansion at the lava vent within a naturally roofed rock basin, accounts for the constant refrigeration.

Other interesting questions pertain to the development of great caverns in limestone formations, with successive chapters distinctly punctuated by changes in the rate or in the manner of channel enlargement, or by alternate solution and deposition of calcareous material—as if token payments were an essential part of the plan. Every travertine deposit around thermal springs surely represents material transported from underground caverns or solution channels in the making. Also, the successive horizons in great caverns such as the Carlsbad Caverns in New Mexico, where fossilized lily forms predominate, must represent partial repayment in kind for materials previously removed by solution processes to be precipitated later in pools which either cooled or shrank in volume to the saturation point.

The surface drainage channel terminating in the original bat-cave discovery portal of Carlsbad Caverns affords a striking illustration of the interlocking of physical sciences such as ecology (dealing with the adaptation of plants and organisms to their surroundings) and the restricted meteorologic and hydrologic resources prevailing on top of the Guadalupe Mountains in New Mexico. There, on the naked, shattered outcroppings of highly soluble, laminated limestone, practically all vestiges of soil are in the interstices and crevices of ledge rock, and the mean annual rainfall of some 13 in. is subject to extreme variations of 10 in. above or below the mean. Nevertheless, the familiar prickly pear of the desert is apparently at its best, yielding heavy fruitage in spite of the dearth of both soil and moisture and the remote depths to ground-water tables. In the same location, the native American black walnut, normally dependent on access to abundant ground water, has degenerated to the form of a bush or shrub. It bears perfectly formed walnuts, septum divisions and all, which retain the characteristic flavor but have pygmy dimensions, comparable with those of service berries and wild currants.

Perhaps the conservation habits of the most distinctive desert vegetation may indicate at once the more positive replenishment, and more nearly complete utilization, of ground-water storage. The stomata seem to be ever ready to receive the slightest trickle of moisture from the smooth, wax-like surfaces of desert foliage, and storage is maintained with minimum losses for weeks or even months between showers. As is well known, the changes made by man on the land surface, whether by deforestation, overgrazing, or exposing cultivated soil to ruinous erosion through wasteful practices, have all tended toward accelerated runoff through fouling or sealing the porous soil structure. The counterpart and inevitable result of such accelerated runoff is decreased infiltration opportunity, as well as capacity, thus robbing underground storage of its rightful quota.

Ground-Water Conservation.—Wasteful practices with regard to ground-water resources are best illustrated by reference to artesian areas. While engaged in assembling basic material for an official report (44), the writer noted a definite reluctance on the part of landowners to turn off artesian wells for periods of nonuse. The objections seemed to be well founded, from the viewpoint of the landowners, since a definite decrease in discharge had been observed upon resumption of flow from a well that had been idle. The item that was almost universally disregarded was the inertia of the water column. Some time is required for available pressure to start the entire column within a deep well and to accelerate motion until a steady normal discharge approaches the maximum attainable.

Artesian wells were left uncapped and continued to waste the underground storage and incidentally to waterlog the adjacent farm or pasture areas, in spite of the earnest pleas of advisers for agricultural college extension courses and other officials. Three decades were required before the Utah State Legislature enacted the necessary regulatory laws to correct the situation for coming years, after most of the pressures and the artesian reservoirs had become seriously depleted.

Similarly, the great artesian basin in Australia declined 40% in the 30-yr period, from 1909 to 1939, despite the fourfold increase in the number of wells—with capacities of from 563 million gal to 340 million gal daily, or from 870 cu ft per sec to 525 cu ft per sec (45). What opportunities await the diversion of the surplus waters that escape to the sea, practically unused except for occasional power developments, to rejuvenate the depleted aquifers of the interior (46)!

One of the deterrents against direct diversion of storm runoff into porous underground formations, such as gravel, sand, or checked and laminated sedimentary ledges, using open cuts, pits, and well shafts may be the fear of pollution. The replenishment of dwindling underground storage must be weighed against possible contamination. No one is likely to claim that flood runoff from ordinary land surfaces could escape the pollution which is generally associated with pathogenic bacteria. It is yet to be determined under what conditions and at what distances from a well site the diversion of surface runoff directly into an aquifer would be safe.

Recently it was required to test the safe yield of an underground storage reservoir with a view to increasing the output through new wells included in the municipality's plans for the coming season. The novel test imposed was to release the accumulated storage from one of the city's large elevated tanks directly into some of the existing wells. Since this added volume did not change the elevation of the water table perceptibly, it was concluded that the underground reservoir was extensive and capable of supplying the increased demand. This fact was later demonstrated. More of these practical, understandable, and direct approaches to hydrologic and ground-water problems are needed.

Conclusion.—The author has contributed much to the formulation and simplification of the fundamental concepts of ground-water occurrence, safe yield, depletion with overdraft, and replenishment by either the necessary reduction of output or a corresponding increase of input. Finally, he has stressed the importance of long-range planning to insure utmost beneficial use, as well as security against serious shortage of supply. Increased recognition awaits wisely planned developments and research in this field. The field was never more inviting.

JOHN R. CHARLES,[5] ASSOC. M. AM. SOC. C. E.—The author has restricted consideration to the unconsolidated and sand gravel deposits of the "mountain and valley" ground-water province and to the Tertiary and Quaternary alluviums in the Great Plains area, under or near the major streams. However, the fundamental concepts are equally applicable to unconsolidated sands and gravels wherever these occur. The writer will attempt to illustrate how the fundamental equation can be applied to most ground-water supplies obtained from unconsolidated deposits. The discussion is restricted to a consideration of unconsolidated deposits, which constitute the sources of a large proportion of all ground-water supplies in the United States.

It has not been generally appreciated (see heading, "Summary") that

> "* * * ground water is merely water beneath the earth's surface and that surface water is merely water on the earth's surface. The same drop of water moves from above the surface to below the surface and vice versa. Whether above or below, it follows the same physical laws at all times."

If the author has done nothing more than to help establish the fact that surface water may not be wholly disconnected from the ground water, his contribution has been immense. If the author has illustrated how underground reservoirs can be used for storage in the same manner as surface reservoirs, his contribution will have been still greater.

Lowering the ground-water table may be only evidence of better development and use of the ground-water unit. Such lowering of the water table would be serious only when the level approached the practical limit on the basis of pumping cost, or in areas where there might be danger of encroaching salt water from the sea or from a connate salt water body.

However, there has been a recently evidenced tendency to indulge in propaganda concerning the lowering of the water table, wherever this may occur, and whether or not it is merely evidence of better development and use. Recent articles in nonscientific publications indicate this lack of understanding. To pump water from a well, the water level must be lowered to produce a flow into the well; to use the underground storage reservoir, the water level must be lowered in dry periods, as would be done with any other kind of reservoir.

The author has shown that the loss from large surface reservoirs in some areas may be 50% to 60% of available supply, lost in evaporation, whereas the loss from evaporation for underground reservoirs may be practically nil. Writers have used Ohio as an alarming illustration of the result of lowering water tables during the present drought in that area. However, the most notable water failures were the surface water supplies for the cities of Columbus and Lima, Ohio.

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The author states (see heading, "Safe Yield: Determination of Safe Yield"):

> "In many cases the supply is also modified by this [the elevation of the water table] because the natural water table may be so high as to preclude percolation from parts of the streams. When pumping is begun the new and additional draft causes the water table to lower and natural outflow decreases. At the same time the lowered water table may induce a greater supply because of greater percolation opportunity. In other words, any reservoir, when half full, can and will impound more of the water that reaches it than it could if it were three fourths full or entirely full. Also, discharge through the uncontrolled outlets of a reservoir or over the spillway decreases as the water level in the reservoir decreases."

C. F. Tolman (*8a*) has stated the same fundamental principle:

> "It must not be concluded that a considerable lowering of the water table is serious or detrimental to the water supply. Just as a surface reservoir must be drawn down in order to catch and preserve flood flow, so the subsurface-reservoir level (water table) must be lowered sufficiently to prevent loss by effluent seepage. A decrease in the area of effluent seepage increases the area of influent seepage (absorptive area) and in turn increases the rate of ground-water recharge. A depleted reservoir at the end of the dry season or cycle of dry years is necessary if the water is to be salvaged in the following wet season or cycle."

The author states (see heading, "Characteristics of Alluvial Valleys"):

> "It is probable that the water supply of none of the stream systems of the west can be fully developed without utilizing some part of the storage possibilities of the unconsolidated alluvium * * *."

The writer would simply add that this statement is true of any stream system containing alluviums—east or west, north or south. Furthermore, development and use of any ground-water reservoir will require the lowering of the water table.

To induce additional percolation or surface inflow, to increase available ground water, or to use the ground-water reservoir, the water table must be lowered. The possibilities of ground-water storage utilization are immense. These reservoirs can be used to catch and preserve flood flow; and also in many stream systems they might be developed for flood control.

The tendency among some engineers, hydrologists, and members of federal government bureaus has been to consider ground water similar to other underground natural resources such as oil and gas, which are not constantly replenished. On this basis, any lowering of the water table has been considered a sign of depletion and loss, when it may be only an indication of normal useful withdrawal.

Where replenishment from rainfall is admitted, ground water has often been considered a stream from some far-off source, completely separated from the surface water. Attempts have been made to predict "safe yield" from the formula:

$$Q = P\,I\,A \qquad (1)$$

in which: Q is the "safe yield"; P is the coefficient of permeability for the formation; I is the hydraulic gradient; and A is the cross-sectional area of the formation under consideration.

The explanatory text indicates carefully how the coefficient of permeability may be determined, but stipulates that the existing hydraulic gradient of the ground water be used in making the calculations of safe yield. This concept is obviously in error since it does not consider water source. Assuming the source to be infinite, it can readily be seen that, in doubling the hydraulic gradient by pumping to lower the water table, the "safe yield" can similarly be doubled by Eq. 1.

Conversely, the "safe yield" determined under the existing hydraulic gradient will be only that quantity of water that is presently flowing through the formation. The amount of increased percolation or surface inflow that may be obtained by lowering the water table is completely disregarded.

Safe yield has been defined by O. E. Meinzer as "* * * the practical rate of withdrawing water for human use." The writer prefers this definition to the one used by the author (see heading, "Safe Yield: Definitions"). The word "practical" includes the three items of the author's definition, and eliminates the use of the words, "permissible cost" and "undesirable quality."

"Safe yield" is such an empirical term that the writer suggests the use of just plain "yield" with all the reservations that engineers may care to make, fully outlined in the text. Obviously, it is "safe" to induce additional percolation and surface inflow by lowering water tables, in most cases, but even these might be subject to various opinions. "Safe" yield indicates the engineer's opinion of what is safe and what may not be safe, based on his knowledge and the facts at his disposal.

Fairly exhaustive tests have been made on the filtering qualities of unconsolidated sands and gravels. It is generally accepted that properly constructed wells more than 200 ft from a source of pollution will produce potable water. Therefore, except for the danger of waters with a high dissolved solids content and the method of well construction, full development and use of the ground-water reservoirs should not change the quality of the water, and should not be detrimental to the quality, except possibly in terms of ground-water temperatures.

Development of water supplies drawing on ground water near Meadville, Pa., has demonstrated the fallacy of foretelling the yield of such areas, by formulas involving a permeability coefficient (Eq. 1).

The formula method of determining permeability has been applied in the Meadville district—first by the U. S. Geological Survey, with a test in 1929, on the wells northwest of the city. A pumping test requiring two observation holes at different distances from the pumped well was made. To solve for permeability, a simplified form of the equation was used as follows:

$$P = \frac{527.7\,Q \log \frac{L}{L_1}}{H\,(h - h_1)} \qquad (2)$$

in which: P is the coefficient of permeability—that is, the number of gallons per day under prevailing conditions that are conducted laterally through each mile of water-bearing bed under investigation (measured at right angles to direction of flow), for each foot of thickness of the bed and for each foot per mile of hydraulic gradient; Q is the rate of pumping in gallons per minute at distances L_1 and L between the two observation wells and the pumped well; H is the average thickness of the water-bearing bed, in feet; and h_1 and h represent the drawdown of the two observation holes in feet.

The coefficient of permeability was determined by Eq. 2. L. K. Wenzel, Assoc. M. Am. Soc. C. E. (47), has outlined this method and several other methods of making permeability determinations. All these formulas seem to arrive at about the same value for permeability with the exception of the recovery method which Mr. Wenzel indicates will require an empirical correction. The margin of error may be reduced by installing additional observation wells, but the method is superior to laboratory methods and in general is correct.

However, when the value of 14,264 for permeability in the area northwest of Meadville had been determined—and the following data were known: Width of the valley, 0.7 miles; thickness of water-bearing bed, 53 ft; and existing hydraulic gradient, 6 ft per mile—then the product of 3,175,000 gal per day was obtained by multiplication in Eq. 1. This quantity was arbitrarily called safe yield. Perennial pumping has proved that the yield for this area is 10,200,000 gal per day, without exceeding the annual recharge. Annual recharge in this area restores the ground water removed from storage during the spring flood season, with water levels returning to those noted in 1903. This recharge is a function of the stream banks and flood plains, not mathematically predictable from Eq. 1.

By a similar method, it was predicted in 1942 that the first three wells installed for the Keystone Ordnance Works, near Geneva, Pa., southwest of Meadville, would have a combined capacity of 1,000,000 gal per day, at a pumping level below the bottoms of the wells, within a period of one month. However, constant pumping has proved that the safe yield is 2,200,000 gal per day at levels above the tops of the screens. Additional percolation from rainfall was induced by lowered water levels following pumping.

Similarly, the capacity of well No. 10 of the ordnance works in the French Creek Valley near Shaws, Pa., was predicted to be 1,860,000 gal per day, and the possibility of surface inflow for a time was overlooked. The safe yield of this well, on a yearly pumping basis, is actually 4,300,000 gal per day, obviously induced when the water table was lowered below the level of French Creek.

In the Meadville district, induced additional percolation and surface inflow by developing well fields has increased the available ground water by several times the amount indicated to be flowing through the virgin areas.

Negative predictions based on formulas, without careful investigation of the influence of near-by tributary seepage, and other important factors affecting yield, might lead to the abandonment of an adequate ground-water project in ng of less desirable and more expensive construction of dams, diversion ms, and cooling plants with added expense in construction costs. In the achella district, in three instances, the safe yield of the formations has oved to be several times the quantity predicted by the ordinary approach in Fig. 3. In each case, had these predictions been followed, a great natural source *would have remained undeveloped.*

The writer agrees with the author that it would be almost impossible to tain safe yield for a single area. In the Coachella district, all the water bles were above, or equal to, stream level. In one case, northwest of [illegible] la, annual recharge in large volume during the spring flood season would not have occurred if the water table had not been drawn down, and the ound-water reservoir had not been utilized. In the other two areas, it was cessary to lower the levels to permit surface inflow, and to induce additional rcolation of rainfall. For undeveloped areas, or ground-water units, estiates of yield must be matters of judgment based on long experience with milar problems. Careful consideration must be given to all the complex ctors that make up the ground-water inventory.

The author's equation (see heading, "Safe Yield: The Hydrologic Equation") n be rewritten to include safe yields from unconsolidated deposits, expressed terms of the yield that may be used for consumptive use, or for exportation ther than in runoff below the unit, or in discharge after industrial or municipal es, thus:

Yield equals:
- Percolation of precipitation on the surface or intake
- plus
- Surface inflow minus surface outflow
- plus
- Underflow in minus underflow out
- plus
- Percolation of irrigation or other water spread on surface or into wells
- plus
- Effective reservoir capacity

It can readily be seen that lowered water levels following pumping may crease yield in any of the following ways:

1. Drawing down a water table which is at or near the surface makes possible percolation of precipitation which otherwise would flow away as noff, and may enlarge the [illegible] area.

2. Lowering the water level in the ground will obviously decrease or stop urface outflow; surface inflow may be increased by lowering the water level hich would otherwise preclude that possibility, and may enlarge the area of rface infiltration.

3. Reducing the water level will obviously decrease or stop underflow from eas not drained at the former level and may produce additional underflow to these areas.

4. Lowered water level may make possible percolation of water spread on the surface or into wells, which was impossible at former levels.

5. The water levels must be lowered to make possible, or to increase, the use of the formations as an underground reservoir.

The author has outlined two direct methods for determining "safe yield" of a ground-water unit which has been highly developed, and concerning which there is a wealth of data. The writer believes that, under the conditions outlined by the author (which are very special conditions), the determinations may be fairly exact.

In the area under consideration, percolation of precipitation on the surface is believed to be negligible; the water table is far below the surface, underflow in or out is considered negligible, and the total source is the surface inflow minus the surface outflow.

The Tule River-Deer Creek area was most carefully considered, and the drawbacks to making the safe yield determinations in this area were carefully outlined by the author. Irrigation was continuous from 1921 to 1939, inclusive, at a practically constant rate. The water table dropped a total of about 36 ft in 18 years; the quantity of water pumped to cause this drop was determined; and the specific yield of the formation was called 9.5%. However, and as the author states, the amount of underflow, in or out, might change with the changed elevation of the water table. Similarly, the writer believes that at some point in the mountains, the water table will meet the surface of the ground or the igneous rocks, so that, with the changed water table, surface inflow may have been increased during the 18-yr period, by the lowered water table. In other words, lowering may not indicate an overdraft, but only a development of the supply.

Although the same number of acres have been irrigated by surface water augmented by ground-water pumping, for the 18-yr period, the author has not made clear whether the amount of ground-water pumping has been constant during the 18-yr period.

To use the direct method (Case II, under the heading, "Direct Methods of Safe Yield Determination"), it should be understood that the draft on the supply, whether surface or underground, must be measured carefully. It is often most difficult to measure outward underflow, or the changes produced by lowering the water table.

If Case I, for which supply to the ground-water unit must be uniform year by year, is used, supply to the unit may become variable because of the changed water-level elevation which induces additional percolation or surface inflow and decreases surface outflow or outward underflow.

The author has also made some observations on the determination of "specific yield," a concept that usually has been expressed in percentages. The author shows specific yield as a percentage obtained by subtracting percentage of specific retention from percentage of porosity. The writer suggests the definition of specific yield in terms of gallons per cubic foot.

Considerable confusion has originated from calling "yield" a percentage. One engineer estimated yield for a ground-water unit as follows:

Description	Percentage	Quantity (inches)
Rainfall	(100)	(40)
Percolation	25	10
Minus specific retention	10	4
Yield	15	6

Then he arrived at the conclusion that the yield of the area would be 6 in. per yr over the basin. In other words, 10% per yr of the percolation would be lost in specific retention. A formation that could retain 10% of the percolation per year would soon have the water built up in blocks above the ground higher than the derrick.

In areas subject to the encroachment of salt water, dissolved solids content of the water produced should be carefully noted, and some measure of local control instituted to prevent this serious detriment to the ground-water supplies. In all other areas, lowering the water table may produce additional water which was formerly lost in runoff. Therefore, until lowering approaches the practical limit imposed by the cost of pumping and the use of water, control (either local or federal) based on insufficient information or unreliable methods might lead to loss of great natural resources. Every ground-water unit is unique in some of its features. Possibly in time some method—other than the practical limit of pumping costs—may be developed for determining the limit to which the water table should be drawn down.

In areas that are highly developed in ground-water use, careful study of the records of pumping, use and exportation of water, comparisons with water-table decline, and annual precipitation should indicate the solution. A considerable amount of water-table lowering may develop additional available ground water. With all the facts carefully recorded, over a period of a year or two it could be demonstrated, to the satisfaction of all concerned, that further lowering the water table would not produce any additional water, and that some measure of cooperation or control was necessary to increase the supply available. Such measures will be beneficial and will lead to cooperation and wise control of the quantity of water used.

Such control could be best directed by a state water authority, for any affected ground-water area. Its action should be based upon considerable study over a sufficient period of time to have the effect of average rainfall. This will insure the maximum practical use without excessive lowering of the water table, and the employment of every feasible method to augment the total ground-water supply.

Of most serious consequence would be federal control by a department that might autocratically apply some of the theories that have been proved fallacious in the determination of probable ground-water yield. Such negative type of control, with the bureaucratic power to impose regulations, might lead to a loss of great natural resources.

Each state should make available for its citizens, immediately, a department on ground water to study those areas in which there may be danger of a local ground-water shortage. Subsequently, there should be state control, through authorized water authorities, over those areas that require control. Only in this manner will centralized bureaucratic power be avoided, and the development of the ground waters of the United States brought to full realization.

Mr. Conkling has added much knowledge to this relatively new and little understood field. The paper has indicated the unique character of each ground-water problem. The author has demonstrated the difficulty of determining values correctly, the need for exercising extreme care in the general engineering approach to the problem of full ground-water utilization, and the necessity of insuring eventual state control.

C. W. Sopp,[6] M. Am. Soc. C. E.—Ground water is of vast importance in the economic development of the midwestern and western states. That ground-water storage is an important factor for preserving water resources is recognized by artificial recharge in certain localities and by limitation of the pumping draft to the safe yield in other areas.

The author mentions the recharge of ground-water basins by diverting water to wells which are not being used. This practice generally has not been successful because of reduction in the intake capacity of the wells. The wells act as a sand filter in reverse; and, if they are not equipped with pumps, there is no opportunity for backwashing. Bacterial growth in the wells and in the surrounding alluvium may also reduce the rate of infiltration. It might be necessary to filter and chlorinate water to be infiltrated into basins by means of wells. Near Lytle Creek, in California, a shaft was dug to a depth of 250 ft, all of which was through gravel except 8 ft. Openings were left in the cribbing throughout the length of the shaft. Initially, the shaft had an infiltration capacity of from 5 cu ft per sec to 6 cu ft per sec, but after seven years this capacity had been reduced to 2 cu ft per sec.

The total storage capacity of the ground-water unit may not be available for economic use for the following three reasons: (*a*) The "specific yield" or available porosity decreases with depth due to consolidation of the alluvium; (*b*) where storage is developed by pumping, only the storage within the range of pumping lifts is usable; and (*c*) all storage should not be exhausted, thus making the annual supply in the ground-water unit fluctuate in accordance with its annual recharge.

In one basin studied by the writer the "specific yield" decreased from 15% to 7% at 150 ft greater depth. Mr. Conkling lists many factors that may enter the hydraulic equation to determine the safe yield of a ground-water unit. The safe yield equals the water entering the unit minus water losses, including leakage. Records of the requisite data may not exist, and the data are often difficult to estimate. The safe yield, being the difference between fairly large quantities, is subject to large error if this method of computation is followed.

Mr. Conkling also gives a method of determining the safe yield by using the factors of draft and change in storage but mentions the difficulty of evaluating change in storage due to the obscure factor of "specific yield." Nevertheless, this method has the advantage of making many records or estimates unnecessary as it gives the safe yield from all sources without the necessity of segregation. Satisfactory computation of the safe yield can be made by this method in many cases.

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In the use of ground-water storage, the true occurrence of ground water as a subterranean stream, or as in a basin, is of great importance under the laws of certain states.

The underflow of a river may follow the course of the surface channel; but part of the underflow may leave the surface channel at some point and may not rejoin it. The interrelation between a surface stream and its underflow is more evident when the stream is traversing a mountain valley, where it may be in contact with or support the surface flow, than after the stream has debouched upon a broad plain. In this latter case, part of the underflow may follow an ancient, buried channel of the stream, and water moving in such an ancient underground channel may conform to the classification of ground water flowing in a defined channel. Other ground water flowing in the general region and perhaps in the same direction may not, strictly, be traversing a definite underground channel and may more properly conform to the definition of percolating water. Water flowing in an ancient channel cannot be developed independently of other overlying ground water.

The San Gabriel River and basin cited by the author illustrate certain differences in occurrence of ground water in a subterranean stream with a known and definite channel and basin water. In the canyon the underflow is confined between the impervious canyon walls and is clearly a subterranean stream with a known and definite channel. Below the canyon mouth in San Gabriel basin, but upstream from the Paso de Bartolo (Whittier Narrows), the ground water is far below the stream bed. The boundaries of the ground-water body are somewhat indefinite and are several miles apart—in fact, the width is about eight tenths of the length. The direction of movement of ground water in the basin generally follows that of the surface stream, but the ground water in the basin clearly occurs as percolating water. Immediately above the Paso de Bartolo the movement is again like that of a subterranean stream (see Fig. 3). D. G. Thompson (48) states that he

> "* * * does not know whether any Court decision has classified the movement of ground-water in the San Gabriel Basin as 'percolation,' but he believes that it is a proper classification according to the customary conception of that term as it is commonly used except perhaps for minor parts of the basin where certain geological conditions produce special conditions * * *."

Beneath the water table in a basin there may be distributary arteries of principal movement and of water in a buried channel, but the whole may be properly classified as percolating water.

The flow of water through the alluvium in a ground-water stream or through a basin as percolation is caused and controlled by the same hydraulic laws that govern percolation through a porous medium. It should be well recognized that ground water in a basin is never at rest but moves from points of recharge to points of extraction or discharge. The courts have merely applied a name, "percolating water," to classify ground water moving through a basin to distinguish it from ground water moving in a channel. The author seeks to remove the mystery of movement of ground water from the minds of laymen, and some courts by stating (see heading, "Characteristics of Alluvial Valleys") that "* * * it is moving as a stream or group of streams through the unit * * *." Such a mystery does not exist in the minds of hydrologists and this comparison may cause legal confusion. The comparison and any extension to classify all ground waters as streams lead to legal difficulties in certain states depending on the various state laws.

The rate of movement of ground water is important if a large portion of the recharge to a basin is localized as percolation from a stream overlying the basin. Ground-water movement is usually a combination of displacement and the actual movement of a particle of water. A study of water-table fluctuation and of chemical constituents of water in wells in the Raymond basin indicates that a displacement wave travels at the rate of about 124 ft per day, whereas the actual movement of a particle of water was only 8 ft per day.

Storage, rather than flow, is the dominant feature in the occurrence of ground water, whereas, for a stream, flow is the principal feature. The subflow of canyons is illustrative of the small amount of water flowing in ground-water streams. In San Gabriel canyon, the Edison Company's diversion dam developed 50 miner's inches of subflow; and the subflow at the Morris Dam of the City of Pasadena was 20 miner's inches. The subflow at the dams of the Los Angeles County Flood Control District on other streams was measured during construction as follows:

Dam	Miner's inches
Pacoima	2.3
Sawpit	5.3
Dalton	1.4
Tujunga	12.0

Contrasting with the flow of ground-water streams is storage in basins such as the Raymond basin. Here the specific yield is about 2,000 acre-ft per foot of depth; and, assuming 100 ft of fluctuation of the water table, the total usable storage is 200,000 acre-ft, whereas the annual movement in and through the basin is about 31,000 acre-ft. This large storage capacity equates the supply from wet and dry periods and has supported an overdraft since about 1917–1918.

In nature a ground-water basin is in equilibrium—that is, the water entering the basin is discharged as a regulated flow at the outlet or by evaporation and transpiration in areas where the water table is reached by the root zone of vegetation. In the natural state there is no safe yield; the outflow equals the inflow. Disturbance of natural conditions by pumping from the ground water creates a safe yield by reducing the outflow and facilitating percolation from overlying rainfall and streams. The safe yield of a basin can be increased by ground-water drafts, limited by economical lifts, up to the point where the outflow ceases, thereby utilizing the complete water supply of the hydrologic system, provided that the basin storage capacity is adequate. Development of the safe yield of a basin is accompanied by increased pumping lifts; but, with the present art of pumping and return from crops, water can be produced from lifts of several hundred feet with resulting profit to growers of crops. In utilizing the storage capacity of a ground-water basin, the water table will fluctuate over wet and dry periods. A greater fluctuation is more feasible where the pumping lift is small than where it is large.

In certain states ground water must be divided into the following classes: (*a*) Water flowing in defined subterranean streams; and (*b*) percolating water.

To judge the water storage capacity in alluvium, it is necessary to distinguish between these two classes of ground water. The reasons for such distinction are largely legal, pertaining to water rights. Distinction is unnecessary in certain states, such as Utah, Idaho, and Nevada, where permits are required for well drilling. Section 42 of the Water Commission Act of California provides that:

> "* * * whenever the terms 'streams,' 'stream system,' 'lake' or 'other body of water,' or 'water' occur in sections of this act relating to applications to appropriate water or permits issued pursuant to such applications * * * such terms shall be interpreted to refer only to surface water and subterranean streams flowing through known and definite channels."

C. S. Kinney (49) states:

> "* * * that the word 'defined' means a contracted and bounded channel, though the course of the stream may be undefined by human knowledge; and the word 'known' refers to the knowledge of the course of the stream by reasonable inference."

The presumption that ground waters are percolating runs through many court decisions. The burden of proof is upon the party who asserts that a defined underground stream exists. The rules of law in certain states pertaining to surface or subterranean streams, flowing in known and definite channels and percolating ground water in a basin, differ as follows:

A. Application of the Water Commission Act of California would require permits from the State Division of Water Resources for drilling wells to ground-water streams, whereas no permits are required for wells drawing percolating water from a ground-water basin.

B. Owners of wells drawing from a ground-water stream can acquire rights adverse to downstream producers but the contrary is not true. This rule is not applicable to percolating water in a basin where all pumping is mutually adverse.

C. Under section 20*a* of the California Water Commission Act, 3 years of nonuse of water from a ground-water stream would be interpreted as constituting a forfeiture of the right, whereas under court decisions 5 years of nonuse of water from a basin is required for a forfeiture.

The maximum supply from a stream system requires use of the large storage capacity of the underflow and of any ground-water basin to which the stream is contributory. Such storage requires no dam and is not subject to evaporation losses although the leakage may be large.

J. A. BRADLEY,[7] ASSOC. M. AM. SOC. C. E.—A comprehensive and excellent discussion of ground-water reservoirs on certain stream systems in the western United States and of the determination of their utilization from an engineer's standpoint is presented by Mr. Conkling. It is common knowledge that man's need for supplemental water in most of the alluvium ground-water provinces shown in Fig. 1 is increasing. No better evidence can be found than the already constructed works and further contemplated developments for the control of flood waters and the prevention of uneconomic "evapo-transpiration" losses from areas of high ground water which frequently occur at the lower ends of ground-water basins. These flood and wasted waters must be controlled and conserved if additional supplies are to be obtained. As Mr. Conkling has stated (see heading, "Characteristics of Alluvial Valleys"), full utilization of the water supply of the stream systems of the west cannot be attained without balancing the advantages and disadvantages of surface and underground storage, which will ultimately result in the use of "some part of the storage possibilities of the unconsolidated alluvium."

The terms, "ground-water reservoir" and "ground-water basin," are generally used synonymously. As Mr. Conkling explains, the term, "basin," is used very loosely; but it is a convenient term, nevertheless. However, the term, "ground-water basin," may be considered univocal. A surface stream has a bed and banks; a lake has a bottom and shores; but a basin is a depressed land form and has a "bottom" and a marginal boundary or "rim." Geologically, basins may have resulted from the operation of subterranean forces or may have originated in the constructional or destructional processes of nature (50). If the term, "basin," is applied to a ground-water basin, the "bottom" is the foundation upon which a more porous valley fill material rests—generally considered the basement complex underlying that porous material. The "rim" may be an elevated extension of the bottom; but, in any event, it is a physiographic or geologic feature acting like an obstruction to the movement of ground water. The material composing the bottom and rim will have at least one or more characteristics to distinguish it from the porous material which it surrounds or at least partly surrounds (the rim may be nonexistent along parts of an "open" type basin). The degree of imperviousness of bottom and rim is, or may be, only a relative matter. However, the term, "reservoir," indicates an extra supply or storage detained for future usefulness and, in the writer's opinion, should be preferred in describing the performance of a ground-water body.

According to the writer's conception of essential requirements, a ground-water reservoir in alluvium should have the following characteristics:

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(*a*) Water in the porous material everywhere in contact and moving in compliance with the physical laws of seepage and percolation;

(*b*) Capacity sufficient to yield water of good quality, in commercial quantities, by pumping at feasible cost;

(*c*) Capability of being recharged either naturally or artificially; and

(*d*) Some escape as outflow (surface or subsurface or both), to prevent the water from becoming so saline as to be unsuitable for use.

Since the introduction of the deep-well turbine pump, the development of ground-water supplies in Southern California basins has increased rapidly. The water table in these various ground-water reservoirs has shown considerable annual, as well as cyclic, fluctuation. One inference that might be gained from Mr. Conkling's paper is that, once a ground-water reservoir reaches a stage of development in which the water table is lowered, the supply may actually be increased because of greater percolation opportunity and improved recharge conditions caused by change from native to cultivated vegetation. In consequence, more emphasis has been placed recently on studies of methods for replenishment by ground-water investigators.

As the author has indicated, a ground-water basin may consist of two parts —a pressure area (containing confined water), and a forebay or intake area (containing a free water table). However, changes in storage are possible only in the intake area. Where this combination occurs, it appears to be an inherent characteristic of an underground basin in alluvium that the intake area is around the higher boundaries of the basin. Only a small portion of the intake area is generally used by nature at normal flood times for distributary channels to carry runoff. In intermittent streams, crossing alluvium water is lost underground from the beds of the distributary channels: First, by the physical laws of seepage; and second, after field capacity has been reached, by those laws attendant upon percolation. In any event, the movement of water is predominantly downward to the water table (unless intercepted by impervious strata and thence carried laterally and downward to the water table). Naturally, an attempt to increase the wetted area of these channels was made; hence, off-stream spreading was resorted to where topographically feasible on adjacent porous lands having sufficiently pervious soil types.

Spreading water for storage in underground reservoirs has already been described by A. T. Mitchelson and Dean C. Muckel (5). However, the writer wishes to add to the experience of recharging ground-water reservoirs through the use of wells drilled solely for that purpose by illustrating the performance (see Fig. 9) of a typical well drilled in the quaternary alluvium of the Orange County, California, coastal plain. The well is one of six, driven in a peripherally-leveed spreading ground of 70 acres, constructed and operated by the Orange County Flood Control District for the purpose of sinking water underground. This spreading ground overlies a part of the intake area of one of the ground-water reservoirs of the South Coastal Basin of California (27). The supply to the spreading grounds consists wholly of the storm-water runoff collected from an entire city area (which formerly had no definite outlet channels) by a system of storm drains. It is delivered by gravity to the spreading area, where it is permitted to flood over the entire surface. The water is absorbed and cleared of silt by the porous sandy soil, and then drains into a system of 6-in. concrete tile drains laid at an average depth of 10 ft below the surface in a rock-filled trench. These drains empty into each side of 4-ft square redwood intake cribs constructed around each well. The rotatory motion of the water is induced as it passes through the intake perforations in the well tube to the underground strata.

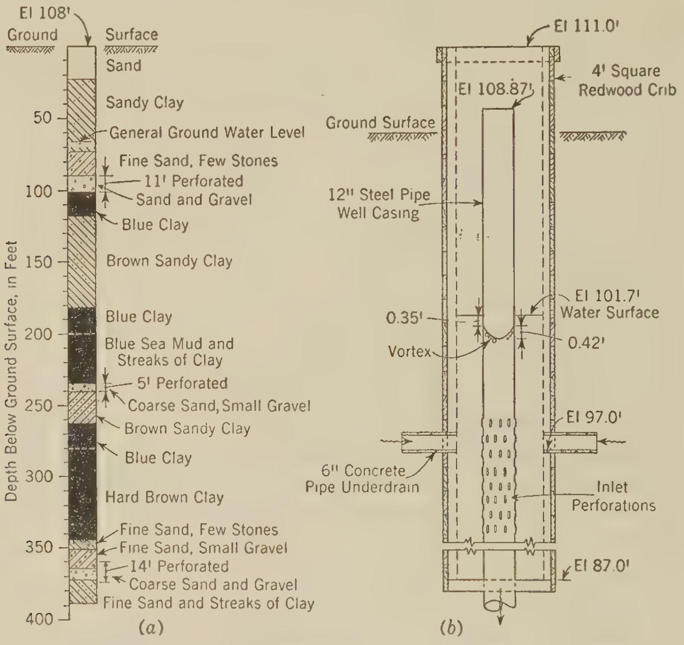
El 108'
Ground
Surface
Sand
Sandy Clay
50
General Ground Water Level
Fine Sand, Few Stones
11' Perforated
100
Sand and Gravel
Blue Clay
150
Brown Sandy Clay
Depth Below Ground Surface, in Feet
Blue Clay
200
Blue Sea Mud and Streaks of Clay
5' Perforated
250
Coarse Sand, Small Gravel
Brown Sandy Clay
Blue Clay
300
Hard Brown Clay
Fine Sand, Few Stones
350
Fine Sand, Small Gravel
14' Perforated
Coarse Sand and Gravel
Fine Sand and Streaks of Clay
400
(a)
El 111.0'
4' Square Redwood Crib
El 108.87'
Ground Surface
12" Steel Pipe Well Casing
El 101.7' Water Surface
0.35'
0.42'
Vortex
El 97.0'
6" Concrete Pipe Underdrain
Inlet Perforations
El 87.0'
(b)

Fig. 9.—Graphic Log and Operation of Typical Recharging Well

In discussing the performance of a ground-water reservoir, Mr. Conkling has justly stressed the necessity for determining the specific yield of the water-bearing material—an important factor in estimating storage capacity. The writer believes that, in those underground reservoirs of considerable extent where pumps are drawing from confined water (and it is feasible to salvage wasted waters from areas of high ground water), knowledge of the permeability of the underlying aquifers is equally desirable if the long-time average safe yield is to be estimated and the ground-water reservoir is to be managed efficiently. Transmissibility is generally determined as the coefficient of permeability and, as stated by Leland K. Wenzel, Assoc. M. Am. Soc. C. E. (51), and C. F. Tolman (8) but as credited to O. E. Meinzer, is defined as the rate of flow in gallons a day through a square foot of cross section of material, under a hydraulic gradient of 100%, at a temperature of 60° F. In 1940 the writer had charge of some pumping tests on drilled wells for determining permeability and specific yield (effective porosity) of water-bearing materials in a ground-water reservoir on the stream system of the Santa Ana River in California. Values computed for the coefficient of permeability from some of the tests varied from 4,880 to 6,070, and those for the specific yield varied from 16.3% to 18.7%, respectively. The Thiem method (51), as described by Mr. Wenzel, was used. Provided that free water-table conditions prevail in the material tested, both the coefficient of permeability and the specific yield may be computed from data obtained during a single pumping test. Although it is expensive, the method is advantageous because the results obtained are for undisturbed material lying in the natural state.

Well-developed underground reservoirs operated on a sound management basis will generally experience considerable fluctuation in elevation of the water table. Consequently, after consideration of the views of some geologists and pedologists (soil scientists), it appears that the specific yield of the water-bearing material may not remain constant throughout time, because the effects of compaction, weathering, and chemical action result in cementation. These effects may extend more or less throughout the zone of aeration.

Mr. Conkling has ably outlined the methods so far developed and the objectives to be attained in ground-water reservoir investigation. Continuity of proper records and wise management are essential to avoid depleted ground-water supplies if the many planned works for future construction are to accomplish their purpose.

IRVING B. CROSBY,[8] AFFILIATE AM. SOC. C. E.—The scope of this paper is deliberately restricted to the arid or semiarid regions of the United States west of the 97th meridian. The paper is largely confined to a discussion of the use of ground water for irrigation. This tends to give the impression that ground-water storage is more highly developed in the west than elsewhere and that the most important use of ground-water storage is for irrigation. Mr. Conkling also restricts his paper to ground-water storage in alluvium. It should be interesting and valuable, therefore, to cite some instances of the development of ground-water storage in other regions, for other purposes, and in other geological formations.

The use of ground-water storage for irrigation, its development, and climatic conditions are stressed by the author; but he includes little concerning the geologic conditions which make possible the accumulation and use of ground water in the valleys of the west. The two regions for which ground-water storage is discussed are the Great Plains and the "mountain and valley province." These two regions cover a vast area with widely varying conditions. Although the term, "mountain and valley province," is mentioned in connection with references to C. F. Tolman (8) and O. E. Meinzer (11), it does not appear to be used by them. Since there are already two classifications of ground-water provinces in the west, this attempt to introduce a different term is unfortunate and unnecessary. The region included in the author's term, "mountain and valley province," is subdivided into five provinces in Mr. Tolman's classification (8*b*). These are: (*A*) The Great Basin province covering parts of California, Nevada, Utah, and Idaho; (*B*) the Columbia Plateau province in Idaho, Washington, Oregon, and California; (*C*) the Great Valley of California; (*D*) the Coast Ranges of Central and Southern California; and (*E*) the northern Coast Ranges. The author's examples are largely from province (*D*), the Coast Ranges of Central and Southern California. The geologic conditions of this province are very different from those of other parts of the region and the attempt to lump these five ground-water provinces into one province oversimplifies the case.

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Although the author's discussion of ground-water storage is restricted to alluvium, the principles he presents apply to pervious rocks and are important in many places. Some geologic formations, in addition to alluvium, in which ground-water storage may be significant are: (*a*) Very pervious sandstones, (*b*) beds of volcanic ash, (*c*) beds of fractured lava, (*d*) cavernous limestones, and (*e*) highly fractured zones in any type of rock. In the last three cases, in which the water occupies fractures and cavities rather than pores, wells for recharging the ground water may be important.

A peculiar case of underground storage in limestone was encountered by the writer in the West Indies. The water supply of Port Au Prince, Haiti, was formerly obtained entirely from springs at the base of Morne l'Hopital, a long limestone ridge which rises 3,000 ft directly behind the city. In 1938 and 1939 an investigation of possible additional water resources was made by the writer, acting as consulting geologist for the J. G. White Engineering Corporation, and it was decided to drive a tunnel into the mountain directly back of the city. A large flow of water was encountered 1,500 ft from the entrance, with a volume, at first, of 2.75 mgd. Later this volume gradually decreased to 1 mgd and became constant. Subsequent observations indicated that a large subterranean cavity filled with water had been tapped, that the large temporary flow was caused by the draining of this stored water, and that the smaller constant flow was the average yield resulting from rain on the mountain. During wet seasons this underground storage reservoir fills; the flow increases; and, during dry seasons, the reservoir is drawn down.

The two areas discussed in detail by the author—the Platte River Valley in the Great Plains and the valleys of the Coast Ranges in Southern California—are both outside the glaciated region, but similar conditions exist in the glaciated regions. In the valleys of Southern California the ground water occurs in alluvial fans at the base of the mountains. The alluvial material filling these valleys grades from very coarse gravel at the base of the mountains to finer sand at some distance but is all pervious in varying degree and is generally very thick. The alluvial fill of the valleys of the Great Plains, on the contrary, is generally thin.

Valleys with deep alluvial fill are common in New England and in other parts of the glaciated regions (52). In contrast with the unglaciated valleys, however, there is no regular gradation from coarse alluvium in the upper parts of the valleys to finer alluvium in the lower part. Because of the irregularities of glacial deposition, there may be an impervious dam of fine alluvium across a valley with basins of pervious gravel above and below it. The pattern of surface drainage is often very different from that of the buried preglacial valleys, and ground-water basins in the buried valleys may drain in different directions from surface streams. Therefore, it is more difficult to determine the size of ground-water reservoirs in glaciated regions. Such reservoirs exist in many places, however, and are used for municipal and industrial water supplies. Generally, no attempt is made to recharge these ground-water reservoirs artificially, but one system which is so operated will be described briefly.

The first public water supply of the City of Cambridge (Mass.) was Fresh Pond in the western part of the city, and the pond is still used as part of the supply. Fresh Pond occupies a glacial kettle hole at the southern end of a glacial sand plain in a buried preglacial valley. No surface stream follows the entire length of this valley; but the upper part is occupied by the Mystic River, and the Charles River crosses the valley a mile south of Fresh Pond. The buried valley is filled with alluvium to a depth of about 170 ft. Immediately south of Fresh Pond is a glacial moraine which forms a ground-water dam and isolates the ground-water basin of Fresh Pond from the Charles River Valley. Another ground-water divide, a mile and a quarter north of the pond, separates the Fresh Pond ground-water basin from that part of the buried valley occupied by the Mystic River. Fresh Pond has an area of 85 acres and is 55 ft deep. The total surface drainage area tributary to it is 1.25 sq miles, but the area tributary to the sand plain is about 5 sq miles. Since 1887 there has been an additional water supply by conduit from Stony Brook Reservoir in Waltham, Mass.

In 1941 Cambridge needed additional water to supply war industries, and an investigation of ground-water possibilities was made by the writer who recommended that greater use should be made of ground-water storage, that Fresh Pond should be lowered much more in the summer to draw upon more ground-water storage, and that Fresh Pond should be filled in the spring to replenish the storage. Since then Fresh Pond has been drawn down in the summer, obtaining a large additional yield from the ground, and both pond and ground are filled in the spring by surplus water from Stony Brook which would otherwise be wasted over the dam.

Artificial recharge by water spreading is practiced at several plaecs in New Jersey, both in and outside the glaciated region. Artificial recharge has been practiced for more than 30 years at the Perth Amboy Water Works where rates of recharge ranging from 4,000 to 90,000 gal per acre per day have been measured. Ground-water recharge is also practiced in Long Island where it is recharged into glacial beds by means of wells and pits.

One of the most intensely developed rivers in the world is the Ruhr River in Germany. Although the climate is different, the geology somewhat different, and the water use very different, from the conditions in Southern California, there are interesting similarities in the developments. The Ruhr River flows west into the Rhine River at Duisburg-Ruhrort and has a drainage basin of 1,715 sq miles. The average natural flow before the construction of reservoirs or water works was 1,989 mgd, or 2,800 cu ft per sec. The Ruhr River is just outside the glaciated region, and it flows in an alluvial valley between low bluffs of slate and other relatively impervious rocks. The valley is floored with clay beneath which is a relatively thick layer of coarse, pervious gravel. This gravel layer forms a very long, narrow ground-water storage reservoir in which the water table is frequently lower than the river. The river is then influent to the ground water. The Ruhr River cuts through the clay layer into the gravel, but the river bed is generally coated with sludge which hinders infiltration of river water into the ground.

The Ruhr River is the only source of water supply for the Ruhr industrial district, which includes a large area north of the Ruhr Valley. The total area served is 2,500 sq miles, with a population of 3,500,000. The average consumption, all obtained from the gravel, is 250 mgd, or more than the low-water flow of the river (53). This supply was obtained from ninety different water works. In 1929, the year of greatest consumption and lowest flow, 476 mgd were used. During September, 1929, no water flowed into the Rhine River from the Ruhr River.

An elaborate series of planned developments has been completed to use the limited water resources (54) to the greatest possible extent. Water is obtained from batteries of wells and from infiltration galleries near the riverbanks. To replenish the ground-water reservoir, river water is conducted to off-stream spreading grounds, which are basins excavated through the layer of clay into the gravel. These basins are floored with sand which is cleaned occasionally. Thus, there is an extensive underground reservoir protected from evaporation losses. The only loss is seepage through the gravel down the Ruhr Valley to the Rhine River. To develop the water resources further, a series of storage reservoirs was built on tributaries of the Ruhr River. Excessive deposition of sludge was caused by the reaction of the acid water from the Lenne River, which flows through an iron mining district, with the alkaline water from the Ruhr River, which comes from a paper mill district. To precipitate this sludge, a series of lakes was formed by low dams on the Ruhr River. The sludge is flushed out of the lakes during high water. This combined development of the surface and ground-water resources of the Ruhr River provides water supply, river regulation, navigation, water power, and recreation.

Another example of intensively developed ground-water storage is the water supply of the City of Haltern at the junction of the Stever and Lippe rivers in Germany. The Lippe River, the next large stream north of the Ruhr River, flows west and enters the Rhine River at Wesel. Salt springs and drainage from mines make the water of the Lippe River saline. The Stever River comes from the north and joins the Lippe River at Haltern. The Stever River Valley is filled to a depth of 400 ft with sand, whose grain size is 1 mm or less. The permeability of the sand is 340 ft per day at unit head. The drainage area of the Stever River is 395 sq miles. The annual precipitation on this area is 547,100 acre-ft and the annual runoff is 186,400 acre-ft (55). To obtain the maximum safe yield without drawing in saline water from the Lippe Valley, a dam with a sheet-pile cutoff was built on the Stever River, raising the water level 20 ft and forming two reservoirs (56). From these reservoirs, water is conducted to spreading grounds from which it seeps into the ground. The surface storage is 7,629 acre-ft, and the underground storage is estimated at 4,621 acre-ft. By making full use of the underground storage, an annual supply of 64,850 acre-ft, or 58 mgd, is available—even in extremely dry years. The water is obtained by a battery of gravel-packed wells arranged in an east-west line across the valley.

These scattered examples of the uses of ground-water storage in the intensive development of stream systems show that the principle is not closely restricted by climatic or geologic conditions or by types of water use but that its application is feasible and valuable in a wide variety of conditions. By his detailed discussion of conditions and controlling factors in the semiarid western states, Mr. Conkling has made a valuable contribution to the subject.

G. B. DRUMMOND,[9] ASSOC. M. AM. SOC. C. E.—People of the semi-dehydrated west read with interest anything about water, particularly a paper as able as that prepared by Mr. Conkling. However, it is to be remembered that the most comprehensive understanding of the phenomena of waters is of little avail unless administrative action conforms to correct scientific knowledge.

In New Mexico the laws covering ground water have been written on the basis of a scientific knowledge of hydrology. The State Constitution defines the doctrine of "appropriation" when it states:

> "The unappropriated water of every natural stream, perennial or torrential, within the State of New Mexico, is hereby declared to belong to the public and to be subject to appropriation for beneficial use, in accordance with the laws of the state. Priority of appropriation shall give the better right."

The Constitution continues: "Beneficial use shall be the basis, the measure and the limit of the right to the use of water."

The statute of the State of New Mexico passed in 1927 authorizing the appropriation of ground water was declared unconstitutional because of a technical error. In this case (Yeo vs. Tweedy, 286 Pacific 970), the New Mexico Supreme Court discussed in detail the common law principle that an owner in fee has dominion to the center of the earth, but declared that a sensible pursuit of that theory required reasonable use of any right and that the use shall be of such a nature as to avoid injury to others. On the basis of that decision, the present statute was passed in 1931. By this statute, "Waters of underground streams, channels, artesian basins, reservoirs or lakes, having reasonably ascertainable boundaries" are public waters subject to appropriation for beneficial use.

The law is administered by the State Engineer to whom persons desiring to drill wells for irrigation or industrial purposes make application. After publication, if an interval passes during which no objections are filed and if the

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State Engineer ascertains that there are available unused waters in the area, a drilling permit is issued subject to the rights of previous appropriators from the source. If protests are filed, a hearing may be held. Appeals from the decision of the State Engineer may be taken to the courts. The State Engineer can establish ground-water districts, can set limits on the quantity of water appropriated in a district, and can exercise police power to prevent waste. "Beneficial use" being the basis of the right to use water, an appropriation is effective only as long as it is used beneficially, and water rights not exercised for a period of four consecutive years are forfeited.

The careful administration of this law has prevented the overexpansion of certain ground-water areas; and, in other areas where serious depletion of the water supply was threatened, rehabilitation has commenced.

Raphael G. Kazmann,[10] Assoc. M. Am. Soc. C. E.—A distinct contribution to ground-water hydrology is presented in this paper because it discusses, from an engineering viewpoint, a subject that has been all too neglected in the past by the majority of civil engineers. However, the self-imposed restrictions on the scope of the paper are too severe—many of the concepts outlined are applicable east of the 97th meridian and are useful to industry as well as to agriculture. Limitation 2 (see "Introduction") deserves to be quoted again in full, as it expresses conditions which are also necessary for the successful use of ground water by industry.

> "2. Only these situations in which the following conditions exist are considered:
>
> "(*a*) Ground water in the alluvium is readily accessible to extraction by pumps;
>
> "(*b*) The alluvium is sufficiently permeable to yield water in commercial quantities;
>
> "(*c*) The alluvium is naturally charged with water or is susceptible of being charged artificially to such an extent that heavy commercial drafts can be sustained; and
>
> "(*d*) If artificial charging is necessary it can be done at feasible cost."

In the eastern part of the United States, many deposits of alluvium furnish large supplies of ground water; and, in the northern states, many river valleys are filled with glacial outwash that may be a productive source of water. Many of these valleys are largely or wholly untouched by industrial or agricultural ground-water developments, thus making the problem of the safe yield from a "virgin valley" a very real one.

With regard to the industrial development of ground water, the major limiting factors are hydrologic and not economic. The utility of ground water for industrial purposes is so great that the cost of lifting the necessary quantities an additional distance is generally insignificant as compared with the cost of replacing the ground-water supply. Furthermore, this water is either discharged in sewers or, when uncontaminated, is returned to water-bearing sand through recharge wells, as in Long Island, New York (57).

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The methods used by Mr. Conkling in the determination of the safe yield from a valley are based on the sampling of material followed by laboratory work. As is recognized in the paper, this procedure is subject to large errors.

The most fundamental difficulty in applying the results of laboratory analyses to field conditions is the tacit assumption that the sum of the properties of the parts of a water-bearing formation is equal to the properties of the formation as a whole. This is seldom, if ever, true. The sum of the properties of the parts of any complex unit is usually something different from the properties of the entire unit. This can be illustrated by an analogy: An automobile is far more than the equivalent of the sum of the unassembled parts—it has properties gained from the relation of the parts acting together that the parts, unassembled, do not have. In much the same way an aquifer, as a unit, has properties different from the properties of the materials comprising it—when the individual properties are determined and added in any of a number of complicated ways. This basic difficulty remains no matter how carefully the samples are taken. There are other difficulties, such as the problem of obtaining representative samples and the disturbance of the sample as it is brought into the laboratory for analysis. This latter condition is especially true of unconsolidated materials.

A questionable procedure should not be justified merely by stating that it is followed in determining other physical properties. The procedure may be satisfactory for determining some physical properties and yet be unsatisfactory for determining hydrologic coefficients. To carry the previous analogy a step further: The weight of the car is equal to the sum of the weights of the parts. However, without testing the car in an assembled condition, how can one determine how far the car will go or what horsepower will be developed at 50 miles per hour?

There are methods for testing materials in place and determining hydrologic coefficients such as permeability and specific yield without relying on samples for quantitative determinations. Such methods can be applied profitably to alluvial deposits. For example, the problem of the safe yield of an essentially "virgin valley" was encountered in 1942 by the writer and associates in the valley of the Miami River south of Hamilton, Ohio (58). As the work was done under circumstances typifying the industrial utilization of ground water in the east, details may be of interest. The estimates of the safe yield were based on determinations of hydrologic coefficients of the aquifer in place, resulting in an increase in the reliability of the results as compared with those based on laboratory determinations.

The problem started in the Mill Creek Valley near Cincinnati, Ohio, where large industrial plants were in operation—partly because of the plentiful supply of ground water. By 1942 the industries and the municipalities in suburban Cincinnati were pumping more than 16 mgd from the water-bearing alluvium in the valley. The cumulative effect of the pumping was to lower ground-water levels in some parts of the area as much as 90 ft over a period of 50 years. By 1942 only 40 ft of saturated, water-bearing sand remained in some of the most heavily pumped areas. In others a saturated thickness of 60 ft or 70 ft remained.

In 1941 a new plant for the manufacture of airplane engines was built in the Mill Creek Valley just north of Lockland, Ohio. Even as originally designed, the plant was a giant one. Then, under the forced draft of war, its capacity was doubled. The water requirements, already high when the available ground-water supply was considered, also climbed. Before the attack on Pearl Harbor, Hawaii, in 1941, the water source for the big engine plant was a battery of large diameter wells tapping the valley fill underlying the plant property.

As water demands rose, water levels in the vicinity of the plant declined rapidly; and, partly on the basis of previous work by the U. S. Geological Survey, it was decided to install a new well field in the outwash-filled valley of the Miami River south of Hamilton, Ohio, and to abandon the well field at the plant except for emergency service. This plan necessitated the construction of about 20 miles of 42-in. and 36-in. pipe line, which, together with smaller pipe lines, wells, pumps, and motors, cost nearly three million dollars. This sum indicates the industrial value of cool, clean, ground water.

The Federal Works Agency, which was to construct the well field and pipe line, requested the Geological Survey to determine the effects of the new well field on existing ground-water supplies in the near-by Hamilton area and to make estimates of the perennial safe yield of the well-field area. In this connection, "safe yield" was defined as in Mr. Conkling's definition (see heading, "Safe Yield: Definitions"):

> "* * * the annual extraction from a ground-water unit which will not, or does not—
>
> "1. Exceed the average annual recharge * * *."

During the course of the investigation, thirty-two new wells were drilled in the area of the well field, including twelve production wells, and the geologic information obtained was analyzed carefully to delineate the area and the thickness of the ground-water reservoir. A number of farm and industrial wells that had been located and measured during an earlier investigation yielded valuable water-level information, as well as some geologic data (59)(60).

Water levels in the Miami Valley south of Hamilton had been measured periodically since 1939 and hydrographs of a number of observation wells were available, as illustrated in Fig. 10. A study of these graphs revealed an average annual water-level fluctuation of 8 ft.

The ground-water reservoir is recharged each spring by water from rain and snow, following which the water levels slowly decline as the water is evaporated and transpired while moving toward the Miami River. In 1941 and 1942, which were very dry years, the water levels did not recover normally, but in 1943 they rose to about the 1940 levels.

Maps of the piezometric surface were prepared and these showed conclusively that ground water south of Hamilton was feeding the Miami River. Fig. 11 is such a contour map. The slope of the water table is toward the Miami River. There was no heavy pumping in the area concerned, although arge quantities of water have been pumped for many years in Hamilton, and his pumping undoubtedly has had some effect on the water levels in the area o the south. In the latter area, the inhabitants used the ground-water supply nly for domestic purposes and for stock. At the time of the investigation, here was no development for irrigation.

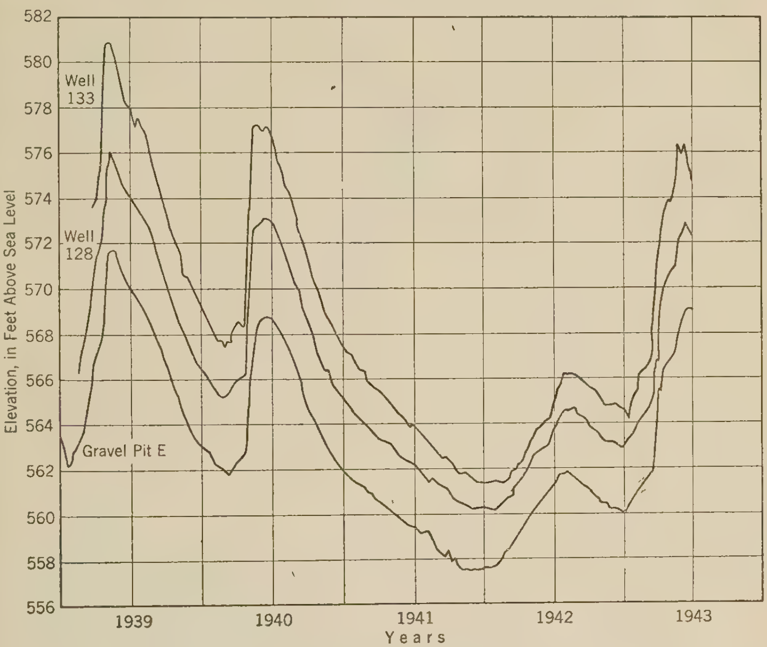

FIG. 10.—TYPICAL HYDROGRAPHS OF WELLS AND GRAVEL PITS IN THE AREA SOUTH OF HAMILTON, OHIO

To translate the available information into useful computations of safe yield, two pumping test investigations were conducted using methods developed by C. V. Theis (61) and described by L. K. Wenzel, Assoc. M. Am. Soc. C. E. (62). The method of investigation consists of sinking observation wells on a line from a well that is being pumped and observing the drawdowns that occur after pumping begins. From the discharge rate, the shape of the cone of depression at different times, and the rate of decline of the piezometric surface at the several observation wells, it is possible to compute the transmissibility of the water-bearing formation and the specific yield, or coefficient of storage, of the aquifer.

Using the quantities obtained from such pumping tests, two independent methods of computing safe yield were available:

1. By use of maps of the piezometric surface, together with the related geologic information, the average gradient and cross-sectional area were determined. The average gradient multipled by the cross-sectional area and the

coefficient of transmissibility on any line perpendicular to the direction of ground-water flow gave the daily rate of flow through the cross section.

2. The preceding computation should give results similar to those obtained by using the product of the specific yield times the average annual rise of the water level, multiplied by the up-gradient area between the given cross-section line and the ground-water divide, divided by 365 to convert the result to a daily basis. Such an assumption is based on the theory that precipitation is the source of the ground water and that, after reaching the water table, the water flows toward the Miami River, as indicated by contour maps.

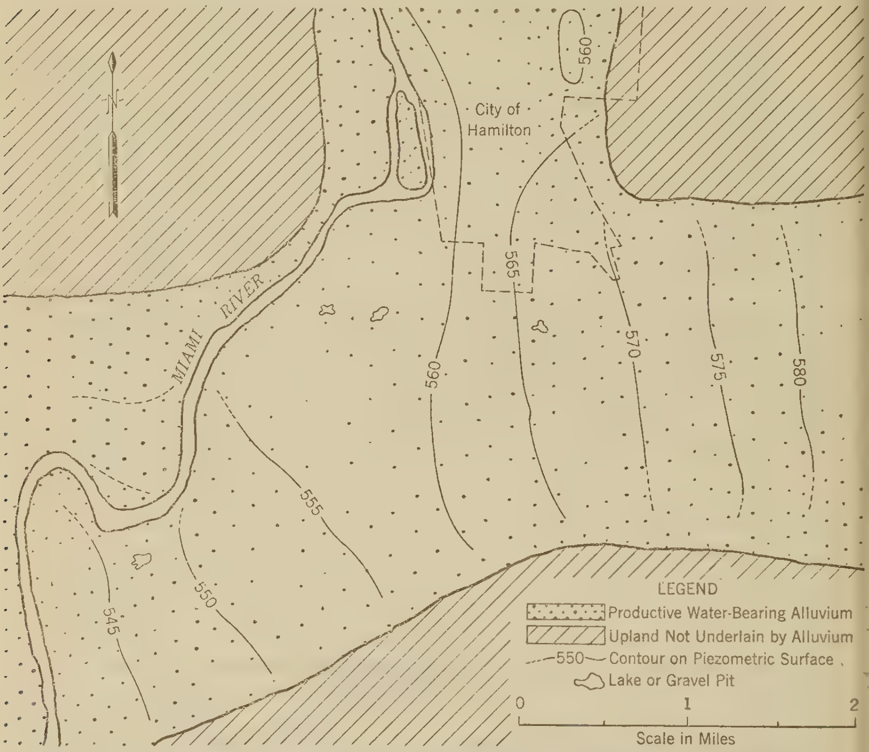
City of
Hamilton
MIAMI RIVER
560
565
570
575
580
555
550
545
LEGEND
Productive Water-Bearing Alluvium
Upland Not Underlain by Alluvium
550 Contour on Piezometric Surface
Lake or Gravel Pit
0
1
2
Scale in Miles

Fig. 11.—Piezometric Contour Map of Area of the Proposed Well Field South of Hamilton, Ohio, in April, 1943

The results were in reasonable agreement, and it was found that an average of about 13 mgd would be available from precipitation alone. However, the results of one of the pumping tests showed that, by lowering the ground-water level near the Miami River, water would infiltrate from the river into the aquifer, thus increasing appreciably the indicated safe yield of the area. From the geometry of the new well field, computations were made which showed that a minimum of 6.5 mgd could be expected to infiltrate from the river over a ong period of well-field operation. This value was undoubtedly conservative s it did not include the effects of occasional floods which cover part of the welleld area—undoubtedly helping to replenish the ground-water reservoir.

The quantity of water recharged from the Miami River was found to deend, in large part, on the pumping distribution within the well field. Thus, n unwise pumping distribution would reduce the recharge from the river and ause unnecessary declines in ground-water levels in the Hamilton area.

Inasmuch as the water was to be used for industrial cooling, it was important hat river infiltration should be encouraged in the winter when the temperature f the river water is low, and should be discouraged in the summer when the iver temperature reaches from 85° F to 90° F. A schedule was devised for wellield operation which would cause the smallest decline in water levels in the Hamilton area and would produce the coolest water for use by industry.

The methods used in this investigation can be applied, with appropriate modifications, to other alluvial deposits.

Donald M. Baker,[11] M. Am. Soc. C. E.—A method of water conservation long known and exercised has been clarified by the author. However, this procedure has just recently been considered in the planning of comprehensive stream system developments. An important point in the paper is the reference to the fact that, in the past, ground-water storage in such developments has occurred not as a result of systematic foresight and planning, but rather as a by-product of direct utilization of waters. As these comprehensive developments become more common, greater reliance upon underground storage in certain sections of the United States for complete conservation can be expected; but, before this is possible to the fullest extent, rights to the use of water so conserved must be susceptible of reasonably exact determination.

Nearly 1,000 decisions of importance with respect to the rights to the use of surface and underground water have been handed down by the supreme courts of the seventeen western states and by the federal courts, since the admission of California into the Union in 1850. All these states, except Montana, have centralized agencies for the administration of surface waters, and in some states, of underground waters. These decisions, plus constitutional and legislative enactments, as well as practices and policies developed by state administrative officials, have fairly well crystallized the legal principles that relate to rights to the use of both surface and underground waters, when such waters occur as the result of the natural phenomena of precipitation and runoff.

The doctrines of appropriation and of riparian rights for surface waters and underground waters in known and defined channels are well established. The former is recognized in some states, whereas both doctrines exist side by side in others. In the case of percolating underground water, the "common law" doctrine, under which the landowner is presumed to own all water under his land as part and parcel of the soil thereof, the doctrine of correlative rights, more or less analogous to that of riparian rights in surface flow, and the doctrine of appropriation—all have been reasonably clearly defined.

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[11] (Ruscardon Engrs.), Los Angeles, Calif.

When water occurs in underground storage as a result of man's activities and when it is extracted for beneficial purposes or finds its way into a natural stream, augmenting the flow thereof before diversion, the rights to use of such waters are not so clearly established. Most cases that have reached the higher courts as a result of controversies over the foregoing types of waters have dealt with return flow from irrigated lands, which has reached either an artificial or a natural drainage channel. In some states, those responsible for the occurrence of such waters are allowed to dispose of them to others. In other states, once the waters reach a natural stream—particularly when they have passed beyond the limits of the area from whose irrigation they result—they are considered part and parcel of the stream. Nearly all the decisions relating to this type of controversy have arisen in those states where the riparian doctrine does not exist.

During the next decade or two, however—particularly when a period of deficient precipitation and stream flow occurs—situations resulting from developments initiated since 1925 may be expected to cause serious controversies. In some instances, difficulties may be obviated by legislative enactments or constitutional amendments, whereas in others they must be settled by court decisions.

One factor to which the courts in many states appear to give consideration is the intent of the agency that is responsible for the occurrence of water in underground storage. If the agency has ignored the existence of such water—usually return flow, although sometimes surface waste—as an augmented supply to an underground basin or to a surface stream, the tendency has been to allow others to assert and secure rights to the supply. On the other hand, if the agency responsible for the occurrence of the water has consistently claimed the right to the use of the water, it has usually been allowed. Another principle, which has been favored with but little recognition by the courts, is that of the "foreignness in time" of return waters which augment natural stream flow.

Waters, originating outside a drainage basin and entering a stream, are considered as "foreign." In some states, riparian rights are not attached to the use of these "foreign" waters, since such waters are not a part of the natural flow of the stream. Somewhat similarly, an appropriator, when he made his appropriation and planned his development and use of water, might consider as available to him such waters of the natural flow of the stream as existed over and above existing prior rights. If the natural flow were augmented by artificial means, it would seem that such water should not inure to the appropriator, but should be subject to appropriation by the first comer. The return water, appearing and augmenting the natural flow, although water of the stream in question, nevertheless is no more a part of the natural flow occurring at the time than water from a distant drainage basin is part of the natural flow of the stream. However, most of the higher courts where the subject has been considered do not agree with this opinion.

The following problems may be considered as representative of many that may be expected during the next decade or so:

1. A flood control district gradually releases flood waters from its reservoirs, a considerable portion of these seeping from the stream channel into an under-

ground basin, thus resulting in a substantial increase in the normal replenishment of such a basin. Prior to the construction of the flood control reservoirs, replenishment of the basin had been in excess of draft; and, after adjudication, the pumping rights had been substantially reduced. In the adjudication proceeding, a substantial area of land within the basin was enjoined from ever using ground water. Does this augmented supply benefit those pumpers whose pumping rights were reduced in the adjudication? Does it inure to those others who were enjoined because of an inadequate supply? In other words, how should the supply be apportioned?

2. In a situation similar to that in problem 1, the increased replenishment caused a substantial increase in surface flow in a lower constricted section of the stream during the period of low flow. Does this augmented supply in the stream benefit existing appropriators in this section, or riparians, or may any one secure an appropriative right by filing a claim?

3. In a given underground basin, pumping draft over a period of years equals replenishment, and pumping rights have been adjudicated and restricted upon this basis. Gravity water is brought into the basin from a distant source, and return water from this source materially increases the replenishment. Can the original pumpers from the basin increase their draft because of the augmented supply, or are they limited to a draft equal to previous natural replenishment? In the latter case, could owners of overlying land, previously precluded from pumping because of deficiency in supply, utilize the return water, or would the agency importing the water have the first right to its recovery and use?

4. An irrigation district stores winter and early spring flood waters and uses them for irrigation. At the time plans were being prepared for the project, it was expected that the summer flow of the stream, which is inadequate to supply existing lower appropriative rights, would be augmented by return flow. Therefore, the district filed a claim on such augmented flow at the time of the original storage filing, for use on lands in the lower reaches of the project. Can the district secure a prior appropriative right to this augmented flow, or does such flow inure to existing and prior lower appropriators or to riparians to augment deficient summer supply?

5. Water from a surface stream is "spread," or placed in underground storage artificially, by an agency that expects to recover it later from wells. Because of physical conditions, not all the water so spread can be recovered by the agency doing the spreading. A party who has been exporting water from the basin is physically able to secure the remainder not recovered by the agency spreading it. However, certain overlying pumpers also claim this augmented supply. Who has the right to such water?

These and similar problems will be before the courts in the future. Sound legislation, passed to settle such controversies before they arise, or intelligent decisions, based upon an informed knowledge of the technical and economical features involved in the cases as well as upon legal precedent, will do much to crystallize the legal principles applying to water occurring in underground storage as a result of man's activities, and will encourage conservation. Much knowledge has become available concerning the occurrence and behavior of ground water since 1925 or 1930, and the fact of the existence of ground water should allow legislatures and courts to act with far more assurance than they were able to do shortly after the turn of the century.

HYDE FORBES,[12] M. AM. SOC. C. E.—This discussion of ground-water utilization provides an excellent over-all view of the objectives and accomplishments of such work in the west. The geologically modern alluvial fans built up by the major streams of the California Central Valley, southwestern Arizona, western Nevada, parts of Oregon, and south-central Washington and the alluvium filling of geological structural basins of mountain valleys and coastal plains of the west provide underground storage reservoirs whose utilization in connection with the development of the surface water resources should be a requirement in drainage basin development. Such reservoirs aid in the irrigation of less favorably situated lands and, in some instances, in the control of contamination of soils through alkali concentrations caused by the excessive accumulation of ground water.

In the "Summary," the author states:

> "This paper has presented, briefly, some of the values found and methods so far achieved, keeping in mind always that a solution of any water problem of the nature discussed in this paper [ground water] is accomplished by merely substituting correct values in a simple equation. It is the values which are often difficult to determine correctly; but as data accumulate this difficulty decreases."

Such a statement is the essence of all investigation involving underground conditions; and the writer finds, as the data accumulate, that preconceived simple relationships are not in reality simple and that the complexity of factors and of values increases. However, simple relationships or simple equations are a good starting point, subject to modification as more is learned about each specific problem.

In connection with the California State Wide Plan (later known as the Central Valley Project), in 1930 the writer undertook an investigation of the geology and underground water storage capacity of the Sacramento (63) and San Joaquin valleys (22*a*). Drainage factors for the ground-water reservoirs were assigned on the basis of grain size of the alluvial materials comprising broad areas as determined from penetration records of wells bored in those areas. The total capacity of underground storage was based on the extent of materials of a water-absorptive character delimited through an areal geological survey and on the practical limitations of water-table fluctuations. The recharge values were assumed as average seepage contributions, determined by test and measurement.

The writer's conception of the problem was that ground-water reservoirs (inflow and yield) could be treated like surface reservoirs—once the factors relative to capacity and yield had been assigned. However, the years since 1930 have produced data which indicate that the storage capacity, as measured by yield of most basins, is greatly reduced with depth below ground surface due to compaction under overburden load. The storage capacity is further reduced by the compaction of the material taking place after the first dewatering, and it is to be expected that further compaction will occur with each substantial lowering of water levels after recharge.

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[12] Cons. Engr. and Geologist, San Francisco, Calif.

The increase in density of the water-bearing materials and decrease in yield as the depth below ground surface increases were determined quantitatively during the drilling of the plant 6 well for the City of Lodi in the Mokelumne River area of the California Central Valley. A continuous core was obtained and examined for compaction density and cementation in connection with pump tests and water level measurements made in 1937. When the well was 92 ft deep and had a water column of 66.6 ft, it produced an average of 65.6 gal per min of water per foot of drawdown, at discharge rates varying from 618 gal per min to 2,280 gal per min. The zone of pumping influence extended over a maximum area of 0.315 sq miles. The yield factor per foot of water column drawn upon $\left(\frac{66.6}{65.6}\right)$ was close to 1.00.

The well was driven to 270 ft and the pumping tests were repeated, extending the area of the zone of pumping influence to 0.546 sq miles and reducing the drawdown, but giving a resultant yield factor of 0.40. The final depth was 450 ft with a maximum pumping influence of 0.641 sq miles and a yield factor of 0.30—or less than one third of the yield in gallons per minute per foot of drawdown per foot of well column obtained at shallow depth. A current meter in an orifice was lowered into the well during the pumping tests at the 270-ft and 450-ft depths to measure the water yield of each section of the coarse sand developed. The results showed that the yield per foot of sand diminished rapidly with depth. More than half of the total well yield came from the top 66.6 ft of the column which contained eleven lenses of sand totaling 29 ft in thickness. The well bore hole between 92 ft and 270 ft intercepted twenty-six lenses of sand, totaling 46 ft in thickness, and the bottom 180 ft of the bore hole contained a total thickness of 58 ft of coarse sand lenses.

It is probable that yields from underground reservoirs will diminish with each foot of water-table lowering. The degree of diminishing return will depend on the character of the material comprising the reservoir. Each ground-water basin is a problem in itself and it is doubtful that values obtained from one area or combination of geological conditions are applicable to other areas. The question is: How much compaction, if any, will take place in the upper portion of the column where overburden load is not great and where seasonal fluctuation of the water table due to recharge and draft is effective?

The example of the subsidence of the Santa Clara Valley around San Francisco Bay in California due to ground-water drainage has been cited frequently—namely, 1 in. at Redwood City, 0.5 ft at Palo Alto, and 4.1 ft at San Jose in the 12 years from 1920 to 1932. Further subsidence was measured in the 6 years from 1932 to 1938 during which time conservation dams were constructed and spreading recharge was practiced (for the last 3 years) over the upper stream fans. This subsidence was measured as 0.15 ft at Palo Alto and 1.6 ft at San Jose.

The water-table levels of well-field area for the City of Palo Alto had declined to a critical elevation; and, by 1937, the water drawn upon was of poor quality. Water was purchased from the municipal supply of San Francisco and pump draft was reduced more than half. The wet years from 1940 through 1942 rapidly raised the well-field water levels. The writer has obtained the seasonal fluctuation of water levels in numerous wells in and surrounding the city well field since 1933. The plotted hydrologic pattern of these wells has been closely repetitive year after year, declining from 1933 through 1937 and rising in point of elevation from 1938 through 1942 until the fall of 1943 when the pattern broke and a decline started which has continued through the dry years of 1944 and 1945.

Pumping was further reduced in 1944 but the decline continued. It is not possible to obtain an exact quantitative measure; but, during the 5 years preceding October, 1937, the Palo Alto pump draft averaged 819.7 million gal per season and the average seasonal replenishment was 70.8% of normal—the water level of the ground-water reservoir was lowered 4 ft, or only 0.8 ft per season (about 0.1 ft per 100 million gal pumped). During the following 5 years, when the supplemental supply was brought in, the average seasonal pumpage amounted to 425 million gal per season and the average seasonal replenishment was 110% of normal. The basin water level rose 48.5 ft, or an average of 9.7 ft per season. During the season of 1943–1944 replenishment was 68.5% of normal, comparable to the first 5-yr period of record, and pump draft was reduced to 388 million gal. The basin water level lowered an average of 5.5 ft or 1.4 ft per million gal pumped as against 0.1 ft per million gal of pump draft when the basin was first dewatered. These data indicate that there is a substantial loss of storage capacity due to compaction upon drainage from ground-water storage. This capacity may not be recovered when the water levels rise.

A considerable area in California has been devoted to ground-water recharge through spreading, particularly since 1930, and a large amount of data has been accumulated relative to specific test plots, methods, and effects. The author has mentioned some of the more general results and difficulties that have been encountered. Recharge is readily effected in the ground-water areas most intimately connected with and adjacent to large surface streams. The flow of such streams, possibly needing regulation, is adequate to serve all the lands and purposes. On the other hand, ground-water basins provided by the limited alluvial cones of small intermittent streams, or interstream areas are those wherein ground-water resources are of the most value, have been most heavily drawn upon, and, in many localities, are critically exhausted. The problem of recharge of such areas is far from solution at present.

The ramifications of the general problem of ground-water utilization, draft, and recharge, cover many scientific fields and economic and legal considerations. In stream system development, an exchange of water—service from surface water over areas of ground-water depletion and to a considerable extent from wells for areas bordering major streams of the west—should be the answer. The economic adjustment and legal difficulties, when property or water rights are involved, are probably insurmountable.

Harold Conkling,[13] M. Am. Soc. C. E.—The scope of the ten discussions of this paper is broad. Mr. Jarvis has contributed an illuminating discussion on underground water developments and problems throughout the world. Mr. Crosby has drawn attention to the artificial recharge of ground-water aquifers in Europe antedating similar projects in the United States. Mr. Hill has developed the method of determining safe yield from pumpage records which was mentioned briefly as Case I under the heading, "Direct Methods of Safe Yield Determination." Mr. Kazmann has discussed the permeability method used by the U. S. Geological Survey, the same method which is criticized by Mr. Charles in his broad discussion. Mr. Bradley has made an interesting contribution drawn from his own experience in Southern California. The method of causing recharge by means of a well which he describes is ingenious and worthy of note. Mr. Bradley also sounds a note of caution as to deterioration of the storage capacity of alluvium with the passage of time. Mr. Sopp has gone into certain phases of the legal aspects, and his statements as to his findings as to certain physical conditions should be noted. Mr. Baker and Mr. Drummond have confined themselves to legal phases. Mr. Forbes gives some startling calculations on decrease in storage capacity after the first unwatering of an aquifer.

Several have noted that the scope of the paper was too narrowly delimited inasmuch as the same principles apply to humid regions as well as to arid, and to industrial use as well as to irrigation use. This is true, of course, but even with the limitation imposed the paper was long and, to the writer, seemed to "take in a large amount of territory." Had there been no such limitation the paper, conceivably, could have filled four times as many pages. Presumably the space would not have been available. In addition, by such delimitation the writer had the considerable advantage of discussing situations with which he is more or less familiar.

This closure is limited to discussion of methods of estimating safe yield, of changes in storage capacity of aquifers due to unwatering, and of the legal phases which were brought into the picture by the discussers. There are methods of determining safe yield other than those mentioned in the paper or in the discussions, and it is a disappointment that those most familiar with them did not contribute a discussion. One of the purposes of the paper was to provoke a symposium, as it were, of recent experience particularly in that aspect of the matter. Records of such experience other than by permeability methods are generally not given much publicity.

In the paper, in discussion of the Hill method of direct determination of safe yield under Case I, it was stated that the recharge of the ground water must be fairly uniform from period to period, whether the period were a year in length or longer, for the method to be usable in any particular investigation.

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[13] Cons. Engr., Los Angeles, Calif.

It was also stated that large annual variations in supply might be "ironed out" by using moving averages embracing sequences of years. In other words, it would be necessary to use a sufficient number of years in obtaining the averages so that the recharge in each period would be approximately uniform. In Southern California, especially, and in most of arid California, in general, the periods required to obtain an approximately average supply are from twenty to thirty years inasmuch as the weather moves in "cycles" of about that length in which somewhat more than half of the years are deficient in precipitation and somewhat less than half have a surplus. In the wet phase of the cycle almost all the years have excess precipitation. In the dry phase almost all have deficient precipitation. Therefore, an inordinately large number of years must be taken to reach the average, and the period of record is not long enough to give a sufficient number of averages. In addition, development has been so rapid, and so many changes have taken place during the period of record, that recharge of a quarter century ago may not be comparable with present recharge. Probably somewhat the same conditions are found in some other parts of arid America.

Regardless of this, if the recharge to the particular portion of the acquifer for which safe yield is to be determined is by lateral percolation from a more or less distant source, so that inequalities in recharge for different periods are averaged; or, if the deep percolation of direct precipitation on the overlying surface is sufficiently impeded by impervious lenses of material so that inequalities from that source are also "ironed out," the method is usable for that particular zone.

In a large part of the coastal area of California, precipitation is large as compared to that of the southern part of Central Valley, and as compared to that of most of arid America. In much of the coastal area, also, the water table is free. In other words, recharge is largely vertical, whether from streams or from direct precipitation, so the necessary condition of equated recharge cannot exist. In some parts, however, notably for some distance from the ocean in the flat coastal plains and also in some parts of the interior valleys, the prerequisite conditions do exist.

If attempt is made to use the Case I method described by Mr. Hill in the areas with large direct vertical recharge or with lateral recharge from a not sufficiently distant source, the result will be a series of points on the graph which obviously can bear no relation to one another because they give the change in elevation of water table with the recharge of that particular period. Through each of these points, parallel curves may be drawn but there is no criterion to guide the slope of the curves unless it should happen that some two of them represent average recharge.

The point is well illustrated by Fig. 8 in Mr. Hill's discussion, in that recharge from 1918 through 1926 was approximately uniform. Then conditions began to change and a series of curves are drawn through individual points with slope gradually changing from that which obtained in the above period to that labeled "present conditions" which began in 1932 and presumably still exists. Something changed the recharge after 1926 and it did not become static again until 1932. Mr. Hill states that the change was presumably aused by movement along a major fault in 1927. The point is that if it were ot for the antecedent uniform recharge, and the present uniform recharge, here would have been nothing to indicate the direction of the intermediate urves. In Fig. 7, a successful application of the method is shown; but this is ue to the fact that, in the area to which the curve applies, the recharge is airly uniform as Mr. Hill suggests. This is because of the impedance to novement of water brought about by many clay strata formed as the original ediments changed with age.

It would seem that the restrictions on the use of this method suggested in he paper are demonstrated to be valid by Fig. 8. That does not mean, however, that the method is not a very useful one where the necessary conditions xist, as they probably do over much of arid America.

Mr. Kazmann discusses the permeability method of determining safe yield vhich is the subject of several bulletins of the U. S. Geological Survey. This onsists of determining the permeability of an aquifer by pumping tests, that is, letermining by application of Darcy's law the rate at which water will flow hrough the cross section of an aquifer. The method may be used to estimate he yield of a single proposed well, or of an entire aquifer, together with the ecessary spacing of wells to extract it.

This method seems peculiarly suited to an aquifer which has been little xploited, and, particularly, to one in which the water table is close to the ground surface so that observations on the drawdown caused by pumping the test well can be measured by small test holes. It would also be necessary for the tests to be made at the least permeable cross section of the aquifer.

It would be difficult to use it in a large part of California or in other parts of the arid states where the water table is far below the surface, because of the xpense of drilling the observation wells. This might not be necessary if sufficient wells drilled for production existed in proper locations; such a condition would be rare. Another difficulty would be the variation in specific yield as illustrated in Fig. 5. However, it would be useful in many other situations nvolving the confined aquifers of the coastal plains of California and the central parts of many valleys not on the coast, where the ground water is confined. In such cases, where pumping has already reached considerable proportions, the Hill method also is applicable and the answer secured by it might be more lependable. However, in this case, probably it would be better to use both methods plus the hydraulic equation mentioned subsequently which gives the yield as a differential.

Mr. Kazmann cites an interesting illustration of results by the permeability method in Miami Valley, Ohio, where new drafts on ground water were suddenly mposed by the war.

On the other hand, Mr. Charles gives three illustrations of predictions of safe yield by this method for water production zones in Pennsylvania as compared to results actually achieved. Actual safe yield as demonstrated by xperience was from two to three times the prediction. Had industrial levelopment been limited to that which could use the predicted yield, production of war material might have been seriously hampered. The fact that actual yield far exceeded estimated yield is perhaps corroborative of the writer's belief that it is difficult to estimate the safe yield of a little exploited territory. Probably in many cases, if not in most, exploitation increases the recharge because greater opportunity exists after the water table has been lowered by pumping.

The method of determining safe yield (that is, determination of drainable voids primarily) was described by the writer at considerable length. Mr. Kazmann seems to feel that the permeability method is preferable because it is more accurate. However (leaving aside the moot question of greater accuracy), as mentioned in a preceding paragraph it would be difficult to apply the permeability method in much of the arid West because of depth to water table, changes in permeability of the aquifer, and (not previously mentioned) fault zones across the aquifer.

The calculation of safe yield by substituting values in the hydraulic equation is laborious and, as both Mr. Hill and Mr. Sopp indicate, errors may accumulate in the residual which is the answer sought. A variation of this used by some is to calculate the deep percolation for each rainstorm and for each irrigation, and to measure the stream percolation. This is even more laborious but good results have been achieved.

It appears that for most situations some one of the methods of determining safe yield is preferable but several may give almost equally good results. Absolute merit does not lie in any one. It also appears that the method is not the only determinant. Judgment enters into the matter to such an extent that it requires considerable experience to apply any method. An important point to consider is that the books should balance. Calculated supply and disposal should be equal, and no estimate can be considered valid unless the balance is made.

Under the general direction of the writer, attempts were made in the early 1930's to determine the effect of the weight of the overlying alluvium on the porosity of various classes of sediments (27). The results are not conclusive because the range in values is large. Nevertheless, curves were plotted through the scatter points. The values in Table 3 are taken from them. These data indicate far less compaction with depth than those cited by Mr. Forbes in the case of the well of the City of Lodi if the decrease in that well is all charged to

TABLE 3.—PERCENTAGE POROSITIES AT THREE LEVELS (GROUND SURFACE EQUALS 0)

Description	0	−50	−1,000
Unweathered silts and sedimentary clays	43	37	32
Unweathered fine sands	43	38	32[a]
Nongritty residual clays	34	29	26
Reddish-brown gritty residual clays	30	27	21

[a] For this material there were no tests at depths greater than − 800 ft; at this depth porosity was 32.5%. The value shown was found by projecting the curve to the 1,000-ft depth.

compaction due to overburden. Mr. Forbes, however, does not state that the sands at depth had the same make-up as those at a higher level.

Certain data concerning the Palo Alto situation from Mr. Forbes' discussion are cited in Table 4 for better comparison. On its face this would indicate a yield per foot of recession, in 1943–1944, only 7% as great as the yield when the aquifers were first drawn on. Such diminution seems impossible and the real cause should be sought along other trails. However, diminutions of much less magnitude have been found in isolated cases. It would seem impossible to have diminution of this type without a settling of the earth's crust. Mr. Forbes cites one example. Three others come to the writer's mind—one in Central Valley and two in Southern California. These are quite local; and there is no indication so far of any widespread subsidence although the water table in most of the ground-water basins of California has varied widely in elevation since about 1930.

TABLE 4.—CHANGE IN RECESSION OF WATER TABLE AT PALO ALTO, CALIF., PER MILLION GALLONS PUMPED

Period	Normal replenishment (%)	Pump draft (million gallons)	RECESSION OF WATER TABLE	
			Ft	Ft per million gal
Average, 1932–1937..................	70.8	819.7	0.8	0.1
Total, 1943–1944....................	68.5	38.8	5.5	1.4

Legal Phases.—Mr. Baker very ably outlines the legal problems which may occur in ground-water developments of the future. It is unfortunate that the courts have developed many of their precedents from the inadequate hydrological knowledge of a half century ago. Solutions of the problems noted by Mr. Baker, and other problems which will arise, will be sadly handicapped by these precedents. There is in this circumstance, material for a paper which might be of aid to jurists.

Mr. Sopp discusses the legal phases of rights to the use of ground water, and indicates the differences between law as applied to ground water and as applied to surface water. These differences are unfortunate and were made because of lack of knowledge. Of late years, the legislatures of various western states have rescued the situation by placing all water under administrative control, that is, under the same kind of law. In California, the courts have applied the same law to both surface and underground water. Under the fundamental law of California, which is the riparian doctrine, access to a water supply gives right of use without application to the state. The individual with such access takes water under his vested right and is not termed an appropriator. The individual without such access who takes water from a source is termed an appropriator. The legislature requires that the individual who does not have access (the would-be appropriator) must apply to the state for a permit to appropriate surface water; but the legislature has not made such application a prerequisite to appropriation of ground water. Even if the legislature made application to the state a prerequisite to appropriation of ground water it would apply only to the exporter to distant lands. The overlying owner would continue to take it without application to the state.

In addition, the test of "adverse user" is not whether the invader is upstream or downstream but whether his taking limits the taking of another user. If a ground-water stream is deep enough and sluggish enough the downstream user might invade the right of the upstream user.

The foregoing short discussion is made in the fear that Mr. Sopp's able discussion might be misinterpreted in certain minor respects.

Mr. Drummond discusses briefly the excellent results secured in New Mexico by careful administration of the 1931 Act which makes underground water subject to appropriation in the same way that surface water is. This law has been assumed to be a model; and passage of a similar one has been urged on the several legislatures of the western states.

The law is an improvement over conditions which existed before its passage. It could probably be improved still further but with wise administration it should be workable as it is, for the present at least. Such a law would not be effective in California unless the constitution were changed.

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CONVERSION OF KINETIC TO POTENTIAL ENERGY IN FLOW EXPANSIONS

By A. A. Kalinske,[1] Assoc. M. Am. Soc. C. E.

With Discussion by F. T. Mavis, J. C. Stevens, Boris A. Bakhmeteff, E. R. Van Driest, A. R. Thomas, and A. A. Kalinske.

Synopsis

Information on the energy changes that occur when fluids flow in expanding conduits is presented in this paper. Specific data are presented on mean velocity distribution, turbulence, and pressure changes for 7.5°, 15°, 30°, and 180° expansions from 2.75-in. to 4.75-in. pipes. The velocity and turbulence measurements were made by photographing, on motion-picture film, the flow in transparent conduits. The water therein had suspended in it small, immiscible droplets which formed streaks on the pictures, from which the direction and magnitude of the velocity at any instant in two dimensions could be obtained.

The analysis of the data for mean velocity distribution, variation of turbulence, and pressure changes gives considerable information on the mechanics of the energy conversion and dissipation processes. It is found that the total loss of energy is considerably greater than the total energy of turbulence that is produced, thus indicating that a major portion of the energy is lost by direct conversion into heat at the regions of high, local shear in the fluid. The turbulence created is a by-product of the energy dissipation process; however, it plays an important rôle in the shaping of the mean velocity distribution curve in and beyond the expansion. Data are presented on the energy loss and efficiency of the energy conversion for two rates of flow through the four expansions studied.

A. Introduction

Examples of fluid flow in conduits where the cross-sectional area increases, with a resultant decrease in velocity and increase in pressure, are common wherever fluids are carried and controlled. Specific examples range all the

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way from small pipe fittings, known as "reducers," to draft tubes on turbines, and the loss of head occurring in any such expansion may be only of minor importance to the engineer; or it may be of considerable importance. Throughout the years, considerable data have been accumulated, relating to losses in various types of such expansions, and in general hydraulic engineers have a fair idea of the pressure losses involved. Therefore, this paper is not concerned with the mere presentation of further pressure loss data, but rather with supplying data that will shed some light on the internal mechanics of the energy changes that occur in the fluid flowing in and beyond expansions.

The experimental data presented herein have been an outgrowth of a study relating to the general problem of the hydraulic conversion of kinetic to potential energy, which was started by the Society's Committee on Hydraulic Research in 1935 at the Iowa Institute of Hydraulic Research. In order that quantitative information might be available relating to the conversion process, it was necessary that all the elements entering into the process be measured. Therefore, for each expansion studied the mean velocity distribution was determined at several sections in and beyond the expansion. Since the conversion process involves transformation of a part of the kinetic energy into turbulence energy, it was desirable to have some measurements relating to the intensity of turbulence at various sections. This, of course, proved to be the most difficult part of the experimental study; however, some very significant data were secured. The change in the pressure along the conduit in which the expansion was installed was also determined.

Before the experiments and results are discussed in detail, a brief review will be given of some of the fundamental facts known about flow in expansions and pertinent past work on this subject. Also, since certain items relating to fluid turbulence are basic to this particular study, a short description will be given of the relation of turbulence to the energy changes that occur in flowing fluids.

Notation

The letter symbols, used in this paper, conform essentially to American Standard Letter Symbols for Hydraulics,[2] prepared by a Committee of the American Standards Association, with Society representation, and approved by the Association in 1942.

B. Mechanics of Fluid Flow in Expansions

From a practical standpoint the principal item of interest is the pressure loss that occurs when fluids flow through expansions. However, in order to understand exactly what causes this extra pressure loss and where it takes place along the conduit, it is necessary that the flow mechanism be examined in detail. This paper is concerned only with incompressible fluids; therefore, density and temperature changes will be neglected.

Visual observations of flow in conical expansions reveal that there is an appreciable increase in the angularity of the velocity of fluid particles near the boundaries as they pass through an expansion. In other words, the flow becomes more eddying, and the eddies created near the boundaries gradually "eat" into the center stream which for a while after leaving the smaller conduit is relatively free of turbulence. The eddies coming from the wall naturally have relatively low forward velocities, and, as they intermingle with the rapidly moving central part of the stream, the fast-moving fluid is gradually slowed down. As the mean velocity decreases there is a gradual pressure rise, and the efficiency of the conversion process of kinetic to potential energy depends on how much of the initial kinetic energy is used in forming eddies and in direct frictional loss.

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[2] ASA—Z10.2—1942.

The loss in any expanding section can be divided into two parts—that due to friction on the boundaries and the extra loss due to the shape and form of the conduit. Except for slow expansions (those having total angles less than about 10°) the frictional loss is negligible compared to what might be called "form loss." Both analytical work and experimental data indicate that the total expansion angle giving the least loss varies between 5° and 10°, the exact value being dependent on the velocity ratio and the initial velocity distribution. As the expansion angle is increased, the form loss increases rapidly, as it is associated with the flow separation that occurs at the boundaries; that is, as the expansion angle becomes large, the fluid near the boundaries does not have sufficient kinetic energy to move against the pressure gradient; instead a backward flow occurs. There is thus created a discontinuity inside the fluid with the result that high shearing stresses occur in the fluid and violent turbulence is generated.

Some extensive data on the loss in expansions varying from 3° to 180° (sudden expansion) between circular pipes of 1.5 and 3.0 in. in diameter were obtained by A. H. Gibson.[3] He expressed the loss in terms of the Borda loss for sudden expansions, and showed that the total loss was a minimum for an expansion of 5.5°, being about 13.5% of the Borda loss; it reached a maximum of 121% when the angle was 63°, and then diminished to 102% for a 180° expansion. Extensive experiments on the loss in sudden expansions were reported by W. H. Archer,[4] who expressed the loss as

$$h_i = \frac{1.098\,(\bar{U}_{m1} - \bar{U}_{m2})^{1.919}}{2\,g} \quad \text{...................} (1)$$

in which $\bar{U}_{m1}$ and $\bar{U}_{m2}$ are the mean velocities in the upstream and downstream conduits, respectively.

In his work on conversion of energy in cross-sectional divergences, H. Peters[5] emphasized the influence of the conditions of inflow on the efficiency of the conversion process. He stated that complete conversion requires a discharge length beyond the expansion which depends on the angle and the velocity distribution at the entrance to the expansion. He defined the efficiency of conversion as the ratio of the actual rise in pressure, corrected for wall friction, to the difference in the initial and final kinetic energies. The kinetic energy was calculated by integration of the actual velocity distribution across any section. Mr. Peters showed that superimposing a spiral motion on the flow entering any expansion increased the efficiency markedly. It appears that the rotational motion delays separation, and thus decreases the energy loss.

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[3] "Conversion of Kinetic to Potential Energy in the Flow of Water Through Passages Having Divergent Boundaries," by A. H. Gibson, *Engineering*, Vol. 93, 1912, p. 205.

[4] "Experimental Determination of Loss of Head Due to Sudden Enlargement in Circular Pipes," by W. H. Archer, *Transactions*, Am. Soc. C. E., Vol. LXXVI (1913), p. 999.

[5] "Conversion of Energy in Cross-Sectional Divergences Under Different Conditions of Inflow," by H. Peters, *Technical Memorandum No. 737* (Translation), National Advisory Committee for Aeronautics, 1934.

A. P. Kratz and J. R. Fellows[6] conducted detailed experiments on pressure losses resulting from changes in the cross-sectional area of rectangular air ducts, and among their findings is the conclusion that diverging sections having included angles greater than 60° give losses at least as great as those in a sudden expansion, and that no material advantage is gained by use of angles greater than 30°.

J. Nikuradse[7] made velocity distribution measurements in two-dimensional diverging sections up to angles of 5°, and it is apparent from his data that the shape of the velocity curves depends on the angle of divergence, the curves becoming more pointed for the larger angles. He noted that separation occurred at a 5° divergence and that, for slightly larger angles, there was a tendency for asymmetrical velocity distribution and a switching of the flow from wall to wall.

Some analytical work has been done in regard to retarded and accelerated flows by extension of the knowledge relating to turbulent boundary layers to conditions where a pressure gradient exists. Calculations for such conditions indicate whether a given velocity distribution and pressure gradient are possible without separation of the boundary layer. A detailed discussion of this is given by L. Prandtl.[8]

C. CERTAIN CONCEPTS RELATING TO TURBULENCE

Since the energy dissipation in conduit expansions is closely related to the fluid turbulence, an understanding of some of the basic ideas relating to turbulence is essential. Certain aspects of the statistical theory of turbulence have been presented in a previous paper.[9] The important items relating to turbulence are its origin, designation of its intensity and energy, its diffusing power, and its dissipation.

In general, for various dynamic conditions, regions of high shearing stress, where instability develops, occur in moving fluids, with the result that "rollers" or "eddies" are created. Such regions can occur near solid boundaries in a moving fluid, at boundaries of jets entering a slower-moving or still fluid, and at surfaces between two fluids of different density when a sufficient relative velocity exists. As the vortices or eddies develop, they move away and enter other parts of the fluid. Although the motion of an isolated vortex, such as of the ring or helical type, might be analyzed, when large numbers of such vortices move into a mass of fluid their mutual interactions become so complex that it becomes impossible to predict the behavior of a particular eddy. When a mass of fluid has a large number of eddies of varying size and strength, it is said to be turbulent. It has been accepted by various workers in the field of fluid turbulence that the term would be used only to describe fluid motion that is entirely haphazard in character and which can only be described quantitatively in the statistical sense. Thus the well-ordered system of vortices one sees immediately behind a cylinder immersed in a moving fluid stream is not considered as true turbulence.

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[6] "Pressure Losses Resulting from Changes in Cross-Sectional Area in Air Ducts," by A. P. Kratz and J. R. Fellows, *Bulletin No. 52*, Univ. of Illinois, Urbana, 1938.

[7] "Untersuchungen über die Strömungen des Wassers in konvergenten und divergenten Kanälen," by J. Nikuradse, *Forschungsarbeiten des Vereines Deutscher Ingenieure*, Heft 289, 1929.

[8] "Aerodynamic Theory," W. F. Durand, Editor, Julius Springer, Berlin, Vol. 3, 1935, p. 155 (reprinted by California Inst. Tech.).

[9] "Relation of the Statistical Theory of Turbulence to Hydraulics," by A. A. Kalinske, *Transactions*, Am. Soc. C. E., Vol. 105 (1940), p. 1547.

Intensity and Energy of Turbulence.—The notation to be used for velocity is as follows: The velocity at any instant will be designated by the three rectangular components U, V, and W. The direction of U will be in the direction of mean flow, or, in this paper, along the conduit axis. The components V and W will be at right angles to U. The value of U can be considered as equal to $\bar{U} + u$, in which $\bar{U}$ is the mean axial velocity with respect to time at any point, and u is the fluctuating part due to the turbulence. Since the mean flow is in the direction of $\bar{U}$, $\bar{V}$ and $\bar{W}$ are zero. The turbulence components at any instant perpendicular to the u-component are v and w. Borrowing the concept of "standard deviation" from statistics, the intensity of the turbulence in each of the three directions will be defined as $\sqrt{\overline{u^2}}$, $\sqrt{\overline{v^2}}$, and $\sqrt{\overline{w^2}}$.

The total kinetic energy of the fluid at any point in the section, expressed in foot-pounds per pound of fluid, will then be equal to

$$e = \frac{\bar{U}^2}{2\,g} + \frac{\overline{u^2} + \overline{v^2} + \overline{w^2}}{2\,g} \qquad (2)$$

The first term is the kinetic energy per pound due to the mean velocity (referred to in hydraulics as velocity head), and will be designated by e_m. The sum of the other terms is the kinetic energy of the turbulence and will be designated by e_t.

An item to keep in mind is that similar values of $\overline{u^2}$, $\overline{v^2}$, and $\overline{w^2}$ for two flow conditions do not necessarily mean that the turbulence is identical. The intensity of the turbulence only characterizes the rotational velocity of the eddies. For a complete description of the turbulence, some measure of the average eddy size is necessary. The diffusing power of the turbulence is directly proportional to both the intensity and the scale.

Turbulence Energy Diffusion and Dissipation.—For a fluid moving under shear, the energy producing the flow is continually being transformed into heat. The work done by the shear, per unit volume of fluid, per unit time, for parallel flow, is equal to $\frac{\tau\, d\bar{U}}{dy}$, in which τ is the unit shear stress, and $d\bar{U}/dy$ is the mean velocity gradient transverse to the flow. For viscous flow this rate is equal to $\mu \left(\frac{d\bar{U}}{dy}\right)^2$, in which μ is the coefficient of dynamic viscosity. In turbulent flow the rate at which the mean shear does work is not necessarily equivalent to the rate of energy conversion into heat. Part of the work done by the shear results in the creation of turbulence; that is, the energy producing flow is in part transformed into energy of turbulence, which is eventually converted into heat through the action of viscosity. Of course, in the laminar boundary layer of smooth conduits, an appreciable portion of the energy is directly converted into heat since the value of $d\bar{U}/dy$ is very high in this region. Since turbulence is so diffusive in character, the turbulence energy created at one point is not necessarily dissipated into heat at that point.

The dissipation of turbulence energy is an extremely complicated phenomenon from an analytical standpoint because of its three-dimensional character. A general expression for the dissipation of turbulence energy by viscosity has been derived by G. I. Taylor[10] and it contains terms of the type $\overline{(\partial u/\partial x)^2}$, $\overline{\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right)^2}$, etc. These terms are the mean of the squares of the instantaneous velocity gradients due to the fluctuating velocity components. For isotropic turbulence, ($\overline{u^2} = \overline{v^2} = \overline{w^2}$), such as that generated in the throat of a wind tunnel due to a regular system of grids, the rate of turbulence energy dissipation, as shown by Mr. Taylor, reduces to a simple expression because of the simple relation that exists between the fluctuating velocity gradients in the different directions. The rate of dissipation per unit volume is then equal to:

$$e_d = 7.5\,\mu\,\overline{\left(\frac{\partial u}{\partial y}\right)^2} \qquad \text{(3)}$$

Of course, in conduit flow the turbulence is far from isotropic. The practical significance of Eq. 3 is that it gives a clue as to what factors affect the rate of turbulence energy dissipation. The term $\overline{(\partial u/\partial y)^2}$ can be set proportional to $\overline{u^2}/\lambda^2$, in which λ is a scale of turbulence associated with the dissipation process. Thus, as the size of the eddies decreases, for a given intensity of turbulence, the rate of dissipation increases. Therefore, turbulence can be dissipated more rapidly by breaking up the larger eddies. There is experimental evidence that as turbulence decays, owing to dissipation of its energy, the average scale tends to increase, since the smaller eddies are dissipated more rapidly. The larger eddies are "ground down" in a process that is not as yet clearly understood. There seems to be a stretching of the vortex filaments; for instance, Mr. Taylor[11] shows that the rate of dissipation by viscosity of the vorticity is several times greater than the rate at which vorticity actually disappears. This means that vorticity is being produced through the process of stretching the vortex filaments with a resultant decrease in size of eddies but an increase in the angular velocity, which follows from the law of conservation of angular momentum.

The foregoing ideas will serve as a background for analyzing the data on turbulence that were obtained in the flow through conduit expansions.

D. EXPERIMENTAL AND ANALYTICAL PROCEDURES

A complete study of the energy conversion and dissipation processes in the flow in expanding conduits involves a study of the mean pressure change along the conduit, the distributions of mean velocity at various sections, and measurement of the turbulence. So far as the turbulence is concerned, the obtaining of complete information regarding it presents formidable experimental difficulties. It was decided that, if its intensity could be measured at various points in several sections along the conduit, this would be all that could be attempted.

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[10] "Statistical Theory of Turbulence," by G. I. Taylor, *Proceedings*, Royal Soc. of London, Vol. 151A, 1935, p. 430.

[11] "Production and Dissipation of Vorticity in a Turbulent Fluid," by G. I. Taylor, *ibid.*, Vol. 164A, 1938, p. 15.

Since this entire investigation extended over several years, changes in regard to general experimental procedures and apparatus occurred as information was obtained relating to experimental difficulties, and also as the general emphasis of the study was altered. The first studies were made on a 15° expansion between pipes of 3-in. and 5-in. diameters. The conical section was constructed of transparent pyralin and the straight pipe of pyralin sheets. Studies on this expansion perfected the experimental techniques, and some useful data were secured. However, after this expansion was in use about a year, it warped and had to be abandoned, thus preventing the collection of a complete set of data for several rates of flow. Since at that time transparent lucite pipe and sheet material had become available, it was decided to complete the studies using various expansions between lucite pipes of 2.75-in. and 4.75-in. diameters. Time permitted obtaining velocity and turbulence data for at least two rates of flow for expansions of 7.5°, 30°, and 180°. Pressure data were obtained on these expansions and also on a 15° expansion between 2.75-in. and 4.75-in. pipe.

The general setup used in the experimental work consisted of a constant-level supply tank, calibrated orifice, approach piping, and regulating valve. After considerable preliminary work, it was decided to obtain the velocity data photographically by use of motion pictures. The technique was as follows: Into the flowing water, about 30 ft upstream from the expansion, a mixture of carbon tetrachloride and benzine was injected, the proportions being adjusted so that the mixture had a density equal to that of the water. Powdered anthracene was suspended in the mixture to give it a milky appearance. The mixture was injected into the flowing water by means of a water-pressure operated injector. The mixture was thus broken up into droplets the size and number of which could be controlled by adjusting the rate of injection and by varying the water pressure on the injector. The size of the particles considered suitable for this work varied between one and two millimeters. The water temperature in these experiments varied from 50° to 65° F.

At sections where velocity data were desired, the flow with the suspended droplets was illuminated with a carbon-arc light by focusing an intense beam through a double-slit barrier, which gave a plane of light about $\frac{1}{8}$ to $\frac{1}{4}$ in. thick. This horizontal plane of light was directed through the center line of the conduit, and particles entering this plane of light were brilliantly illuminated. A 16-mm, motion-picture camera was mounted above the conduit and focused on the plane of light in the conduit. The shutter opening and speed of the camera were adjusted for any run, so that for the given water speed the particles would, on an average, produce streaks about $\frac{1}{2}$ in. long when the film was projected to the actual size of the conduit. A scale was inserted next to the conduit in the field of view to give the linear dimension. The field of view was only large enough to take in the width of the conduit and the scale. For each run, about 15 ft of motion-picture film was taken, giving about eight hundred individual frames. The time interval for this footage varied with the camera speed, but on the average was about 1.5 min. For each shutter opening and camera speed used, the camera was calibrated for exposure time by photographing bright particles attached to a belt which was run by a constant-speed motor.

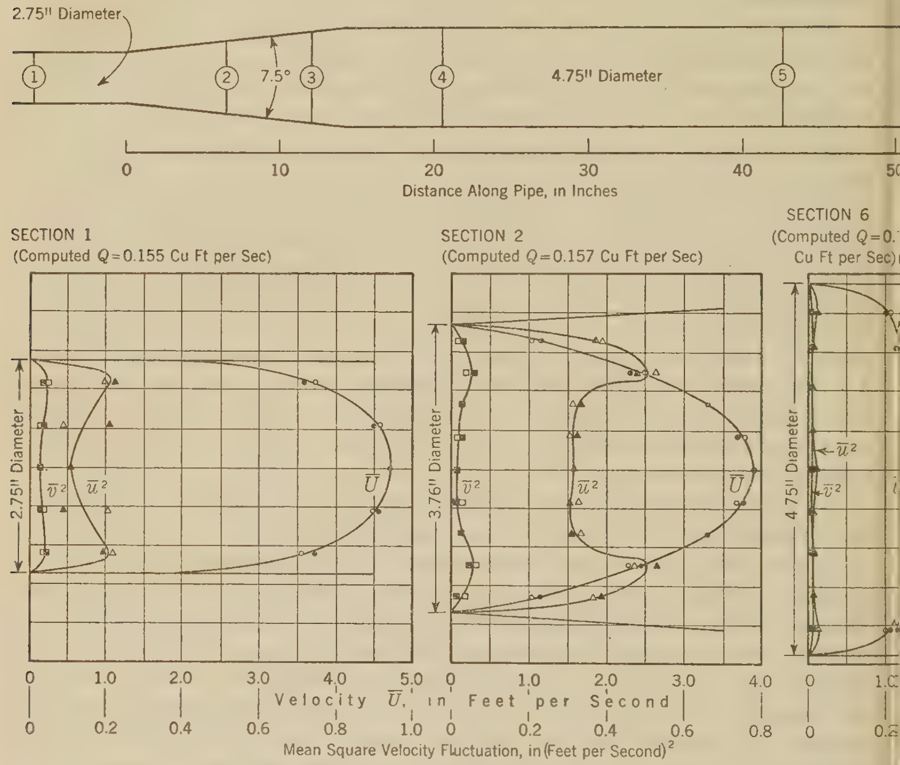
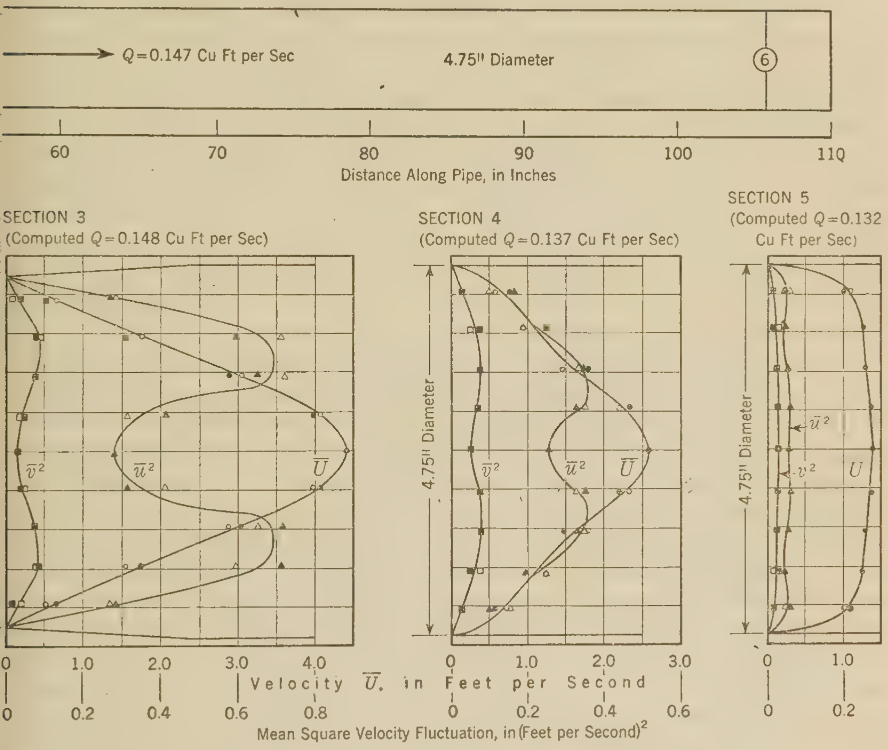


FIG. 1.—DISTRIBUTIONS OF MEAN VELOCITY AND MEAN SQUAR

The pictures were analyzed by projecting the film to somewhat larger than full scale and measuring the length of the streaks in the following manner: At the section of the conduit where velocity data were required, some seven to nine points were selected across the conduit. At these points on the screen, squares were drawn about 1/25th of the conduit diameter in size. Streaks falling on these squares were taken as giving the velocity at that point at that instant, or, more accurately, the average velocity for the instant of time required for the droplet to move the distance of the streak. This time varied with the water speed, but was between 1/20th and 1/80th of a second. Three measurements were taken for each streak—the length components along and perpendicular to the conduit, and the width. The length of the streak in the direction of the conduit minus the streak width (droplet size) gave the velocity component in that direction at that instant. The average of such lengths gave the mean axial velocity at any point. This is designated by the symbol $\overline{U}$. Subtracting $\overline{U}$ from the individual velocity values at various instances gave u, and from these the value of $\overline{u^2}$ could be obtained.



ocity Fluctuations for 7.5° Expansion (Q = 0.147 Cu Ft Per Sec)

The length component of the streak perpendicular to the conduit gave the velocity component, V, and then, from $(\overline{V} - V)$, values of the turbulent component v could be obtained, and finally $\overline{v^2}$. Thus the streak data permitted the calculation, at each point, of the values of $\overline{U}$, $\overline{u^2}$, and $\overline{v^2}$. The number of motion-picture frames analyzed varied from 500 to 1,000. The number of readings obtained averaged about 300. As can be imagined, the taking of the data from the pictures and their evaluation were very time consuming. However, since such data had never been obtained before, it was thought worth while to do the necessary work, since it seemed that some significant information could be obtained.

The pressure-variation data were obtained by installing wall piezometers along the conduit and connecting these to a tube bank. Colored water was put into the top of the columns so as to make them more visible, and a motion picture was taken of the water levels in all the tubes. The average value of the pressure was determined by projecting the picture and reading the water-column heights from several hundred frames distributed over a time interval of about 2 min.

E. MEAN VELOCITY AND VELOCITY FLUCTUATION DATA

The distribution of the mean velocity and velocity fluctuations across several sections of a conduit was first obtained for a flow of 0.082 cu ft per sec through the 15° conical expansion between 3-in. and 5-in. pipes. A detailed plotting or tabulation of the values of $\overline{U}$, $\overline{u^2}$, and $\overline{v^2}$ for all the tests would require considerable space; therefore only typical detailed data will be shown for a few of the runs. Figs. 1 and 2 contain data obtained for the 7.5° and 180° expansions for a flow of 0.147 cu ft per sec. From visual observations it appeared that on the average the flow was symmetrical; therefore, the actual values obtained on one side of the conduit were also reflected to the other side to assist in drawing in the average curve. The actual measured values are shown by solid symbols, and the points reflected from the opposite side are open symbols.

Similar data were obtained for the 30° expansion and also for a rate of flow of 0.210 cu ft per sec in the 7.5°, 30°, and 180° expansions, and in addition for a flow of 0.082 in the 180° expansion. All these data will be used in calculating the turbulence energy and in studying its variation along the conduits.

From an examination of the data in Fig. 1 for the 7.5° expansion it is apparent that, at a distance of 100 in. beyond the start of the expansion, normal flow conditions had definitely been attained. All the data indicated that the $\overline{u^2}$-values are always larger than the $\overline{v^2}$-values. The highest $\overline{u^2}$-values and $\overline{v^2}$-values occur beyond the sudden expansion. Relative changes from section to section of $\overline{u^2}$-values and of $\overline{v^2}$-values for any expansion are not comparable; in other words, one of the turbulence components may change differently from the other. The highest values of turbulence intensity occur near the boundaries in the expansion, whereas beyond the expansion the turbulence gradually diffuses into the center of the stream. For the sudden expansion (Fig. 2), backflow near the walls is apparent. It was also observed at the end of the 30° expansion. Only a slight backflow very near the walls was visible for the 15° expansion, and none was noted for the 7.5° expansion.

Values of the discharge were computed by numerical integration of the mean velocity curves and are shown in Figs. 1 and 2. In analyzing the large amount of data it was necessary to use different people at various times, and, since considerable personal error can enter into the readings of the streak lengths from the motion-picture film, it is believed that this explains the discrepancy between the actual and calculated values of Q. In comparing values of $\overline{u^2}$ and $\overline{v^2}$ from section to section, it would be well if these values were corrected by the square of the ratio of calculated discharge to actual discharge. This will be done when the values for the turbulence energy are calculated.

Whether the values of $\overline{u^2}$ and $\overline{v^2}$ that were obtained are correct in absolute magnitude is problematical; however, in any case the relative values should be as indicated. The actual values obtained for normal flow in the straight

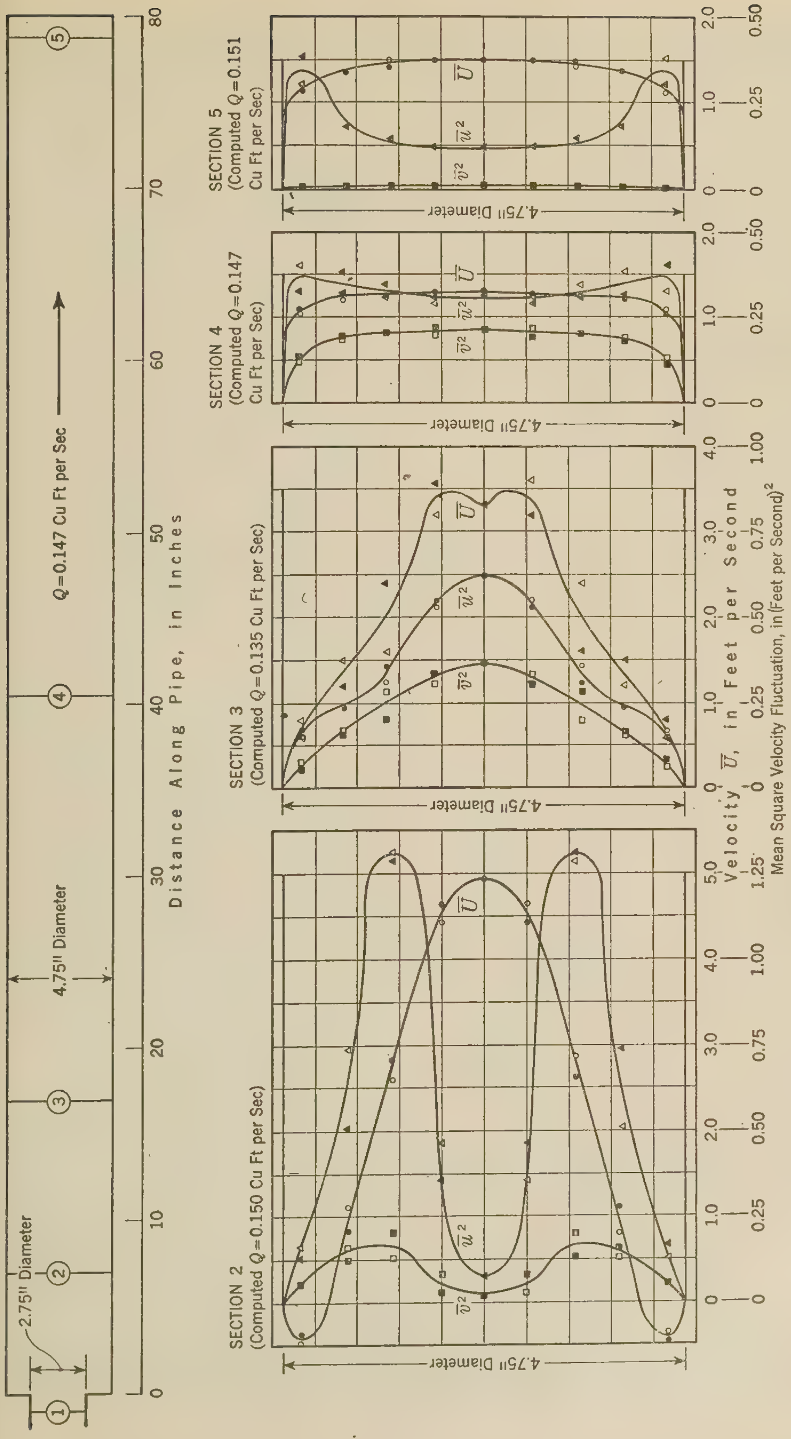


FIG. 2.—DISTRIBUTIONS OF MEAN VELOCITY AND MEAN SQUARE OF VELOCITY FLUCTUATIONS FOR 180° EXPANSION ($Q = 0.147$ CU FT PER SEC)

conduits seem to be of the order of magnitude obtained by H. C. H. Townend[12] for flow in a square pipe. Also, as the present data indicate, the value of $\overline{u^2}$ tended to be larger than that of $\overline{v^2}$ in Mr. Townend's studies. He measured the w-component and found it to be of the order of the v-component, except near the wall, where the v-component (perpendicular to the wall) tended toward zero, whereas the w-component did not.

TABLE 1.—MAXIMUM VALUES OF $\sqrt{\overline{u^2}}/\overline{U}_c$

Location of section[a] (see Figs. 1 and 2)	FLOW Q, IN CUBIC FEET PER SECOND:		
	0.147	0.210	0.082
(*a*) 7.5° EXPANSION; FROM 2.75-IN. TO 4.75-IN. PIPE			
0[a]	0.094	0.110	....
6.5	0.181	0.200	....
12.0	0.189	0.197	....
20.5	....	0.322	....
20.6	0.228	....	....
42.7	0.185	0.188	....
64.8	....	0.123	....
105.5	0.112	0.117	....
127.0	....	0.110	....
(*b*) 30° EXPANSION; FROM 2.75-IN. TO 4.75-IN. PIPE			
0[a]	0.094	0.110	....
1.6	....	0.228	....
1.8	0.220	....	....
9.9	0.437	0.495	....
17.8	....	0.420	....
22.5	0.278	....	....
64.0	0.142	0.145	....
81.0	....	0.140	....
(*c*) 180° EXPANSION; FROM 2.75-IN. TO 4.75-IN. PIPE			
0[a]	0.094	0.110	0.110
7.0	0.231	0.234	0.254
17.0	0.372	0.347	0.284
40.5	0.112	....	0.181
41.0	....	0.150	....
78.8	0.120	....	0.124
79.0	....	0.117	....
(*d*) 15° EXPANSION; FROM 3.00-IN. TO 5-IN. PIPE			
0[a]	....	....	0.160
3.0	....	....	0.217
4.0	....	....	0.200
6.0	....	....	0.232
15.5	....	....	0.292
30.5	....	....	0.224

[a] The location of the section is given as the distance in inches from the beginning of the expansion. The values given at zero distance are in the small conduit, although not necessarily just at the end of the small conduit.

Significant items to be noted from Fig. 2 are the manner in which the mean center velocity is reduced and how the turbulence gets into the center stream in the 180° expansion. In the case of the 30° expansion at a distance of 0.5 ft beyond the end of the expansion, the center velocity was reduced from a maximum of 4.7 ft to 2.5 ft per sec. Also, it was noted that at this distance the turbulence created at the boundaries of the expansion had already permeated into the center of the stream. The foregoing reduction in center velocity and permeation of the turbulence do not occur, for the 180° expansion, until a distance of 1.5 ft beyond the expansion (see Fig. 2). Evidently the turbulence created in the 30° expansion is of a larger scale and has greater diffusive power, and thereby penetrates and slows down the center portion of the stream in a shorter distance than in the 180° expansion.

In order to indicate the variation of the relative intensity of the turbulence in the conduits with the various expansions, Table 1 contains values of the ratio of maximum value of $\sqrt{\overline{u^2}}$ to the center velocity in the cross section, $\overline{U}_c$. Data shown in this table are for all the measurements made in these experiments. It is noted that the maximum value of this ratio occurs for the 30° expansion and is about 0.50 for the flow of 0.210 cu ft per sec. This ratio seems to have about the same value in the 2.75-in. pipe and in the 4.75-in. pipe, when normal flow is again developed. It seems that it took a distance of about 125 in. or about twenty-five diameters from the start of the expansions for this ratio to attain a constant value in the large pipe. For any expansion and at any given section this ratio, in general, has a smaller value for the smaller rate of flow.

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[12] "Statistical Measurements of Turbulence in the Flow of Air Through a Pipe," by H. C. H. Townend, *Proceedings*, Royal Soc. of London, Vol. 145A, 1934, p. 180.

An examination of the typical detailed data shown in Figs. 1 and 2 and the values in Table 1 offers a fairly complete picture of the variation of the turbulence in conduit expansions.

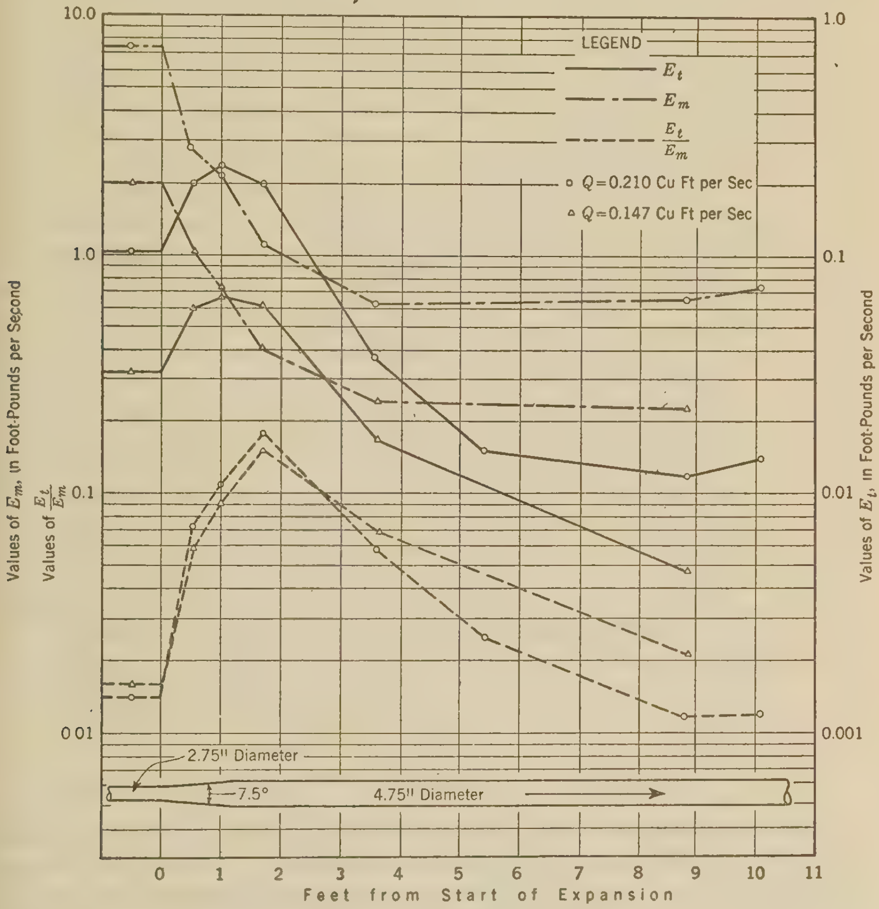

Fig. 3.—Kinetic Energy of Mean Flow and of Turbulence for 7.5° Expansion

F. Mean Kinetic Energy and Turbulence Energy

To indicate the changes occurring in the kinetic energy of the mean flow and of the turbulence energy along the conduit for the several expansions, the total value of these energies was calculated by numerical integration of data such as those shown in Figs. 1 and 2. The total kinetic energy of the mean flow in foot-pounds per second was obtained from the following expression:

$$E_m = \frac{\pi \gamma}{g} \int_0^r \overline{U}^3 \, y \, dy \ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

in which γ is the weight of water per unit volume, r is the radius of the pipe section, and y is the distance from the center. If the distribution of the mean velocity was uniform, E_m could be expressed as $\frac{Q \gamma \overline{U}^2_m}{2g}$, in which $\overline{U}_m$ is the mean velocity in a cross section. For normal turbulent flow in a circular conduit, the values given by Eq. 4 are several per cent greater than the values obtained by using the mean velocity in the cross section. However, for velocity distribution in and immediately beyond expansions, the correct value of E_m may be several times the value obtained by using the mean velocity in the cross section. The total kinetic energy of the turbulence per pound of fluid flowing is equal to

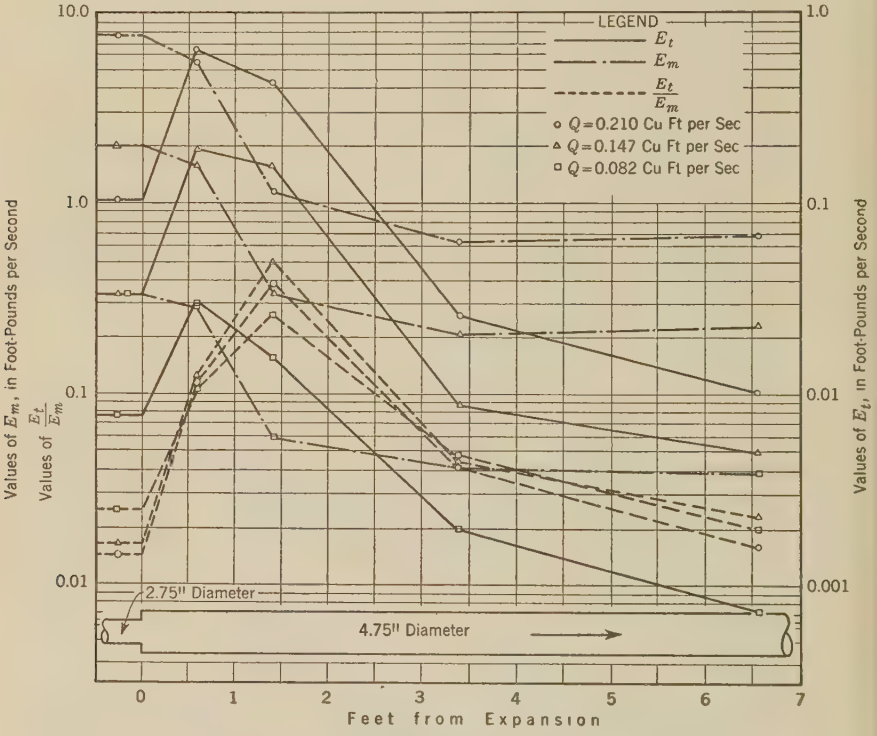

FIG. 4.—KINETIC ENERGY OF MEAN FLOW AND OF TURBULENCE FOR 180° EXPANSION

$$e_t = \frac{\overline{u^2} + \overline{v^2} + \overline{w^2}}{2g} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

The total kinetic energy of the turbulence in the water flowing per unit of time past any section is then given by:

$$E_t = \frac{\pi \gamma}{g} \int_0^r \overline{U}(\overline{u^2} + \overline{v^2} + \overline{w^2})\, y\, dy \ldots\ldots\ldots\ldots\ldots (6)$$

In the present studies the value of $\overline{w^2}$ was not determined; however, in the evaluation of E_t, $\overline{w^2}$ was taken equal to $\overline{v^2}$. This appears to be a fairly good approximation except near the walls of the conduit. The values of E_t were obtained by numerical integration of data such as those shown in Figs. 1 and 2.

The ratio of calculated to actual discharge at any section was assumed to indicate the error in the streak length or in the length of velocity scale used. The calculated values of E_m or E_t were therefore corrected by the cube of this ratio. In Figs. 3 and 4 and in Table 2 are shown the values of E_m, E_t, and the ratio E_t/E_m for all the data obtained in these studies. Several pertinent items might be mentioned regarding these data: The maximum value of E_t occurs right at the end of the 7.5° expansion; for the 15° expansion it occurs slightly beyond; and, for the 30° and 180° expansions, the maximum value of E_t is about 0.60 ft beyond the expansion. The maximum value of the ratios E_t/E_m always occurs beyond the end of the expansions; the distances beyond the various expansions were 1.4 ft for 180°, 0.65 ft for 30°, and 0.50 ft for 7.5°. The turbulence energy appears to have ceased changing in the 4.75-in. conduit at a distance of 9 ft or about twenty-three diameters from the start of the 7.5° expansion. However, the change in E_m is fairly small beyond about ten diameters from the start of the expansions.

TABLE 2.—KINETIC ENERGY OF MEAN FLOW AND OF TURBULENCE

Location of section[a] (see Figs. 2 and 3)	Kinetic Energy, in Foot-Pounds Per Second		Ratio, E_t/E_m
	Mean flow, E_m	Turbulence, E_t	
(a) 15° Expansion from 3-In. to 5-In. Pipe; $Q = 0.082$ Cu Ft Per Sec			
0[a]	0.221	0.0067	0.030
0.25	0.157	0.0106	0.068
0.33	0.125	0.0110	0.088
0.50	0.101	0.0122	0.121
1.30	0.042	0.0122	0.291
2.55	0.038	0.0042	0.110
(b) 30° Expansion from 2.75-In. to 4.75-In. Pipe; $Q = 0.147$ Cu Ft Per Sec			
0[a]	2.00	0.032	0.016
0.15	1.42	0.110	0.078
0.85	0.481	0.230	0.480
1.90	0.221	0.040	0.182
5.35	0.220	0.006	0.027
(c) 30° Expansion from 2.75-In. to 4.75-In. Pipe; $Q = 0.210$ Cu Ft Per Sec			
0[a]	7.56	0.101	0.014
0.15	3.85	0.382	0.099
0.85	1.40	0.781	0.513
1.40	0.765	0.248	0.325
5.35	0.622	0.031	0.050
6.75	0.570	0.013	0.023

[a] The location of the section is given as the distance, in feet, from the beginning of the expansion. Values given at zero distance are in the small conduit.

The approximate maximum values of E_t/E_m for the various expansions tested are as follows: 7.5°, 0.17; 15°, 0.28; 30° and 180°, 0.50. The values appear to be somewhat higher for the lower rates of flow. The actual values of E_t for the 30° expansion are slightly greater than those for the 180° expansion. The rate of decrease of E_t beyond the expansion is apparently most rapid for the 180° expansion, indicating that the turbulence is probably finer grained for the 180° than for the 30° expansion and thus dissipates faster. This was mentioned in Section E in connection with another item. The fact that the 30° expansion appears to have as much total turbulence energy as the 180° expansion does not necessarily mean that the head loss through it is as great. The intensity of the turbulence at a point in the fluid corresponds somewhat to the magnitude of the friction or shear stress. If this intensity is high, even if it is quite localized, the energy loss due to friction is certain to be large. It was noted that the maximum intensity of the turbulence for the 180° expansion is about 1.3 times greater than that for the 30° expansion for any given rate of flow.

TABLE 3.—PRESSURE VARIATION ALONG CONDUITS, IN FEET OF WATER

(Expansion from 2.75-In. Pipe to 4.75-In. Pipe)

Location of section[a]	$Q = 0.147$	$Q = 0.210$
	(a) 15° EXPANSION	
−1.50	0.100	0.100
0.00	0.080	0.051
0.35	0.162	0.215
0.75	0.177	0.250
0.90	0.186	0.265
1.20	0.202	0.285
1.92	0.208	0.308
3.35	0.212	0.314
7.20	0.202	0.305
	(b) 30° EXPANSION	
−1.50	0.100	0.100
0.00	0.080	0.051
0.42	0.131	0.175
0.60	0.150	0.210
0.86	0.172	0.250
1.62	0.191	0.288
3.05	0.194	0.294
6.92	0.185	0.282

[a] The location of the section is given as the distance in feet from the beginning of the expansion (see Fig. 5). The flow Q is in cubic feet per second.

The aforementioned items seem to be the principal information that can be obtained from an examination of the E_m-curves and E_t-curves. These quantitative data give some information relating to the changes in the mean kinetic and turbulence energy that take place in the flow through expanding conduits.

G. PRESSURE DATA

To complete the picture of the energy conversion process for flow through expanding conduits it is necessary to have data on the variation of the pressure. Pressures were measured for the 7.5°, 15°, 30°, and 180° expansions in the manner described in Section *D*. The pressure variation data for these expansions are shown in Fig. 5 and Table 3.

The very appreciable drop in the wall pressure at the start of the larger conduit for the sudden expansion, as shown in Fig. 5(*b*), is quite characteristic of this type of expansion. The maximum pressure is reached at distances beyond the start of the expansions which seem to increase with decrease in angle; thus, for the 7.5° expansion this distance is about 4.0 ft, for 15° it is 3.3 ft, for 30° it is 2.8 ft, and for 180° it is 2.5 ft.

In order to indicate the change in the total energy, exclusive of the turbulence energy, along the conduits, the quantity $(E_p + E_m)$ was calculated and is plotted in Fig. 6. The value of E_p was obtained by taking the values of pressure head, h, and multiplying by $62.4\,Q$. It is worth noting that the change that occurs in this total energy in the 180° expansion is much more gradual than that in the 30° expansion. In other words, as was mentioned previously, the energy conversion occurs more gradually in the sudden expansion than in the 30° expansion due to the fact that the relatively small-scale turbulence that is generated at the edges of the high-velocity stream is not so effective in "eating into" the stream and slowing it down. The diffusive power of the turbulence depends on both its intensity and scale and evidently

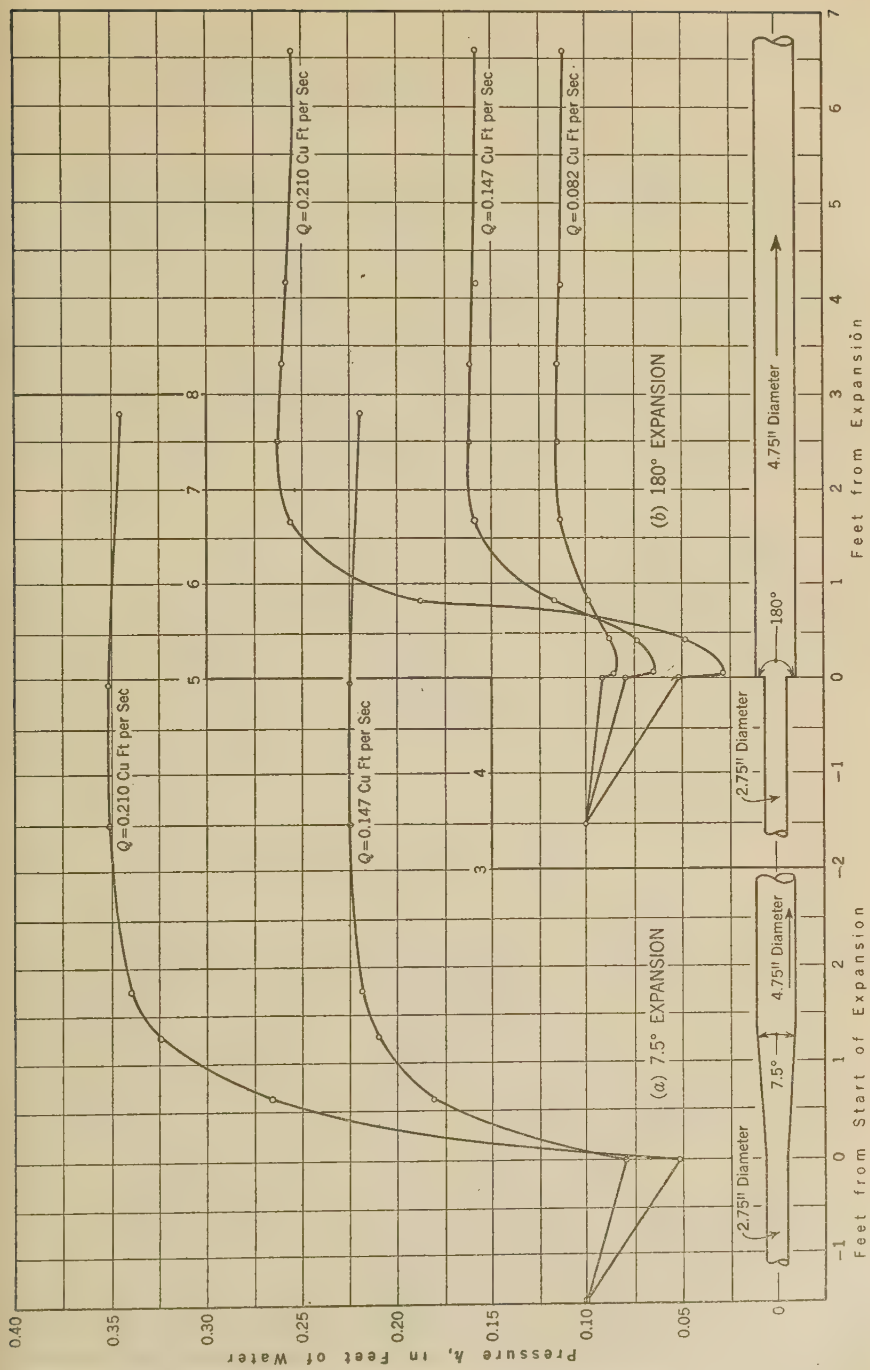

FIG. 5.—CURVES OF PRESSURE VARIATION

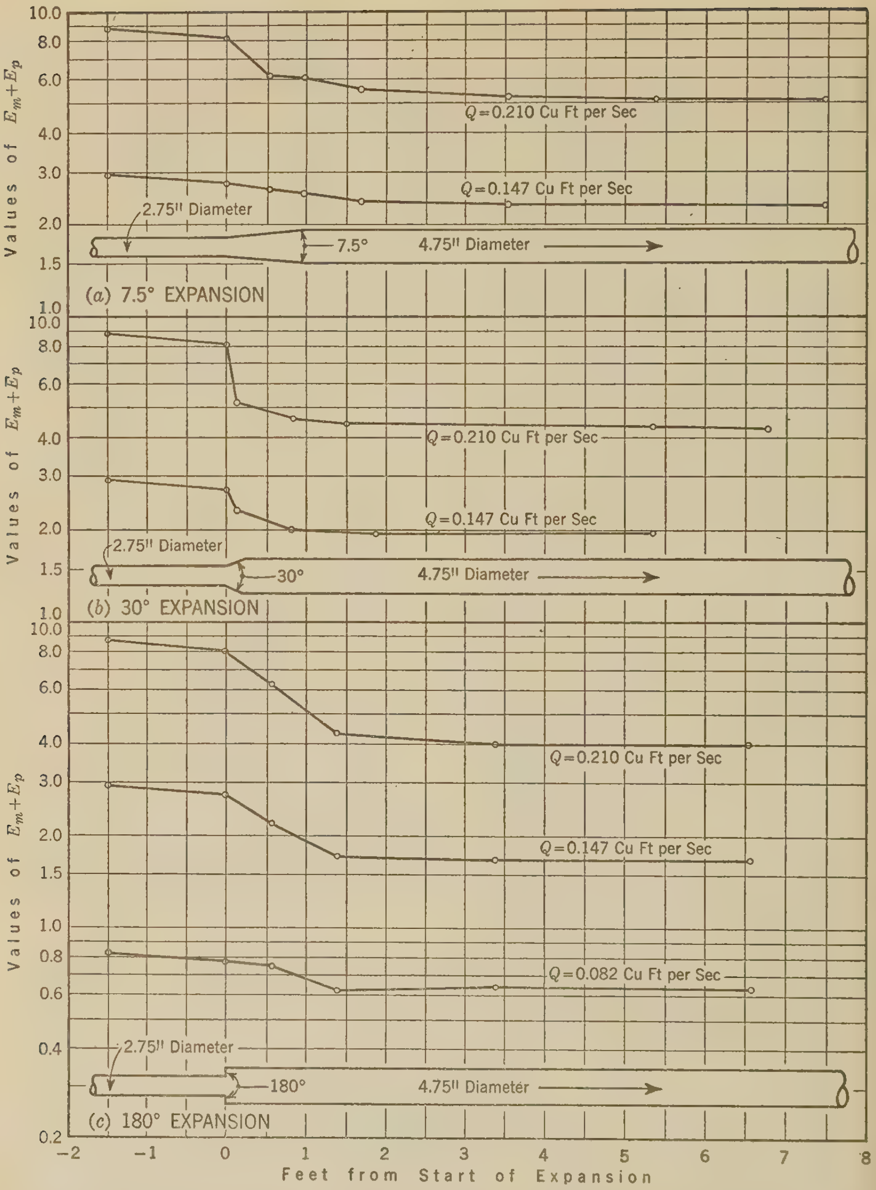

FIG. 6.—VARIATION OF KINETIC PLUS POTENTIAL ENERGY OF MEAN FLOW

the turbulence generated in the more gradual 30° expansion is of larger scale and thus gets into the main stream more quickly.

If the values of E_t are added to $(E_m + E_p)$ it will be seen that changes in the turbulence energy are minor in comparison to the changes in $(E_m + E_p)$. In other words, the total energy loss occurring is several times the maximum amount of turbulence that is generated in any of the expansions. This appears to be one of the most significant facts coming out of these studies. What, then, is the exact rôle played by the turbulence? It appears that the major portion of the kinetic energy of mean flow is lost directly due to the locally high, and quite variable, shear stresses that are set up in the fluid on the outside of the high-velocity jet. The turbulence created is a kind of by-product which, of course, uses up some of the kinetic energy of the mean flow. Although a by-product of the major energy-loss process, the turbulence, none the less, has an important influence on slowing down the high-velocity water and in shaping the velocity distribution curve.

From the values of $(E_m + E_p)$ it was possible to calculate the actual loss of energy. This was done by taking the difference between the values of $(E_m + E_p)$ at the start of the expansion and at about twenty to thirty diameters beyond the expansion and subtracting from this the normal friction in the conduit between the two points. In the conical expansions the friction loss was calculated by summing the loss for increments along the expansion. The values of the energy loss, expressed in feet of water, are shown in Table 4. Also, the efficiency of the energy conversion is given, which is expressed as:

$$\eta = \frac{(h_2 - h_1)\, Q\, \gamma}{E_{m1} - E_{m2}} \qquad \text{(7)}$$

in which $(h_2 - h_1)$ is the rise in pressure head (after correction for friction loss), and the denominator is the difference in the mean kinetic energy.

TABLE 4.—ENERGY LOSS, h, AND EFFICIENCY, η, OF VARIOUS EXPANSIONS FROM 2.75-IN. PIPE TO 4.75-IN. PIPE

Flow (cu ft per sec)	7.5° Expansion		15° Expansion		30° Expansion		180° Expansion	
	h (ft)	η (%)	h (ft)	η (%)	h (ft)	η (%)	h (ft)	η (%)
0.082	....	....	....	....	....	....	0.027	51
0.147	0.037	81	0.058	70	0.081	59	0.108	45
0.210	0.211	60	0.238	54	0.277	48	0.296	43

The energy-loss values, as given in Table 4, do not appear to vary consistently with rate of flow. Checking the loss figures given for the 180° expansion with the values obtained from the Borda formula indicates that for the higher rates of flow the figures given are appreciably larger. The reason for these apparent discrepancies is that the kinetic energy of the mean flow in the small pipe is considerably greater than that which would be obtained using the mean velocity in the cross section. For instance, for the flow of 0.210 cu ft per sec, the ratio of the actual to approximate total kinetic energy in the 2.75-in. pipe is 1.42; for the flow of 0.147 cu ft per sec, it is 1.11; and, for 0.082 cu ft per sec, it is 1.02. The variation in this ratio accounts for the fact that the loss does not vary as the square of the rate of flow. For 0.082 cu ft per sec, the loss in the sudden expansion checks that obtained from the Borda equation very well, and the reason is apparent. It is thus obvious that the energy loss in an expansion will depend on the velocity distribution at the inlet side, and this distribution must be taken into account if the actual loss is to be correctly evaluated. It is noted that the efficiency of energy conversion in the 30° expansion is not much better than in the 180° expansion.

H. CONCLUSIONS

Analysis of experimental data obtained in this investigation leads to the following conclusions relating to the energy conversion and dissipation processes in circular pipe expansions:

1. The largest energy changes occurring in the flow through any expansion are those due to the reduction in velocity and change in shape of the mean velocity distribution curve.

2. The maximum total turbulence energy is a small part of the total energy change taking place. However, at any particular section the turbulence energy may be a considerable part of the total kinetic energy. For the 30° expansion and 180° expansion, the maximum value of the ratio of turbulence energy to mean kinetic energy, E_t/E_m, rises to 0.50. For the 7.5° expansion it is only 0.17.

3. The maximum ratio between the root-mean-square value of the axial velocity fluctuation, $\overline{u^2}$, and the mean center velocity occurs in the 30° expansion, and is 0.50. The transverse velocity fluctuation is always appreciably less than the axial.

4. The loss of energy occurs more rapidly in the 30° expansion than in the sudden expansion.

5. The principal loss of energy occurs at the outside of the high-velocity stream as it passes into the expansion due to the very high shear developed. This loss is represented by a direct conversion into heat of the mean kinetic energy of the flow. The turbulence produced is but a by-product of this energy conversion process. Of course, the turbulence created also represents lost energy, since eventually it is dissipated into heat.

6. The turbulence created in the expansion plays an important rôle in helping to reduce the velocity of the center portion of the stream. Larger scale turbulence is more effective in this than in fine-grained turbulence.

7. The energy lost in an expansion and the efficiency of the conversion process are influenced considerably by the distribution of the mean velocity at the inlet to the expansion. High ratios of center velocity to the mean velocity in the cross section produce poor efficiencies and large energy losses.

I. ACKNOWLEDGMENTS

Since the experimental work reported herein extended over a period of six years, it is natural that many individuals should have contributed their efforts. This project was sponsored by the Hydraulic Research Committee of the Society, and financial aid was provided by the Engineering Foundation. The work was done in the hydraulics laboratory of the Iowa Institute of Hydraulic Research.

F. T. Mavis, M. Am. Soc. C. E., formerly head of the Department of Mechanics and Hydraulics at the University of Iowa, at Iowa City, is responsible for starting this investigation, and for his advice, suggestions, and encouragement the writer is very grateful. Graduate students who assisted at various times with the laboratory and analytical work were: E. R. Van Driest and C. L. Pien, Assoc. Members, Am. Soc. C. E., A. R. Luecker, J. M. Robertson, and Claude Lomax, Jr., Juniors, Am. Soc. C. E., and W. M. Wachter. The National Youth Administration provided a considerable number of assistants for the statistical work.

DISCUSSION

F. T. Mavis,[13] M. Am. Soc. C. E.—In a direct fashion by patient compilation and analysis of statistical data, Professor Kalinske has presented a paper which is based on a staggering number of photographs.

Whenever water decelerates in a conduit under conditions of steady flow, there is a conversion of kinetic to potential energy. This conversion is not made without loss in excess of that induced by the surface of the conduit. The losses which accompany this conversion of energy have been studied intensively and the methods of study can be classified broadly into two categories:

(1) Quantitative experiments on a particular installation or model from which over-all losses incident to the conversion can be deduced; and
(2) Mathematical analyses of general and often abstract motions of an idealized fluid.

Category (1) is the usual engineering method. It can lead to valid engineering deductions and can contribute much directly to the fundamental understanding of the mechanism of energy conversion. Both approaches to the study of the conversion of kinetic to potential energy have yielded significant contributions.

The engineering approach is usually to seek a particular solution of a problem—leaving to others the related problems of correlating, generalizing, and rationalizing the data into a general pattern. The mathematical approach usually begins with an idealized problem, leads to a general hypothesis, and leaves to others the task of applying these generalizations to particular cases. Too often these two approaches to the problem—springing from opposite sides of a chasm of unknowns—do not meet to form a well-integrated structure of theory and experiment. The investigation reported by Professor Kalinske was directed toward studying the internal mechanism of energy conversion in conduits with straight axial expansions and toward forging another link in this integrated rational structure.

The purpose of this discussion is (*a*) to classify the elementary phenomena of conversion of kinetic to potential energy in hydraulic engineering, and (*b*) to present selected bibliographic data which may suggest the course of further laboratory studies and illumine the weakness or strength of the mathematical theory.

Table 5 classifies the fundamental problems of the hydraulic conversion of kinetic to potential energy and the significant geometrical variables for each case. Common to all the problems listed are such physical variables as rate of discharge, roughness of conduit, state of motion, head loss, and velocity distribution. There are but two major classes of flow. Within the reach of the conduit under consideration the flow is either fully confined by the walls of the conduit (Table 5(*b*)), or not fully confined (Tables 5(*c*) and 5(*d*)), in which case there is a free water surface in the reach. The interplay of unbalanced forces and accelerations of liquid elements is maintained in the first case by changing pressures along fixed boundaries, and in the second case by an aperiodic pulsating free water surface or by a complex zone of air-entrained water.

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[13] Prof. and Head, Civ. Eng. Dept., Carnegie Inst. Tech., Pittsburgh, Pa.

A conduit which fully confines the flow within a reach under consideration may induce deceleration of the liquid by reducing the velocity of flow or by changing its direction. There are four primary elements:

1. A straight axial expansion which is represented in its simplest form by the lower leg of a venturi meter, by a straight, conical draft tube, and by similar diffusers;[14]

2. An axial-to-radial expansion of which the spreading draft tube is an example;[15,16,17,18,19,20,21]

3. An elbow in which the flow can be decelerated by changing the direction of flow only, or by a combination of changes in direction and changes in average velocity. Elbows may, therefore, be subdivided further into—

   (*a*) Prismatic elbows in which the cross section and shape of the conduit remain unchanged;

   (*b*) Bisymmetrical elbows in which the cross section or shape is symmetrical with reference to two planes—one in the plane of the elbow axis and the other normal to that plane and bisecting the straight approach and outlet conduits; and

   (*c*) Expanding elbows represented by elbow draft tubes.

4. A manifold which may be represented by the filling and emptying conduit system of the more common American type of navigation locks. (In the sense that the water is discharged into a lock chamber through submerged ports during the filling stage, the manifold may also be classified as an example of a partly confined jet.[22])

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[14] "Report of the Committee on Hydrodynamics," by Hugh L. Dryden, Francis D. Murnaghan, and H. Bateman, *Bulletin No. 84*, National Research Council, Washington, D. C., 1932, pp. 469–481 (bibliography, 68 entries).

[15] "Report of the Committee on Hydrodynamics," by Hugh L. Dryden, Francis D. Murnaghan, and H. Bateman, *Bulletin No. 84*, National Research Council, Washington, D. C., 1932, pp. 481–492 (bibliography, 58 entries).

[16] "A Study of the Hydrodynamics of Spreading Draft Tubes," by Andreas Luksch, thesis presented to the State Univ. of Iowa, Iowa City, in June, 1935, in partial fulfilment of the requirements for the degree of Doctor of Philosophy (bibliography, 52 entries).

[17] "Hydraulic Tests of Small Diffusers," by F. T. Mavis, Andreas Luksch, and Hsi-Hou Chang, *Bulletin No. 13*, Studies in Engineering, Univ. of Iowa, Iowa City, 1938.

[18] "Flow Characteristics in Elbow Draft-Tubes," by C. A. Mockmore, *Transactions*, Am. Soc. C. E., Vol. 103 (1938), pp. 402–464.

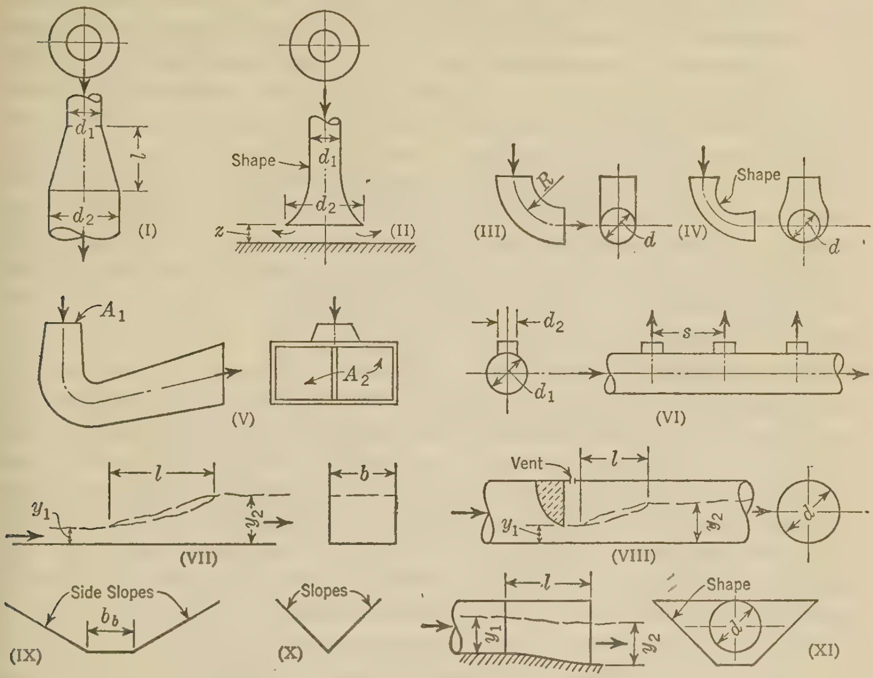
[19] "Bibliography of Important Papers on Draft Tubes Since 1913," National Bureau of Standards, U. S. Dept. of Commerce, Appendix I to report on investigation of draft tubes conducted for the Tennessee Valley Authority, June 30, 1935 (bibliography, 78 entries).

[20] "Flow of Water Around Bends in Pipes," by David L. Yarnell and Floyd A. Nagler, *Transactions*, Am. Soc. C. E., Vol. 100 (1935), pp. 1018–1043.

[21] "Flow of Water Around 180-Degree Bends," by David L. Yarnell and Sherman M. Woodward, *Technical Bulletin No. 526*, U. S. D. A., 1936 (bibliography, 19 entries).

[22] "Lock Manifold Experiments," by Edward Soucek and E. W. Zelnick, *Transactions*, Am. Soc. C. E., Vol. 110 (1945), p. 1357.

TABLE 5.—Elements of Hydraulic Conversion of Kinetic to Potential Energy



(*a*) Sketches of Pipe and Channel Sections

Element		Geometrical variables (see sketches in Table (*a*))
Description	No.	
		(*b*) Flow Fully Confined
Straight axial expansion	i	Diameters d_1 and d_2, length l, and the shape
Axial to radial expansion	ii	Diameters d_1 and d_2, the shape, clearance z, and the inserts or the vaning
Prismatic elbow	iii	Diameter d, radius r_b, and the central angle
Bisymmetrical elbow	iv	The shape and the vaning
Expanding elbow	v	Areas A_1 and A_2
Manifold	vi	Diameters d_1 and d_2, spacing s, and the shape of the passages
		(*c*) Flow Not Fully Confined; Supercritical Initial Velocity, V_o
Hydraulic Jump—		
Rectangular conduits	vii	Depths y_1 and y_2, breadth b, and length l
Circular conduits	viii	Diameter d and the venting
Trapezoidal conduits	ix	Bottom width b_b and the side slopes
Triangular conduits	x	Side slopes (special case of sketch ix)
Jets	....	(Surface, plunging, and partly confined)
		(*d*) Flow Not Fully Confined; Subcritical Initial Velocity, V_o
Transitions	xi	Depths y_1 and y_2, diameter d, length l, and the shape

Decelerating flow in a conduit with a free surface (flow not fully confined by the walls of the conduit) has two major subdivisions:[23,24,25,26,27]

I. If the velocity in a reach is supercritical (velocity head greater than half the hydraulic mean depth), the water either passes through an hydraulic jump, or continues in some form of jet. The hydraulic jump has been studied intensively in rectangular conduits, circular conduits, trapezoidal conduits, and conduits of other sections. Surface waves and plunging jets have been studied in open channel flow and submerged weirs chiefly.

II. If the approach velocity in an expanding conduit is subcritical (velocity head less than half the hydraulic mean depth), the hydraulic problem of the transition becomes primarily a matter of backwater curves. This topic has received extensive experimental and mathematical study. Practical problems in this category can be handled quite satisfactorily by combinations of laboratory study and conduit analysis.

Essentials in the study of energy conversion in flowing water are Newton's laws of motion, a complete physical concept or hypothesis, exact definitions of terms, rational induction and deduction, and objective comparisons and experiments. The engineer has usually approached the study with the objective of determining over-all losses of energy rather than with the purpose of studying the internal mechanism of energy conversion. Many guide posts to engineering judgment have been established on the basis of so-called one-dimensional solutions of problems in hydraulics. Five illustrations of "one-dimensional" approaches to problems involving conversion of kinetic to potential energy in flowing water are sketched in the following paragraphs:

A. Newton's second law of motion expresses the relationship between the unbalanced force acting on a particle and its corresponding acceleration. Precisely consistent relationships can be expressed between the work done on a particle and the change in its kinetic energy, or between the impulse of a force acting on a particle and the change in its linear momentum. Consistently applied, the principles of force and acceleration, work and energy, and impulse and momentum lead to identical conclusions. Evidence that this fact is sometimes misunderstood is the inference that Bernoulli's theorem (essentially work and energy) "applies" in some cases and that the momentum principle "applies" in others. When all factors are considered in any given problem, identical conclusions follow the logical application of the principle of work and energy or the principle of impulse and momentum. Students of hydraulics sometimes do not realize fully that apparent "differences" are but logical deductions which follow from differences in viewing the complete physical picture or concept.

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[23] "Hydraulics of Open Channels," by B. A. Bakhmeteff, McGraw-Hill Book Co., Inc., New York, N. Y., 1932.

[24] "The Hydraulic Design of Flume and Siphon Transitions," by Julian Hinds, *Transactions*, Am. Soc. C. E., Vol. 92 (1928), pp. 1422–1485.

[25] "Hydraulic Jump in Trapezoidal Channels," by C. J. Posey and P. S. Hsing, *Engineering News-Record*, December 22, 1938, pp. 797–798.

[26] "Determining the Energy Lost in the Hydraulic Jump," by J. C. Stevens, *ibid.*, June 4, 1925, pp. 928–929.

[27] "Hydraulics of Steady Flow in Open Channels," by Sherman M. Woodward and Chesley J. Posey, John Wiley & Sons, Inc., New York, N. Y., 1941.

B. The principle of work and energy applied to the flow of a liquid in a stream tube leads directly to Bernoulli's theorem. If real water flows in a real conduit, the walls of the conduit exert a drag on the flowing liquid which opposes the motion. In mathematical derivations and algebraic manipulations, if the physical pictures are incomplete or inconsistently simple, "other factors" are sometimes introduced presumably as logical conclusions rather than as hypotheses whose validity must eventually be tested by experiments.

C. The principle of impulse and momentum applied to declerating flow of water in a level open conduit leads to an expression for depths of flow adjacent to the hydraulic jump. Simplé assumptions lead to an expression for head loss in terms of alternate or sequent depths of flow which is in substantial agreement with laboratory and field observations. Suffice it to state that, if the principle of impulse and momentum is logically applied to accelerating flows of water in a level open channel, the valid conclusion must be reached that the water surface promptly becomes level as a consequence of the basic definition of a fluid. On any horizontal plane the fluid cannot indefinitely resist shearing stresses which must be set up by unbalanced pressures parallel to the plane.

D. The principle of impulse and momentum applied to decelerating flow in a 90° expansion leads to differences in pressure above and below the expansion which depend on the assumed pressure distribution on the face of the expanding section. Reasonable assumptions of this pressure distribution lead to computed head losses which are in substantial agreement with laboratory observations.

E. The principle of impulse and momentum applied to "nonresilient impact" of two particles moving in the same direction with different initial velocities leads to an expression for head loss which is identical with the expression for head loss in a 90° expansion in a closed conduit. In the over-all analysis of head loss in a sudden expansion, nothing need be assumed as to the detailed mechanism of conversion. Particles at the boundary of the jet approaching the expansion come in contact with slower moving particles to emerge—after mixing—with reduced velocity. The term "inelastic impact," often used to designate this phenomenon, implies an inexact concept since a confined liquid is "elastic" in the sense that it has a well-defined bulk modulus of elasticity. Individual globules of an unconfined liquid are not resilient, however, in the sense that mechanical energy can be stored in an unconfined globule by virtue of its elastic deformation. Hence, the term nonresilient impact seems more descriptive. This nonresilient impact of liquid masses may be viewed as a physical basis for analyzing the conversion of kinetic to potential energy, and it may throw added light on the author's second conclusion that "The maximum total turbulence energy is a small part of the total energy change taking place."

Professor Kalinske has outlined briefly certain concepts relating to turbulence and he has referred to selected literature. The engineer will doubtless feel a void between these concepts and their application to even an elementary engineering situation. If statistical methods of studying the phenomena of turbulence and the nonrecoverable components of kinetic energy in a stream are to be applied to a given conduit, one must know the primary pattern of flow through the conduit, the intensity of turbulence at any given point in the conduit, and the scale of turbulence at the corresponding point. At present, if not inherently, these data must be obtained experimentally and at the expense of considerable refinement in laboratory techniques. If the data are obtained from model tests with the intention of translating results from one conduit to another, it follows that the two conduits must be geometrically similar, the intensity of turbulence at corresponding points in the two conduits must be identical, and the scale of the turbulence in the two conduits must be geometrically similar.

The practical significance of the statistical approach to fluid turbulence in the study of the conversion of kinetic to potential energy has not yet reached far beyond the laboratory and the computer's desk. The engineer certainly cannot look to the analyst of turbulence problems for the design of a diffuser, a draft tube, or the hydraulic system for a lock. However, if the basic theory of fluid turbulence prompts the experimenter and the engineer to look more critically into the effect of baffles and the internal similarity of flow patterns, the mathematical approaches—as yet of limited direct application—may eventually make their effects felt in more important problems of hydraulic engineering analysis and design.

The study described in this paper was initiated by the writer in 1936 at the invitation of J. C. Stevens, President, Am. Soc. C. E., chairman of the Society's Committee on Hydraulic Research, and progress reports have been abstracted periodically in *Civil Engineering*. Between 1936 and 1939 a bibliographic study was completed by Andreas Luksch, graduate assistant in hydraulics at the State University of Iowa, and selected tests were conducted by Mr. Luksch; by Edward R. Van Driest, Assoc. M. Am. Soc. C. E., and Arthur R. Luecker, Jun. Am. Soc. C. E., research assistants in mechanics and hydraulics; and by Hsi-Hou Chang, graduate student. In 1939 and subsequently the work on the project was directed by Professor Kalinske.

Credit for prior development of the photographic technique which played such a significant rôle in the studies reported by Professor Kalinske is due to Edgar E. Ambrosius, John C. Reed, and Henry F. Irving[28] who pursued a related program of basic studies in the hydraulics laboratory of the University of Illinois at Urbana which was under the direction of M. L. Enger, M. Am. Soc. C. E.

J. C. STEVENS,[29] PRESIDENT, AM. SOC. C. E.—Some of the mental confusion that has surrounded the particular problem of converting kinetic energy to potential energy in flow expansions has been dispelled by Professor Kalinske. He has shown that turbulence, heretofore looked upon as the principal source of ultimate energy loss, is merely a by-product of the direct loss (energy conversion into heat) in the zone of shear surrounding the jet.

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[28] "Study of the Flow of Water Through a Glass Pipe," by Edgar E. Ambrosius, John C. Reed, and Henry F. Irving, preprinted papers and program, Aeronautic and Hydr. Divs., A.S.M.E., Summer Meeting, June 19, 20, and 21, 1934, George Reproduction Co., San Francisco, Calif., pp. 83-89.

[29] Cons. Hydr. Engr. (Stevens & Koon), Portland, Ore.

The author found that the maximum energy of turbulence was only half the maximum energy of the mean flow; and, of course, only a part of this turbulent energy is converted into heat as the intensity diminishes downstream. For example, in the 30° expansion, the total energy lost was 4.3 ft-lb per sec for 0.210 cu ft per sec (Fig. 6); and the maximum energy of turbulence was 0.781 ft-lb per sec (Table 2). Even if all this turbulence were lost (converted into heat), it would represent only 18% of the total energy similarly lost in the expansion.

The efficiency of an expansion is subject to various interpretations. The author has used the ratio of the gain in potential energy to the loss in kinetic energy (both in foot-pounds per second) as a measure of the efficiency of the expansion. This is primarily a measure of the recovery of velocity head. It seems to the writer that efficiency is something more than this. If the pipe length from the beginning of the expansion to the point where normal flow is restored in the larger pipe (say, sixteen diameters) is considered as a mechanism for converting from one state of flow to another, the efficiency becomes the ratio of the output energy to the input energy as is the case with any other device.

Curiosity caused the writer to prepare Table 6, comparing the orthodox concepts of efficiencies with those shown by the author in Table 4. The input and output energies were taken from Fig. 6. In either expansion, and by either method, the lower the flow, the higher the efficiency would be—approaching 100% for zero flow.

The tedium of analyzing so many movie-camera frames prevented the extension of the experiments to 60° expansions and 90° expansions. It would be interesting to know what rate of expansion yields the highest rate of energy loss—in other words, the maximum scale of turbulence.

TABLE 6.—Comparison of Efficiencies

Flow (cu ft per sec)	7.5° Expansion				30° Expansion				180° Expansion			
	Input, Station 0 ft $\left(\frac{E_{m1}}{E_{p1}}\right)$	Output, Station 6.5 ft $\left(\frac{E_{m2}}{E_{p2}}\right)$	Efficiency (%)		Input, Station 0 ft $\left(\frac{E_{m1}}{E_{p1}}\right)$	Output, Station 6.5 ft $\left(\frac{E_{m2}}{E_{p2}}\right)$	Efficiency (%)		Input, Station 0 ft $\left(\frac{E_{m1}}{E_{p1}}\right)$	Output, Station 6.5 ft $\left(\frac{E_{m2}}{E_{p2}}\right)$	Efficiency (%)	
			Orthodox (η')	Table 4 (η)			Orthodox (η')	Table 4 (η)			Orthodox (η')	Table 4 (η)
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
0.082	..	..	..	..	..	..	..	..	0.78	0.63	81	51
0.147	2.8	2.4	85	81	2.8	1.9	68	59	2.8	1.7	61	45
0.210	8.2	5.1	62	60	8.2	4.3	52	48	8.0	4.0	50	43

Professor Kalinske used the only method so far devised, of which the writer is aware, for measuring the components of filament velocities. It is hoped some method will be devised whereby such components can be measured with less time-consuming labor. Perhaps the method evolved at Columbia University,[30] in New York, N. Y., might be extended to the measurement of filament velocities in pipes of nonmagnetic material.

Research into these fundamental concepts should not end with this paper; this study is just a good beginning. A most excellent approach to determining the nature of hydraulic friction is that presented by Boris A. Bakhmeteff, M. Am. Soc. C. E., and William Allan, Assoc. M. Am. Soc. C. E., in February, 1945.[31] The findings in the researches by Professor Kalinske and by Professors Bakhmeteff and Allan dovetail most admirably and open new vistas for the hydraulician.

As noted by the author (see heading, "Acknowledgments"), the project reported in the paper was sponsored by the Hydraulic Research Committee of the Society which, in 1938, became the Committee on Hydraulic Research of the Hydraulics Division. The Committee is convinced that little further progress may be expected in the solution of many hydraulic problems until the internal mechanics of hydraulic friction and energy conversions in flowing water are better known. In this respect the committee is justly proud of the work Professor Kalinske has done and of the results he has obtained.

BORIS A. BAKHMETEFF,[32] M. AM. SOC. C. E.—Professor Kalinske's paper marks a serious advance in the science of fluid mechanics. It reveals facts, relating to turbulence in "form resistance" patterns, wholly unknown before and which substantially further present-day knowledge in this obscure and generally uncharted realm. The most significant result is the disclosing of the quantitative inner relations between the over-all energy loss E_r, occurring in the course of expansion, and the turbulent energy E_t, apparent and measurable in the turbulent fluctuations. Although the paper deals with flow expansions only, the facts, as established, seem to be typical in all probability of other "form" or "local" resistances.

The aforesaid terms are meant to designate the concentrated losses of energy known to accompany the abrupt variations of flow geometry, such as the losses in bends, knees, valves, back of bluff bodies, etc. It is generally recognized that the generic cause conducive to "form" losses is "separation," with the highly strained and labile "discontinuity sheets" serving as seats of concentrated turbulence engendering activity. It is accepted further that the turbulent eddying composite, featured by the energy E_t, represents the final phase of a complex process by the force of which flow energy is withdrawn from the store inherent to the streaming, and is subjected to "conversion" into vortical eddying form. It has been surmised, finally, that, in the course of the cyclic deformations of the free discontinuity sheet which accompany "transition" from the laminar to the turbulent regimen, only a portion of the used up flow energy reappears in the ostensible turbulent manifestations, and that the remainder of such energy is dissipated in loco in the process of conversion, constituting what may be termed a "conversion loss."[33] However, all prior inferences regarding the possible energy balance in the turbulent process were necessarily conjectural, as no factual evidence existed to permit reliable quantitative interpretations. It is only now, in the light of Professor Kalinske's findings, that the actual situation has been ascertained reliably.

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[30] "Electromagnetic Velomentry I. A Method for the Determination of Fluid Velocity Distribution in Space and Time," by Alexander Kolin, *Journal of Applied Physics*, Vol. 15, No. 2, February, 1944, pp. 150–164.

[31] "The Mechanism of Energy Loss in Fluid Friction," by Boris A. Bakhmeteff and William Allan, *Proceedings*, Am. Soc. C. E., February, 1945, p. 129.

[32] Prof., Civ. Eng., Columbia Univ., New York, N. Y.

[33] "The Mechanism of Energy Loss in Fluid Friction," by Boris A. Bakhmeteff and William Allan, *Proceedings*, Am. Soc. C. E., February, 1945, pp. 143–147.

It would seem that the situation is best illustrated by expressing the different pertinent quantities in terms of the initial kinetic energy E_{m1}, at the mouth of the small conduit. In fact, since the denominator in Eq. 7 may be represented with sufficient approximation by

$$E_{m1} - E_{m2} = E_{m1}\left[1 - \left(\frac{D_1}{D_2}\right)^4\right] \dots\dots\dots\dots\dots\dots (8)$$

the over-all relative loss E_r/E_{m1} may be expressed by

$$(1 - \eta)\,[D_1 - (D_1/D_2)^4] \dots\dots\dots\dots\dots\dots\dots\dots (9)$$

—or, for the 2.75-in. to 4.75-in. expansion, by 0.89 $(1 - \eta)$. The data required for the calculations may be taken from Tables 2 and 4, as well as from Fig. 4. One also finds therein the information for establishing the ratio E_t (max)$/E_{m1}$ of the maximum measured turbulence energy to the initial E_{m1}, and with this the final ratio of E_t (max)$/E_r$. The values are assembled in Table 7.

TABLE 7.—VALUES OF ENERGY RATIOS

Flow (cu ft per sec)	30° EXPANSION			180° EXPANSION		
	$\frac{E_r}{E_{m1}}$	$\frac{E_t\text{ (max)}}{E_{m1}}$	$\frac{E_t\text{ (max)}}{E_r}$	$\frac{E_r}{E_{m1}}$	$\frac{E_t\text{ (max)}}{E_{m1}}$	$\frac{E_t\text{ (max)}}{E_r}$
0.082	....	....	....	0.425	0.090	21%
0.147	0.365	0.115	31%	0.470	0.095	19%
0.210	0.474	0.105	22%	0.506	0.082	16%

With all possible allowances for experimental uncertainties devolving (as the author states) from the colossal observational difficulties, the order of magnitude is established beyond doubt. The turbulent energy E_t at its maximum is but a small portion of the total loss E_r. No wonder the author is prompted to refer to the turbulence created as a "kind of by-product," attributing the major loss to the intense and variable local shearing stresses, set up on the outside of the high-velocity jet. The description fits the rather striking circumstances, provided the wording is not misinterpreted. The fact is, of course, that the mechanism of energy loss is an entity, a composite whole, no constituent portion of which should be considered separate and alone. Turbulence is engendered ultimately for the very reason that separation originates intensive local stresses, initially of viscous nature. In the inevitable sequence of events, these local stresses lead to vehement deformations, transforming the discontinuity sheets into eddying patterns, with the latter in turn breaking up into inordinate turbulent "swarming." In other words, the first and the last phases in this process, characterized respectively by the highly strained viscous sheets and by the ostensible turbulent fluctuations, constitute inseparable links of an organic whole. In this light, the findings of the paper, in so far as the small numerical ratios of E_t/E_r are concerned, could be qualified as having disclosed the surprisingly low efficiency of energy exchange in the turbulence producing mechanism.

In reflecting on these properties, one gains the impression that perhaps the feature of having relatively small manifestations reflect the presence of momentous processes, deeply affecting the essence of things, is rather typical of turbulence phenomena in general. For instance, consider the case of so-called established turbulence, in uniform motion in conduits. The measured turbulent energy in pipes, etc., constitutes on the average only a few per cent of the kinetic energy of the main motion. Nevertheless, the transverse mixing, concomitant to such turbulence, increases the stress structure, and with that the over-all energy loss, to many hundreds if not thousands of times the shearing forces and the losses, which would prevail if the motion at the same flow rate were to remain laminar. Again in this case, the measurable E_t is only a part of the total energy E_r withdrawn from the flow and spent on conversion.[34] In fact, the writer feels that the possible (E_t/E_r)-ratios in uniform motion, when actually measured, may prove to be even lower than the values shown in the paper for flow expansions.

In uniform motion the production of turbulence and all other features of the energy spending process unfold in equal measure in each of the successive cross sections. In other words, all the functions of the energy-loss mechanism are uniformly distributed along the length of the conduit. In the case of form resistances, on the other hand, the consecutive functions, started by separation, unfold in successive fashion over a certain distance range along the axis. Naturally one is tempted to seek, in the factual evidence presented in the paper, possible clues which would help to disclose, even if conjecturally, the physical nature of the energy-loss mechanism. Note, in this connection, that in Fig. 4, for the 180° expansion, turbulence reaches its maximum in the very first measured station at a distance of about 0.55 ft from zero. It is true that, in the case of the 30° and particularly with the 15% expansion (see Table 2), the locus of maximum E_t is shifted somewhat downstream. This could be explained, however, by the fact[35] that, in divergent cones, the separation is somewhat delayed. These facts are quite significant. Obviously, in the initial stage of separation, when the live jet first comes into contact with the "roller," friction in the discontinuity sheet is viscous, so that the shearing stresses are shaped by the Newton law $\left(\frac{\mu\, du}{dy}\right)$. Furthermore, as these viscous discontinuity sheets are highly unstable, the rolling up process to form eddying patterns is begun almost at once. In other words, the over-all distance from the beginning of separation to transition must be rather short. Beyond the transition reach, where the turbulent regimen has established itself, the high local stress may continue by force of convective transverse mixing. The pertinent question however is whether, in this post-transition stage, there is any further production of turbulence, in the sense of further investment of flow energy into vortical turbulent form; or, perhaps the conversion process, with the inherent concentrated absorption of flow energy, is confined to the short pre-transition stage. Within the latter, of course, flow energy is spent by way of viscous deformation in accordance with the generic Stokes formulation $\mu \zeta^2$, in which ζ denotes vorticity. The abrupt drop of the energy line in Fig. 6 would seem to point in the direction of the latter hypothesis. If such is the case, and if no further turbulence is engendered beyond the transition reach, then subsequent losses of flow energy would occur by reason of the transverse turbulence mixing. The dynamics of this particular loss, concomitant to "turbulent diffusion," have been discussed elsewhere.[36] In uniform flow such diffusion loss is comparatively low. The rather gentle slope of the total energy lines in Fig. 6, beyond the first abrupt drop, seems to point toward analogous conditions in the case of flow expansions.

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[34] "The Mechanism of Energy Loss in Fluid Friction," by Boris A. Bakhmeteff and William Allan, *Proceedings*, Am. Soc. C. E., February, 1945, p. 151.

[35] "Versuche über die Strömungsvorgänge in erweiterten und verengten Kanälen," by H. Hochschild, *Forschungsarbeiten des Vereines Deutscher Ingenieure*, Heft 114, 1912.

Another point, to which the writer would like to draw attention, is the prevailing custom of qualifying the losses in flow expansions, as losses of kinetic energy. Actually, in a closed conduit the mean velocities U_1 and U_2 in the initial and final sections are prescribed by continuity. Accordingly, except for the secondary effect of local velocity distributions, the corresponding kinetic energies of the average flow are predetermined and cannot be either increased or reduced. What is actually lost is pressure, for the eventual gain in potential energy, which theoretically should accompany the reduction in velocity, is consummated only partly, to the extent of $\eta\,(E_{m1} - E_{m2})$. One may take account of what happens by resorting to the basic Euler equation

$$\frac{dp}{ds} = -\rho \frac{d}{ds}\left(\frac{u^2}{2}\right) - F_r \qquad (10)$$

featuring the dynamic balance along a filament s. The eventual pressure increase developing from the inertial force, $-\rho\, u \frac{du}{ds}$, is reduced to the extent of the resultant friction traction F_r, forthcoming from the high local stresses. Note, in particular, that the F_r resultant is determined not by the absolute value of the local stress but by the transverse gradient of the latter; and, of course, it is typical of separation sheets to evidence highly concentrated stress changes. If and when separation (which is the source of high local shear) can be reduced by that or other means, the process of restoring pressure is correspondingly remedied. The paper indicates certain means to that effect, cited by H. Peters.[5] The writer would like to supplement the reference by drawing attention to some earlier research by K. Andres[37] and by H. Hochschild.[35] Both of these exceptionally complete and valuable monographs on flow expansion deal in detail with the physical and dynamical features of the phenomenon.

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[36] "The Mechanism of Energy Loss in Fluid Friction," by Boris A. Bakhmeteff and William Allan, *Proceedings*, Am. Soc. C. E., February, 1945, p. 153.

[37] "Versuche über die Umsetzung von Wassergeschwindigkeit in Druck," by K. Andres, *Forschungsarbeiten des Vereines Deutscher Ingenieure*, Heft 76, 1909.

In particular, the Andres experiments reveal the extent to which pressure restoration is affected by the degree of turbulence in the oncoming streaming. Restoration is at its lowest when the initial flow is smooth and the inherent turbulence intensity small. The higher the disturbances, imposed by artificially increasing the turbulence, the smaller will be the effects of the forthcoming separation and with that the higher the rate of the gained pressure. At the time when the Andres report was written (1909), the science of fluid mechanics was in its infancy and the aforesaid results might have appeared paradoxical. At present, one sees at once the analogy with the well-known effect of a turbulent boundary layer delaying separation and thus reducing the form resistance of an immersed sphere or of a cylinder. The best remedy for counteracting pressure losses, as found by Mr. Andres, was to impose a rotary motion on the expanding jet. Through such means he raised the restoration rate to more than 90%. A similar effect is achieved, of course, by removing the reversed flow film next to the solid boundary, by artificial suction.[38]

The Hochschild experiments are especially valuable in that they offer detailed velocity and pressure distribution profiles in the diverging sections. Characteristic is the lack of symmetry in the expanding flow, and the fact that the actual expansion of the jet extends in many instances far beyond the end of the diverging tube. On the whole, the different Hochschild diagrams, featuring the energy distribution in flow expansions, complement most usefully the over-all flow pictures presented in Figs. 1 and 2.

The sole intention prompting these remarks is to possibly add toward clarifying the different topics, presented in pioneer fashion in Professor Kalinske's excellent paper. Indeed, the engineering profession is deeply indebted to the author for a contribution which raises so many new and fascinating questions to be attacked by future research.

E. R. Van Driest,[39] Assoc. M. Am. Soc. C. E.—The chief advantage of this paper is that it gives a quantitative description of the distribution of turbulence in, as well as the variation of pressure along, a fluid as it moves through an expanding conduit. Such information is of great concern to the fluid flow analyst.

The author's findings concerning the relative amounts of molecular internal energy and turbulence energy are very interesting and prompt one to calculate the rise in temperature if no turbulence energy appeared. For the high flow through the sudden expansion, a simple calculation leads to a temperature change of about 0.0004 F. The measurement of such a small change would require a most sensitive thermocouple. Although the author states that temperature changes are to be neglected during the expansion process, these changes do occur for a real fluid in spite of the incompressibility.

That turbulence is a by-product of shearing action is a fact that all fluid technicians should know. It is only through shear that instability of flow and resulting turbulence arise. The effect of turbulence on shaping the ensuing velocity distribution by diffusion is well exemplified by the change in the distribution through the transition region in flow along a flat plate.

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[38] For reference to Ackeret's work on diffusers and other experiments on boundary layer control, see "Applied Hydro- and Aeromechanics," based on lectures by L. Prandtl, by O. G. Tietjens, McGraw-Hill Book Co., Inc., New York, N. Y., 1934, p. 81.

[39] Asst. Prof., Mech. Eng., Mass. Inst. Tech., Cambridge, Mass.

Although, as the author states, it was not the purpose of the paper to present further pressure-loss data, nevertheless, owing to its practical importance, it may be of interest to compare the efficiencies of the energy conversion as computed from the author's data with efficiencies determined by other experimenters. The conventional efficiency formula is

$$\eta = \frac{p_2 - p_1}{\frac{\gamma \bar{U}^2_{m_1}}{2g}\left[1 - \left(\frac{A_1}{A_2}\right)^2\right]} \qquad \text{(11a)}$$

which differs from Eq. 7 in that the velocity distribution is assumed uniform across the initial and final sections. For sudden expansions, Eq. 11*a* becomes (upon insertion of the Borda loss formula):

$$\eta = \frac{2}{1 + \frac{A_2}{A_1}} \qquad \text{(11b)}$$

From the very comprehensive paper by G. N. Patterson,[40] the writer has replotted, in Fig. 7, data obtained by H. Peters and A. H. Gibson. Calculated efficiencies are plotted in Fig. 7, using Eq. 11*a* and the author's data in Table 3 and Fig. 5. It is seen that the author's efficiency data are somewhat higher in general than would be expected from Eq. 11*b* (with which the Peters and Gibson results agree well) and somewhat lower than the test data of Messrs. Peters and Gibson at small angles of divergence.

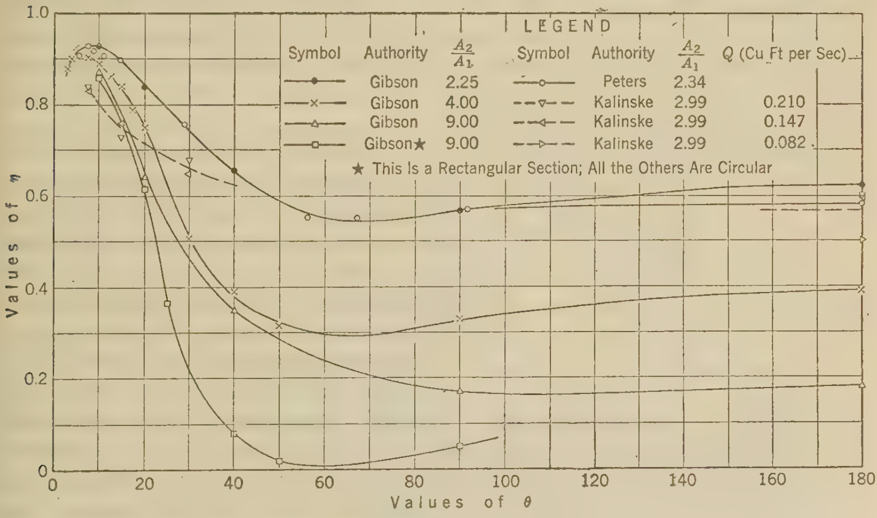

FIG. 7

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[40] "Modern Diffuser Design," by G. N. Patterson, *Aircraft Engineering*, September, 1938, p. 267.

Dimensional analysis shows that

$$\eta = f\left(\frac{\bar{U}_{m1} D_1 \rho}{\mu}, \frac{A_2}{A_1}, \theta\right) \ldots\ldots\ldots\ldots\ldots\ldots (12)$$

if the density ρ and the viscosity of the fluid are assumed to be effective. In Eq. 12, D_1 is the diameter of the small pipe and θ is the total angle of divergence. However, Mr. Gibson[41] stated that he found no systematic variation of his results with the Reynolds number, which was always greater than about 50,000 in his experiments. It is probably true that the variation of the author's data with discharge is also not an effect of the Reynolds number which was greater than 50,000 for the two highest discharges.

The writer is very happy to have assisted in the development of the photographic technique used in these studies and to have helped in the collection of some of the turbulence data.

A. R. THOMAS,[42] ESQ.—The need for data on the characteristics of turbulence in flow expansions has long been evident and Professor Kalinske's observations and analysis are of great interest and value.

The data presented refer to circular pipes, but the problem of designing expansions in open channels is even more complex and is of considerable importance where the channel downstream of the expansion is erodible, because a poor design often results in the creation of a return eddy which plays havoc with the side banks. The main principles are the same in open channels as in circular pipes, and, although there are added complexities due to cross-sectional shape and a free water surface, many of the conclusions drawn for the one case should be applicable to the other.

In the experiments described, the flow appears to have been symmetrical about the central axis. The existence of an adverse pressure gradient in the expansion, however, may result in instability of flow pattern, due to which the main stream moves to one side, with the formation of slack water or a return eddy on the other, as shown in Fig. 8.

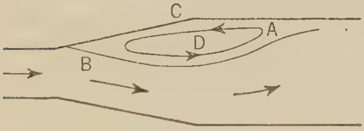

FIG. 8.

The maintenance of the return eddy depends on a balance of forces which may be expressed in a simplified form as follows:[43] Because of the conversion of kinetic energy into pressure energy, the pressure is greater at point A than at point B, creating conditions favorable to flow from points A to B. Considering the eddy as a whole, its equilibrium is maintained by the balance between the force due to this pressure gradient, acting upstream, the force due to the lateral influx of momentum from the main stream between points A and B, due to turbulence, acting downstream, and the resistance of the side wall and bed, acting against the flow. Local pressure differences are created by the curvature of flow, determined to some degree by the shape of expansion, and the shape of eddy adjusts itself to maintain the balance.

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[41] "On the Flow of Water through Pipes and Passages Having Converging or Diverging Boundaries," by A. H. Gibson, *Proceedings*, Roy. Soc. of London, Series A, Vol. 83, 1910, p. 366.

[42] Secretary, Central Board of Irrig., Simla, India.

[43] "Flow in Expansions in Open Channels," by A. R. Thomas, *Proceedings*, Punjab Eng. Cong., Lahore, Paper No. 236, 1940, p. 179.

The influx of momentum has greatest effect on the outside of the eddy, decreasing toward the side wall. If its effect at point D, the center of the eddy, is not sufficient to outbalance the adverse pressure gradient, flow at that point will be upstream; and the width of eddy will increase, contracting the main stream, raising its velocity, and so increasing the influx of momentum until equilibrium is reached. On the other hand, if the influx more than outbalances the adverse pressure gradient at point D, flow at that point will be downstream, and the eddy will contract until equilibrium is reached. If the influx of momentum is so great that it outbalances the adverse pressure gradient even at the side wall, the eddy will disappear and the entire flow will be in the downstream direction.

This is another way of describing how the lateral transmission of momentum determines the velocity distribution, but it facilitates the drawing of certain conclusions; namely: (1) The greater (*a*) the angle of divergence, (*b*) the expansion ratio, (*c*) the depth-width ratio, or (*d*) the Froude number ($U^2/2\,g\,D$), the greater is the pressure gradient—hence the larger is the eddy; and (2) the greater the turbulence, the greater is the influx of momentum—hence the smaller is the eddy.

As conclusion (1) assumes turbulence to be independent of the variables considered (which, as shown in the paper, is not generally true) it may have limitations. Conclusion (2) explains why measures taken to increase turbulence (such as a cross grid, or blocks on the bed) improve the flow. The effect of such measures is increased by the loss of head caused, which reduces the pressure gradient. It also explains why, turbulence being subject to scale effect, models of flow expansions do not accurately represent larger prototypes.

It is also clear that, in addition to the angle of divergence, the expansion ratio, depth : width ratio, Froude number, and intensity of turbulence determine the behavior of expanding flow, so that results obtained with a given angle of divergence cannot be applied indiscriminately to other cases.

These conclusions, however, are only qualitative and do not give much assistance in practical design. For example, they do not indicate the maximum permissible angle of expansion in a given case or the most suitable side curves. A quantitative expression in terms of the discharge and channel dimensions can be derived for pressure gradient, modified by loss of head based on empirical data, but it has not so far been possible to derive an expression for the lateral transfer of momentum due to turbulence in expanding flow. It is here that experimental and analytical work, such as is presented by Professor Kalinske, will be of value, and it is hoped that he will be able to extend his researches to other cases.

A. A. KALINSKE,[44] ASSOC. M. AM. SOC. C. E.—The data presented in this paper were given in considerably abbreviated form because of space requirements; and, therefore, careful study of the paper is necessary in order that the reader may obtain a proper picture of the variations in turbulence, mean velocity, pressure, and energy for the different expansions. Complete graphical representation of all the data would have been desirable. Most of the discussers were acquainted with the general nature of the investigation, and the results obtained, through personal correspondence and contacts and progress reports. Doubtless, the practical hydraulic engineer concerned solely with over-all pressure losses in conduits may not have appreciated completely some of the basic facts uncovered in this study.

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[44] Director of Fluid Mechanics, Appliance Corp. of America, Milwaukee, Wis.

The author would like to reemphasize what he considers the most important fact revealed by this study. Professor Bakhmeteff's discussion very effectively elucidates this item—namely, that, although form resistance in fluids is accompanied by the generation of considerable visual turbulence, the energy represented by this turbulence is relatively small compared to the total loss. The major portion of the energy is lost by direct conversion into heat through viscous action in the so-called "surfaces of discontinuity" where the turbulence originates. It appears that this phenomenon also occurs for flow in straight pipes and in turbulent boundary layers along straight surfaces. However, although no direct evidence is available to prove this theory definitely, it is hoped that the fundamental idea, as to how energy is converted into heat (lost) in fluid flow, may form the basis for other studies and analyses that will increase knowledge of fluid flow under turbulent conditions still further.

The paper revealed some information relating to the over-all efficiency of the energy conversion process in conical expansions. Messrs. Bakhmeteff, Stevens, and Van Driest commented on this particular item. The significant fact seems to be that the shape of the mean velocity distribution across the section of the entrance to the expansion has considerable influence on the efficiency of the energy conversion process, and must be taken into account in any efficiency calculations. Since roughness of the approaching conduit influences the velocity distribution, it appears that this is another variable that must be added to those given in Eq. 12. The author does not agree with Mr. Stevens that the efficiency of an expansion is subject to various interpretations. The efficiency must be calculated on the basis of what proportion of the initial kinetic energy is converted into pressure. The ratio of total "output energy" to "input energy" would depend on the total pressure on the system which, of course, does not control the flow phenomena in the expansion, unless cavitation conditions are approached.

Professor Mavis has outlined very completely the many hydraulic problems which concern expansion of flow. His discussion and the literature he cites form a most excellent review of practical hydraulic phenomena. In this field further knowledge relating to conversion of kinetic to potential energy would be helpful. Mr. Thomas stresses some items that must be considered if engineers are to understand flow expansion phenomena in open channels more fully. He states correctly that similarity of Froude numbers does not necessarily mean similar turbulence conditions, since the formation of eddies is not controlled by gravity forces.

The writer thanks the discussers for their comments and the additions to the subject covered by the paper. He hopes that basic research of this type will occupy a more prominent place in the activities of hydraulic research laboratories.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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Paper No. 2274

INTERRELATION OF CERTAIN STRUCTURAL CONCEPTS

By Camillo Weiss,[1] M. Am. Soc. C. E.

With Discussion by Messrs. Ralph W. Stewart, R. C. Brumfield, Max W. Strauss, Leon Beskin, Alexander Dodge, William C. Spiker, T. F. Hickerson, Dean F. Peterson, Jr., A. A. Eremin, and Camillo Weiss.

Synopsis

A study in restraint as it affects engineering structures is presented in this paper. Physical interpretation is given to such structural terms as "fixed point," "carry-over factor," and "degree of restraint," and their mutual relations are established.

Introduction

The analysis of stresses and bending moments in any indeterminate structure is dependent upon the sections of the members comprising this structure. Because of the rapid increase in the number and variety of structures involving indeterminate frames there has been a correspondingly rapid increase in the methods for the computation of the stresses and moments in such frameworks.

The method of moment distribution introduced by Hardy Cross,[2] M. Am. Soc. C. E., offers a relatively rapid and precise means of computation and, in the writer's opinion, is as good a method as can be devised. It permits the analysis of a framework from a unique point of view. Whereas the method of least work is based on the concepts of work and energy; the slope-deflection method on relations governing deformations comprising the geometry of flexure; the Cross method may be considered as based on a complex of concepts that can be summed up under the term "restraint."

Work and deformation imply the action of a force. Restraint, however, is entirely a function of the structure itself. In this study an attempt has been made to correlate some of the component parts that characterize restraint. It is a purely academic study and no new methods are proposed.

Note.—Published in January, 1945, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

[1] Designer, Bethlehem Steel Co., Fabricated Steel Constr., Eng. Dept., Bethlehem, Pa.

[2] "Analysis of Continuous Frames by Distributing Fixed-End Moments," by Hardy Cross, *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 1.

The structural concepts primarily considered are the "fixed point," the "carry-over factor," and the "degree of restraint." "Fixed points" have been defined as points of zero bending moment, or fixed points of contraflexure, for unloaded spans.[3] Usually such points are thought of in connection with beams continuous over three or more supports; but two such "fixed points" exist in every member of any frame. They are called "fixed" because their position on the elastic curve is stationary. In this study the shorter term "node" has been used to designate these characteristic points.

For the purpose of distributing fixed-end moments, Professor Cross defined the carry-over factor as the ratio of the moment induced at the fixed end to the moment applied at the rotating end of a member which is on unyielding supports at both ends. In this study the term "carry-over factor" denotes the ratio of the moment induced at the restrained, but not necessarily fixed, end to the moment applied at the other end of a member which is on unyielding supports at both ends.

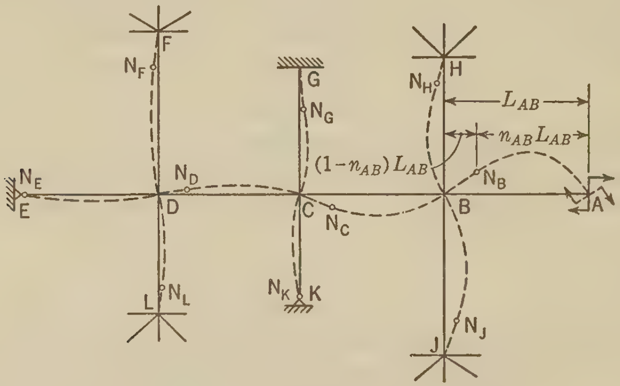

FIG. 1

"Degree of restraint" is a term which has been used loosely. A general definition of the concept is offered in this study and a law is established which states that, for m members rigidly connected to form a joint, the sum of their degrees of restraint equals $(m - 1)$. A "degree of fixation" has been defined by Thomas F. Hickerson,[4] M. Am. Soc. C. E. It is a useful concept and is a special case of this general degree of restraint. The interrelation of these three concepts is derived analytically and, to a limited extent, illustrated graphically.

NOTATION

The letter symbols used in this paper are defined where they first appear, in text or illustration, and are assembled for convenience of reference in Appendix I.

FIXED POINTS AND CARRY-OVER FACTORS

When one end of a restrained member is rotated back and forth, the wave-like elastic curves will pass through a certain node. In the special case of the beam continuous over several supports, this node is one of the "fixed points" of the given span. A similar node will occur in a member AB of any framework when one end A is rotated as illustrated in Fig. 1. Its location on the axis of the member is stationary and is a function of the stiffness of the member and of the restraint at the far end. The node (points N, Fig. 1) caused by end rotation is at the far end of the member when that end is not restrained, or at some point between the far end and the third point nearest that end, this latter position being the position for full fixity at that end. Hence the nodal intercept measured from the generating end (end A in member AB; end B in members BC, BH, and BJ, etc.) is generally more than twice the nodal intercept measured from the restrained end.

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[3] "Continuous Bridges," by D. B. Steinman in "Movable and Long-Span Bridges," by G. A. Hool and W. S. Kinne, 1923, pp. 250–252.

[4] "Statically Indeterminate Frameworks," by Thomas F. Hickerson, Univ. of North Carolina Press, Chapel Hill, 1937, pp. 8–10.

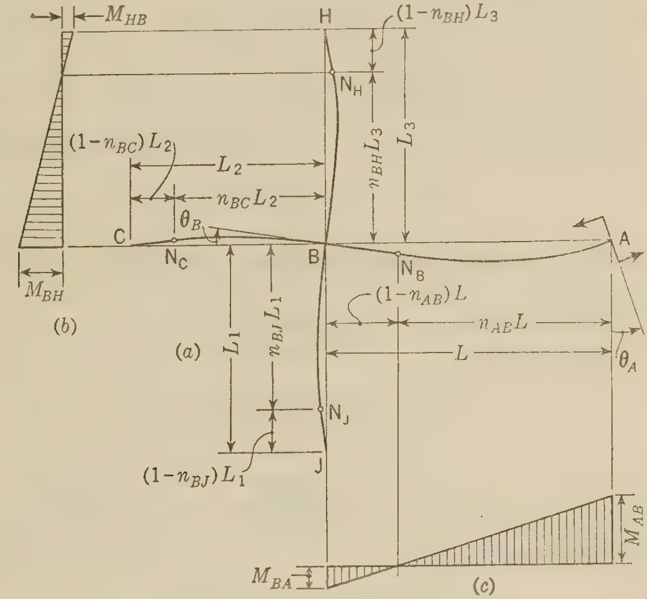


FIG. 2

The node divides each member in the ratio (n) which the generating moment bears to the induced moment. This property of the node furnishes a physical concept for a general carry-over factor f; that is, the carry-over factor is the ratio of the shorter nodal intercept to the longer nodal intercept. From end A to end B the carry-over factor is

$$f_{AB} = \frac{1 - n_{AB}}{n_{AB}} \quad \text{.........................(1)}$$

When the induced moment occurs at a fixed end, $n_{AB} = \frac{2}{3}$ and the ratio is $\frac{1}{2}$, which is defined as the carry-over factor for beams of constant cross section by Professor Cross.[2]

NODAL INTERCEPT RATIOS

The unloaded beam AB, Fig. 2(a), is restrained at point B by unloaded members BJ, BC, and BH. A moment applied at end A causes the joint to rotate through an angle θ_A. Elastic distortion of member AB induces rotations θ_B, θ_J, θ_C, and θ_H of joints B, J, C, and H, respectively, and distortion of the members meeting at those points. Assuming that no relative displacement occurs—

$$M_{BH} = 2\,E\,K_{BH}\,(2\,\theta_B + \theta_H) \qquad (2a)$$

and

$$M_{HB} = 2\,E\,K_{BH}\,(2\,\theta_H + \theta_B) \qquad (2b)$$

Dividing Eq. 2*b* by Eq. 2*a* gives the relation

$$\frac{M_{HB}}{M_{BH}} = \frac{2\,\theta_H + \theta_B}{2\,\theta_B + \theta_H} \qquad (3a)$$

Referring to Fig. 2(*b*) it follows from the similarity of triangles that

$$\frac{M_{HB}}{M_{BH}} = \frac{1 - n_{BH}}{n_{BH}} \qquad (3b)$$

By combining Eqs. 3*a* and 3*b* it is possible to express θ_H in terms of θ_B:

$$\theta_H = -\,\theta_B\,\frac{3\,n_{BH} - 2}{3\,n_{BH} - 1} \qquad (4)$$

Likewise, the slopes of the other joints adjacent to joint B can be expressed in terms of θ_B:

$$\theta_J = -\,\theta_B\,\frac{3\,n_{BJ} - 2}{3\,n_{BJ} - 1} \qquad (5a)$$

$$\theta_C = -\,\theta_B\,\frac{3\,n_{BC} - 2}{3\,n_{BC} - 1} \qquad (5b)$$

and

$$\theta_H = -\,\theta_B\,\frac{3\,n_{BH} - 2}{3\,n_{BH} - 1} \qquad (5c)$$

Similarly (compare Eqs. 3 and 4):

$$\frac{M_{BA}}{M_{AB}} = \frac{2\,\theta_B + \theta_A}{2\,\theta_A + \theta_B} = \frac{1 - n_{AB}}{n_{AB}} \qquad (6a)$$

and

$$\theta_A = -\,\theta_B\,\frac{3\,n_{AB} - 1}{3\,n_{AB} - 2} \qquad (6b)$$

The condition for static equilibrium of joint B requires that

$$M_{BJ} + M_{BC} + M_{BH} + M_{BA} = 0 \qquad (7)$$

Expressing the moments in terms of slopes similar to Eqs. 2 and utilizing Eqs. 4 and 6*b*,

$$2\,E\,K_{BJ}\,\frac{3\,n_{BJ}}{3\,n_{BJ} - 1} + 2\,E\,K_{BC}\,\frac{3\,n_{BC}}{3\,n_{BC} - 1} + 2\,E\,K_{BH}\,\frac{3\,n_{BH}}{3\,n_{BH} - 1} + 2\,E\,K_{AB}\,\frac{3\,n_{AB} - 3}{3\,n_{AB} - 2} = 0 \qquad (8)$$

Dividing by 6 E and rearranging Eq. 8,

$$\frac{1 - n_{AB}}{3\,n_{AB} - 2} = \frac{K_{BJ}}{K_{AB}} \times \frac{n_{BJ}}{3\,n_{BJ} - 1} + \frac{K_{BC}}{K_{AB}} \times \frac{n_{BC}}{3\,n_{BC} - 1} + \frac{K_{BH}}{K_{AB}} \times \frac{n_{BH}}{3\,n_{BH} - 1} + \cdots \qquad (9)$$

Eq. 9 may be considered a general equation applying to any joint of the type shown in Fig. 3. It gives the location of the node which characterizes the restraint at end B of a member AB with constant moment of inertia if the remote nodes of the restraining members are known.

Stiffness Factors and Distribution Factors

For a member with a constant ratio $K = \frac{I}{L}$, the term 4 E K is a measure of its maximum stiffness. It expresses the resistance to rotation of one end of a member if the other end is rigidly fixed. If the restraint at the far end is relaxed until it is free to turn, the resistance to rotation of the near end will diminish from 4 E K to 3 E K.

In general the resistance to rotation of the end of a member will be E times a stiffness factor or restraint factor "r." For a member with constant moment of inertia, r then varies between 3 K and 4 K.

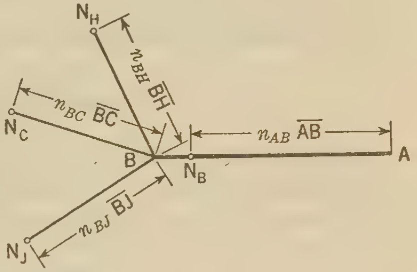


Fig. 3

Referring again to Eqs. 7 and 8 and to Figs. 2 and 3, it is evident that, provided lateral displacement of joints is prevented, each member BX which restrains end B of member AB contributes to the restraint a quantity E times r, in which the stiffness factor or restraint factor r equals

$$r_{BX} = 6\,K_{BX}\,\frac{n_{BX}}{3\,n_{BX} - 1} \qquad (10)$$

Assuming that all members are made of the same material, E is a constant. It will be omitted hereafter but should not be forgotten. Relatively, then, each member contributes to the restraint of end B of member AB a quantity:

$$r_{BJ} = \frac{6\,n_{BJ}}{3\,n_{BJ} - 1}\,K_{BJ} \qquad (11a)$$

$$r_{BC} = \frac{6\,n_{BC}}{3\,n_{BC} - 1}\,K_{BC} \qquad (11b)$$

and

$$r_{BH} = \frac{6\,n_{BH}}{3\,n_{BH} - 1}\,K_{BH} \qquad (11c)$$

The total restraint R_{BA} exerted upon end B of member AB equals the sum of the restraint factors:

$$R_{BA} = r_{BJ} + r_{BC} + r_{BH} \qquad (12)$$

The quantity

$$d_{BA} = \frac{r_{BA}}{R_{BA} + r_{BA}} \qquad (13)$$

measures the ratio in which an external moment applied at the joint B will be distributed, and has been called a "distribution factor."

Eqs. 1, 9, and 10 give the carry-over factor, the nodal intercept ratio, and the stiffness factor or restraint factor in terms of the nodal intercept ratio. Nodal intercept ratio, restraint factor, and carry-over factor can be expressed also in terms of the restraint, R, as follows: By combining Eqs. 9, 10, and 12 and solving for n_{AB}, the nodal intercept ratio is equal to

$$n_{AB} = \frac{2}{3}\frac{R_{BA} + 3\,K_{AB}}{R_{BA} + 2\,K_{AB}} \qquad (14)$$

By combining Eqs. 10 and 14, the stiffness factor or restraint factor is equal to

$$r_{AB} = 4\,K_{AB}\frac{R_{BA} + 3\,K_{AB}}{R_{BA} + 4\,K_{AB}} \qquad (15)$$

By combining Eqs. 1 and 14, the carry-over factor is equal to

$$f_{AB} = \frac{1}{2}\frac{R_{BA}}{R_{BA} + 3\,K_{AB}} \qquad (16)$$

If end B is hinged, $R_{BA} = 0$ and Eqs. 14 to 16 furnish $n_{AB} = 1$; $r_{AB} = 3\,K_{AB}$; and $f_{AB} = 0$. If end B is fixed, $R_{BA} = \infty$, and Eqs. 14 to 16 furnish $n_{AB} = \frac{2}{3}$; $r_{AB} = 4\,K_{AB}$; and $f_{AB} = \frac{1}{2}$.

Degree of Restraint

The term "degree of restraint" is generally applied very loosely. Based on the preceding discussion the degree of restraint Q_{AB} will be defined as the ratio of the restraint exerted upon the end of a member to the resistance against rotation of the entire joint:

$$Q_{AB} = \frac{R_{AB}}{R_{AB} + r_{AB}} \qquad (17a)$$

The degree of restraint can be defined also as the difference between unity and the distribution factor, because

$$Q_{AB} = \frac{R_{AB}}{R_{AB} + r_{AB}} = 1 - \frac{r_{AB}}{R_{AB} + r_{AB}} \qquad (17b)$$

It can be shown that for any rigid joint of m members the summation of their degrees of restraint as defined by Eqs. 17 must equal $(m - 1)$; that is:

$$\sum Q = m - 1 \qquad (18)$$

For instance, if four members form a rigid joint, the summation $\sum Q = 4 - 1 = 3$, and the average restraint equals 75%. If any of the four members have a degree of restraint greater than 75%, the restraint on one or more of the other three members is correspondingly reduced, and vice versa. If two members form a joint and one member restrains the other 100% (as, for instance, a member rigidly embedded in the earth), the second member exerts no restraint on the first member.

In terms of nodal intercept ratios Eq. 17*a* may be written

$$Q_{AB} = \frac{(1 - n_{BA})\,(3\,n_{AB} - 1)}{(1 - n_{BA})\,(3\,n_{AB} - 1) + n_{AB}\,(3\,n_{BA} - 2)} \quad \text{........} (19)$$

Obviously, as both n_{AB} and n_{BA} are contained in Eq. 19, the degree of restraint at one end is a function also of the degree of restraint at the other end of a member.

Degree of Fixity

"Degree of fixity" (or "degree of fixation") is another term commonly used in the study of indeterminate structures. It has been defined[4] in terms of end slopes as follows:

> Applying a rotation θ_B to end B of an unloaded member AB, the degree of fixity at end A is the difference between unity and the ratio of the slope at the restrained end A to the slope at A if the end were hinged.

From this definition the expressions for the carry-over factor, stiffness factor, nodal intercept ratio, and degree of restraint in terms of the degree of fixity are as follows:

$$f_{AB} = \frac{2\,F_{BA}}{3 + F_{BA}} \quad \text{........................} (20a)$$

$$r_{AB} = K_{AB}\,(3 + F_{BA}) \quad \text{...................} (20b)$$

$$n_{AB} = \frac{1}{3}\,\frac{3 + F_{BA}}{1 + F_{BA}} \quad \text{.....................} (20c)$$

and

$$Q_{AB} = \frac{4\,F_{AB}}{4\,F_{AB} + (1 - F_{AB})\,(3 + F_{BA})} \quad \text{............} (20d)$$

Some Mutual Relations

By definition the degree of fixity F_{BA} may vary between zero and unity. From Eq. 20*d* it can be seen that the degree of restraint, Q_{AB}, may vary, therefore, between $\dfrac{4\,F_{AB}}{3 + F_{AB}}$ and F_{AB}. In other words, $Q'_{AB} = F_{AB}$ when $F_{BA} = 1$. In terms of the nodal intercept ratio this means that $Q'_{AB} = F_{AB}$ when $n_{AB} = \frac{2}{3}$ (see Eq. 20*c*) and, from Eq. 19, it follows that

$$Q'_{AB} = F_{AB} = \frac{3 - 3\,n_{BA}}{3\,n_{BA} - 1} \quad \text{..................} (21)$$

It is further interesting to note that, if end B is hinged and $n_{AB} = 1$, Eq. 19 reduces to

$$Q''_{BA} = \frac{2\,(1 - n_{BA})}{n_{BA}} \quad \text{...................} (22a)$$

Referring to Eq. 1 it is seen that

$$Q''_{AB} = 2 f_{BA} \qquad \text{(22b)}$$

This can be demonstrated also as follows: Let C_{AB} be the fixed-end moment at end A due to an external moment M_B at end B. Then $C_{AB} = \frac{1}{2} M_B$. The end moment at end A is $M_{AB} = Q_{AB} C_{AB}$ and by definition

$$f_{BA} = \frac{M_{AB}}{M_B} = \frac{Q_{AB} C_{AB}}{2 C_{AB}} \qquad \text{(23)}$$

from which $Q_{AB} = 2 f_{BA}$.

Eqs. 21 and 22 have been plotted in the chart in Table 1, which shows graphically the restraint characteristics of the joint. If for a given member AB the nodal intercept ratio n_{BA} is determined, which is a function of the characteristics of joint A only, the following information is known:

(1) The degree of fixity at joint A, as this lies on the lower curve;

(2) The carry-over factor from joint B to A, which is one half the value of the ordinate of the upper curve; and

(3) The fact that the degree of restraint (and, therefore, the distribution factor) for end A of a member AB for any external moment at joint A is the ordinate of a point A somewhere between the two curves depending on the restraint at end B. This very narrow range demonstrates the well-known fact that (especially with increasing remoteness) adjacent restraints do not affect, materially, the distribution of an external moment at a joint.

MOMENT FLOW

An external moment (such as an unbalanced fixed-end moment) is held in equilibrium by all the members forming the joint at which it is acting and is distributed among all of these members in the ratio of their distribution factors. A moment that has been carried over to a joint by any one member is held in equilibrium by the other members of that joint and is distributed to these other members.

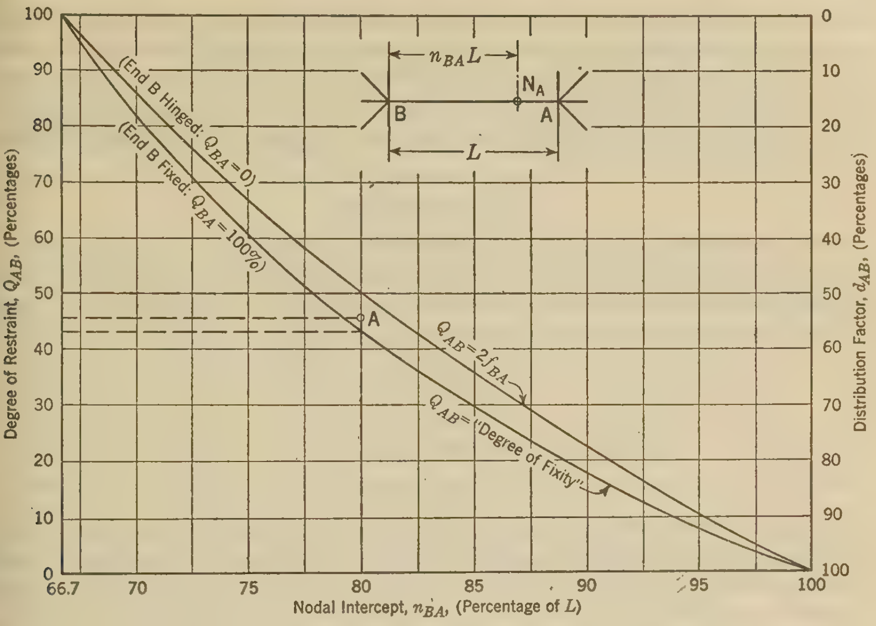
An unbalanced fixed-end moment or an external moment thus flows from joint to joint, diminishing as it spreads and leaving each joint in equilibrium. In a closed frame the result is an infinite series of waves. It is well known that convergence is so rapid, however, that for all practical purposes it is permissible to insolate a part of a frame by assigning arbitrary restraints to the marginal joints. In an open frame the necessary constants can generally be determined and the moments obtained by one distribution.

Some of the relations established in this paper are illustrated in Table 1. Attention is called to the fact that all functions are based on constant moments of inertia and on absence of joint displacements.

SUMMARY

In an attempt to generalize and correlate certain concepts used in the analysis of structures, and assuming constant cross sections, simple algebraic relations were found to exist between some of these terms which in effect reduce them to a "common denominator."

TABLE 1.—Interrelation Between the Node, Degree of Restraint, Degree of Fixity, and Carry-Over Factor



Structural concept	Symbol	Structural Concept in Terms of:		
		Degree of fixity	Restraint factors	Nodal intercept ratio
Nodal intercept ratio	n_{AB}	$\frac{1}{3}\frac{3+F_{BA}}{1+F_{BA}}$	$\frac{2}{3}\frac{R_{BA}+3K_{AB}}{R_{BA}+2K_{AB}}$	(See Eq. 9 and Fig. 3)
Distribution factor	d_{AB}	$\frac{(1-F_{AB})(3+F_{BA})}{(1-F_{AB})(3+F_{BA})+4F_{AB}}$	$\frac{r_{AB}}{R_{AB}+r_{AB}}$	$\frac{n_{AB}(3n_{BA}-2)}{n_{AB}(3n_{BA}-2)+(1-n_{BA})(3n_{AB}-1)}$
Carry-over factor	f_{AB}	$\frac{2F_{BA}}{3+F_{BA}}$	$\frac{1}{2}\frac{R_{BA}}{R_{BA}+3K_{AB}}$	$\frac{1-n_{AB}}{n_{AB}}$
Degree of restraint	Q_{AB}	$\frac{4F_{AB}}{4F_{AB}+(1-F_{AB})(3+F_{BA})}$	$\frac{R_{AB}}{R_{AB}+r_{AB}}$	$\frac{(1-n_{BA})(3n_{AB}-1)}{(1-n_{BA})(3n_{AB}-1)+n_{AB}(3n_{BA}-2)}$
Degree of fixity	F_{AB}	....	$\frac{R_{AB}}{R_{AB}+4K_{AB}}$	$\frac{3-3n_{BA}}{3n_{BA}-1}$
Restraint factor (or stiffness factor)	r_{AB}	$K_{AB}(3+F_{BA})$	$4K_{AB}\frac{R_{BA}+3K_{AB}}{R_{BA}+4K_{AB}}$	$K_{AB}\frac{6n_{AB}}{3n_{AB}-1}$

The approach to the subject matter being essentially mathematical, a problem incidental thereto, but important none the less, presents itself. It is the question of adopting some form of uniform mathematical language—algebraic letter symbols—for the expression of commonly recognizable terms. In Appendix I, the writer has attempted to conform essentially with standard lists now in process of formulation. It is acknowledged that the effort is imperfect, and that there is need for objective thought in the compilation of such a "dictionary."

In the hope that some readers may have suggestions to offer in discussion, a list of Tentative American Standard Letter Symbols for Structural Analysis, being perfected currently under the chairmanship of Albert Haertlein, M. Am. Soc. C. E., is added to the paper as Appendix II. It is not intended to open for discussion the entire field of structural symbols, but any suggestions for adapting the symbols of the present paper to the proposed standard will be a contribution to this subject.

ACKNOWLEDGMENT

This study was submitted in draft form to W. H. Jameson and J. L. Walmsley, Members, Am. Soc. C. E., and George L. Gray, Assoc. M. Am. Soc. C. E., and their helpful comments are gratefully acknowledged, as is the assistance given by Sidney Liu in preparing the copy.

APPENDIX I

NOTATION

The following letter symbols, used in this paper, conform essentially with American Standard Letter Symbols for Mechanics, Structural Engineering and Testing Materials[5] prepared by a Committee of the American Standards Association, with Society representation, and approved by the Association in 1932.

Subscripts.—Capital letter subscripts are used to specify a given location on the frame. For example, M_{AB} denotes the moment at end A of a beam AB; or K_{AB} denotes the stiffness ratio I/L for the beam AB.

d = distribution factor for an unbalanced moment;
E = modulus of elasticity in tension and compression;
F = degree of fixity at the points indicated by subscripts;
f = carry-over factor = the ratio of the shorter nodal intercept to the longer nodal intercept (Eq. 1);
I = moment of inertia;
K = a constant ratio = I/L, or stiffness ratio;
L = length; total span length between joint intersections;
M = external moment;
n = ratio of the longer of two nodal intercept distances, to the length L; for brevity this ratio is called "the nodal intercept ratio";
Q = degree of restraint (see Eq. 17a);
R = summation of restraint or stiffness factors r, of all beam ends exerting a restraint on the end of a given beam (see Eq. 12);
r = restraint factor or stiffness factor at the end of a given beam (see Eq. 10);
θ = change in the slope of the tangent to the elastic curve at the end of a given beam.

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[5] ASA—Z10a—1932.

APPENDIX II

Proposed American Standard Letter Symbols for Structural Analysis

In the following the general principles of Letter Symbol Standardization were formulated by a Committee of the American Standards Association, with Society representation, and have been adopted by the Association as a guide for the selection of letter symbols in all fields. The list of structural symbols is a tentative one, approaching final action by the Association.

General Principles of Standardization

1. Manuscript.—In preparing manuscripts, it is suggested that authors give careful attention to the use of symbols from this and other standard lists and to the principles here given. Symbols used should be defined clearly. When a table of symbols is not given, it is desirable to make reference to the standard lists from which the symbols are taken. The many numbers, letters, and signs which are similar in appearance should be distinguished carefully.

2. Letter Symbol.—A letter symbol is a single character, with subscript or superscript if required, used to designate a physical magnitude in mathematical equations and expressions. Two or more symbols together always represent a product.

Letter symbols are to be distinguished from abbreviations, mathematical signs and operators, graphical symbols, and chemical symbols:

(*a*) Abbreviations are shortened forms of names and expressions employed in texts and tabulations and should not be used as symbols in equations.
(*b*) Mathematical signs and operators are characters used with letter symbols to denote mathematical operations and relations.
(*c*) Graphical symbols are conventionalized diagrams and letters used on plans and drawings.
(*d*) Chemical symbols are letters and other characters designating chemical elements and groups.

3.—The same symbol should be used for the same physical magnitude regardless of the units employed and regardless of special values occurring for different states, points, parts, times, etc. Units or special values may be distinguished when necessary by subscripts, superscripts, or by upper-case and lower-case letters when both are specifically included as symbols in a standard list. The units used should be indicated when necessary. Sometimes different symbols are used for the components of a vector.

4. Subscript.—A subscript preferably should be a simple character. It is commonly employed to indicate constancy of a particular physical magnitude, such as pressure or temperature, when there are other variables. A multiple subscript, sometimes divided by a comma, refers to more than one state, point, part, time, etc. A subscript should not be attached to a subscript. Further uses of subscripts are listed in Principles 3 and 6.

5. Superscript.—A symbol with a superscript, such as a prime (′) or a second (″), should be enclosed in parentheses, braces, or brackets before affixing an exponent. A complicated exponent (or any other expression frequently repeated) may be replaced by a single symbol selected to represent it. Reference marks should not be attached to symbols. Further uses of superscripts are listed in Principles 3 and 6.

6. Conflicts.—Conflicts which would occur when different physical magnitudes are assigned the identical symbol in the same or different standard symbol lists may be resolved in one of the following ways:

(*a*) For one or more of the conflicting uses, the given symbol may be employed with subscript or superscript selected by the author.
(*b*) If one of the magnitudes has an alternate symbol in a standard list, it may be used.
(*c*) A slight change in the name of the magnitude may remove the conflict. For instance, one may use L for "length of radius" when r for "radius" conflicts with r used for another magnitude.

7. Unlisted Magnitudes.—To symbolize a special value of a listed magnitude, see Principle 3. The symbol chosen by an author for a physical magnitude not appearing in any standard list should be one that does not have already a different meaning in the field of the text.

8. Typography.—Letter symbols, letter subscripts, and letter superscripts, whether upper case or lower case, should be printed with italic type unless definitely specified otherwise. On manuscript this is indicated by underlining each symbol which is to be italicized. Special types, such as Old English and type that is currently used for vector magnitudes, should be avoided for scalar magnitudes. When special type is used for vector magnitudes, the same italic letter should be used for the corresponding scalar magnitude. Vertical Arabic numerals should be used as coefficients in equations and in subscripts and exponents. Bars, dots, and other modifying signs and operators should be used in the manner currently recognized by mathematicians.

TENTATIVE LIST OF SYMBOLS

General Letter Symbols.—

A = area;
b = breadth or width;
D = diameter;
E = modulus of elasticity (Young's modulus);
e = eccentricity;
F = force (see also P);
f = coefficient of sliding friction;
G = modulus of elasticity in shear, or rigidity;
g = acceleration of gravity;
h = height, depth, or thickness;
I = inertia, rectangular moment of (area); I_{xy} = product moment of inertia;

J = polar moment of inertia.
k = radius of gyration.
l = length.
M = moment of forces, bending moment.
P = forces or loads, concentrated (see also P).
p = pressure, normal force per unit area.
Q = moment of area, statical, or first moment.
R = modulus of rupture.
r = radius.
s = normal stress per unit area.
s_s = shear stress per unit area.
T = torque.
t = temperature.
V = volume.
W = weight, total.
w = load, or weight per unit distance.
x = coordinate distance measured parallel to an x-axis.
y = coordinate distance measured parallel to a y-axis; deflection, or ordinate to the elastic line of a beam at distance x.
Z = section modulus.
α = coefficient of linear expansion, linear.
γ = weight per unit volume.
δ = total elongation.
ϵ = unit linear deformation, normal strain.
η = efficiency.
μ = Poisson's ratio.
ρ = density; mass per unit volume, $\frac{\gamma}{g}$;
ϕ = total angular deformation.

her Symbols *Primarily for Reinforced Concrete.*—

C = total compressive force.
c = distance from the neutral axis to the extreme fiber.
d = depth from the compression face of a beam or slab to the center of the longitudinal tensile reinforcement.
f = unit stress, f_c = compressive stress in concrete and f_s = tensile stress in steel reinforcement.
j = ratio of the distance between the resultant compressive stress and the resultant tensile stress to the distance from the outer compression fiber to the resultant tensile stress in a reinforced concrete beam.
n = ratio, modulus of elasticity of steel to modulus of elasticity of concrete.
p = ratio, area of steel reinforcement to area of concrete.
T = total tensile force on the steel reinforcement.
u = unit bond stress.
v = unit shearing stress.

Letter Symbols Primarily for the Analysis of Frameworks, Arches, and Cables.—

c = distance from the neutral axis to the extreme fiber;
h = rise of an arch or sag of a cable;
K = stiffness factor = $\frac{I}{\text{span}}$;
M = moment of a force; bending moment;
m = moment at any section of a beam due to a unit load or to a unit moment applied at a given point;
R = reaction;
s = linear distance, or length measured along an arc;
u = force at any bar of a framed structure due to a unit load applied at a given point in a given direction;
V = shearing force in a beam section;
y = deformation; total displacement of a panel point of a truss or a point on the elastic line of a beam;
Z = section modulus;
γ = shear strain;
δ = total elongation;
ϵ = unit linear deformation; normal strain;
θ = angular distance in slope deflection;
ϕ = total angular deformation.

Letter Symbols Primarily for Mechanics of Solid Bodies.[6]—

a = linear acceleration;
E = energy, E_k being kinetic energy and E_p being potential energy (use T and V, respectively, when there is a conflict with E = modulus of elasticity);
f = frequency;
K = stress concentration factor;
T = kinetic energy (E_k is preferred except when there is a conflict with E = modulus of elasticity);
t = time;
V = potential energy (E_p is preferred except when there is a conflict with E = modulus of elasticity);
v = linear velocity;
W = work;
α = angular acceleration;
ω = angular velocity.

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[6] See also American Standards Association Letter Symbols for Mechanics of Solid Bodies, ASA—Z10.3—1942.

DISCUSSION

Ralph W. Stewart,[7] M. Am. Soc. C. E.—The flexure of a continuous structure exhibits a number of interrelated phenomena both in the structure as a whole and in the individual spans. These phenomena comprise what are known as the "beam constants" and the "load constants," as well as a variety of other constants or properties that can be derived from the beam constants and the load constants. Each of the various methods for computing the bending moments in a continuous structure is contrived by selecting a limited number of these properties and using the properties chosen in a more or less regimented manner. A conversion table showing the interrelationship between the constants used in a number of systems of analysis has been presented by Fang-Yin Tsai.[8] The table covers the general case—namely, members with variable cross sections.

The final test of a structural concept is its usefulness in computing the stresses in a structure. The author's concept features nodal distances, restraint factors, and final carry-over factors. These terms may be applied to the solution of a continuous structure such as that shown in Table 2, which gives the computations for the relative moments at end E of the frame analyzed. For any moment applied at point E, 2/347 of the moment will be transmitted to end A, 3/347 of it to end G, etc. This frame was selected because a very thorough exposition of its analysis by the concept of end moment distribution is available for reference.[2] Fig. 4 shows the additional work required to determine the effect at all joints of an applied moment at point D. Restraint factors are computed as follows:

$$r_{DC} = \frac{6 \times 0.7666}{3 \times 0.7666 - 1} \times 5 = 17.7$$

and

$$r_{DE} = \frac{6 \times 1}{3 \times 1 - 1} \times 3 = 9.0$$

The moment M_{DE} equals $9/17.7 \times 115 = 58.5$; and the total fixed-end moment is $58.5 + 115 = 173.5$. Fig. 4 then indicates that 2/173.5 of a moment applied at point D is transferred to end A, etc.

The solutions of Eq. 9 to obtain the nodal intercepts for members CD and DE require an appreciable length of time. Also, the nodal distances in this structure and other continuous structures are different for each member and are different for the ends of the same member. This makes the nodal distances inferior to the moment-area constants as bases for analyzing a structure.

Fig. 5 shows a solution which uses the moment-area constants delineated in the form of an elastic curve traverse. All angles of flexure are at the one-third points of the span. They do not move in response to changes in restraint as do the nodal distances. Thus, any labor or difficulties in computing the locations of the angles are eliminated.

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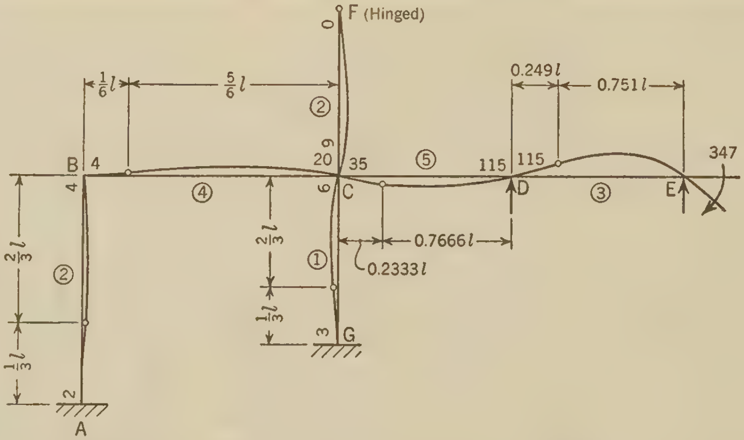
[7] Bridge Engr., City of Los Angeles, Los Angeles, Calif.

[8] *Transactions*, Am. Soc. C. E., Vol. 102 (1936), Table 3, p. 44.

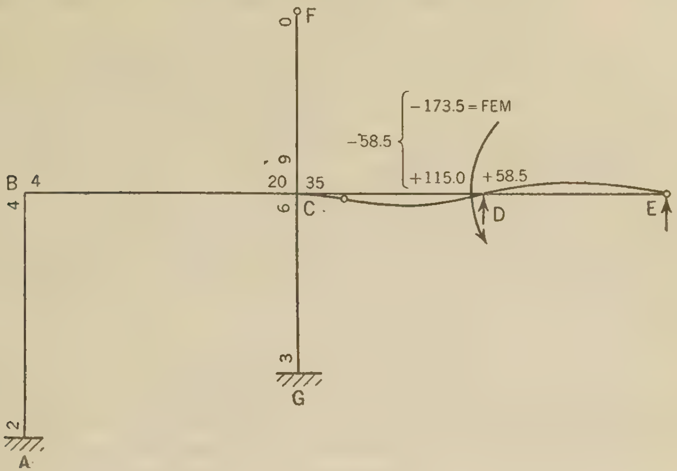
TABLE 2.—ILLUSTRATIVE EXAMPLE

(Successive Steps Shown by Numerals in Parentheses)


F (Hinged)
0
$\frac{1}{6}l$
$\frac{5}{6}l$
②
9
20
35
⑤
115
115
0.249l
0.751l
347
B
4
4
④
6
C
D
③
E
$\frac{2}{3}l$
②
$\frac{2}{3}l$
①
0.7666l
0.2333l
$\frac{1}{3}l$
$\frac{1}{3}l$
3
G
2
A

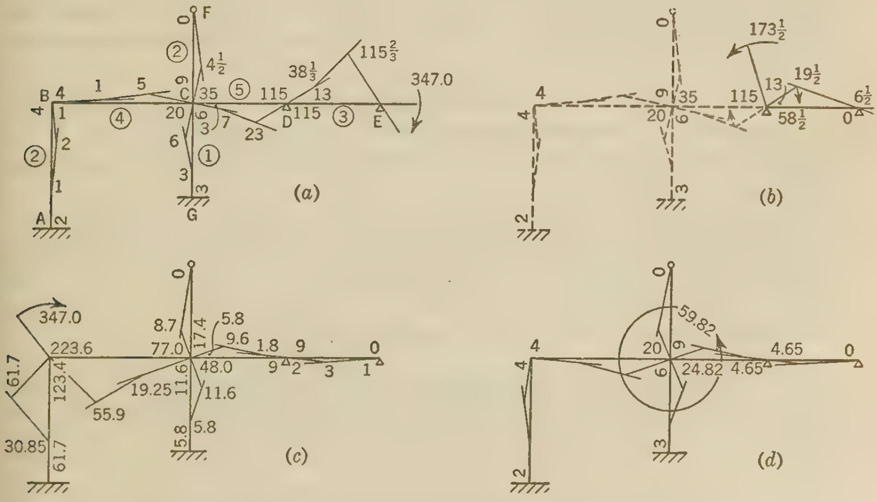
Line	Concept	Member AB	Member BC	Member CD	Member DE	Member CF	Member CG
1	Nodal intercept	(1) (Known) 1/3	(3) (Eq. 9) 5/6	(5) (Eq. 9) 0.7666	(7) (Eq. 9) 0.751	(9) (Known) 0	(10) (Known) 1/3
2	Carry-over factor	(2) (Known) 1/2	(4) $\frac{1/6}{5/6} = \frac{1}{5}$	(6) $\frac{0.2333}{0.7666} = 0.304$	(8) $\frac{0.249}{0.751} = 0.332$	(Not needed)	(Not needed)
3	Restraint factor	(Not needed)	(11) (Eq. 10) 13.33	(Not needed)	(Not needed)	(12) (Eq. 10) 6	(13) (Eq. 10) 4
4	Relative moments	(14) AB = 2 (assumed) BA = 2/0.5 = 4	(15) BC = BA = 4 CB = 4/0.20 = 20	(See line 5)	(See line 5)	(16) CF = 6/13.33 × 20 = 9	(17) CG = 4/13.33 × 20 = 6
5	Relative moments	....	....	(18) CD = 20 + 9 + 6 = 35 DC = 35/0.304 = 115	(19) DE = DC = 115 ED = 115/0.332 = 347	....	....

An angle of flexure multiplied by the stiffness of the member involved gives a moment, and a moment divided by the stiffness of the member gives the appurtenant flexure angle. By using these properties of Fig. 5 in connection with simple geometry, the arithmetical computations for Fig. 5(*a*) (which yields the same result as Table 2) can be performed in much less time than that required for the example in Table 2. This condition is also true for Fig. 5(*b*) which corresponds to Fig. 4. Fig. 5(*c*), which shows the final carry-over factor to all joints for an applied moment at point B, furnishes an automatic check of the computations for Figs. 5(*a*) and 5(*c*) by noting that Fig. 5(*a*) gives an applied moment of 347 at end E for a joint rotation of unity at point



0
F
−173.5 = FEM
−58.5
+115.0
+58.5
9
B
4
20
35
4
6
C
D
E
3
G
2
A

FIG. 4



0
F
②
4½
38⅓
115⅔
347.0
B
4
1
5
9
C
35
⑤
115
13
4
1
④
20
6
7
3
D
115
③
E
23
6
①
②
2
3
1
3
A
2
G
(a)
0
173½
13
19½
4
9
35
115
6½
4
20
6
58½
0
3
2
(b)
347.0
8.7
17.4
5.8
9.6
223.6
77.0
1.8
9
0
61.7
123.4
6
48.0
9
2
3
1
11.6
19.25
11.6
55.9
5.8
30.85
5.8
61.7
(c)
0
59.82
4
20
9
4.65
0
4
6
24.82
4.65
3
2
(d)

FIG. 5

B and that the computations for Fig. 5(*c*) for a rotation of unity at end E yield a moment of 347 at point B, as required by Maxwell's theorem of reciprocal deflections. The computations for Fig. 5(*d*) require only one setting of a slide rule from which two readings are taken, followed by a moment summation about joint C. Fig. 5(*d*) gives the final carry-over factors at all joints for a moment applied at joint C. All the necessary arithmetical work is shown directly in Fig. 5. For a solution based on nodal distances the additional work represented by Table 2, and scratch-paper work not shown, are necessary.

The foregoing solutions demonstrate how the structural concepts which the author presents can be appraised as tools for practical analytical work. An extension of the study to structures having tapering members would show even greater relative adaptability for the moment-area constants.

The paper is well written and will be of value to computers who make analyses of structures based on the position of the points of contraflexure of the members.

R. C. BRUMFIELD,[9] M. AM. SOC. C. E.—In this paper Mr. Weiss has discussed primarily five terms which are applicable in connection with the solution of indeterminate structures composed of interlocking beams, the distortions of which structures are essentially due to bending moment. The five terms, with the definitions assigned to them in the paper, are given by the author as follows:

Carry-Over Factor.—See "Introduction":

> "In this study the term 'carry-over factor' denotes the ratio of the moment induced at the restrained, but not necessarily fixed, end to the moment applied at the other end of a member which is on unyielding supports at both ends."

(By unyielding supports it is obvious that the author means supports that allow no movement of translation although the beam may rotate.)

Stiffness or Restraint Factor.—See "Stiffness Factors and Distribution Factors":

> "* * * the resistance to rotation of the end of a member will be *E* times a stiffness factor or restraint factor '*r*.' For a member with constant moment of inertia, *r* then varies between 3 *K* and 4 *K*."

Distribution Factor.—See sentence containing Eq. 13:

> "[The distribution factor] measures the ratio in which an external moment applied at the joint B will be distributed [to a particular member of the joint]"

Degree of Restraint.—

> "* * * the degree of restraint Q_{AB} will be defined as the ratio of the restraint exerted upon the end of a member to the resistance against rotation of the entire joint."

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Degree of Fixity.—

> "Applying a rotation θ_B to end B of an unloaded member AB, the degree of fixity at end A is the difference between unity and the ratio of the slope at the restrained end A to the slope at A if the end were hinged."

It seems clear from the context that the author has in mind the use of these terms largely in connection with the Cross method[2] of solving indeterminate structures by distribution through successive approximations. This appears to be evident from the statement in the "Introduction" and from the fact that, in deriving an expression for the distribution factor (Eq. 13), only that one is derived for an artificial moment applied to the joint externally, this being the one required in the Cross method of distribution.

The justification for such terms as those listed must lie wholly in the relative number of cases in which they are found useful, such as (*a*) the services that they may be made to render in computing the bending effects of loads applied to an indeterminate structure; (*b*) the allocation of these effects to the separate members of the structure; and (*c*) the evaluation of the distortions induced in the structure. Of these purposes the first two will ordinarily be the more important. Concepts that serve these purposes adequately and with a maximum of simplicity should be adopted and used. Those which do not meet these tests eventually will be discarded.

As a basis for deriving his various formulas, the author uses the concept of the "fixed point," or "node point" as he terms it. This concept has long been in existence and is used to express the fact that, in any beam whose bending-moment diagram consists exclusively of the effects of the bending moment introduced at one end of a span and that induced by it at the opposite end of the same span, the point of zero bending moment remains at a fixed point along the beam regardless of the size or sense of the inducing moment.

The selection of the term "node point" for this position of zero bending moment is unfortunate, particularly when taken in connection with the context of the paper in which the term "elastic curve" is used to identify this point. In vibratory motion, the most important characteristic of a node is that there is no movement of the vibrating body at the node. This is not true of the "fixed point" of a beam. In fact, there is always lateral movement at its fixed point, when the beam is only under the influence of a moment at one end and its induced moment at the opposite end. The casual reader is likely to form quite an opposite impression in the paper from the statement (see heading, "Fixed Points and Carry-Over Factors"): "* * * the wave-like elastic curves will pass through a certain node."

The idea of fixed point might well be eliminated entirely from the considerations, in connection with the five definitions listed at the beginning of this discussion. Although the author has used the concept as a basis for his derivations, it need not have been so used. Every equation in the paper could have been derived with equal or greater simplicity if the concept had not been introduced, using as a basis well-established ideas that have long existed. Judging the term, then, from the standpoint of service it seems to be superfluous.

The author uses the terms "stiffness factor" and "restraint factor" interchangeably as measures of the bending moment required to be applied to the end of a beam, resting on a free support, in order to rotate that end through a unit angle under the influence of the restraints that may exist at the opposite end of the same beam. Such a beam might be span AB, Fig. 2, the restraints that influence the beam being those at joint B.

The writer would suggest a change in this concept. Stiffness is characterized by movement or, rather, capacity to resist movement. The author measures this movement by the bending moment required to cause it. This has one virtue and one fault. It represents a stiff beam in terms of a large number and so appeals to the natural instinct that this should be so. Its fault lies in the fact that the stiffness is not measured in terms of movement, but rather, in an auxiliary fashion, in terms of the moment required to cause movement.

If the movement itself is used as a measure of stiffness, the term "stiffness factor" or preferably "stiffness index" may be defined as a measure of the angle of rotation caused at the end of a beam by the application there of a unit couple. Such a definition allows stiffness to be measured directly in terms of movement or rotation. The fact that it is now an inverse measure of the movement (that is, a large "stiffness index" represents a relatively flexible member) does not mitigate against the definition since by it a true concept is obtained of the movement of the beam end. Thus the word "index" is used instead of the word "factor" which ordinarily is thought of as a multiplier.

One should also distinguish sharply between the terms "stiffness factor" (or "stiffness index") and "restraint factor" (or "restraint index") which terms the author uses interchangeably. Stiffness is a function that is (and should be) influenced largely by the characteristics of the beam itself. Restraint, on the other hand, is something applied from without and beyond the beam, which is applied only to the beam end. In such a beam as span AB, Fig. 2, the term "stiffness index" should be applied to end A, whose rotation is influenced largely by beam AB, and the term "restraint factor" (or "index") to the far end of beam B, where rotation is influenced by the three restraining beams, BJ, BC, and BH. Thus, when a moment is transmitted from point A to point B, the rotation of end A of the beam is measured by the stiffness index, and that of the same beam at end B, by its restraint index. Such a concept leads to simple expressions for the various terms under discussion.

Referring to Fig. 2, if a unit couple is applied to span AB at end A, the angle of rotation at end A is:

$$6\,E\,\theta_A = \frac{2 - f_{AB}}{K} \quad \ldots\ldots\ldots\ldots\ldots\ldots (24)$$

From Eq. 24, one may define "stiffness factor" (or "index") by the equation:

$$i_{AB} = \frac{2 - f_{AB}}{K} \quad \ldots\ldots\ldots\ldots\ldots\ldots (25)$$

Thus, the index is shown to be a measure of angular rotation. Without giving the derivations that lead to the conclusion, the expression for the restraint index for span AB, at B, is given by the expression:

$$R_{AB} = \frac{1}{\dfrac{1}{i_{BJ}} + \dfrac{1}{i_{BC}} + \dfrac{1}{i_{BH}}} \qquad (26a)$$

or

$$R_{AB} = \frac{i_{BJ}}{1 + \dfrac{i_{BJ}}{i_{BC}}\left(1 + \dfrac{i_{BC}}{i_{BH}}\right)} \qquad (26b)$$

The right-hand quantity of either Eq. 26*a* or Eq. 26*b* is usable, but Eq. 26*b* is preferable since it is adaptable to slide-rule use without any other mathematical work except mental additions. The usefulness of this restraint index is apparent from the fact that the bending moment arriving at joint B, from end A, divides to the three restraining members according to the following expressions:

$$M_{BJ} = M_{BA}\frac{R_{AB}}{i_{BJ}} \qquad (27a)$$

$$M_{BC} = M_{BA}\frac{R_{AB}}{i_{BC}} \qquad (27b)$$

and

$$M_{BH} = M_{BA}\frac{R_{AB}}{i_{BH}}, \qquad (27c)$$

or, in general:

$$M_n = M\frac{R}{i_n} \qquad (28)$$

It is seen that Eq. 28 has a close relationship to the distribution factor given in Eq. 13. It differs from this factor in that it gives the division, to the restraining members, of moment arriving at the joint through the transmitting member or actuating member AB, rather than the division of the moment to the members when an artificial torque is applied to the joint and all radiating members are acting as transmitting members.

From Eqs. 24 and 28 it is clear that, if the bending moment which arrives at joint B, through span AB, is unity, the angle of rotation of joint B induced thereby is

$$6\,E\,\theta_{BJ} = 1 \times \frac{R_{AB}}{i_{BJ}} \times i_{BJ} = R_{AB} \qquad (29)$$

Thus R_{AB} becomes a direct measure of the rotation of the end B of beam AB, just as i_{AB} was a direct measure of the rotation at the opposite end. The expression R_{AB} involves only the characteristics of the restraining members and thus becomes a true restraint index, whereas i_{AB} involves mostly the characteristics of the beam, restraint being only included through the expression f_{AB}. Thus "stiffness index" is a proper name for this term.

With these concepts in mind the author's expression for the carry-over factor, freed from its special application in the Cross method of analysis as explained in the paper, becomes:

$$f_{AB} = \frac{1}{2 + K_{AB}\, R_{AB}} \qquad (30)$$

which compares with Eq. 16.

In the writer's opinion the term "degree of restraint" discussed in the paper is not especially useful. In terms of the foregoing concepts and the author's definition it would be the ratio of the restraint index of member AB to the restraint index of the joint as a whole, against the artificial torque applied to it. Thus

$$Q_{AB} = \frac{\dfrac{1}{\dfrac{1}{i_{BJ}} + \dfrac{1}{i_{BC}} + \dfrac{1}{i_{BH}}}}{\dfrac{1}{\dfrac{1}{i_{BA}} + \dfrac{1}{i_{BJ}} + \dfrac{1}{i_{BC}} + \dfrac{1}{i_{BH}}}} = 1 + \frac{R_{AB}}{i_{BA}} \qquad (31)$$

The degree of fixity, using the definition of the paper, may be derived as follows, using the beam noted in the definition and assuming a unit couple applied at joint B: With the beam as restrained, Eq. 24 yields—

$$6\, E\, \theta_A = \frac{2\, f_{BA} - 1}{K} \qquad (32a)$$

With end A assumed resting on a support that permits perfect freedom of rotation—

$$6\, E\, \theta'_A = \frac{2 \times 0 - 1}{K} \qquad (32b)$$

Then:

$$F_{AB} = 1 - \left(\frac{2\, f_{BA} - 1}{K} \times \frac{K}{-1}\right) = 2\, f_{BA} \qquad (33)$$

These examples serve to show that the introduction of the idea of "restraint index" for the beam as differentiated from "stiffness index" permits the mathematical delineation, in quite simple forms, of all the concepts listed in the paper. The elimination of the idea of "fixed point" (or "node point" as it is called) in no way hinders the work, and the derivations leading to the results are, in most cases, simpler than those listed in the paper.

The author's project of codifying the nomenclature used in connection with indeterminate structures is praiseworthy. It should lead to beneficial results if properly handled. Such standardization, if well done, will be an assistance in clarifying thought and mutual understanding in the engineering profession. There is one danger to be noted, however: If too rigidly applied and if there is too great insistence upon the letter of the standard rather than the spirit, the result may be that engineering thought is put into a "strait jacket" and initiative stifled. The number of concepts involved is large when indeterminate structures are to be considered in their broadest sense. A noncoplanar frame, involving not only beams of variable section but also curved beams, is a case in point. In such a structure, the number of interrelated factors is sufficient to exhaust the letters in the ordinary alphabet as well as those in the Greek alphabet. Every one in a position to influence standardization should be patient and move cautiously until the literature on these matters is much more nearly crystallized than at present.

Max W. Strauss,[10] Jun. Am. Soc. C. E.—In the "Introduction," the author makes the statement that "The method of moment distribution introduced by Hardy Cross,[2] M. Am. Soc. C. E., * * * is as good a method as can be devised." That this is open to question is suggested by the author himself by the fact that he introduces the concept of an exact carry-over factor. This concept is not a new one. The writer has seen it traced back to Maurice Lévy who developed it about 1880.[11] Since then it has frequently been employed by others. However, beyond implying this concept, the development of the thesis of the paper seems to fall short of its logical conclusion. For instance, if Eq. 1 is substituted in Eq. 9, the latter may be rewritten:

$$f_{AB} = \frac{1}{2 + \dfrac{K_{AB}}{\dfrac{K_{BJ}}{2 - f_{BJ}} + \dfrac{K_{BC}}{2 - f_{BC}} + \dfrac{K_{BH}}{2 - f_{BH}}}} \qquad (34)$$

Thus, to determine the carry-over factor for any span toward a joint, it is only necessary to know the carry-over factors away from that joint for all other spans. Starting at a fixed or free-end support, all other carry-over factors may be determined rather easily. This is the basic principle long advocated by R. C. Brumfield, M. Am. Soc. C. E.[12]

For continuous beams, Eq. 34 reduces to:

$$f_X = \frac{1}{2 + \dfrac{K_X}{K_{X-1}}(2 - f_{X-1})} \qquad (35a)$$

which, for continuous beams of constant moment of inertia and equal spans throughout, becomes:

$$f_X = \frac{1}{4 - f_{X-1}} \qquad (35b)$$

Because of the very rapid convergence, f_X approaches f_{X-1}; and, starting at either a free or fixed support, $f_X \approx f_{X-1} \approx 2 - \sqrt{3} = 0.268$. For all practical purposes, this convergence can be assumed to take place in three or four spans. The writer's attention has been called to a short table compiled by Bertrand de Fontviolant which illustrates this effect.[13]

Moment Transmission and Distribution.—Eq. 13 is an expression for the distribution of a moment applied externally at joint B to the various spans at that joint. The terms in the right-hand side of the expression are defined by Eqs. 11 and 12. Instead of an external moment, consider a moment in span BA at joint B, arriving from joint A, which is to be distributed to the other spans at joint B. This moment will be distributed to each span in the ratio of its distribution factor to the sum of the distribution factors of the receiving members. Thus,

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[10] Stress Analyst, Consolidated-Vultee Aircraft Corp., San Diego, Calif.

[11] "La Statique Graphique et Ses Applications Aux Constructions," by Maurice Lévy, Gauthier-Villars & Co., Paris, France, 1886–1888.

[12] "Moving Loads on Restrained Beams and Frames," by R. C. Brumfield, *Proceedings*, Am. Soc. C. E., May, 1945, p. 627.

[13] "Resistance des Matériaux," by Bertrand de Fontviolant, J. B. Bailliere & Fils, Paris, France, 1923, p. 452.

$$M_r = M_s \frac{d_r}{1 - d_s} = M_s \frac{r_r}{R_s} \ldots\ldots\ldots\ldots\ldots\ldots (36)$$

in which the subscripts r and s indicate the receiving member and the sending member, respectively.

Referring to Fig. 3, consider member AB loaded in such a manner that the fixed-end moments are FEM_{AB} and FEM_{BA}. Then the moment in span BJ at joint B is

$$M_{BJ} = \text{FEM}_{\text{BA}}\, d_{BJ} + \text{FEM}_{\text{AB}}\, d_{AB}\, f_{AB} \frac{d_{BJ}}{d_{BJ} + d_{BC} + d_{BH}} \ldots\ldots (37)$$

Factoring out the moment split[12] for member BJ:

$$M_{BJ} = \frac{r_{BJ}}{R_{BA}} [\text{FEM}_{\text{BA}}\, (1 - d_{BA}) + \text{FEM}_{\text{AB}}\, d_{AB}\, f_{AB}] \ldots\ldots (38)$$

Comparing Eq. 38 with Eq. 36:

$$M_{BA} = \text{FEM}_{\text{BA}}\, (1 - d_{BA}) + \text{FEM}_{\text{AB}}\, d_{AB}\, f_{AB} \ldots\ldots\ldots (39a)$$

Similarly,

$$M_{AB} = \text{FEM}_{\text{AB}}\, (1 - d_{AB}) + \text{FEM}_{\text{BA}}\, d_{BA}\, f_{BA} \ldots\ldots\ldots (39b)$$

Illustrative Problem.—To illustrate the foregoing, the writer has chosen a frame (see Fig. 6) that has been solved previously by the Cross method of successive approximations.[14] The expressions for the carry-over factor and the distribution factor are given in Table 3 as are the moments at the supports due to the various loads. A usual sign convention is followed. A positive moment indicates a clockwise resisting moment acting on the span and a negative moment indicates a counterclockwise resisting moment. Thus, a moment transmitted from one end of a span to the other will not change sign; but, in passing through a joint, a moment will change sign before it is distributed to the other spans.

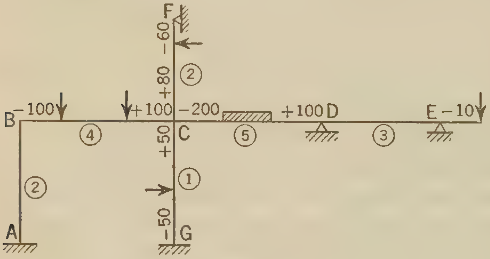

Fig. 6

The K-values are shown circled on each span in Fig. 6 and the fixed-end moments are indicated at the end of each span. Since $f_{BA} = \frac{1}{2}$, $r_{BA} = \frac{2}{1.5}$

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[14] "Analysis of Continuous Frames by Distributing Fixed-End Moments," by Hardy Cross, *Transactions*, Am. Soc. C. E., Vol. 96 (1932), Fig. 1, p. 5.

$= R_{BC}$, which results in $f_{CB} = \frac{1}{5}$ and $r_{CB} = \frac{4}{1.8}$. Similarly, $r_{CF} = \frac{2}{2}$ and $r_{CG} = \frac{1}{1.5}$, so that $R_{CD} = \frac{4}{1.8} + \frac{2}{2} + \frac{1}{1.5} = \frac{7}{1.8}$ and $f_{DC} = \frac{7}{23}$. Therefore,

TABLE 3.—COMPUTATION OF FRAME CHARACTERISTICS AND MOMENTS IN FIG. 6

Member	$K = \frac{I}{L}$ (1)	$r = \frac{K}{2 - f}$ (2)	$R = \Sigma r_R$ (3)	$f = \frac{1}{2 + K/R}$ (4)	MOMENT SPLIT $= \frac{r}{R_R}$ (5)	(6)	(7)	(8)
Point A—								
AB	....	....	....	....	....	....	....	....
Joint B—								
BA	2	1.3333	2.4160	0.3536 $= f_{AB}$	....	1.0000	....	....
BC	4	2.4160	1.3333	0.2000 $= f_{CB}$	1.0000	....	....	....
Joint C—								
CG	1	0.6667	5.9808	0.4614 $= f_{GC}$	....	0.1507	0.1181	0.1714
CB	4	2.2222	4.4253	0.3444 $= f_{BC}$	0.3716	....	0.3935	0.5714
CF	2	1.0000	5.6475	0.4248 $= f_{FC}$	0.1672	0.2260	....	0.2572
CD	5	2.7586	3.8889	0.3043 $= f_{DC}$	0.4612	0.6233	0.4884	....
Point F—								
FC	....	....	....	....	....	....	....	....
Point G—								
GC	....	....	....	....	....	....	....	....
Joint D—								
DC	5	2.9486	1.5000	0.1875 $= f_{CD}$	....	1.0000	....	....
DE	3	1.5000	2.9486	0.3314 $= f_{ED}$	1.0000	....	....	....
Joint E—								
ED	....	....	....	....	....	....	....	....

TABLE 3.—(*Continued*)

Member	$d = \frac{1}{1 + R/r}$ (9)	ACTUAL MOMENTS AT JOINTS, DUE TO LOADS IN THE GIVEN SPAN: BC (10)	CF (11)	GC (12)	CD (13)	Joint E (14)	Total (15)
Point A—							
AB	....	+21.12	+3.68	+1.67	−7.84	−0.06	+18.57
Joint B—							
BA	0.3556	+42.24	+7.35	+3.34	−15.67	−0.12	+37.14
BC	0.6444	−42.24	−7.35	−3.34	+15.67	+0.12	−37.14
Joint C—							
CG	0.1003	−13.38	−11.03	+44.98	+23.51	+0.18	+44.26
CB	0.3343	+88.76	−36.77	−16.71	+78.37	+0.61	+114.26
CF	0.1504	−20.06	+93.45	−7.52	+35.29	+0.27	+101.43
CD	0.4150	−55.32	−45.65	−20.75	−137.17	−1.06	−259.95
Point F—							
FC	....	0	0	0	0	0	0
Point G—							
GC	....	−6.69	−5.52	−52.51	+11.76	+0.09	−52.87
Joint D—							
DC	0.6628	−10.37	−8.56	−3.89	+49.28	−3.31	+23.13
DE	0.3372	+10.37	+8.56	+3.89	−49.28	+3.31	−23.13
Joint E—							
ED	....	0	0	0	0	+10.00	+10.00

$r_{DC} = \frac{115}{39} = R_{DE}$ and $f_{ED} = \frac{115}{347}$. Beginning at the other end, $f_{DE} = 0$ and $r_{DE} = \frac{3}{2.0} = R_{DE}$, so that $f_{CD} = \frac{3}{16}$. Also, $r_{CD} = \frac{80}{29}$ and $R_{CB} = r_{CD} + r_{CG}$

$+ r_{CF} = \frac{80}{29} + \frac{1}{1.5} + \frac{2}{2} = \frac{385}{87}$ and $f_{BC} = \frac{385}{1{,}118}$. Therefore, $r_{BC} = \frac{4{,}472}{1{,}851} = R_{BA}$ and $f_{AB} = \frac{4{,}472}{12{,}646}$.

The moment splits (m) are obtained by Eq. 36. At a joint such as C, they are computed separately for each member acting as a transmitting member. Thus, for example, for the moment transmitted from joint B to joint C: $m_{CG} = \frac{r_{CG}}{R_{CB}} = \frac{1}{1.5} + \frac{385}{87} = \frac{174}{1{,}155}$; $m_{CF} = \frac{r_{CF}}{R_{CB}} = \frac{2}{2} \times \frac{87}{385} = \frac{87}{385}$; and $m_{CD} = \frac{80}{29} \times \frac{287}{385} = \frac{6{,}960}{11{,}165}$.

The distribution factors are obtained by Eq. 13. For example, $d_{CD} = \frac{1}{1 + R_{CD}/r_{CD}} = \frac{1}{1 + 7/1.8 \times 29/80} = \frac{144}{347} = 0.4150$.

Actual end moments of each loaded span due to the loads in the span are obtained by Eqs. 39; thus:

$$
\begin{aligned}
M_{BC} &= -\,(100 \times 0.3556 + 100 \times -\,0.3343 \times 0.200) = -\,42.24\\
M_{CB} &= +\,(100 \times 0.6657 + 100 \times 0.6444 \times 0.3444) = +\,88.76\\
M_{GC} &= -\,(50 \times 1 + 50 \times 0.1003 \times 0.500) = -\,52.51\\
M_{CG} &= +\,(50 \times 0.8997 + 50 \times 0) = +\,44.98\\
M_{CF} &= +\,(80 \times 0.8496 + 60 \times 1 \times 0.4248) = +\,93.45\\
M_{CD} &= -\,(200 \times 0.585 + 100 \times 0.6628 \times 0.3043) = -\,137.17\\
M_{DC} &= +\,(100 \times 0.3372 + 200 \times 0.4150 \times 0.1875) = +\,49.28
\end{aligned}
$$

These moments are distributed to the frame by the calculated carry-over factors and distribution factors as shown.

Conclusions.—In view of the obvious similarity of the Cross method to the method of successive approximations and the fact that the Cross method is derived from the basic slope-deflection equations, it seems fair to question the author's assertion that the Cross method is based on a concept of "restraint" whereas the slope-deflection method is based "on relations governing deformations comprising the geometry of flexure." Actually, use of the Cross method is equivalent to solving the simultaneous slope-deflection equations by successive approximations. On the other hand, the method indicated by the author is truly based on the concept of "restraint." By recognizing that "restraint" is a function solely of the structure and is entirely independent of the loading and by proper evaluation of "restraint," a method is provided for the distribution of moments in a framework of restrained beams by a single operation.

LEON BESKIN,[15] ASSOC. M. AM. SOC. C. E.—The structural concepts reviewed by the author cover the efforts of engineers, over a period of seventy years, to simplify the analysis of frameworks. By restricting the problem to that of a framework with no joint displacement, the unknowns are limited to joint rotations and to beam-end moments. Two expressions can be written defining the relation between the moments and the rotations at the two ends of each beam of a framework. The analysis of the framework is equivalent to the solution of the system of simultaneous equations represented by these expressions. All the so-called "methods of analysis" are only various procedures for the solution of the system of simultaneous equations. This explains why there is so little fundamental difference between all existent methods, and why the coefficients introduced by one writer can be reduced so easily to those derived by another writer. Practically every stress analyst has computed such relationships at one time or another, and it is improbable that, in this paper, any new physical interpretation is given to structural terms. The relationship between fixed points, carry-over factors, and stiffness factors, known in principle since the fundamental works of O. Mohr and Maurice Lévy,[11] can be found in various forms in more recent papers.[16]

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The concept of node (fixed point) seems to have been introduced by Mr. Mohr for the special case of continuous beams. Later, it was extended to frameworks. A common practice—especially in Europe—is the analysis of frameworks by determining the locations of the fixed points.[17,18] For instance, Professor Campus has taught these methods at Liége University in Liége, Belgium, for many years. These facts challenge the remark in the "Introduction" that "Usually such points are thought of in connection with beams * * *." The author's definitions of the fixed points are misleading. It is incorrect, for example, to state (see "Introduction") that "their position on the elastic curve is stationary," and (see heading, "Fixed Points and Carry-Over Factors") that: "When one end of a restrained member is rotated back and forth, the wave-like elastic curves will pass through a certain node." The amplitude of displacement of a node is different from zero, and, characteristically, a node is a point of inflection. From the principle of superposition, this point is fixed when there is only one applied end moment.

The relation between intercept ratios is well known in the form of an expression for carry-over factors. For continuous beams it is:

$$f_{BC} = \frac{K_{BC}}{2\,(K_{BA} + K_{BC}) - f_{AB}\,K_{BC}} \dots\dots\dots\dots\dots (40a)$$

or

$$\frac{K_{BA}}{2 - f_{AD}} + \frac{K_{BC}}{2 - \dfrac{1}{f_{BC}}} = 0 \dots\dots\dots\dots\dots\dots\dots (40b)$$

The corresponding relation for frameworks is also well known.[17,18] The writer, for example, has shown[18] that a simple derivation of the formula can be established, and that the formula can be written in a symmetric form:

$$\sum_X K_{BX} \frac{1}{2 - f_{BX}} = 0 \dots\dots\dots\dots\dots\dots\dots (41a)$$

or, using the author's notation and defining all the intercept ratios from the common joint:

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[16] "Moments in Restrained and Continuous Beams by the Method of Conjugate Points," by L. H. Nishkian and D. B. Steinman, *Transactions*, Am. Soc. C. E., Vol. 90 (1927), p. 1.

[17] "Analysis of the General Two-Dimensional Framework," by Yves Nubar, *Transactions*, Am. Soc. C. E., Vol. 110 (1945), p. 905.

[18] *Transactions*, Am. Soc. C. E., Vol. 110 (1945), p. 933.

$$\Sigma K_{BX} \frac{n_{BX}}{3\, n_{BX} - 1} = 0 \ldots\ldots\ldots\ldots (41b)$$

In general, the use of carry-over factors leads to simpler formulas than those written in terms of nodal intercept ratios. For instance (compare Table 1):

$$d_{AB} = \frac{2 - f_{AB}}{2 + f_{BA}\,(1 - f_{AB})} \ldots\ldots\ldots\ldots (42a)$$

$$Q_{AB} = 1 - d_{AB} \ldots\ldots\ldots\ldots (42b)$$

$$F_{AB} = \frac{3\, f_{BA}}{2 - f_{BA}} \ldots\ldots\ldots\ldots (42c)$$

$$r_{AB} = \frac{6\, K_{AB}}{2 - f_{AB}} \ldots\ldots\ldots\ldots (42d)$$

and

$$n_{AB} = \frac{1}{1 + f_{AB}} \ldots\ldots\ldots\ldots (42e)$$

In the light of these remarks, it is fair to state, contrary to the author, that the so-called "Cross method" does not permit the analysis of frameworks from a different or novel point of view. The basic relations between end moments and end rotations can be established, either by geometric considerations based on the relation between moment and curvature, or by strain energy methods. All methods have the same basis, and, although the wording is different, the mathematical process is the same. For instance, consider the formula giving the end rotation of a beam as a function of the moments in the beam:

$$\theta_{AB} = \int_A^B \frac{(L - x)\, M}{E\, I}\, dx; \ldots\ldots\ldots\ldots (43)$$

From geometric considerations, it appears that the curvature $\frac{M}{E\,I}$ produces a rotation $\frac{M\,dx}{E\,I} = d\phi$ between the two ends of an infinitesimal element dx, so that the deflection at one joint B, in reference to the tangent at the other joint A, is:

$$\delta_B = \int_A^B (L - x)\, d\phi \ldots\ldots\ldots\ldots (44)$$

If the reference line is rotated by an angle θ_{AB}, equal to the rotation at point A, the deflection at point B is equal to zero, which defines θ_{AB}:

$$\theta_{AB} = \frac{\delta_B}{L} \ldots\ldots\ldots\ldots (45a)$$

By application of strain energy methods, a dummy couple equal to 1 is placed at A, which produces moments $m = (L - x)/L$ at each point of the beam, and the rotation is:

$$\theta_{AB} = \int_A^B m\, d\phi \qquad (45b)$$

Eqs. 45*a* and 45*b* lead to the same relation (Eq. 43). In the "Introduction," Mr. Weiss states:

> "Whereas the method of least work is based on the concepts of work and energy * * * the Cross method may be considered as based on a complex of concepts that can be summed up under the term 'restraint.'"

This statement ignores the fact that Eq. 43 is the starting point for the Cross method as well as for other methods. Continuing, the author states: "Work and deformation imply the action of a force. Restraint, however, is entirely a function of the structure itself." This seems to imply that the concept of restraint can be defined without introducing into it the concept of force. Paradoxically, the author defines restraint thus (see heading, "Stiffness Factors and Distribution Factors"): "In general the resistance to rotation * * * will be E times a stiffness factor or restraint factor 'r.'" In other words, far from being exclusively a function of the structure, restraint is nothing but the ratio of an externally applied load (generally a couple) to a displacement (generally a rotation), multiplied by E.

The author's opinion (see "Introduction") that "The method of moment distribution * * * is as good a method as can be devised," accepted without question by many structural engineers, should nevertheless be challenged. The following summary is intended to clarify the comparison between various methods.

Methods of solving simultaneous equations can be divided into three main groups—succession elimination of unknowns, determinants, and successive approximations. Determinants require an excessive amount of computations when there are more than three or four unknowns, and this method is of little practical interest. The best solution by successive elimination was that presented by Karl F. Gauss, and it has been applied to the problem of continuous beams as the method of conjugate points.[11,16] The relationship between these two methods is easy to establish. To make the problem more general, beams of variable sections are considered. Numbering the supports by even numbers $2\,j$, and the spans by odd numbers $2\,j + 1$, the three-moment equation is:

$$B_{2j-1}\, M_{2j-2} + C_{2j}\, M_{2j} + B_{2j+1}\, M_{2j+2} = H_{2j} \qquad (46)$$

The quantities B, C, and H are defined by the following relations, in which x is the abscissa in each span, measured in the direction of increasing j; ζ is the ratio x/L, and m is the statical moment in each span:

$$B_{2j-1} = L_{2j-1} \int_0^1 \frac{\zeta\,(1-\zeta)\, d\zeta}{I_{2j-1}} \qquad (47a)$$

$$C_{2j} = L_{2j-1} \int_0^1 \frac{\zeta^2\, d\zeta}{I_{2j-1}} + L_{2j+1} \int_0^1 \frac{(1-\zeta)^2\, d\zeta}{I_{2j+1}} \qquad (47b)$$

and

$$H_{2j} = \int_0^1 \frac{\zeta\, m_{2j-1}\, d\zeta}{I_{2j-1}} + \int_0^1 \frac{(1-\zeta)\, m_{2j+1}\, d\zeta}{I_{2j+1}} \ldots\ldots\ldots\ldots (47c)$$

Eliminating unknowns, successively, to the moment M_{2j-4}, inclusive, Eq. 46 (written for the support $2j - 2$) is transformed into:

$$C'_{2j-2}\, M_{2j-2} + B_{2j-1}\, M_{2j} = H'_{2j-2} \ldots\ldots\ldots\ldots (48a)$$

The moment M_{2j-2} is eliminated by the linear combination of Eqs. 46 and 48*a* and leads to an equation similar to Eq. 48*a*:

$$C'_{2j}\, M_{2j} + B_{2j+1}\, M_{2j+2} = H'_{2j} \ldots\ldots\ldots\ldots (48b)$$

which shows that quantities C'_{2j} and H'_{2j} are defined by the following relations of recurrence:

$$C'_{2j} = C_{2j} - \frac{B^2_{2j-1}}{C'_{2j-2}} \ldots\ldots\ldots\ldots (48c)$$

and

$$H'_{2j} = H_{2j} - \frac{B_{2j-1}}{C'_{2j-2}}\, H'_{2j-2} \ldots\ldots\ldots\ldots (48d)$$

The first member of Eq. 48*b* defines a linear relation between M_{2j} and M_{2j+2}, which shows that the carry-over factor f_{2j+1} is:

$$f_{2j+1} = \frac{B_{2j+1}}{C'_{2j}} = \frac{B_{2j+1}}{C_{2j} - f_{2j-1}\, B_{2j-1}} \ldots\ldots\ldots\ldots (49)$$

Eq. 49 can be identified easily with Eq. 40*a*, since, in the case of beams with constant sections, the following relations hold:

$$B_{2j+1} = \frac{1}{6\, K_{2j+1}}; \ldots\ldots\ldots\ldots (50a)$$

and

$$C_{2j} = \frac{1}{3\, K_{2j-1}} + \frac{1}{3\, K_{2j+1}} \ldots\ldots\ldots\ldots (50b)$$

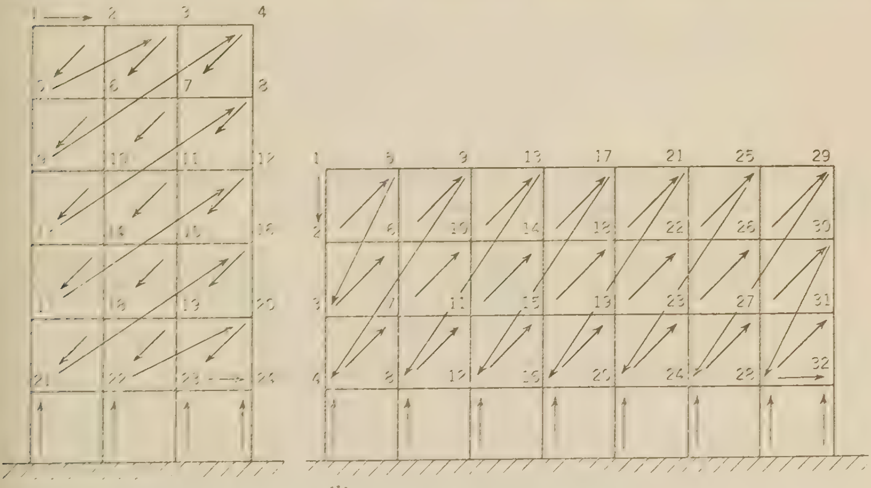
Eq. 48*b* can be written for all the spans, and the equation for the last span contains only one unknown moment, which is thus directly determined. Using Eq. 48*b* in reversed sequence, all the moments are defined, successively, by relations such as:

$$M_{2j} = -\frac{B_{2j+1}}{C'_{2j}}\, M_{2j+2} + \frac{H'_{2j}}{C'_{2j}} \ldots\ldots\ldots\ldots (51)$$

Thus, the method of successive eliminations leads to the use of Eqs. 48*c*, 48*d*, and 51, representing eight numerical operations per span, three being independent of the loading conditions (determination of the coefficients C'_{2j}).

The Cross method requires five numerical operations per span, per cycle—two distributions, two carry-over operations, and one summation. When the cycles are completed, an over-all addition, equivalent to at least one operation per cycle, is required. Thus, there are six operations per cycle, instead of a total of eight operations by the direct method. If checks are made, it can be shown that the direct method requires thirteen operations per span (additional calculation of coefficients C'' and H' from the second end of the beam), whereas the Cross method is equivalent to nine operations per span, per cycle. The preliminary operations are equivalent in both methods. For instance, the two carry-over factors in a span are $\frac{B_{2j-1}}{C_{2j-2}}$ and $\frac{B_{2j-1}}{C_{2j}}$; and the determination of the factors H is equivalent to the determination of the fixed-end moments. Thus, as two or three cycles, and very often more, are required by the Cross method, it is more expeditious, in the case of beams, to apply the direct method. In addition, three operations per span (or six operations, including checks) are independent of the loading in the direct method. Thus, for additional loading conditions, seven operations per span (including checks) are required by the direct method, whereas nine operations per span, per cycle, are required by the Cross method. In this case, the Cross method is definitely inefficient.

The direct method can be used in the case of frameworks. However, because there is no initial equation of the type of Eq. 48*a* along a closed circuit, the direct method is generally transformed into a method of successive approximations. In the initial form, complications increase with the number of parallel beams (stories) or columns (spans). For instance, for four parallel rows (spans or stories), the elimination should be made in the sequence (see Fig. 7): 1, 2, 5, 3, 6, 9, 4, 7, 10, 13, 8, 11, 14, 17, · · ·. In the case of two rows of joints, and especially when the joints are not fixed (Vierendeel beams such as high buildings with lateral load), the direct method is more advantageous



(*a*) (*b*)

FIG. 7

than methods of successive approximation, but this advantage disappears for a greater number of rows of joints. Finally, contrary to general opinion, the accuracy of the direct method does not decrease when there are many spans, because the coefficients, such as $\frac{B^2_{2j-1}}{C'_{2j-2}}$ (see Eq. 48*c*), are of the order of magnitude of one quarter of the coefficients C_{2j} so that small errors tend to disappear, instead of accumulating. This fact is precisely the cause of the rapid convergence of methods of successive approximations.

Methods of successive approximations have been developed by physicists and mathematicians, in connection with various problems in physics. The first systematic work in this field can be attributed to J. Fourier,[19] who developed his theories around 1820. However, it was not until between 1900 and 1910 that various mathematicians, principally J. Fredholm and D. Hilbert,[20] established the theory in its present form. Their work dealt mainly with equations containing an infinite number of unknowns, because physical problems generally lead to such equations. Thus, the theories are greatly simplified when applied to structural problems with a finite number of unknowns.

Methods of successive approximations can be presented as follows (see Eqs. 47). A system of linear equations can be written in the form:

$$\sum_j X_j\, a_{ij} = Y_i \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (52)$$

in which X_j represents the unknowns, a_{ij} the coefficients (defined by the structure only), and Y_i the second members (defined by the structure and the loading). The matrix form of Eq. 52 is:

$$|X_j|\,|a_{ij}| = |Y_i| \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (53a)$$

and the solution is:

$$|X_j| = |Y_i|\,|a'_{ij}| \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (53b)$$

in which the matrix $|a'_{ij}|$ is the reciprocal of the matrix $|a_{ij}|$; thus:

$$|a'_{ij}| = \frac{1}{|a_{ij}|} = |a_{ij}|^{-1} \ldots\ldots\ldots\ldots\ldots\ldots (54a)$$

Eq. 54*a* has no significance until the numerical computation which it represents is defined. For this purpose, the matrix $|a_{ij}|$ is written as:

$$|a_{ij}| = |a_{ii}|\left|\frac{a_{ij}}{a_{ii}}\right| = |a_{ii}|\left[|E| + |b_{ij}|\right] \ldots\ldots\ldots\ldots (54b)$$

The first equality in Eq. 54*b* signifies that each column of the square matrix $|a_{ij}|$ is divided by its diagonal term a_{ii} and the resulting matrix $\left|\frac{a_{ij}}{a_{ii}}\right|$ can be multiplied by the column matrix $|a_{ii}|$ to find the matrix $|a_{ij}|$. In $\left|\frac{a_{ij}}{a_{ii}}\right|$, the diagonal terms $\frac{a_{ii}}{a_{ii}}$ are equal to unity. The second equality in Eq. 54*b* signifies that $\left|\frac{a_{ij}}{a_{ii}}\right|$ can be written in the form of a sum of the identical matrix $|E| = |e_{ij}|$ (with: $e_{ii} = 1$ and $e_{ij} = 0$), and of the matrix $|b_{ij}|$ $\Bigg($with: $b_{ii} = 0$ and

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[19] "Théorie Analytique de la Chaleur," by J. Fourier, Didot, Paris, France, 1822.

[20] "Methoden der Mathematischen Physik," by R. Courant and D. Hilbert, Julius Springer, Berlin, 1931.

$b_{ij} = \dfrac{a_{ij}}{a_{ii}}$). It can be shown that the relation—

$$(1 + r)^{-1} = \frac{1}{1 + r} = 1 - r + r^2 - r^3 \cdots \pm r^n \cdots \ldots\ldots\ldots (55)$$

—holds for the matrix $|E| + |b_{ij}|$; thus:

$$(|E| + |b_{ij}|)^{-1} = |E| - |b_{ij}| + |b_{ij}|^2 \cdots \pm |b_{ij}|^n \cdots \ldots\ldots (56)$$

The conditions of convergence of the series in the right-hand member of Eq. 56 are satisfied in the case of equations corresponding to redundant structures. Thus, Eq. 53*b* can be written in the form:

$$|X_j| = |Y_i|\,|a_{ij}|^{-1} = \left|\frac{Y_i}{a_{ii}}\right| (|E| - |b_{ij}| \cdots \pm |b_{ij}|^n \cdots) \ldots (57)$$

Eq. 57 has a definite significance, since multiplication and addition of matrices are defined operations. This equation is the basis of the "method of relaxation" developed by R. V. Southwell,[21] of which the Cross and the Maney-Goldberg[22,23,24,25] methods are only special applications. In the case of frameworks, the latter method can be reconciled with the method of relaxation as follows:

In the Cross method, quantities that can be called "total unbalanced moments" (M_j) are the principal unknowns. These quantities are related to the fixed-end moments $M^o_{j,i}$ (moment at joint j, in beam ji), by

$$M_j + \sum_i M_i\, b_{i,j} = \sum_i M^o_{j,i} = M^o_j \ldots\ldots\ldots\ldots\ldots (58)$$

Factors $b_{i,j}$ are the products of distribution factors by carry-over factors:

$$b_{i,j} = d_{i,j} f_{i,j} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (59)$$

The final-end moments $M_{i,j}$ are defined by:

$$M_{i,j} = M^o_{i,j} - M_i\, d_{i,j} - M_j\, b_{j,i} \ldots\ldots\ldots\ldots\ldots (60)$$

The method of successive approximation can be expressed by the following relations between successive unbalanced moments:

$$M^1_j = - \sum_i b_{i,j}\, M^o_i \ldots\ldots\ldots\ldots\ldots\ldots\ldots (61a)$$

and

$$M^{n+1}_j = - \sum_i b_{i,j}\, M^n_i \ldots\ldots\ldots\ldots\ldots\ldots (61b)$$

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[21] "Systematic Relaxation of Constraints," by R. V. Southwell, *Proceedings*, Royal Soc. of London, Vol. CLI, Series A, 1935, pp. 56–95; Vol. CLIII, Series A, 1935–1936, pp. 41–76.

[22] "Wind Stresses in the Steel Frames of Office Buildings," by W. M. Wilson and G. A. Maney, *Bulletin No. 80*, Univ. of Illinois Eng. Experiment Station, Urbana, 1915.

[23] "Vertical-Load Analysis of Rigid Building Frames Made Practicable," by John E. Goldberg, *Engineering News-Record*, November 12, 1931, pp. 770–772.

[24] "Simplified Methods for the Analysis of Multiple Joint Rigid Frames," by G. A. Maney and John E. Goldberg, *Bulletin*, Northwestern Univ., School of Eng., 1932.

[25] "Wind Stresses by Slope Deflection and Converging Approximations," by John E. Goldberg, *Transactions*, Am. Soc. C. E., Vol. 99 (1934), pp. 962–985.

so that:

$$M_j = \sum_{n=1}^{\infty} M^n{}_j \tag{62}$$

In the Cross method, Eqs. 61 are written in the following detailed form: Distribution—

$$\Delta M^{n+1}{}_{i,j} = - M^n{}_i \, d_{i,j} \tag{63a}$$

carry over—

$$\partial M^{n+1}{}_{j,i} = \Delta M^{n+1}{}_{i,j} \, f_{i,j} \tag{63b}$$

unbalance—

$$M^{n+1}{}_j = \sum_i \partial M^{n+1}{}_{j,i} \tag{63c}$$

Eq. 60 is written in the form:

$$M_{i,j} = M^o{}_{i,j} + \sum_{n=1}^{\infty} \Delta M^n{}_{i,j} + \sum_{n=1}^{\infty} \partial M^n{}_{i,j} \tag{64}$$

The method has been perfected by elimination of Eq. 63*a*, so that:

$$\partial M^{n+1}{}_{j,i} = - M^n{}_i \, b_{i,j} \tag{65}$$

whereas Eq. 64 is written as:

$$M_{i,j} = M^o{}_{i,j} - M_i \, d_{i,j} + \sum_{n=1}^{\infty} \partial M^n{}_{i,j} \tag{66}$$

The method thus developed is easily identified with Eq. 57, written as:

$$|M_j| = |M^o{}_i| \, (|E| - |b_{i,j}| + \cdots \pm |b_{i,j}|^n + \cdots) \tag{67a}$$

$$|M_j| = |M^o{}_j| - |M^o{}_i| \, |b_{i,j}| + \cdots \pm |M^o{}_i| \, |b_{i,j}|^n + \cdots \tag{67b}$$

and

$$|M_j| = |M^o{}_j| + |M^1{}_j| + \cdots + |M^n{}_j| + \cdots \tag{67c}$$

Eq. 67*c* is the result of the repeated substitution of Eq. 61*b*, written in the form—

$$|M^{n+1}{}_j| = - |M^n{}_i| \, |b_{i,j}| \tag{68}$$

—in Eq. 67*b*.

Thus, it appears that the Cross method is an application, to structural problems, of a conventional method of mathematical analysis. The slope-deflection method, developed by G. A. Maney, M. Am. Soc. C. E., and John E. Goldberg,[22,23,24,25] Assoc. M. Am. Soc. C. E., is equivalent in its initial form to the improved Cross method (elimination of Eq. 63*a*). The difference is in the choice of unknowns which are the slope deflections instead of the "total unbalanced moments." The final moments are defined by a relation similar to Eq. 60, in which $r_{i,j}$ is a restraint factor:

$$M_{i,j} = M^o{}_{i,j} - E \, \theta_i \, r_{i,j} - E \, \theta_j \, r_{j,i} \, f_{j,i} \tag{69}$$

From the relation

$$d_{i,j} = \frac{r_{i,j}}{\sum_j r_{i,j}} \tag{70}$$

it is found that

$$M_i = E\,\theta_i \sum_j r_{i,j} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (71)$$

which shows the relationship between the two methods. The Maney-Goldberg method appears to be the simpler, since it makes use of the elementary concept of joint rotation instead of the complex concept of total unbalanced moment which, in final analysis, is the external couple required to produce the actual joint rotation when the other joints are fixed and when no other load is applied.

The original Maney-Goldberg method has been improved by its authors in the following manner: Dividing the joints into two groups i and j defined in such a manner that every span connects two joints belonging to the two groups, Eq. 67*a* can be written in the form—

$$|M_j| = (|M^{\circ}_j| - |M^{\circ}_i|\,|b_{ij}|)\,(|E| + |b_{i,j}|^2 + \cdots + |b_{i,j}|^{2n} \cdots) \,.\,. (72a)$$

and

$$|M_i| = |M^{\circ}_i| + (|M^{\circ}_j| - |M^{\circ}_i|\,|b_{i,j}|) \times |b_{i,j}|\,(|E| + \cdots + |b_{i,j}|^{2n} \cdots) \ldots\ldots\ldots\ldots (72b)$$

Since the sum $|M^{\circ}_j| - |M^{\circ}_i|\,|b_{i,j}|$ forms one matrix, the number of operations required by Eqs. 72 is half of the number necessitated by Eq. 67*a*, to obtain the same approximation. This short cut is essential when the convergence is slow (as, for example, for structures with sidesway).

The Maney-Goldberg method may appear longer than the Cross method, because it necessitates the computation of moments equivalent to rotations. However, since a check of the results is indispensable, it is necessary to calculate the rotations when the Cross method is applied, to find whether the relations between rotations and end moments are satisfied by the solution. The additional operation is included in the Maney-Goldberg method which is self-checking. When this operation is added to the Cross method, the latter becomes self-checking, and the number of operations is the same by both methods, provided the Maney-Goldberg short cuts are applied to the Cross method.

Another method of successive approximations results from a different interpretation of Eq. 67*a*: If the matrix,

$$|B_{i,j}| = |E| - |b_{i,j}| + |b_{i,j}|^2 \cdots \pm |b_{i,j}|^n \cdots\ldots\ldots\ldots (73)$$

is determined, the total unbalanced moments are defined simply by:

$$|M_j| = |M^{\circ}_i|\,|B_{i,j}| \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (74)$$

This method can be applied to slope deflection as well. The numerical calculation of the matrix $|B_{i,j}|$ is about as complex as the determination of one actual solution by Eq. 67*c*. On the other hand, this operation is done only once for a given structure, after which Eq. 74 defines all the total unbalanced moments with a minimum of operations for any loading condition, avoiding the tedious repetition of successive approximations in each case. This method can easily be identified with the method of conjugate points, since, in both methods, the moments at any joint are defined directly in terms of a linear combination of the fixed-end moments at all the joints.

In conclusion, the direct method of successive elimination is always the most advantageous in the case of one row of spans, and it is generally the most advantageous for two rows of spans. In all the other cases, methods of successive approximations are more powerful. When only one condition or very few loading conditions are examined, the Maney-Goldberg method is the most expeditious, although the Cross method can be improved to obtain the same efficiency. When many loading conditions are examined, the method of successive elimination, transformed into a method of successive approximations (conjugate points in frameworks), is generally the best. The section under the heading, "General Principles of Standardization," in Appendix II, will help to avoid the use of such cumbersome notations as FEM for fixed-end moments and $\overline{\text{DL}}$ and $\overline{\text{LL}}$ for dead load and live load.

ALEXANDER DODGE,[26] ASSOC. M. AM. SOC. C. E.—The slope-deflection method has been used in this paper to derive equations for members of constant moment of inertia. In appraising the work done by Mr. Weiss, it will be instructive to compare it with parallel studies by the writer[27] in 1939, which involved the development of the moment-area principle for members of varying moments of inertia. The writer's studies included the effect of sidesway and the construction of influence lines. The definition of fixity constitutes the main difference in the papers. In the writer's concept, the "end fixity" is defined as the ratio of $\frac{Z'}{Z}$, in which Z' is the exact moment at point B due to unit moment at point A, and Z is the moment at point B due to a unit moment at point A if end B is fixed. For members with a constant moment of inertia, $Z = 0.5$. Referring to Fig. 2, the "end fixity" is

$$f_{ba} = \frac{\Sigma S_p}{\Sigma S_p + S_{ba}} \qquad (75a)$$

in which S_p is the exact stiffness corresponding with the stiffness factor r in the author's concept, or

$$f_{ba} = \frac{R_{BA}}{R_{BA} + 3\,K_{BA}} \qquad (75b)$$

Eq. 75*b* will be recognized as a part of Eq. 16. The reversed subscript for the symbol K denotes that the stiffness at end B of member AB is used, which is an important distinction in analyzing a member of varying moment of inertia. The carry-over factor may be looked upon as the exact moment at joint B due to unit moment at end A. With these changes in the subscript notations as explained in Appendix I, Eq. 16 would be changed to

$$f_{BA} = \tfrac{1}{2}\,\frac{R_{BA}}{R_{BA} + 3\,K_{BA}} \qquad (76)$$

thus simplifying most of the equations derived from it. In itself, this is not an important change, but the fact that all terms in Eq. 76 refer to the same point

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[26] Engr., U. S. Engrs., Portland, Ore.

[27] "Solution of Rigid Frames by Coefficients of Unit Moment," by Alexander Dodge, thesis presented to the University of Washington, Seattle, Wash., in 1939 in partial fulfilment of the requirements for the professional degree of Civil Engineer.

(joint B) will certainly be appreciated by engineers who may have practical use for the concepts. The definition of end fixity expressed by Eqs. 75 has several advantages: (1) It is a linear function of the carry-over factor and, therefore, has a physical significance that can be visualized easily and clearly; and (2) it makes the expression for the exact stiffness of the member, S_p, very simple, namely:[27]

$$S_p = \frac{C'}{C} S \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (77)$$

in which S is the stiffness of the member at joint B if end A is hinged; C is the ratio of stiffness S to the stiffness of the member at joint B if end A is fixed; and[27]

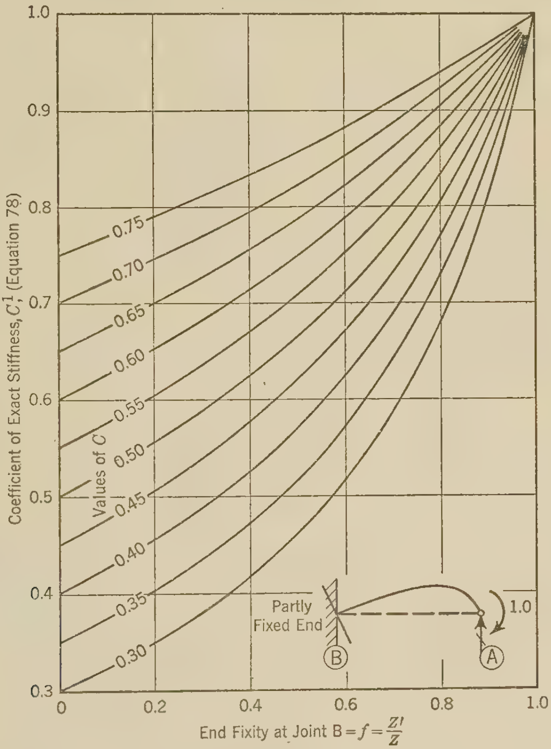

FIG. 8

$$C' = \frac{C}{1 - f(1 - C)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (78)$$

Charts for the constants S and C have been prepared and published by Thorbjörn Germundsson,[28] Assoc. M. Am. Soc. C. E. Values of C' for any variation of moment of inertia can be selected from Fig. 8. For a constant moment of inertia, Eq. 77 becomes:

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[28] "Design of Continuous Frames Having Variable Moment of Inertia," by Thor Germundsson, *Civil Engineering*, October, 1932, p. 647.

$$S_p = \frac{C'}{0.75} \times \frac{3\,I}{L} = \frac{4\,C'\,I}{L} \ldots\ldots\ldots\ldots\ldots\ldots (79)$$

In general, referring to Eq. 23, the end moment at end A is not equal to $Q_{AB}\,C_{AB}$; but, by Eq. 16, it is equal to $\frac{R_{AB}}{R_{AB} + 3\,K_{AB}}\,C_{AB}$, which is a constant that is independent of the characteristics of joint B. In Eq. 17*a*, r_{AB} has a limit of $3\,K_{AB} \leqq r_{AB} \leqq 4\,K_{AB}$, which shows that r_{AB} is a function of the characteristics of joint B also. Therefore, using Fig. 1 as an example, it should be kept clearly in mind that, for this particular frame, the relation $Q_{XY} = 2\,f_{YX}$ applies to no member except span BA, because $Q_{XY} \neq \frac{R_{XY}}{R_{XY} + 3\,K_{XY}}$.

In order to generalize the subject, the effect of varying moment of inertia should be examined. Using charts prepared by Mr. Germundsson,[28] with Eqs. 16 and 17*a*, the values in Table 4 were computed for the ratio $b = \frac{I}{I'} = 0.08$, as applying to four designs of beam AB. The values common to all beams were assumed to be: $R_{AB} = 2.0$; the stiffness factor at end A of the member AB, with end B hinged, is equal to $S_{ab} = 1.0$, and $Q_{AB} = 67\%$ when $Q_{BA} = 0$. If the values of Table 4 were plotted on the chart in Table 1, they would be found scattered over a wide area. This indicates that, in general, no simple relation exists between values of Q and n. An important modification would also be noted in Eq. 22*b*, the constant of which would be changed from 2 to 1.333, 0.908, and 3.33, respectively, for the first three beams in Table 4.

TABLE 4.—CONSTANTS FOR BEAM AB AND VARIOUS COMBINATIONS OF STRAIGHT HAUNCHES

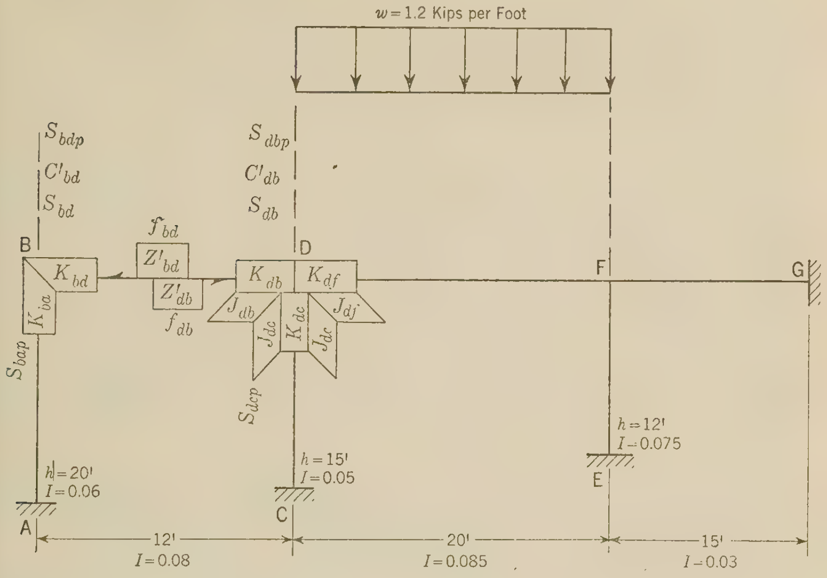
Haunches at:	Ratio, haunch to span length[a]	f_{BA}	n_{BA}	Q_{AB} (end B fixed)[b]
A and B..............	0.4	0.5	0.667	0.462
End A................	0.8	0.734	0.572	0.568
End B................	0.8	0.2	0.833	0.568
Uniform section........	....	0.333	0.75	0.6

[a] The ratio of minimum to maximum moment of inertia is 0.08 in each case. [b] The value of Q_{AB} with end B hinged is 0.67 in each case.

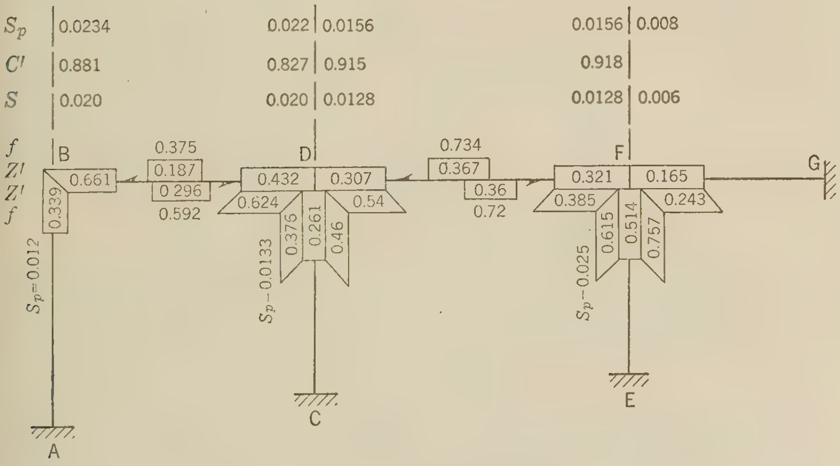
The basic structural concepts are expressed fully by Eqs. 13, 16, 75, and 77, and their application to the solution of various structures, including frames with sidesway and varying moment of inertia as well as to the construction of influence lines has been demonstrated previously.[27] A typical example is demonstrated in Fig. 9. The method of tabulating the needed values in terms of symbols is shown in Fig. 9(*a*), and the numerical solution is given in Fig. 9(*b*). Recalling that constants for a beam of uniform moment of inertia are: $S = \frac{3\,I}{L}$, $C = 0.75$, and $Z = 0.5$, the computations are made starting from the left end of the frame and proceeding to the right end. By Eqs. 75, $f_{bd} = 0.375$. With this value enter Fig. 8 and read $C'_{db} = 0.827$. By Eq. 77, $S_{dbp} = 0.022$; by Eqs. 75, $f_{df} = 0.734$; $C'_{fd} = 0.918$; and $S_{fdp} = \frac{0.918 \times 0.0128}{0.75} = 0.0156$.

Beginning at the right end of the frame and proceeding to the left end:

$$f_{fd} = \frac{0.008 + 0.025}{0.008 + 0.025 + 0.0128} = 0.72;\quad C'_{df} = 0.915;\quad S_{dfp} = \frac{0.915 \times 0.0128}{0.75}$$


w = 1.2 Kips per Foot
S_bdp
C'_bd
S_bd
S_dbp
C'_db
S_db
f_bd
Z'_bd
Z'_db
f_db
B
D
F
G
K_bd
K_ba
K_db
K_df
J_db
J_dc
K_dc
J_dc
J_df
S_bap
S_dcp
h = 20'
I = 0.06
h = 15'
I = 0.05
h = 12'
I = 0.075
A
C
E
12'
I = 0.08
20'
I = 0.085
15'
I = 0.03

(a) GENERAL DATA


S_p
C'
S
f
Z'
Z'
f
0.0234
0.881
0.020
0.022
0.0156
0.827
0.915
0.020
0.0128
0.0156
0.008
0.918
0.0128
0.006
B
D
F
G
0.375
0.187
0.296
0.592
0.661
0.339
0.432
0.307
0.624
0.54
0.376
0.261
0.46
0.734
0.367
0.36
0.72
0.321
0.165
0.385
0.243
0.615
0.514
0.757
S_p = 0.012
S_p = 0.0133
S_p = 0.025
A
C
E

(b) COMPUTATIONS

FIG. 9.—SOLUTION OF A RIGID FRAME

$= 0.0156$; $f_{db} = \dfrac{0.0156 + 0.0133}{0.0156 + 0.0133 + 0.02} = 0.592$; $C'_{bd} = 0.881$; and $S_{bdp} = \dfrac{0.02 \times 0.881}{0.75} = 0.0234$. By Eq. 13, $K_{ba} = \dfrac{0.012}{0.012 + 0.0234} = 0.339$. It should be noted that factor K_{ba} corresponds to factor d_{BA} in the author's concept.

All other K-values are computed similarly. In practice, the computations are made on a slide rule and recorded directly in Fig. 9(a). It will be noted that, in addition to the foregoing computations, the coefficients J are also recorded in Fig. 9. These coefficients indicate the proportion in which the coefficient Z' received at the joint is divided between the remaining members; for example, $J_{db} = \frac{K_{db}}{K_{db} + K_{dc}} = \frac{0.432}{0.432 + 0.261} = 0.624$, etc. Noting that the coefficients K and J always carry a minus sign and that coefficients Z' have a plus sign, and assuming clockwise moments to be positive, for the loading conditions of Fig. 9(a), the final moments will be as follows: Fixed-end moments $= \frac{w\,L^2}{12} = \frac{1.2 \times 20^2}{12} = 40$ kip-ft; $M_{DF} = -40\,(1 - K_{df}) + 40\,(-K_{fd} \times Z'_{df}) = -32.43$ kip-ft; $M_{DB} = M_{DF}\,J_{db} = 20.24$; $M_{BD} = M_{DB}\,Z' = 3.78$; $M_{AB} = 3.78 \times -1 \times 0.5 = -1.89$; $M_{DC} = -32.43 \times -0.376 = 12.19$; $M_{CD} = 12.19 \times 0.5 = 6.10$; $M_{FD} = 40\,(1 - 0.321) + (-40 \times -0.307 \times 0.36) = 31.58$; $M_{FE} = 31.58 \times -0.757 = -23.91$; $M_{EF} = -23.91 \times 0.5 = -11.96$; $M_{FG} = 31.58 \times -0.243 = -7.67$; and $M_{GF} = -7.67 \times 0.5 = -3.84$. If a frame is built up of members with varying moments of inertia, the same procedure is used, except that the coefficient C' would be selected from Fig. 8 to correspond with the respective beam C-values.

The author's paper, being essentially mathematical, might be improved if its definitions and symbols were adapted to a more general case. For example, for the third beam in Table 4, the node caused by end rotation is closer to the generating end A if end B is fixed. Therefore, the definition of the carry-over factor as the ratio of the shorter nodal intercept to the longer nodal intercept would be in error. For these reasons, the definition of the "carry-over factor" as originally stated by Hardy Cross, M. Am. Soc. C. E., and so well rooted in engineering literature, should not be altered. There is an objection also to symbol r being used to designate the restraint or stiffness factor. The symbol r, designating the word "ratio," has been used to designate Professor Cross' "carry-over" factor, and tables have been prepared, published,[28] and broadly used in structural analyses. The symbol S, abbreviating a word stiffness, also has a good logical reason to be retained to represent the "stiffness" of a beam, and S_p "the exact stiffness," where the subscript p denotes "a partly fixed or partly restrained beam." As noted in Appendix II, the letter symbol $K = \frac{I}{L}$ equals the stiffness factor or a factor of proportionality of stiffness. Sometimes the letter K also denotes "the relative stiffness of a member" when compared with other members. Therefore, to avoid confusion, r_{BX} in Eq. 10 should not be referred to as "the stiffness factor," but as "the stiffness" or rather, "the exact stiffness." The meaning of "exact stiffness" may be clarified by referring to the "unit stiffness" concept which is defined thus: If a unit moment applied at the hinged end of a beam is causing unit rotation of the same end (for a given degree of restraint at the other end) the beam is said to have a "unit stiffness" at the end where the unit moment is applied. The "exact stiffness," then, is an inverse function of the true rotation angle at the end of the beam at which the unit moment is applied.

WILLIAM C. SPIKER,[29] M. AM. SOC. C. E.—A possible method is suggested by the author whereby engineers may eventually come nearer to agreement as to the simplest possible procedure for computing stresses in rigid frames. If engineers can agree on fundamental concepts and uniform symbols, progress will have been made.

The author and the writer agree that the "point of inflection" is an important and fundamental concept. Apparently they do not agree as to the importance of θ as a measure of resistance although they do agree as to the importance of resistance as a concept. The writer especially does not accept the author's symbols "d," "r," and "n" because they do not suggest what they mean and are in conflict with other symbols already well established.

Very strongly, the writer agrees with the author's idea of charting the location of points of inflection; but he is of the opinion that Table 1 is either not clear or is incorrect; and that the formulas of the paper are unnecessarily abstruse.

For computing tables and charts of θ-values, resistances, points of inflection, etc., the writer uses principles and formulas that can be deduced from the equations of the slope-deflection method and from the routine of its application. For convenience, the fixed-end moment FEM is taken equal to 100 or multiples of 100. Values of L are then equal to the square root of 1,200, when the unit load (w) is equal to 1, or multiples of 1.

Instead of using $I/L = K$ for all spans, that ratio is used only where one end of a member is actually fixed and where reference is made to the particular member along which "carry overs" are being made and for which equivalent K-values are being computed.

Reference can be made to the short cut or special case of the slope-deflection equation for beams hinged at one end; thus,

$$M_{A-B} = 3\,E\,K\,\theta_A - 3\,E\,K\,R - (\text{FEM})_1 \ldots\ldots\ldots\ldots (80)$$

in which $(\text{FEM})_1$ is the fixed-end moment of the beam when only one end of the beam is fixed or assumed to be fixed. It is the algebraic sum of FEM and the carry over. In Eq. 80, the quantity $3\,E\,K\,\theta$ is actually $4\,E\,(\frac{3}{4}\,K)\,\theta$. In other words, K would be $\frac{3}{4}\,K$ if conditions remained the same except that the other end of the beam were fixed. This reduced K may be called the true, effective, or equivalent, value of K; but whatever it is called it designates a measurable, computable (or determinable) amount of resistance to the rotation of the end of a member. It is always less than the ratio I/L except when the other end of the member is actually fixed; or when the other end of the member is being rotated in the same direction as the end in question, and carry overs are omitted. The formula for computing the reduction of effectiveness of I/L

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[29] Cons. Engr., Atlanta, Ga.

at end of any member is taken from the slope-deflection method; thus:

$$K_{A-B}\text{ (equivalent value)} = K_{A-B} - \left[\frac{(0.5\,K)^2}{K_{A-B} + \Sigma K_B\text{ (effective values)}}\right] \ldots (81)$$

In Eq. 81, ΣK_B (effective values) is the K-value for members that are actually fixed at the other end, plus K_B (equivalent values) for members that are not fixed at the other end. The true, or effective value of K is always I/L or a percentage of that ratio.

The total carry over is always the product of $\dfrac{0.5\,K}{K_{A-B} + \Sigma K_B\text{ (effective values)}}$ and the algebraic sum of the fixed-end moment for beam AB at end B and the values of $(\text{FEM})_1$ for members meeting AB at end B.

The product $4\,\theta$ in the slope-deflection equation is always the algebraic sum of $(\text{FEM})_1$'s divided by ΣK (effective values) at the joint. Then the application of the slope-deflection equation to the known values of θ determines the moments and checks them for continuity. Also, it detects an error in the location of the point of contraflexure in columns in case the points of contraflexure in the columns have been approximated.

T. F. Hickerson,[30] M. Am. Soc. C. E.—This is an interesting and valuable paper; but these structural concepts and the interrelated exact carry-over factors do not constitute an integral part of the Cross method of moment distribution, as the casual reader might infer from the author's introduction.

To the list of algebraic relations in Table 1, one more column might be added for completeness, as indicated in Table 5. Here certain concepts are expressed in terms of "carry-over factors." The term "degree of restraint" as introduced by the author might well be omitted. It serves no useful purpose except as an approximate check on the "degree of fixity"; the K-values alone could serve the same purpose in a much simpler manner. Thus, in Fig. 3:

$$F_{BA} = \frac{K_{BJ} + K_{BC} + K_{BH}}{K_{BJ} + K_{BC} + K_{BH} + K_{BA}} \ldots\ldots\ldots\ldots\ldots (82)$$

approximately.

The letter symbols in Appendix I appear, in the main, to be well chosen. Subscripts must be added to avoid confusion. The idea of nodal intercept tends to confuse rather than clarify the concept of "carry-over factor," and should be omitted from the definition. Instead of defining $K\ (= I/L)$ as "stiffness ratio," the writer prefers "stiffness coefficient" or "nominal stiffness" inasmuch as the real stiffness (or restraint) of a structural member is dependent not only on its K-value but also on the degree of fixity at the far end of the member.

A symbol should be added for "moment split." This symbol indicates how a moment that is carried over to the end of a member is to be balanced into, or absorbed by, each of the other members at the joint. Evidently this induced moment divides itself among the receiving members in accordance

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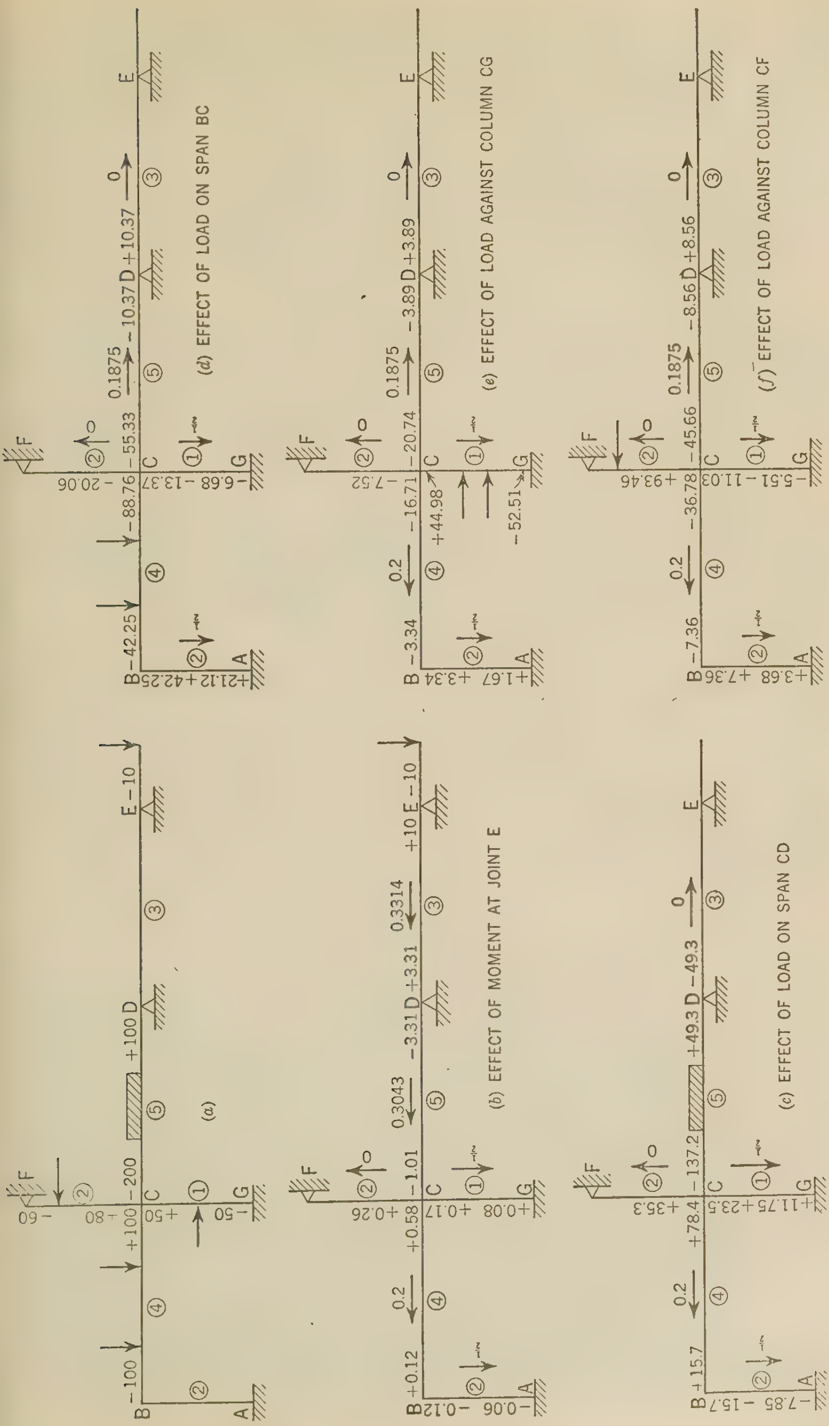
[30] Prof. of Applied Math., Univ. of North Carolina, Chapel Hill, N. C.


B
A
C
D
E
F
G
① ② ③ ④ ⑤
−100
+100
−200
+100 D
E −10
−60
+80
+50
−50
+0.12
−0.12
−0.06
0.2
+0.58
+0.26
+0.17
+0.08
−1.01
0.3043
−3.31 D +3.31
0.3314
+10 E −10
B +15.7
−15.7
−7.85
+78.4
+35.3
+23.5
+11.75
−137.2
+49.3 D −49.3
0
B −42.25
+42.25
+21.12
−88.76
−20.06
−13.37
−6.68
−55.33
0.1875
−10.37 D +10.37
B −3.34
+3.34
+1.67
−16.71
−7.52
+44.98
−52.51
−20.74
−3.89 D +3.89
B −7.36
+7.36
+3.68
−36.78
+93.46
−11.03
−5.51
−45.66
−8.56 D +8.56

(a)

(b) EFFECT OF MOMENT AT JOINT E

(c) EFFECT OF LOAD ON SPAN CD

(d) EFFECT OF LOAD ON SPAN BC

(e) EFFECT OF LOAD AGAINST COLUMN CG

(f) EFFECT OF LOAD AGAINST COLUMN CF

Fig. 10.

with their relative stiffness values. This "moment split" ratio is likely to be confused with the "distribution factor." As a rule, the writer prefers not even to compute the distribution factor; that is, instead of assuming the "near end" fixed—a condition that does not actually exist—just use the real degrees of fixity at each end of the loaded member as arguments (in a table compiled by the writer[31]) and write down the coefficients giving final moments in the member. These end moments are then balanced into the restraining members, and carried on to remote joints in greatly reduced quantities.

TABLE 5.—STRUCTURAL CONCEPTS IN TERMS OF THE CARRY-OVER FACTOR

Structural concept	Symbol	Carry-over factor
Nodal intercept ratio	n_{AB}	$\dfrac{1}{f_{AB}+1}$
Distribution factor	d_{AB}	$\dfrac{1-2f_{BA}}{1-f_{AB}f_{BA}}$
Degree of fixity	F_{AB}	$\dfrac{3f_{BA}}{2-f_{BA}}$
Restraint factor (or stiffness factor)	R_{AB}	$\dfrac{6f_{BA}K_{AB}}{1-2f_{BA}}$

Instead of Eqs. 39, a parallel set is as follows:

$$M_{BA} = [\mathrm{FEM_{BA}} + \mathrm{FEM_{AB}}\,\tfrac{1}{2}\,(1 - F_{AB})]\,(1 - d_{BA})\ldots\ldots(83a)$$

and

$$M_{AB} = [\mathrm{FEM_{AB}} + \mathrm{FEM_{BA}}\,\tfrac{1}{2}\,(1 - F_{BA})]\,(1 - d_{AB})\ldots\ldots(83b)$$

The only advantage that can be claimed for Eqs. 83 in comparison to Eqs. 39 is that they contain two variables, in addition to the fixed-end moments, instead of three variables.

Instead of Eqs. 83 the writer, as a rule, prefers to use the following:

$$M_{AB} = \frac{4\,F_{AB}\,H_{AB}}{D}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(84a)$$

$$M_{BA} = \frac{4\,F_{BA}\,H_{BA}}{D}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(84b)$$

in which: $H_{AB} = \mathrm{FEM_{AB}} + \frac{1}{2}\,\mathrm{FEM_{BA}}\,(1 - F_{BA})$; $D = 4 - (1 - F_{AB})\,(1 - F_{BA})$; and $H_{BA} = \mathrm{FEM_{BA}} + \frac{1}{2}\,\mathrm{FEM_{AB}}\,(1 - F_{AB})$.

If either end is fixed, $D = 4$, and Eqs. 84 are greatly simplified; but the real advantage of Eqs. 84 is their adaptability to a table of coefficients, which has been compiled.[31] Thus, with F_{AB} and F_{BA} as arguments, the moments at the ends of the loaded member can be written down most easily.

To illustrate the foregoing the writer has solved the frame shown in Fig. 10(*a*). The results are written on the members in Figs. 10(*b*) to 10(*f*). This method of solution serves also as an influence diagram indicating which combination of loads will produce maximum effects. If all the members are loaded simultaneously, the total moments are as follows: $M_A = +18.56$; $M_{BA} = M_{BC} = +37.13$; $M_{CG} = +44.25$; $M_{CB} = +114.25$; $M_{CF} = +101.44$; $M_{CD} = -259.94$; $M_{DC} = -M_{DE} = +23.17$; $M_{ED} = -M_{DE} = +10$; and $M_G = -52.87$.

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[31] "Statically Indeterminate Frameworks," by Thomas F. Hickerson, Univ. of North Carolina Press, Chapel Hill, 1937, pp. 90–93.

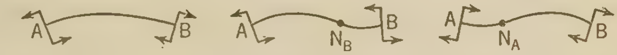
The fact that the "degree of restraint" and "carry-over factors" for interior spans of continuous beams of constant moment of inertia and equal spans converge toward a fixed quantity was discovered by the writer many years ago. Thus the "degree of restraint" produced by three or more equal spans is 46% whether the far terminal be hinged or fixed.

Dean F. Peterson, Jr.,[32] Assoc. M. Am. Soc. C. E.—Using the slope-deflection equations, Mr. Weiss has developed the algebraic interrelationships between the various "structural concepts" commonly used in the method of moment distribution. The "common factor" used to derive the equations and to illustrate the interrelationship is the concept of the "fixed point."

In the "Introduction," Mr. Weiss states:

> "Usually such points [points of zero bending moment] are thought of in connection with beams continuous over three or more supports; but two such 'fixed points' exist in every member of any frame. They are called 'fixed' because their position on the elastic curve is stationary."

The position of the "fixed point" is actually stationary in only one direction since it is free to deflect transversely. The choice of the term "nodal" further implies that this condition is not true. The number of "fixed points" occurring at any time depends upon the moments acting on the member. Either none, or one, will actively occur in each of the unloaded spans (Fig. 11). When distributing a single fixed-end moment to a frame, all the unloaded members will have one and only one fixed point, which will occur either at N_A or N_B. The distance measured along the span to this point always will be the same regardless of the magnitude or sense of the moment.



(a) NO FIXED POINT (b) ONE FIXED POINT (c) ONE FIXED POINT

Fig. 11

The solution of Eq. 9 for n_{AB} in the frame of Fig. 1 requires the use of n_{BC} which is unknown since it depends on the restraining properties of joint C. Similarly, the restraining properties of joint C depend upon those of joint D, etc. Presumably it is mathematically necessary to start the solution at some joint where the restraining properties at the far ends of all of the concurrent members, except one, are known. Having solved for n at such a joint, it is possible to solve the equations at the other joints progressively. If no such joint exists the equations must be written for each joint and solved simultaneously. This is merely a solution of the slope-deflection equations for the entire structure with an applied moment at A in terms of the parameter n and the amount of computation involved is about the same as if the conventional equations had been used.

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[32] Melbourne Beach, Fla.

The solution of n may be made directly from the geometry of flexure by using an elastic curve traverse.[33] For example, in Fig. 12, relative stiffness factors have been assigned to a part of the frame of Fig. 1 and joints F and L are assumed to be fully restrained. To find n_{CD}, assume that a bending moment induced from joint C acts to rotate joint D. Draw the traverse. Since the traverse angles marked 19/3 and 56/3 are a measure of the moments existing at joints D and C, respectively, $n_{cD} = \dfrac{\frac{56}{3}}{\frac{19}{3} + \frac{56}{3}} = \dfrac{56}{75}$. The traverse may be continued for the entire frame and the fixed points for each member can be found as easily, if desired.

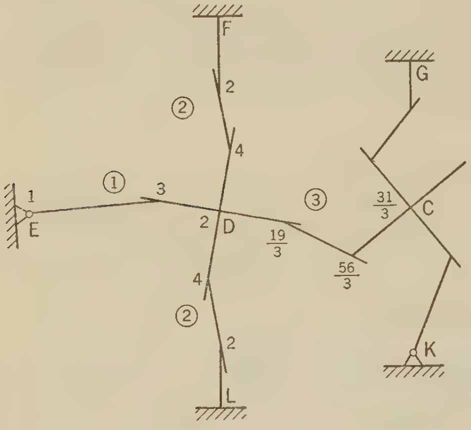

FIG. 12

However, moments might just as well have been determined in the first place. As an example, suppose the moment to be carried over from end C is 100; then all of the moments in the members about joint D may be computed immediately since they are proportional to the traverse angle and the relative stiffness: $M_{DC} = \dfrac{100}{\frac{56}{3}} \times \dfrac{19}{3} = 34.0$; and $M_{DF} = \dfrac{100}{\frac{56}{3}} \times 4 \times \dfrac{2}{3} = 14.3$. Similarly: $M_{FD} = 7.2$; $M_{DE} = 5.4$; $M_{DL} = 14.3$; and $M_{LD} = 7.2$. The traverse could easily be extended through the frame to point A and all moments due to the moment at A computed.

In the "Introduction" Mr. Weiss states:

> "The method of moment distribution introduced by Hardy Cross,[2] M. Am. Soc. C. E., offers a relatively rapid and precise means of computation and, in the writer's opinion, is as good a method as can be devised. It permits the analysis of a framework from a unique point of view. Whereas the method of least work is based on the concepts of work and energy; the slope-deflection method on relations governing deformations comprising the geometry of flexure; the Cross method may be considered as based on a complex of concepts that can be summed up under the term 'restraint.'"

It is the writer's desire to point out that the concepts summed up under the term "restraint" are basically associated with the relations comprising the geometry of flexure. The solution for n, Fig. 12, is based on an extension of these relationships. It is further noted that the slope-deflection equations (Eqs. 2) are the basic equations of the author's mathematical development.

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[33] "Relative Flexure Factors for Analyzing Continuous Structures," by Ralph W. Stewart, *Transactions*, Am. Soc. C. E., Vol. 104 (1939), p. 521.

From a consideration of Eqs. 9, 10, 11, and 12, it may be stated that the "restraint" imposed by any joint, not fully fixed or hinged, in a continuous structure is dependent upon every member and upon every other joint in the frame. Prior to the introduction of the Cross method this complete interdependency was stated in the form of a set of simultaneous equations involving, as unknowns, rotations or moments at each joint. In order to avoid the usually arduous task of solving these equations, Professor Cross developed a means of "distributing" unbalanced moments to the concurrent members of a joint assuming the far ends of the members to be fixed. By successively releasing each joint the effects of the "restraint" of remote joints and members were added as increments to the moment at the original joint. In this method the conceptions of "stiffness factor," "carry-over factor," and "distribution factor" are based on the assumption that the remote ends of the members are fully fixed. In Mr. Weiss' paper these factors are based on the true "restraint" of the joints at the far ends. Therefore, in the general case, none of them can be evaluated except after, or in connection with, some kind of a complete analysis for a moment imposed at some joint on the frame. Since such an analysis will also determine the resulting moments (which is the information that interests the designer), the various concepts set forth by this paper, although interesting from an academic point of view, appear to be of secondary importance in practice. For example, while, from the solution of the frame of Fig. 12, it is possible to determine n_{CD}, the moments resulting from the loading were also determined by the same analysis. This would generally be true regardless of the method of analysis. The use of the terms "stiffness factor," "carry-over factor," and "distribution factor" to describe physical conceptions somewhat different than those which they describe in the method of moment distribution may lead to some confusion.

A. A. EREMIN,[34] ASSOC. M. AM. SOC. C. E.—It is true, as Mr. Weiss has stated, that a uniform mathematical language, composed of commonly recognized terms, simplifies the presentation and development of computation methods. This is true regardless of how often such "standard" symbols may need to be revised. As knowledge of a subject increases and new symbols are introduced, the alphabet is often insufficient to cover the required terms. Furthermore, normal progress will require new interpretations of the physical properties of a structure. For instance, the term, "fixed point," is not defined conclusively at present. This term may be applied to three pairs of fixed points—central points, inflection points, and elastic points, as explained by the writer elsewhere.[35] The position of all these characteristic points is controlled by the physical and geometric properties of structures. With doubtful justification, Mr. Weiss introduces the term "node point" to denote an inflection point. The term "inflection point" is more directly associated with the characteristic variations in the shape of a member, its neutral axis, and moment diagram. Less effort is required to memorize the various steps in the moment-distribution method when thought of in terms of inflection points.

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[34] Associate Bridge Engr., Bridge Dept., Div. of Highways, State Dept. of Public Works, Sacramento, Calif.

[35] *Transactions*, Am. Soc. C. E., Vol. 110 (1945), p. 1542.

By a simple change in the notation the authors' algebraic expressions for the restraining factors may be converted to those derived by the writer in 1943.[36] These restraining factors also may be obtained by simple graphical methods, resulting in a saving of time and effort. The member AB in Fig. 13 has a constant section, its basic stiffness being $K_{AB} = I/L$. The stiffness factor of AB at B, with end A fixed, may be determined as follows: Draw line A′B′ through the third point of the span and through the ordinate 2 K_{AB} at the third point of the span near end B. The ordinate BB′ is equivalent to the stiffness factor r_{AB}. If the restraint at end A is reduced to zero by introducing a hinge the stiffness line will be revolved about the ordinate 2 K_{AB} to the hinge at end A. The ordinate BB″ is equivalent to the stiffness factor of AB with the opposite end hinged. If the restraint at end A is reduced to one half of the full restraint, the inflection point in this case will occur midway between end A and the third point of the span, and the stiffness line will be revolved about the ordinate 2 K_{AB} to the inflection point. The ordinate BB‴ is equivalent to the stiffness factor of AB with the opposite end 50% restrained. The dashed parallel lines near end A in Fig. 13 illustrate the graphical construction of the inflection-point location according to the given degree of restraint.

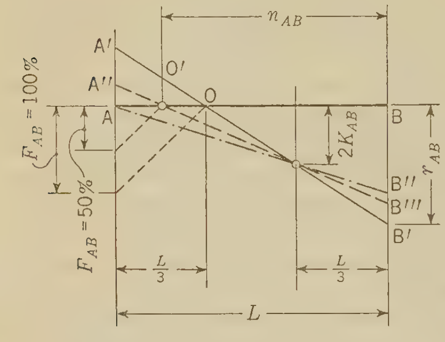

FIG. 13

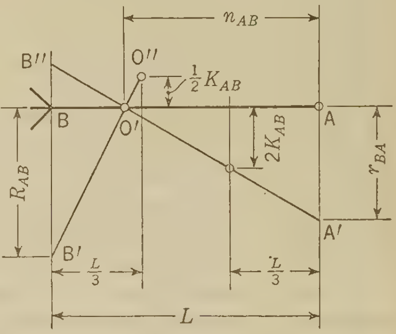

FIG. 14

The graphic construction for locating the inflection point in a rigid frame is shown in Fig. 14. Member BA in Fig. 14 is the same as that shown in Fig. 3. The inflection point O′ is determined by intersecting the member axis BA with line BO″ drawn through the ordinate of the restraint R_{AB} at end B and the ordinate 1/2 K_{AB} at the third point of the span. The stiffness factor r_{BA} is determined as shown in Fig. 13.

The inflection points are used with a considerable advantage in the graphical distribution of moments. Therefore, it is gratifying that Mr. Weiss has adopted the inflection points also in his algebraic formulas. Obviously the author's restraining factors will facilitate development of various semigraphical methods of moment distribution.

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[36] "Analysis of Continuous Frames by Graphical Distribution of Moments," by A. A. Eremin, published by the author, Sacramento, Calif., 1943.

Camillo Weiss,[37] M. Am. Soc. C. E.—Discussion of the interrelation of certain structural concepts elicited interesting comment and instructive criticism. It also revealed a certain amount of disagreement among the discussers with respect to the relative merits of the concepts presented. Several new and additional concepts were proposed and methods of analysis compared.

The concepts treated in the paper were not selected with a view to advocating their use, but were included as they appeared in natural sequence during the course of the study. Carried further, Mr. Stewart's interesting and useful angle traverse relations might well have been added. Equally intriguing is Mr. Eremin's graphical representation of restraining factors.

Several of the discussers brought up the question of methods. Although outside the scope of the paper, some of the problems presented illustrate how the algebraic interrelations can be used. Mr. Stewart's problem serves particularly well and shows his complete familiarity with the various concepts and their interrelation.

Professor Hickerson is right in stating that the "correct" interrelations do not constitute an integral part of the Cross method. They belong rather to the various methods of "direct moment distribution." On the other hand, Hardy Cross, M. Am. Soc. C. E., must be credited with conceiving and introducing such terms as "carry-over factor"—thus giving structural analysis an entirely new angle of approach. Practical use of the relations in combination with the Cross method would be limited, in the writer's opinion, to cases in which part of the frame was to be isolated for investigation. For such purposes it is often customary to make assumptions of a "degree of restraint"—the usefulness of which concept has been questioned by several writers.

In Professor Brumfield's opinion the distribution factor as defined and derived holds only for the Cross method, but this is not the case. The sharp distinction which he makes between the moment applied to a joint by some external means, such as a crank, and the moment delivered to the joint by one of its members (carry-over moment) may be helpful but is not required. The carry-over moment may be considered as an external moment applied to the joint. By considering the joint as consisting of the restraining members only, the formulas give the correct distribution factors for a carry-over moment as well.

Professor Brumfield states that the terms "stiffness factor" and "restraint factor" were used interchangeably. Under the given definitions the two concepts are identical. The writer agrees with Professor Brumfield that "restraint factor" is preferable. Unfortunately, "stiffness factor" is in common usage whereas "restraint factor" is not. The terms "stiffness index" and "restraint index" were introduced by Professor Brumfield but have not been interpreted as concepts that can be visualized.

Mr. Strauss presents a general equation for the carry-over factor and also demonstrates a direct method of solution by the use of moment splits. Mr. Strauss takes issue with the statement that the Cross method is based on a concept of restraint. He considers it equivalent to solving the simultaneous slope-deflection equations by successive approximations. This is quite a legitimate view to take; but, on the other hand, Professor Cross himself presented his method as a dramatic sequence of alternately locking and unlocking the joints of a framework—quite obviously a systematic control of restraints rather than a solution of equations.

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[37] Designer, Bethlehem Steel Co., Fabricated Steel Constr., Eng. Dept., Bethlehem, Pa.

Mr. Beskin, in tracing some of the concepts back to their origin, emphasizes their importance and calls attention to their close relationship. In his opinion it is improbable that any new physical interpretation is given to structural terms in this paper. He challenges the statement that "fixed points" are usually thought of in connection with beams and makes the counterclaim that the analysis of frameworks by determining the locations of the fixed points is a common practice. In support of this contention he refers to papers by Yves Nubar,[17] M. Am. Soc. C. E., and Professor Brumfield [12] and to lectures of Professor Campus of Liége University, in Liége, Belgium. In his paper Mr. Nubar does not state or imply that a generalized theory of fixed points is common practice, but he indicates that such a theory could be based on a geometric interpretation of one of his findings. Mr. Nubar refers to the paper by L. H. Nishkian and D. B. Steinman,[38] Members, Am. Soc. C. E., on the conjugate points of the continuous beam. Professor Brumfield, far from lending support to Mr. Beskin's contention, does not mention fixed points at all in his paper.[12] Indeed, in his discussion of structural concepts, Professor Brumfield states that he would eliminate the idea of fixed points from consideration entirely. Thus, only the lectures of Professor Campus remain, of which unfortunately there are no records available. However, even if there were occasional instances of a more general application, the statement that fixed points are usually thought of in connection with continuous beams would not be refuted.

In Mr. Beskin's opinion the "so-called" Cross method does not permit the analysis of framework from a different or novel point of view. However, when the method was first proposed it was greeted with such expressions as "ingenious," "an outstanding contribution to the field of structural analysis," "a refreshingly new and eminently useful avenue of approach," "quite unique," "the most convenient and expeditious method yet brought forward," "the most powerful and versatile tool presented in many years," and "the greatest single contribution to the theory of stress analysis in a generation."

With reference to Mr. Beskin's Eqs. 41, it should be stated that they apply to a moment acting at joint B. Eqs. 42 correspond to those given by Professor Hickerson in Table 5 except Eq. 42*a* which is in error as comparison with Table 5 will show.

The statement made in the paper (see "Introduction") that "The method of moment distribution * * * is as good a method as can be devised" was rather irrelevant to the main theme of interrelations, but Mr. Beskin challenges the statement and for this purpose presents a very scholarly comparison of various methods. Although distantly related to the topic of the paper, the subject of methods does not come within its modest scope.

Mr. Dodge has contributed instructive comment and made valuable suggestions. His statements concerning symbols are well taken, and his remarks concerning the introduction of the proper subscripts for the general case of varying moments of inertia are pertinent. His adaptation of the interrelations to more general application is a welcome contribution.

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[38] "Moments in Restrained and Continuous Beams by the Method of Conjugate Points," by L. H. Nishkian and D. B. Steinman. *Transactions*, Am. Soc. C. E., Vol. 90 (1927), pp. 1–206.

To Mr. Spiker the interrelations suggest a possible method, but he believes that the formulas of the paper are unnecessarily abstruse and that Table 1 is not clear. The writer is in sympathy with Mr. Spiker and wishes he could have expressed himself more clearly. Mr. Spiker's discussion shows, however, that he is quite familiar with the subject of interrelations. He favors the use of the "point of inflection" and tables and charts for this and other concepts. He objects to certain symbols used in the paper because of conflict with others. This conflict does exist and is admittedly confusing to both reader and writer. Appendix II was added in the hope of stimulating discussion for the remedy of just such conflicts.

In Table 5 Professor Hickerson presents a welcome supplement to Table 1 and contributes some valuable interrelations of his own including some which modify advantageously certain relations presented by Mr. Strauss. Since Professor Hickerson was a pioneer in the study of restraint, his comments carry special weight.

Mr. Peterson is correct in stating that, when a single moment is applied to a joint of a frame, only one "fixed point" in each member will be a point of inflection: It will be the left-hand point for unloaded members to the left of the joint and the right-hand point for unloaded members to the right of the joint. In general, it will always be the far point. One of the fixed points may be a hinged end in which case it may not appear to be a point of inflection. These are well-known characteristics of the fixed points. Mr. Peterson states that it is mathematically necessary to start the solution of the nodal intercept at some joint where the restraining properties of the far ends of all the concurrent members, except one, are known. This is theoretically quite correct. In practice, however, it is not necessary. The restraining properties of members several panels removed from the joint under investigation may be arbitrarily assumed. In most cases their influence will be found negligible.

Special attention is called to Mr. Eremin's interesting graphical representation of restraining factors.

Among the various terms used, the term "node" has drawn the heaviest fire. Professor Brumfield holds that there is no movement at a "node" although the "fixed point" does move laterally. Mr. Beskin correctly states that the amplitude of displacement of a node is different from zero; and, characteristically, a node is a point of inflection. Mr. Peterson believes that the "fixed point" is stationary in only one direction since it is free to deflect transversely.

Actually, the "fixed point" may move in any direction. It is "fixed" only longitudinally relative to the restrained end of the unloaded member and moves with it in any direction, but it remains stationary on the elastic curve, or neutral axis, of the unloaded member. The term "node" is admittedly not an ideal choice. It was used primarily for brevity and for want of a better or more descriptive term. The term "characteristic point" is cumbersome without being descriptive. "Fixed point" is better, except that the point is not really fixed and "point of inflection" is a general mathematical term. "Grove's Dictionary of Music and Musicians" [39] defines "node" essentially as follows:

> "When a string is plucked it does not vibrate in any one of the simple forms, but in several of them at once. There the node is no longer a point of complete rest, but a point where the amplitude of vibration is least."

In general, adverse criticism of the paper appears to have been directed chiefly against terminology, definitions, and method of derivation. Constructive comment has been offered to supplement and generalize the interrelations presented. Therefore the conclusion is justified that the substance of the study, the stated algebraic interrelations of the various concepts, has been accepted as being essentially correct.

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[39] "Grove's Dictionary of Music and Musicians," Theodore Presser Co., Philadelphia, Pa., 1918.

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Paper No. 2275

STRESSES IN THE LININGS OF SHIELD-DRIVEN TUNNELS

By Anders Bull,[1] M. Am. Soc. C. E.

With Discussion by Messrs. J. A. Van den Broek, Nathan D. Brodkin, M. A. Drucker, Sigvald Johannesson, Jacob Feld, Leon Beskin, D. P. Krynine, A. A. Eremin, G. N. Alexander, Louis Balog, and Anders Bull.

Synopsis

A rational method for determining the amounts and distribution of the soil reactions (passive pressures) developed around a shield-driven tunnel lining is presented in this paper.

Considering the lining supported by a hypothetical diaphragm, the thrust, shear, and moment produced at the support are first determined. From the values obtained, formulas are next derived, and tables prepared, permitting the deflections produced at various points of the lining under the combined influence of active loads and soil reactions to be expressed as simple product sums. However, the deflections may also be expressed in a different way, as functions of the soil reactions and the settlement. By equating the two sets of expressions a number of simultaneous equations are obtained from which the soil reactions are determined.

Applying the method to an actual case, the values obtained for the dimensions of the lining are compared with those obtained by the customary method of analysis, and the economic aspects thus raised are discussed, as are the comparative merits of steel as a lining material.

Present State of Art

1. *Introductory Remarks.*—With few exceptions, shield-driven tunnels are of circular cross section. The shell or "lining" resisting the pressure of the surrounding ground is usually made of a succession of rings of iron, steel, or concrete, each of which is built up inside the shield from a number of segments. For purposes of design, the axial cross section through a ring at a joint is considered equivalent to one taken between joints as the latter is supposed to have the same strength as the segments proper. Although a tunnel lining is thus one

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Note.—Published in November, 1944, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

[1] Civ. Engr., Board of Transportation, New York, N. Y.

of the simplest structures imaginable, its rational design is complicated by the highly indeterminate character of the pressure distribution.

Among the pressures acting on a lining may first be mentioned those due to the weight of the surrounding ground. They are referred to as "active pressures" and are determined with sufficient accuracy by means of certain broad rules roughly fitting the average soil condition anticipated within a given length of tunnel, it being obviously impracticable to make provision for minor variations. The accepted rule in the typical case of a sandy soil is to take the vertical pressure at a given point as the unit weight of the soil times the depth of the point below the surface, and the corresponding horizontal pressure as one third of that amount, additional allowance being made for the weight of the lining and the hydrostatic pressure of any water present.

Wherever the lining, as a result of these active pressures, is displaced in a direction away from the center of the tunnel, it causes the abutting ground to be compressed, and secondary pressures are developed which tend to resist the displacement. In sandy soils the latter pressures, according to the rules, are proportional to the amount of displacement, regardless of the depth of the point considered. They are generally referred to as "passive pressures." The word "passive" is misleading in this connection, however, because in a more widely used sense it denotes the maximum pressure that a soil is capable of exerting without being ruptured, a pressure which increases with the depth; so in order to preclude any confusion on this score, the term "soil reactions" will be used for pressures and forces due to displacement.

To narrow the scope of the paper, it will be assumed that the soil considered is of a type to which the foregoing rules may be applied.

2. Original Theory.—As stated, the active pressures exerted against a tunnel lining vary from point to point, depending on the depth below ground surface and water table, respectively. If the vertical pressures are plotted along a horizontal diameter, and vice versa, the pressure distribution is somewhat as indicated by the outlines of the unshaded diagrams in Fig. 1.

The thrust, shear, and moment which these pressures produce in the lining at some principal points along the periphery are found by treating the lining as an arch spanning a complete circle. In order to facilitate the mathematical work involved, the diagrams are divided into elementary shapes 1 to 11, Fig. 1, the effects due to each shape being determined separately, after which the combined effects are found by algebraic summation. The active pressures are: The downward pressure due to weight of soil (areas 1 and 2) and water (areas 3 and 4) above the horizontal diameter of the tunnel; the downward pressure due to weight of lining (area 5); the upward pressure due to hydrostatic action (areas 6 and 7); and the horizontal pressure due to soil (areas 8 and 9) and water (areas 10 and 11).

The horizontal pressures are usually symmetrical with respect to the vertical diameter of the lining, the corresponding forces being, therefore, in static equilibrium. Such is generally not the case with the active vertical pressures, the corresponding downward force being either larger or smaller than the upward one. If the downward force predominates (as assumed in this paper) the lining will settle, compressing the ground below until soil reactions in the amount required for static equilibrium have been developed.

By the original theory,[2] the vertical soil reactions are assumed to be uniformly distributed over the horizontal diameter of the lining, as indicated by the shaded rectangle 12 in Fig. 1. By adding their effects to those produced by the active pressures 1 to 11, the total thrust, shear, and moment at each principal point are obtained, from which the required strength of the lining is determined.

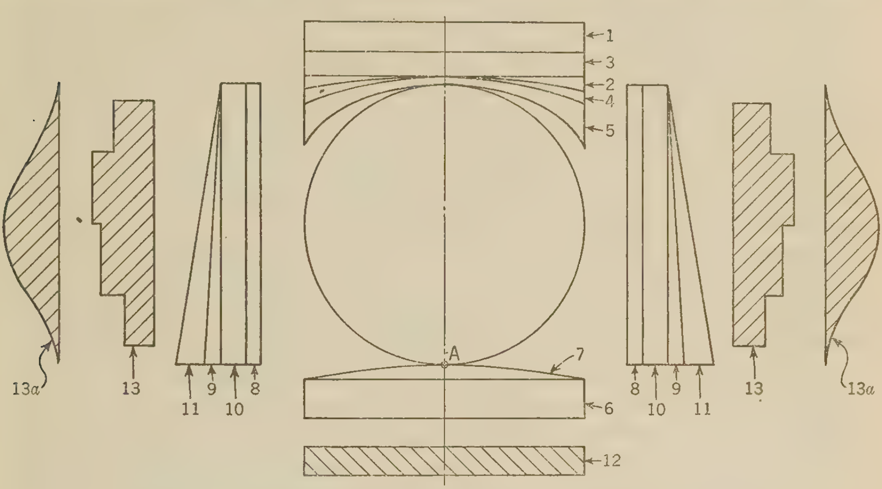

FIG. 1.—ASSUMED SOIL PRESSURE DISTRIBUTION AROUND A LINING

It is clear at once that the theory disregards a most important feature—the deflection of the lining—since only for a perfectly rigid lining could the soil reactions be distributed in the manner claimed. Although the deflection may amount to only a few hundredths of an inch, the change it causes in the pressure distribution has a profound effect on the moments that to a large extent govern the stresses in the lining. This is due to the sensitive state of moment balance prevailing in a ring-shaped structure. As will be shown, the resultant moment at any point is the outcome of positive and negative moments whose magnitude may each be several hundred times that of the former, so that even a slight error in determining the component moments may upset the final result completely.

The failure to take the deflection into consideration leads to excessive values for the moments computed and thereby to designs which are very uneconomical. It seems unlikely, in fact, that a tunnel was ever built in strict compliance with the original theory, as a comparison with other tunnels, already long in service, would immediately have revealed that something must be radically wrong.

3. Customary Method of Analysis.—Not until the middle 1920's was serious attention paid to the relieving influence which the deflection has upon the moments in the lining. As a result of these investigations a theory was developed which may be considered the foundation of present-day tunnel design.

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[2] "Shield and Compressed Air Tunneling," by B. H. M. Hewett and S. Johannesson, 1st Ed., McGraw-Hill Book Co., Inc., New York, N. Y., 1922, p. 74.

The problem was not fully grasped at the outset, however, and the erroneous conception of a uniform distribution of the vertical soil reactions was thus inherited by the newer theory. As a novel feature, horizontal soil reactions were introduced which are subject to the following considerations:

Under the pressures 1 to 12, Fig. 1, the lining deflects, the horizontal components of the deflection being directed outward, compressing the ground horizontally. Due to the restraining influence of the resulting soil reactions the deflection is reduced; this, in turn, causes the soil reactions to be also reduced, until a balance is established. The problem is simplified by assuming a step-by-step variation of the horizontal soil reactions, as indicated by diagrams 13. The values required for a balance may be determined mathematically, but the further details are not open to discussion as no account of the procedure seems as yet to have been published.

A simple method for attaining the same end has been described by M. A. Drucker,[3] who determines the horizontal soil reactions as continuous functions of the depth, the resulting pressure distribution being shown by diagrams 13a. The procedure would be an improvement except for the fact that also in this case the vertical soil reactions are assumed to be uniformly distributed.

Because of the restraint exerted by the horizontal soil reactions, the modified methods will yield lower values for the moments than does the original theory; but the obvious inconsistency of treating the lining as if it were, at the same time, both flexible and perfectly rigid makes it a foregone conclusion that the results obtained on such premises cannot be correct. The error incurred is found to increase with the flexibility of the lining.

In this paper the problem is approached from a different angle, permitting a method to be developed by which the complete pressure distribution due to a given loading condition may be determined in accordance with accepted mechanical laws and engineering practice. The important bearing which the flexibility of the lining has upon the economy of design—a feature which, so far, has been obscured because of the assumed fixed distribution of the vertical soil reactions—is thereby revealed.

Notation.—The letter symbols in this paper are defined where they first appear, either in the text or by diagram, and those used in solving specific problems are assembled for convenience of reference in the Appendix, where the adopted sign convention is also defined.

Reactions in a Tunnel Lining

4. Basic Principle.—The thrust, shear, and moment produced at any point of a tunnel lining may be found by the same principle as the one on which the method of structural model analysis introduced by the late George E. Beggs,[4] M. Am. Soc. C. E., is based, and which may be briefly stated as follows: In an elastic structure, if a support is displaced by an infinitesimal amount in a given direction the resulting deflection line is identical with the influence line for the similarly directed reaction at that support.

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[3] "Determination of Lateral Passive Soil Pressure and Its Effect on Tunnel Stresses," by M. A. Drucker, *Journal of the Franklin Institute*, May, 1943, p. 499.

[4] *Transactions*, Am. Soc. C. E., Vol. 88 (1925), p. 1208.

For a proof of the truth of the foregoing statement it is customary to refer to Maxwell's theorem, from which it was originally deduced. However, the same conclusion is reached more directly by considering that the work performed, respectively, by a load and the corresponding reaction during an infinitesimal displacement of the support must be of equal amount and opposite sign, since the work spent on the elastic deformation of the structure, being a function of the square of the displacement, becomes negligible. The latter method, which has the advantage of greater clarity, will be followed.

A circular tunnel ring lends itself particularly well to this type of analysis because of its assumedly uniform cross section and perfectly regular shape, as the result of which the deflection lines due to given displacements are of fixed shapes, comparable to the deflection lines for simple beams.

The ring in Fig. 2 is a 1-ft slice cut from a tunnel lining, the neutral circle of which is marked c. The ring is assumed to be cut at the invert point A and to be supported by a thin, inflexible diaphragm D, made up of two interlocking and detachable halves of which the upper half is rigidly connected to the left-hand end of the ring, while the lower half is rigidly connected to the right-hand end.

The reactions to be determined are those exerted by the diaphragm as a whole upon the right-hand end when a single force P is applied to the ring at a point p, located on the neutral circle at an angular distance α from point A.

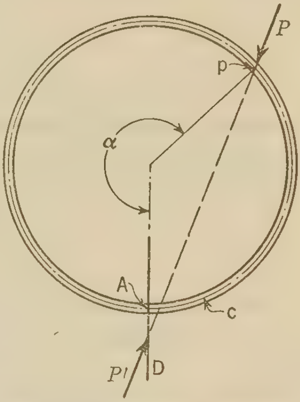


FIG. 2.—HYPOTHETICAL SUPPORT OF A TUNNEL RING

5. *Reactions at the Support.*—Referring to Fig. 3, the curved dashed lines represent the neutral circle of the ring, whereas the solid curves indicate the shapes assumed when the right-hand end is displaced in the various ways shown.

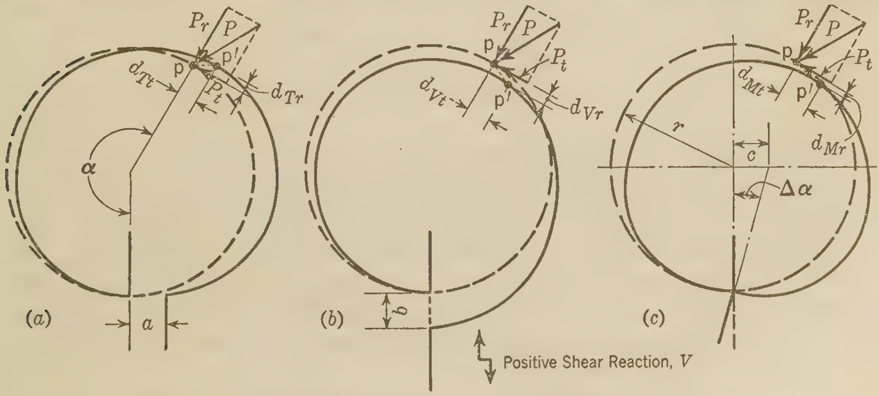


FIG. 3.—INFLUENCE LINES OBTAINED BY DISPLACING THE SUPPORT

For finding the thrust reaction T, the right-hand end is assumed to be displaced by an infinitesimal amount a in the direction of positive thrust reaction (see Fig. 3(a)). The point p will then move to the new position p′. Resolving the deflection pp′ into its radial and tangential components d_{Tr} and d_{Tt}, respectively, and the force P, similarly, into the components P_r and P_t, one may write the work equation:

$$T\,a = P_r\,d_{Tr} + P_t\,d_{Tt} \qquad \text{(1)}$$

a deflection component being considered positive when its direction is opposite to that of the force component by which it is multiplied (work negative). From Eq. 1 it follows that

$$T = P_r\frac{d_{Tr}}{a} + P_t\frac{d_{Tt}}{a} \qquad \text{(2)}$$

For the shear reaction V, the right-hand end of the ring is assumed to be displaced downward by an amount b in the direction of positive shear reaction. Resolving the deflection pp′ into its components d_{Vr} and d_{Vt}, the work is expressed by: $V\,b = P_r\,d_{Vr} + P_t\,d_{Vt}$ or

$$V = P_r\frac{d_{Vr}}{b} + P_t\frac{d_{Vt}}{b} \qquad \text{(3)}$$

For the moment reaction M, the right-hand end of the ring is rotated in the direction of positive moment reaction (so as to stretch the outer fibers in the right-hand end). This is done by tilting the lower half of the diaphragm clockwise by an angle $\Delta\alpha$ about the invert point. The deflection pp′ is resolved into the components d_{Mr} and d_{Mt}, and the work equation becomes:

$$M\,\Delta\alpha = P_r\,d_{Mr} + P_t\,d_{Mt} \qquad \text{(4a)}$$

The infinitesimal angle $\Delta\alpha$ is equal to c/r, in which c is the intercept made by the produced lower half of the diaphragm on the horizontal diameter of the ring, and r is the radius of its neutral fibers. Substituting this value gives:

$$M = r\left(P_r\frac{d_{Mr}}{c} + P_t\frac{d_{Mt}}{c}\right) \qquad \text{(4b)}$$

Eqs. 2, 3, and 4 show that the three reactions T, V, and M, due to a given force P, are obtained, simply, by multiplying the radial and tangential force components P_r and P_t by the quotients formed in dividing the respective deflection components d_r and d_t by the displacements to which they are due. For the small displacements considered, the numerator, in each case, is proportional to the denominator, which means that the six quotients obtained for any given point p are fixed in value. Denoting the quotients by the letter A, Eqs. 2 to 4 take the forms, respectively:

$$T = P_r\,A_{Tr} + P_t\,A_{Tt} \qquad \text{(5a)}$$

$$V = P_r\,A_{Vr} + P_t\,A_{Vt} \qquad \text{(5b)}$$

$$M = r\,(P_r\,A_{Mr} + P_t\,A_{Mt}) \qquad \text{(5c)}$$

the subscripts being self-explanatory. As the deflection pp′ changes from point to point along the periphery, both in amount and direction, the six factors A (to be referred to henceforth as A-constants) will assume different values, each of them typical of the point considered.

When the ring is subjected to several forces, acting at different points, the thrust, shear, and moment reactions are determined separately for each force, and commensurable reactions are added up. The total reactions produced at the right-hand side of the support are, accordingly, expressed by the following general equations:

$$T = \sum(P_r A_{Tr}) + \sum(P_t A_{Tt}) \ldots\ldots (6a)$$

$$V = \sum(P_r A_{Vr}) + \sum(P_t A_{Vt}) \ldots\ldots (6b)$$

$$M = r\,[\sum(P_r A_{Mr}) + \sum(P_t A_{Mt})] \ldots\ldots (6c)$$

6. Evaluating the A-Constants.—In the case of a circular ring, formulas for the six A-constants at any point p may be derived once for all, the operation involving the following steps: (1) Determine the forces and moments required at the right-hand end of the ring to reproduce the displacements a, b, and c ($\Delta\alpha$) in Fig. 3; (2) determine the radial and tangential deflections d_r and d_t simultaneously produced at any point by such forces and moments; and (3) divide the deflections by the corresponding displacements in accordance with Eqs. 2 to 4.

For the computations, which are omitted for the sake of brevity, the following standard deflection equations are used:

$$\Delta x = + \int \frac{M_o}{E\,I}(Y - y)\,ds \ldots\ldots (7a)$$

$$\Delta y = - \int \frac{M_o}{E\,I}(X - x)\,ds \ldots\ldots (7b)$$

$$\Delta\alpha = + \int \frac{M_o}{E\,I}\,ds \ldots\ldots (7c)$$

in which X and Y are the rectangular coordinates of point p, as referred to the vertical diameter of the ring and a tangent to the neutral circle at point A (positive to the right and upward, respectively); x and y are the coordinates of a moving point; M_o = moment at point x, y; s = length of arc along the neutral circle from point A to point x, y; E = modulus of elasticity; and I = moment of inertia of lining per foot of tunnel. The details of these computations are contained in the original complete manuscript, filed for reference in Engineering Societies Library.

The formulas thus derived are:

$$\left.\begin{aligned} A_{Tr} &= -\frac{\alpha \sin\alpha}{2\,\pi}; & A_{Tt} &= \mp\frac{\sin\alpha - \alpha\cos\alpha}{2\,\pi} \\ A_{Vr} &= -\frac{\sin\alpha - \alpha\cos\alpha}{2\,\pi}; & A_{Vt} &= \mp\frac{2\,(1 - \cos\alpha) - \alpha\sin\alpha}{2\,\pi} \\ A_{Mr} &= -\frac{\alpha\sin\alpha + 1 - \cos\alpha}{2\,\pi}; & A_{Mt} &= \mp\frac{\alpha\,(1 - \cos\alpha)}{2\,\pi} \end{aligned}\right\}\ldots (8)$$

In the right-hand column of Eqs. 8, the upper sign is used with angles α between 0 and π, the lower sign with angles between π and $2\,\pi$.

7. Load Concentrations.—The loads on tunnel rings are usually distributed loads, as indicated in Fig. 1. By replacing these loads with equivalent load concentrations, acting at points spaced a uniform distance apart, any desired approximation to the true loading condition may be obtained by making the spacing close enough. A spacing equal to one sixteenth of the periphery will be suitable for present purposes.

Accordingly, the ring is considered composed of sixteen rigidly joined segments of equal size, with the respective load concentrations acting at the middle point of each segment. (These segments should not be confused with the segments into which a ring is divided for construction purposes.) For identifying the various segments, joints, middle points, forces, etc., the markings shown in Fig. 4 are used. Subscripts are used when necessary for specifying the location or character with which a quantity is identified. Thus, A_{Mt5} would denote the A-constant by which the tangential component of a force at point 5 is multiplied in determining its share of the moment reaction at the support, whereas A_5 would denote any or all of the six A-constants typical of point 5.

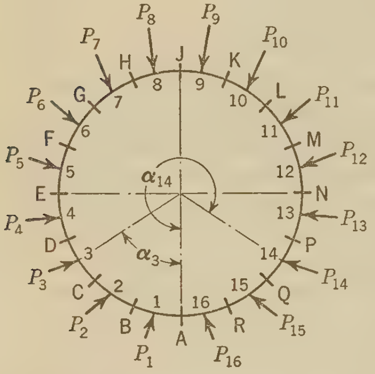


FIG. 4.—IDENTIFICATION MARKINGS

The location of the points to be considered having thus been fixed, the numerical values of the corresponding A-constants are next computed from Eqs. 8, and arranged, as shown in Table 1.

TABLE 1.—A-CONSTANTS FOR THRUST, SHEAR, AND MOMENT REACTIONS AT THE INVERT JOINT

Force	A_{Tr}	A_{Vr}	A_{Mr}	Force	A_{Tt}	A_{Vt}	A_{Mt}
P_{r1}	−0.00610	−0.00040	−0.00915 −0.00822	P_{t1}	−0.00040	−0.00002	−0.00060 ....
P_{r2}	−0.05208	−0.01047	−0.07891	P_{t2}	−0.01047	−0.00156	−0.01580
P_{r3}	−0.12992	−0.04552	−0.20065	P_{t3}	−0.04552	−0.01155	−0.06944
P_{r4}	−0.21455	−0.11342	−0.34265	P_{t4}	−0.11342	−0.04166	−0.17607
P_{r5}	−0.27585	−0.21097	−0.46605	P_{t5}	−0.21097	−0.10456	−0.33612
P_{r6}	−0.28582	−0.32331	−0.53339	P_{t6}	−0.32331	−0.20934	−0.53473
P_{r7}	−0.22570	−0.42621	−0.51719	P_{t7}	−0.42621	−0.35727	−0.74403
P_{r8}	−0.09145	−0.49079	−0.40670	P_{t8}	−0.49079	−0.53905	−0.92849
P_{r9}	+0.10364	−0.48999	−0.21161	P_{t9}	+0.48999	+0.73415	+1.05229
P_{r10}	+0.32987	−0.40526	+0.03838	P_{t10}	+0.40526	+0.91284	+1.08744
P_{r11}	+0.54565	−0.23226	+0.29808	P_{t11}	+0.23226	+1.04081	+1.02084
P_{r12}	+0.70494	+0.01588	+0.51473	P_{t12}	−0.01588	+1.08535	+0.85897
P_{r13}	+0.76624	+0.30851	+0.63813	P_{t13}	−0.30851	+1.02245	+0.62884
P_{r14}	+0.70155	+0.60109	+0.63082	P_{t14}	−0.60109	+0.84302	+0.37499
P_{r15}	+0.50349	+0.84194	+0.47666	P_{t15}	−0.84194	+0.55713	+0.15273
P_{r16}	+0.18899	+0.98119	+0.18594 −0.00822	P_{t16}	−0.98119	+0.19511	+0.01861

8. Adjusting for Polygonal Effect.—Because of the delicate balance of positive and negative moments in a tunnel ring, the substitution of forces for distributed loads may give rise to an appreciable error in the value obtained for the moment reaction M in Eqs. 6, making an adjustment necessary. For an illustration, assume that a ring is subjected to a radial pressure of a uniform intensity along its entire periphery. The pressure line then takes the shape of a circle coinciding with the neutral fibers, as indicated in Fig. 5. However, if the uniform pressure is replaced by sixteen equivalent forces P_r, acting at the middle points of the segments, the pressure line changes into a polygon, a moment being produced at each of the sixteen joints, amounting to

$$M = +0.01644\,P_r\,r \ldots\ldots (9)$$

in which r is the radius of the neutral circle. To compensate for this moment, which is essentially a local effect due to the two forces P'_r and P''_r straddling the joint, a negative moment of equal amount must be added. For the invert joint, P'_r and P''_r become P_{r1} and P_{r16}, respectively; therefore the moment to be added to the product sum $\sum(P_r\,A_{Mr})$ in Eqs. 6 is:

$$-M = -0.01644\,\frac{P'_r + P''_r}{2} = -0.00822\,(P_{r1} + P_{r16}) \ldots\ldots (10)$$

as indicated in Table 1. The adjustment does not pretend to be accurate when a load distribution is not uniform, but it meets all practical requirements and is easy to apply. No great error results from the concentration of tangential pressures; nor is any adjustment required in the thrust or shear reaction.

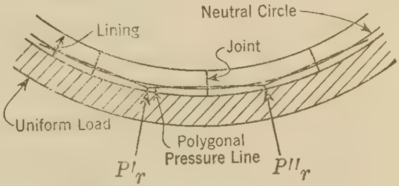


FIG. 5.—PRESSURE LINE AS AFFECTED BY TYPE OF LOADING

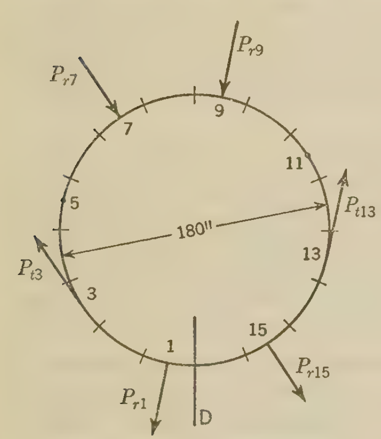


FIG. 6.—LOADING OF THE RING

9. Example.—A 180-in. ring (Fig. 6) is supported by the diaphragm D and subjected to forces P_1 to P_{15}, Table 2. The radial forces are concentrations of distributed loads on the respective segments. The problem is to determine the resulting reactions at the right-hand side of the support. The forces, multiplied by the corresponding A-constants in Table 1, yield the products entered under the respective headings. The products are added algebraically, as in Eqs. 6, and the moment product sum is multiplied by the radius: $T = -1.08317 - 1.77015 = -2.85332$ kips; $V = -4.01595 + 5.05450 = +1.03855$ kips; and $M = \frac{180}{2}(-3.14551 + 2.79700) = -31.3659$ in-kips.

10. Forces in Static Equilibrium.—Along with the reactions T, V, and M, conjugate reactions T', V', and M' are set up simultaneously at the left-hand side of the diaphragm D. With only a single force P applied to the ring, the sum of each pair of conjugate reactions represents, respectively, the total thrust, shear, and moment that must be exerted on the ring by the support as a whole in order that static equilibrium be maintained. In order to be able to exert those reactions, the support must be restrained by subjecting it to an outside force P', equal and opposite to the force P, as shown in Fig. 2.

TABLE 2.—DETERMINING THE REACTIONS FOR A SPECIFIED LOADING

Force		Reactions Caused by Radial Forces			Reactions Caused by Tangential Forces		
Fig. 6	Kips	$P_r A_{T_r}$	$P_r A_{V_r}$	$P_r A_{M_r}$	$P_t A_{T_t}$	$P_t A_{V_t}$	$P_t A_{M_t}$
P_{r1}	−2	+0.01220	+0.00080	+0.01830 +0.01644	....	....	....
P_{t3}	+5	....	....	....	−0.22760	−0.05775	−0.34720
P_{r7}	+4	−0.90280	−1.70484	−2.06876	....	....	....
P_{r9}	+3	+0.31092	−1.46997	−0.63483	....	....	....
P_{t13}	+5	....	....	....	−1.54255	+5.11225	+3.14420
P_{r15}	−1	−0.50349	−0.84194	−0.47666	....	....	....
Total	....	−1.08317	−4.01595	−3.14551	−1.77015	+5.05450	+2.79700

However, if the ring is subjected to a system of forces which are by themselves in static equilibrium, it is obvious that there can be no reaction left for the support to exert, in which case the support can have no influence on the stresses in the ring.

The forces acting on a tunnel ring must always be in static equilibrium as the ring would otherwise be subject to accelerating motion. The three reactions found from Eqs. 6 are then simply the reactions exerted by the ring portion to the left of the invert point upon the portion immediately to the right of it, being numerically equal to the thrust, shear, and moment produced in the ring at that point.

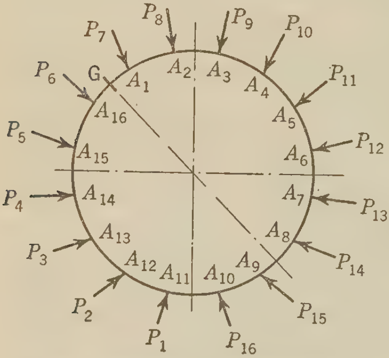


FIG. 7.—SHIFTING OF A-CONSTANTS

11. Reactions at Other Joints.—When the reactions at the invert joint, as well as all the forces acting on the ring, are known, the reactions at any other joint may be determined by treating the ring as a simple beam; but the procedure involves a lot of arithmetical work. An easier method will be explained by an example. In Fig. 7, the ring is assumed to be cut and supported at joint G instead of at the invert joint. Turning the diagram until joint G is at the bottom, one may imagine that the ring end to the right is displaced in the same manner and by the same amounts as was the right-hand end in Fig. 3.

It is evident that the resulting deflection lines will be identical with those obtained in the former case, yielding the same values for the A-constants, except that the latter are now referred to joint G. The effect of this procedure is that the constants are shifted with respect to the forces, as shown in Fig. 7. However, since the tangential components of the forces are considered positive when directed upward—that is, away from the invert joint—the trend of the components P_{t1} to P_{t6} and P_{t9} to P_{t14} with respect to joint G is reversed. This has the same effect on the products $P_t\,A_t$ as if the signs of the constants A_t for points 1 to 6, and 9 to 14, were reversed. Thus, for the moment reaction at joint G,

$$\begin{aligned} M_G = r\,[&P_{r1}\,0.29808 + P_{r2}\,0.51473 \cdots - P_{r15}\,0.21161 + P_{r16}\,0.03838 \\ &- (P_{r6} + P_{r7})\,0.00822 - P_{t1}\,1.02084 - P_{t2}\,0.85897 \cdots \\ &- P_{t6}\,0.01861 - P_{t7}\,0.00060 - P_{t8}\,0.01580 + P_{t9}\,0.06944 \\ &+ P_{t10}\,0.17607 \cdots + P_{t14}\,0.92849 + P_{t15}\,1.05229 + P_{t16}\,1.08744] \quad (11) \end{aligned}$$

The reactions found are those exerted by the support on the ring portion extending counterclockwise from joint G. When the forces are in static equilibrium, the reactions are numerically equal to the thrust, shear, and moment in the ring at that joint, for the reason explained in Section 10.

12. Special Case of Symmetrical Loading.—The loading on tunnel linings is mostly symmetrical with respect to the vertical diameter, which means that $P_1 = P_{16}$; $P_2 = P_{15}$; $\cdots$ $P_8 = P_9$. For use in such cases, Table 1 may be put into a more convenient form by combining the A-constants, by which the radial and tangential components of any two equal forces would have to be multiplied, into a single constant, thereby reducing the number of multiplications. The factor $-\,0.00822$ in Eq. 10 is likewise included in the new constants, which will be referred to as B-constants and identified by the letter B. The B_M-constants for the invert joint, for example, are computed as shown in Table 3.

TABLE 3.—COMPUTATION OF B_M-CONSTANTS FOR INVERT JOINT A

Constant	For radial forces	For tangential forces
B_{M1}	$-0.00915 + 0.18594 - 2 \times 0.00822 = +0.16035$	$-0.00060 + 0.01861 = +0.01801$
B_{M2}	$-0.07891 + 0.47666 = +0.39775$	$-0.01580 + 0.15273 = +0.13693$
....	....	....
....	....	....
....	....	....
B_{M8}	$-0.40670 - 0.21161 = -0.61831$	$-0.92849 + 1.05229 = +0.12380$

The B-constants for any other joint are obtained by the same process of shifting as was described in Section 11, due attention being paid to the signs of tangential components. Again taking joint G for an example (see Fig. 7), the combination of the A_M-constants for equal forces is demonstrated by Table 4.

TABLE 4.—COMPUTATION OF B_M-CONSTANTS FOR JOINT G

Constant	Radial constants, B_r	Tangential constants, B_t
B_{M1}	$+A_{Mr11} + A_{Mr10} = +0.29808 + 0.03838 = +0.33646$	$-A_{Mt11} + A_{Mt10} = -1.02084 + 1.08744 = +0.06660$
....	....	....
....	....	....
....	....	....
B_{M6}	$+0.18594 - 0.46605 - 0.00822 = -0.28833$	$-0.01861 + 0.33612 = +0.31751$
B_{M7}	$-0.00915 - 0.34265 - 0.00822 = -0.36002$	$-0.00060 + 0.17607 = +0.17547$
B_{M8}	$-0.07891 - 0.20065 = -0.27956$	$-0.01580 + 0.06944 = +0.05364$

The B-constants for moment and thrust reactions at the joints A to J are arranged in Table 5, permitting the reactions at any joint to be determined with equal ease, it being noted that the reactions are alike at symmetrical joints, such as B and R in Fig. 4. No B-constants are given for shear reactions; in the rare cases where they are of interest, the loading is usually asymmetrical, and Table 1 must be used.

TABLE 5.—B-CONSTANTS FOR USE WITH SYMMETRICAL LOADS

Force	JOINT (SEE FIG. 4)								
	A	B	C	D	E	F	G	H	J
(*a*) FOR MOMENT REACTIONS CAUSED BY RADIAL COMPONENTS									
P_{r1}	+0.1603	+0.6544	+1.1075	+1.2690	+1.1529	+0.8128	+0.3365	−0.1732	−0.6183
P_{r2}	+0.3978	+0.6134	+0.8158	+0.9914	+0.9289	+0.6765	+0.3031	−0.1086	−0.4788
P_{r3}	+0.4302	+0.5592	+0.4974	+0.4758	+0.5150	+0.4192	+0.2314	−0.0025	−0.2353
P_{r4}	+0.2955	+0.3141	+0.2192	+0.0210	−0.0339	+0.0700	+0.1136	+0.1047	+0.0487
P_{r5}	+0.0487	−0.0446	−0.1603	−0.2905	−0.4241	−0.3395	−0.0567	+0.1648	+0.2955
P_{r6}	−0.2353	−0.4277	−0.5543	−0.6074	−0.5961	−0.5508	−0.2883	+0.1340	+0.4302
P_{r7}	−0.4788	−0.7450	−0.8728	−0.8598	−0.7340	−0.5450	−0.3600	−0.0229	+0.3978
P_{r8}	−0.6183	−0.9239	−1.0506	−0.9994	−0.8087	−0.5433	−0.2796	−0.0963	+0.1603
(*b*) FOR MOMENT REACTIONS CAUSED BY TANGENTIAL COMPONENTS									
P_{t1}	+0.0180	+0.1341	+0.2223	+0.2538	+0.2301	+0.1619	+0.0666	−0.0351	−0.1238
P_{t2}	+0.1369	+0.3744	+0.6102	+0.7062	+0.6459	+0.4586	+0.1933	−0.0924	−0.3434
P_{t3}	+0.3055	+0.6130	+0.8584	+1.0022	+0.9347	+0.6773	+0.2997	−0.1149	−0.4861
P_{t4}	+0.4528	+0.7895	+1.0050	+1.0868	+1.0337	+0.7758	+0.3690	−0.0941	−0.5229
P_{t5}	+0.5229	+0.8448	+1.0180	+1.0365	+0.9279	+0.7254	+0.3820	−0.0389	−0.4528
P_{t6}	+0.4861	+0.7513	+0.8762	+0.8591	+0.7282	+0.5341	+0.3175	+0.0233	−0.3055
P_{t7}	+0.3434	+0.5176	+0.5924	+0.5680	+0.4653	+0.3203	+0.1755	+0.0508	−0.1369
P_{t8}	+0.1238	+0.1845	+0.2093	+0.1986	+0.1600	+0.1066	+0.0536	+0.0152	−0.0180
(*c*) FOR THRUST REACTIONS CAUSED BY RADIAL COMPONENTS									
P_{r1}	+0.1829	+0.6925	+1.2050	+1.4678	+1.4712	+1.2506	+0.8755	+0.4335	+0.0122
P_{r2}	+0.4514	+0.6955	+0.9552	+1.2084	+1.2472	+1.0961	+0.8086	+0.4542	+0.1042
P_{r3}	+0.5716	+0.7142	+0.6988	+0.7346	+0.8334	+0.8052	+0.6748	+0.4792	+0.2598
P_{r4}	+0.5517	+0.5750	+0.4936	+0.3238	+0.2926	+0.4120	+0.4759	+0.4804	+0.4291
P_{r5}	+0.4291	+0.3311	+0.2000	+0.0516	−0.0975	−0.0367	+0.2177	+0.4257	+0.5517
P_{r6}	+0.2598	+0.0540	−0.1109	−0.2214	−0.2778	−0.2919	−0.0869	+0.2889	+0.5716
P_{r7}	+0.1042	−0.1822	−0.3673	−0.4402	−0.4157	−0.3279	−0.2206	+0.0591	+0.4514
P_{r8}	+0.0122	−0.3171	−0.5115	−0.5617	−0.4904	−0.3445	−0.1820	−0.0582	+0.1829
(*d*) FOR THRUST REACTIONS CAUSED BY TANGENTIAL COMPONENTS									
P_{t1}	−0.9816	+0.1392	+0.2408	+0.2926	+0.2926	+0.2481	+0.1730	+0.0847	+0.0008
P_{t2}	−0.8524	−0.6015	+0.6727	+0.8261	+0.8334	+0.7138	+0.5059	+0.2585	+0.0209
P_{t3}	−0.6466	−0.3190	−0.0163	+1.2134	+1.2472	+1.0911	+0.7993	+0.4421	+0.0910
P_{t4}	−0.4219	−0.0614	+0.2218	+0.4049	+1.4712	+1.3327	+1.0273	+0.6318	+0.2268
P_{t5}	−0.2268	+0.1188	+0.3597	+0.4795	+0.4904	+1.4074	+1.1652	+0.8121	+0.4219
P_{t6}	−0.0910	+0.1943	+0.3766	+0.4453	+0.4157	+0.3229	+1.1922	+0.9554	+0.6466
P_{t7}	−0.0209	+0.1667	+0.2798	+0.3128	+0.2778	+0.2005	+0.1130	+1.0267	+0.8524
P_{t8}	−0.0008	+0.0646	+0.1029	+0.1123	+0.0975	+0.0679	+0.0351	+0.0101	+0.9816

Table 5 is used in the same way as Table 1: Each force component acting on the left half of the ring is multiplied by the B-constant given in the same line under the proper heading, and the products are added. The moment reaction at any joint may thus be expressed symbolically by the formula:

$$M_N = r\left[\sum(P_r B_N) + \sum(P_t B_N)\right] \dots\dots\dots\dots\dots (12)$$

in which the subscript N stands for any of the letters A to J, identifying the joint, and B_N stands for the corresponding sixteen B-constants in Tables 5(a) and 5(b).

DEFLECTION OF A TUNNEL LINING

13. The next step involves the evaluation of the radial deflection d (Fig. 8) produced at any point q of a ring by a single force P applied at a point p and having the radial and tangential components P_r and P_t, respectively. This deflection is in no way related to the deflections d_{Tr} to d_{Mt} referred to in discussing Fig. 3, which are due to the displacement of one end of the cut ring at the support and which are independent of the stiffness of the ring.

Formulas for d may be derived by means of Eqs. 7, expressions being first obtained for the rectangular deflections Δx and Δy. The calculations (on file in Engineering Societies Library) yield the following equations:

$$\begin{aligned} d = \frac{r^3}{EI}\Bigg\{ & T\left(1 - \cos\beta - \frac{\beta\sin\beta}{2}\right) + V\left(\frac{\beta\cos\beta}{2} - \frac{\sin\beta}{2}\right) \\ & + \frac{M}{r}(\cos\beta - 1) + P_r\left[\frac{\cos\alpha\sin\beta}{2} - \frac{\beta\cos(\alpha-\beta)}{2}\right] \\ & \pm P_t\left[\frac{\sin\alpha\sin\beta}{2} - \frac{\beta\sin(\alpha-\beta)}{2} + \cos\beta - 1\right]\Bigg\} \quad \ldots\ldots(13a) \end{aligned}$$

and

$$\begin{aligned} d = \frac{r^3}{EI}\Bigg\{ & T\left(1 - \cos\beta - \frac{\beta\sin\beta}{2}\right) + V\left(\frac{\beta\cos\beta}{2} - \frac{\sin\beta}{2}\right) \\ & + \frac{M}{r}(\cos\beta - 1) + P_r\left[\frac{\sin\alpha\cos\beta}{2} - \frac{\alpha\cos(\beta-\alpha)}{2}\right] \\ & \pm P_t\left[\cos\beta - \cos(\beta-\alpha) + \frac{\sin\alpha\sin\beta}{2} + \frac{\alpha\sin(\beta-\alpha)}{2}\right]\Bigg\} \quad ..(13b) \end{aligned}$$

The upper sign is used with P_t for values of α between 0 and π, the lower sign for values between π and $2\,\pi$. Eq. 13a covers the ring portion extending clockwise from the invert joint to point p, and Eq. 13b covers the remainder of the ring. In order not to make the formulas too bulky, the deflection is expressed as a function of the reactions at the invert joint.

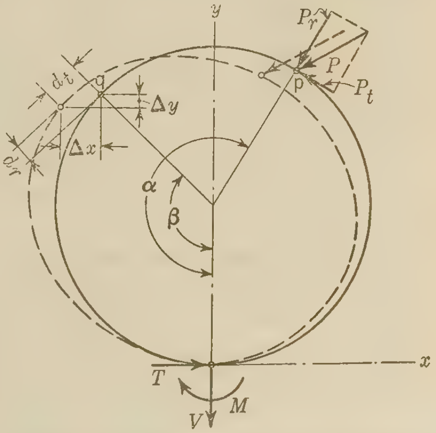


FIG. 8.—RING DEFLECTED BY A SINGLE FORCE

14. C-Constants.—With the aid of tables, the deflection d may be determined with the same ease as are the reactions T, V, and M, by multiplying the components P_r and P_t by a set of constants, which are, in this case, functions of two angles—α, which locates point p at which the force is applied; and β, which locates point q for which the deflection is sought. The new constants will be named C-constants. Their numerical values for all possible combinations of the sixteen values of α and of β identified with points 1 to 16 are given in Table 6. Because of symmetry, the deflections produced at points 9 to 16 by a unit radial or tangential force applied, say, at point 3, must equal those produced at points 8 to 1, respectively, by a unit radial or tangential force applied at point 14. It therefore suffices to register the C-constants for the first eight points, the forces being simply used in reversed order with the constants C_9 to C_{16} (see Fig. 4).

TABLE 6.—C-CONSTANTS FOR RADIAL DEFLECTIONS

(Use Forces in Parentheses for Determining Deflection at Points (9) to (16))

Force	POINT (SEE FIG. 4)							
	1 (16)	2 (15)	3 (14)	4 (13)	5 (12)	6 (11)	7 (10)	8 (9)
(a) C-CONSTANTS FOR RADIAL COMPONENTS P_r								
P_{r1} (P_{r16})	−0.002328	−0.008070	−0.011322	−0.011827	−0.009827	−0.005986	−0.001226	+0.003457
P_{r2} (P_{r15})	−0.008070	−0.050167	−0.083099	−0.093640	−0.082771	−0.055201	−0.018184	+0.020051
P_{r3} (P_{r14})	−0.011322	−0.083099	−0.168932	−0.211681	−0.200916	−0.145894	−0.063050	+0.027728
P_{r4} (P_{r13})	−0.011827	−0.093640	−0.211681	−0.303417	−0.316388	−0.251262	−0.132122	+0.009298
P_{r5} (P_{r12})	−0.009827	−0.082771	−0.200916	−0.316388	−0.373064	−0.330918	−0.206106	−0.037813
P_{r6} (P_{r11})	−0.005986	−0.055201	−0.145894	−0.251262	−0.330918	−0.340739	−0.254397	−0.100355
P_{r7} (P_{r10})	−0.001226	−0.018184	−0.063050	−0.132122	−0.206106	−0.254397	−0.242681	−0.152198
P_{r8} (P_{r9})	+0.003457	+0.020051	+0.027728	+0.009298	−0.037813	−0.100355	−0.152198	−0.160835
P_{r9} (P_{r8})	+0.007192	+0.051947	+0.107239	+0.140507	+0.130721	+0.074008	−0.014939	−0.106020
P_{r10} (P_{r7})	+0.009388	+0.072035	+0.160709	+0.234881	+0.261469	+0.223190	+0.123209	−0.014939
P_{r11} (P_{r6})	+0.009825	+0.077856	+0.180446	+0.276721	+0.329285	+0.313377	+0.223190	+0.074008
P_{r12} (P_{r5})	+0.008673	+0.070251	+0.166946	+0.263851	+0.326537	+0.329285	+0.261469	+0.130721
P_{r13} (P_{r4})	+0.006428	+0.052964	+0.128279	+0.207307	+0.263851	+0.276721	+0.234881	+0.140507
P_{r14} (P_{r3})	+0.003790	+0.031681	+0.077950	+0.128279	+0.166946	+0.180446	+0.160709	+0.107239
P_{r15} (P_{r2})	+0.001500	+0.012698	+0.031681	+0.052964	+0.070251	+0.077856	+0.072035	+0.051947
P_{r16} (P_{r1})	+0.000175	+0.001500	+0.003790	+0.006428	+0.008673	+0.009825	+0.009388	+0.007192
(b) C-CONSTANTS FOR TANGENTIAL COMPONENTS P_t								
P_{t1} (P_{t16})	−0.000173	−0.000547	−0.000756	−0.000783	−0.000646	−0.000389	−0.000073	+0.000236
P_{t2} (P_{t15})	−0.002287	−0.011539	−0.017765	−0.019406	−0.016745	−0.010801	−0.003080	+0.004743
P_{t3} (P_{t14})	−0.006183	−0.038410	−0.067897	−0.079142	−0.071661	−0.049261	−0.018078	+0.014774
P_{t4} (P_{t13})	−0.010817	−0.073854	−0.144437	−0.182216	−0.174231	−0.127367	−0.055838	+0.023037
P_{t5} (P_{t12})	−0.015142	−0.109136	−0.227102	−0.306739	−0.312428	−0.243253	−0.122540	+0.018251
P_{t6} (P_{t11})	−0.018293	−0.136665	−0.296410	−0.420424	−0.453890	−0.378094	−0.214367	−0.008739
P_{t7} (P_{t10})	−0.019723	−0.151250	−0.338029	−0.496975	−0.561451	−0.497878	−0.314464	−0.059196
P_{t8} (P_{t9})	−0.019267	−0.150790	−0.344894	−0.521272	−0.610055	−0.568995	−0.394272	−0.122610
P_{t9} (P_{t8})	+0.017133	+0.136340	+0.317747	+0.491036	+0.591141	+0.574045	+0.427859	+0.176840
P_{t10} (P_{t7})	+0.013822	+0.111550	+0.264109	+0.415789	+0.512391	+0.514218	+0.405920	+0.201125
P_{t11} (P_{t6})	+0.009992	+0.081647	+0.195979	+0.313498	+0.394100	+0.406558	+0.336157	+0.188889
P_{t12} (P_{t5})	+0.006315	+0.052173	+0.126772	+0.205674	+0.263090	+0.277910	+0.238795	+0.147288
P_{t13} (P_{t4})	+0.003325	+0.027747	+0.068179	+0.112033	+0.145553	+0.156986	+0.139383	+0.092448
P_{t14} (P_{t3})	+0.001317	+0.011100	+0.027559	+0.045821	+0.060381	+0.066352	+0.060619	+0.042647
P_{t15} (P_{t2})	+0.000301	+0.002559	+0.006419	+0.010791	+0.014408	+0.016106	+0.015094	+0.011157
P_{t16} (P_{t1})	+0.000012	+0.000098	+0.000250	+0.000425	+0.000573	+0.000651	+0.000625	+0.000482

For finding the deflection d produced at any of the sixteen points by a system of forces applied at some or all of the same sixteen points, the respective force components are multiplied by the C-constants in the corresponding column and lines, after which the products are added and their sum is multiplied by the factor $\frac{r^3}{EI}$. It should be noted that the deflection thus found is the one produced in a ring supported as shown in Fig. 2. Only when the forces are in static equilibrium is the restraining influence of the support eliminated, as was explained in Section 10.

15. Example. To Determine the Amount by Which the Length of a Diameter Through Points 3 and 11 on the Ring in Fig. 6 Has Been Changed as a Result of Radial Deflections Due to the Forces Shown.—Assuming $E = 30{,}000$ kips per sq in. and $I = 4$ in.^{4}, then $\frac{r^3}{E\,I} = \frac{729{,}000}{120{,}000} = 6.075$. The computations, given in Table 7, show that the deflections at points 3 and 11 are both positive. Therefore, the diameter is lengthened by the amount $+\,0.376 + 1.957 = 2.333$ in. The lengthening would be the same if the ring were not supported, the forces in the example being in static equilibrium.

TABLE 7.—COMPUTATION OF CHANGE IN DIAMETER CAUSED BY RADIAL DEFLECTIONS

FORCE		$P\,C_3$	$P\,C_{11}$
Fig. 6	Kips		
Radial Force P_r—			
P_{r1}	−2	+0.022644	−0.019650
P_{r7}	+4	−0.252200	+0.892760
P_{r9}	+3	+0.321717	−0.301065
P_{r15}	−1	−0.031681	+0.055201
Tangential Force P_t—			
P_{t3}	+5	−0.339485	+0.331760
P_{t13}	+5	+0.340895	−0.636835
Total	....	+0.061890	+0.322171
Total times 6.075...	....	+0.376	+1.957

16. D-Constants.—In the most common case of a symmetrically loaded tunnel lining, Table 6 may be simplified by combining, in each column, the two constants used with symmetrical and equal forces, just as was done in deriving the *B*-constants. The constants thus obtained will be referred to as *D*-constants. Their numerical values are given in Table 8, which is used in the same manner as Table 6. For symmetrical loading, the radial deflection d at any of the eight points in the left half of the ring may be expressed by the formula:

$$d_n = \frac{r^3}{E\,I}\left[\sum(P_r\,D_n) + \sum(P_t\,D_n)\right] \dots\dots\dots\dots\dots (14)$$

in which the subscript n refers to any of the numbers 1 to 8 designating the point, and D_n stands for the sixteen *D*-constants given for that point in Table 8.

SOIL REACTIONS

17. Effects of Displacement.—When a plane surface of area A' square feet, abutting on soil, is displaced in a normal direction by an amount ρ inches, the soil will resist the displacement with a force F which, on the assumptions made in Section 1 is determined by the formula:

$$F = \rho\,K\,A' \dots\dots\dots\dots\dots\dots\dots\dots (15)$$

in which K stands for the "soil compression constant"; that is, the resisting force in kips exerted by the soil on a surface of 1 sq ft which is displaced normally by an amount of 1 in. The resistance offered by the soil to a lateral displacement of the surface must be presumed to increase, at first, in accordance with a similar formula, the soil being stressed elastically in shear. There is the difference, however, that when the shearing stress at any point reaches a certain limit a slip takes place between the surface and the soil particles, or between the soil particles themselves, causing the stress at that point to be relieved suddenly. Instead of an elastic reaction the result is what is known as friction, representing the integrated value of the resistance offered at all the points of the surface. The shearing stress between soil and surface, therefore, cannot exceed a certain limit determined by the physical characteristics and the normal force pressing them together. The critical value at which the individual slips set in may drop almost to zero, in sandy soils, when the soil is vibrated, the hold of the particles being thereby relaxed momentarily.

TABLE 8.—D-CONSTANTS FOR RADIAL DEFLECTIONS

Force	Point (See Fig. 4)							
	1	2	3	4	5	6	7	8
	(*a*) *D*-Constants for Radial Components P_r							
P_{r1}	−0.00215	−0.00657	−0.00753	−0.00540	−0.00115	+0.00384	+0.00816	+0.01065
P_{r2}	−0.00657	−0.03747	−0.05142	−0.04068	−0.01252	+0.02266	+0.05385	+0.07200
P_{r3}	−0.00753	−0.05142	−0.09098	−0.08340	−0.03397	+0.03455	+0.09766	+0.13497
P_{r4}	−0.00540	−0.04068	−0.08340	−0.09611	−0.05254	+0.02546	+0.10276	+0.14980
P_{r5}	−0.00115	−0.01252	−0.03397	−0.05254	−0.04653	−0.00163	+0.05536	+0.09291
P_{r6}	+0.00384	+0.02266	+0.03455	+0.02546	−0.00163	−0.02736	−0.03121	−0.02635
P_{r7}	+0.00816	+0.05385	+0.09766	+0.10276	+0.05536	−0.03121	−0.11947	−0.16714
P_{r8}	+0.01065	+0.07200	+0.13497	+0.14980	+0.09291	−0.02635	−0.16714	−0.26686
	(*b*) *D*-Constants for Tangential Components P_t							
P_{t1}	−0.00016	−0.00045	−0.00051	−0.00036	−0.00007	+0.00026	+0.00055	+0.00072
P_{t2}	−0.00199	−0.00898	−0.01135	−0.00862	−0.00234	+0.00531	+0.01201	+0.01590
P_{t3}	−0.00487	−0.02731	−0.04034	−0.03332	−0.01128	+0.01709	+0.04254	+0.05742
P_{t4}	−0.00749	−0.04611	−0.07626	−0.07018	−0.02868	+0.02962	+0.08354	+0.11549
P_{t5}	−0.00883	−0.05696	−0.10033	−0.10106	−0.04934	+0.03466	+0.11625	+0.16554
P_{t6}	−0.00830	−0.05502	−0.10043	−0.10693	−0.05979	+0.02846	+0.12179	+0.18015
P_{t7}	−0.00590	−0.03970	−0.07392	−0.08119	−0.04906	+0.01634	+0.09146	+0.14193
P_{t8}	−0.00213	−0.01445	−0.02715	−0.03024	−0.01891	+0.00505	+0.03359	+0.05423

18. Tangential Forces.—Lateral shear between a tunnel lining and the abutting soil would evidently give rise to tangential forces. However, in tunneling by the shield method, some special features must be considered in connection with shearing stresses. As the shield is being shoved ahead for the erection of another ring, an annular void is formed around the lining, and the soil, which before the shove surrounded the upper tail end of the shield, is thereby left without a support; but this does not necessarily lead to collapse, as arch action and cohesion combine to keep the bulk of the soil in place until the void has been filled with gravel which is injected from the working chamber by air pressure. As an additional precaution, grout is next forced into the gravel. The mixture of gravel and grout is in a more or less plastic state for a day or two and is thus incapable of resisting shearing forces. For some distance back of the shield the tunnel rings, therefore, are subjected to the major onset of the soil pressure at a time when only radial pressures can be transmitted through the gravel and grout filling. Moreover, any tangential forces developed later eventually spend themselves because of their distribution around the lining, being directed clockwise as well as counterclockwise. During the slips induced by earth tremors and vibrations from the numerous small cave-ins taking place in the surrounding ground as it gradually settles, the forces will thus impel the soil particles in opposite directions. Through these tiny movements the soil particles are crowded together on some parts of the periphery, the pressure between them increasing until it balances the impelling force, at which point the movements come to a stop. The final outcome is that the shearing forces between the soil and the ring surface are replaced by tangential compressive forces in the adjacent ground, the lining at this stage performing a function similar to that of the scaffolding or "centering" used in erecting a masonry arch.

On the basis of the foregoing reasoning, one is justified in disregarding the tangential components of forces due to soil pressure, the only tangential components to be considered being those due to the weight of the lining.

19. Soil Reactions Due to Settlement and Deflection.—Loading a lining embedded in homogeneous soil has the effect of displacing it in the direction of the resultant of the active forces. With the assumed preponderance of downward forces, a symmetrically loaded lining is thus displaced vertically downward, the underlying soil being thereby compressed until soil reactions in sufficient amount to balance the downward forces have been developed.

The soil reactions are not determined by the settlement alone, however. Under the influence of all the forces, active as well as reactive, the lining deflects. The radial deflection at any point will either increase or reduce the compression of the soil at that point, and thus add to or detract from the soil reaction due to settlement.

In Fig. 9, let the solid circle c represent the position of a ring before it is loaded, and the line d its position and shape upon completion of the downward movement. Wherever line d is on the outside of circle c the soil is being compressed; at all other points voids are created which are presently filled as the soil closes in upon the ring under the push of active forces.

Settlement is defined in this paper as the distance δ by which the invert joint A has been displaced. In the imaginary case of a perfectly rigid ring there could be no deflection; the soil reactions would here be due to settlement only and would be confined to the lower half of the ring (see dashed circle in Fig. 9), the middle point of each of the lower segments being displaced radially by an amount $\delta \cos \alpha$. As the flexibility increases, however, deflection becomes more and more of a contributing factor,

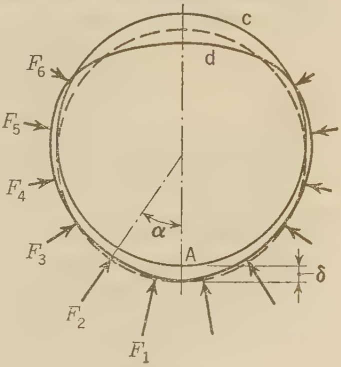


FIG. 9.—DISTRIBUTION OF SOIL REACTIONS

and the range within which the soil is being compressed gradually extends over the upper half of the ring, giving rise to soil reactions of a downward trend, such as F_5 and F_6 in Fig. 9. Far from resisting the active force resultant, such forces add to its intensity, thereby increasing the settlement.

Referring to the left-hand half of a symmetrically loaded ring, let u be the number of segments in which soil reactions are developed (that is, those whose middle points are outside of the circle c in Fig. 9). In most practical cases these segments will be consecutive from the invert point up. For a very stiff ring $u = 4$, whereas for a perfectly flexible one it would be 8. For the rings used in tunnels it is usually 5 or 6; the value is not known definitely beforehand but is tentatively assumed, the result of the computations being relied on to show whether the choice made was a correct one.

Like the active pressures (see Section 7), the soil reactions are replaced by forces F applied at the middle points of the u segments, the forces being considered proportional to the radial displacement at those points. Since the displacement at point n is equal to the amount due to settlement, $\delta \cos \alpha_n$, plus the radial deflection d_n, Eq. 15 yields, for the soil reaction developed in any of the u segments:

$$F_n = (\delta \cos \alpha_n + d_n)\, K\, A' \dots\dots\dots\dots\dots\dots (16)$$

in which A' is the area of the segment as projected on its chordal plane, the width of the segment being, as before, 1 ft.

20. Evaluation of Soil Reactions.—Solving for d_n in Eq. 16 gives, for each of the u segments:

$$d_n = \frac{F_n}{K\, A'} - \delta \cos \alpha_n \dots\dots\dots\dots\dots\dots (17a)$$

Taking into account both active forecs and soil reactions, the same deflection d_n, according to Eq. 14, may be expressed as:

$$d_n = \frac{r^3}{E\, I}\left[\sum_1^8 (P_r\, D_n) + \sum_1^8 (P_t\, D_n) + \sum_1^u (F\, D_n)\right] \dots\dots (17b)$$

If the amounts of the u forces F were known, their substitution in Eqs. 17a and 17b would yield identical values for the deflection d_n. Conversely, by equating the right-hand side of the two equations for each one of the deflections d_n and solving the u simultaneous equations thus obtained, the particular values of the forces F may be determined by which such equality is brought about.[5]

Assuming soil reactions to be developed in segments 1 to 5 only (that is, $u = 5$), the equation for point 2, say, is: $\frac{F_2}{K\, A'} - \delta \cos \alpha_2 = \frac{r^3}{E\, I}[\sum (P_r\, D_2) + \sum (P_t\, D_2) - 0.00657\, F_1 - 0.03747\, F_2 \cdots - 0.01252\, F_5]$; or, by rearranging the terms:

$$- 0.00657\, F_1 - \left(0.03747 + \frac{E\, I}{r^3\, K\, A'}\right) F_2 \cdots - 0.01252\, F_5$$
$$= - \frac{E\, I}{r^3} \cos \alpha_2\, \delta - \sum (P_r\, D_2) - \sum (P_t\, D_2) \dots\dots\dots\dots (18)$$

---

[5] Compare: "Soil Pressure Distribution Along Flexible Foundations," by Anders Bull, *Journal of the Franklin Institute*, June, 1942, p. 562.

Denoting by Q the quantity $\frac{E\,I}{r^3\,K\,A'}$, and by $-\sum(P\,D_n)$ the terms $-\sum(P_r\,D_n) - \sum(P_t\,D_n)$, for brevity, one may write for the complete set of five simultaneous equations:

$$\left.\begin{aligned}
&-(0.00215+Q)\,F_1 - 0.00657\,F_2 - 0.00753\,F_3\\
&-0.00540\,F_4 - 0.00115\,F_5 = -\frac{E\,I}{r^3}\cos\alpha_1\,\delta - \sum(P\,D_1)\ldots\ldots\\
&-0.00657\,F_1 - (0.03747+Q)\,F_2 - 0.05142\,F_3\\
&-0.04068\,F_4 - 0.01252\,F_5 = -\frac{E\,I}{r^3}\cos\alpha_2\,\delta - \sum(P\,D_2)\ldots\ldots\\
&-0.00753\,F_1 - 0.05142\,F_2 - (0.09098+Q)\,F_3\\
&-0.08340\,F_4 - 0.03397\,F_5 = -\frac{E\,I}{r^3}\cos\alpha_3\,\delta - \sum(P\,D_3)\ldots\ldots\\
&-0.00540\,F_1 - 0.04068\,F_2 - 0.08340\,F_3\\
&-(0.09611+Q)\,F_4 - 0.05254\,F_5 = -\frac{E\,I}{r^3}\cos\alpha_4\,\delta - \sum(P\,D_4)\ldots\\
&-0.00115\,F_1 - 0.01252\,F_2 - 0.03397\,F_3 - 0.05254\,F_4\\
&-(0.04653+Q)\,F_5 = -\frac{E\,I}{r^3}\cos\alpha_5\,\delta - \sum(P\,D_5)\ldots\ldots\ldots
\end{aligned}\right\}\ldots(19)$$

On the left-hand side of the equality signs the coefficients for the unknown forces F are simply the D-constants given in the first five lines and columns of Table 8(a), except that the quantity $-Q$ has been added to the D-constants forming the diagonal row (underscored in Table 8(a)).

On the right-hand side all the quantities, except the settlement δ, are fixed by the dimensions and other data given in the specific case. Hence, in solving for the forces F expressions are obtained having the form:

$$F_n = C'_n\,\delta + C''_n \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(20)$$

in which C'_n and C''_n are numerical quantities. From these expressions the unknown δ is determined by the conditions of equilibrium requiring that the sum of the vertical components of the soil reactions, $\sum_1^u(F\cos\alpha)$, must balance the sum of the vertical components of the active forces, or: $\sum_1^u(C'\,\delta\cos\alpha) + \sum_1^u(C''\cos\alpha) = -\sum_1^8(P_r\cos\alpha) - \sum_1^8(P_t\sin\alpha)$; from which:

$$\delta = \frac{-\sum_1^u(C''\cos\alpha) - \sum_1^8(P_r\cos\alpha) - \sum_1^8(P_t\sin\alpha)}{\sum_1^u(C'\cos\alpha)}\ldots\ldots\ldots(21)$$

Substituting this value for δ in Eq. 20, the forces F are determined. The active forces P being known, the moments and thrusts in the lining are found by means of Table 5, and from the values thus obtained the stresses are computed.

PRACTICAL APPLICATION

21. Specified Conditions.—The proposed method will now be applied to an actual case. A tunnel lining will be designed for the identical conditions prevailing for a short stretch of tunnel, forming part of the Sixth Avenue line of the Independent Subway System of New York, N. Y. As the latter was designed by the customary method the results obtained by the two methods may be compared.

The crown of the lining is 36 ft below the ground surface and 27 ft below the ground-water level, as shown in Fig. 10. The governing data are:

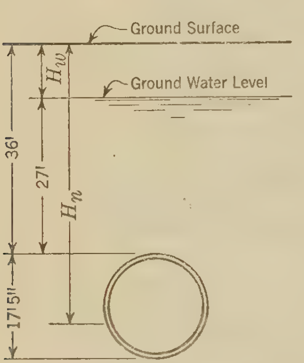


FIG. 10.—CROSS SECTION OF TUNNEL

Radius r_e of the outer surface, in feet.	8.71
Radius r of the neutral circle, in inches	102.1
Assumed Weight W, in Kips per Cubic Foot of—	
Soil above ground-water level	0.100
Soil below ground-water level	0.125
Water	0.063

Denoting by H_w and H_n, respectively, the depth below ground surface of the water table and any of the points 1 to 16 in Fig. 4, the vertical and horizontal pressures p, in kips per square foot at points 1 to 8 (or 16 to 9), are determined by the following formulas, pressures directed upward being considered positive:

Vertical pressure at points 1 to 4—

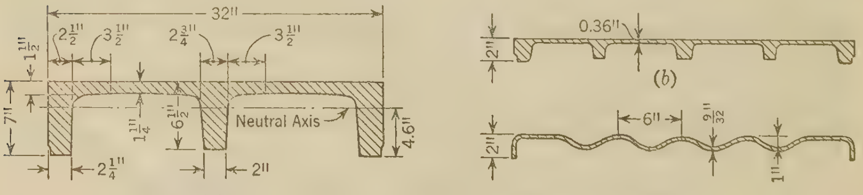
$$p_V = +0.063\,(H_n - H_w) \qquad (22a)$$

vertical pressure at points 5 to 8—

$$p_V = -0.100\,H_w - 0.125\,(H_n - H_w) \qquad (22b)$$

and horizontal pressure at points 1 to 8—

$$p_H = +\frac{0.100\,H_n}{3} + 0.063\,(H_n - H_w) \qquad (22c)$$



(a) (c)

FIG. 11.—AXIAL CROSS SECTION OF TUNNEL SEGMENTS

The lining is of cast iron for which a modulus E of 12,000 kips per sq in. is assumed. It has an inner covering of concrete which, however, is disregarded as to strength. Fig. 11(*a*) shows an axial cross section of the segments used whose physical properties per foot of tunnel are:

Area A, deducting area of holes (square inches)	27.6
Gross moment of inertia I (in.^{4})	136
Net Section Modulus S (in.^{3})—	
Intrados	23.4
Extrados	55.0
Weight W, including concrete (kips per square foot of lining surface)	0.22
Soil compression constant K (assumed)	12

22. Active Forces.—By means of Eqs. 22, the vertical and horizontal pressures at points 1 to 8 of the lining (Fig. 4) are determined as in Cols. 3 and 4, Table 9. The pressures are multiplied by the projected area of each segment (1 ft wide) in a horizontal and vertical plane, respectively, and the amount $-\frac{2\pi \times 8.71 \times 0.22}{16} = -0.752$ kip is added to each vertical force for the weight of the lining, yielding the values given in Cols. 7 and 8, Table 9. The forces P_r (Col. 9) are the sums of the radial components of each force pair, P (vertical) $\cos\alpha + P$ (horizontal) $\sin\alpha$, whereas the forces P_t, as stated in Section 18, are the tangential components of the weight of the lining only, $-0.752 \sin\alpha$. Points 1 to 8 being located as in Fig. 4, the respective values of $\cos\alpha$ and $\sin\alpha$ are:

TABLE 9.—COMPUTATIONS FOR DETERMINING RADIAL AND TANGENTIAL FORCES

Point	Depth H_n, in ft	Pressure, in Kips per Sq Ft		Projected Area, in Sq Ft		Forces, in Kips			
		Vertical	Horizontal	Horizontal	Vertical	Vertical	Horizontal	P_r	P_t
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
1	53.25	+2.788	4.563	3.333	0.663	+8.541	3.025	8.966	−0.147
2	51.95	+2.706	4.438	2.826	1.888	+6.894	8.379	10.388	−0.418
3	49.55	+2.555	4.206	1.888	2.826	+4.071	11.886	12.144	−0.625
4	46.41	+2.357	3.904	0.663	3.333	+0.811	13.012	12.920	−0.738
5	43.01	−5.151	3.576	0.663	3.333	−4.167	11.921	12.505	−0.738
6	39.87	−4.759	3.274	1.888	2.826	−9.737	9.251	13.102	−0 625
7	37.47	−4.459	3.040	2.826	1.888	−13.351	5.741	14.290	−0.418
8	36.17	−4.296	2.917	3.333	0.663	−15.071	1.934	15.159	−0 147

$$\cos\alpha_1 = -\cos\alpha_8 = \sin\alpha_4 = \sin\alpha_5 = 0.98079$$
$$\cos\alpha_2 = -\cos\alpha_7 = \sin\alpha_3 = \sin\alpha_6 = 0.83147$$
$$\cos\alpha_3 = -\cos\alpha_6 = \sin\alpha_2 = \sin\alpha_7 = 0.55557\ .$$
$$\cos\alpha_4 = -\cos\alpha_5 = \sin\alpha_1 = \sin\alpha_8 = 0.19509$$

23. Simultaneous Equations.—The values found are substituted for P_{r1} to P_{t8} in the first column of Table 8, and each force is multiplied by the eight D-constants in the same line, the products being entered, in Table 10, in the same order as the D-constants. The products are next added up vertically, the sums representing the terms $\sum(P\,D_n)$ on the right-hand side of Eqs. 19.

The coefficients for δ are found by multiplying the quantity $\dfrac{E\,I}{r^3} = \dfrac{12{,}000 \times 136}{1{,}064{,}331} = 1.533$ by the successive values, $\cos\alpha_1$, $\cos\alpha_2$, $\cdots$.

TABLE 10.—DEFLECTION PRODUCTS DUE TO ACTIVE FORCES

Force	Point							
	1	2	3	4	5	6	7	8
Radial:								
8.966	−0.0193	−0.0589	−0.0675	−0.0484	−0.0103	+0.0344	+0.0732	+0.0955
10.388	−0.0682	−0.3892	−0.5342	−0.4226	−0.1301	+0.2354	+0.5594	+0.7479
12.144	−0.0914	−0.6244	−1.1049	−1.0128	−0.4125	+0.4196	+1.1860	+1.6391
12.920	−0.0698	−0.5256	−1.0775	−1.2417	−0.6788	+0.3289	+1.3277	+1.9354
12.505	−0.0144	−0.1566	−0.4248	−0.6570	−0.5819	−0.0204	+0.6923	+1.1618
13.102	+0.0503	+0.2969	+0.4527	+0.3336	−0.0214	−0.3585	−0.4089	−0.2452
14.290	+0.1166	+0.7695	+1.3956	+1.4684	+0.7911	−0.4460	−1.7072	−2.3884
15.159	+0.1614	+1.0914	+2.0460	+2.2708	+1.4084	−0.3994	−2.5337	−4.0453
Tangential:								
−0.147	0.0000	+0.0001	+0.0001	+0.0001	0.0000	0.0000	−0.0001	−0.0001
−0.418	+0.0008	+0.0038	+0.0047	+0.0036	+0.0010	−0.0022	−0.0050	−0.0066
−0.625	+0.0030	+0.0171	+0.0252	+0.0208	+0.0071	−0.0107	−0.0266	−0.0359
−0.738	+0.0055	+0.0340	+0.0563	+0.0518	+0.0212	−0.0219	−0.0617	−0.0852
−0.738	+0.0065	+0.0420	+0.0740	+0.0746	+0.0364	−0.0256	−0.0858	−0.1222
−0.625	+0.0052	+0.0344	+0.0628	+0.0668	+0.0374	−0.0178	−0.0761	−0.1126
−0.418	+0.0025	+0.0166	+0.0309	+0.0339	+0.0205	−0.0068	−0.0382	−0.0593
−0.147	+0.0003	+0.0021	+0.0040	+0.0044	+0.0028	−0.0007	−0.0049	−0.0080
$\Sigma(P\,D)$	+0.0890	+0.5532	+0.9434	+0.9463	+0.4909	−0.2917	−1.1096	−1.6291

Since each of the sixteen segments is spanning an angle of 22° 30′, its chordal projection has an area of: $A' = 2\,r_o \sin\dfrac{22.5°}{2} = 3.40$ sq ft, from which $K\,A' = 12 \times 3.40 = 40.80$; and $Q = \dfrac{E\,I}{r^3\,K\,A'} = \dfrac{1.533}{40.80} = 0.03758$.

The lining shown in Fig. 11(a) is considered relatively stiff and, therefore, the low value of 5 is assumed tentatively for u. Consequently there are five simultaneous equations for determining the soil reactions F, the equations, according to the foregoing, taking the form:

$$\left.\begin{array}{l}
-\,0.03973\,F_1 - 0.00657\,F_2 - 0.00753\,F_3 - 0.00540\,F_4 - 0.00115\,F_5 \\
\qquad = -\,1.50390\,\delta - 0.08900 \ldots\ldots\ldots\ldots \\
-\,0.00657\,F_1 - 0.07505\,F_2 - 0.05142\,F_3 - 0.04068\,F_4 - 0.01252\,F_5 \\
\qquad = -\,1.27500\,\delta - 0.55320 \ldots\ldots\ldots\ldots \\
-\,0.00753\,F_1 - 0.05142\,F_2 - 0.12856\,F_3 - 0.08340\,F_4 - 0.03397\,F_5 \\
\qquad = -\,0.85190\,\delta - 0.94340 \ldots\ldots\ldots\ldots \\
-\,0.00540\,F_1 - 0.04068\,F_2 - 0.08340\,F_3 - 0.13369\,F_4 - 0.05254\,F_5 \\
\qquad = -\,0.29920\,\delta - 0.94630 \ldots\ldots\ldots\ldots \\
-\,0.00115\,F_1 - 0.01252\,F_2 - 0.03397\,F_3 - 0.05254\,F_4 - 0.08411\,F_5 \\
\qquad = +\,0.29920\,\delta - 0.49090 \ldots\ldots\ldots\ldots
\end{array}\right\}\ldots(23)$$

24. Soil Reactions.—The F-values satisfying Eqs. 23 are listed in Cols. 4 and 5, Table 11, which also shows the steps by which they are reduced to numerical terms. Proceeding in accordance with Eq. 21 and selecting proper values from

Table 11, δ is computed as: $\delta = \frac{-5.156 + 9.769 + 3.010}{+48.857} = +0.156$ in.; and finally, by substitution for δ in Col. 6, the soil reactions are determined as given in Col. 10, Table 11. The sum of their vertical components $F \cos \alpha$ (Col. 11, Table 11) is $+12.778$, which tallies with $-\sum(P_r \cos \alpha) - \sum(P_t \sin \alpha) = +9.769 + 3.010$, as required for equilibrium (see derivation of Eq. 21).

TABLE 11.—COMPUTATIONS FOR SOIL REACTIONS

Point	$P_r \cos \alpha$ (P_r from Table 9)	$P_t \sin \alpha$ (P_t from Table 9)	F (Eq. 20) = $C'\delta + C''$		$F \cos \alpha$ = $C'\delta \cos \alpha + C'' \cos \alpha$		F = $C'\delta + C''$		= F	$F \cos \alpha$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
1	+8.794	−0.029	+35.565 δ	+0.606	+34.882 δ	+0.594	+5.548	+0.606	6.154	+6.036
2	+8.638	−0.232	+15.099 δ	+2.799	+12.555 δ	+2.327	+2.355	+2.799	5.154	+4.285
3	+6.747	−0.520	+1.702 δ	+3.597	+0.946 δ	+1.998	+0.266	+3.597	3.863	+2.146
4	+2.521	−0.724	−2.800 δ	+3.184	−0.546 δ	+0.621	−0.437	+3.184	2.747	+0.536
5	−2.440	−0.724	−5.230 δ	+1.970	+1.020 δ	−0.384	−0.816	+1.970	1.154	−0.225
6	−7.279	−0.520	....	....	....	....	....	....	....	....
7	−11.882	−0.232	....	....	....	....	....	....	....	....
8	−14.868	−0.029	....	....	....	....	....	....	....	....
Σ	−9.769	−3.010	....	....	+48.857 δ	+5.156	....	....	....	+12.778

It remains to be ascertained whether the tentative assumption, $u = 5$, is correct. For that purpose the D-products of the soil reactions are determined in Table 12, the F-values being substituted for the forces P in Table 8. To the sums of these products in line 6, Table 12, are added the amounts obtained for $\sum(P\,D)$ in Table 10, and the totals (line 8) are multiplied by the factor $\frac{r^3}{E\,I} = \frac{1{,}064{,}331}{12{,}000 \times 136} = 0.6522$, yielding the deflections, in inches, at points 1 to 8 (line 9, Table 12). Adding to each deflection the radial displacement due to settlement, $\delta \cos \alpha_n$, the amounts are found (line 10) by which the points have moved out from their neutral position on circle c in Fig. 9. Only at points

TABLE 12.—CHECK COMPUTATIONS FOR THE CONSTANT, u

Line No.	Soil reaction	Point 1	2	3	4	5	6	7	8
1	6.154	−0.0132	−0.0404	−0.0463	−0.0332	−0.0071	+0.0236	+0.0502	+0.0655
2	5.154	−0.0339	−0.1931	−0.2650	−0.2097	−0.0645	+0.1168	+0.2775	+0.3711
3	3.863	−0.0291	−0.1986	−0.3515	−0.3222	−0.1312	+0.1335	+0.3773	+0.5214
4	2.747	−0.0148	−0.1117	−0.2291	−0.2640	−0.1443	+0.0699	+0.2823	+0.4115
5	1.154	−0.0013	−0.0144	−0.0392	−0.0606	−0.0537	−0.0019	+0.0639	+0.1072
6	Total, lines 1–5	−0.0923	−0.5582	−0.9311	−0.8897	−0.4008	+0.3419	+1.0512	+1.4767
7	$\Sigma(P\,D)$	+0.0890	+0.5532	+0.9434	+0.9463	+0.4909	−0.2917	−1.1096	−1.6291
8	Line 6 + line 7	−0.0033	−0.0050	+0.0123	+0.0566	+0.0901	+0.0502	−0.0584	−0.1524
9	Line 8 × 0.6522	−0.0022	−0.0033	+0.0080	+0.0369	+0.0588	+0.0327	−0.0381	−0.0994
10	$\delta \cos \alpha$	+0.1530	+0.1297	+0.0867	+0.0304	−0.0304	−0.0867	−0.1297	−0.1530
11	Line 9 + line 10	+0.1508	+0.1264	+0.0947	+0.0673	+0.0284	−0.0540	−0.1678	−0.2524
12	Line 11 × 40.8	6.153	5.157	3.863	2.746	1.159	....	....	....

where this amount is positive are soil reactions developed. Table 12 shows that the displacement (line 11) is positive at points 1 to 5 only, proving that the tentative assumption, $u = 5$, is correct. This table provides a rigorous check on the correctness of the arithmetical work since, according to Eq. 16, the radial displacement at any of the points 1 to 5, multiplied by the factor $K A' = 40.8$, should give the soil reaction at that point. As shown in line 12, Table 12, this is the case within the limits of unavoidable error.

25. Bending Moments and Thrusts.—The bending moments and thrusts produced in the ring may now be determined, as shown in Table 13, active forces and soil reactions having been treated separately. The procedure is the same as in finding the deflections: Each force is multiplied by the nine B_M-constants appearing in the same line in Table 5(a), and the products, entered in the order of the constants, are added. The product sums $\sum(P\ B)$ due to the active forces are then added to those due to soil reactions $\sum(F\ B)$ (Table 13(b)), and the sums are multiplied by the radius $r = 102.1$ in. Thus, for the moment at the various joints, the amounts given in the last line of Table 13(b) are obtained.

TABLE 13.—COMPUTATION OF BENDING MOMENTS AND THRUSTS IN THE TUNNEL RING

Force	MOMENTS AT JOINTS									THRUSTS	
	A	B	C	D	E	F	G	H	J	Joint F	Joint J
(*a*) ACTIVE FORCES											
Radial:											
8.966	+1.437	+5.867	+9.930	+11.378	+10.336	+7.288	+3.017	−1.553	−5.544	+11.213	+0.109
10.388	+4.132	+6.371	+8.475	+10.299	+9.649	+7.027	+3.149	−1.128	−4.974	+11.386	+1.082
12.144	+5.224	+6.791	+6.040	+5.778	+6.254	+5.091	+2.810	−0.030	−2.857	+9.778	+3.155
12.920	+3.818	+4.058	+2.832	+0.271	−0.438	+0.904	+1.468	+1.353	+0.629	+5.323	+5.544
12.505	+0.609	−0.558	−2.030	−3.633	−5.303	−4.245	−0.709	+2.061	+3.695	−0.474	+6.899
13.102	−3.083	−5.604	−7.262	−7.958	−7.810	−7.217	−3.777	+1.756	+5.636	−3.824	+7.489
14.290	−6.842	−10.646	−12.472	−12.287	−10.489	−7.788	−5.144	−0.327	+5.685	−4.686	+6.451
15.159	−9.373	−14.005	−15.926	−15.150	−12.259	−8.236	−4.238	−1.460	+2.430	−5.222	+2.773
Tangential:											
−0.147	−0.003	−0.020	−0.033	−0.037	−0.034	−0.024	−0.010	+0.005	+0.018	−0.036	0.000
−0.418	−0.057	−0.156	−0.255	−0.295	−0.270	−0.192	−0.081	+0.039	+0.144	−0.298	−0.009
−0.625	−0.191	−0.383	−0.537	−0.626	−0.584	−0.423	−0.187	+0.072	+0.304	−0.682	−0.057
−0.738	−0.334	−0.583	−0.742	−0.802	−0.763	−0.573	−0.272	+0.069	+0.386	−0.984	−0.167
−0.738	−0.386	−0.623	−0.751	−0.765	−0.685	−0.535	−0.282	+0.029	+0.334	−1.039	−0.311
−0.625	−0.304	−0.470	−0.548	−0.537	−0.455	−0.334	−0.198	−0.015	+0.191	−0.202	−0.404
−0.418	−0.144	−0.216	−0.248	−0.237	−0.194	−0.134	−0.073	−0.021	+0.057	−0.084	−0.356
−0.147	−0.018	−0.027	−0.031	−0.029	−0.024	−0.016	−0.008	−0.002	+0.003	−0.010	−0.144
$\Sigma(P\ B)$	−5.515	−10.204	−13.558	−14.630	−13.069	−9.407	−4.535	+0.848	+6.137	+20.159	+32.054
(*b*) SOIL REACTIONS AND TOTALS											
6.154	+0.987	+4.027	+6.816	+7.809	+7.095	+5.002	+2.071	−1.066	−3.805	+7.696	+0.075
5.154	+2.050	+3.161	+4.205	+5.110	+4.788	+3.487	+1.562	−0.560	−2.468	+5.649	+0.537
3.863	+1.662	+2.160	+1.921	+1.838	+1.989	+1.619	+0.894	−0.010	−0.909	+3.110	+1.001
2.747	+0.812	+0.863	+0.602	+0.058	−0.093	+0.192	+0.312	+0.288	+0.134	+1.132	+1.179
1.154	+0.056	−0.051	−0.187	−0.335	−0.489	−0.392	−0.065	+0.190	+0.341	−0.042	+0.637
$\Sigma(F\ B)$	+5.567	+10.160	+13.357	+14.480	+13.290	+9.908	+4.774	−1.158	−6.707	+17.545	+3.432
$\Sigma(P\ B)$	−5.515	−10.204	−13.558	−14.630	−13.069	−9.407	−4.535	+0.848	+6.137	+20.159	+32.054
Total	+0.052	−0.044	−0.201	−0.150	+0.221	+0.501	+0.239	−0.310	−0.570	+37.704	+35.486
× 102.1	+5.3	−4.5	−20.5	−15.3	+22.6	+51.2	+24.4	−31.7	−58.2	....	....

The thrusts are determined only for joints F and J, where the stresses reach critical values. The last two columns of Table 13 are used for that purpose, the products being those of the forces in the B_T-constants of Tables 5(*c*) and 5(*d*).

Table 13 illustrates what was stated in Section 2 in regard to the sensitive state of moment balance in a ring. At joint B (Fig. 4), for example, the B_M-products due to forces P_{r3} and P_{r8} are 6.791 and $-$ 14.005, respectively, as compared to the total of 0.044.

26. Critical Stresses.—The design of a lining is governed by the maximum stresses caused by bending and thrust combined, the negative thrust due to the increased air pressure in the working chamber during construction being also considered. With a cross section like the one in Fig. 11(*a*), the greater part of the maximum stresses are due to bending, the fibers most highly stressed being those in the ribs at the intrados.

According to Table 13(*b*), the moment in the present case reaches a positive maximum at joint F, and a negative maximum at joint J. For the properties given in Section 21, therefore, the respective bending stresses σ_b, expressed in kips per square inch, are: $\sigma_{bF} = \frac{51.2}{23.4} = -2.19$; and $\sigma_{bJ} = \frac{58.2}{23.4} = +2.49$—a positive sign indicating tension and a negative sign compression.

The thrusts at joints F and J are shown in Table 13(*b*) to be 37.7 kips and 35.5 kips, respectively, which yield, for the corresponding stresses σ_t: $\sigma_{tF} = \frac{37.7}{27.6} = -1.37$; and $\sigma_{tJ} = \frac{35.5}{27.6} = -1.29$.

The internal air pressure is usually made equal to the hydrostatic pressure of the ground water at the level of the tunnel axis. For a pressure head of 35.7 ft and an effective radius of 8.5 ft, the tensile force produced in the lining is $35.7 \times 0.063 \times 8.5 = 19.1$ kips, resulting in a stress of $\sigma_a = \frac{19.1}{27.6} = +0.69$.

Accordingly, the maximum tensile stress, occurring at joint J, is $\sigma_J = +2.49 - 1.29 + 0.69 = +1.89$ kips per sq in. The compressive stress at joint F does not reach its maximum until the internal air pressure is removed, when $\sigma_F = -2.19 - 1.37 = -3.56$ kips per sq in.

As the usual working stresses for cast iron are 4.0 kips per sq in. in tension and 20.0 kips per sq in. in compression, the stress at joint J is only 47% and that at joint F only 18% of the respective allowable values.

The obvious conclusion is that the lining, as designed, is too heavy; but to what extent its cross section should be reduced in order to meet the requirements of economy is not immediately apparent. A reduction in area also involves the moment of inertia, and as the latter enters into Eqs. 19 by which the soil reactions are determined, the pressure distribution will not remain the same as before. This affects the moments in particular because of their extreme sensitiveness to changes in the pressure distribution.

27. Economic Cross Section.—For investigating the question raised in the preceding section assume the moment of inertia of the lining, per foot of tunnel, to be reduced in a geometric progression to 1/4, 1/16, 1/64, etc., of its original value of 136 in.^{4}, the reduction being effected by shrinking the radial dimensions of the cross section, Fig. 11(*a*). Such shrinking causes the section modulus and area to be reduced correspondingly to $(1/4)^{2/3}$, $(1/16)^{2/3}$, $(1/64)^{2/3} \cdots$ and $(1/4)^{1/3}$, $(1/16)^{1/3}$, $(1/64)^{1/3} \cdots$ of the original amounts of 23.4 in.^{3} and 27.6 in.^{2}, respectively. The soil reactions, moments, thrusts, and critical stresses are then determined for each of the reduced sections by means of the method explained.

TABLE 14.—EFFECTS OF VARIATIONS IN THE MOMENT OF INERTIA OF THE TUNNEL LINING

Description	MOMENT OF INERTIA I, IN INCHES^4:											
	136		34		8.5		2.13		0.53		0.13	
	F	M	F	M	F	M	F	M	F	M	F	M
(*a*) PHYSICAL PROPERTIES												
S (in.^{3})	23.4		9.29		3.69		1.46		0.580		0.227	
A (in.^{2})	27.6		17.4		11.0		6.90		4.34		2.72	
(*b*) SOIL REACTIONS, IN KIPS, AND MOMENTS, IN INCH-KIPS												
Joint A		+5.3		+4.3		+0.9		+0.2		0		0
Point 1	6.154		6.632		7.068		7.232		7.286		7.295	
Joint B		−4.5		+0.9		+0.6		−0.1		−0.1		−0.2
Point 2	5.154		5.199		5.546		5.674		5.715		5.735	
Joint C		−20.5		−6.6		−1.7		−0.9		−0.4		−0.5
Point 3	3.863		3.481		3.639		3.793		3.841		3.840	
Joint D		−15.3		−10.1		−2.7		−0.5		+0.2		+0.1
Point 4	2.747		2.713		2.633		2.643		2.674		2.698	
Joint E		+22.7		+1.0		−1.6		−0.8		0		−0.1
Point 5	1.154		2.588		2.982		2.957		2.908		2.884	
Joint F		+51.2		+27.7		+8.0		+1.2		−0.5		−0.9
Point 6	....		....		1.282		1.941		2.171		2.235	
Joint G		+24.4		+24.5		+12.6		+7.0		+5.2		+4.6
Joint H		−31.7		−16.7		−4.9		−0.6		+0.5		+0.9
Joint J		−58.2		−38.3		−17.8		−10.9		−8.7		−8.0
(*c*) SETTLEMENT δ, IN INCHES												
....	0.156	....	0.175	....	0.206	....	0.292	....	0.624	....	1.991	....
(*d*) THRUST, IN KIPS												
Joint F		37.7		37.8		....		....		....		....
Joint G		....		....		38.0		38.3		38.4		38.5
Joint J		35.5		36.2		37.2		37.6		37.7		37.8
(*e*) MAXIMUM STRESS AT JOINT F OR G, IN KIPS PER SQUARE INCH												
σ_b	51.2/23.4 = −2.19		27.7/9.29 = −2.98		12.6/3.69 = −3.42		7.0/1.46 = −4.79		5.2/0.580 = −8.97		4.6/0.227 = −20.26	
σ_t	37.7/27.6 = −1.37		37.8/17.4 = −2.17		38.0/11.0 = −3.45		38.3/6.90 = −5.55		38.4/4.34 = −8.85		38.5/2.72 = −14.15	
Total	−3.56		−5.15		−6.87		−10.34		−17.82		−34.41	
(*f*) MAXIMUM STRESS AT JOINT J, IN KIPS PER SQUARE INCH												
σ_b	58.2/23.4 = +2.49		38.3/9.29 = +4.12		17.8/3.69 = +4.82		10.9/1.46 = +7.47		8.7/0.58 = +15.00		8.0/0.227 = +35.24	
σ_t	35.5/27.6 = −1.29		36.2/17.4 = −2.08		37.2/11.0 = −3.38		37.6/6.90 = −5.45		37.7/4.34 = − 8.70		37.8/2.72 = −13.90	
σ_a	19.1/27.6 = +0.69		19.1/17.4 = +1.10		19.1/11.0 = +1.74		19.1/6.90 = +2.77		19.1/4.34 = + 4.40		19.1/2.72 = +7.02	
Total	+1.89		+3.14		+3.18		+4.79		+10.70		+28.36	

Omitting the arithmetical work for the sake of brevity, the results are tabulated for ready comparison in Table 14. The substantial change in the moments attending a relatively slight variation in the soil reactions should be noted.

The effects of a successive reduction in cross section are more strikingly revealed in Fig. 12, where the moments obtained for the several cross sections have been plotted against the developed periphery of the left half of the lining, each graph being marked with the corresponding value of *I*. The thrusts, which vary but slightly, have been plotted for the largest and smallest cross section only.

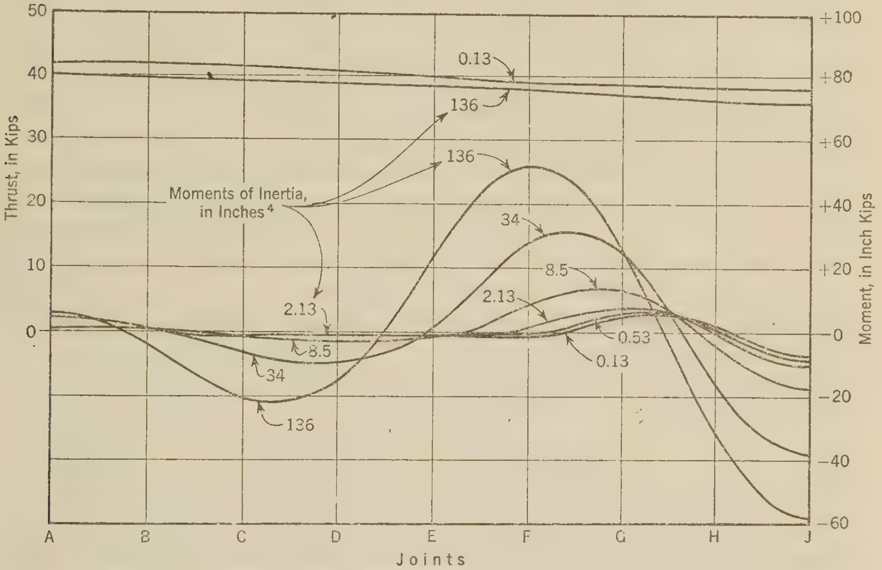


FIG. 12.—MOMENTS AND THRUSTS RELATED TO THE MOMENT OF INERTIA OF A TUNNEL LINING

Fig. 12 shows that, when the cross section is reduced beyond a certain point, the moments in the lower portion of the lining practically disappear, the only moments remaining being those produced in the arch formed by the top portion, where there are no soil reactions. In this respect the results differ sharply from those obtained by the customary method of analysis, according to which the moment at the invert joint is only slightly smaller than that at the crown.

When the reduction is accomplished, as in this case, through radial shrinking of a cross section of a given shape the relatively most economic section is evidently that at which one of the critical stresses reaches its allowable limit, although this does not necessarily mean that it is the most economic section attainable. For determining the relatively most economic section, the critical stresses, section moduli, and cross section areas in the last four columns of Table 14 have been plotted against the corresponding moments of inertia, as shown in Fig. 13, a logarithmic scale being used.

Referring to the curves drawn in solid lines, representing the case of a cast-iron lining, in order to bring the tensile stress at joint J up to the allowable 4 kips per sq in. the moment of inertia must be reduced to 3.3 in.^{4}, corresponding to an area of 8.0 sq in. This would indicate that with a lining as actually designed more than 70% of the cast iron is wasted.

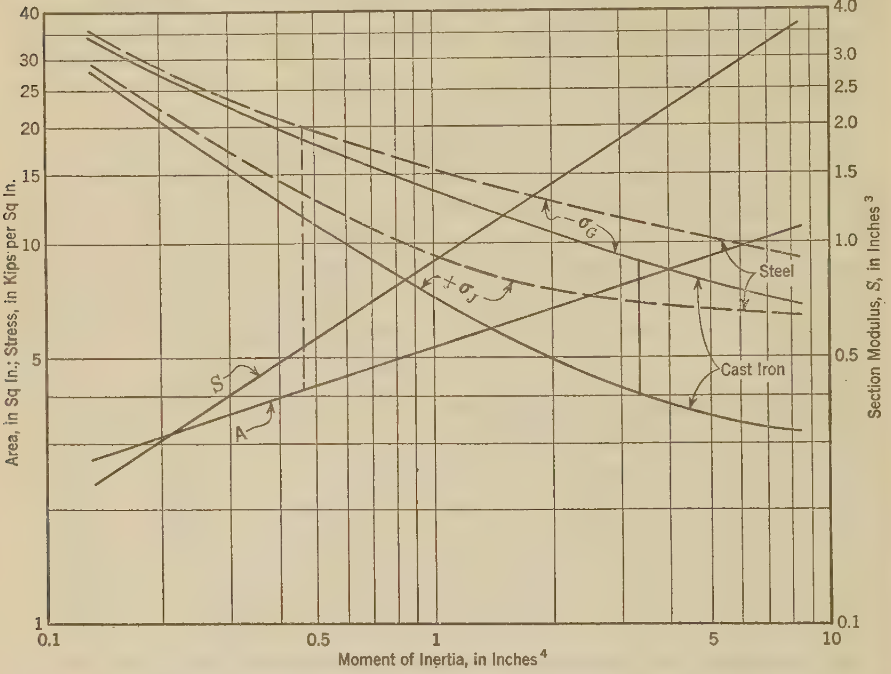


FIG. 13.—ECONOMIC SECTION DETERMINED BY A GRAPHIC INTERPOLATION

Theoretically, the cross section shown in Fig. 11(*b*) would meet requirements as regards strength. Practically, however, the casting of large segments of such a slender shape is apt to present difficulties, the cylindrical portion or "skin" being less than 3/8 in. thick.

28. Steel Versus Cast Iron.—By making the lining of steel, not only is the fabrication of the segments simplified, but a still further reduction of the cross section is made possible because of the greater tensile strength of that material. The consequent lighter weight of the segments will make them easier to handle. They may be made to span wider angles, so that the number of joints per ring may be reduced and erection facilitated. Because of the high ductility of steel, cracking of segments, always a source of serious trouble, is eliminated. Although cast iron is less apt to rust, a number of inexpensive treatments are now available by which steel surfaces are rendered inert toward oxygen. Steel has proved entirely satisfactory as a lining material in many instances, both in the United States and elsewhere, so one may rather wonder at the continued use of cast iron for shield-driven tunnels, when for most other structures it has long ago been replaced by steel.

On account of the higher modulus of elasticity of steel (30,000 kips per sq in., as compared to 12,000 for cast iron), a steel lining will be stiffer than a cast-iron one of the same dimensions, the effect being the same as if its moment of inertia were increased to 30,000/12,000 = 5/2 times its actual value, resulting in higher bending moments and slightly lower thrusts, as shown in Fig. 12. The stresses produced are easily determined from the data in Table 14, observing that the actual moment of inertia corresponding to the moments and thrusts in any column is thus only 2/5 = 0.4 times the value given for I in that column and that, therefore, the section modulus S is to be multiplied by $0.4^{2/3} = 0.542$, and the area A by $0.4^{1/3} = 0.737$ when computing the stresses.

The critical stresses thus obtained for a steel lining are represented by the two curves drawn in dashed lines in Fig. 13. If working stresses of ± 20 kips per sq in. are allowed, the strength of the lining is no longer governed by the tensile stress at joint J, but by the compressive stress at joint G, the value − 20 corresponding to a moment of inertia of 0.47 in.^{4}, a section modulus of 0.53 in.^{3}, and an area of 4.1 sq in. per foot of tunnel.

The cross section shown in Fig. 11(*c*) approximately meets these requirements. It has a moment of inertia of 0.58 in.^{4}, a section modulus of 0.79 in.^{3}, and an area of 3.8 sq in. per ft. The maximum stresses produced under the loading conditions assumed will be − 18.0 kips per sq in. at joint G, and + 7.6 kips per sq in. at joint J. The higher economy attained as compared to the section derived from Fig. 13 is due to a more favorable distribution of the material about the neutral axis.

The foregoing results are in good agreement with the sections used in large corrugated steel culverts under comparable loading conditions, details of which have been published. A culvert of 15-ft diameter was thus installed in 1939 under an embankment of the Denver and Salt Lake Railway.[6] It has served satisfactorily under a 42-ft cover of dry fill and superimposed train load. The embankment is 20 ft wide at the top, with slopes 1 on 1.5. According to data furnished by M. G. Spangler,[7] M. Am. Soc. C. E., the corrugations have a depth of 2 in. and a spacing of 6 in., the thickness of the plates varying from 3/16 in. at the intake and outlet to 9/32 in. at the center of the embankment. The latter thickness gives a moment of inertia of 2.3 in.^{4}, a section modulus of 2.0 in.^{3}, and an area of 4.3 sq in. per ft.

Other corrugated steel culverts of 13.75-ft and 15-ft diameters with covers of 57 ft and 27 ft, respectively, were built in connection with the Lake Mead Road project.[8] For the smaller diameter the same type of plates was used as for the aforementioned railroad culvert; for the larger diameters, plates of somewhat lighter size were used.

A paper by Karl Terzaghi, M. Am. Soc. C. E., "Liner-Plate Tunnels on the Chicago Subway,"[9] may also be referred to in this connection. Discussing cases in which the soil conditions as well as the shape of the tunnel are different from those considered in the present paper, Professor Terzaghi, on empirical premises, arrives at the same conclusion in regard to the excessive strength of the linings required by the customary method of design because of its failure to make proper allowance for the soil reactions.

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[6] *Engineering News-Record*, November 7, 1940, p. 74.

[7] "The Structural Design of Flexible Pipe Culverts," by M. G. Spangler, *Bulletin No. 153*, Eng. Experiment Station, Iowa State College, Ames, 1941.

[8] "Drainage Problems on New Lake Mead Road," by George D. Whittle, *Civil Engineering*, April, 1942, p. 196.

[9] *Transactions*, Am. Soc. C. E., Vol. 108 (1943), p 970.

Lieutenant-Commander Spangler[10] calls attention to what he terms "deferred deflection," representing the slow, additional deflection observed in culverts after the loading has reached its maximum. The writer considers this phenomenon as being in accord with the views expressed in Section 18 regarding the gradual transfer of the tangential soil pressures from the lining to the surrounding ground. As the pressures are directed outward, both in the upper and lower half of the lining, such transfer would naturally cause an outward yielding of the soil at either end of the horizontal diameter and a sagging at the crown, without a corresponding increase in radial loads, the flexible lining adjusting itself to these movements.

29. Construction Features.—In view of the light cross section required when a lining is made of steel, segments of the rolled or pressed type would be the most suitable ones. The skin and the curved ribs are here formed from a single plate, the straight, axial ribs being subsequently welded on in the shop. Bolts (which are now used for joining the segments to one another) may conceivably be dispensed with, as the small thickness of the flanges makes them particularly well adapted to resistance welding. With the welding transformer mounted at the end of the erector arm a series of "shot welds" may be produced through the flanges in a fraction of the time needed for bolting them together. By overlapping the welds, the flanges may be joined by a continuous seam of fused metal insuring a waterproof joint requiring no calking, except at corners. As no arc or flame is used with this kind of welding, no fire hazard is involved.

One objection that may be raised to reducing the weight of the lining is that a large cross section is necessary for resisting the pressure of the shoving jacks. There seems to be no good reason, however, for making material that is needed only during the erection of a tunnel an integral part of it, something that is being done in no other type of structure. A more satisfactory solution would be to use scaffolding, as it could be reclaimed after having served its purpose. The scaffolding should be tied in with the lining so as to effect a gradual transfer of the jack pressure to the surrounding ground; some distance back of the shield it would then no longer be stressed during a shove, so that it could be detached and used over again. Furthermore, it should leave the joints in the ring to be welded free and easily accessible.

Although it is true that such a type of scaffolding has yet to be developed, it may also be stated that, thus far, there has been no demand for it, as a heavy lining has been considered essential on account of the soil pressures. Once the fallacy of this contention is recognized, a solution of the problem presented by the scaffolding may be confidently expected.

Conclusions

In designing the linings of shield-driven tunnels, the erroneous assumption is currently made that the vertical soil reactions are uniformly distributed over the horizontal diameter of the lining. When, instead, their distribution is determined by rational methods it is found that the stresses in the lining are much lower than heretofore presumed, permitting the cross section to be substantially reduced. Such reduction, because of the consequent increase in the flexibility of the lining, has the effect of still further reducing the stresses. By successive reductions and interpolation of the values obtained for the stresses, an economic cross section may be determined.

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[10] "The Structural Design of Flexible Pipe Culverts," by M. G. Spangler, *Bulletin No. 153*, Eng. Experiment Station, Iowa State College, Ames, 1941, p. 61.

Steel is more economical than cast iron as a lining material because of its higher tensile strength. Steel segments are simpler to manufacture and may be joined to one another by means of resistance welding, thus facilitating erection and waterproofing of the tunnel.

In order that the prospective savings in material and labor may be realized, however, a suitable form of scaffolding must first be developed.

APPENDIX

NOTATION

The letter symbols, used in this paper, conform essentially to American Standard Symbols for Mechanics, Structural Engineering and Testing Materials[11] prepared by a Committee of the American Standards Association, with Society representation, and approved by the Association in 1932. Symbols for soil characteristics conform to "Soil Mechanics Nomenclature."[12] Subscript n, standing for any number from 1 to 16, is used for identifying any one of the sixteen imaginary segments of a tunnel ring; and subscript N, standing for any letter from A to R, serves to identify the joint between any two such segments (see Fig. 4). Subscripts T, V, and M signify "thrust," "shear," and "moment," respectively; and r and t signify "radial component" and "tangential component," respectively. Symbols are defined where they first appear; only those entering the formulas used in solving specific problems are listed in this Appendix.

A_N = a constant used in determining reactions at the invert joint A;

A = axial cross section of lining per foot of tunnel, in square inches;

A' = area of a segment as projected on its chordal plane, in square feet;

B_N = a constant used with symmetrical loading in determining reactions at any of the joints A to J;

C_n = a constant used in determining radial deflection at any of the points 1 to 16;

D_n = a constant used with symmetrical loading in determining radial deflection at any of the points 1 to 8;

d, d_r = radial deflection, in inches; positive when directed away from center of tunnel;

E = modulus of elasticity, in kips per square inch;

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[11] ASA—Z10a—1932.

[12] *Manual of Engineering Practice No. 22*, Am. Soc. C. E., 1941.

F = soil reaction, in kips; positive when directed toward center of tunnel;
H_n = depth of any point n below ground surface, in feet;
H_w = depth of water table below ground surface, in feet; considered negative when ground surface is submerged;
I = average moment of inertia of lining per foot of tunnel, in inches^4;
K = soil compression constant, in kips per square foot per inch;
M = moment reaction, in inch-kips; positive when stretching outer fibers of lining;
P = active load, in kips; the radial component P_r is positive when directed toward center of tunnel; the tangential component P_t is positive when directed upward;
Q = a constant = $\frac{E\,I}{r^3\,K\,A'}$;
r = radius of neutral circle of lining, in inches;
r_o = radius of outer surface of lining, in feet;
S = section modulus of lining per foot of tunnel, in inches^3;
s = arc measured on neutral circle from point A to point x,y (see Eqs. 7);
T = thrust reaction, in kips; positive when due to compression;
u = number of segments in which soil reactions are developed;
V = shear reaction, in kips; positive when it tends to produce clockwise rotation;
X,Y = rectangular coordinates of a given point; X is positive to the right of the vertical diameter of the lining; Y is positive above a tangent to the neutral circle in point A;
x,y = rectangular coordinates of a moving point;
α, β = angles measured from point A to points p and q on neutral circle;
δ = settlement, or vertical displacement of invert point A due to loading, in inches; positive when directed downward.

DISCUSSION

J. A. VAN DEN BROEK,[13] M. AM. SOC. C. E.—Mr. Bull's impressive paper deals with different subjects, three, to be exact—soil mechanics, elasticity, and theory of strength—all in relation to his main topic, that of lining stresses in shield-driven tunnels. The writer wishes to confine his comments to only one of these subjects, that of theory of strength, in Sections 4 to 12, inclusive.

For some fifty years or more designers have been familiar with the fact that an elastic curve, suitably interpreted, is an influence line. Much depends, however, on the "suitable interpretation." The use of elastic curves as influence lines for closed rings is new, at least to the writer, and in itself constitutes a worthy contribution to the theory of strength. In addition to presenting a difficult subject simply, lucidly, and in very readable language, Mr. Bull has done a vast amount of work, very little of which appears on the surface. All his computations, which must be voluminous, are on file in the Engineering Societies Library. This is eminently proper. Had they been published few would have checked them, and even then few would have been absolutely sure of the results. Mr. Bull offers tables which are of the greatest value. They are classic in the sense that they seem to provide the last word in regard to the analyses of circular rings of uniform cross section.

Mr. Bull has apparently neglected one thing, inclusion of which some of his readers would have appreciated. He has failed to call attention to some obvious partial checks of his tables which appear readily at hand. This may be understandable. Apparently Mr. Bull set himself the task of condensing enough material for a book within the space of only thirty pages. No doubt he applied numerous checks as he progressed. Possibly he regards some of them as self-evident.

The writer has applied Mr. Bull's tables to the solution of certain problems, the answers for which were known in advance. This, to him, serves to inspire confidence both in Mr. Bull's logic and in the correctness of the values given in the tables. Not that Mr. Bull's logic needs confirmation. On the contrary, it is as clear as a crystal and rings true. Especially, Section 10 seems a master stroke. If Mr. Bull will realize that the writer has full faith in his own logic, yet questions all his own computations, he will pardon him, he trusts, when he refuses to accept any tabular data on faith. Typographical errors sometimes occur, and, if tables are to stand the test of time, a great deal of independent checking appears justified.

Below are offered a few such simple checks:

1. The formula for the bending moment in a ring subject to a number of equal and evenly spaced radial forces P is

$$M_A = \frac{P\,r}{2}\left(\frac{1}{\sin\phi} - \frac{1}{\phi}\right) \quad \ldots\ldots\ldots\ldots (24)$$

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[13] Prof., Eng. Mechanics, Univ. of Michigan, Ann Arbor, Mich.

in which ϕ is one half the angle between the forces. If this formula is applied to Fig. 4, with $\phi = 11° 15'$, $M_A = 0.0164\,P\,r$. Adding Col. A_{Mr}, Table 1 (without including the two terms $-$ 0.00822—apparently Mr. Bull's solution for the same problem), the result is also $M = 0.0164\,P\,r$. To the writer, this constitutes one partial check. Mr. Bull's correction factor of 0.00822 seems sound and is very ingenious indeed.

2. The addition of all the entries in Col. A_{Mr}, Table 1, including the two correction factors $-$ 0.00822, gives zero for an answer. This was explained in the text and is somewhat of a check. When it is considered, however, that Table 5 was obtained from Table 1, that it involves a regrouping of the constants in Table 1 nine separate times, and that each time the columns for moment reactions caused by radial loading add up to zero, then it is hard to conceive how any typographical or other error may be hidden without being detected.

3. As a third independent check that the values in the tables are numerically correct, apply them to the evaluation of the moment at the bottom of a circular pipe of constants E and I, flowing full with water and supported radially by two reactions, each 33° 45′ removed from the vertical diameter and therefore corresponding to P_2 and P_{15} in Fig. 4. The so-called exact solution may be obtained by superimposing Case V and Case VIII[14] in the writer's book, "Elastic Energy Theory." This solution involves the assumptions of elasticity, constants E and I, and constant circular shape which are the same assumptions as those involved in Mr. Bull's theory.

Substituting $\phi = \pi$ in Case V, $M_A = +\,0.23873\,W\,r$, in which W is the weight of the water.

Substituting $\phi = \pi$ and $\phi_1 = \frac{3}{16}\pi = 0.58905$ radians in Case VIII,

$M = -\,0.23919\,W\,r$. Superposing these two values, $M_A = -\,0.00046\,W\,r$. This value checks the value shown[15] in the right half of Fig. 14, in which M_A is shown as substantially zero for the case of $\phi_1 = 33° 35'$.

In evaluating, by means of the tables, the moment at the bottom point of a ring (point A, Fig. 4) loaded with hydrostatic pressure and concentrated radial reactions R applied in the middle of Sections 2 and 15, Fig. 4, the value of the P-forces was computed as follows: The hydrostatic pressure, say, at the midpoint of segment 6, is $w\,r\,(1 + \cos\phi_6) = w\,r\,(1 + \cos 123° 45') = 0.44443\,w\,r$ (the relation between the unit weight of water, w, and the total weight of water, W, is $W = \pi\,r^2\,w$). Considering this pressure to act uniformly over segment 6, $P = 0.44443\,w\,r \times \frac{\pi\,r}{8} = \frac{0.44443\,W}{8}$. At segment 2 the reaction R is of opposite sense to that of the P-forces and equals $\frac{W}{2\cos 33° 45'} = \frac{W}{1.66294}$. The hydrostatic force P_2 acting on segment 2 would be $w\,r\,(1 + \cos 33° 45')$

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[14] "Elastic Energy Theory," by J. A. Van den Broek, 2d Ed., John Wiley & Sons, Inc., New York, N. Y., 1942, pp. 128–129.

[15] "Elastic Energy Theory," by J. A. Van den Broek, 2d Ed., John Wiley & Sons, Inc., New York, N. Y., 1942, p. 136, Fig. 57.

$\times \frac{\pi r}{8} = \frac{1.83147\ W}{8}$. The resultant force acting on segment 2 would be $R - P_2 = 0.372410\ W$. The other P-forces are computed similarly. Multiplying these values for P-forces, thus obtained, by their corresponding values listed in Col. A, Table 5, for moment reactions caused by radial components, and adding this entire column, with due regard for signs, $M_A = 0.000446\ W\,r$. This is in substantial agreement with the value of $0.00046\ W\,r$ obtained by the so-called exact analysis.

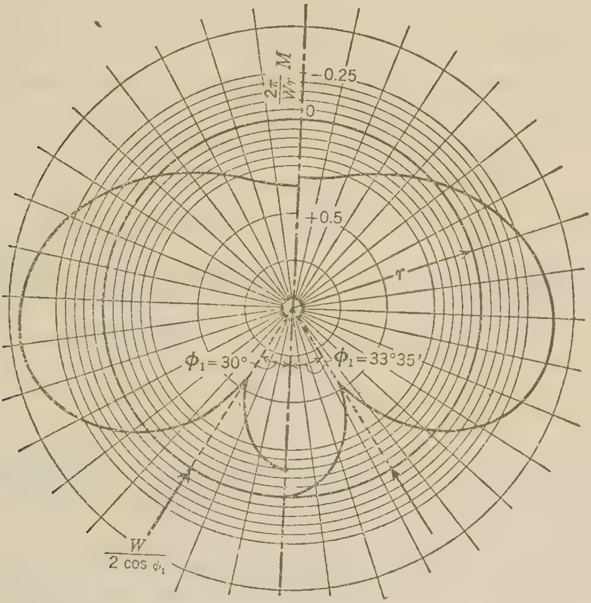

FIG. 14.—MOMENT DIAGRAMS, SYMMETRICAL ABOUT CENTER LINES, FOR CYLINDRICAL SHELLS FLOWING FULL WITH RADIAL SUPPORTS

4. If a circular pipe full of water, supported at its bottom, point A, is considered, force P_2 is included in the tabulation and the reaction R is disregarded. Table 5 then yields $M_A = 0.23877\ W\,r$, which compares with the value of $M_A = 0.23870\ W\,r$ obtained by the so-called exact analysis developed previously.

5. Fig. 15 represents a ring subject to sixteen uniformly distributed tangential forces P. The equation for the moment in this ring, at any point, is

$$M = \frac{T}{2\pi}(\phi - 2\sin\phi) \qquad \text{.....................(25)}$$

Thus, at point A the moment is

$$M_A = \frac{16\ P\,r}{2} = 8\ P\,r \qquad \text{.....................(26)}$$

Adding all the coefficients in Col. A_{Mt}, Table 1, provided they all are given the same sign, and multiplying by $P\,r$, the moment M_A is 7.99999 $P\,r$, which again is a beautiful check. As to signs, the writer knows no rule in regard to elastic curves interpreted as influence lines other than the one which states that, when force and deflection are of the same direction and sense, their product is positive. When they are of the same direction but opposite sense, their product is negative. It would seem that Mr. Bull followed some other rule which does not appear to be explained in the paper. Some more light on this very crucial matter would be much appreciated.

6. The shear stresses in an aeroplane fuselage act tangential to the surface (see Fig. 16) and are given by the expression $\frac{W \sin \phi}{\pi\, r\, t}$, in which t is the thickness of the ring. The equation for the bending moment at any point [16] is

$$M = \frac{W\,r}{2\,\pi}\left(1 - \phi \sin \phi - \frac{\cos \phi}{2}\right) \quad \ldots\ldots\ldots\ldots\ldots (27)$$

Therefore,

$$M_A = \frac{3\,W\,r}{4\,\pi} = 0.2387\,W\,r \quad \ldots\ldots\ldots\ldots\ldots\ldots (28)$$

The shear stress at the midpoint of segment 6, for example, is $\frac{W \sin}{\pi\, r\, t}$ (56° 15′) $= \frac{0.83147\,W}{\pi\, r\, t}$. Assuming this stress to act uniformly over the entire segment,

$$P_{6t} = \frac{0.83147\,W}{\pi\, r\, t} \times \frac{\pi\, r\, t}{8} = \frac{0.83147\,W}{8} \quad \ldots\ldots\ldots\ldots (29)$$

Computing the other P_t forces in similar manner, multiplying them by the coefficients for A in Table 5(*b*), and adding the resulting numbers, $M_A = 0.2387\,W\,r$.

These few checks have greatly increased the writer's confidence in the accuracy of Mr. Bull's tables. Possibly they may serve the same purposes for others.

Mr. Bull nominally crowds enough material for a book into a limited number of pages, dealing strictly with but one issue—tunnel stresses. To the writer he has done a great deal more than that. He has offered a sound bit of engineering philosophy. Section 8 is clever, and Section 10 is a stroke of genius. He has offered a very simple method which has applications far beyond tunnel analyses. Aeroplane fuselages, dirigible airships, submarines, large pipes, and various kinds of tanks are all closed rings. Last but not least, there is the large size sewer. The analysis of a sewer on the assumption that it may be regarded either as a fixed arch or as a hinged arch is so palpably beside the truth as to be worthless. Mr. Bull, it seems to the writer, offers a method which lends itself to any degree of precision. Mr. Bull has chosen to compute the mathematical values found in his tables to an accuracy of five significant places. All this was natural and necessary. The accuracy of five significant places is fictitious, of course. It was necessary, however, if Mr. Bull wanted to apply checks such as the foregoing and, furthermore, if he wanted some of his columns to add up exactly to zero. The procedure is also well justified in view of the fact that Mr. Bull's tables may be used several generations hence in their present form. However, to apply Mr. Bull's method of analysis to a sewer section with a variable moment of inertia, and other than perfect circular outline, would require about double the amount of work represented by Mr. Bull's complete paper in the Engineering Societies Library. Thus, although possible and accurate, it may not be feasible. Furthermore, such work could probably not be generalized as Mr. Bull has done for the circular ring of constants E and I and would have to be repeated for each new section considered in any one design problem. One way out of this dilemma, it seems to the writer, would be to obtain the elastic curve experimentally instead of mathematically. This would not result in an accuracy such as is implied in numbers of five significant places. The writer has already called such an accuracy fictitious, as probably Mr. Bull will agree. However, provided the technique in obtaining elastic curves is properly under control, the results would be more reliable—that is, philosophically more sound—than results obtained by conventional mathematical analyses such as Mr. Bull and the writer have used.

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[16] "Elastic Energy Theory," by J. A. Van den Broek, 2d Ed., John Wiley & Sons, Inc., New York, N. Y., 1942, Appendix II, p. 271, and Example 31, p. 123.

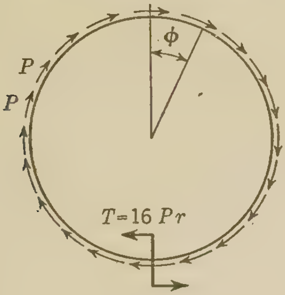

FIG. 15

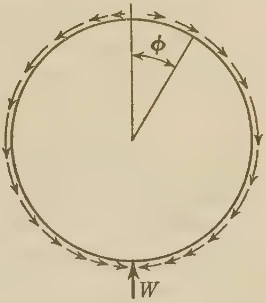

FIG. 16

The so-called exact analyses used in checks Nos. 1 to 6 were based on the conventional assumption that the principle of superposition applies; namely, that the rings maintain their original shape even in an analysis made for the purpose of obtaining changes in shape. Consider that the model from which the elastic curve is obtained is not perfectly elastic and thus violates elastic theory. How about applying an elastic theory to a structure as imperfectly elastic as reinforced concrete? In such a case, is the structure wrong or is the theory wrong? The philosophy of elastic curves as influence lines is sounder than any other mathematical theory of elasticity known to the writer because it is not restricted by assumptions of superposition and perfect elasticity as other theories of elasticity commonly are. Even the extremely small deformations, which often are stipulated, are philosophically not essential.

To some engineers who are not overly familiar with the science of soil mechanics, it may appear difficult to realize how tangential forces may be transmitted from earth to a smooth steel shell. However, in the shells of the aeroplane fuselages the shear forces are very real and are all directed tangentially. This leads the writer to surmise that Mr. Bull's carefully composed tables may well find wider application in other fields than merely the field of tunnel stresses for which they were originally computed.

The writer takes pleasure in expressing his admiration for, and his sense of indebtedness to, Mr. Bull's contribution. He expects to see Mr. Bull's theory and methods applied to a variety of problems, some of them possibly of greater importance, economically, than linings of shield-driven tunnels.

NATHAN D. BRODKIN,[17] ASSOC. M. AM. SOC. C. E.—The primary value of this paper is that it presents a rational method of determining the soil reactions for shield-driven tunnels and dispels much of the "fog" hitherto surrounding an analytical treatment of the subject. The high cost of modern, shield-driven, vehicular and rapid transit tunnels makes it of prime importance to attain the utmost economy of design consistent with reasonably assumed loading conditions. The author's proposal for reducing the weight of metal in the linings of such tunnels deserves the careful thought of tunnel builders.

An elastic tunnel ring subjected to a system of active loads will deform at all points in a definite manner; and, consistent with these deformations, soil reactions of definite magnitude and direction will appear in the surrounding material, dependent only on the elastic properties of the soil and the flexibility of the tunnel lining. The customary practice of assuming arbitrarily that the vertical soil reactions are of uniform intensity is basically erroneous and may lead (as indicated by the author) to uneconomical designs.

Selecting the invert joint as a point of reference, the author has derived expressions for A-constants (Eqs. 8), which, when multiplied by the external radial or tangential forces, yield the moment, thrust, and shear at that joint, provided the system is in static equilibrium. When referred to the developed periphery of the ring, the A-constants represent the ordinates to the influence lines for moment, thrust, and shear at the invert joint due to a unit load acting radially or tangentially at sixteen points on the circumference of a ring of unit radius.

For example, the constants A_{Mr} and A_{Mt}, when so plotted, yield the influence diagrams shown in Fig. 17. To locate the maxima and minima of the influence curves, first derivatives of the expressions in Eqs. 8 are equated to zero, yielding the angular values of these points. The influence ordinates corresponding to these values may then be found by resubstitution in Eqs. 8.

In Table 1 the A-constants have been determined for intervals of 22° 30′. Where force concentrations are applied anywhere between the sixteen tabulated points, a linear interpolation of the A-values might lead to inaccurate results. Influence diagrams plotted to a large scale give a truer indication of point-to-point variation, permit the intermediate values to be determined more accurately, and furnish a means for determining graphically the reactions for any system of loads in static equilibrium. Since any joint may be assumed as a point of origin similar to the invert joint, the same influence diagrams may be used for any joint by shifting the joint and force notation—in a manner similar to the analytical procedure described by the author in Section 11.

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[17] Civ. Engr., Board of Transportation, City of New York, New York, N. Y.

Eqs. 8 are basic for any ring-shaped structure, greatly simplifying the mathematical work of deriving equations for moment, thrust, and shear in tunnel rings due to any given loading, as an example will serve to illustrate.[18]

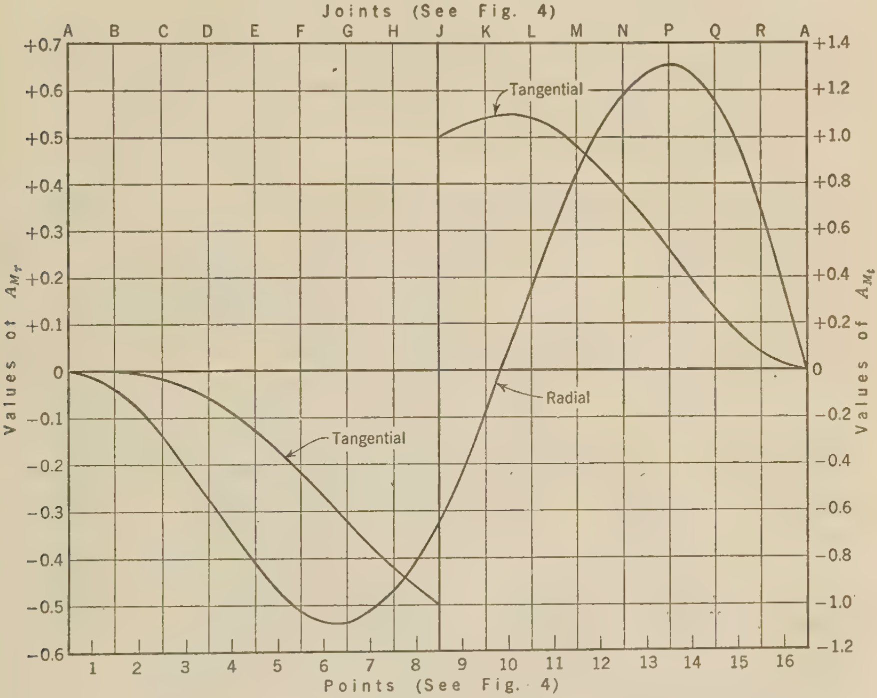

FIG. 17.—MOMENT INFLUENCE LINE FOR RADIAL FORCES AND TANGENTIAL FORCES

The ring, shown in Fig. 18, is subjected to two equal and opposite horizontal forces P concentrated at θ and $2\pi - \theta$. An equation must be written for moment at point A.

In Fig. 18, the horizontal forces P have been replaced by their radial and tangential components, $P \sin\theta$ and $P \cos\theta$, respectively. Substituting in Eq. 6c for A_{Mr} and A_{Mt} from Eqs. 8 and replacing α by θ and $2\pi - \theta$, respectively, the separate effects of radial and tangential forces may be written as follows: Moment produced by radial forces—

$$M'_A = -\frac{P r}{2\pi}\sin\theta\,(\theta\sin\theta + 1 - \cos\theta) - \frac{P r}{2\pi}\sin\theta\,[-(2\pi - \theta)\sin\theta + 1 - \cos\theta] = -\frac{P r}{\pi}[-(\pi - \theta)\sin^2\theta + \sin\theta - \sin\theta\cos\theta] \;..(30a)$$

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[18] "Stress Coefficients for Large Horizontal Pipes," by James M. Paris, *Engineering News-Record*, November 10, 1921, p. 768.

and moments produced by tangential forces—

$$M''_A = +\frac{P\,r}{2\,\pi}\cos\theta \times \theta\,(1 - \cos\theta) - \frac{P\,r}{2\,\pi}\cos\theta\,(2\,\pi - \theta)\,(1 - \cos\theta)$$

$$= -\frac{P\,r}{\pi}\left[(\pi - \theta)\cos\theta - (\pi - \theta)\cos^2\theta\right] \ldots\ldots\ldots\ldots (30b)$$

Adding the separate effects:

$$M_A = M'_A + M''_A = -\frac{P\,r}{\pi}\left[-(\pi - \theta)(\sin^2\theta + \cos^2\theta) + (\pi - \theta)\cos\theta\right.$$

$$\left. + \sin\theta - \sin\theta\cos\theta\right] = +\frac{P\,r}{\pi}(\pi - \theta - \sin\theta)(1 - \cos\theta) \ldots . (31)$$

The writer would like to call attention to the effect caused by a change in the value of the soil compression constant K. For this purpose the moments produced in the tunnel lining shown in Fig. 11(*a*) for successive values of K from $K = 3$ to $K = 15$ were computed using the author's method of analysis. The results are summarized in Table 15, and moment diagrams corresponding to the various values of K have been plotted against the developed ring circumference in Fig. 19. Values for $K = 12$ in Fig. 19 are taken from Table 14.

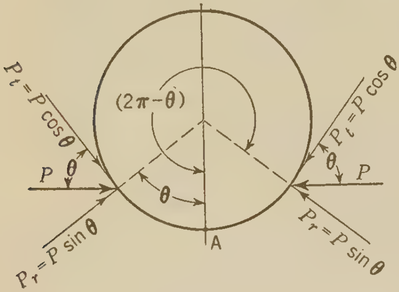

Fig. 18

Fig. 19 shows the progressive reduction in crown moment with an increase in the value of K. If this moment is plotted against the soil compression constant as in Fig. 20, it is seen that the variation in this reduction is nonlinear. Mr. Drucker,[3] although treating the problem of soil reactions from a different standpoint, obtains a curve of similar nature.

Assuming, as a basis, a sandy soil having a K-value of 12 to which the example given by the author applies, it is seen from Table 15 that, for decreases in K to 9, 6, and 3, the crown moment increases 9.5%, 23.7%, and 48.5%, respectively. Similarly, the increase in crown tensile fiber stress is 12.7%, 32.3%, and 65% for the respective values of K. When it is considered that a soil having a K-value as low as 3 might be somewhat plastic, and would exert a greater active lateral pressure than that assumed herein, the percentages for $K = 3$ are perhaps too high. Unless conclusive evidence is at hand as to the nature of the soil, it would appear prudent to assume a conservative value for K.

From the foregoing, it is evident that increasing the soil compression constant has the same effect as increasing the flexibility of the lining. The simple relationship existing between I and K may be shown in the following manner: Eq. 18 may be written in the form,

$$\Sigma(F\,D_1) - \Sigma(P\,D_1) = +\frac{E\,I}{r^3}\cos\alpha_1\,\delta - \frac{E\,I}{r^3\,K\,A'}F_1 \ldots\ldots (32a)$$

or

$$\frac{[\Sigma(F\,D_1) - \Sigma(P\,D_1)]\,r^3}{E} = I \cos \alpha_1\, \delta - \frac{I\,F_1}{K\,A'} \quad \text{.........} (32b)$$

Denoting by F_A the soil reaction on a segment extending from point 1 to point 16, Eq. 15 takes the form $F_A = \delta\,K\,A'$ or $\delta = \dfrac{F_A}{K\,A'}$. Substituting,

TABLE 15.—EFFECTS OF THE VARIATION OF THE SOIL COMPRESSION CONSTANT ON THE TUNNEL LINING

Description (see Fig. 4)	SOIL COMPRESSION CONSTANTS, K									
	3		6		9		12		15	
	F	M	F	M	F	M	F	M	F	M
(*a*) PHYSICAL PROPERTIES ($I = 136$ IN.^{4}; $S = 23.4$ IN.^{3}; AND $A = 27.6$ SQ. IN.)										
(*b*) SOIL REACTIONS, IN KIPS, AND MOMENTS, IN INCH-KIPS										
Joint A		−14.0		− 1.3		+ 2.4		+ 5.3		+ 6.0
Point 1	5.941		5.986		6.071		6.154		6.227	
Joint B		−22.3		−11.7		− 7.0		− 4.5		− 3.0
Point 2	5.205		5.179		5.164		5.154		5.149	
Joint C		−28.9		−25.8		−22.9		−20.5		−18.3
Point 3	3.937		3.964		3.920		3.863		3.807	
Joint D		−10.6		−10.7		−14.1		−15.3		−15.8
Point 4	2.239		2.548		2.680		2.747		2.783	
Joint E		+57.9		+39.5		+29.2		+22.7		+17.7
Point 5	....		0.499		0.871		1.154		1.382	
Joint F		+76.6		+64.6		+56.8		+51.2		+46.9
Joint G		+21.2		+23.3		+24.0		+24.4		+24.5
Joint H		−53.3		−42.1		−35.7		−31.7		−28.6
Joint J		−86.4		−72.0		−63.7		−58.2		−54.2
(*c*) SETTLEMENT δ, IN INCHES										
....	0.594		0.301		0.204		0.156		0.127	
(*d*) THRUST, IN KIPS										
Joint F	+37.4		+37.5		+37.6		+37.7		+37.8	
Joint J	+34.7		+35.1		+35.3		+35.5		+35.6	
(*e*) MAXIMUM STRESS AT JOINT F, IN KIPS PER SQUARE INCH										
σ_b	76.6/23.4 = −3.27		64.6/23.4 = −2.76		56.8/23.4 = −2.43		51.2/23.4 = −2.19		46.9/23.4 = −2.00	
σ_t	37.4/27.6 = −1.36		37.5/27.6 = −1.36		37.6/27.6 = −1.36		37.7/27.6 = −1.37		37.8/27.6 = −1.37	
Total	−4.63		−4.12		−3.79		−3.56		−3.37	
(*f*) MAXIMUM STRESS AT JOINT J, IN KIPS PER SQUARE INCH										
σ_b	86.4/23.4 = +3.69		72.0/23.4 = +3.08		63.7/23.4 = +2.72		58.2/23.4 = +2.49		54.2/23.4 = +2.32	
σ_t	34.7/27.6 = −1.26		35.1/27.6 = −1.27		35.3/27.6 = −1.28		35.5/27.6 = −1.29		35.6/27.6 = −1.29	
σ_a	19.1/27.6 = +0.69		19.1/27.6 = +0.69		19.1/27.6 = +0.69		19.1/27.6 = +0.69		19.1/27.6 = +0.69	
Total	+3.12		+2.50		+2.13		+1.89		+1.72	

therefore, for δ in Eq. 32*b*:

$$\frac{[\Sigma(F\,D_1) - \Sigma(P\,D_1)]\,r^3}{E} = I \cos \alpha_1 \frac{F_A}{K\,A'} - \frac{I\,F_1}{K\,A'}$$

$$= \frac{I}{K\,A'}\,(F_A \cos \alpha_1 - F_1) \quad \ldots\ldots\ldots\ldots\ldots\ldots (33a)$$

or

$$\frac{I}{K} = \frac{[\Sigma(F\,D_1) - \Sigma(P\,D_1)]\,r^3\,A'}{E\,(F_A \cos \alpha_1 - F_1)} \quad \ldots\ldots\ldots\ldots\ldots (33b)$$

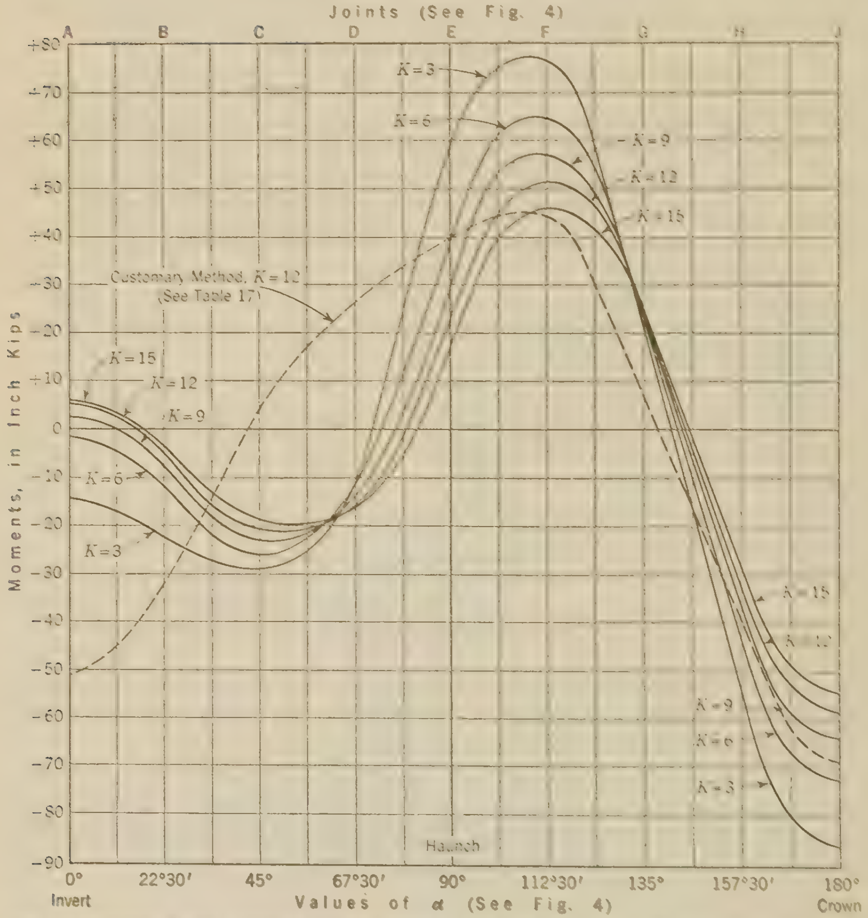

FIG. 19.—MOMENTS IN THE TUNNEL LINING RELATED TO THE SOIL COMPRESSSION CONSTANT (I = 136 IN.^{4})

Since there is a definite relationship between the soil reactions F_1, F_2, $\cdots$, F_5 for any given loading, F_A must likewise bear a fixed relationship to F_1.

From Eq. 33*b*, it is seen that, if the value of K is multiplied by a certain factor, it has the same effect as if the value of I were divided by the same factor.

Stated in other words, if the soil compression constant is increased, it has the same effect as a corresponding increase in the flexibility of the lining.

It is interesting to note in Table 15 that, for the various values of K for which analyses were made, the reactive forces F vary only slightly at similar points on the ring for different values of K. These small variations, however, are sufficient to cause large changes in the moments.

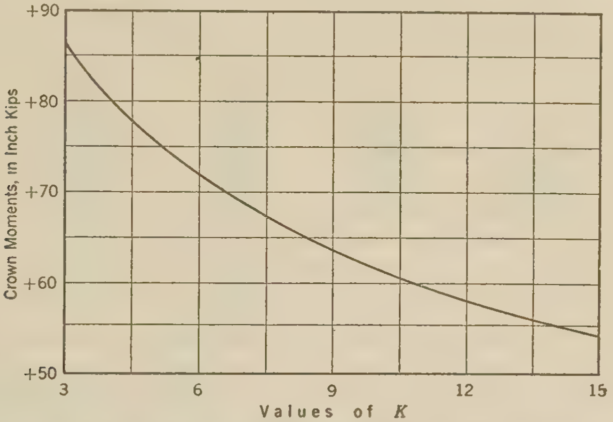

Fig. 20.—Crown Moment in the Tunnel Lining Related to the Soil Compression Constant (I = 136 In.^{4})

It is further noted in Table 15 that, as the soil compression constant is decreased, the effects of soil reaction diminish on the upper parts of the tunnel. For low values of K and a moment of inertia of 136 in.^{4}, the reactive forces are confined to the lower half of the tunnel periphery.

Contrary to the usual conception of a flattening at the invert of the ring with a predominance of downward loads, the results obtained using the author's method of analysis indicate that it is quite possible for the curvature to sharpen at the invert. The computed radial deformations of the lining due to combined active loads and soil reactions, corresponding to the assumed values of K, have been listed in Table 16. The final displacements are due to three factors—active loads, soil reactions, and the settlement. Actual deformation of the ring is a function of the first two factors only. A negative radial deformation means that the point has moved inward and a positive value indicates an outward movement with respect to the position of the ring before it is loaded. Table 16 shows quite clearly that, for high values of K, points 1 and 2 move inward with respect to joint A. As the soil compression constant is decreased, the soil, becoming more yielding, allows the lining to flatten at points near the invert under the effects of active loads, until a value of K is reached where all points from 1 to 6, inclusive, move outward. This is the case for an assumed value of K = 3. Thus, for high values of K it is possible to obtain moments at the invert causing tension on the outer fibers.

With an increase in the soil compression constant, the soil reactions tend to reduce the moments in the lower half of the lining, and, with a simultaneous reduction in the amount of the displacement at the horizontal diameter as shown in Table 16, they cause this part of the ring to act somewhat as an abutment for the upper half. The upper part of the ring then behaves much as an arch subject mainly to the effects of active loads.

TABLE 16.—DEFORMATION, IN INCHES, SHOWING THE EFFECT OF A VARIATION IN THE SOIL COMPRESSION CONSTANT ON THE RADIAL DEFORMATION OF THE LINING (I = 136 IN.^{4})

Point	$K = 3$	$K = 6$	$K = 9$	$K = 12$[a]	$K = 15$
1	+0.0002	−0.0014	−0.0020	−0.0022	−0.0023
2	+0.0168	+0.0041	−0.0011	−0.0033	−0.0045
3	+0.0562	+0.0273	+0.0147	+0.0080	+0.0041
4	+0.1036	+0.0663	+0.0477	+0.0369	+0.0297
5	+0.1092	+0.0830	+0.0682	+0.0588	+0.0517
6	+0.0275	+0.0325	+0.0331	+0.0327	+0.0320
7	−0.1084	−0.0687	−0.0428	−0.0381	−0.0308
8	−0.2114	−0.1496	−0.1187	−0.0994	−0.0867

[a] See line 9, Table 12.

For purposes of comparison, the writer has analyzed the tunnel chosen by the author as an example in Section 21 (for which I = 136 in.^{4}) by the customary method of analysis described in Section 3, assuming (1) the vertical soil reactions to be distributed uniformly over the horizontal diameter (Fig. 1, loading 12); and (2) an induced lateral passive pressure of an intensity dependent on the increase in the horizontal diameter under active loads. The effects of lateral passive pressure are in accordance with the pressure diagram shown in Fig. 1, loading 13a, and are based on the theory developed by Mr. Drucker,[3] using a value of K = 12. A lateral displacement of 0.51 in. at joint E (Fig. 4) due to active loads was applied to Mr. Drucker's formulas for the passive pressure and the resulting moments were determined utilizing the author's B-constants from Table 5. The results are shown in Table 17 and the corresponding moment diagram has been plotted in Fig. 19. The moments as determined by the author are also given in Table 17 for purposes of comparison. Comparing the two sets of values, the difference in results is apparent at once, especially in the lower parts of the lining. The customary analysis yields a crown moment 18% greater than that obtained by the author. The invert

TABLE 17.—COMPARISON OF MOMENTS, IN INCH-KIPS (K = 12 AND I = 136 IN.^{4})

Moments	JOINT (SEE FIG. 4)								
	A	B	C	D	E	F	G	H	J
Customary method—									
Caused by active forces...	−230.2	−166.5	− 7.6	+161.1	+242.2	+179.0	+ 4.5	−173.8	−247.9
Caused by passive forces..	+179.2	+134.6	+11.8	−134.8	−202.5	−134.8	+11.8	+134.6	+179.2
Net moment..........	− 51.0	− 31.9	+ 4.2	+ 26.3	+ 39.7	+ 44.2	+16.3	− 39.2	− 68.7
Author's method (net moment)	+ 5.3	− 4.5	−20.5	−15.3	+ 22.7	+ 51.2	+24.4	− 31.7	− 58.2

moment is about 74% of the crown moment obtained by the customary method, whereas the author's analysis shows a very small moment at this point amounting to about 9% of that at the crown. From the foregoing, it is evident that the customary assumptions relative to soil reactions result in moments in the lining varying considerably from those obtained by a more rational method.

M. A. Drucker,[19] Esq.—A fundamental method of complete stress analysis for tunnels is presented in this paper including the effects of both the active loading and the passive soil resistance, or, as the author designates the latter, "soil reactions." It should be of interest to ascertain the effects of the active loading separately in order to study the effect, on the final stresses, of considering the passive pressures. This would also show to what extent reliance is placed on the capability of the passive pressures to keep the stresses in the tunnel from exceeding the maximum permissible stress. In the author's analysis, however, the two sets of stresses are inseparably combined. Therefore, to form a clear idea of the effect of the proposed method, as compared with what may be termed the ordinary method, it would be necessary first to obtain the stresses due to the active loading according to general practice in tunnel design.

For the purpose of comparison, of course, the active loading and pressures used by the author will have to be followed. However, attention may be called to what appear to be several discrepancies. In considering the effect of the weight of the tunnel ring, 220 lb per sq ft is used. This weight consists of 110 lb for the weight of the cast-iron ring having a sectional area of 27.6 sq in. and of 110 lb for the weight of the concrete lining; but, as the concrete lining is poured in normal air, a considerable time after the cast-iron ring is erected, the weight of the cast iron alone should have been considered in conjunction with the effect of the internal air pressure used to keep out the water while erecting the cast-iron lining. In obtaining the moment of inertia and section modulus of the tunnel ring, the author correctly disregards the concrete lining.

Another point that may be noted is that the author apparently neglects the buoyant effect of the water on the soil when obtaining the lateral active pressures against the tunnel. Using Eq. 22*c* he takes a unit active horizontal pressure equal to one third the weight of overlying soil—100 lb per cu ft. This corresponds to an active pressure due to a dry soil having an angle of repose of 30°. If the buoyant effect of the water were considered, the unit active lateral earth pressure, of course, would be less than 33.3 lb. The variation in stress resulting from this difference in pressure will be considered subsequently.

Tunnel design analysis, for active loading, based on the ordinary method, resulted in a simplification by employing coefficients for bending moments and thrusts for the component parts of the total loading and pressures. These coefficients, for the stresses at point J, are given in Table 18, Cols. 2 and 4. In the problem considered by the author (see Section 21), the outside diameter of the tunnel is 17 ft 5 in. and the radius at the neutral axis is 8.5 ft. Loaded on the tunnel section are an earth cover of 36 ft and 27 ft of water. The net sectional area of the ring is 27.6 sq in., its gross moment of inertia is 136 in.^{4}, and the intrados section modulus is 23.4 in.^{3}, all per foot of tunnel length. Applying the coefficients of Table 18 to this case, the stress at point J is $20.57 \times \frac{12}{23.4} - \frac{28.36}{27.6} = 10.53 - 1.03 = 9.50$ kips per sq in. The author finds the final stress at this point to be + 1.89 kips per sq in., including + 0.69 kip per sq in. due to the air pressure, which is independent of the effect of the external loading. Therefore, the remaining stress, without air pressure, would be + 1.2 kips per sq in., so that, in accordance with the method presented in the paper, the tensile stress reduction due to the passive pressures may be said to be 9.5 − 1.2 or 8.3 kips per sq in.

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[19] Structural Civ. Engr.—in Chg., Board of Transportation, City of New York, New York, N. Y.

TABLE 18.—COMPUTATION OF STRESSES FOR POINT J (SEE FIG. 4) (BASED ON INDICATED COEFFICIENTS FOR BENDING MOMENT AND THRUST)

Loading No.[a]	Loading	MOMENTS, IN FOOT-POUNDS		THRUSTS, IN POUNDS	
		Formula	Moment	Formula	Thrust
	(1)	(2)	(3)	(4)	(5)
1	$w_1 = 36 \times 100 = 3,600$	$-0.25\, w_1 r^2$	− 65,000	0	0
2	$w_2 = 100$	$-0.027\, w_2 r^2$	− 1,650	$-0.021\, w_2 r^2$	− 152
3	$w_3 = 27 \times 25 = 675$	$-0.25\, w_3 r^2$	− 12,300	0	0
4	$w_4 = 25$	$-0.027\, w_4 r^2$	− 410	$-0.021\, w_4 r^2$	− 38
5	$w_5 = 220$	$-0.345\, w_5 r^2$	− 5,480	$-0.167\, w_5 r$	− 312
7	$w_7 = 63$	$-0.009\, w_7 r^2$	− 360	$-0.021\, w_7 r^2$	− 96
8	$w_8 = \frac{1}{3} \times 3,600 = 1,200$	$+0.25\, w_8 r^2$	+ 21,650	$+1.00\, w_8 r$	+ 10,200
9	$w_9 = 33.3$	$+0.208\, w_9 r^2$	+ 4,250	$+0.625\, w_9 r^2$	+ 1,510
10	$w_{10} = 27 \times 63 = 1,700$	$+0.25\, w_{10} r^2$	+ 30,700	$+1.00\, w_{10} r$	+ 14,400
11	$w_{11} = 63$	$+0.208\, w_{11} r^2$	+ 8,030	$+0.625\, w_{11} r^2$	+ 2,850
....	Summation	....	− 20,570	....	+ 28,362

[a] The loading numbers correspond to those shown in Fig. 1; loading 6 is omitted because its effect is included in loadings 1 to 5.

On the loading diagram, Fig. 1, the author represents by loading 13*a* the distribution of passive pressures as considered by the writer.[3] He states (see Section 3) that these pressures are "continuous functions of the depth." It would be more correct to describe this method as one in which the horizontal passive pressures are assumed proportional to the lateral deflections caused by the active loading. In conjunction with this method, the excess of the downward vertical loading over the upward water pressure is assumed to be uniformly distributed below the horizontal diameter in accordance with general practice. The author states several times that such assumed vertical pressure distribution is the cause of high calculated stresses. From the following computations it should become clear that such pressure distribution does not result in large computed stresses.

To determine the stress reduction due to the passive pressures in accordance with the writer's method, the horizontal deflection at the horizontal tunnel axis due to all the active loading and pressures will first be found from the resulting bending moment at point J—the assumption being that the deflections due to all the active loadings will be the same as the deflection due to a uniform vertical loading causing the same moment at this point.

For a uniform vertical loading of w lb per sq ft, $M_J = 0.25\, w\, r^2$. Table 18 gives M_J as 20,570 ft-lb. Therefore, $0.25\, w\, r^2 = 20{,}570$ and $w = \dfrac{20{,}570}{0.25\,(8.5)^2}$ $= 1{,}140$ lb per sq ft. For such uniform load the deflection d_1 at the horizontal diameter is

$$d_1 = \frac{144\, w\, r^4}{E\, I} \quad \text{(34)}$$

For the problem under consideration, $d_1 = \dfrac{144 \times 1{,}140 \times (8.5)^4}{12{,}000{,}000 \times 136} = 0.526$ in.

The effect of the passive pressures may be found in the following manner: $p = 1.28\, w = 1.28 \times 1{,}140 = 1{,}460$ lb. In this computation p is the maximum intensity of a pressure, varying as $\sin^3 \alpha_J$ (see Fig. 21), that would be capable of deflecting the tunnel back to its original position. This is an extreme pressure and is obtained only as a necessary intermediate step.

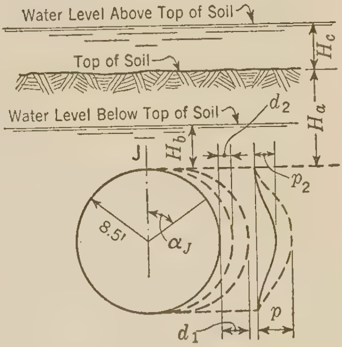

FIG. 21

With the foregoing information and the given value of 12,000 lb for the soil constant, K, the reduction in stress, σ_x, at point J, due to the passive pressures may be found from the following formula derived by the writer:

$$\sigma_x = \frac{12 \times 0.182\, K\, r^2\, p\, d_1}{\dfrac{I}{c}\,(K\, d_1 + p)} + \frac{0.59\, K\, r\, p\, d_1}{A\,(K\, d_1 + p)} \quad \text{(35)}$$

Substituting the proper values in Eq. 35,

$$\sigma_x = \frac{2.18 \times 12{,}000 \times (8.5)^2 \times 1{,}460 \times 0.526}{23.4\,(12{,}000 \times 0.526 + 1{,}460)}$$

$$+ \frac{0.59 \times 12{,}000 \times 8.5 \times 1{,}460 \times 0.526}{27.6\,(12{,}000 \times 0.526 + 1{,}460)} = 8.22 \text{ kips per sq in.}$$

As previously indicated, the author's method gives a stress at point J of 8.3 kips per sq in. less than that obtained by the ordinary method for active loading only. Comparing this value with the foregoing stress reduction of 8.22 kips per sq in. due to the passive pressures, the difference in the results obtained by the two methods is found to be only about 1%. It is gratifying, indeed, to find that the writer's method, which includes approximations for the sake of simplicity and which requires only simple slide-rule computations, gives results in such close agreement with the author's more extensive analysis.

With the reduction in stress of 8.22 kips per sq in., due to the developed passive pressures, the stress at point J would become 9.5 − 8.22 = 1.28 kips per sq in. without considering the stress due to air pressure, and 1.28 + 0.69 = 1.97 kips per sq in. considering the stress due to air pressure, as against the author's 1.2 kips per sq in. and 1.89 kips per sq in., respectively.

At times it may be more desirable to determine the value of the soil constant, K, necessary to keep the stress from exceeding the maximum allowable value, than to begin with an assumed value for this constant to find the resulting stress. In the present stage of knowledge of soil mechanics, it would be rather difficult to defend any definite K-value even if it were based on the best possible available information as to the nature of the soil through which the tunnel is to be constructed. Surely, one could not contend for a value of $K = 12$ while another argued for a value of, say, 10 or 8.

Knowing the stress at point J due to the active loading, computed as indicated in Table 18, and the maximum permissible tensile stress, the necessary value of the soil constant may be obtained from the following formula derived by the writer:

$$K = \frac{\sigma_x}{d_1 \left[\frac{2.18\, r^2}{\frac{I}{c}} + \frac{0.59\, r}{A} - \frac{\sigma_x}{p} \right]} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (36)$$

in which, as before, σ_x is the necessary stress reduction to be provided by the passive pressures.

For a maximum allowable tensile stress of 4 kips per sq in. for cast iron and taking into account the stress due to air pressure, σ_x, for the problem under discussion, would have to be $9.50 + 0.69 - 4.00 = 6.19$ kips per sq in. Substituting this and other known values in Eq. 36,

$$K = \frac{6.19}{0.526 \left(\frac{2.18\,(8.5)^2}{23.4} + \frac{0.59 \times 8.5}{27.6} - \frac{6{,}190}{1{,}460} \right)} = 4.37.$$

This result shows that, for the tunnel section considered, there would be no reason for urging a soil constant of $K = 12$ in checking as to whether the tunnel ring is of sufficient strength for a certain location, when one of only $K = 4.4$ would keep the stress from exceeding 4 kips per sq in.

In this discussion, reference has been made to the stresses at point J only, because it is the point of greatest tensile stress and, therefore, governs for cast iron. Theoretical computations, including those by the author, as well as tests on rings used for the London tubes,[20] show this to be the case. The loading during these tests (which were conducted under conditions approximating those affecting a tunnel after construction) was increased until cracks appeared in the rings—at the crown of the tunnel. Also, if the stress is known at one point, the stress at any other point can be determined readily, if for any unusual reason it should be desired to do so, by considering the bending and thrust at the point of known stress in conjunction with the known or assumed loading, including the passive pressures.

As previously noted, the author assumed the weight of the tunnel ring as 220 lb per sq ft whereas he should have used only 110 lb for the ring having a sectional area of 27.6 sq in. He also neglected to consider the buoyant effect of the water on the surrounding soil. Making the correction for the weight of the tunnel ring, the final stress at J for this ring was found to be 1.71 kips per sq in. as indicated in Table 19 for condition C. Correcting for the effect of buoyancy, assuming 40% voids, and taking the angle of repose of the soil as 30° gave the effective weight of the soil below water as 100 − 0.60 × 63 = 62.2 lb per cu ft and the unit active earth pressure as $62.2 \times \frac{1 - \sin 30°}{1 + \sin 30°} = 62.2 \times \frac{1 - 0.5}{1 + 0.5} = 20.7$ lb. For this reduced active earth pressure below water the stress at J was found to be 2.71 kips per sq in. for condition D.

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[20] "Tunnel Linings with Special Reference to a New Form of Reinforced Concrete Lining," by G. L. Groves, *Journal*, Institution of Civ. Engrs., London, England, March, 1943, p. 29.

TABLE 19.—COMPARISON OF STRESSES AT POINT J

Condition	Description	Stress[a] σ_J	
		$A = 27.6$	$A = 11.0$
A	Author's specified conditions (see Section 21) and method	1.89	3.18
B	Conditions A, with passive pressures based on the writer's method	1.97	3.70
C	Conditions B, with corrected tunnel weights[b]	1.71	2.95
D	Conditions C, taking buoyant effect of water into account, with angle of repose = 30°	2.71	4.90
E	Conditions D, with angle of repose = 45°	3.87	7.50
F	Conditions C, except that all active side earth pressure is disregarded	5.10	9.40

[a] Stresses are in kips per square inch, for section areas of 27.6 sq in. and 11.0 sq in., respectively. [b] For $A = 27.6$ sq in., the weight was corrected to 110 lb; and, for $A = 11.0$ sq in., the weight was corrected to 50 lb.

The angle of repose was taken to be 30° as assumed by the author. It may not be out of place, however, to give this assumption a little thought. An angle of repose of 30° is considered a reasonable basis for obtaining side pressures on structures, whose safety requires that the highest possible active pressure be assumed to provide sufficient resistance against it. In the case of tunnels, however, as the active side earth pressures reduce the tensile stress at the governing point, or at the crown of the tunnel, it becomes necessary to decide on an angle of repose that would give active earth pressures which may be counted on to exist at all times. This may be a rather difficult question to decide; but the angle of repose for maximum active earth pressure should not be used when the minimum pressure is desired. In this connection, it may be noted that Lieutenant-Commander Spangler[7] assumes no active side earth pressure whatever although he analyzes fills at culverts. Therefore, for such a soil as that to which the author assigns a value of $K = 12$ for the soil constant, which is compact and undisturbed, an angle of repose of 45° would not be too large. For such angle of repose, and taking buoyancy into account, the unit active earth pressure would be $62.2 \times \frac{1 - \sin 45°}{1 + \sin 45°} = 10.7$ lb. Based on this active earth pressure the stress at J was found to be 3.87 kips per sq in. for the heavier tunnel section as indicated for condition E, Table 19. Neglecting all active side earth pressure, σ_J was found to be + 5.10 kips per sq in. for condition F. As the angle of repose of the undisturbed soil would very likely be greater than 45°, the final stress, even for so large a soil constant as $K = 12$, would be between + 3.87 and + 5.10 kips per sq in.

The low stresses determined by Mr. Bull prompted him to state (see Section 27) "* * * that with a lining as actually designed more than 70% of the cast iron is wasted." That these low stresses are not inherent in Mr. Bull's method of analysis is demonstrated in Table 19 which shows that, for the same tunnel ring, the same low stresses were obtained by the writer's method under the conditions assumed by the author. The low stresses were due to two factors: (1) The author neglected the buoyant effect of the water on the soil when computing the active horizontal soil pressures; and (2) he assumed a high value for the soil constant without assuming a correspondingly large angle of repose. A more correct basis is that represented by condition E which includes the effects of buoyancy and assumes an angle of repose of 45°. For this condition σ_J was found to be 3.87 kips per sq in. which is close enough, for all practical purposes, to the limiting design stress of 4 kips per sq in. for cast iron.

To compare the writer's method with that of the author for lighter ring sections, a ring having a moment of inertia of 8.5 in.^4, or one sixteenth of that so far considered, was analyzed. For condition B, Table 19, the stress at J was found to be 3.70 kips per sq in. as against 3.18 kips per sq in. given by the author. The difference between the two stresses should not be surprising in view of the fact that, due to the active loading, the stress for this light ring was found to be 66.0 kips per sq in. so that the stress of 3.70 kips per sq in. represents a reduction in stress of 62.3 kips per sq in. due to the passive pressures. For the other conditions, after making corrections for the weight of the ring, the stresses increase somewhat more than for the heavier section (see Table 19).

For the tunnel ring having an area of 27.6 sq in. the stress at J, computed by the writer's method, was found to be 0.08 kip higher than that given by the author. For the ring having an area of 11.0 sq in. the stress obtained by the writer is about 0.5 kip higher than that given by the author. However, for the area of 17.4 sq in. the writer's value for the stress was found to be 0.9 kip less than that given by the author. In an effort to interpret these varying and irregular differences both sets of stresses were plotted in Fig. 22. As may be seen the stresses computed by the writer (identified by the letter D in Fig. 22) could be joined by a fairly smooth curve marked L_D whereas the author's stress points, due to the stress of 3.14 kips for the area of 17.4 sq in., cannot be so joined except for the stress points for 11.0 sq in. and less, through which a curve L_B could be passed. The irregularity of the other points may be explained by the fact that, as indicated in Table 14, for areas of 11.0 sq in. and less the stresses were obtained on the basis of having six ring segments in contact with the soil whereas for heavier sections only five segments on each side of the tunnel were considered pressing against the soil.

Assuming that all stress values given by the author are correct, in accordance with his method, the question arises as to what the stress might be for areas between 11.0 and 17.4 sq in. Between these areas there should be one for which, if six segments be assumed in contact with the soil, the pressure on the sixth segment would prove to be zero or nearly so. For such sectional area, which may occur about halfway between those for which the stresses are given, and which may be represented by line A–A in Fig. 22, five segments in contact with the soil could then be assumed. It would seem reasonable that by extending line L_B toward the right the stress for this area would be at point a or not far from it. Similarly, by extending line F toward the left, the stress for this area of ring would be represented by point b or b_1 if the stress line be assumed to curve upward as suggested by the other stress curves. Although the computed stress values for the area at section A–A may be somewhat different from those indicated in Fig. 22, it would appear that for the same sectional area and loading a consideration of six segments and then five segments would give stresses differing by more than a kip for an average of two stresses of about 3.0 kips. A break in stress values such as at line A–A suggests that the accuracy of computed stress values, by the author's method, for areas on either side of this line would also be questionable. If this be so it would indicate that an accurate method of tunnel design, including the effects of passive pressures, has yet to be developed.

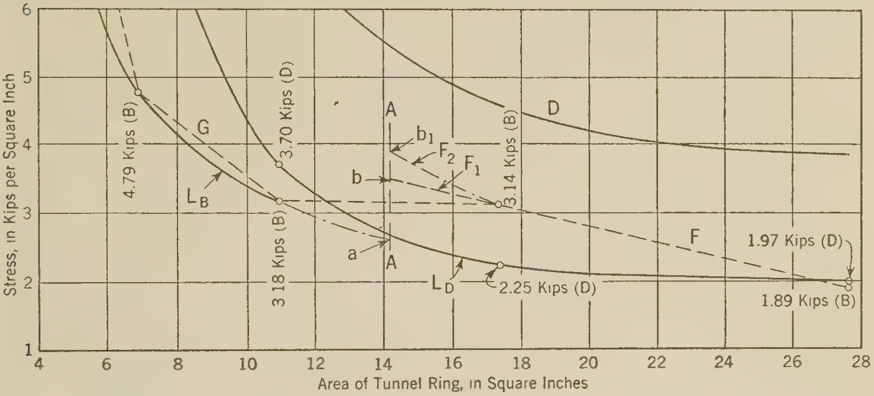

FIG. 22

It should also be noted that the stresses represented by lines L_B, L_D, and F are based on the author's loading assumptions; line D shows the stresses that could be expected based on the writer's method, as obtained for condition E, Table 19, for the assumed loading and active soil pressures which, in his opinion, should have been used.

Mr. Bull asserts that the large critical moments obtained by methods other than his occur because (see "Conclusions") "* * * the erroneous assumption is currently made that the vertical soil reactions are uniformly distributed over the horizontal diameter * * *." That this is not the cause of large moments may be noted from the close agreement between the stresses for the same conditions based on the writer's analysis (which assumes uniform pressure distribution below the horizontal diameter) and those obtained by the author. In fact, from the vertical components of reactions in Col. 11, Table 11, the pressure distribution obtained on the horizontal projections of the corresponding segments is greatest at the vertical axis of the tunnel and less toward the sides. Such pressure distribution causes greater moments at J than does uniform upward pressure for the same total load. Therefore, the relatively small stresses, obtained by either the author's method or the writer's method, are due to the horizontal passive pressures involved in both methods.

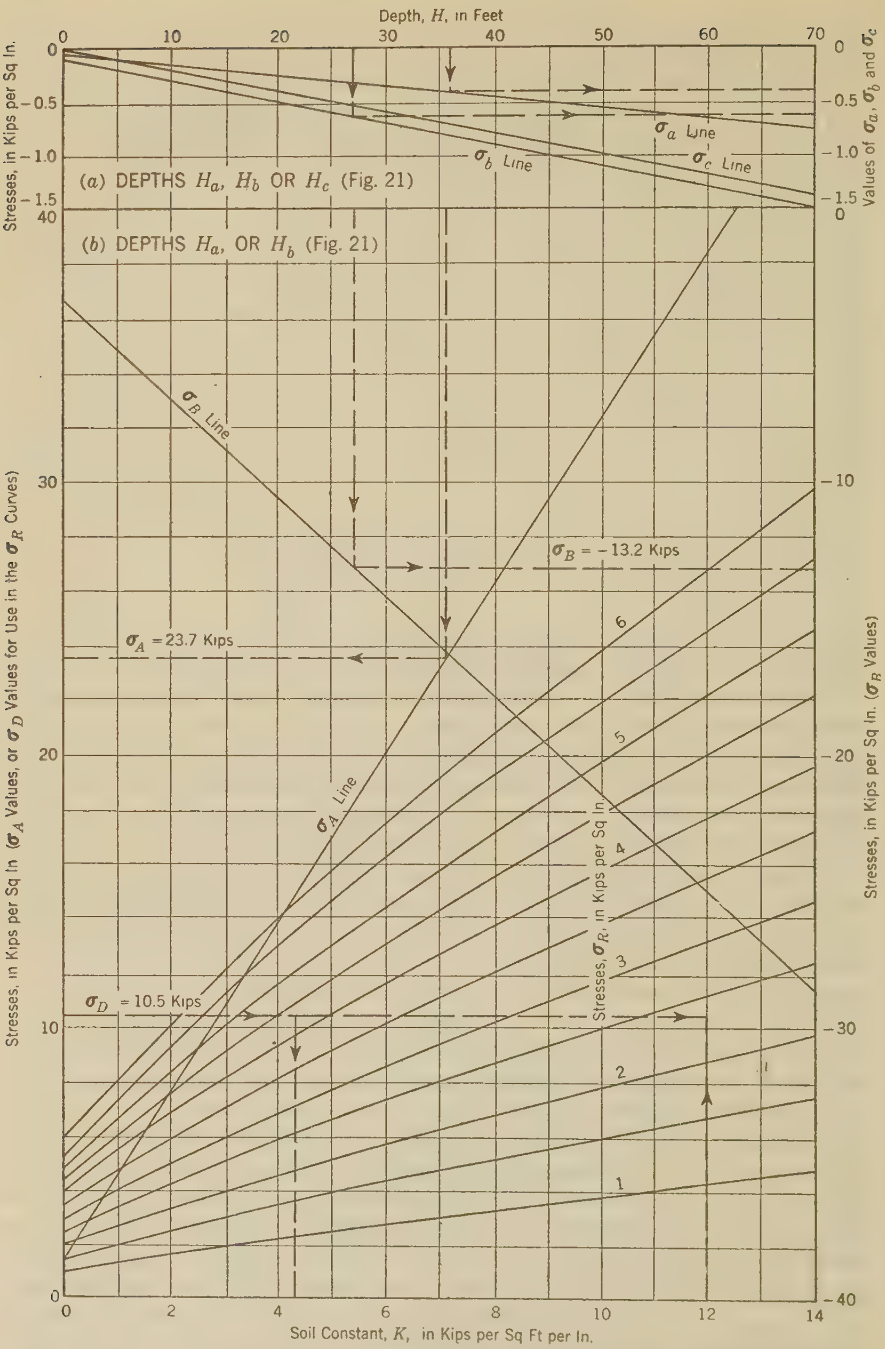

FIG. 23.—COMPREHENSIVE CHART FOR DETERMINING THE STRESS AT JOINT J FOR A TUNNEL RING HAVING AN AREA OF 27.6 SQ IN.

When a tunnel section is considered or finally decided upon for a project, it may be necessary to obtain the stresses for various conditions of head of water, height of earth cover, and nature of soil, for which different soil constant values may have to be assigned. To further simplify the process of obtaining the stress at point J for various conditions, a chart similar to Fig. 23 would be useful. This chart was prepared for the ring having a sectional area of 27.6 sq in. and, on the basis of the author's loading assumptions, without corrections.

The notation is defined as follows: σ_A = bending stress for active loading, including side earth pressure, cover = H_a (see Fig. 21); σ_B = bending stress caused by the water load, without considering buoyancy (to conform with the author's assumption), head of water = H_b; σ_D = bending stress due to all active loading ($\sigma_D = \sigma_A - \sigma_B$); σ_a = compressive stress due to the thrust caused by active earth pressure only; σ_b = compressive stress in addition to σ_a, due to the thrust caused by water, head of water = H_b; and σ_c = compressive stress in addition to σ_a, due to the thrust caused by water, head of water = H_c.

The σ_R-lines, based on Eq. 36, are used to obtain the final bending stresses after taking account of the bending stresses due to the passive pressures corresponding to the indicated soil constants. The other lines in Fig. 23 were obtained by selecting the proper moment and thrust coefficients from Table 18. From Fig. 23 the final stress can be obtained readily after assuming a soil constant, or the soil constant necessary to limit the stress to a certain amount may be determined.

Problem 1.—Under the conditions stipulated by Mr. Bull in Section 21 (see conditions B, Table 19), let H_a (Fig. 21) = 36 ft and H_b = 27 ft. Entering Fig. 23 at H = 36 ft, the vertical dropped to the σ_a-line in Fig. 23(*a*) intersects at $\sigma_a = -0.4$ kip per sq in.; and, extended vertically to the σ_A-line in Fig. 23(*b*), intersects at $\sigma_A = 23.7$ kips per sq in. Again entering Fig. 23 at H = 27 ft, the vertical dropped to the σ_b-line in Fig. 23(*a*) intersects at $\sigma_b = -0.6$ kip per sq in.; and, extended vertically to the σ_B-line in Fig. 23(*b*), intersects at $\sigma_B = -13.2$ kips per sq in. The bending stress σ_D due to all active loading is $23.7 - 13.2 = 10.5$ kips per sq in. In Fig. 23(*b*), the intersection of a horizontal from $\sigma_D = 10.5$ and a vertical from $K = 12$ gives $\sigma_R = 2.3$ kips per sq in. The total of $\sigma_a + \sigma_b = -0.4 + -0.6 = -1.0$ kip per sq in., and σ_J, without considering air pressure, is $2.3 - 1.0 = 1.3$ kips per sq in. Adding the air-pressure effect, + 0.69 kip per sq in., the final value of $\sigma_J = +1.3 + 0.69 = 1.99$ kips per sq in., which checks the stress given in Table 19. For a total stress σ_J limited to 4.0 kips per sq in., σ_R could be as great as $4.0 + 1.0 - 0.69 = 4.31$ kips per sq in. Entering Fig. 23(*b*) at $\sigma_D = 10.5$ kips per sq in. and drawing a line horizontally to $\sigma_R = 4.31$ yields (vertically) $K = 4.4$, which demonstrates how the necessary soil constant may be obtained so that the stress shall not exceed a certain value.

SIGVALD JOHANNESSON,[21] M. AM. SOC. C. E.—The use of cast iron for tunnel linings was conceived by Marc Isambard Brunel in 1818, but it was not until the Tower Subway under the Thames River in London, England, was built in 1869 that a tunnel was constructed with cast iron as lining material.

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[21] Chf., Highway Planning Bureau, State Highway Dept., Trenton, N. J.

Since then cast-iron tunnel lining has been used extensively for shield-driven tunnels, particularly in Great Britain and the United States.

Until comparatively recently cast-iron linings have been proportioned almost entirely by judgment, checked perhaps by static stress analysis. Since about 1920, however, various attempts have been made to develop a rational method for determining the stresses in tunnel linings, and the author is to be congratulated for having made a most important step in this development.

Based on his formulas the author finds that an excessive volume of metal is used in the present cast-iron linings. Although this may be true theoretically, it is still questionable whether cast-iron linings could be materially decreased in weight. The author gives two reasons why this may not be done: The first is the difficulty of casting large segments of slender shape; and the second is the need for a large cross section to resist the pressure of the shoving jacks. Also, cast-iron linings are cast in segments bolted together at the horizontal flanges, the deflections of which, under circumferential tension, confound the theoretical conclusion; and forces originating in the interior of the tunnels, such as an explosion or a collision, might cause the tunnel to collapse if the cast-iron lining were as thin as the theoretical stress computations would indicate to be safe. These conditions have been, and must be, considered in proportioning the lining as long as the present type of cast-iron lining is used.

However, with the development of the art of making, shaping, and fabricating steel, the present type of cast-iron lining should, and will, become obsolete, and it will be superseded by linings of steel (as suggested by the writer in 1922),[22] provided that the natural hesitancy of tunnel designers to break with precedent may be overcome. One of the greatest obstacles to change has been the uncertain knowledge of the stresses to which a tunnel lining may be subjected. The author's contribution to the determination of these stresses will greatly facilitate the solution of this problem.

JACOB FELD,[23] M. AM. SOC. C. E.—A complete mathematical analysis of stresses in tunnel sections, based upon assumed loadings and resulting deformations of a circular ring, has been developed in this paper. The method of attack and the resulting tabular data provide useful tools for a rigid solution of the design problem. However, the value of the results is dependent upon the validity of the assumptions as to the soil materials and as to the relationship between acting loads or resisting reactions and deformations of the ring.

All the forces acting on a shield-driven tunnel can be divided into two classes: (*a*) Those acting in planes normal to the axis of the tunnel in the form of radial forces and tangential forces; and (*b*) those acting parallel to the longitudinal axis. It must be kept in mind that the problem is a space, or three-dimensional, problem and that the longitudinal forces will affect the radial forces and vice versa.

The science of tunnel design is dependent upon the art of tunnel construction. It is a serious mistake to design a tunnel without considering the feasibility of construction and also the conditions under which construction can proceed safely.

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[22] "Shield and Compressed Air Tunneling," by B. H. M. Hewett and S. Johannesson, 1st Ed., McGraw-Hill Book Co., Inc., New York, N. Y., 1922.

[23] Cons. Engr., New York, N. Y.

In an indeterminate structure of the type described in this paper, the distribution of stresses is dependent on the strain or shape of the ring. If the construction of the ring is progressing in a perfectly fluid or liquid medium, the reactions are determinate. Even in this medium, the shape of the ring does not remain circular, although the balancing effect of the air pressure tends to eliminate the required vertical resultants acting outside the ring. If it were possible to vary the amount of air pressure at various heights within the ring, a perfect balance could be obtained and the only deformation would be a radial squeezing of the ring.

If the tunnel is being constructed in a solid material, of sufficient internal resistance strength to permit immediate transfer of loads beyond the limits of the excavation, there is no reaction on the ring, and, of course, in this case no ring is necessary. It should be remembered that, in some solid materials, tunnels bored without shields and without rings do encounter trouble from the lateral, and sometimes also from the upward, squeeze of the rock or earth into the previously excavated tunnel section.

The case considered by the author is that of a tunnel in a saturated soil material. In this case the author makes the basic assumptions that the active pressure acting at any point in the ring is independent of the amount of ring deformation, but that the magnitude of the soil reaction at any point in the ring is directly proportional to the amount of the outward deformation from the theoretical circular shape. Information on soil reactions does not warrant such an assumption. There is no doubt that there can be no soil reaction without a strain. The value of the active pressure at any point is dependent upon the deflection at that point. This phenomenon was described a number of times by the late James C. Meem, M. Am. Soc. C. E., and termed by him "arching action over the tunnel." Mr. Meem stressed that the effects of the compressed air would be (1) to dry the soil locally above the shield, and (2) to form localized arches of rigid soil which would relieve the vertical pressure. The distribution of active pressure depends upon the change in shape of the upper portion of the circular ring. In general, those portions which deflect least will carry the greater part of the total load; and, conversely, those portions which deflect most will carry less than their share of the total load.

The amount of soil reaction at any point depends upon the amount of displacement into the soil at that point. It is questionable whether the relationship is linear. Of course, even if the relationship is linear for small deflections, the materials encountered in shield-driven tunnels are usually quite plastic, and a redistribution of soil reactions will occur in time. Just what the actual conditions will be at any one time and just how long the conditions will remain static are items of necessary information, which are lacking.

However, it seems necessary to design a ring for the worst condition of each series in the various stages of construction. When the shield is shoved forward, there must be an annular ring of soil mixed with water and air surrounding the entire perimeter which is substantially fluid. Just how long the material will remain fluid depends upon the nature of the soil in which the tunnel is built. The less clay the material contains, the sooner the liquid state will disappear. However, if the soil has a sufficient clay content to form a colloidal gel as a result of the shield compression and movement, a liquid contact ring may exist for a considerable length of time. The tunnel design must take such a possible state into account.

Farther back from the shield, the soil will be in more intimate contact with the ring. If the soil has sufficient solid characteristics, the change in the shape of the top portion may relieve the center section of the expected load and will then transfer practically all the active pressures to the sides of the tunnel ring, and possibly a large portion also to the adjacent soil. This is the condition termed "arching" by Mr. Meem.

As the shield is pushed forward, with partial removal of the soil in the front of the shield, the shear strength of the material penetrated is overcome by compression against the compressed rings. If these rings are in intimate contact with the outside soil, a portion of such force is transformed into the soil in the form of a longitudinal compressive strain. Such a strain will disappear into the soil farther away from the ring. This action shows as a relative tendency of the ring sections to separate, as if the longitudinal length increased or expanded. The result is the formation of hair cracks or separations in the tunnel lining, having the appearance usually resulting from the shrinkage of the lining.

When some methods are developed for measuring the distribution of radial forces and tangential forces during the various stages of construction, as well as for measuring the actual values of such acting forces, the necessary data will be available for a proper use of the author's design procedure. Until such information is available, it seems very doubtful to the writer that a radical decrease in tunnel lining weight and thickness can be, or should be, proposed.

Leon Beskin,[24] Assoc. M. Am. Soc. C. E.—The "basic principle" on which this paper is based has been used since the beginning of the twentieth century and is probably due to H. F. B. Müller-Breslau. It was taught by Bertrand de Fontviolant for at least twenty years before publication of his book on strength of materials.[25] The author has expended considerable energy on the redevelopment of conventional formulas for rings, which can be found in textbooks on stress analysis. These equations appear in various forms, such as the charts devised by Joseph A. Wise,[26] M. Am. Soc. C. E., and reproduced by Alfred S. Niles, Assoc. M. Am. Soc. C. E., and Joseph S. Newell.[27] Lieutenant-Commander Wise's formulas are more convenient than the author's expressions, which can be derived from them. Whereas Commander Wise uses a supporting fictitious shell, which has shearing reactions defined by a conventional beam distribution, the author deals with diaphragm reactions, in such a manner that the shell reactions vanish if the loads applied to the ring are in equilibrium. Influence factors are obtained in both methods.

A great advantage of the fictitious shell over the diaphragm is that the shell does not introduce any dissymmetry with respect to the load. Thus, the tables need be computed only for one half of a ring. It is a simple matter to transform Commander Wise's formulas into those of the paper. For instance, the function A_{Mr} is obtained by the superposition of the following loads on the ring shown in Fig. 2; thus, for a load P_r at any point p:

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[24] Design Specialist, Consolidated Vultee Aircraft Corp., San Diego, Calif.

[25] "Resistance des Materiaux," by Bertrand de Fontviolant, J. B. Baillere & Fils, Paris, France, Vol. I, 1923, pp. 388 and 538; Vol. II, 1927, p. 628.

[26] "Analysis of Circular Rings for Monocoque Fuselages," by Joseph A. Wise, *Journal of the Aeronautical Sciences*, September, 1939, p. 460.

[27] "Airplane Structures," by Alfred S. Niles and Joseph S. Newell, John Wiley & Sons, Inc., New York, N. Y., 3d Ed., Vol. II, 1943, p. 272 et seq.

$$P'_r = P_r \cos \alpha \qquad (37a)$$

and

$$P'_t = P_r \sin \alpha \qquad (37b)$$

At the invert joint A,

$$M = P_r\, r \sin \alpha \qquad (38)$$

Defining W_1, W_4, and W_7 as the three coefficients of moment due to unit radial and tangential forces and unit moments—

$$A_{Mr} = -W_1(\alpha) + W_1(0) \cos \alpha + W_7(0) \sin \alpha - W_4(0) \sin \alpha \qquad (39)$$

The values found by Commander Wise are:

$$\left.\begin{aligned} 2\,\pi\, W_1(\alpha) &= 0.5 \cos \alpha - (\pi - \alpha) \sin \alpha + 1 \\ 2\,\pi\, W_4(\alpha) &= 1.5 \sin \alpha - (\pi - \alpha)(\cos \alpha - 1) \\ 2\,\pi\, W_7(\alpha) &= 2 \sin \alpha + \alpha - \pi \end{aligned}\right\} \qquad (40)$$

which, substituted into Eq. 39, yields the expression for A_{Mr} given by the author in Eqs. 8. The A-functions in this paper are no more informative than the charts presented by Commander Wise. In addition, the author's method is extremely sensitive to small computational inaccuracies, because the fictitious reaction is concentrated, instead of being distributed, as in the case of the Wise formulas. For instance, if applied forces are not balanced by a small couple, the total unbalance appears at the diaphragm location, so that the total error is repeated at each station, successively. When a shell is used as a fictitious support, the couple is absorbed by a constant shear flow and the error is distributed on the entire ring, instead of being concentrated at each section.

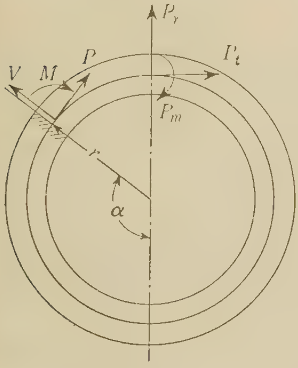

Fig. 24

The coefficients of moment can be established in a straightforward manner. The equations of equilibrium of a ring are:

$$\frac{dM}{d\alpha} = r\, V_p \qquad (41a)$$

$$\frac{dV_p}{d\alpha} = P \qquad (41b)$$

and

$$\frac{dP}{d\alpha} = -r\, V_s - V_p \qquad (41c)$$

in which M, P, and V_p represent the moment and the axial and shearing forces at a section p; and V_s represents the shear from the fictitious shell. This shear is defined by the conventional beam formulas applied to the shell:

$$V_s = -\left(\frac{P_m}{2\,\pi\,r^2} + \frac{P_t}{2\,\pi\,r}\right) + \frac{P_t}{\pi\,r}\cos\alpha - \frac{P_r}{\pi\,r}\sin\alpha \quad \text{........} (42)$$

in which P_t, P_r, and P_m are the concentrated tangential and radial loads and the concentrated moment applied at the point $\alpha = \pm\,\pi$ in the shell (Fig. 24). Elimination of P and V_p in Eqs. 41 leads to:

$$\frac{dM}{d\alpha} + \frac{d^3M}{d\alpha^3} + r^2\,V_s = 0 \quad \text{....................} (43)$$

Considering a tangential load P_t, the solution for M is the general antisymmetrical solution of Eq. 43, in which V_s has the value defined by Eq. 42, in which $P_r = P_m = 0$. For $\alpha > 0$:

$$M = r\,P_t\left[\,k\sin\alpha + \frac{\alpha}{2\,\pi}(1 + \cos\alpha)\,\right] \quad \text{.............} (44)$$

The constant k is defined by the condition of minimum strain energy which gives:

$$k = -\frac{\displaystyle\int_0^\pi \alpha\,(1 + \cos\alpha)\sin\alpha\,d\alpha}{\displaystyle 2\,\pi\int_0^\pi \sin^2\alpha\,d\alpha} = -\frac{3}{4\,\pi} \quad \text{............} (45)$$

Thus, the expression for M is:

$$M = r\,P_t\,\phi(\alpha) \quad \text{........................} (46a)$$

in which

$$\phi(\alpha) = \frac{1}{2\,\pi}\left[\,\alpha\,(1 + \cos\alpha) - \frac{3}{2}\sin\alpha\,\right] \quad \text{............} (46b)$$

Applying Eqs. 41, V_p and P are obtained by differentiation:

$$V_p = P_t\,\phi'(\alpha) \qquad \text{with} \qquad \phi'(\alpha) = \frac{1}{2\,\pi}\left(1 - \alpha\sin\alpha - \frac{\cos\alpha}{2}\right) \quad \text{..} (47a)$$

and

$$P = P_t\,\phi''(\alpha) \qquad \text{with} \qquad \phi''(\alpha) = -\frac{1}{2\,\pi}\left(\frac{\sin\alpha}{2} + \alpha\cos\alpha\right) \quad \text{..} (47b)$$

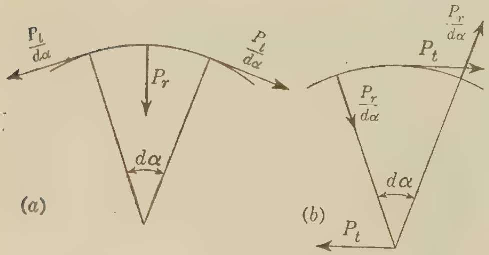

FIG. 25

Considering two loads $P_t = \pm\,P_r/d\alpha$, placed at a distance $r\,d\alpha$ from one another (Fig. 25(a)), which are equivalent to a load P_r, it appears that the coefficients for a unit load P_r are obtained by differentiating the coefficients corresponding to P_t; thus:

$$M = r\,P_r\,\phi'(\alpha) \quad \text{........................} (48a)$$

$$V_p = P_r\,\phi''(\alpha) \quad \text{........................} (48b)$$

and

$$P = P_r \phi'''(\alpha) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (48c)$$

Applied couples may be considered as the limit of two loads $\pm P_r/d\alpha$ and a load $P_t = P_r$ (Fig. 25(*b*)), since this combination of loads is equivalent to a couple, $P_m = P_r r$. Thus, combining Eqs. 46, 47, and 48:

$$M = P_m (\phi + \phi'') = P_m \psi \ldots\ldots\ldots\ldots\ldots\ldots (49a)$$

$$V_p = \frac{P_m \psi'}{r} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (49b)$$

and

$$P = \frac{P_m \psi''}{r} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (49c)$$

The *A*-constants in Table 1 are computed for very large increments (22° 30′); and the curves presented by Commander Wise do not always yield values of sufficient accuracy for practical use. Using the latter, the writer prepared[28] a table of ϕ-functions and ψ-functions for 5° increments of α, in which four ϕ-functions replace the author's six *A*-functions.

Adjusting for Polygonal Effect.—The adjustment proposed by the author in Section 8 can be avoided if the concentrated loads are replaced by uniformly distributed loads on an arc $2\,\epsilon$ defined by the angles $\alpha - \epsilon$, $\alpha + \epsilon$. For instance, the coefficient A_{M_r} would be (the angle ω varying between $\alpha - \epsilon$, $\alpha + \epsilon$):

$$A_{M_r} = -\frac{1}{2\,\pi \times 2\,\epsilon} \int_{\alpha-\epsilon}^{\alpha+\epsilon} (\omega \sin \omega + 1 - \cos \omega)\, d\omega$$

$$= -\frac{1}{2\,\pi} \left(\alpha \sin \alpha \frac{\sin \epsilon}{\epsilon} + 1 - \cos \alpha \cos \epsilon \right) \ldots\ldots\ldots\ldots (50)$$

which yields values that differ but slightly from the values given by the formulas in Eqs. 8.

Symmetrical Loading.—The *B*-constants (see Section 12) require an extremely extended table if the functions are to be computed by small increments. The time saved by these coefficients is of relatively minor importance in any case.

Deflections of a Tunnel Lining.—The expressions given by the author contain two angles, α and β, and their computation by sufficiently small increments is a laborious operation. When a shell is used instead of a diaphragm, the method is greatly simplified. Coefficients of deflection can be defined by expressions of the type:

$$k_{(ti)(rj)} = \int_0^{2\pi} m_{ti}\, m_{rj}\, d\alpha \ldots\ldots\ldots\ldots\ldots\ldots (51)$$

in which the subscripts denote values caused, respectively, by a unit load P_t at point I and by a unit load of P_r at point J—that is, the first of two subscripts shows the location of the load and the second shows the nature of the load. Replacing coefficients *m* by the functions computed by Eqs. 46 to 49 and integrating, coefficients *k* are obtained. The deflections can be computed by means of unit dummy loads. Using radial and tangential dummy loads, the radial and tangential components of the displacement of a point B (designated d_{rb} and d_{tb}, respectively) are

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[28] "General Circular Ring Analysis," by Leon Beskin, *Aircraft Engineering*, Vol. XVII, No. 195, May, 1945, p. 126.

$$d_{rb} = \frac{r^3}{E\,I}\left(\Sigma k_{(rb)(ti)}\, P_{ti} + \Sigma k_{(rb)(ri)}\right) P_{ri} \quad \text{(52a)}$$

and

$$d_{tb} = \frac{r^3}{E\,I}\left(\Sigma k_{(tb)(ti)}\, P_{ti} + \Sigma k_{(tb)(ri)}\right) P_{ri} \quad \text{(52b)}$$

The summations in Eqs. 52 are extended to all the loads on the ring. These displacements are relative and, to define absolute displacements, a point must be considered fixed. For instance, if the invert joint A is fixed, it is necessary to cancel its displacement d_{ra} which, under symmetrical loading, is expressed by Eq. 52*a* (except that subscript *b* becomes *a*). Adding $-d_{ra}$ to all the displacements, the final components of the displacement of point B (separated by the angle α from point A) are:

$$\bar{d}_{rb} = d_{rb} - d_{ra} \cos \alpha \quad \text{(53a)}$$

and

$$\bar{d}_{tb} = d_{tb} + d_{ra} \sin \alpha \quad \text{(53b)}$$

Coefficients k are functions only of the angle α between the point where the load is applied and the point where the deflection is calculated.

There are six different coefficients $k(\alpha)$:

$$\left.\begin{aligned}
k_{tt}(\alpha) &= \Gamma(\alpha) = \frac{1}{2\,\pi}\left[\cos\alpha\left(\frac{\pi^2}{6} - \frac{23}{8} - \frac{\pi\,\alpha}{2} + \frac{\alpha^2}{4}\right)\right.\\
&\qquad \left. + \frac{3}{2}\sin\alpha\,(\pi - \alpha) + \frac{\pi^2}{3} - 2 - \alpha\,\pi + \frac{\alpha^2}{2}\right]\\
k_{tr}(\alpha) &= \Gamma'(\alpha) = \frac{1}{2\,\pi}\left[(\pi - \alpha)(\cos\alpha - 1)\right.\\
&\qquad \left. - \sin\alpha\left(\frac{\pi^2}{6} - \frac{11}{8} - \frac{\pi\,\alpha}{2} + \frac{\alpha^2}{4}\right)\right]\\
k_{rr}(\alpha) &= -\Gamma''(\alpha) = \frac{1}{2\,\pi}\left[\frac{1}{2}(\pi - \alpha)\sin\alpha - 1\right.\\
&\qquad \left. + \cos\alpha\left(\frac{\pi^2}{6} - \frac{3}{8} - \frac{\pi\,\alpha}{2} + \frac{\alpha^2}{4}\right)\right]\\
k_{tm}(\alpha) &= \Delta(\alpha) = \Gamma(\alpha) + \Gamma''(\alpha) = \frac{1}{2\,\pi}\left[-\frac{5\cos\alpha}{2}\right.\\
&\qquad \left. + (\pi - \alpha)\sin\alpha + \frac{\pi^2}{3} - 1 - \pi\,\alpha + \frac{\alpha^2}{2}\right]\\
k_{rm}(\alpha) &= -\Delta'(\alpha) = \phi(\pi - \alpha)\\
&= \frac{1}{2\,\pi}\left[(\pi - \alpha)(1 - \cos\alpha) - \frac{3}{2}\sin\alpha\right]\\
k_{mm}(\alpha) &= \mathrm{E}(\alpha) = \Delta(\alpha) + \Delta''(\alpha)\\
&= \frac{1}{2\,\pi}\left[-2\cos\alpha + \frac{\pi^2}{3} - \alpha\,\pi + \frac{\alpha^2}{2}\right]
\end{aligned}\right\} \quad \text{(54)}$$

It is feasible then to prepare tables of deflection[28] coefficients from Eqs. 54.

Example.—To illustrate the foregoing argument the example given by Mr. Bull in Section 9 and Fig. 6 is solved by the Wise method. Since only four radial loads are applied, the adjustment for polygonal effect is not justified, and the comparison is made without adjustment (if necessary, the same adjustment can be made for both methods). The author's values are:

$$\begin{aligned}
&\text{Thrust } T = -1.08317 - 1.77015 &&= -2.85332\\
&\text{Shear } V = -4.01595 + 5.05450 &&= +1.03855\\
&\text{Moment } M = -3.14551 + 2.79700 - 0.01644 &&= -0.36495
\end{aligned}$$

The values resulting from the use of the Wise method have been computed with coefficients read from the original graphs, Table 20(*a*),[26] and with coefficients interpolated from a seven-place table (see Table 20(*b*)).

TABLE 20.—SOLUTION OF AUTHOR'S EXAMPLE, SECTION 9, BY THE WISE METHOD

FORCE		UNIT COEFFICIENTS			TOTAL VALUES		
Fig. 6	Kips	T	V	M	T	V	M
(*a*) USING COEFFICIENTS READ FROM GRAPHS							
P_{r1}	+2	+0.325	+0.445	+0.145	+0.650	+0.890	+0.290
P_{t3}	+5	+0.125	−0.080	−0.045	−0.625	−0.400	−0.225
P_{r7}	−4	−0.145	−0.120	+0.040	+0.580	+0.480	−0.160
P_{r9}	−3	−0.225	+0.045	+0.075	+0.675	−0.135	−0.225
P_{t13}	−5	+0.020	−0.100	−0.010	−0.100	+0.500	−0.050
P_{r15}	+1	+0.420	−0.295	−0.000	+0.420	−0.295	−0.000
Totals	...	....	....	....	+2.850	+1.040	−0.370
(*b*) USING COEFFICIENTS READ FROM A SEVEN-PLACE TABLE							
P_{r1}	+2	+0.3255754	+0.4442074	+0.1457640	+0.6511508	+0.8884148	+0.2915280
P_{t3}	+5	+0.1248151	−0.0824400	−0.0457225	+0.6240755	−0.4122000	−0.2286125
P_{r7}	−4	−0.1464023	−0.1221527	+0.0408998	+0.5856092	+0.4886108	−0.1635992
P_{r9}	−3	−0.2280333	+0.0461712	+0.0750035	+0.6840999	−0.1385136	−0.2250105
P_{t13}	−5	+0.0231697	−0.1011569	−0.0077650	−0.1158485	+0.5057845	−0.0388250
P_{r15}	+1	+0.4241794	−0.2935702	−0.0003671	+0.4241794	−0.2935702	−0.0003671
Totals	...	....	....	....	+2.8532663	+1.0385263	−0.3648863

The author's values and the Wise values are seen to be in very close agreement; and the charts originally prepared by Commander Wise are shown to be sufficient for all practical purposes when only a few loads are applied on the ring.

The differences in signs result from different sign conventions. The change in diameter (see Section 15) can also be determined by means of a table[28] giving the coefficients k_{tr} and k_{rr} (functions $\Gamma'(\alpha)$ and $\Gamma''(\alpha)$).

Table 21 indicates that the agreement between both methods is excellent, which simply means that the numerical tables have been computed without error.

Comparison of Table 21 with Table 7 shows that the author's method necessitates more numerical computations than the method derived from Commander Wise's concepts, because of the dissymmetry introduced by the diaphragm in Mr. Bull's method, which does not occur in the Wise method.

Another advantage resulting from Commander Wise's concepts appears from comparison of Tables 20 and 21 with the tables in the paper: The Wise coefficients have much smaller values than the author's coefficients, so that, for the same degree of final accuracy, less laborious computations are required.

TABLE 21.—COMPUTATION OF CHANGE IN DIAMETER

Forces	Angle	Kips	Function	Value	Product
P_{r7}, P_{r8}	90°	4 − 1 = 3	$\Gamma''(\alpha)$	+0.034160 × 2	+0.20496
P_{r1}, P_{r9}	45°	3 − 2 = 1	$\Gamma''(\alpha)$	+0.005140 − 0.00245	+0.00269
P_{t3}	45°	5	$\Gamma'(\alpha)$	+0.016560 + 0.018726	+0.17643
P_{t13}	90°	5	$\Gamma'(\alpha)$	0	0
Total coefficient	....	....	....	....	+0.38408
Author's value	....	....	....	+0.061890 + 0.322171	(+0.384061)

For instance, the moment coefficients are approximately seven times smaller, and the deflection coefficients are five times smaller. This disparity may mean the difference between using a slide rule or a computing machine in many practical problems.

Finally, it can be remarked that, if the C-constants were calculated by 5° increments, it would be necessary to calculate 5,184 entries instead of the 72 entries required for the $\Gamma'(\alpha)$-functions and $\Gamma''(\alpha)$-functions.[28] Also, the interpolation in a table with two independent variables is extremely laborious.

Effects of Displacement.—Mr. Bull states in Section 17 that

> "* * * when the shearing stress at any point reaches a certain limit a slip takes place between the surface and the soil particles, or between the soil particles themselves, causing the stress at that point to be relieved suddenly."

The slip between the lining and the soil particles or between soil particles actually occurs under constant shearing stress (provided the normal force is not changed), which is practically independent of the rapidity of displacement and is equal to the product of the normal pressure and the coefficient of friction.

Tangential Forces.—It is very improbable (see Section 18) that the region between the lining and the soil is free from tangential forces. It is true that an arch effect exists near the shield, but this arch effect is certainly helped by the shears developed between the arch and the soil in front of the shield as well as by the shears between the arch and the soil that have already settled at some distance behind the shield. Thus, the equilibrium cannot be examined as a plane equilibrium in sections perpendicular to the direction of the tunnel—at least when the effect of tunneling is being felt. It is highly probable that a stress redistribution occurs when the shield has moved to some distance from a given section, since the aforementioned effects disappear, and a plane equilibrium reappears. The best assumption is that of the uniform equilibrium defined by Rankine's formulas, which imply the existence of shear in all directions, except the vertical and the horizontal. A state of equilibrium without shear along the lining may be found, but that cannot be the condition assumed by the author because the normal component acting on the lining is not in equilibrium with the horizontal and vertical normal forces defined by Rankine's formulas. This is demonstrated by comparing Fig. 26(*a*), showing the loads on the lining, and Fig. 26(*b*), representing Mohr's circle of stress at any point of the lining (at an arbitrary scale). The maximum (vertical) and minimum (horizontal) pressures P_M and P_m are correlated by the formula,

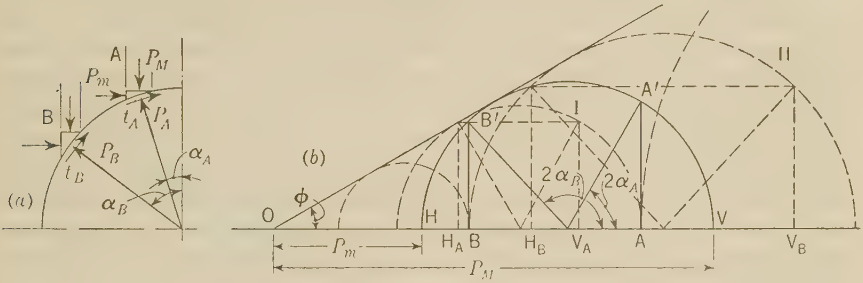

Fig. 26

$$P_m = P_M \tan^2\left(\frac{\pi}{4} - \frac{\phi}{2}\right) \qquad (55)$$

In this case ϕ is the angle of friction. The author has used the conventional values $P_m/P_M = 1/3$, or $\phi = 30°$.

At points A and B (Fig. 26(*a*)), the normal stresses are represented by OA and OB; the shears are represented by AA′ and BB′ (see Fig. 26(*b*)). With the author's assumption that $\overline{AA'} = \overline{BB'} = 0$, Mohr's circles must be modified, so that their diameters end at point A or B. There are two possible Mohr's circles at each point. Near the crown, circle I must be chosen for reason of continuity; and, near the skewback, circle II must be chosen. Thus, neglecting the dead weight between points A and B, it appears that the pressures at point B are greater than the pressures at point A. This seems to indicate that there is an arch effect—a conclusion which contradicts the fact that the pressure at the skewback is defined without arch effect. The latter (arch effect) shows that the pressure distribution does not correspond to a possible equilibrium. Another aspect of this contradiction results from the inspection of the values of the vertical and horizontal pressures at points A and B (represented by OV_A, OV_B, OH_A, and OH_B, respectively). It appears that the vertical pressure, when moving along the lining, first decreases (points V_A); then, at some unknown arbitrary point, it jumps to the values represented by points V_B, instead of remaining constant and equal to P_M (point V). Such a state of stress appears extremely improbable. In addition, the reactions of the lining must be in over-all equilibrium with the loads above the lining. More precisely, a vertical elemental prism limited to the surface must be in equilibrium with the lining reactions and the shears along its vertical faces. The author has not attempted to show that this equilibrium is consistent with the lining reactions he has chosen, although proof is essential to give the method a sound basis. Because of the aforementioned considerations, the method appears to be highly questionable. In addition, if considered approximate, the method is not conservative, since the vertical components of the lining reactions are smaller than the loads above the lining.

Soil Reactions Due to Settlement.—The author disregards tangential reactions, which must exist, since the displacement δ has the components $\delta \cos \alpha$ (normal) and $\delta \sin \alpha$ (tangential). For this reason, the directions of the forces F_1 to F_6 (Fig. 9) are incorrect, and the subsequent analysis is extremely questionable. Especially, the direction of forces F_5 and F_6 cannot be downward because the main component of displacement in the vicinity of these forces is tangential. Furthermore, the vertical component of the forces above the ring is determined by the dead weight—for example, if the direction of a force F_6 were downward, the soil reaction at point 7 would be decreased. On the basis of the foregoing reasoning, it is conclusive that Eq. 16 does not represent the soil reactions correctly.

It is impossible to obtain soil reactions on the segments near the crown since the ring settles as a whole and does not expand and since the section at its crown settles more than the other sections. The settled ring is obviously below the unsettled ring, and it is extremely probable that the maximum u (number of segments in which soil reactions are developed) is equal to 6, a fact confirmed by Table 14.

Specified Conditions.—Contrary to Eq. 22*c*, the horizontal pressure is equal to one third of the net vertical pressure (minus the hydrostatic pressure), plus the hydrostatic pressure:

$$p_H = \frac{0.100\, H_w + (0.125 - 0.063)\, (H_n - H_w)}{3} + 0.063\, (H_n - H_w)$$

$$= \frac{0.038\, H_n + 0.062\, H_w}{3} + 0.063\, (H_n - H_w) \dots\dots\dots\dots (56)$$

Eq. 56 defines a smaller pressure than does Eq. 22*c*, and the difference is especially important when H_w is small. Below the skewback, there is a vertical soil reaction, equal to one third of the net horizotnal pressure, thus, contrary to Eq. 22*a*:

$$p_v = \frac{0.038\, H_n + 0.062\, H_w}{9} + 0.063\, (H_n - H_w) \dots\dots\dots\dots (57)$$

Because of the foregoing, it is very dubious that the active forces are computed correctly. For example, if Eq. 22*c* is applied to a completely immersed material which has the density of water, a horizontal pressure greater than the hydrostatic pressure is indicated. This is an obviously incorrect result, since the material is in suspension and cannot produce an additional pressure.

If Eq. 22*a* is applied to a case in which the water level is just above the point under consideration, it indicates a pressure practically equal to zero, which is also incorrect.

Perhaps these formulas can be used when crude methods of analyses are used, but not when a refined analysis is sought.

Active Forces.—The calculations indicated by the author can be simplified somewhat. For example, the tangential forces due to the lining weight, which are sinusoidally distributed, can be replaced by radial forces following a cosine distribution of the same amplitude, without introducing any additional bending in the lining. This fact can be shown by considering the differential equation of equilibrium of the ring. The radial forces can be divided into a constant pressure, producing thrust exclusively, and a variable pressure, giving bending moments and thrust.

Conclusions.—In this paper, applied active forces, as well as reactive forces, have been estimated by methods that cannot be considered satisfactory, and errors may result in the values of the final moments. There is no doubt that the author is entirely correct when he states that conventional methods are highly conservative, but it seems nonconservative to use the author's results without an additional factor of safety that would consider the inaccuracies of the method. Happily, it seems probable that errors in the basic data do not have an excessive influence on the final moments. For instance, a variation of the moment of inertia (or of the soil compression constant) in the ratio of 4 to 1 generally produces a variation of bending moments in a ratio of less than 2 to 1.

Thus, it is probable that the additional factor of safety would be so small that it could be omitted if the active forces were recomputed more conservatively. Because of the lack of precision in the basic data, the author's method seems too intricate. However, the numerical results presented may be considered a basis for estimating the reduction of moments due to the flexibility of the lining. This estimate can be made by computing the dimensionless coefficient $\frac{K\,r^4}{E\,I}$ and by using the results obtained by the author for the same value of this coefficient. For that purpose, it would be very helpful if the author would add to his data the values of the bending moments in the case where the flexibility of the lining is disregarded.

Notation.—The author specifies that the tangential component P_t is positive when directed upward. This is contrary to the conventional positive direction, which is either clockwise (or opposite), with reference to the center of the ring. Also, this convention is not consistent with the use of the author's tables, because they thus become restricted to the invert joint. If the tables are to be applied to other points, the sign convention must be changed so that the tangential component is positive when directed away from the diaphragm. Thus, when the diaphragm is moved, the sign of a given tangential force must be changed according to the location of the diaphragm. Such cumbersome notation, which is a source of error, should be rejected, and the signs in the tables corrected in order to make them consistent with the stipulation that positive tangential forces are clockwise.

D. P. Krynine,[29] M. Am. Soc. C. E.—In describing the present state of the art of designing and building shield-driven tunnels (circular in cross section), the author of this excellent paper presents Fig. 1 showing assumed vertical and horizontal pressures on the lining. Judging from the measurements of pressure on the lining of the Lincoln Tunnel in New York City,[30,31] the shape of the vertical downward pressure diagram seems to be qualitatively correct, whereas the shapes of other diagrams shown in Fig. 1 should be considered temporarily as approximations only, until more field measurements and analyses along these lines are available. Purely theoretical attempts of establishing the shape of such diagrams are not sufficient.

Eq. 15 and the "Soil Compression Constant."—In a general case, the value of K, termed the "soil compression constant," depends on the size and the shape of the plane area abutting the soil and displaced in a normal direction. According to Eq. 15, a given unit load $\frac{F}{A'}$ abutting a given soil (constant K) always produces the same displacement of ρ inches. It is a well-known experimental fact, however, that from the two areas loaded with the same unit load, the larger area will settle more than the smaller one.

In the particular case of a shield-driven tunnel, the value of K changes also according to where the load acting on the soil is applied. The value of K will be smaller in the upward direction than in the downward direction, especially if the tunnel is shallow. The value of K in a sidewise direction, perhaps, would be between the two; also it may be smaller than the other two. The writer cannot even imagine how the value of K in the case of a shield-driven tunnel could be determined experimentally. It is impossible to organize such a test and the value of K must be guessed.

Eq. 15 means implicitly that the soil mass obeys Hooke's law which, in general, is not the case.

Reversible and Nonreversible Displacements in the Soil Mass.—If a rigid loaded plate is placed on the surface of a previously unloaded soil, a displacement occurs which consists of a reversible and a nonreversible part, the reversible (elastic) part being generally much smaller than the nonreversible. Assume that the experimental plate is first loaded with a certain unit load and thereupon a new unit load of the same magnitude is superimposed on the given acting load. The additional displacement thus caused will be much smaller than that in the original case. The reversible (elastic) part of the displacement may be the same in both cases, but the nonreversible one will decrease substantially. This is an additional reason why the concept of the "soil compression constant" should be used with extreme care.

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[29] Research Associate in Soil Mechanics, Dept. of Civ. Eng., Yale Univ., New Haven, Conn.

[30] "Lincoln Tunnel. The Field Measurements and Study of Stresses in Tunnel Lining," by G. M. Rapp and A. H. Baker, The Port of New York Authority, July, 1937 (lithoprinted).

[31] "Pressure on the Lining of Circular Tunnels in Plastic Soils," by D. P. Krynine, *Proceedings*, Am. Soc. C. E., May, 1944, p. 629.

Stressed Condition of the Lining During and After Shoving.—If the tunnel is driven in a plastic mass (organic silt or plastic clay), the vertical diameter of the lining first increases, apparently due to shoving. In the case of the Lincoln Tunnel, this increase was about 0.4% of the diameter of the lining. It is not clear from the paper in what way the stresses caused in the lining by the shoving process should be estimated or computed. Apparently these stresses should be fully or partly superimposed on those caused by the loads, as in Fig. 1.

Decrease in the Weight of the Lining.—Mr. Bull proposes using scaffolding in order to decrease the weight of the lining. Granted that the weight of the lining can be decreased in thát manner, two conditions still remain: (*a*) The maximum acceptable deflections of the lining should be controlled by the shearing strength of the overburden; and (*b*) a rather heavy lining in soils possessing small shearing resistance still may be required to balance the effect of buoyancy. To clarify the statement made under item (*a*), the well-known trap-door experiment should be recalled. The bottom of a box filled with sand is provided with a horizontal trap door at the center. Some downward movement of the trap door causes the vertical pressure on it to decrease, but excessive downward movement of that door causes an increase in pressure.

The writer disagrees with the statement at the end of the paper:

> "There seems to be no good reason, however, for making material that is needed only during the erection of a tunnel an integral part of it, something that is being done in no other type of structure."

On the contrary, many types of foundations are built with excessive material which is useful only during the erection of the structure. One of such examples is the use of pre-cast concrete piles which are designed to furnish excess resistance to both handling stresses and driving stresses. A caisson (for instance, a metallic compressed air caisson) is nothing more than an excessively heavy shell of the structure, useful only during construction, but which, however, remains buried in the earth forever.

A. A. EREMIN,[32] ASSOC. M. AM. SOC. C. E.—By considering the elastic properties of soil the author has developed an interesting method of computing stresses in tunnel linings. However, the conclusion that linings designed by current methods are too heavy seems premature. Actually, the tunnel lining carries a more complicated loading than that described by Mr. Bull. The final shape of a tunnel lining often is established after pressure grouting was applied. For example, the Posey Tube[33] in Oakland, Calif., was moved laterally by the forces transmitted through the backfill. The final stresses in the tunnel lining are also influenced by the loading on the roadway slab and by the forces transmitted through the tie rods. These influences were not mentioned by the author.

The coefficient of compressibility, K, has been used widely in the past. Recent experiments, however, have revealed that this coefficient is variable.

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[32] Associate Bridge Engr., Bridge Dept., Div. of Highways, State Dept. of Public Works, Sacramento, Calif.

[33] *Transactions*, Am. Soc. C. E., Vol. 109 (1944), p. 747.

Therefore, it cannot be used in computing stresses by the principle of superposition of stresses and deformation.

The design loading to be adopted is often influenced by the method of erection of structure. In Section 29, Mr. Bull has stated that no other type of structure requires that erection material be made an integral part, as in tunnel lining. Unfortunately, extra material must be provided for erection stresses in several other types. Pre-cast tunnel tubes are proportioned for the maximum stresses that may be developed during the launching and while shipping the tube sections to their final location. The cantilever method of erecting continuous bridge trusses requires that various members and connections, that are not fully stressed in the completed structure, be reinforced to resist erection stresses. Extra heavy, reinforcing armature sometimes is provided in reinforced concrete arches to support the centering.

Mr. Bull believes it is erroneous to assume that the vertical soil reactions are uniformly distributed along a horizontal diameter of tube (see Section 19). The accuracy of this assumption varies with the ratio of the horizontal soil pressure to the vertical soil pressure. When this ratio is approaching unity, the error of the adopted assumption is negligible. The assumption of uniform distribution of the vertical reactions simplifies the computation of stresses in the tunnel lining.[34] Of course, deviation from this assumption is quite common when local conditions require it—for instance, when the tube is supported on a cushion or on piers. When the ratio of horizontal soil pressure to vertical soil pressure is less than unity, a simplified graphical expression of soil pressure exerted by a prism is generally adopted.[35] Such a diagrammatic expression of loading gives a convenient criterion for the stresses in the lining. The diagrammatic loading is easy to visualize. It can be compared with the actual loading encountered during erection and, later, when the structure is carrying the actual superimposed load.

G. N. Alexander,[36] Esq.—As there is no generally accepted method for designing tunnel linings, the author's analysis is very welcome in providing a rational method for determining what are generally termed passive pressures. Although there may be good grounds for maintaining that successful precedent, where such exists, is the best guide to tunnel lining design, progress depends in part on the promulgation of such theories. However, until knowledge of active pressures advances, elaborate theories regarding passive pressures tend to be academic.

The author rightly criticizes the "classical" theory[2] given by S. Johannesson and the late B. H. M. Hewett, Members, Am. Soc. C. E., because it does not consider that the deflection of the lining is sufficient to give rise to stresses. This theory depended on an assumption that, provided the total passive pressure determined from Rankine's theory is not exceeded, bending stresses due to earth pressure need not be considered. This may lead to unsafe rather than conservative designs. The late Walter C. Parmley,[37] M. Am. Soc. C. E., directed attention to the weakness of their contentions and made an analysis based on an assumed elliptical deflection curve with passive pressures depending on the deflections in the lateral quadrants. Such methods require what is variously termed the "soil constant" (M. A. Drucker), "soil compression conconstant" (Anders Bull, M. Am. Soc. C. E.), and "coefficient of horizontal soil reaction" (Karl Terzaghi, M. Am. Soc. C. E.). Unfortunately, experimental evidence concerning the value of the "soil constant" K is extremely meager and any designer using it must do some "guestimating." It therefore seems desirable to derive an expression to show rapidly the effect of varying the factors that influence passive pressures. Some consideration must also be given to the assumptions made.

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[34] *Transactions*, Am. Soc. C. E., Vol. 103 (1938), p. 271.

[35] *Ibid.*, Vol. 95 (1931), p. 376.

[36] Designing Engr., Melbourne and Metropolitan Board of Works, Melbourne, Victoria, Australia.

[37] Discussion by Walter C. Parmley of "Notes on Tunnel Lining for Soft Ground," by S. Johannesson and B. H. M. Hewett, *Transactions*, Am. Soc. C. E., Vol. LXXXIII (1919–1920), p. 1880.

The author criticizes Mr. Drucker[3] (see Section 3) and other exponents of theories (such as M. G. Spangler,[10] M. Am. Soc. C. E., in Section 28) who use a uniform vertical load to determine passive pressures. For analytical investigations loading may be expressed by using rectangular or polar coordinates. It would appear that the loading assumptions made may depend on the system chosen—a scarcely defensible procedure. If rectangular coordinates are used it is generally assumed that the horizontal pressure is equal to k times vertical pressure, which may be termed case I. If such a loading is expressed in polar coordinates tangential tractions are involved, but the author, by deductive processes, disposes of these which may be termed case II. Fig. 27 illustrates the two systems and the relative magnitude of the tangential tractions, at θ. Of the radial pressure OC, only the variable part EC is effective in producing distortion and hence effective passive pressures. As $\overline{CB} = \overline{CE} \cot \theta$ it cannot be neglected on the score of the relative magnitude.

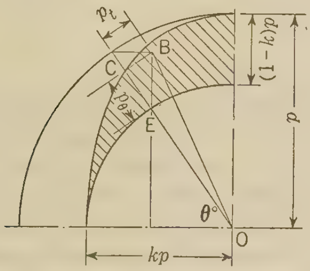

FIG. 27.—ACTIVE PRESSURE

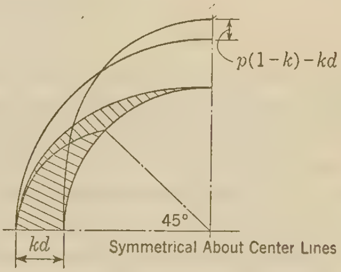

FIG. 28.—PASSIVE PRESSURE

To determine the relative importance of radial and tangential loadings it may be shown that, for a constant vertical pressure of p_v, the value of M_θ is given for case I by

$$M_\theta = \frac{p_v\, r^2\, (1 - k) \cos 2\,\theta}{4} \qquad (58)$$

For case II, for a radial pressure only, given by

$$p_\theta = p\,(1 - k) \sin^2 \theta \qquad (59)$$

the corresponding moment can be determined most readily by considering a pair of equal but opposite diametrical loads. Thus, if $P_\theta = p_\theta \, r \, d\theta$, M'_ϕ due to P_θ, with $\theta > \phi$, is given by[38]

$$M'_\phi = \frac{P_\theta \, r}{2} \left(\sin \overline{\theta - \phi} - \frac{2}{\pi} \right) \dots\dots\dots\dots\dots\dots (60)$$

Hence the bending moment at ϕ due to distributed load is given by

$$M_\phi = \frac{p \, (1 - k) \, r^2}{2} \int_{\theta=\phi}^{\theta=\pi+\phi} \sin^2 \theta \left(\sin \overline{\theta - \phi} \, \frac{2}{\pi} \right) d\theta = \frac{p \, (1 - k) \, r^2 \cos 2 \phi}{6} \quad (61)$$

Thus, the tangential tractions produce half as much moment as the radial loads and clearly should not be neglected unless more evidence can be produced to show that such tangential loadings as implied by the use of the relation $p_h = k \, p_v$ do not exist. To determine the deflections the differential curve of the ring may be used,[39] thus:

$$\frac{d^2 d}{ds^2} + \frac{d}{r^2} = - \frac{M}{E \, I} \dots\dots\dots\dots\dots\dots\dots\dots (62)$$

Substituting the value for M given by Eq. 61 and solving, the deflection at any point is

$$d_\phi = \frac{p \, r^4 \, (1 - k) \cos 2 \phi}{18 \, E \, I} \dots\dots\dots\dots\dots\dots\dots (63)$$

That is, for case *I* the value is increased by 50%.

Passive Pressures.—Deflections and moments calculated on active pressures alone will not generally be realized due to the effect of passive pressures. Various assumptions are made as to how, where, and when these passive pressures act. The use of a soil constant K seems necessary but its use involves crude approximations, and consolidation effects may mask any elastic effect. Grouting also may have considerable significance. The author assumes that passive pressures are directly proportional to the radial deflections outside the settled position of the undeflected ring. The writer also assumes that the active pressure is unchanged at the crown but the movement of the tunnel in a vertical direction may be disregarded in determining passive pressures due to deflection of the lining. In this case the active pressure $p \, k$ (Fig. 27) of the horizontal diameter is increased by a passive pressure, $K \, d$ (Fig. 28), in which d is the net maximum radial deflection. For the analytical determination of moments this passive pressure $K \, d$ can be subtracted over the entire circumference from the net active pressure causing distortion used previously, leaving a pressure difference between the crown and sides of $p \, (1 - k) - K \, d$ (Fig. 28). At first it may appear safest to consider that passive pressures are limited to the two lateral quadrants. However, such pressures can be maintained only by the deflection of the lining due to the excess of vertical over horizontal active pressures. This requires continuous contact between earth and lining and it is presumed that the final pressure curve is continuous. Therefore, the writer assumes a final pressure distribution curve of the same form as that for active pressure. The passive pressures additional to those which may undoubtedly be assumed to occur over the lateral quadrants will cause no significant distortion to the lining, as each of the four quadrants is equally loaded with these additional pressures (Fig. 28).

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[38] "Strength of Materials," by S. Timoshenko, D. Van Nostrand Co., Inc., New York, N. Y., 2d Ed., 1940–1941, Pt. II, p. 82.

[39] *Ibid.*, p. 103.

Hence, the net deflection d due to active and passive pressures, assuming the net loading is due to radial loads only, is as follows:

$$d = [p\,(1 - k) - K\,d]\,\frac{r^4}{18\,E\,I} \quad \ldots\ldots\ldots\ldots\ldots (64a)$$

Therefore,

$$d = \frac{p\,(1 - k)\,r^4}{18\,E\,I}\,\frac{1}{1 + \frac{K\,r^4}{18\,E\,I}} = d_a \times \frac{1}{1 + n} \quad \ldots\ldots\ldots\ldots (64b)$$

In Eq. 64*b*, $n = \frac{K\,r^4}{18\,E\,I}$ and the maximum active deflection is $d_a = \frac{p\,(1 - k)\,r^4}{18\,E\,I}$;

or

$$d = \frac{p\,(1 - k)\,n}{K\,(1 + n)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (65)$$

Thus the dimensionless term $\frac{K\,r^4}{18\,E\,I}$ may be evaluated to give the effect of the passive pressures. The factors in this term deserve consideration. Clearly the most significant is r and large diameter tunnels may make much more use of passive pressures unless tie rods and roadways reduce the deflection. The moment of inertia I may have only a fraction of its theoretical value due to flexibility at the joints, especially in a cast-iron tunnel where longitudinal joints are not staggered. This flexibility (if strength is not impaired) therefore may be an advantage. It would be advisable to determine the value of $E\,I$ for a ring by experiment if Eq. 65 were to be used.

There appear to be good grounds for the author's contention concerning the advantages of steel, especially as the governing stresses in the conventional cast-iron lining are tensile. The provision for jacking stresses and the apparent unwillingness of designers to depart from customary methods would seem to be the major reason for the continued use of cast iron.

Therefore, it appears that, although it is desirable that more rational methods be applied to tunnel design than heretofore, much more experimental evidence is necessary to enable the variables to be determined before such an elaborate theory as that proposed by the author is necessary. The writer suggests that for all cases, except large shallow tunnels, the ratio of passive to active pressures, deflections, and moments is given approximately by the ratio, $\frac{K r^4}{K r^4 + 18 E I}$.

The author is to be congratulated on his extensive mathematical work and it is to be hoped that his paper will accelerate the accumulation and dissemination of data on soil pressure on tunnels to enable a theory to be developed which can be substantiated by experimental data.

LOUIS BALOG,[40] ESQ.—In the design of underground cylindrical structures the analysis can often be limited to a ring of unit length under the action of transverse forces. The method of ring analysis applied to the solution of this problem by Mr. Bull was developed for structures of circular section with concentrated supports. Inasmuch as the support conditions of the tunnel ring are continuous and elastic, it would be advantageous to begin the analysis by the theory of curved beams on continuous elastic support.

The first step in the solution of this problem by this approach—the formulation of a differential equation of the elastic line of the circular ring—was published in 1883.[41] The general solution of this differential equation for the circular beam on continuous elastic support (inverted foundation arch) was published in 1900;[42] another solution appeared in 1921;[43] and the solution of the author's problem (equations of influence lines for stresses in an elastic ring embedded in an elastic medium) was published by Adolf Voellmy in 1936.[44] Mr. Voellmy's work, including experimental verification, was completed in 1934.[45]

The carrying capacity of an elastic cylinder embedded in the soil is greatly influenced by the deformations. "Safe" design assumptions which disregard this important fact may result in weaker structures at considerably increased expense. Since the thinner cylinder is able to undergo larger deformations without failure, such deformations advantageously modify the pressure distribution on the cylinder surface. Thus, the internal stresses will be smaller than those in a structure with thicker walls. This explains the great carrying capacity of thin ductile metal cylinders and shows the importance of designing underground cylindrical structures as flexible as practicable. Therefore, a basic distinction should be made between pressures and stresses derived with or without considering the effect of deformations in the ring and the surrounding medium. The segregated determination of these two kinds of pressure and stress distributions facilitates clear visualization of the behavior of the structure as influenced by deformability characteristics—a great advantage in judging the effect of the various factors that enter into the design and in selecting the proper solution. From the design point of view, the inseparable consideration of these effects impairs the practical value of the author's method.

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[40] Cons. Engr., New York, N. Y.

[41] "Résistance d'un anneau à la flexion, quand sa surface extérieure support une pression normale constante par unité de longueur de sa fibre moyenne," by J. Boussinesq, *Comptes Rendus*, Vol. 97, Paris, 1883, pp. 843 and 1131.

[42] "Einiges über Grundbögen," by A. Francke, *Schweizerische Bauzeitung*, Vol. 36, 1900, p. 71.

[43] "Theorie des Trägers auf elastischer Underlage," by Keiichi Hayashi, Julius Springer, Berlin, 1921, p. 145.

[44] "Erddruck auf elastisch eingebettete Rohre," by A. Voellmy, *Publications*, Vol. IV, International Assn. for Bridge and Structural Eng., Zürich, 1936, pp. 591–611.

[45] "Eingebettete Rohre," by Adolf Voellmy, Leemann & Co., Zürich, 1937, p. 151.

As is the case for the analysis of suspension bridges, the difference between the stresses that refer to the original shape of the structure, and those that refer to the deformed shape, will be small if the ring is rigid. In designing suspension bridges, the continuous change of the deformability characteristics along the span generally makes it impracticable to use the principle of superposition, whereas the constant deformability characteristic of a circular ring facilitates design by superposition. In the latter case superposition is also best suited to the nature of the design problem under consideration. Two kinds of influence lines are required, therefore, for design purposes: (*a*) Influence lines which are generally valid for a circular ring and (*b*) one influence line for the consideration of the effects of the deformations of a given ring.

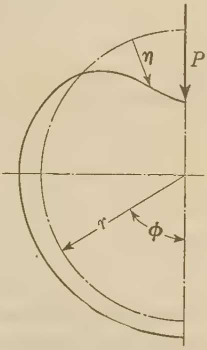

FIG. 29

The differential equation of the elastic line of the circular ring on elastic support (see Fig. 29) is:[44]

$$\eta'''' + \eta'' + \left(1 + \frac{r^4 K}{B}\right)\eta = \frac{C_1 r^2}{B} \quad \ldots\ldots\ldots\ldots\ldots(66)$$

and the general solution of Eq. 66 is:

$$\eta = \frac{C_1 r^3}{B a^2} + (C_2 \sinh v_1 \phi + C_3 \cosh v_1 \phi) \sin v_2 \phi$$
$$+ (C_4 \sinh v_1 \phi + C_5 \cosh v_1 \phi) \cos v_2 \phi \quad \ldots\ldots\ldots\ldots(67)$$

For the case of a radial load at $\phi = \pi$ (see Fig. 29), constants C_1 to C_5 are determined by the following five conditions: (*a*) $\eta' = 0$ for $\phi = 0$; (*b*) $\eta' = 0$ for $\phi = \pi$; (*c*) $V = -\frac{B}{r^3}(\eta' + \eta''') = 0$ for $\phi = 0$; (*d*) $V = -\frac{B}{r^3}(\eta' + \eta''') = -\frac{p}{2}$ for $\phi = \pi$; and (*e*) $\frac{1}{B}\int_0^\pi M\, d\phi = -\frac{1}{r^2}\int_0^\pi (\eta + \eta'')\, d\phi = 0$. Introducing the derivatives of Eq. 67 into conditions (*a*) to (*e*), the following values are obtained for the constants C_1 to C_5: From conditions (*a*) and (*c*), $C_3 = C_4 = 0$; from conditions (*b*) and (*d*), $C_2 = \frac{p\, r^3}{4\, B\, v_1\, v_2}\Phi$ and $C_5 = -\frac{p\, r^3}{4\, B\, v_1\, v_2}\Psi$; and, from condition (*e*), $C_1 = \frac{p}{2\,\pi}$. Introducing these values of the constants in Eq. 67, the influence line of the radial deflections in section $\phi = \pi$ becomes:

$$\eta_1 = \frac{r^3}{4\, B\, v_1\, v_2}\left(\frac{2\, v_1\, v_2}{a^2\, \pi} + \Phi \sinh v_1 \phi \sin v_2 \phi - \Psi \cosh v_1 \phi \cos v_2 \phi\right) \ldots(68)$$

in which

$$a = \sqrt{1 + \frac{r^4 K}{B}}; \quad B = \frac{m^2}{m^2 - 1} E I; \quad v_1 = \sqrt{\frac{a - 1}{2}};$$

$$\text{and} \quad v_2 = \sqrt{\frac{a + 1}{2}} \ldots\ldots\ldots\ldots (69)$$

Furthermore,

$$\Phi = \frac{v_1 \sinh v_1 \pi \cos v_2 \pi - v_2 \cosh v_1 \pi \sin v_2 \pi}{a (\sinh^2 v_1 \pi + \sin^2 v_2 \pi)} \ldots\ldots\ldots\ldots (70a)$$

and

$$\Psi = \frac{v_1 \cosh v_1 \pi \sin v_2 \pi + v_2 \sinh v_1 \pi \cos v_2 \pi}{a (\sinh^2 v_1 \pi + \sin^2 v_2 \pi)} \ldots\ldots\ldots\ldots (70b)$$

In Eq. 69 m is Poisson's number (10/3 for metals). If m is neglected, $B = E I$. Because the deflections and pressures are assumed to be proportional (Eq. 15), the expression for the influence line of passive pressures becomes:

$$p_1 = K \eta = \sqrt{\frac{a^2 - 1}{2 r}} \left(\frac{2 v_1 v_2}{a^2 \pi} + \Phi \sinh v_1 \phi \sin v_2 \phi \right.$$

$$\left. - \Psi \cosh v_1 \phi \cos v_2 \phi \right) \ldots\ldots\ldots\ldots (71)$$

The evaluation of this influence line for the pressures and reactions acting on the ring, assumed to be rigid, gives the passive pressures on the elastic ring. The algebraic summation of the values thus obtained and of the pressures and reactions producing them constitutes the actual load distribution on the deformed ring. The actual stresses in the deformed ring are obtained by applying this actual load distribution to influence lines which are generally valid for a rigid circular ring. The generally valid influence lines are the boundary values of Eqs. 68 and 71 and of the expressions for moment, shear, and thrust that are similar to them. These stresses refer to the conditions: $K = 0$, $B = \infty$, $a = 1$, $v_1 = 0$, and $v_2 = 1$; and, for radial loads, they become

$$M = \frac{r}{2 \pi} \left(\phi \sin \phi + \frac{\cos \phi}{2} - 1 \right) \ldots\ldots\ldots\ldots (72a)$$

$$V = \frac{1}{2 \pi} \left(\phi \cos \phi + \frac{\sin \phi}{2} \right) \ldots\ldots\ldots\ldots (72b)$$

and

$$T = \frac{1}{2 \pi} \left(\phi \sin \phi + \frac{\cos \phi}{2} \right) \ldots\ldots\ldots\ldots (72c)$$

In Section 28, Mr. Bull states that the critical stresses and cross-sectional areas indicated in Fig. 13 are in good agreement with actual sections used in large corrugated steel culverts.[6] This agreement is due only to the circumstance that the reduction of the stresses by the deformations of the culverts received consideration in the design. The support and loading conditions as well as the deformations of, and the moment distribution in, these culverts are different from those on which Figs. 12 and 13 are based. The culverts are cradled in a firm bed of soil, and the backfill around their sides is compacted to a desired density. The maximum moment in the culverts occurs at the invert and not at the crown; and it is opposite in sign to that indicated in Fig. 12.

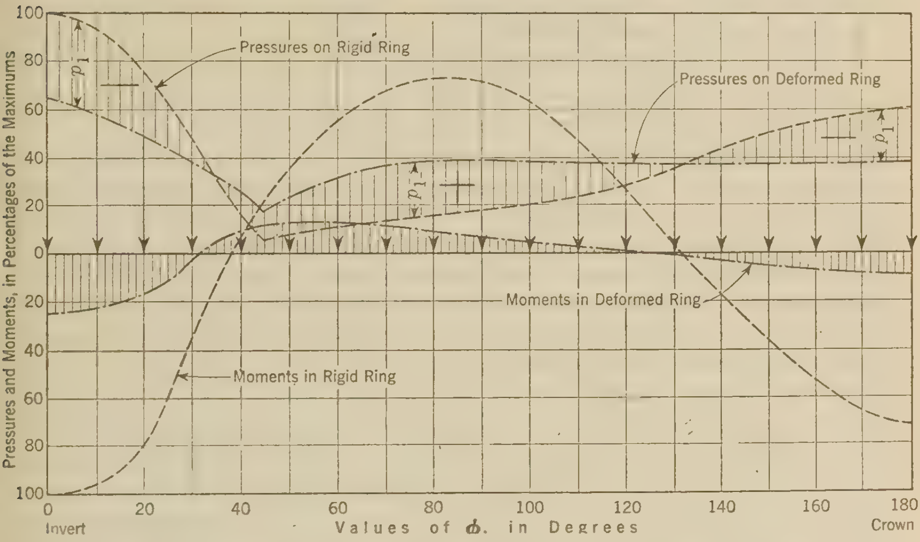

FIG. 30.—PRESSURE AND MOMENT DISTRIBUTION OF RIGID AND DEFORMED RING (FOR DEFINITION OF ϕ, SEE FIG. 29)

In Fig. 30 the pressures on, and the corresponding moments in, a ring that was first assumed to be rigid and the modification of the pressures by the deformations of the ring and of the actual pressures on, and moments in, the deformed ring are indicated in percentages of the maximum values. This diagram facilitates the visualization of the effect of the deformations. The wide-shaded areas between the pressure diagrams indicate the deformations of the ring and the pressure modifications resulting from these deformations. The close-shaded areas indicate the actual moments. The distribution of the pressures on the deformed ring, as obtained by the use of Eq. 71, is identical with the experimental results of M. G. Spangler,[7] M. Am. Soc. C. E.

The functional relations between the forces acting and internal stresses of an elastic ring embedded in an elastic medium are thus shown to be well established. Although he approaches the problem indirectly, Mr. Bull contributes to this knowledge. However, the paper does not help to dispel the uncertainties concerning the magnitude and variation of the forces acting on the tunnel ring. Until methods are found by which these forces can be determined or controlled as reliably as in the case of the culvert type of structure, claims for the rational design of the shield-driven tunnel lining are not warranted.

ANDERS BULL,[46] M. AM. SOC. C. E.—The discussions touch upon several points that were not properly emphasized or, because of space limitations, were not sufficiently elaborated in the paper. The writer welcomes this opportunity to clarify some of his statements.

The aim of the paper is to call attention to the inconsistency of the assumptions underlying the customary method of lining design, and to explain the principles of a method resting on a sounder theoretical foundation. To demonstrate the practical consequences involved, a comparison was made of the results obtained by the two methods.

Designing a tunnel lining is largely a problem in soil mechanics. To enter upon such a moot theme might have led the investigation far afield, however, and would have diverted attention from the main topic. It was stated at the outset, therefore, that the paper would be confined to the consideration of an ideal soil, subject to certain simple rules. To criticize these rules, as has been done by some of the discussers, is thus strictly outside the scope of the paper. They are engineering rules, with no pretentions to exact scientific truth; they have been used by the Board of Transportation of the City of New York and its predecessors for many years in designing subways and tunnels, including the one chosen for an example in the paper. Only by adhering to these rules could results be compared. What effect the substitution of other rules will have remains to be seen; but, although the numerical results may differ somewhat, it seems doubtful that this should materially affect the final conclusions.

The writer is grateful to Professor Van den Broek for his friendly comments on the first part of the paper, which are all the more appreciated because of his thorough familiarity with the subject under discussion. The proposed method depends on the use of numerical tables, so it is of great importance that the printed values be absolutely reliable; with the sensitive moment balance to be dealt with in a lining, a single error is likely to upset the results completely. Professor Van den Broek's numerous checks on the A-constants and B-constants should dispel any doubt on that score. The value of such checks might have been questioned had they been part of the paper, as there is always the possibility of an error in typesetting. As far as the C-constants and D-constants are concerned, the writer, after comparing the printed values with the original ones, is pleased to state that no typographical error has been found. In preparing the tables, the utmost care was exerted, the constants being computed to seven decimal places by two different methods and subjected to rigorous tests.

Raising a question of signs, Professor Van den Broek states that by the ordinary rule the product of force and deflection is considered positive when both factors are of the same direction and sense.

The statement following Eq. 1, by which a deflection component is considered positive when its direction is opposite to that of the force component, may seem to violate this rule, but is qualified by the term "(work negative)." By the ordinary rule one would write, for the condition shown in Fig. 3(a)

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[46] Cons. Engr., Forest Hills, N. Y.

(since the total work performed by reaction and force components equals zero):

$$T\,a + (-\,P_r\,d_{Tr} - P_t\,d_{Tt}) = 0 \ldots\ldots\ldots\ldots\ldots\ldots (73)$$

which is tantamount to Eq. 1. The writer has consistently made use of this form in model analysis[47] and has found it convenient. Strange to say, the late George E. Beggs,[48] M. Am. Soc. C. E., failed to lay down any rule for signs, leaving this to the judgment of the reader.

Convenience has also been aimed at in the other sign conventions of the paper, which are stated for each symbol in the Appendix. By thus considering tangential forces positive when directed upward their sign is apparent at a glance.

Professor Van den Broek suggests that influence lines be obtained experimentally from models in analyzing rings of noncircular shape, and variable moment of inertia, conditions which render a satisfactory mathematical treatment too complicated for practical use.

By using precision equipment for distorting the model and for measuring the deflections it should be entirely feasible to determine the equivalents of the A-constants to three significant places and, similarly, also load-deflection coefficients corresponding to the C-constants. Structures of this type are usually made of reinforced concrete, for which material the customary assumptions regarding location of the neutral axis and tension in the concrete are far from true in any case; so that extreme accuracy would be of no real use.

Mr. Brodkin's proposal, to use large-scale influence diagrams, such as Fig. 17, for obtaining more accurate results when concentrated loads are applied between the sixteen tabulated points, may be of advantage when all the forces acting on a ring are known. In the case of a tunnel lining, however, the many conjectural factors involved in determining the soil reactions make such refinements futile. Furthermore, if an A-constant were to be measured between points, the corresponding sixteen C-constants would logically have to be treated the same way; but this could not be expressed by the basic Eqs. 19 which depend on a uniform spacing. To avoid this difficulty, intermediate concentrated loads are assumed to be apportioned between the nearest two of the tabulated points, considering the latter as simple supports.

Mr. Brodkin presents instructive data proving the simple relationship between the soil compression constant K and the moment of inertia I of a lining. The latter behaves in this respect much like a flat slab resting on an elastically yielding soil. Since reducing K at a given ratio has the same effect on the moments as increasing I at the inverse of that ratio, it may be shown that a flexible lining is less affected by changes in the value of K than is a stiffer one. In the lining for which $I = 0.53$ in Table 14, a reduction of K from 12 to 3 would thus change the moments to the values given for $I = 2.13$, causing the crown moment to be increased by $\frac{10.9 - 8.7}{8.7} = 25\%$, whereas in the lining referred to in Table 15 ($I = 136$), the increase is $\frac{86.4 - 58.2}{58.2} = 48\%$, or nearly twice as much.

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[47] *Engineering News-Record*, December 8, 1927, p. 920; *Civil Engineering* December, 1930, p. 181.

[48] *Transactions*, Vol. 88 (1925), p. 1208.

The constant K and the moment of inertia I, being of equal importance in designing a lining, Mr. Brodkin suggests that, as a matter of prudence, a conservative value be used for K. The question then arises as to what value may be considered safe in the present case of a sandy soil. Of much interest in this connection are the results obtained from an extensive investigation of the compressive qualities of the soil underlying a 1.3-mile stretch of the Houston Street branch of the Independent Subway System of New York City. The investigation was undertaken on the initiative of the late George Paaswell, M. Am. Soc. C. E., chief engineer for the contractors (Corson Construction Corporation), for the purpose of predetermining the settlement that would be caused by future building loads. A special penetration device designed by Karl Terzaghi,[49] M. Am. Soc. C. E., was used for the tests, which were conducted by Glennon Gilboy, Assoc, M. Am. Soc. C. E.

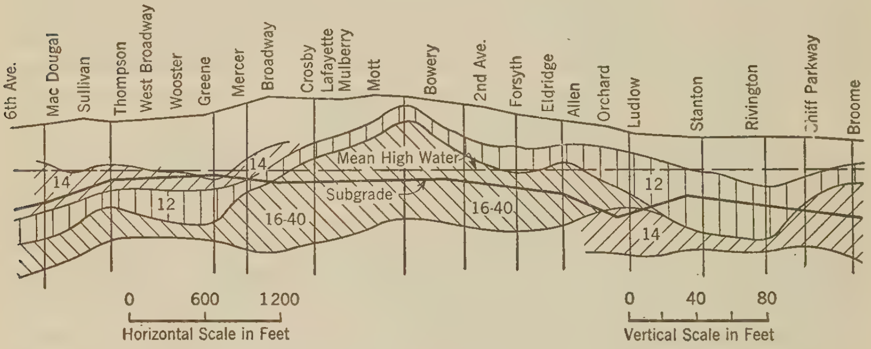
6th Ave.
Mac Dougal
Sullivan
Thompson
West Broadway
Wooster
Greene
Mercer
Broadway
Crosby
Lafayette
Mulberry
Mott
Bowery
2nd Ave.
Forsyth
Eldridge
Allen
Orchard
Ludlow
Stanton
Rivington
Chiff Parkway
Broome
14
12
14
16-40
Mean High Water
Subgrade
16-40
12
14
0
600
1200
Horizontal Scale in Feet
0
40
80
Vertical Scale in Feet

FIG. 31.—PROFILE ALONG HOUSTON STREET, SHOWING VARIATION OF SOIL COMPRESSION CONSTANT

In Fig. 31 (a diagram prepared by Mr. Paaswell based on the tests), the vertical lines indicate the sites at which measurements were made. The soil encountered was sand of almost uniform character throughout, with no deposits of silt or clay. Although the grain size varied considerably, this was found to have no appreciable influence on the compressibility of the soil. The numbers in each of the differently shaded areas (which vary between 12 and 40) indicate the respective values determined for K, in kips per square foot per inch.

A résumé of measurements made by other investigators[50] may also be mentioned. For railroad ties embedded in stone ballast (Kiesbettung) K was found to vary between 16 and 42, depending on the nature of the subgrade; and, for plates 11 in. square, resting on ballast varying in depth from 6 in. to 22 in., the respective K-values ranged from 96 to 162. In a test made on garden soil of unspecified character, Föppl[51] found K to be as high as 208.

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[49] *Engineering News-Record*, Vol. 106, 1931, p. 570.

[50] "Handbuch für Eisenbetonbau," Band 8, Ernst und Sohn, Berlin, 1922, p. 38.

In view of the aforementioned figures, the value $K = 12$ assumed in the paper would appear conservative.

Mr. Drucker considers it a discrepancy to include the weight of the concrete lining in the weight of the tunnel ring, arguing that, since the concrete is poured in normal air, its weight should not be considered in conjunction with the effect of the internal air pressure.

A newly erected ring carries various temporary loads for which no other provision is made, such as working platform, equipment for graveling and grouting, and sometimes part of the excavated soil. For this reason it is customary to include the weight of the concrete in the design load.[52] The same weight (0.22 kip per sq ft) was assumed for the lighter linings, Figs. 11(*b*) and 11(*c*), in order to provide for the weight of the scaffolding.

Another point criticized by Mr. Drucker is that the buoyant effect of the water on the soil was neglected. This practice is implicit in design specifications of the Board of Transportation which, for reasons stated previously, were accepted as a norm for this paper. There is absolutely no justification for his relating the soil compression constant K, with the angle of repose.

Mr. Drucker suggests that there may be a break in the computed stress value in passing from one side to the other of a line A–A, Fig. 22 (which represents the cross-section area at which u changes from 5 to 6), implying that the stress curve may have two branches at A–A which would yield different stresses, a and b_1. This view is easily disproved. Denoting by D the D-constants in Table 8, and considering first the case of $u = 5$, Eqs. 19 may be written in the following form which corresponds to the sequence of operations performed in lines 1 to 12 of Table 12:

$$\left.\begin{array}{r}\Big\{[D_{11}\,F_1 + D_{12}\,F_2 + D_{13}\,F_3 + D_{14}\,F_4 + D_{15}\,F_5 \qquad\qquad\qquad\qquad \\ + \Sigma(P\,D_1)]\,\dfrac{r^3}{E\,I} + \delta\cos\alpha_1\Big\}\,K\,A' = F_1 \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ \Big\{[D_{51}\,F_1 + D_{52}\,F_2 + D_{53}\,F_3 + D_{54}\,F_4 + D_{55}\,F_5 \qquad\qquad\qquad\qquad \\ + \Sigma(P\,D_5)]\,\dfrac{r^3}{E\,I} + \delta\cos\alpha_5\Big\}\,K\,A' = F_5\end{array}\right\}\ldots(74)$$

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[51] "Vorlesungen über technische Mechanik," Teubner, Leipzig, 1927, p. 279.

[52] "The Queens Midtown Tunnel," by Ole Singstad, *Transactions*, Am. Soc. C. E., Vol. 109 (1944), p. 688.

According to Section 24, the sign of the radial displacement at point 6 (line 11 of Table 12) is the criterion of whether u is equal to, or greater than, 5. At A–A, Fig. 22, therefore, where u changes from 5 to 6, the displacement must be zero, or:

$$\left\{[D_{61} F_1 + D_{62} F_2 + D_{63} F_3 + D_{64} F_4 + D_{65} F_5 + \Sigma(P\, D_6)] \frac{r^3}{E\, I} + \delta \cos \alpha_6\right\} K\, A' = F_6 = 0 \quad \text{..(75)}$$

In the case of $u = 6$, there would be six equations of a structure similar to Eqs. 74, each equation containing one more term, $D\, F_6$; but, since $F_6 = 0$, these additional terms may be dropped, leaving six equations identical with Eqs. 74 plus Eq. 75. Since the F-values in Eq. 75 thus satisfy both sets of equations there can be no splitting up of the stress curve.

It so happens that, for $A' = 17.4$, Eq. 75 gives a value for F_6 which is close to zero ($F_6 = -0.037$ kip), placing line A–A at an area about 17 sq in. in Fig. 22. Similarly, F_5 will be zero for an area of about 40 sq in., so that the stress curve for joint J will be about as shown in Fig. 32.

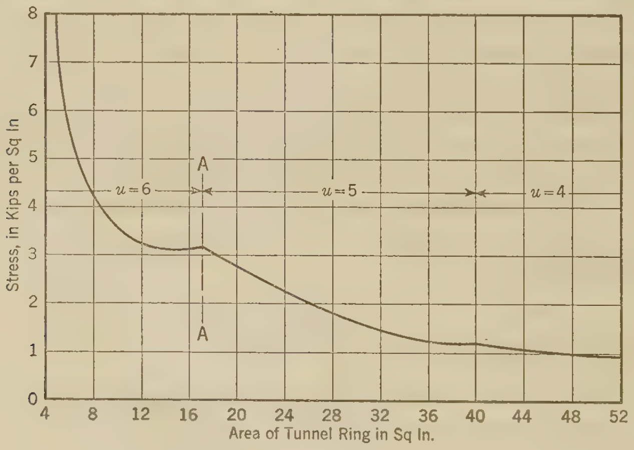

FIG. 32.—STRESS AT JOINT J

Its jagged shape results from replacing the distributed loads and soil reactions by force concentrations applied at sixteen points. If, instead, the analysis had been based on thirty-two force concentrations the resulting stress curve would have had twice the number of jags, but these would have been smaller, and the curve would have conformed more closely to the true stress curve. However, with so many uncertain factors entering the problem, an increase beyond sixteen points does not appear warranted, the arithmetical work increasing rapidly with the number of points.

Mr. Johannesson questions whether cast-iron linings could be decreased in weight materially, citing the need for a large cross section to resist the pressure of the shoving jacks. There have been cases when this was not the deciding consideration, as appears from the following statement by Ole Singstad,[53] M. Am. Soc. C. E., referring to the cast-iron linings used for the shield-driven river sections of the Queens Midtown Tunnel:

> "The heavier ring * * * with 14-in. flanges and weighing approximately 22.7 tons per ring, was used in soft ground and mixed face or where the character or thickness of rock cover required it; the lighter ring with 11-in. flanges and weighing approximately 12.5 tons was used in rock face where the cover was adequate."

The writer ventures the opinion that, since, in this case, the lighter lining was evidently strong enough to resist the pressure from the shoving jacks, a lining weighing no more might safely have been used for the entire river section of the tunnel, at a saving of more than 18,000 tons of cast iron, representing a reduction in cost of more than one million dollars.

Mr. Johannesson rightly states that the deflection of the horizontal joints under circumferential tension confounds the theoretical conclusions; but as yielding of the joints has the same effect as if the flexibility of the lining were increased it would cause a reduction in the critical moment. The stresses obtained by considering the joints stiff are, therefore, on the safe side.

Mr. Feld stresses the fact that designing a lining is a three-dimensional problem and that the forces acting in the three planes affect one another.

Longitudinal forces, apart from the temporary ones due to the shoving process, arise from axial bending of the tunnel. They may result in appreciable tangential shearing stresses in linings built up of segments of the usual shape, which makes for great axial stiffness, but are generally neglected because of the difficulty of estimating them. With a cross section like the one in Fig. 11(*c*), longitudinal forces would have less effect since the axial flexibility of the lining is greater.

According to Mr. Feld:

> "It is a serious mistake to design a tunnel without considering the feasibility of construction and also the conditions under which construction can proceed safely."

The writer agrees, of course, but questions the relevancy of the observation as he is not conscious of having advocated impracticable or unsafe methods.

Mr. Feld further states that, if it were possible to vary the air pressure at different heights within a lining surrounded by a fluid medium, a perfect balance could be obtained between the forces and the only deformation would be a radial squeezing of the ring. The same kind of balance is obtained at uniform air pressure when the weight of a lining equals its buoyancy.[54] The latter condition, which likewise results in zero moments throughout the ring, is of considerable practical interest as it is approximated in tunnels driven through river silt.

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[53] "The Queens Midtown Tunnel," by Ole Singstad, *Transactions*, Vol. 109 (1944), p. 687.

Referring to the views of the late J. C. Meem, M. Am. Soc. C. E., on arching in soil, Mr. Feld claims that the active pressure at any point is dependent on the deflection at that point. Until more definite information is available on this subject, it seems safer to go by Rankine's theory which is still the one most widely accepted. Although Mr. Meem's views are substantiated in some degree by the behavior of temporary structures, such as sheeting, their application to permanent structures is open to serious criticism as the soil arches eventually fail.[55]

Mr. Feld concludes his comments with the statement:

> "When some methods are developed for measuring the distribution of radial forces and tangential forces during the various stages of construction, as well as for measuring the actual values of such acting forces, the necessary data will be available for a proper use of the author's design procedure. Until such information is available, it seems very doubtful to the writer that a radical decrease in tunnel lining weight and thickness can be, or should be, proposed."

Contemplating the uncertain values of soil characteristics and the great differences in the moments M attending comparatively small differences in the forces F (Table 14), it is difficult to understand what conclusions could be drawn from such measurements, even if they could be made. What is needed are not measurements (which are almost impossible to achieve under construction conditions), but experiments, undertaken with imagination and courage, as exemplified by the culverts referred to in Section 28. Following the beaten path is safe policy but it does not contribute much to the advancement of engineering art.

According to Mr. Beskin, the basic principle referred to in Section 4 is "probably" due to Müller-Breslau. It should rather be credited to Mohr, whose "Beitrag zur Theorie des Fachwerks"[56] is fundamental in that it presents the first complete theory of statically indeterminate trusses. In this work the law of reciprocity of forces and deflections is deduced independently, and, for the first time, elastic lines are used as influence lines for reactions, their practical advantage being illustrated by examples. Müller-Breslau's contribution, made some ten years later,[57] consisted in expanding the principle, thus making it applicable to moment reactions as well. In deriving Eqs. 2 to 4, the writer has availed himself of Mohr's approach by which work performed, rather than deflection, is the leading concept.

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[54] "Elastic Energy Theory," by J. A. Van den Broek, John Wiley & Sons, Inc., New York, N. Y., 1931, p. 114.

[55] *Engineering News-Record*, April 19, 1934, p. 514; and October 25, 1934, p. 535.

[56] *Zeitschrift des Architekten- und Ingenieur-Vereins zu Hannover*, 1874, p. 510; and 1875, p.18.

[57] "Die neueren Methoden der Festigkeitslehre," Baumgärtner, Leipzig, 1886.

One is tempted to ask whether some of Mr. Beskin's comments are meant to be taken seriously, as, *for instance*, when he asserts that the writer has been "redeveloping conventional formulas," presented in an article by Joseph A. Wise, M. Am. Soc. C. E., and then proceeds to explain how the latter formulas may be transformed into those of the writer. Actually, the Wise formulas are for thrust, shear, and moment diagrams, whereas the writer's formulas are for influence factors, the two treatments having nothing in common.

Mr. Beskin announces the pending publication of tables of coefficients for 5° increments, intended to replace the A-constants and C-constants. He judges it to be a labor-saving feature that his coefficients are from five to seven times smaller, whereas the time saved by the writer's B-constants is considered of minor importance. The computation of the deflection coefficients is stated to have been "greatly simplified" by using only one angle instead of the writer's two (α and β in Fig. 8). Two angles are necessary for establishing the basic Eqs. 19. As far as the problem in hand is concerned, it serves no useful purpose to use a smaller increment than 22° 30′, as by so doing the number of simultaneous equations (Eqs. 19) is increased.

Mr. Beskin further endeavors to prove by means of Mohr's circle that the tangential forces referred to in Section 18 cannot be disregarded, as the radial soil reactions are not in equilibrium with the Rankine pressures assumed. Mohr's circle is not applicable to the boundary condition existing between the soil and the plastic layer of gravel and grout. Outside the boundary there is a soil mass for which the circle HA′V, Fig. 26(*b*), may be used; inside the boundary there is a semifluid mixture corresponding to a Mohr circle of zero radius inscribed in an angle of 0°.

A hopper with perfectly smooth sides (Fig. 33) presents an analogous case. When the trap door at the bottom is opened the pressures P_M and P_m are not in equilibrium with the normal pressure N, the unbalanced pressures causing the grain to slide down the sides. On closing the trap door, equilibrium is re-established by the pressure T, produced within the grain mass through the crowding of the grains.

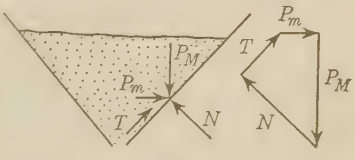

FIG. 33

In discussing Eq. 15, Professor Krynine calls attention to the experimental fact that under the same unit load a larger area will settle more than a smaller one. This is true of foundations buried to the same depth, the difference being the result of friction along their vertical faces, which is comparatively larger for the smaller area. It seems to be generally accepted, however, that in sand, with friction eliminated, the settlement by and large is independent of the area loaded. Only on that assumption could the results of penetration tests be used for predetermining the settlement of buildings, as illustrated by Fig. 31.

The K-value to be used for the design of a lining is determined by what Professor Krynine refers to as the "reversible part" of the displacement whereas the "nonreversible part," or initial consolidation of the soil, is assumed to be overcome by the pressure under which the annular space is grouted. Actually, there is no reversal of the displacements anywhere in a lining, as the pressures producing them are applied only once.

Of special interest is the minimum value that K attains in a given stretch of tunnel, this being generally the value for which the stresses are highest. It is true that this crucial value of K must be guessed. If it could be shown to be related to other soil characteristics it might be predicted more definitely. Research, thus narrowed down to minimum values, may prove fruitful of results and such research is much needed.

Some of the conclusions drawn by Professor Krynine from the field measurements on the Lincoln Tunnel are not applicable to a lining in sandy soil. Thus, in sand, the radial stresses resulting from the shoving process are negligible. Buoyancy is likewise a factor of less importance, as, in order to prevent excessive escape of pressure air, the tunnel must be kept at a low level, or blankets of clay must be provided, adding to the shearing strength of the overburden.

Mr. Eremin calls attention to the influence exerted on the final stresses in the lining by the loading on the roadway slab and by the forces transmitted through tie rods. In dealing with structures of this type it is not customary to consider internal live loads, as their transient effects are assumed to be absorbed by the inertia of the lining and the surrounding soil. The influence of tie rods may be determined easily by first assigning a value to the horizontal restraining force transmitted; this force is resolved into its radial and tangential components which are then treated as other active forces, and the resulting moment curve is plotted. By changing the assigned value successively, a series of moment curves is obtained from which the restraining force producing the greatest stress relief may be selected.

Fig. 34 shows the results of a theoretical study made along these lines; a very flexible lining ($I = 0.53$, Table 14) being assumed, which has a tie rod mounted between points 7 and 10 (Fig. 4). The curves represent the total moments produced in the lining under the combined influence of active loads, soil reactions, and tie rod forces. The latter were successively assumed to be 1, 2, and 3 kips, the corresponding curves being marked accordingly, whereas the curve marked O represents the lining without a tie rod.

An interesting feature is here revealed in that the curves intersect one another to form a number of nodal points. The moments at these points will remain unaffected by the force exerted by the tie rod and, therefore, are to be considered as irreducible minima for the particular position of the rod. Of the three cases considered, a tie rod force of 2 kips seems the most suitable, as the corresponding maximum moments—both positive and negative—are only slightly larger than the moments at the nearest nodal points.

Although the statement in Section 29, in regard to erection material being made an integral part of the structure, might have been somewhat less sweeping, the fact remains that the percentage of idle material in tunnels by far exceeds that in any other type of structure.

Mr. Alexander suggests the use of a simple expression for showing, rapidly, the effect of varying the factors that influence soil reactions. However, the loading conditions assumed in his example, Figs. 27 and 28, differ to such an extent from those applying to a tunnel lining that results are hardly comparable. Mr. Alexander thus assumes the vertical and horizontal active pressures to be uniform and symmetrical about the axes, thereby dispensing with the most important source of soil reactions—the settlement. Furthermore, he considers only reactions of a horizontal direction, acting on the two lateral quadrants of the lining.

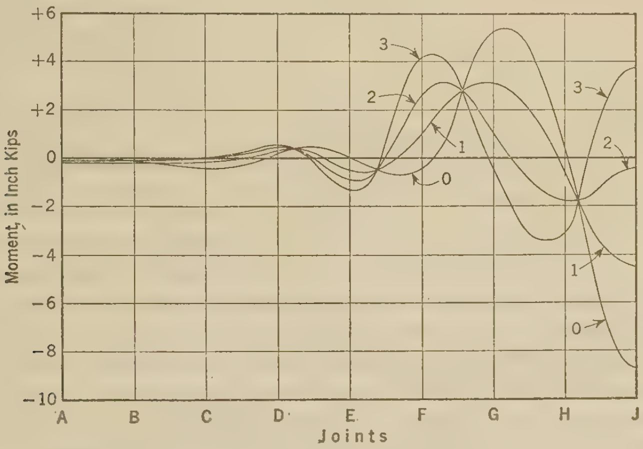

Fig. 34.—Effect of Tie Rod on Moment in Lining

The sensitiveness with which the moments M respond to small changes in the soil reactions F (see Table 14) is a feature calling for more accurate treatment. In bridge, subway, or building design, where a great number of members subjected to varying loads are to be proportioned, short cuts and slide rule computations are in order; a tunnel lining, however, constitutes a single member for the design of which a staff of specialists is usually available. To them, an elaborate treatment presents no obstacle. As several of the commentators have objected to the laboriousness of the proposed method, it seems only fair to mention here that the method by which some of the tunnels in the New York Metropolitan District have been designed is very much more cumbersome.

Mr. Alexander claims that tangential forces exert considerable influence on the moments in the lining, and that, therefore, they should not be neglected unless more evidence can be produced to show that they do not exist. Such forces are neglected, not because of their low value, but because they are prevented from reaching the lining (see Section 18).

Mr. Balog quotes at length from a treatise published in 1936 by Adolf Voellmy in which a solution of the lining problem is attempted by means of a differential equation (Eq.66). A similar equation of a somewhat simpler form—

$$\eta'''' + \frac{K}{EI}\eta = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (76)$$

—has long been used[58] for the analysis of flat slabs resting on elastically yielding soil. As far as practical application is concerned, both equations are of limited scope, as will first be demonstrated for the simpler form.

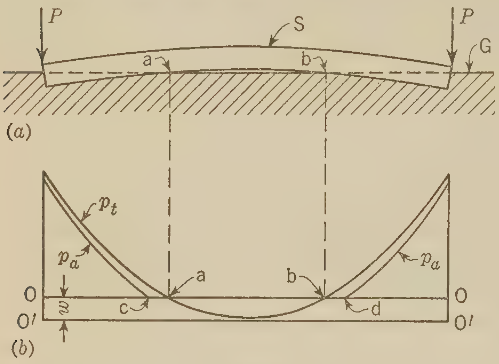

FIG. 35

Fig. 35(*a*) shows a slab S resting on the ground G and supporting two equal loads *P* at its ends. When the flexibility of the slab exceeds a certain value its middle part ab rises above ground level. Eq. 76 requires in this case that negative soil reactions be produced stressing the soil under the part ab in tension, thereby causing it to expand and exert a downward pull on the slab. As such action is incompatible with the behavior of soil the equation becomes invalid.

The theoretical pressure distribution along the slab, as obtained from Eq. 76, is shown in Fig. 35(*b*) by the curve p_t, the part ab below line OO representing the negative pressures, whereas the actual distribution would be about as shown by the curve p_a. The latter curve, which is discontinuous at points c and d, cannot be expressed mathematically, but may be approximately determined by means of a model.[59]

The scope of Eq. 76 may be extended, provided the unit weight w of the slab equals or exceeds the amount of negative soil pressure at the center of span. This weight will place the soil under an initial compression, lowering the zero line OO by an equal amount w to position O′O′ in Fig. 35(*b*).

A reversal of the sign of the soil reactions occurs wherever the deflected elastic line intersects its neutral position. Whereas this may happen in a flat slab, it is unavoidable in a lining, as it would not be possible for some parts of a ring to move outward from the center unless, at the same time, other parts were moving inward. In his treatise,[60] Voellmy states in this connection:

> "Equation (1) is assumed to hold for positive as well as for negative displacements of the lining surface. This assumption is justified when the tensile stresses due to one loading condition are neutralized by the compressive stresses due to another loading condition, as is the case in most practical applications."

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[58] "Die Lehre von der Elasticitat und Festigkeit," by E. Winkler, Dominicus, Prague, 1867, p. 182.

[59] "Soil Pressure Distribution Along Flexible Foundations," by Anders Bull, *Journal of the Franklin Institute*, June, 1942, p. 579.

[60] "Erddruck auf elastisch eingebettete Rohre," by A. Voellmy, *Publications*, Vol. IV, International Assn. for Bridge and Structural Eng., Zürich, 1936, p. 596.

In the foregoing statement, which is the writer's translation of the original German, Eq. 1 is the equivalent of Eq. 15 in the paper.

In Fig. 30, the pressures on the rigid ring represent one loading condition and the soil reactions p_1, due to deflection, the other. Their algebraic sum should then, according to Voellmy, represent the total resulting pressure distribution. The fallacy in this reasoning becomes evident when the condition at the crown is considered, where the negative soil reaction is supposed to reduce the pressure by about 40%. By the same kind of logic it could be contended that the pressure at any point of a flat slab S (Fig. 36), carrying a soil load, is reduced by the amount δK, in which δ is the deflection and K the soil compression constant; or that the pressure could be relieved by anchors A, embedded in the overlying soil.

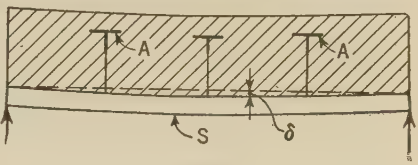

FIG. 36

The only way in which the negative soil reactions yielded by Eq. 66 can be neutralized is to provide additional positive soil reactions of a uniform intensity over the entire perimeter, such as would be produced, for instance, by a uniform radial expansion of the ring by an amount $\geqq -\frac{F'}{K}$, in which F' is the maximum negative soil reaction and is comparable to the additional pressure w in Fig. 35(b). No such measure being contemplated, the Voellmy equations are valid only for a ring embedded in a medium capable of resisting tension in accordance with Hooke's law. They do not hold for a lining in soil.

It might be argued that a uniform pressure over the entire perimeter can have no effect on the moments in the ring and might as well be omitted, but that is not the point. The pressure has a mechanical effect in that it forestalls a separation between the soil and the lining at any point, just as the pressure w in Fig. 35(b) prevents the slab and soil from separating. It is this separation, followed by a cave-in of the soil over the top portion of the lining, that upsets the mathematical conclusions.

As explained in Section 19, the extent of the ring portion on which soil reactions are produced (the angular distance between the points of intersection of lines c and d in Fig. 9, represented roughly by the number u) is a function of all the forces acting on the ring, including the soil reactions themselves. The principle of superposition does not apply, therefore, and as a consequence it is impossible to establish a generally valid influence line by which the effects of the deformation of a ring embedded in soil might be determined.

Despite the different methods of construction, there is no essential difference between the load distribution on a tunnel lining and that on a culvert. Mr. Balog's contention, that the maximum moment in culverts occurs at the invert and not at the crown, may be in accord with the Voellmy theory, but it is not confirmed by the observed fact that culverts fail by caving in at the crown.[61] No general rule can be given for the sign of the moment at the invert point, as is illustrated by Mr. Brodkin's Table 15 and Fig. 19, where a change in the value of K is shown to reverse the sign.

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[61] "The Structural Design of Flexible Pipe Culverts," by M. G. Spangler, *Bulletin No. 153*, Eng. Experiment Station, Iowa State College, Ames, 1941, p. 9.

It should be noted that the same assumptions are made by Voellmy as are made by the writer, in using the same basic Eq. 15, and in neglecting tangential forces. The totally different character of the moment curves obtained by him is thus traceable to the negative soil reactions. The absence of appreciable moments in the lower portion of a flexible lining is a typical feature of the proposed method and may furnish a criterion of its soundness. It may also be of important economic value in the design of certain structures, such as culverts, as the plates stressed only in compression may be made of correspondingly lighter weight.

In closing, the writer wishes to express his sincere thanks to the discussers for their interest and valuable contributions.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2276

FLOW IN A CHANNEL OF DEFINITE ROUGHNESS

By Ralph W. Powell,[1] M. Am. Soc. C. E.

With Discussion by Messrs. Joe W. Johnson and E. A. LeRoux, Garbis H. Keulegan, C. J. Posey, and Ralph W. Powell.

Synopsis

Although the effect of definite roughness of the wall upon the resistance to the flow of fluids in pipes has been thoroughly studied, there seems no equally satisfactory study of the effect of known wall roughness on the flow in open channels. This paper reports the results of some experiments bearing on this question which were performed by the writer during the summer of 1940 in the laboratory of the Institute of Hydraulic Research at Iowa City, Iowa. A total of two hundred runs on a rectangular flume, with eleven different roughnesses and four different slopes, indicated that Chézy's C could be expressed as $40 \log_{10} R/\epsilon + c_s$, in which R is the hydraulic radius, ϵ is a measure of the roughness, and c_s is a small shape correction. The mean of the deviations of the formula from the data was 2.1, and the mean percentage deviation was 4.4%.

In the same flume without roughness twenty-one runs of tranquil flow gave the formula $C = 40 \log_{10} \mathbf{R}/C + c_s + 4.8$, in which $\mathbf{R}$ is the Reynolds number. The mean deviation of this formula from the data was 2.32 or about 2.0% of the value of C. Shooting flow in the same smooth flume gave the formula $C = 60 \log_{10} \mathbf{R}/C + 30\,\mathbf{F} + c_s - 175$, in which $\mathbf{F}$ is the Froude number. The mean deviation of this formula from the data was 2.44, or about 2.1% of the value of C.

If these data are correct, two surprising results follow: (1) The "universal constant," $b = 2.5$, changes from 2.5 for pipes to about 3.0 in open channels; and (2) the law of resistance to shooting flow is quite different from the law of resistance to tranquil flow.

Note.—Published in December, 1944, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

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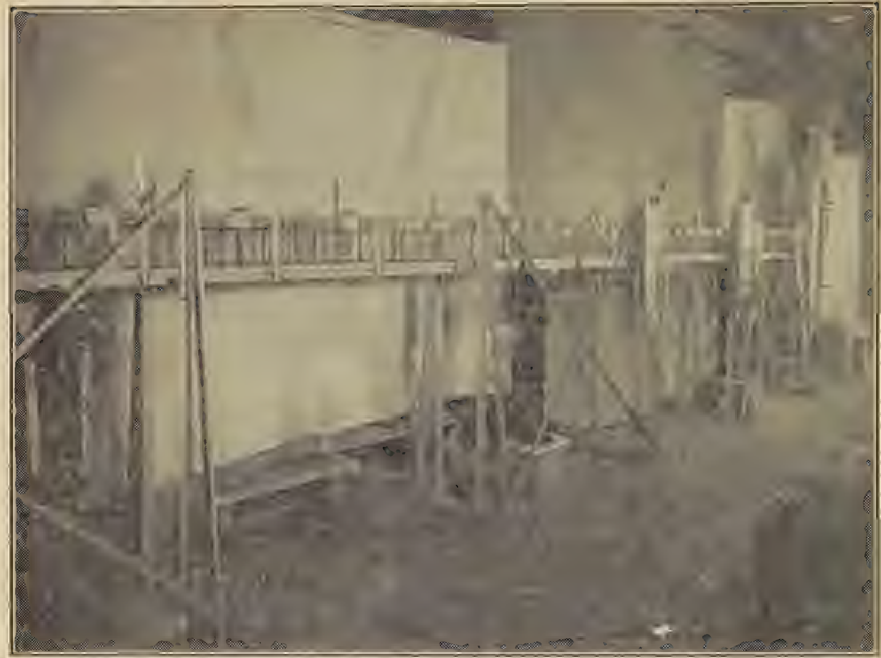

Fig 1.—General View of Apparatus, Facing Upstream

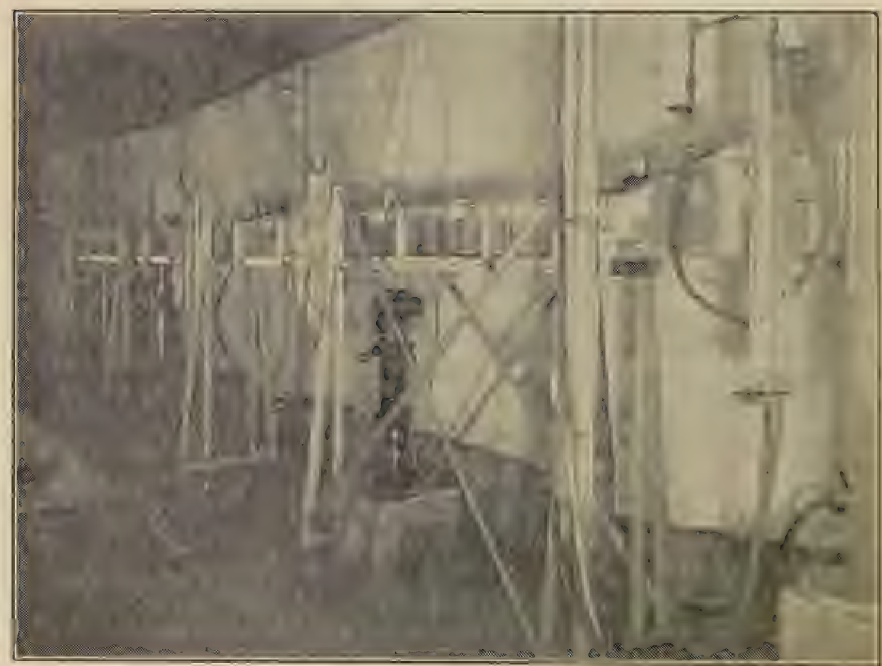

Fig. 2.—General View of Apparatus, Facing Downstream

HISTORICAL

The pioneer research on the flow of water in open channels with definite artificial roughness was made by H. Bazin[2,3] between 1855 and 1860. These tests were made in a channel 2 m wide and nearly 600 m long, and the measurements were quite accurate; but only a few of the roughnesses used were so definitely described that they could be accurately duplicated today, and apparently Bazin made no attempt to reduce his results to a formula that included a definite roughness figure. In fact, the idea of a definite linear unit of roughness seems not to have originated until the twentieth century.[4]

Measurements of the resistance to flow in pipes of definite roughness seem to have been first made by L. Hopf[5] and K. Fromm[6] in 1923. A rational theory of velocity distribution and, from it, formulas for resistance to flow were later developed by Theodor von Kármán,[7] M. Am. Soc. C. E., and Ludwig Prandtl.[8] In 1933, J. Nikuradse[9] published his epoch-making paper combining Prandtl's theory with definite experimental measurements of the resistance in pipes lined with sand grains of definite size. Since then a number of studies of flow in pipes of definite roughness have been made, of which may be mentioned those by V. L. Streeter,[10] Assoc. M. Am. Soc. C. E., H. Schlichting,[11] C. F. Colebrook[12,13] and C. M. White.[13]

The corresponding problem of the effect of definite roughness on flow in open channels does not seem to have received much attention. Mr. Keulegan[3] has published a good discussion of the problem, but with no experimental values other than Bazin's. The experiments described in this paper are a preliminary attempt to do for open channels what Nikuradse did for pipes.

APPARATUS AND EXPERIMENTAL PROCEDURE

The flume used was 50 ft long, 8 in. wide, and 7 in. deep. The side-walls were of lucite and the base was a structural steel channel with its back upper-

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[2] "Recherches hydrauliques," by H. Bazin, Paris, 1865. His results reduced to English units are given in "Flow of Water in Rivers and Other Channels," by E. Ganguillet and W. R. Kutter, translated into English by Rudolph Hering and John C. Trautwine, John Wiley & Sons, Inc., New York, N. Y., 1893. (The temperatures are not given in the translation, but they are evidently in the original as they are given by G. H. Keulegan.)

[3] "Laws of Turbulent Flow in Open Channels," by Garbis H. Keulegan, *Research Paper RP 1151*, National Bureau of Standards (U. S.), Vol. 21, December, 1938, pp. 707–741.

[4] "Elemente der technischen Hydromechanik," by Richard von Mises, Leipzig, 1914, p. 50, et seq.

[5] "Die Messung der hydraulischen Rauhigkeit," by L. Hopf, *Zeitschrift für angewandte Mathematik und Mechanik*, Vol. 3, 1923, pp. 329–339.

[6] "Strömungswiderstand in rauhen Rohren," by K. Fromm, *ibid.*, pp. 339–358.

[7] "Mechanische Ähnlichkeit und Turbulenz," by Theodor von Kármán, *Proceedings*, 3d International Cong. for Applied Mechanics, Stockholm, 1930, pp. 85–93.

[8] "Zur turbulenten Strömungen in Rohren und längs Platten," by Ludwig Prandtl, Ergebnisse der Aerodynamischen Versuchsanstalt zu Göttingen, Vol. IV (1932), pp. 18–29; or "Neuere Ergebnisse der Turbulenzforschung," by L. Prandtl, *Zeitschrift des Vereines Deutscher Ingenieure*, Bd. 77, No. 5 (1933), pp. 105–114.

[9] "Strömungsgesetze in rauhen Rohren," by J. Nikuradse, *Forschungsarbeiten auf dem gebiete des ingenieur-wesens*, Heft 361, Berlin, 1933.

[10] "Frictional Resistance in Artificially Roughened Pipes," by Victor L. Streeter, *Transactions*, Am. Soc. C. E., Vol. 101 (1936), pp. 681–713.

[11] "Experimentelle Untersuchungen zum Rauhigskeitsproblem," by H. Schlichting, *Ingenieur-Archiv*, Vol. VII, No. 1 (February, 1936), pp. 1–34. An abridged translation of the paper is given in *Proceedings*, Am. Soc. C. E., November, 1937, pp. 16–31 (following p. 1834).

[12] "Turbulent Flow in Pipes, with Particular Reference to the Transition Region between the Smooth and Rough Pipe Laws," by C. F. Colebrook, *Journal*, Inst. of Civ. Engrs., Vol. 11, 1939, pp. 133–156.

[13] "Experiments with Fluid Friction in Roughened Pipes," by C. F. Colebrook and C. M. White, *Proceedings*, Royal Soc. of London, Series A, Vol. 161, 1937, pp. 367–381.

most, painted with aluminum paint. The flanges of the channel offered attachment for the lucite side panels and for the vertical light steel angles which supported these panels at 20-in. intervals. The lucite panels were also stiffened along the upper edge by a lucite flange. A light steel strap was placed across the top at the middle of each panel and bolted to these flanges. At six points along its length the flume rested on blocks clamped to vertical trestles, as shown in Figs. 1 and 2.

The definite wall roughness was provided by square steel strips which extended down the sides and across the bottom so as to act as cleats or battens. The arrangements of the battens to create the various roughnesses are shown in Fig. 3, the larger of the two sizes being $\frac{1}{4}$ in. square and the smaller, $\frac{1}{8}$ in. square.

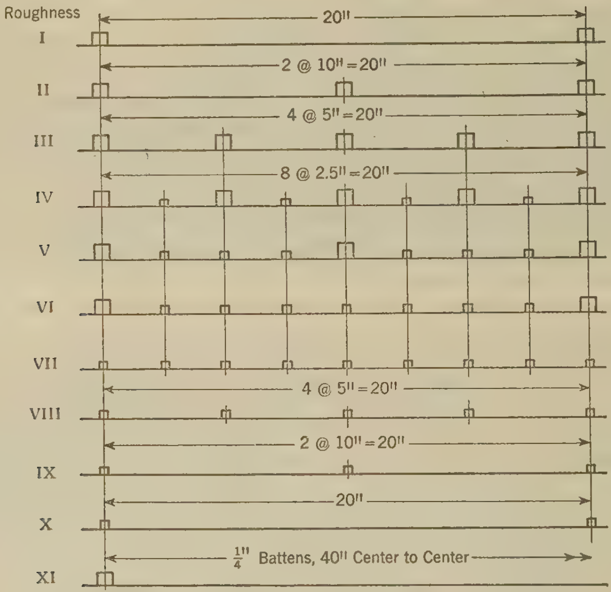

FIG. 3.—SPACING OF BATTENS TO FORM THE VARIOUS ROUGHNESSES

The water was measured by a diaphragm orifice in the 6-in. supply line just visible in the upper right corner of Fig. 1. For the larger flows the differential head on this orifice was measured by the mercury manometer shown at the right of Fig. 2. For most of the flows for which the differential head would have been less than 2 in. of mercury, it was read directly in inches of water in the vertical tubes shown to the left of the mercury manometer.

At the downstream end of the flume a control gate was arranged as shown in Fig. 4(*e*). This allowed part of the flow to pass under and part over, but by adjusting the wing nut (shown at the upper right) the depth at the downstream gage could be adjusted so that the flow throughout the flume was practically uniform. A thermometer is shown at the left in Fig. 4(*e*). The depth was read by hook gages in wells as shown at the right of Fig. 1 and the left of Fig. 2. The term "second slope" in the caption of Fig. 4 means that the bottom slope was 0.0080. The wells were connected by rubber hose with the static pressure tubes shown in Figs. 4(*a*), 4(*c*), 4(*d*), 4(*f*), 5(*a*), and 6(*d*). These were made of copper tubing, 0.187 in. outside diameter, with the upstream end plugged with solder and filed to a "streamlined" contour. Four

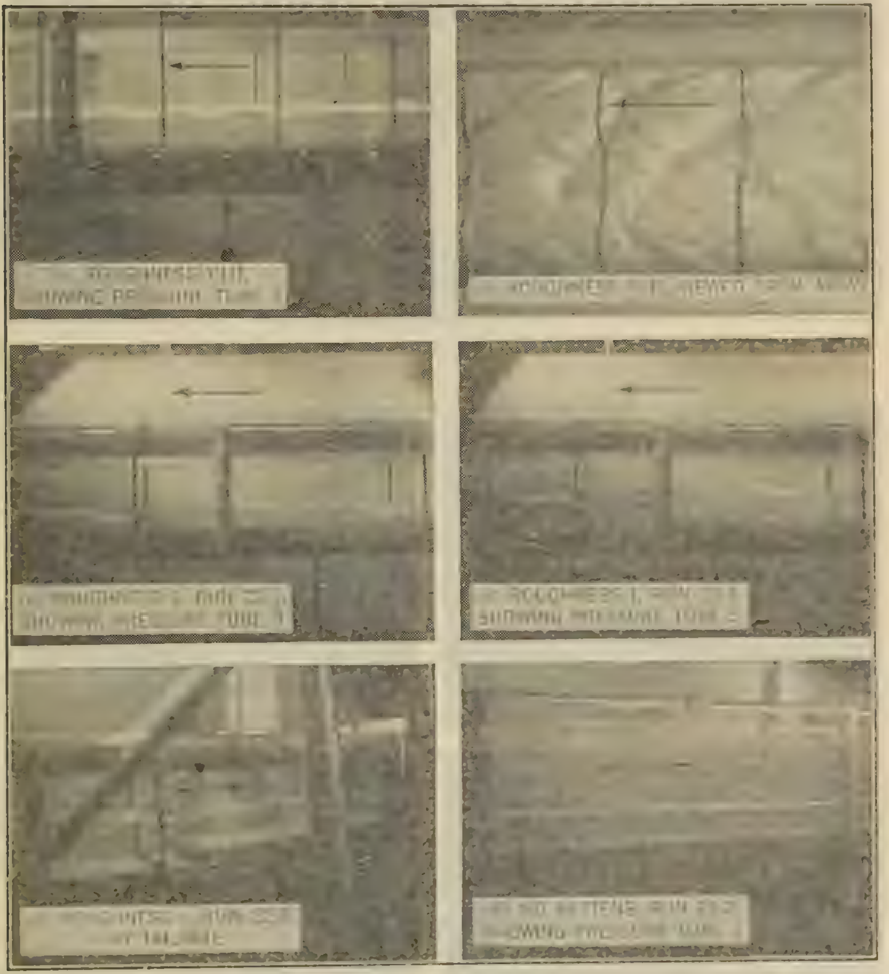

Fig. 4.—Examples of Flow at the Second Slope

holes were drilled with a No. 54 drill, 1 in. from the nose—one on top, one on the bottom, and one on each side. It was felt that some such arrangement was necessary because the pressure at an ordinary flush peizometer opening would undoubtedly have been affected by the proximity of a batten, and, with flows such as those shown in Fig. 5(*a*) (which are similar to the flow in run 5-2), it would have been impossible to estimate the depth visually through the lucite walls. The location of these pressure tubes is shown in Fig. 7. Further details regarding the apparatus and experimental procedure are given in the original draft of this paper on file in the Engineering Societies Library.

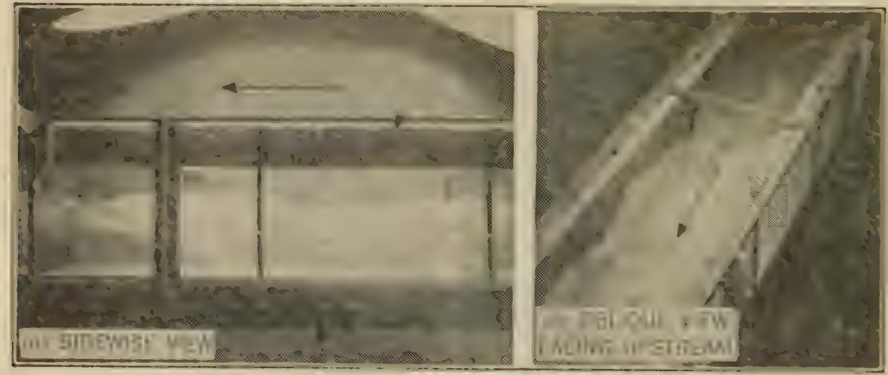
(a) SIDEWISE VIEW
(b) VIEW FACING UPSTREAM

Fig. 5.—Roughness II; Flow at the Steepest Slope

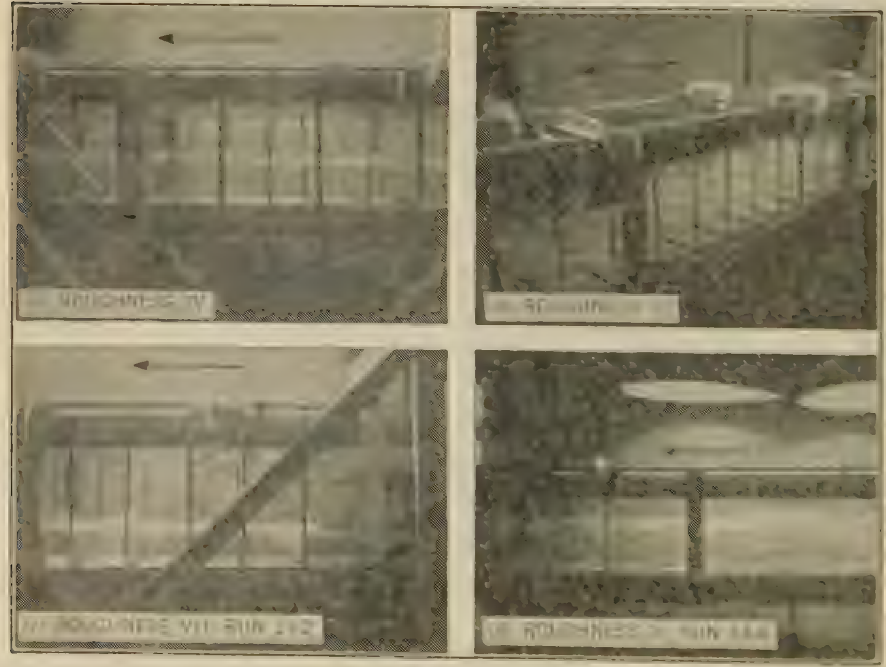
(a) ROUGHNESS IV
(c) ROUGHNESS VII, RUN
(d) ROUGHNESS

Fig. 6.—Examples of Flow at the Steepest Slope

When the work was begun the flume was set with its floor at a slope of $S = 0.0312$. This was steep enough to produce shooting flow in the smooth flume, and would not have been used to start with had it not already been set up. After runs had been made at this slope with the smooth flume and with each of ten roughnesses (with about five different depths for each roughness), the slope was changed successively to 0.0080, 0.0020, and 0.0005, and similar runs were made at each slope. At the flattest slope many of the flows were too small to be measured by the orifice meter, even with the water-air manometer. For these a deflector was arranged at the downstream end to direct the flow into a weighing tank. The time required to run 300 lb of water was measured with a stop-watch, except for a few of the smallest flows where the quantity was reduced to 200 lb, or even only 100 lb; but the time for these smaller weights was never less than 64 sec and was read to fifths of a second, so that these flows were probably determined more accurately than those measured with the orifice.

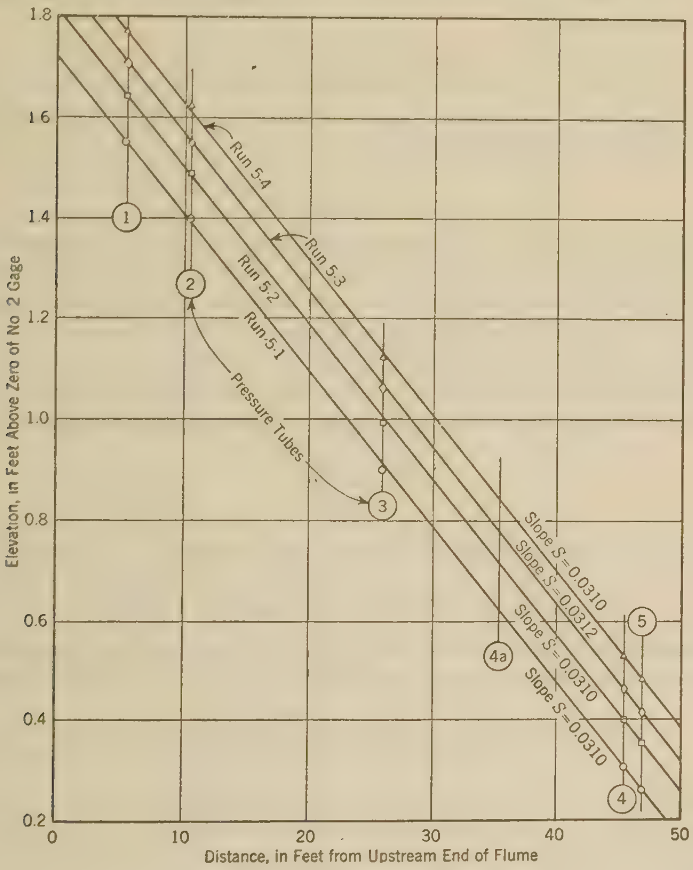

Fig. 7.—Sample of Energy Gradients

Before any runs were taken, the width of the channel was measured at ninety different sections and at two depths in each section with a micrometer gage reading to thousandths of an inch. The width at about 0.3 in. from the floor varied from 8.148 in. to 8.215 in. with a mean of 8.193 ± 0.013 in. At a height of about 4 in. above the floor the width varied from 8.114 in. to 8.210 in. with a mean of 8.166 in. In general, each 20-in. panel was sprung outward so that the channel was about 0.03 in. wider at the middle of the panel than at the junction with adjoining panels; and these junctions sometimes involved an offset of 0.01 in. to 0.02 in. Therefore, the so-called "smooth channel" was not absolutely smooth but had a slight amount of roughness and an appreciable waviness.

The orifice meter had been calibrated a few months before these experiments by a student. The result of this calibration was put into the form of a curve of values of $Q/\sqrt{H}$ plotted against $\sqrt{H}$ to such a scale that the constant could be read to four significant figures.

COMPUTATIONS

The letter symbols in this paper are defined where they first appear and are assembled for reference in the Appendix. The method of computation is described in detail in the original draft of this paper on file in the Engineering Societies Library. The most important data and results are given in Table 1 (Appendix). The slope was determined from plotted energy gradients, a sample of which is shown in Fig. 7. For the steeper slopes there seemed the possibility that the resistance might depend on the Froude number,

$$\mathbf{F} = \frac{V}{\sqrt{g\,y}} \qquad \text{(1)}$$

Therefore, this was computed in addition to the usual R, V, C, and $\mathbf{R}$.

ANALYSIS OF RESULTS WITH SMOOTH CHANNEL

The most fundamental treatment of the resistance to flow in open channels seems to be that of Mr. Keulegan,[14] whose formula for smooth walled channels is

$$\frac{V}{V_f} = a_s - b + b \log_e \left(\frac{R\,V_f}{\nu}\right) + b\,\beta - \bar{\epsilon}\,\frac{V}{V_f} \qquad \text{(2)}$$

in which: a_s and b are constants; ν is the kinematic viscosity of the liquid flowing; β is a function of the shape of the cross section; $\bar{\epsilon}$ measures the resistance of the free surface; and V_f, the so-called friction velocity, is defined as follows:

$$V_f = \sqrt{\frac{\tau_0}{\rho}} = \sqrt{R\,S\,g} = \frac{\sqrt{g}\,V}{C} \qquad \text{(3)}$$

Eq. 2 can be written

$$C = \frac{\sqrt{g}}{1 + \bar{\epsilon}} \left(a_s - b + b\,\beta + b \log_e \frac{\sqrt{g}}{4} + b \log_e \frac{\mathbf{R}}{C}\right) \qquad \text{(4)}$$

in which $\mathbf{R} = 4\,R\,V/\nu$ = the Reynolds number. Mr. Keulegan shows that for a rectangular channel

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[14] "Laws of Turbulent Flow in Open Channels," by Garbis H. Keulegan, *Research Paper RP 1151*, National Bureau of Standards (U. S.), Vol. 21, December, 1938, Eq. 48.

$$\beta = \log_e\left(1 + \frac{2\,y}{B}\right) - \frac{y}{B} \dots\dots\dots\dots\dots\dots\dots\dots (5)$$

in which y is the depth of the flow and B is the width. Assuming that b here has the same value as that which Nikuradse[4] found for pipes, and that $\bar{\epsilon}$ is zero, the shape correction term is equal to

$$c_s = \frac{b\,\beta\sqrt{g}}{1 + \bar{\epsilon}} = 14.18\,\beta \dots\dots\dots\dots\dots\dots\dots\dots (6)$$

Values of this term were calculated for various depths and are plotted against the corresponding hydraulic radius in Fig. 8.[14a] Since this term represents such a small part of the total C, it is felt that the approximation of making $\bar{\epsilon} = 0$ and $b = 2.5$ will introduce no appreciable error. By grouping the remaining terms, Eq. 4 may be rewritten as

$$C - c_s = m \log_{10} \frac{\mathbf{R}}{C} + J \dots\dots (7)$$

in which the logarithm is now to the base 10, and

$$m = \frac{2.3026\,b\sqrt{g}}{1 + \bar{\epsilon}} \dots\dots\dots (8a)$$

The same assumptions as those in Eq. 6 would make $m = 32.65$. Define J as:

$$J = \frac{\sqrt{g}}{1 + \bar{\epsilon}}\left(a_s - b + b \log_e \frac{\sqrt{g}}{4}\right) \dots (8b)$$

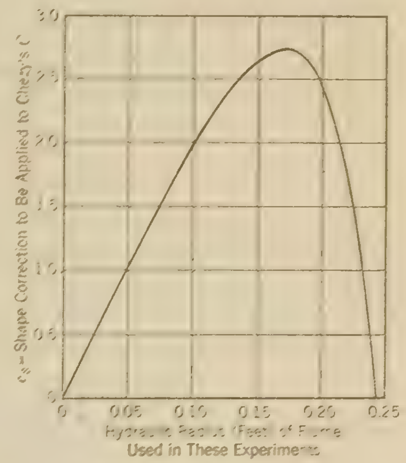

FIG. 8.—SHAPE CORRECTION

Using Nikuradse's value of $a_s = 5.5$ in addition to these assumptions, J would be $\sqrt{32.16}\,(5.5 - 2.5 + 2.5 \log_e \sqrt{2.01}) = 21.96$ and

$$C - c_s = 32.65 \log_{10} \frac{\mathbf{R}}{C} + 21.96 \dots\dots\dots\dots\dots\dots (9a)$$

It may be noted that for a semicircular cross section, Mr. Keulegan shows that $\beta = 0.259$, so that $c_s = 14.18 \times 0.259 = 3.67$, and

$$C = 32.65 \log_{10} \frac{\mathbf{R}}{C} + 25.63 \dots\dots\dots\dots\dots\dots (9b)$$

which is almost identical with the formula for flow in smooth channels derived independently by the writer[15] in 1938.

However, when $C - c_s$ was plotted against $\log_{10} \mathbf{R}/C$, the data for this smooth channel (Fig. 9) failed to agree with Eq. 9a. In the first place the points for the steepest slope, where the flow was definitely shooting (with $\mathbf{F} = 2.49$ or more), fall in a distinct region indicating an entirely different law. The others lie fairly well along a straight line, but its equation, fitted by least squares, is

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[14a] Corrections for Fig. 8 are discussed in the closure.

[15] "Mechanics of Liquids," by Ralph W. Powell, Macmillan Co., New York, N. Y., 1940, p. 202.

$$C - c_s = 39.7 \log_{10} \frac{\mathbf{R}}{C} + 5.6 \ldots\ldots\ldots\ldots\ldots\ldots (10)$$

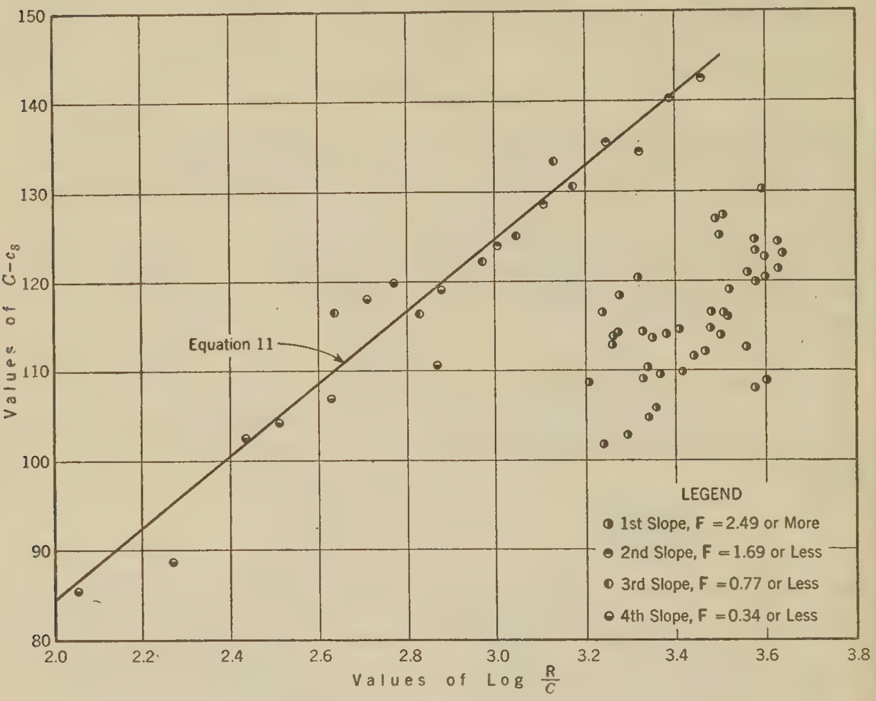

FIG. 9.—VALUES OF CHÉZY COEFFICIENT C FOR A SMOOTH CHANNEL

The probable error of m, also computed by least squares, is 1.4, so that it was decided to replace 39.7 ± 1.4 by 40. This changes the most probable value of J from 5.6 to 4.8, so that Eq. 10 becomes

$$C - c_s = 40 \log_{10} \frac{\mathbf{R}}{C} + 4.8 \ldots\ldots\ldots\ldots\ldots\ldots (11)$$

This line is drawn in Fig. 9. Table 1(a) in the Appendix gives the values of C computed by Eq. 11 (with the help of a table of values of $C + 40 \log_{10} C$), and also the deviations from the observed C. The mean deviation was 2.32 and the "probable error" was 2.0. Since C averaged 117.4, the average error was about 2.0% and the probable error about 1.7%.

If the walls are wavy, the value of J in Eq. 7 would be expected to be less than that for straight walls, but the value of m would be expected to remain the same. However, the value of m found in these experiments is definitely more than that found by Nikuradse. The value of $m = 39.7 \pm 1.4$ corresponds to $b = 3.04 \pm 0.11$ if $\bar{\epsilon} = 0$ and even more if $\bar{\epsilon}$ has an appreciable size. Nikuradse's value of $b = 2.5$ is probably somewhat approximate (notwithstanding the numerous formulas that have been based on it), but it can scarcely be this much in error. Therefore, there are either serious systematic errors in these experiments, or b has a different value for channels than for pipes. The much more extensive tests on rough channels, reported in this paper, also indicated this same value of m. It may also be noted that the study of velocity distribution in open channels by E. H. Taylor,[16] Jun. Am. Soc. C. E., following a modification of a method proposed by M. P. O'Brien,[17] M. Am. Soc. C. E., gave an average value for the "universal constant" of 0.306, which corresponds to $b = 3.27$ and which is even more than the writer's experiments show.

As stated, Eq. 11 does not represent the runs at the steepest slope (setups 1, 2, 3, and 8, in Table 1(*b*) in the Appendix). When the discrepancies from Eq. 11 were plotted against the Froude number **F**, there was no apparent relationship for the flatter slopes (setups 23, 24, and 46, in Table 1(*a*), in which **F** varied from 0.26 to 1.69); but on the steepest slope where **F** varied from 2.49 to 3.28, there was a very definite dependence on **F**, which it seemed might be well expressed by a formula of the type

$$C - c_s = J + z \log_{10} \frac{\mathbf{R}}{C} + x\,\mathbf{F} \ldots\ldots\ldots\ldots\ldots\ldots (12)$$

Solving for the most probable values of J, z, and x by least squares gave: $x = 30.4 \pm 2.2$; $z = 58.7 \pm 3.1$; and $J = -171.5$. In view of the fairly large probable errors it seemed best to round these values to $x = 30$ and $z = 60$, which for the best fit necessitated changing J to -175 and gave the formula

$$C - c_s = 30\,\mathbf{F} + 60 \log_{10} \frac{\mathbf{R}}{C} - 175 \ldots\ldots\ldots\ldots\ldots (13)$$

The values of C computed by Eq. 13 (with the aid of a table of values of $C + 60 \log_{10} C$) and the deviations from the observed C are given in Table 1(*b*) in the Appendix. The mean deviation was 2.44 and the probable error 2.1. Since C averaged 117.9, this makes the average error about 2.1% and the probable error about 1.8%.

Eq. 13 must be regarded as tentative until checked by other investigators and at other slopes. When it is realized how wavy the water surface was in this case (no photograph was taken, but the surface was similar to that in Fig. 4(*d*), it is not surprising that **F** enters into this case although it did not enter when the flow was more tranquil. It is surprising, however, that the coefficient of $\log_{10} \mathbf{R}/C$ should change. It is also surprising that the change from Eq. 11 to Eq. 13 came between $\mathbf{F} = 1.69$ and $\mathbf{F} = 2.49$ instead of at $\mathbf{F} = 1.00$. (Tests at one or more intermediate slopes would be required to determine where the change occurs.) The six runs of setup 23, Table 1(*a*) (Appendix), had Froude numbers **F** varying from 1.45 to 1.69 and agreed with Eq. 11 with an average discrepancy of 0.87, whereas the average discrepancy from Eq. 13 was 53.4.

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[16] "Velocity-Distribution in Open Channels," by E. H. Taylor, *Transactions*, Am. Geophysical Union, 1939, pp. 641–643.

[17] "The Vertical Distribution of Velocity in Wide Rivers," by Morrough P. O'Brien, *ibid.*, 1937, pp. 467–470.

ANALYSIS OF RESULTS WITH ROUGH CHANNEL

Mr. Keulegan's formula for rough walled channels, in the notation of this paper, is

$$\frac{V}{V_f} = a_r - b + b \log_e \left(\frac{R}{\epsilon}\right) + b\,\beta - \bar{\epsilon}\,\frac{V}{V_f} \dots\dots\dots\dots (14a)$$

in which a_r, a constant for rough walls, takes the place of the a_s in Eq. 2, and ϵ is a measure of the roughness of the channel walls (which Messrs. Nikuradse and Keulegan represented by k). Substituting the value of V_f and rearranging Eq. 14a,

$$C = \frac{\sqrt{g}}{1 + \bar{\epsilon}} \left[a_r - b + b\,\beta + b \log_e \left(\frac{R}{\epsilon}\right)\right] \dots\dots\dots\dots (14b)$$

As in the case of smooth channels (see Eq. 6), $\frac{b\,\beta\sqrt{g}}{1 + \bar{\epsilon}}$ can be replaced by c_s and the other terms combined to make

$$C - c_s = J + m \log_{10} \frac{R}{\epsilon} \dots\dots\dots\dots\dots\dots\dots (15)$$

For roughness I, values of $C - c_s$ were plotted against $\log_{10} R$. Since roughness IX (Fig. 3) constituted a half-scale model of roughness I except for width (which was corrected for in each case by the c_s-term), the values for roughness IX, with hydraulic radii twice those observed, were also plotted on the same sheet. Similarly, roughness VIII was combined with roughness II, roughness VII with roughness III, and roughness X with roughness XI. In each case the slope of the line $m = 40$ seemed to give as good a fit as any. This value was obtained visually without a least squares solution.

For roughness III, the equation thus obtained was

$$C - c_s = 72.2 + 40 \log_{10} R \dots\dots\dots\dots\dots\dots\dots (16)$$

If ϵ were taken as 0.25 in. = 0.02083 ft, $1/\epsilon = 48$, and $40 \log_{10} 1/\epsilon = 67.25$, J would be $72.2 - 67.25 = 4.95$. On the other hand, $4.95/40 = 0.12375 = \log_{10} 1.327$ and the reciprocal of 1.327 is 0.754, so that if ϵ had been taken as about three quarters the actual height, J would have been zero. Since the measure of roughness depends not only upon the height of the particle, but also on the spacing, it is obvious that the unit of measurement of roughness is quite arbitrary, and that it would be quite allowable to take it such that $J = 0$. Therefore, the following formula was used:

$$C - c_s = 40 \log_{10} \frac{R}{\epsilon} = 40 \log_{10} R - 40 \log_{10} \epsilon \dots\dots\dots\dots (17)$$

From the aforementioned plottings, and similar ones for roughnesses IV, V, and VI, ϵ was obtained and then C was computed for each run and the discrepancy between the computed and observed values of C was arranged in

Table 1(*c*) to Table 1(*m*) in the Appendix. If there was any serious difference between the sums of the plus and minus discrepancies, the value of ϵ was adjusted to make the discrepancies balance. The final values are listed in Table 2(*a*) in the Appendix. Table 2 also gives the values of ϵ/h, s, and s/h, in which h is the actual dimension of the batten, and s is the spacing of the battens center to center. Since roughnesses IV, V, and VI consisted of both $\frac{1}{8}$-in. and $\frac{1}{4}$-in. battens, no value of h was assigned to them.

PROBABLE ACCURACY

According to probability theory, the percentage probable error of the product or quotient of two observed quantities is the square root of the sum of the squares of the percentage probable errors of the factors. Estimates were made of the probable error in each of the various kinds of readings, and, from them, of the probable errors in the computed quantities, based on the aforementioned probability theory. In the case of the slope, the deviation of the plotted points from a straight profile also gave an indication of the probable error. These estimates gave percentage probable errors, as follows: For the velocity, 1.5%; for the slope, 1.7%; for the hydraulic radius, 1.3%; and for C, 1.9%. These are only estimates and may be too low. At any rate the average deviation of the 200 runs on rough channels from Eq. 17 was 4.37%, which by C. A. F. Peters' formula (probable error is approximately 0.845 times the average deviation) would indicate a probable error of 3.7% in one observation. This includes any error due to the choice of a wrong general form for Eq. 17, any error in the value $m = 40$, and any error due to the wrong choice of ϵ for each particular roughness. As to the value of m for tranquil flow in the smooth channel, it already has been noted that the least squares solution gave a probable value of 39.7 ± 1.4, and as the average value of $\log_{10} \mathbf{R}/C$ was 3.441, the probable error in the term $40 \log_{10} \mathbf{R}/C$ might be estimated at $1.4 \times 3.441 = 4.8$; but the error in the computed value of C would not be this much as it is largely compensated for by choosing a value of J to correspond to the m used. No study was made of the accuracy of the value $m = 40$ in Eq. 17, but from the plotting the writer would judge the probable error to be about 1.5.

The shape correction, c_s, is of the same order of magnitude as the discrepancies, so that the data are insufficient to check its correctness; but a plotting of the discrepancies against values of the depth y showed no systematic relationship, so that there is no evidence against its correctness. It may be mentioned again that the relation $c_s = 14.18\,\beta$ was based on Nikuradse's value of b. The writer's value of $m = 40$, instead of 32.65, is an increase of 22.5% and would increase the coefficient of β from 14.18 to 17.37. On the other hand, $\bar{\epsilon}$, the free surface effect, was taken as zero. If it has an appreciable value, part or all of the difference would be compensated for. At any rate the maximum change in c_s could be only 0.6, so that it was thought unnecessary to recompute all the data.[17a]

In general, it must be admitted that the accuracy of the experiments recorded herein is only of about the same order as those of Osborne Reynolds[18] on pipes in the 1880's. It can only be hoped that, in this case too, they may lead to more accurate measurements by other observers which will clear up the uncertainties.

[17a] This statement is corrected in the closing discussion.

[18] "An Experimental Investigation of the Circumstances Which Determine Whether the Motion of Water Shall Be Direct or Sinuous, and of the Law of Resistance in Parallel Channels," by Osborne Reynolds, *Philosophical Transactions*, Royal Soc. of London, Vol. 174, Pt. III (1883), p. 935.

It might be urged that, since quite large standing waves formed on the surface at the higher velocities in the rough channel, a term involving the Froude number should be included in Eq. 17. However, a plotting of the discrepancies from Eq. 17 against **F** showed no systematic relationship. This was to be expected because, since $\mathbf{F} = V/\sqrt{g\,y}$ and $C = V/\sqrt{R\,S}$, and since R varies nearly as y, resistance depending on **F** could hardly be distinguished from other resistance except by the difference in slopes, and in these tests there was only one slope at which **F** was large enough to have much effect.

Comparison with Bazin's Results

Runs 12 to 17 of Bazin's tests[2] were on rectangular plank channels with wood strips 1 cm thick and 2.7 cm wide nailed crosswise on the bottom and sides. His results, therefore, should be comparable to those herein presented except that the battens were not square. Since it seems that this might make relatively little difference, Table 1(*n*) (Appendix), has been computed. The value of ϵ which gave the best fit in Eq. 17 was selected (and listed in Table 2(*b*) in the Appendix), and the computed C and the deviations from the observed C are given in Table 1(*n*). The mean discrepancy was only 1.6, as compared to 2.1 for the writer's data.

The ratios of the equivalent roughness to the actual height of the "roughness particle" and the ratio of spacing to height, as given in Table 2 in the Appendix, are plotted in Fig. 10. The dashed portion of this curve is admittedly uncertain, but it indicates that the spacing of square battens that would give the maximum resistance is about twelve times the height of the battens, and that the maximum "equivalent roughness" is about 0.83 of the height. It must be remembered that the scale of measurement of "equivalent roughness" was taken arbitrarily to make J in Eq. 15 equal to zero, and that therefore this last result would not necessarily agree with Nikuradse's sand grain size and his formula for pipes. The facts that Schlichting[11] obtained a value of $\epsilon/h = 11.9$ for the flanges of a small angle section placed across his pipe, and that Mr. Streeter[10] found values of ϵ/h as high as 4.57 for square circumferential grooves, both investigators using ϵ equal to Nikuradse's k, indicate that the unit of ϵ used in Eq. 17 may be, say, from five to ten times as large as that used by Nikuradse. Although a standard

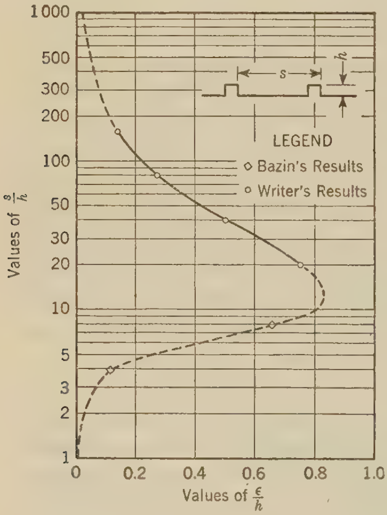

Fig. 10.—Relation of Relative Roughness to Spacing

value would be desirable, it would not seem feasible to adopt one until some one geometrical type of roughness (such as the one used in this study or some other that could be reproduced as easily) has been carefully calibrated by the Schlichting method in a pipe, and by some method comparable to that of the writer, in open channels. Then all other roughnesses could be referred to this scale.

CONCLUSIONS

The discrepancies of Eq. 17 from the observed data average nearly twice as much as would be expected from the probable inaccuracies of the measurements as estimated by the observer; but these are only estimates and it may be that Eq. 17 is entirely correct and the discrepancies entirely due to errors in observation or to errors of observation plus errors in the chosen values of ϵ. It seems more probable that the discrepancies are partly due to these two factors and partly due to wrong values in the constants of Eq. 17, but that the form of the equation is satisfactory, at least for tranquil flow.

For the case of the depth half the width, c_s is 2.74. The resulting formulas for this case are:

For tranquil smooth channels,

$$C = 7.54 + 40 \log_{10} \frac{\mathbf{R}}{C} \quad\text{.................... (18a)}$$

and, for rough channels,

$$C = 2.74 + 40 \log_{10} \frac{R}{\epsilon} \quad\text{.................... (18b)}$$

For laminar flow, assuming that the slope of the hydraulic gradient is the same as for a square pipe flowing full[19]

$$C = \sqrt{\frac{8\,g\,\mathbf{R}}{57.0}} = 2.12\,\sqrt{\mathbf{R}} \quad\text{.................. (18c)}$$

It should be noted that Eqs. 18*a*, 18*b*, and 18*c* apply only when the depth is half the width. For smaller depths the constant in Eq. 18*c* is less, approaching the value

$$C = \sqrt{\frac{8\,g\,\mathbf{R}}{96}} = 1.64\,\sqrt{\mathbf{R}} \quad\text{.................. (18d)}$$

when the width is infinite compared with the depth. Eqs. 18 are plotted in Fig. 11, which sums up most of the results of this paper. It must be understood that, as in the corresponding case for pipes, the same channel may act as rough or smooth. For example, a channel 4 ft wide flowing 2 ft deep, with an absolute roughness (ϵ) of 0.0010 ft, would have $R = 1.0$ ft and $R/\epsilon = 1{,}000$, and C would be 122.7 for any Reynolds number greater than 92,900; but, for Reynolds numbers less than this, it would behave as a smooth channel with C decreasing as $\mathbf{R}$ decreased down to about 67 at $\mathbf{R} = 2{,}000$, when Eq. 18*c* would begin to apply. Of course, the region from $\mathbf{R} = 2{,}000$ to $\mathbf{R} = 8{,}000$ is uncertain. It has been studied by L. Hopf,[20] H. Jeffreys,[21] R. E. Horton, M. Am. Soc. C. E., the late H. R. Leach, Assoc. M. Am. Soc. C. E., and R. Van Vliet,[22] and by L. G. Straub,[23] M. Am. Soc. C. E. Professor Straub, however, used a different definition for Reynolds number and also seems to have used the constant for a round pipe in his equation for laminar flow.

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[19] "A Review of Flow in Pipes and Channels," by S. J. Davis and C. M. White, *Engineering*, Vol. 128 (1929), p. 71. This simply puts into English the results of J. Boussinesq, *Journal de Mathématiques pures et appliquées de M. Liouville* (1868), p. 393.

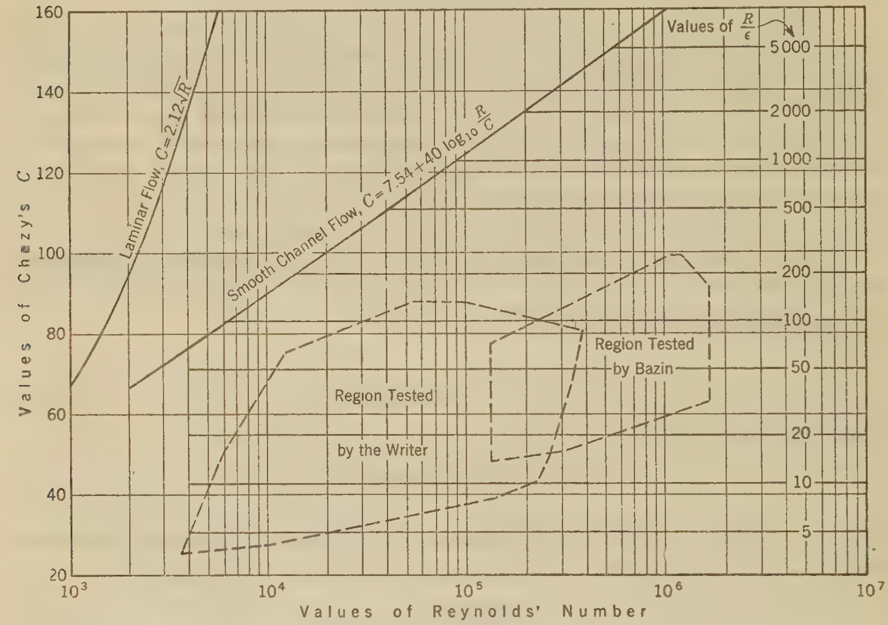

FIG. 11.—RELATION BETWEEN C AND R FOR RECTANGULAR CHANNELS WITH THE DEPTH EQUAL TO HALF THE WIDTH

By analogy with pipes one may suppose also that the region just to the right of the smooth channel line is probably not given correctly by the lines as drawn. One would expect that for uniform roughness there would be an upward bend (just as Nikuradse[9] found a downward bend for the friction coefficient f in pipes) and that for irregular roughnesses the bend would be in the opposite direction, as found by Messrs. Colebrook and White.[13] This transition range in pipes has been discussed quite fully by Mr. Colebrook[12] and by Hunter Rouse,[24] M. Am. Soc. C. E. Unfortunately, the writer's tests did not include any case in which the rough channel acted as smooth. It would be a difficult condition to arrange for water, but very easy with a liquid of the right viscosity.

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[20] "Turbulenz bei einem Flusse," by L. Hopf, *Annalen der Physik* (4) xxxii (1910), pp. 777–808.

[21] "The Flow of Water in an Inclined Channel of Rectangular Section," by Harold Jeffreys, *Philosophical Magazine and Journal of Science*, London, Edinburgh, and Dublin, Vol. 49 (May, 1925), pp. 793–807.

[22] "Laminar Sheet-Flow," by Robert E. Horton, H. R. Leach, and R. Van Vliet, *Transactions*, Am. Geophysical Union, 1934, pp. 393–404.

[23] "Studies of the Transition Region Between Laminar and Turbulent Flow in Open Channels," by Lorenz G. Straub, *ibid.*, 1939, pp. 649–653.

[24] "Evaluation of Boundary Roughness," by Hunter Rouse, *Proceedings*, 2d Hydraulic Conference, Iowa City, Iowa, 1943, pp. 105–116.

In all the computations of this paper the area and hydraulic radius are based on the gross section of the channel without the battens. If the battens were quite close together there certainly would be a question as to whether computation should not be based on the net section, or on some kind of average section. This knotty question is left for further study.

If Fig. 11 correctly represents the flow of liquids in channels, the implications for river model studies are evident. If the prototype acts as a rough channel, it is not sufficient to give the model the same relative roughness, because, at its smaller Reynolds number, it might be acting as a smooth channel. The only solution seems to be to use a sufficiently large model to bring it into the rough channel range.

ACKNOWLEDGMENTS

The writer wishes to express his grateful appreciation of the generosity of the Iowa Institute of Hydraulic Research which placed its facilities at his disposal, and met the incidental expenses of the study; especially to F. M. Dawson, M. Am. Soc. C. E., Director of the Institute, E. W. Lane, M. Am. Soc. C. E., then Associate Director in Charge, and to Hunter Rouse, M. Am. Soc. C. E., and A. A. Kalinske, Assoc. M. Am. Soc. C. E., and to A. M. McLeod. He also wishes to thank Paul Nemenyi and James Robertson, Jun. Am. Soc. C. E., at that time both connected with the Institute, and to express his appreciation of the work of E. W. Thomas and Dale Harris, technicians, who assisted in the construction of the apparatus. Several students, working under provisions of the National Youth Administration, assisted with the computations.

APPENDIX

NOTATION AND TABLES OF OBSERVATIONAL DATA

The following symbols, used in the paper, conform essentially to American Standard Letter Symbols for Hydraulics,[25] prepared by a Committee of the American Standards Association, with Society representation, and approved by the Association in 1942:

a = a constant used by Mr. Keulegan,[3] subscript r denoting rough channels and subscript s denoting smooth channels;
B = channel width;
b = a constant, the reciprocal of von Kármán's "universal constant";
C = the Chézy coefficient in the formula $V = C\sqrt{RS}$;
c = a correction; c_s = a small shape correction;
$\mathbf{F}$ = Froude number = $V/\sqrt{g\,y}$;
f = coefficient of pipe friction in the formula $S = \frac{f\,V^2}{D\,2\,g}$;
g = acceleration of gravity;
H = head on the measuring orifice;
h = height of roughness particles, or battens;
J = a constant defined by Eq. 8b;
k = a symbol for roughness used by Messrs. Nikuradse and Keulegan (see ϵ);
m = a constant in Eq. 7, defined by Eq. 8a;
Q = rate of flow;
R = hydraulic radius;
$\mathbf{R}$ = Reynolds number;
S = slope of the energy gradient;
s = spacing of roughness battens (center to center);
V = mean velocity in the cross section of a flume or channel; V_f = friction velocity as defined by Eq. 3;
x = the unknown coefficient of $\mathbf{F}$ in Eq. 12;
y = depth of flow; that is, vertical measurements or measurements parallel to the Y-axis;
z = an unknown coefficient in Eq. 12;
β = a function of the shape of the cross section as defined by Eq. 5;
ϵ = absolute or equivalent roughness; that is, a measure of the roughness of the channel walls (see k);
$\bar{\epsilon}$ = a measure of the resistance of the free surface;
ν = kinematic viscosity of the flowing liquid;
ρ = density of the flowing liquid;
τ = shearing stress; τ_s = shearing stress at the liquid boundary.

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[25] ASA—Z10.2—1942.

TABLE 1.—OBSERVED TEST DATA AND COMPUTED RESULTS

Run^a	y (ft)	R (ft)	V (ft per sec)	S $\times 10^4$	Observed C	**F**	t (degrees Centigrade)	**R** $\times 10^{-4}$	c_s	Computed C	DISCREPANCY	
											−	+
(a) SMOOTH CHANNEL; TRANQUIL FLOW; DISCREPANCIES BY EQ. 11												
46–1	0.076	0.062	0.407	3.5	86.9	0.26	22.1	0.975	1.3	87.9	....	1.0
46–2	0.103	0.079	0.548	4.6	90.5	0.30	21.9	1.67	1.6	96.2	....	5.7
46–3	0.148	0.103	0.708	4.4	104.6	0.32	22.0	2.82	2.0	104.1	0.5	....
46–4	0.187	0.121	0.727	3.9	106.4	0.30	22.2	3.40	2.3	107.2	....	0.8
46–5	0.239	0.140	0.884	4.4	112.4	0.34	22.1	4.81	2.6	112.6	....	0.2
46–6	0.298	0.159	0.999	4.3	120.9	0.32	22.1	6.14	2.7	116.4	4.5	....
46–7	0.349	0.172	1.080	4.7	122.7	0.32	22.1	7.20	2.7	118.8	3.9	....
46–8	0.410	0.186	1.149	5.5	113.2	0.32	22.1	8.26	2.7	120.9	....	7.7
46–9	0.473	0.198	1.183	4.8	121.7	0.30	22.1	9.05	2.6	122.2	....	0.5
24–1	0.109	0.083	1.227	17	102.3	0.76	27.1	4.40	1.7	110.5	....	8.2
24–2	0.159	0.108	1.667	18	118.6	0.74	27.1	7.85	2.1	119.6	....	1.0
24–3	0.217	0.133	1.990	19	124.7	0.75	27.1	11.5	2.5	125.7	....	1.0
24–4	0.270	0.151	2.16	19	127.6	0.73	27.1	14.1	2.6	129.0	....	1.4
24–5	0.325	0.166	2.51	20	136.1	0.77	27.1	18.1	2.7	132.9	3.2	....
24–6	0.404	0.185	2.55	20	133.3	0.71	24.7	19.4	2.7	133.9	....	0.6
23–1	0.118	0.088	3.30	78	125.8	1.69	27.3	12.6	1.8	126.5	....	0.7
23–2	0.150	0.104	3.68	76	130.6	1.67	27.2	16.6	2.0	130.9	....	0.3
23–3	0.214	0.131	4.22	72	137.7	1.61	27.2	24.1	2.4	137.0	0.7	....
23–4	0.256	0.146	4.50	74	137.0	1.57	27.2	28.6	2.6	139.9	....	2.9
23–5	0.317	0.164	4.91	72	143.1	1.54	27.1	35.0	2.7	143.1	0.0	....
23–6	0.397	0.183	5.18	70	145.0	1.45	27.1	41.2	2.7	145.6	....	0.6
(b) SMOOTH CHANNEL; SHOOTING FLOW; DISCREPANCIES BY EQ. 13												
1–1	0.114	0.085	5.17	293	103.4	2.70	23.6	17.7	1.7	102.1	1.3	....
1–2	0.140	0.099	5.89	281	111.3	2.77	23.7	23.5	2.0	109.9	1.4	....
1–3	0.149	0.104	6.16	274	115.6	2.81	23.9	25.8	2.0	112.8	2.8	....
1–4	0.224	0.135	7.02	258	118.7	2.62	24.0	38.4	2.5	117.0	1.7	....
1–5	0.257	0.146	7.50	252	123.5	2.61	24.0	44.4	2.6	120.0	3.5	....
2–4	0.111	0.084	5.81	288	118.3	3.07	25.4	20.3	1.7	113.9	4.4	....
2–6	0.115	0.086	5.75	293	114.5	2.99	25.4	20.7	1.7	112.4	2.1	....
2–3	0.116	0.086	5.82	293	115.6	3.01	25.1	20.9	1.7	113.0	2.6	....
2–2	0.118	0.088	5.95	299	116.1	3.05	25.1	21.6	1.8	114.7	1.4	....
2–5	0.119	0.088	6.11	294	120.1	3.12	25.4	22.5	1.8	117.3	2.8	....
2–1	0.125	0.091	6.58	318	122.1	3.28	25.1	25.0	1.8	123.6	....	1.5
2–11	0.130	0.094	6.03	286	116.3	2.95	27.1	24.6	1.9	115.2	1.1	....
2–12	0.143	0.101	6.45	306	116.2	3.01	27.0	28.2	2.0	119.6	....	3.4
2–13	0.141	0.100	6.36	300	116.2	2.98	27.0	27.5	2.0	118.3	....	2.1
2–7	0.197	0.125	7.63	280	129.1	3.03	25.4	39.8	2.4	127.9	1.2	....
2–8	0.199	0.126	7.62	285	127.4	3.01	25.5	40.1	2.4	127.6	....	0.2
2–9	0.200	0.126	7.65	287	127.1	3.02	25.5	40.4	2.4	128.0	....	0.9
2–10	0.271	0.151	8.24	255	132.9	2.79	25.6	52.2	2.6	128.0	4.9	....
3–1	0.100	0.077	5.22	291	110.3	2.91	27.4	17.6	1.6	107.0	3.3	....
3–2	0.130	0.094	5.95	298	112.4	2.91	27.2	24.4	1.9	114.0	....	1.6
3–3	0.137	0.098	6.12	308	111.6	2.92	27.3	26.1	1.9	115.7	....	4.1
3–4	0.156	0.107	6.40	281	116.7	2.86	27.4	29.9	2.1	117.3	....	0.6
3–5	0.180	0.118	6.88	284	118.9	2.86	27.4	35.4	2.3	121.0	....	2.1
3–6	0.195	0.124	7.23	298	118.9	2.89	27.5	39.3	2.4	124.1	....	5.2
3–7	0.246	0.143	7.55	247	127.2	2.69	27.5	47.3	2.6	123.3	3.9	....
3–8	0.257	0.146	7.61	252	126.8	2.65	27.6	48.9	2.6	123.0	3.8	....
3–9	0.279	0.153	8.02	261	126.9	2.68	27.6	54.0	2.7	126.0	0.9	....
8–1	0.114	0.085	5.23	302	102.9	2.73	28.7	20.1	1.7	105.4	....	2.5
8–2	0.128	0.093	5.56	292	106.7	2.74	28.6	23.2	1.9	108.8	....	2.1
8–3	0.139	0.099	5.80	292	108.0	2.74	28.6	25.7	2.0	111.1	....	3.1
8–4	0.152	0.105	6.08	280	112.1	2.75	28.5	28.6	2.1	113.7	....	1.6
8–5	0.164	0.111	6.29	276	113.8	2.74	28.6	31.2	2.2	115.3	....	1.5
8–6	0.175	0.116	6.44	275	114.3	2.72	28.6	33.4	2.2	116.3	....	2.0
8–7	0.184	0.119	6.60	266	117.2	2.71	28.6	35.4	2.3	117.3	....	0.1
8–8	0.193	0.123	6.78	257	120.5	2.72	28.6	37.5	2.3	118.8	1.7	....
8–9	0.204	0.128	6.95	256	121.6	2.71	28.7	39.9	2.4	120.0	1.6	....
8–10	0.220	0.134	6.92	270	115.2	2.60	28.7	41.6	2.5	118.2	....	3.0
8–11	0.233	0.138	6.89	281	110.5	2.52	28.7	42.8	2.5	116.9	....	6.4
8–12	0.242	0.141	6.96	275	111.5	2.49	28.7	44.2	2.6	116.9	....	5.4
8–13	0.243	0.142	7.26	247	122.6	2.60	28.8	46.4	2.6	120.7	1.9	....

TABLE 1.—(*Continued*)

Run[a]	y (ft)	R (ft)	V (ft per sec)	S $\times 10^4$	Observed C	$\mathbf{F}$	t (degrees Centigrade)	R $\times 10^{-4}$	c_s	Computed C	DISCREPANCY	
											−	+
(*b*) (*Continued*)												
8–14	0.247	0.143	7.37	240	125.8	2.62	28.7	47.4	2.6	121.7	4.1	....
8–15	0.259	0.147	7.33	242	122.8	2.54	28.7	48.5	2.6	120.2	2.6	....
8–16	0.272	0.151	7.64	252	123.8	2.58	28.7	52.0	2.7	122.8	1.0	....
8–17	0.282	0.154	7.80	250	125.6	2.59	28.7	54.1	2.7	123.8	1.8	....
(*c*) ROUGHNESS XI (FIG. 3); DISCREPANCIES BY EQ. 17												
45–1	0.083	0.067	0.247	3.6	50.4	0.15	21.3	0.626	1.4	55.8	....	5.4
45–2	0.084	0.067	0.308	4.8	54.1	0.19	21.5	0.793	1.4	55.9	....	1.8
45–3	0.145	0.102	0.420	4.2	64.3	0.19	21.6	1.63	2.0	63.7	0.6	....
45–4	0.200	0.126	0.498	4.0	70.1	0.20	21.5	2.39	2.4	67.8	2.3	....
45–5	0.255	0.146	0.578	4.8	69.1	0.20	21.6	3.22	2.6	70.6	....	1.5
45–6	0.309	0.162	0.644	4.4	76.3	0.20	21.6	3.93	2.7	72.5	3.8	....
45–7	0.379	0.179	0.742	5.2	76.9	0.21	21.8	5.11	2.7	74.2	2.7	....
(*d*) ROUGHNESS I (FIG. 3); DISCREPANCIES BY EQ. 17												
44–1	0.098	0.076	0.218	2.6	49.0	0.12	22.5	0.648	1.5	46.7	2.3	....
44–2	0.142	0.100	0.311	3.6	51.7	0.15	22.5	1.22	2.0	51.9	....	0.2
44–3	0.182	0.119	0.412	4.4	57.1	0.17	22.5	1.91	2.3	55.2	1.9	....
44–4	0.254	0.146	0.460	4.6	56.2	0.16	22.5	2.61	2.6	59.0	....	2.8
44–5	0.324	0.166	0.602	5.4	63.6	0.19	21.3	3.79	2.7	61.4	2.2	....
44–6	0.381	0.180	0.583	4.6	64.1	0.17	21.4	3.99	2.7	62.8	1.3	....
25–1	0.150	0.104	0.752	19.8	52.4	0.34	24.2	3.19	2.0	52.6	....	0.2
25–2	0.231	0.138	0.994	20.4	59.3	0.36	24.2	5.56	2.5	57.9	1.4	....
25–3	0.270	0.151	1.06	19.6	61.7	0.36	24.2	6.49	2.6	59.6	2.1	....
25–4	0.345	0.171	1.25	21.4	65.1	0.37	24.3	8.71	2.7	62.0	3.1	....
22–1	0.152	0.105	1.46	78	50.8	0.66	26.7	6.58	2.1	52.9	....	2.1
22–2	0.193	0.123	1.80	79	57.7	0.72	26.7	9.55	2.3	55.8	1.9	....
22–3	0.257	0.146	2.16	79	63.6	0.75	26.7	13.6	2.6	59.1	4.5	....
22–4	0.341	0.170	2.31	79	63.0	0.70	26.7	16.9	2.7	61.8	1.2	....
4–1	0.167	0.112	3.27	306	55.9	1.41	28.1	16.3	2.2	54.1	1.8	....
4–2	0.219	0.133	3.57	309	55.7	1.34	28.4	21.3	2.5	57.4	....	1.7
4–3	0.262	0.148	3.81	312	56.0	1.31	28.3	25.1	2.6	59.3	....	3.3
4–4	0.300	0.159	4.00	312	56.8	1.29	28.3	28.5	2.7	60.7	....	3.9
(*e*) ROUGHNESS II (FIG. 3); DISCREPANCIES BY EQ. 17												
43–1	0.093	0.073	0.198	4.5	34.5	0.11	22.3	0.564	1.5	35.2	....	0.7
43–2	0.130	0.094	0.243	4.0	39.6	0.12	22.3	0.890	1.9	40.0	....	0.4
43–3	0.199	0.126	0.368	5.0	46.5	0.15	22.3	1.80	2.4	45.6	0.9	....
43–4	0.240	0.141	0.345	3.6	48.5	0.12	22.3	1.88	2.6	47.7	0.8	....
43–5	0.317	0.164	0.509	5.5	53.6	0.16	22.4	3.25	2.6	50.4	3.2	....
43–6	0.391	0.182	0.556	5.8	54.0	0.16	22.4	3.94	2.6	52.2	1.8	....
26–5	0.092	0.072	0.410	20.4	33.6	0.24	23.0	1.17	1.5	35.1	....	1.5
26–6	0.134	0.096	0.612	21.6	42.5	0.29	23.0	2.32	1.9	40.4	2.1	....
26–1	0.168	0.112	0.718	21.2	46.3	0.31	22.2	3.13	2.2	43.4	2.9	....
26–2	0.238	0.140	0.874	21.8	49.9	0.32	22.9	4.83	2.5	47.6	2.3	....
26–3	0.277	0.153	0.911	20.8	51.2	0.30	23.0	5.50	2.7	49.3	1.9	....
26–4	0.339	0.170	1.03	21.6	53.4	0.31	23.1	6.90	2.7	51.1	2.3	....
21–5	0.108	0.082	0.959	78	37.9	0.52	26.5	3.37	1.7	37.5	0.4	....
21–6	0.134	0.096	1.11	79	40.3	0.54	26.6	4.60	1.9	40.4	....	0.1
21–1	0.150	0.104	1.18	77	41.7	0.54	26.3	5.25	2.0	41.9	....	0.2
21–7	0.178	0.117	1.33	78	44.0	0.56	26.7	6.69	2.2	44.1	....	0.1
21–2	0.190	0.122	1.35	82	42.7	0.55	26.4	7.04	2.3	45.0	....	2.3
21–8	0.247	0.143	1.56	79	46.5	0.55	26.7	9.62	2.6	48.0	....	1.5
21–3	0.255	0.146	1.60	79	47.2	0.56	26.4	9.96	2.6	48.4	....	1.2
21–9	0.313	0.163	1.75	78	49.1	0.55	26.8	12.3	2.7	50.4	....	1.3
21–4	0.319	0.165	1.93	78	54.0	0.60	26.5	13.6	2.7	50.6	3.4	....

TABLE 1.—(*Continued*)

Run[a]	y (ft)	R (ft)	V (ft per sec)	$S \times 10^4$	Observed C	$\mathbf{F}$	t (degrees Centigrade)	$\mathbf{R} \times 10^{-4}$	c_s	Computed C	DISCREPANCY	
											−	+
(*e*) (*Continued*)												
5–1	0.171	0.114	2.71	310	45.6	1.06	28.4	13.8	2.2	43.6	2.0	....
5–2	0.236	0.139	3.02	310	46.0	1.10	28.6	18.9	2.5	47.5	....	1.5
5–3	0.287	0.156	3.24	312	46.3	1.06	28.7	22.7	2.7	49.6	....	3.3
5–4	0.331	0.168	3.41	310	47.3	1.04	28.6	25.7	2.7	50.9	....	3.6
(*f*) ROUGHNESS III (FIG. 3); DISCREPANCIES BY EQ. 17												
42–1	0.084	0.067	0.160	5.1	27.3	0.10	21.3	0.410	1.4	26.7	0.6	....
42–2	0.139	0.099	0.229	4.9	32.9	0.11	21.6	0.864	2.0	34.0	....	1.1
42–3	0.191	0.122	0.297	4.8	38.7	0.12	22.0	1.40	2.3	38.0	0.7	....
42–4	0.253	0.145	0.389	6.8	39.2	0.14	22.0	2.18	2.6	41.3	....	2.1
42–5	0.328	0.167	0.414	5.6	42.8	0.13	22.2	2.68	2.7	43.8	....	1.0
42–6	0.378	0.179	0.392	4.5	43.7	0.11	22.2	2.72	2.7	45.0	....	1.3
27–1	0.105	0.080	0.447	21.2	34.3	0.24	22.9	1.41	1.6	30.0	4.3	....
27–2	0.154	0.106	0.578	22.7	37.3	0.26	23.0	2.42	2.1	35.3	2.0	....
27–3	0.185	0.120	0.662	22.0	40.8	0.27	23.0	3.14	2.3	37.6	3.2	....
27–4	0.259	0.147	0.800	23.2	43.3	0.28	23.1	4.66	2.6	41.5	1.8	....
27–5	0.307	0.161	0.880	25.0	43.8	0.28	23.0	5.61	2.7	43.2	0.6	....
20–1	0.141	0.100	1.11	77	40.1	0.52	26.1	4.71	2.0	34.1	6.0	....
20–2	0.186	0.120	1.27	78	41.5	0.52	26.1	6.50	2.3	37.7	3.8	....
20–3	0.233	0.138	1.42	77	43.7	0.52	26.2	8.39	2.5	40.3	3.4	....
20–4	0.306	0.161	1.59	79	44.6	0.51	26.2	10.9	2.7	43.2	1.4	....
6–1	0.142	0.100	2.01	309	36.3	0.94	28.5	9.06	2.0	34.2	2.1	....
6–2	0.189	0.122	2.43	309	39.6	0.98	28.5	13.2	2.3	37.9	1.7	....
6–3	0.232	0.138	2.74	309	41.9	1.00	28.5	16.9	2.5	40.3	1.6	....
6–4	0.284	0.155	3.00	309	43.4	0.99	28.6	20.9	2.7	42.5	0.9	....
6–5	0.316	0.164	3.18	309	44.7	1.00	28.6	23.4	2.7	43.5	1.2	....
(*g*) ROUGHNESS IV (FIG. 3); DISCREPANCIES BY EQ. 17												
41–1	0.087	0.069	0.139	4.4	25.2	0.08	22.1	0.372	1.4	26.6	....	1.4
41–2	0.146	0.102	0.255	6.0	32.6	0.12	22.1	1.01	2.0	34.0	....	1.4
41–3	0.196	0.124	0.263	4.3	36.0	0.10	22.0	1.26	2.4	37.8	....	1.8
41–4	0.259	0.147	0.360	5.8	39.0	0.13	22.0	2.04	2.6	40.9	....	1.9
41–5	0.320	0.165	0.333	5.4	35.3	0.10	22.0	2.12	2.7	43.0	....	7.7
41–6	0.381	0.180	0.387	5.2	40.1	0.11	22.1	2.69	2.7	44.5	....	4.4
28–1	0.099	0.077	0.346	21.0	27.3	0.19	21.6	1.01	1.6	28.6	....	1.3
28–2	0.143	0.101	0.496	17.5	37.4	0.24	21.9	1.93	2.0	33.7	3.7	....
28–3	0.197	0.125	0.623	20.4	39.0	0.25	22.0	3.00	2.4	37.8	1.2	....
28–4	0.274	0.152	0.756	21.8	41.6	0.26	22.2	4.45	2.7	41.6	0.0	....
28–5	0.348	0.172	0.816	21.6	42.3	0.24	22.2	5.44	2.7	43.7	....	1.4
19–1	0.131	0.095	1.03	75	38.6	0.50	26.1	4.13	1.9	32.5	6.1	....
19–2	0.189	0.122	1.26	75	41.6	0.51	26.2	6.50	2.3	37.3	4.3	....
19–3	0.242	0.141	1.41	75	43.4	0.51	26.2	8.50	2.6	40.2	3.2	....
19–4	0.342	0.170	1.61	76	44.7	0.48	26.2	11.7	2.7	43.6	1.1	....
7–1	0.203	0.127	2.42	309	38.6	0.95	28.6	13.8	2.4	38.2	0.4	....
7–2	0.235	0.139	2.64	310	40.2	0.96	28.7	16.5	2.5	39.8	0.4	....
7–3	0.263	0.148	2.82	312	41.4	0.96	28.7	18.8	2.6	41.0	0.4	....
7–4	0.290	0.157	2.93	310	42.1	0.96	28.8	20.7	2.7	42.1	0.0	....
7–5	0.316	0.164	3.04	312	42.5	0.95	28.8	22.4	2.7	42.9	....	0.4
(*h*) ROUGHNESS V (FIG. 3); DISCREPANCIES BY EQ. 17												
40–1	0.094	0.074	0.179	4.8	30.1	0.10	22.4	0.514	1.5	30.5	....	0.4
40–2	0.146	0.102	0.238	4.4	35.5	0.11	22.3	0.944	2.0	36.7	....	1.2
40–3	0.226	0.136	0.382	5.8	43.0	0.14	22.2	1.27	2.5	42.1	0.9	....
40–4	0.190	0.122	0.297	4.3	41.0	0.12	22.2	1.40	2.3	40.1	0.9	....
40–5	0.274	0.152	0.384	5.1	43.6	0.13	22.2	2.26	2.7	41.3	....	0.7
40–6	0.319	0.165	0.379	4.4	44.5	0.12	22.2	2.42	2.7	45.7	....	1.2
40–7	0.393	0.182	0.447	5.2	45.9	0.13	22.2	3.16	2.7	47.4	....	1.5

TABLE 1.—(*Continued*)

Run[a]	y (ft)	R (ft)	V (ft per sec)	S $\times 10^4$	Observed C	**F**	t (degrees Centigrade)	R $\times 10^{-4}$	c_s	Computed C	Discrepancy	
											−	+
(*h*) (*Continued*)												
29–1	0.110	0.083	0.379	19.0	30.2	0.20	22.3	1.22	1.7	32.8	....	2.6
29–2	0.148	0.103	0.540	19.6	38.0	0.25	22.4	2.17	2.0	36.8	1.2	....
29–3	0.186	0.120	0.642	20.2	41.2	0.26	22.4	3.01	2.3	39.8	1.4	....
29–4	0.287	0.156	0.765	18.8	44.7	0.25	22.6	4.66	2.7	44.7	0.0	....
29–5	0.330	0.168	0.869	21.0	46.3	0.27	22.6	5.70	2.7	46.0	0.3	....
18–1	0.171	0.114	0.981	71	34.5	0.42	25.8	4.71	2.2	38.7	....	4.2
18–3	0.144	0.101	1.12	78	39.8	0.52	25.9	4.78	2.0	36.5	3.3	....
18–2	0.204	0.128	1.37	79	43.3	0.54	25.8	7.40	2.4	40.9	2.4	....
18–4	0.256	0.146	1.54	78	45.6	0.53	25.9	9.50	2.6	43.5	2.1	....
18–5	0.332	0.168	1.70	79	46.6	0.52	26.0	12.1	2.7	46.0	0.6	....
9–1	0.199	0.126	2.56	309	41.1	1.01	28.5	14.4	2.4	40.7	0.4	....
9–2	0.246	0.143	2.86	309	43.0	1.01	28.4	18.2	2.6	43.1	....	0.1
9–3	0.346	0.172	3.29	312	45.0	0.99	28.5	25.3	2.7	46.4	....	1.4
(*i*) ROUGHNESS VI (FIG. 3); DISCREPANCIES BY EQ. 17												
39–1	0.102	0.078	0.197	4.6	32.8	0.10	22.1	0.600	1.6	35.2	....	2.4
39–2	0.127	0.092	0.238	4.4	37.3	0.11	22.1	0.851	1.8	38.2	....	0.9
39–3	0.211	0.130	0.369	5.2	44.8	0.14	22.0	1.86	2.4	44.8	0.0	....
39–4	0.247	0.143	0.380	5.4	43.2	0.14	22.1	2.10	2.6	46.6	....	3.4
39–5	0.321	0.165	0.494	5.8	50.5	0.15	22.1	3.16	2.7	49.2	1.3	....
39–6	0.405	0.185	0.573	6.9	50.7	0.16	22.1	4.10	2.7	51.2	....	0.5
30–1	0.106	0.081	0.396	19.2	31.8	0.21	22.1	1.24	1.6	35.7	....	3.9
30–2	0.152	0.105	0.604	20.8	40.8	0.27	22.2	2.46	2.1	40.8	0.0	....
30–3	0.205	0.128	0.734	20.8	45.0	0.29	22.3	3.65	2.4	44.5	0.5	....
30–4	0.285	0.155	0.838	20.0	47.6	0.28	22.4	5.06	2.7	48.1	....	0.5
30–5	0.329	0.167	0.919	21.0	49.0	0.28	22.3	5.97	2.7	49.4	....	0.4
17–1	0.155	0.106	1.24	85	41.4	0.56	26.0	5.62	2.1	41.0	0.4	....
17–2	0.195	0.124	1.52	80	48.3	0.61	26.0	7.99	2.3	43.8	4.5	....
17–3	0.245	0.142	1.71	84	49.4	0.61	25.9	10.5	2.6	46.5	2.9	....
17–4	0.316	0.164	1.97	83	53.3	0.62	25.9	13.6	2.7	49.1	4.2	....
10–1	0.187	0.121	2.63	312	42.8	1.07	27.8	14.0	2.3	43.4	....	0.6
10–2	0.250	0.144	3.08	311	46.0	1.09	27.8	19.6	2.6	46.7	....	0.7
10–3	0.325	0.166	3.44	310	48.0	1.06	28.0	25.4	2.7	49.3	....	1.3
(*j*) ROUGHNESS VII (FIG. 3); DISCREPANCIES BY EQ. 17												
38–1	0.107	0.081	0.271	4.4	45.3	0.15	21.7	0.847	1.6	42.2	3.1	....
38–2	0.155	0.106	0.300	4.6	42.9	0.13	21.9	1.23	2.1	47.4	....	4.5
38–3	0.192	0.123	0.339	4.4	46.1	0.13	21.9	1.60	2.3	50.1	....	4.0
38–4	0.242	0.141	0.422	5.4	48.3	0.14	21.9	2.30	2.6	52.8	....	4.5
38–5	0.312	0.163	0.443	4.6	51.2	0.14	21.9	2.78	2.7	55.4	....	4.2
38–6	0.386	0.181	0.558	6.2	52.7	0.16	21.9	3.88	2.7	57.2	....	4.5
31–1	0.098	0.076	0.412	20.2	33.2	0.23	22.6	1.29	1.6	41.1	....	7.9
31–2	0.133	0.096	0.595	20.2	42.8	0.29	22.6	2.23	1.9	45.3	....	2.5
31–3	0.204	0.128	0.818	20.9	50.1	0.32	22.5	4.08	2.4	50.8	....	0.7
31–4	0.268	0.150	0.909	21.4	50.8	0.31	22.6	5.34	2.6	53.8	....	3.0
31–5	0.329	0.167	0.996	21.4	52.6	0.31	22.5	6.51	2.7	55.8	....	3.2
16–1	0.103	0.079	1.20	80	47.5	0.66	25.9	4.00	1.6	41.7	5.8	....
16–2	0.147	0.103	1.47	78	52.0	0.68	25.9	6.39	2.0	46.7	5.3	....
16–3	0.196	0.124	1.72	81	54.1	0.68	26.0	9.05	2.4	50.4	3.7	....
16–4	0.232	0.138	1.90	82	56.4	0.69	26.0	11.1	2.5	52.3	4.1	....
16–5	0.306	0.161	2.09	83	57.2	0.67	26.0	14.3	2.7	55.2	2.0	....
11–1	0.119	0.088	2.12	312	40.5	1.09	27.6	8.23	1.8	43.8	....	3.3
11–2	0.154	0.106	2.56	311	44.6	1.15	27.6	11.9	2.1	47.3	....	2.7
11–3	0.206	0.128	3.08	310	48.9	1.20	27.6	17.4	2.4	50.9	....	2.0
11–4	0.266	0.149	3.49	312	51.1	1.19	27.7	22.9	2.6	53.8	....	2.7
11–5	0.325	0.166	3.78	309	52.7	1.17	27.7	27.6	2.7	55.7	....	3.0

TABLE 1.—(*Continued*)

Run[a]	y (ft)	R (ft)	V (ft per sec)	$S \times 10^4$	Observed C	$\mathbf{F}$	t (degrees Centigrade)	$\mathbf{R} \times 10^{-4}$	c_e	Computed C	Discrepancy	
											−	+
(*k*) Roughness VIII (Fig. 3); Discrepancies by Eq. 17												
37–1	0.083	0.067	0.254	4.6	45.8	0.16	21.2	0.640	1.4	45.6	0.2	....
37–2	0.130	0.094	0.278	3.4	49.2	0.14	21.2	0.992	1.9	52.0	....	2.8
37–3	0.181	0.118	0.422	4.8	56.0	0.18	21.3	1.89	2.3	56.4	....	0.4
37–4	0.257	0.146	0.543	5.6	59.9	0.19	21.3	3.02	2.6	60.4	....	0.5
37–5	0.322	0.165	0.628	6.8	59.2	0.20	21.4	3.95	2.7	62.6	....	3.4
37–6	0.388	0.181	0.724	7.6	61.7	0.20	21.4	4.99	2.7	64.2	....	2.5
32–1	0.097	0.076	0.561	20.8	44.8	0.32	21.1	1.60	1.5	47.8	....	3.0
32–2	0.133	0.096	0.706	20.0	51.0	0.34	21.1	2.55	1.9	52.3	....	1.3
32–3	0.202	0.127	0.926	20.2	57.9	0.36	21.3	4.46	2.4	57.7	0.2	....
32–4	0.245	0.142	0.962	20.5	56.3	0.34	21.8	5.26	2.6	59.9	....	3.6
32–5	0.320	0.165	1.14	20.1	62.7	0.36	21.9	7.25	2.7	62.6	0.1	....
15–1	0.372	0.178	2.41	78	64.8	0.70	25.7	18.1	2.7	63.9	0.9	....
12–1	0.111	0.084	2.60	312	50.8	1.37	27.0	9.43	1.7	49.8	1.0	....
12–2	0.151	0.105	3.20	311	56.1	1.45	27.1	14.5	2.1	54.1	2.0	....
12–3	0.195	0.124	3.72	310	60.0	1.49	27.1	20.0	2.3	57.2	2.8	....
12–4	0.246	0.143	4.12	312	61.7	1.46	27.2	25.6	2.6	60.0	1.7	....
12–5	0.321	0.165	4.55	311	63.4	1.41	27.2	32.7	2.7	62.6	0.8	....
(*l*) Roughness IX (Fig. 3); Discrepancies by Eq. 17												
36–4	0.096	0.075	0.325	4.2	57.9	0.19	21.1	0.92	1.5	58.5	....	0.6
36–5	0.155	0.106	0.466	4.5	67.3	0.21	21.1	1.87	2.1	65.2	2.1	....
36–6	0.200	0.126	0.504	4.3	68.5	0.20	21.1	2.40	2.4	68.4	0.1	....
36–1	0.245	0.142	0.547	4.6	67.6	0.20	21.0	2.93	2.6	70.7	....	3.1
36–7	0.248	0.144	0.571	4.0	75.3	0.20	21.1	3.09	2.6	70.9	4.4	....
36–2	0.334	0.169	0.678	4.4	78.7	0.20	21.1	4.32	2.7	73.8	4.9	....
36–3	0.404	0.185	0.725	5.0	75.4	0.19	21.0	5.04	2.7	75.4	0.0	....
33–1	0.103	0.079	0.682	20.8	53.1	0.38	21.4	2.05	1.6	59.3	....	6.2
33–2	0.132	0.095	0.809	20.6	57.8	0.39	21.7	2.95	1.9	63.0	....	5.2
33–3	0.153	0.106	0.906	19.2	63.6	0.41	21.5	3.65	2.1	65.0	....	1.4
33–4	0.209	0.130	1.13	21.2	68.4	0.44	21.8	5.64	2.4	68.9	....	0.5
33–5	0.263	0.148	1.26	20.4	72.4	0.43	21.8	7.18	2.6	71.4	1.0	....
33–6	0.348	0.172	1.40	19.6	76.4	0.42	21.9	9.28	2.7	74.1	2.3	....
13–1	0.126	0.092	3.51	312	65.5	1.74	27.0	14.0	1.8	62.4	3.1	....
13–2	0.158	0.108	3.84	312	66.1	1.70	27.0	17.9	2.1	65.4	0.7	....
13–3	0.202	0.127	4.20	312	66.7	1.66	27.1	23.1	2.4	68.5	....	1.8
13–4	0.268	0.150	4.60	312	67.2	1.56	27.1	29.9	2.6	71.6	....	4.4
13–5	0.324	0.166	4.90	310	68.3	1.52	26.9	35.2	2.7	73.5	....	5.2
(*m*) Roughness X (Fig. 3); Discrepancies by Eq. 17												
35–1	0.103	0.079	0.422	4.0	75.1	0.23	21.2	1.26	1.6	70.9	4.2	....
35–2	0.139	0.099	0.486	4.4	73.7	0.23	20.8	1.80	2.0	75.2	....	1.5
35–3	0.203	0.127	0.676	5.4	81.5	0.27	20.8	3.22	2.4	80.0	1.5	....
35–4	0.252	0.145	0.745	5.8	81.2	0.26	20.8	4.04	2.6	82.4	....	1.2
35–5	0.320	0.165	0.855	5.8	87.4	0.27	20.9	5.30	2.7	84.8	2.6	....
35–6	0.394	0.182	0.882	5.8	85.7	0.25	20.9	6.05	2.7	86.6	....	0.9
34–1	0.106	0.081	0.746	18.0	61.9	0.40	21.7	2.31	1.6	71.3	....	9.4
34–2	0.161	0.109	1.12	20.4	75.2	0.49	21.8	4.71	2.1	77.0	....	1.8
34–3	0.210	0.130	1.30	19.4	82.0	0.50	21.9	6.51	2.4	80.3	1.7	....
34–4	0.261	0.148	1.44	19.2	84.8	0.50	21.9	8.16	2.6	82.8	2.0	....
34–5	0.326	0.166	1.52	18.3	87.3	0.47	21.9	9.78	2.7	85.0	2.3	....
14–1	0.120	0.089	3.83	305	73.7	1.95	27.0	14.73	1.8	73.1	0.6	....
14–2	0.142	0.100	4.49	309	80.7	2.10	27.0	19.50	2.0	75.4	5.3	....
14–3	0.194	0.124	4.91	310	79.3	1.96	27.0	26.3	2.3	79.4	....	0.1
14–4	0.239	0.140	5.19	308	78.9	1.87	27.0	31.6	2.6	81.9	....	3.0
14–5	0.300	0.159	5.60	304	80.4	1.80	27.1	38.7	2.7	84.2	....	3.8

TABLE 1.—(*Continued*)

Run[a]	y (ft)	R (ft)	V (ft per sec)	S ×10⁴	Observed C	F	t (degrees Centigrade)	R ×10⁻⁴	c_s	Computed C	DISCREPANCY	
											−	+
(*n*) BAZIN'S TESTS ON PLANK FLUME WITH BATTENS (FLUME WIDTHS—SETUPS 12, 13, AND 15, 6.43 FT; SETUPS 14 AND 17, 6.40 FT; AND SETUP 16, 6.44 FT)[a]												
12–1	0.33	0.302	1.65	15	77.4	0.50	8.5	13.5	0.6	77.4	0.0	....
12–2	0.51	0.442	2.17	15	84.5	0.54	8.5	26.6	1.0	84.4	0.1	....
12–3	0.89	0.634	2.86	15	91.0	0.54	8.5	49.3	1.4	91.1	....	0.1
12–4	1.02	0.775	3.33	15	94.0	0.58	8.5	70.1	1.7	94.9	....	0.9
12–5	1.23	0.889	3.68	15	97.0	0.59	8.5	88.9	1.9	97.5	....	0.5
12–6	1.42	0.986	3.98	15	99.0	0.59	8.5	107	2.1	99.5	....	0.5
12–7	1.62	1.706	4.19	15	99.0	0.58	8.5	122	2.2	101.1	....	2.1
13–1	0.22	0.205	2.50	59	71.8	0.94	7.0	13.3	0.4	70.5	1.3	....
13–2	0.33	0.302	3.34	59	79.0	1.03	7.0	26.2	0.6	77.4	1.6	....
13–3	0.51	0.442	4.40	59	86.2	1.09	7.0	50.6	1.0	84.4	1.8	....
13–4	0.67	0.552	5.08	59	89.0	1.10	7.0	72.9	1.2	88.5	0.5	....
13–5	0.80	0.643	5.63	59	91.4	1.11	7.0	94.1	1.4	91.3	0.1	....
13–6	0.92	0.716	6.14	59	94.5	1.13	7.0	114	1.5	93.3	1.2	....
13–7	1.05	0.790	6.48	59	94.8	1.12	7.0	133	1.7	95.2	....	0.4
14–1	0.19	0.182	2.85	89	70.8	1.15	14.0	16.4	0.3	68.3	2.5	....
14–2	0.30	0.273	3.75	89	76.4	1.21	14.0	32.4	0.6	75.6	0.8	....
14–3	0.46	0.403	4.92	89	82.4	1.28	14.0	62.7	0.9	82.7	....	0.3
14–4	0.59	0.499	5.77	89	86.8	1.32	14.0	91.0	1.1	86.6	0.2	....
14–5	0.71	0.582	6.38	89	88.9	1.33	14.0	117	1.3	89.5	....	0.6
14–6	0.83	0.658	6.86	89	89.9	1.34	14.0	143	1.4	91.7	....	1.8
14–7	0.94	0.726	7.26	89	90.5	1.32	14.0	167	1.6	93.6	....	3.1
15–1	0.43	0.378	1.28	15	53.7	0.34	16.5	16.3	0.8	50.5	3.2	....
15–2	0.66	0.550	1.68	15	58.6	0.36	16.5	31.1	1.2	57.4	1.2	....
15–3	1.02	0.777	2.21	15	64.8	0.39	16.5	57.9	1.7	63.9	0.9	....
15–4	1.33	0.942	2.55	15	67.8	0.39	16.5	80.9	2.0	67.6	0.2	....
15–5	1.61	1.073	2.81	15	70.1	0.39	16.5	102	2.2	70.0	0.1	....
15–6	1.91	1.197	2.97	15	70.0	0.38	16.5	120	2.4	72.2	....	2.2
15–7	2.18	1.299	3.11	15	70.5	0.37	16.5	136	2.5	73.7	....	3.2
16–1	0.29	0.264	1.91	59	48.3	0.62	7.5	13.3	0.6	44.1	4.2	....
16–2	0.44	0.384	2.56	59	53.7	0.68	7.5	25.9	0.8	50.8	2.9	....
16–3	0.67	0.553	3.37	59	59.0	0.73	7.5	49.2	1.2	57.5	1.5	....
16–4	0.89	0.686	3.88	59	61.0	0.73	7.5	70.3	1.5	61.6	....	0.6
16–5	1.05	0.791	4.31	59	63.1	0.74	7.5	90.0	1.7	64.3	....	1.2
16–6	1.21	0.882	4.65	59	64.5	0.75	7.5	108	1.9	66.3	....	1.8
16–7	1.38	0.965	4.91	59	65.1	0.74	7.5	125	2.0	68.0	....	2.9
17–1	0.25	0.232	2.21	88.6	48.7	0.78	16.5	17.3	0.5	41.7	7.0	....
17–2	0.39	0.350	2.85	88.6	51.2	0.81	16.5	33.6	0.8	49.2	2.0	....
17–3	0.60	0.509	3.75	88.6	55.8	0.85	16.5	64.3	1.1	56.0	....	0.2
17–4	0.78	0.628	4.37	88.6	58.6	0.88	16.5	92.5	1.3	59.8	....	1.2
17–5	0.94	0.725	4.85	88.6	60.5	0.88	16.5	118	1.6	62.6	....	2.1
17–6	1.09	0.812	5.22	88.6	61.5	0.88	16.5	143	1.7	64.7	....	3.2
17–7	1.22	0.885	5.57	88.6	62.9	0.89	16.5	166	1.9	66.4	....	3.5

[a] The notation 46–1, for example, refers to run 1 in setup 46. In Table 1 (*n*), setups 12 to 17, the run numbers are as actually reported by Bazin.

TABLE 2.—NUMERICAL VALUES OF ROUGHNESSES

Roughness (see Fig. 3)	ϵ (ft)	Actual size h (ft)	$\frac{\epsilon}{h}$	Spacing s (ft)	$\frac{s}{h}$
(*a*) WRITER'S OBSERVATIONS					
XI	0.00292	0.02083	0.140	3.333	160
I	0.00564	0.02083	0.271	1.667	80
II	0.01048	0.02083	0.503	0.833	40
III	0.01568	0.02083	0.753	0.417	20
IV	0.01603	....	....	0.208	....
V	0.01372	....	....	0.208	....
VI	0.01128	....	....	0.208	....
VII	0.00784	0.01042	0.753	0.208	20
VIII	0.00524	0.01042	0.503	0.417	40
IX	0.00282	0.01042	0.271	0.833	80
X	0.00146	0.01042	0.140	1.677	160
(*b*) BAZIN'S DATA (TABLE 1(*n*))					
Setups 12, 13, 14	0.00363	0.0328	0.111	0.121[a]	3.70
Setups 15, 16, 17	0.0216	0.0328	0.659	0.253[b]	7.70

[a] Battens spaced closely. [b] Battens placed wide apart

DISCUSSION

JOE W. JOHNSON,[26] ASSOC. M. AM. SOC. C. E., AND E. A. LEROUX,[27] ESQ.—The author is to be congratulated for a splendid experimental investigation which is a welcome contribution toward a better understanding of the problem of channel roughness. Similar investigations in the University of California Fluid Mechanics Laboratory at Berkeley were made with water in both open and closed channels, and with air in closed channels. Various types of artificial roughnesses were used. Experiments by V. J. Skoglund[28] were made with water in rectangular channels roughened by 60° V grooves transverse to the direction of flow. The grooves were 0.060 in. apart with an average depth of 0.0125 in. The hydraulic radius was varied from 0.030 ft to 0.230 ft. Later experiments were made by J. E. Ogden[29] using this same apparatus but with a groove spacing one half that used by Mr. Skoglund; that is, the roughness grooves were 0.030 in. center to center. Tests were made with the hydraulic radius of the channel equal to 0.019 ft. The results of experiments in a a triangular open channel with the roughness formed by attaching sand grains of constant mean diameter to the channel walls have been published by T. DeF. Rogers.[30]

Other open-channel experiments were made by M. E. Fuller and R. E. Graham, Juniors, Am. Soc. C. E., in a rectangular flume $9\frac{1}{2}$ in. by $9\frac{1}{2}$ in. in cross section and 70 ft long.[31] The type of roughnesses included the following combinations: (*a*) Smooth sides and bottom coated with enamel; (*b*) sides and bottom covered with a 90-lb Flintkote roofing paper with rough side exposed; (*c*) enameled sides with bottom covered with the roofing paper; (*d*) enamel sides with $\frac{1}{2}$-in. mesh hardware cloth tacked over the roofing paper on the bottom. These roughnesses are not "definite" in the sense implied by the author; hence, the results are not summarized herein.

Of particular value in connection with a discussion of the author's data are the results of various investigations on roughnesses, rectangular in cross section. Although most of these data have been discussed elsewhere,[32] a summary (Table 3) in terms of the author's notation is necessary for comparative purposes. For each of the roughnesses tested by the various experimenters, the data in Table 3 include the following terms (see Fig. 12): Center-to-center spacing, s, of the roughness battens; the batten height, h; the batten width, t; the width-height ratio, t/h; and the spacing ratio, s/h. The observa-

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[27] Graduate Student, Univ. of California, Berkeley, Calif.

[28] "Effect of Roughness on the Friction Coefficient of a Closed Channel," by V. J. Skoglund, *Journal of the Aeronautical Sciences*, Vol. 4, November, 1936, pp. 28–29.

[29] "Artificial Roughness," by J. E. Ogden, Univ. of California Fluid Mechanics Laboratory, Berkeley, Calif., 1944 (unpublished report).

[30] "Friction in Hydraulic Models," by Thomas DeF. Rogers, *Civil Engineering*, June, 1939, p. 367.

[31] "A Study of the Distribution of Frictional Energy Losses in Open Channels," by M. E. Fuller and R. E. Graham, thesis submitted to the Univ. of California, Berkeley, Calif., in partial fulfilment of the requirements for the degree of Bachelor of Science, 1942 (unpublished).

[32] "Rectangular Artificial Roughness in Open Channels," by J. W. Johnson, *Transactions*, Am. Geophysical Union, Hydrology Section, 1944.

TABLE 3.—SUMMARY OF INVESTIGATIONS ON RECTANGULAR ROUGHNESSES, UNIVERSITY OF CALIFORNIA

Roughness (see Fig. 13)	Dimensions (Inches)			$\frac{t}{h}$	$\frac{s}{h}$	k (ft)	$\frac{k}{h}$	n_b
	h	t	s					
(a) W. Tripp and F. Jonassen								
....	1/8	1/2	5/8	4	5	0.000652	0.0625	....
1	1/8	3/16	5/16	1.5	2.5	0.00176	0.069	....
2	1/8	1/8	5/8	1	5	0.0264	2.53	....
3	1/8	1/8	1 1/4	1	10	0.0575	5.52	....
4	1/8	1/8	2 1/2	1	20	0.0364	3.49	....
5	1/8	1/8	5	1	40	0.0155	1.49	....
(b) C. A. Smith, Jr., and C. Warren								
A	1/8	5/32	5/16	1.25	2.5	0.0022	0.21	0.0112
B	1/8	5/32	5/8	1.25	4	0.0432	4.15	0.0185
C	1/8	5/32	1 1/4	1.25	10	0.0525	5.04	0.0200
D	1/8	5/32	2 1/2	1.25	20	0.0313	3.00	0.0170
E	1/8	5/32	5	1.25	40	0.0125	1.20	0.0149
(c) J. W. Johnson								
A	1/4	1	2	4	8	0.0202	0.97	0.0163
B	1/4	1	4	4	16	0.1297	6.23	0.0244
C	1/4	1	8	4	32	0.0698	3.35	0.0208
D	1/4	1	2 1/2	4	10	0.0500	2.40	0.0194
E	1/4	1	5	4	20	0.1185	5.69	0.0236
F	1/4	1	3	4	12	0.1050	5.04	0.0229
G	1/4	1	6	4	24	0.0940	4.51	0.0223
H	1/4	1	3 1/2	4	14	0.1255	6.02	0.0240
I	1/4	1	4 1/2	4	18	0.1270	6.10	0.0240
(d) E. A. LeRoux								
J	1/4	3/4	1 1/2	3	3	0.0176	0.846	0.0158
K	1/4	3/4	3	3	12	0.0925	4.44	0.0222
L	1/4	3/4	6	3	24	0.0549	2.64	0.0198
M	1/4	3/4	2 1/4	3	10	0.0641	3.08	0.0204
N	1/4	3/4	4 1/2	3	18	0.0840	4.03	0.0219
O	1/4	3/4	9	3	36	0.0163	0.783	0.0177
P	1/4	3/4	3 3/4	3	15	0.0870	4.18	0.0222

tions of W. Tripp[33] and F. Jonassen[34] were similar to those of Mr. Skoglund, except that air was used in wooden rectangular conduits roughened by saw cuts transverse to the direction of flow (Fig. 13). The investigations by C. A. Smith, Jr., and C. Warren, Juniors, Am. Soc. C. E., and by the writers were made in a wooden flume $9\frac{1}{2}$ in. by $9\frac{1}{2}$ in. in cross section and 70 ft long.[35] In the experiments by Messrs. Smith and Warren the roughness was formed by saw cuts made in removable planks fastened to the flume bottom.

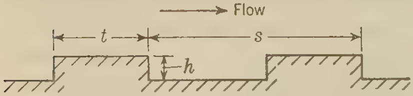

FIG. 12.—DESIGNATION OF DIMENSIONS OF RECTANGULAR ROUGHNESS

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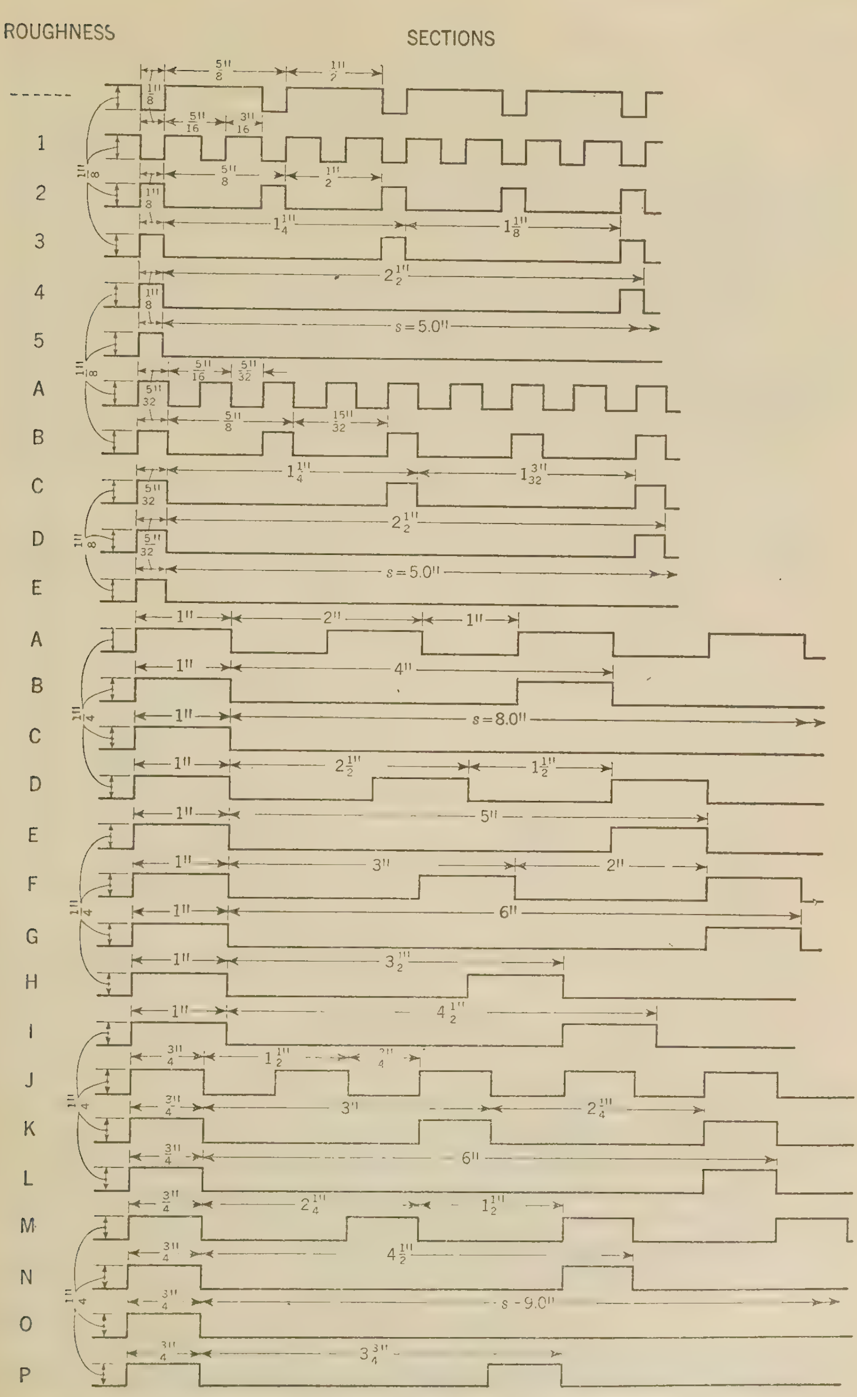
[33] "Friction Losses in an Artificially Roughened Rectangular Channel," by W. Tripp, *Journal of the Aeronautical Sciences*, November, 1936, pp. 10–11.

[34] "Artificial Roughness," by F. Jonassen, Univ. of California Fluid Mechanics Laboratory, Berkeley, Calif. (unpublished).

[35] "Friction Losses in Artificially Roughened Channels," by C. A. Smith and C. Warren, thesis submitted to Univ. of California, Berkeley, Calif., in partial fulfilment of the requirements for the degree of Bachelor of Science, 1943 (unpublished).


ROUGHNESS
SECTIONS

1
2
3
4
5
A
B
C
D
E
A
B
C
D
E
F
G
H
I
J
K
L
M
N
O
P
5"/8
1"/2
1"/8
5"/16
3"/16
1 1"/4
1 1"/8
2 1"/2
s = 5.0"
5"/32
15"/32
1 3"/32
1"
2"
4"
s = 8.0"
2 1"/2
1 1"/2
5"
3"
6"
3 1"/2
4 1"/2
3"/4
2 1"/4
s = 9.0"
3 3"/4
1"/8
1"/4

FIG. 13.—VARIOUS RECTANGULAR ROUGHNESSES TESTED AT THE UNIVERSITY OF CALIFORNIA (NOTE SCALE CHANGES)

Different spacing ratios (s/h) were made by removing alternate ridges between saw cuts (see Fig. 13). It is of interest to note that the average value of Manning's n for the enameled surface was 0.0091, whereas the value of n for the bottom after all roughness ridges had been removed was 0.0104. In the writers' experiments the roughness was made by nailing redwood strips to the bottom of the flume transverse to the direction of flow. Strips 1 in. by $\frac{1}{4}$ in. were used for roughnesses in the Johnson experiments; whereas $\frac{3}{4}$-in. by $\frac{1}{4}$-in. strips were used by Mr. LeRoux (Fig. 13).

The method of calculating the hydraulic radius, Manning's n, and the equivalent sand roughness in the experiments of Messrs. Smith and Warren, and of the writers, have been described elsewhere;[32] however, because of differences in the experimental equipment and the method of analysis of the data a brief review is of interest. In the author's experiments both the bottom and sides of the flume were covered by roughness battens, and the results were analyzed with G. H. Keulegan's formulas as the basis. In the writers' experiments only the channel bottom was roughened. The side-wall effect first was "eliminated" by the method suggested by H. A. Einstein,[36] Assoc. M. Am. Soc. C. E., and then the hydraulic radius of the bed, R_b, Manning's n of the bed, n_b, and the equivalent sand roughness of the bed, k, were computed, the latter term being computed by the Nikuradse equation:

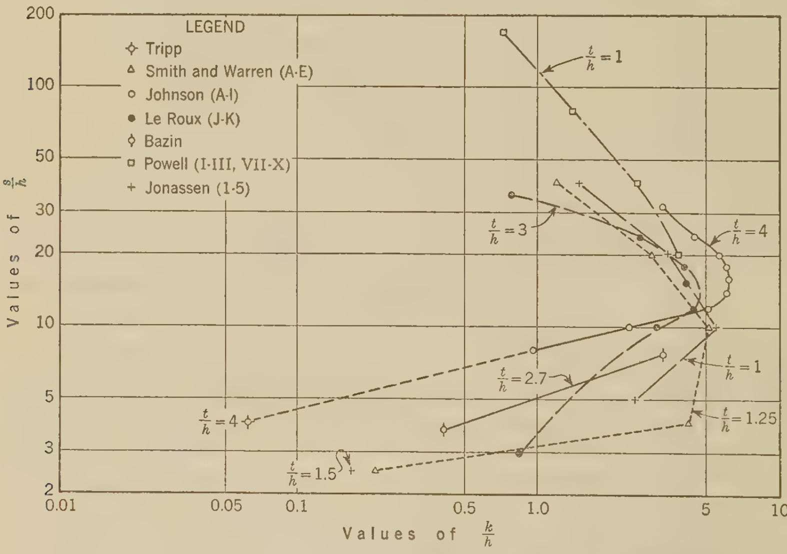


FIG. 14.—RELATION OF RELATIVE ROUGHNESS TO SPACING FOR VARIOUS DATA

$$\frac{1}{\sqrt{f}} = 2 \log_{10}\left(\frac{2\,R_b}{k}\right) + 1.74 \ldots\ldots\ldots\ldots\ldots\ldots (19)$$

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[36] "Formulas for the Transportation of Bed-Load," by H. A. Einstein, *Transactions*, Am. Soc. C. E., Vol. 107 (1942), pp. 575–577.

in which

$$f = \frac{8\,g\,R_b\,S_e}{V^2} \qquad (20)$$

and S_e is the slope of the energy grade line.

The data in Table 3 are of value in giving a more complete picture of the relation of relative roughness to spacing than that given by the author in Fig. 10. Fig. 14 shows a plot of these data and indicates that the author's data and Bazin's results probably are not part of the same curve but appear to be branches of two separate curves. The author has called attention to the uncertainty of a single curve. Fig. 14 indicates that the width-height ratio, t/h, may be of importance. The various curves indicate that the maximum relative roughness occurs in the vicinity of s/h equal to approximately 15 instead of 12. The value of the maximum relative roughness appears to be approximately a constant regardless of the value of t/h. On either side of the point of maximum roughness there is considerable variation in the curves; however, for a particular value of s/h, the roughness appears to be larger with the smaller values of t/h.

It is to be noted that the values of k and n_b (Table 3) are averages. The values of relative roughness for the data of the author and H. Bazin plotted in Fig. 14 are not the values given in Table 2. The appropriate values are:

Roughness	s/h	k/h
Bazin—		
Close spacing	3.7	0.41
Wide spacing	7.7	3.36
The author—		
III and VII	20	3.89
II and VIII	40	2.60
I and IX	80	1.40
X and XI	160	0.72

The author previously has converted the values of ϵ in Table 2 to the corresponding values of the Nikuradse k.[37]

It is important to note that in the author's experiments the water depth was measured from the water surface to the flume bed without battens in place; whereas, in the writers' experiments, the depth was measured to the top of the battens. At small depths of flow relatively large differences in the results from the two experiments may occur if comparisons are attempted. The author has recognized the uncertainty in determining the true depth and has suggested that the problem be given further study.

Garbis H. Keulegan,[38] Esq.—It is appropriate to discuss briefly the meaning of the coefficients appearing in Eq. 2 which form the basis of the author's working formula, Eq. 7. The characteristic waviness of a smooth wall, which is generally due to imperfections in construction, is expressed by a_s.

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[37] Discussion by R. W. Powell of "Rectangular Artificial Roughness in Open Channels," by J. W. Johnson, *Transactions*, Am. Geophysical Union, Section of Hydrology, 1944.

[38] Physicist, National Bureau of Standards, Washington, D. C.

The effect of waviness of the walls may be augmented by standing waves in the flowing water. In any event waviness produces a periodic variation in the thickness of the laminar sublayer. The constant b is a measure of the convection path l at points near the walls. According to experiments the dependence of l upon z (the distance of a point from the wall) is given by $l = z/b$. The Nikuradse value of b for the flow in a circular pipe of constant diameter is 2.5. In divergent channels the value of b varies with the angle of divergence. This fact might suggest that a more rigorous expression for b should contain a term involving the space rate of variation of the shear velocity V_f. In all probability the correction obtained in this manner is small and may be safely ignored. The coefficient β takes the ratio of depth to width into consideration. The coefficient $\bar{\epsilon}$ is introduced to correct, approximately, the deviations in the velocities imposed by the free surface of the water and by the angles between the walls and the bottom of the channel.

In effect, the author's method of treating the observed data consists of plotting the quantity $V/V_f - b\,\beta$ against the logarithm of the Reynolds number, the latter being based on the shear velocity. In the event that the accidental deformation of the walls with increase of water depth is negligible, the surface of the water is smooth, the correction coefficient $\bar{\epsilon}$ is sufficiently small, and the locus of observed points is a straight line, then the slope of that line determines the value of b. The author establishes in Fig. 9 a straight line which yields a value for b of 3.04 ± 0.11. Since this value is at variance with the Nikuradse determinations, the author has concluded that there are either serious systematic errors, or b has values for channels different from those for pipes.

Ignoring the possibility of any systematic errors, the writer desires to examine the effect of a finite value of $\bar{\epsilon}$ on the magnitude of b. The consequences of such an effect are not fully discussed by the author. As was mentioned previously, $\bar{\epsilon}$ is supposed to correct for the dynamic differences produced by the free surface of the water and by the angles between the walls and the bottom of the channel. Since the free surface effect is probably small, consider the nonuniform distribution of wall shear produced by the corners in the channel cross section. The shear is minimum at the corners. Denoting the local shear velocity at any point by v_f,

$$v_f = V_f\,(1 + e) \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (21)$$

and $\bar{\epsilon}$ is given by

$$-\,\bar{\epsilon} = \frac{1}{A}\int e\,dA \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (22)$$

in which dA is an elementary area of the channel cross section and A is the total area.[39]

It is obvious that if y/B is small, the correction due to $\bar{\epsilon}$ may easily be ignored. However, if y/B approaches 0.5, the value of $\bar{\epsilon}$ can be appreciable. By making certain simplifying assumptions the error may be evaluated. Suppose that the width of the channel is 2, the depth of water is 1, and the mean shear velocity is $V_f = 1$ (see Fig. 15). These magnitudes may result by selecting proper units of length and time. The shear velocity distribution along the wetted perimeter may be represented with sufficient accuracy by the expressions

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[39] "Laws of Turbulent Flow in Open Channels," by Garbis H. Keulegan, *Research Paper RP 1151*, National Bureau of Standards (U. S.), Vol. 21, December, 1938, Eq. 46.

$$v_f = a_0 + a_2 \sin\left(\frac{\pi}{2} j\right) \qquad \text{(23a)}$$

and

$$v'_f = a_0 + a_1 \sin\left(\frac{\pi}{2} j'\right) \qquad \text{(23b)}$$

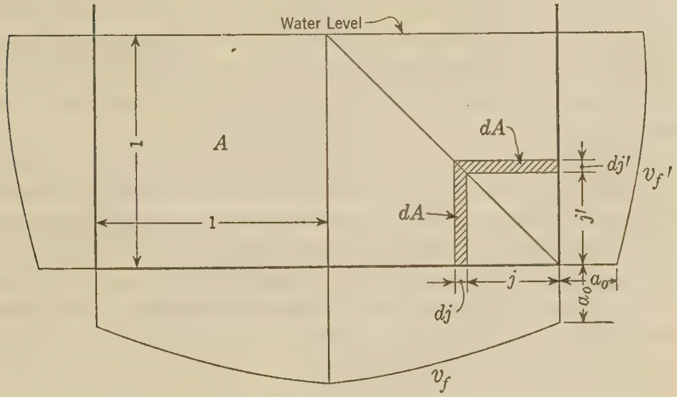

FIG. 15.—DISTRIBUTION OF SHEAR VELOCITY ALONG THE WETTED PERIMETER

of which the first evaluates the shear for the bed, and the second for the side walls. For the meaning of j and j', see Fig. 15. From

$$v_f = 1 + e \qquad \text{(24)}$$

the deviations are

$$e = a_0 + a_2 \sin\left(\frac{\pi}{2} j\right) - 1; \qquad 0 \leqq j \leqq 1 \qquad \text{(25a)}$$

and

$$e' = a_0 + a_1 \sin\left(\frac{\pi}{2} j'\right) - 1; \qquad 0 \leqq j' \leqq 1 \qquad \text{(25b)}$$

Since the average shear velocity along the perimeter is 1, it can be shown readily that

$$\pi a_0 + a_2 + a_1 = \pi \qquad \text{(26)}$$

In the present case, because of symmetry, A may be taken as half of the cross-sectional area and Eq. 22 will be written as

$$-\bar{\epsilon} = \int_0^1 e\, j\, dj + \int_0^1 e'\, j'\, dj' \qquad \text{(27a)}$$

Substituting from Eqs. 25,

$$-\bar{\epsilon} = a_0 - 1 + \frac{4}{\pi^2}(a_2 + a_1) \qquad \text{(27b)}$$

Substituting from Eq. 26,

$$-\bar{\epsilon} = \left(1 - \frac{4}{\pi}\right)(a_0 - 1) \qquad \text{(28a)}$$

or

$$\bar{\epsilon} = 0.274\,(a_0 - 1) \qquad \text{(28b)}$$

Accordingly, the value of $\bar{\epsilon}$ is readily obtained once the shear velocity at a corner is known. Let it be supposed, as a matter of illustration, that, when $y/B = 1/2$, a_0 equals 0.5. From Eq. 28*b* it is then seen that $\bar{\epsilon} = -0.13$. If y/B is made smaller, $\bar{\epsilon}$ becomes correspondingly smaller. When the corrections are introduced into Fig. 9, the following changes become apparent. All the points of the diagram are moved downward. The points that were obtained for the greater depths show the greater displacements. Thus, the new line through the corrected positions of the points will have a smaller slope as the result of the unequal displacements of points. Accordingly, the new determination of *b* may show a smaller disparity from the Nikuradse value. The true modifications can be discussed with certainty only when a_0 is determined experimentally.

The thought expressed by the author on the influence of the Froude number is fruitful. Eq. 12 gives his law of resistance for shooting flow in a smooth channel. There is the possibility, however, that the proposed formula is too restricted and therefore in its present form may be misleading. This misgiving has led the writer to the following simple analysis. It may be assumed, as a matter of hypothesis, of course, that the greater resistance encountered in the tests of shooting flow is due to the waviness of the standing waves in the channel. In the formula for the Chézy coefficient

$$C = J' + m \log_{10} \frac{\mathrm{R}}{C} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (29)$$

m has the same value as in tranquil flow ($m = 40$), but the value of J' is changed through the induced waviness of the surface waters. Values of J' obtained from the data in Table 1(*b*) and computed by Eq. 29 are shown in Fig. 16.

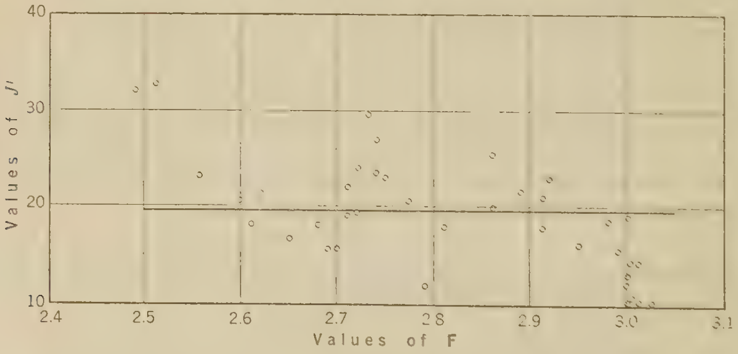

FIG. 16.—RELATIONSHIP OF J' TO FROUDE NUMBER FOR SHOOTING FLOW

As the scatter of the points for the small range of the Froude number considered is pronounced, the line of constant ordinate drawn through the points is judged to be sufficiently representative of the data. This ordinate, which is the average of all the points, has the value of − 19.5. The resulting resistance law

$$C = -20 + 40 \log_{10} \frac{\mathrm{R}}{C} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (30)$$

is offered as an alternate to the author's law as expressed by Eq. 12. A graph of the quantity J', as obtained from all the tests with the smooth channel, is shown in Fig. 17. The interpretation of the diagram is that, at some critical value of the Froude number, J' changes suddenly, but after the change has occurred its new value probably is independent of the Froude number; or, if J' does depend on the Froude number, the relationship between them is of a nature different from that implied by the author's relation given by Eq. 12. The importance of the problem is great enough to require extensive new experiments on the régime of shooting flow.

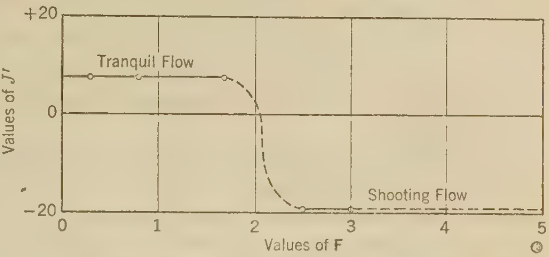

Fig. 17.—Relationship of J' to Froude Number for Tranquil and for Shooting Flow

The need for various studies in the field of flow in open channels is great. Undoubtedly, men of the profession will feel that the author has made a valuable contribution.

C. J. Posey,[40] M. Am. Soc. C. E.—Engineering uses of the science of hydraulics have about reached the point where improvement of the favorite open-channel friction formulas is economically justified. Professor Powell's paper should help to convince the complacent that research on this fundamental problem is badly needed. Although the Kutter coefficient can be predicted to within a thousandth or two, under favorable circumstances, the corresponding uncertainty in the major variables is appreciable. Where long channels form important links in great projects, this lack of precision can be costly.

The two important variables that give trouble, in evaluating friction flow in open channels, are the roughness and the shape of the cross section. Thorough investigation of the effect of the latter has been avoided, in obtaining results for practical use, by testing shapes similar to those used in design. The effect of channel shape has been oversimplified in the "rational" formulas for open-channel flow. A reliable means of evaluating the effect of channel shape, well substantiated by experimentation, would seem to be a necessary part of any satisfactory solution of the roughness problem. It would also free the designer from adherence to traditional shapes, and enable him to take full advantage of new developments in construction methods.

Ralph W. Powell,[41] M. Am. Soc. C. E.—If the writer had known of the work on this subject being done at the University of California, at Berkeley, he would certainly have mentioned it in the paper. Because of this omission he is very grateful to Messrs. Johnson and LeRoux for putting their results (including Mr. LeRoux's hitherto unpublished data) in the form of a discussion of this paper; and the writer agrees heartily with their comments. Fig. 14 is a great improvement over Fig. 10 and shows that the width of the roughnesses cannot be neglected. The writer has already discussed the methods used at the University of California and will not repeat his remarks,[37] except to state that Eq. 17 agrees with his data much better than does Eq. 19. Data are not available to the writer to test this opinion in regard to the investigations listed in Table 3.

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[40] Associate Prof., Hydraulics and Structural Eng.; Research Engr., Iowa Inst. of Hydr. Research, State Univ. of Iowa, Iowa City, Iowa.

[41] Associate Prof. of Mechanics, Ohio State Univ., Columbus, Ohio.

The writer also wishes to thank Mr. Keulegan (whose original paper[14] formed the theoretical basis for much of the writer's paper) for his discussion, especially for his method of obtaining numerical values of $\bar{\epsilon}$. The value of $\bar{\epsilon}$ given in Eq. 28*b* applies, of course, only when $y/B = \frac{1}{2}$. The statement that, if y/B is made smaller, $\bar{\epsilon}$ becomes correspondingly smaller is ambiguous when applied to a negative number, and the writer has therefore investigated the question as follows.

Assuming that Eqs. 23 and 25 apply for any depth of water, y (which is not greater than 1), and that $a_1 = a_2$, and continuing the other assumptions made by Mr. Keulegan:

$$1 + y = \int_0^y \left(a_0 + a_2 \sin \frac{\pi}{2} j' \right) dj' + \int_0^1 \left(a_0 + a_2 \sin \frac{\pi}{2} j \right) dj \ldots (31)$$

which leads to

$$a_2 = \frac{\pi\,(1 - a_0)\,(1 + y)}{2 \left(2 - \cos \frac{\pi}{2} y \right)} \ldots\ldots\ldots\ldots\ldots\ldots (32)$$

Also,

$$-\,\bar{\epsilon}\, y = \int_0^y e'\, j'\, dj' + \int_0^y e\, j\, dj + y \int_y^1 e\, dj \ldots\ldots\ldots\ldots (33)$$

Substituting from Eqs. 25, integrating, and substituting the value of a_2 from Eq. 32:

$$\bar{\epsilon} = \left[1 - \frac{\left(\frac{4}{\pi y} \sin \frac{\pi}{2} y - \cos \frac{\pi}{2} y \right) (1 + y)}{2 - \cos \frac{\pi}{2} y} \right] (1 - a_0) \ldots\ldots (34)$$

For all values of y from 0 to 1, the quantity inside the brackets in Eq. 34 is negative; and, since a_0 is less than 1, $\bar{\epsilon}$ is negative. The values of the ratio of $\bar{\epsilon}$ to $(a_0 - 1)$ are plotted in Fig. 18. As y/B decreases from 0.50 toward zero, the absolute values of $\bar{\epsilon}$ increase slightly to a maximum of about 0.283 $(1 - a_0)$ at $y/B = 0.35$; from this they decrease slowly at first, and then more rapidly, toward zero. The experiments included very few values of y/B less than 0.15, for which $\bar{\epsilon}$ is 0.214 $(a_0 - 1)$.

As Mr. Keulegan states, experiments would be necessary to determine the value of a_0, but it would seem to the writer that it is probably at least 0.6. If $a_1 = a_2$ and $a_0 = 0.6$, Eq. 26 would require a_2 to be 0.628 for $y/B = 0.50$; and v_f would vary from 0.600 at the corner to 1.228 at the middle of the bottom. Since the shear varies as the square of the friction velocity, this would make the shear at the middle more than four times the shear at the corner. For the same case Mr. Keulegan's suggestion of $a_0 = 0.5$ would make the shear at the middle of the bottom 6.6 times the shear at the corner.

The writer had not realized that $\bar{\epsilon}$ would be negative, so that two remarks in the paper are in error, one in the paragraph after Eq. 11 and the other in the second paragraph of the section headed "Probable Accuracy." With b increased 22.5% and $\frac{1}{1 + \bar{\epsilon}}$ increased 12.3%, the maximum value of c_s given by Eq. 6 would be increased from 2.74 to 3.77, making a maximum error of 1.03 instead of 0.6 as stated. Without developing a new curve to replace Fig. 8, it may be stated roughly that the values of c_s should be increased by corrections varying from 0.5 to 1.0. It still seems not worthwhile to recompute the data to eliminate errors of this size when the mean deviations were 2.1, 2.32, and 2.44.

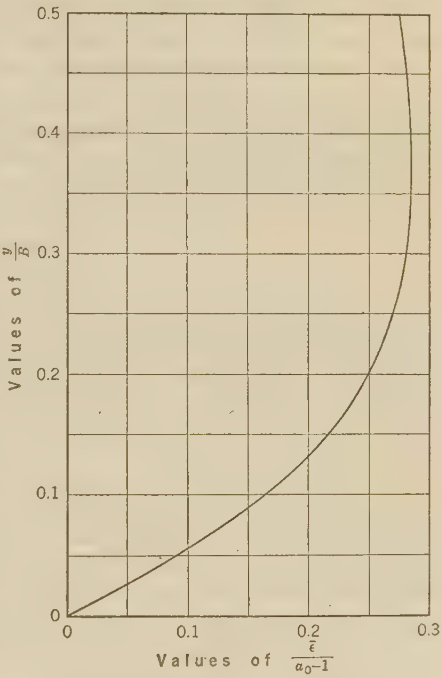

FIG. 18.—VARIATION OF $\bar{\epsilon}$ WITH DEPTH

The apparent drag at the free surface and the resulting depression of the point of maximum velocity puzzled the writer for a long time, but he has come to the conclusion that the late F. P. Stearns,[42] M. Am. Soc. C. E., and Max Möller[43] were right, and that the reduction in surface velocity is due to double spiral flow carrying slow moving liquid from the bottom up along the sides and inward along the surface. The cause of these spirals is still obscure but seems to be tied to the matter of unequal shear around the boundary, since the only case where the spirals are not present—that of the circular pipe flowing full—is also the only case of uniform distribution of boundary shear. The best discussion of the question that the writer knows is by Paul F. Nemenyi.[44]

At first the writer did not understand Mr. Keulegan's suggestion that the more accurate values of $\bar{\epsilon}$ would change Fig. 9 enough to explain the deviation from Nikuradse's results; but, in a subsequent correspondence with the writer, Mr. Keulegan has made his point clear. If, in Eqs. 8, $(1 + \bar{\epsilon})$ is omitted,

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[42] "On the Current Meter, Together with a Reason Why the Maximum Velocity of Water Flowing in Open Channels Is Below the Surface," by F. P. Stearns, *Transactions*, Am. Soc. C. E., Vol. XII (1883), pp. 331–338.

[43] "Studien über die Bewegung des Wassers in Flüssen mit Bezugnahme auf die Ausbildung des Flussprofiles," by Max Möller, *Zeitschrift für Bauwesen*, 1883, p. 201. (A summary in English is given in "Hydraulic Laboratory Practice," edited by J. R. Freeman, A.S.M.E., 1929, p. 70.)

[44] Discussion of "Transportation of Suspended Sediment by Water," by Paul F. Nemenyi, *Proceedings*, Am. Soc. C. E., June, 1945, pp. 865–874.

Eq. 7 is changed to

$$C\,(1 + \bar{\epsilon}) - c_s = C + \bar{\epsilon}\,C - c_s = m \log_{10} \frac{\mathbf{R}}{C} + J \ldots\ldots\ldots (35)$$

Although $\bar{\epsilon}$ is quite small, $\bar{\epsilon}\,C$ is not small, and plotting values of $C + \bar{\epsilon}\,C - c_s$ in Fig. 9 instead of $C - c_s$, Mr. Keulegan finds that a line with a slope of $m = 32.65$ gives a good fit. Therefore, the universal constant remains 2.5 for channels as well as for pipes, and point (1) of the Synopsis is disproved. Eqs. 11, 13, and 17 remain true as empirical formulas representing the results of the experiments, but equations with an ($\bar{\epsilon}\,C$)-term would be more scientific. How these could be adapted for practical use is a matter for further study.

The writer does not agree with the suggestion that, for shooting flow, Eq. 13 should be replaced by Eq. 30. The values of C in Table 1(*b*) were recomputed by Eq. 30, and it was found that the average discrepancy from the observed values was 4.28, as compared with 2.44 given by Eq. 13.

The writer wishes to add one suggestion: Eqs. 18*a* and 18*b* can be merged, following the method used by Hunter Rouse,[45] M. Am. Soc. C. E., for pipes, into

$$C = c_s - 40 \log_{10} \left(\frac{\epsilon}{R} + 0.7586\,\frac{C}{\mathbf{R}} \right) \ldots\ldots\ldots\ldots\ldots (36)$$

When ϵ and $\mathbf{R}$ are large, Eq. 36 practically reduces to Eq. 18*a* for rough channels; and, when the values of ϵ and $\mathbf{R}$ are small, it practically reduces to Eq. 18*b* for smooth channels. In fact, when $\mathbf{R}$ is small enough, the channel acts smooth even if ϵ is not very small. It is believed that Eq. 36 may be found useful for practical work; but, in the transition range between "smooth" and "rough," it would probably apply only to nonuniform roughnesses such as occur in nature. For the uniform roughnesses used in this experiment the fit is not improved by replacing the straight horizontal lines of Fig. 11 by the curves of Eq. 36. In fact, there is an indication in roughness X (the one nearest the smooth channel case) that the lines might turn up slightly before turning down to join the smooth channel curve, just as Nikuradse's uniform sand caused his curves to dip rather than to agree with those in Professor Rouse's formula.

The writer agrees with the remarks of Professor Posey. Even more important than the effect of the shape of the cross section, however, is the question whether the channel is acting as "smooth" or "rough," or whether it is in the transition range between. From analogy with pipes, it can be determined quite accurately that whether a channel acts as "smooth" or not depends not only on the absolute or relative roughness of the surface; but also on the thickness of the boundary layer which depends on the Reynolds number of the flow. It is too much to expect that formulas like Kutter's formula or Manning's formula can handle all the cases that may arise if n is assumed to depend on the roughness only.

It should be noted, however, that the proper value of ϵ for any particular surface in Eq. 36 is only about one fifth of the k-value for that same surface as given by Professor Rouse. The one case for which the writer has determined this relationship[37] gave $k = 5.16\,\epsilon$.

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[45] "Evaluation of Boundary Roughness," by Hunter Rouse, *Proceedings*, 2d Hydr. Conference, Univ. of Iowa, Iowa City, 1942.

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ECONOMICS OF PHOTOGRAMMETRY

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With Discussion by Messrs. Raymond A. Hill, Raymond L. Moore, Ralph S. Pearse, George D. Whitmore, Benjamin A. Wasil, Charles H. Davey, Robert H. Randall, W. J. Ryan, Oliver S. Reading, and Leon T. Eliel.

Synopsis

Contour mapping is rapidly becoming mechanized by the application of photogrammetric methods. Mosaic maps are widely accepted today. Planimetric maps are made and used extensively by many government agencies. There are a variety of photogrammetric processes which are quite controversial. The economics of the various photogrammetric methods and plane table surveys show why the different methods are used.

General

"Aerial photography" encompasses everything in the entire process of taking pictures from the air. When the original picture itself is all that is desired, the work pertaining to it lies entirely in the realm of aerial photography. When this picture is to be transformed into an instrument of precision, however, such as a precise mosaic, a planimetric or contour map, the work is known as "photogrammetry." Fundamentally the word means: "Picture measuring."

Photogrammetry in the United States dates from the early 1920's. In Europe it was practiced prior to World War I, and from the Europeans and, in fact, alas, from the Germans, the United States received most of its original knowledge of photogrammetry.

Only a few years ago most engineers were being disillusioned after their introduction to aerial photography. They had been led to believe that the aerial photograph, which was then procurable, was, in fact, the equivalent of a map. How far from the truth this was they soon found out. They discovered that the picture had certain errors and distortions. They found that the scale of the picture varied with differences in the elevation of the ground. Pictures were assumed distorted near the edges, although the center was known to be reasonably free from these aberrations.

Note.—Published in March, 1945, *Proceedings*. Positions and titles given are those in effect when paper or discussion was received for publication.

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These early undesirable characteristics of the aerial photograph proved to be a blessing in disguise. The distortion is geometric, of course, and the picture is nothing but a conic projection. Thus points are located along radials from the principal point or center of the picture, in accordance with their elevations. Consequently, a vertical object, such as a telegraph pole, an oil derrick, or the Washington Monument, photographs as a radial image, the top being radially displaced in relation to the base.

In fact, it is this so-called parallax of the picture which makes possible the viewing of an over-lapping pair of vertical photographs stereoscopically. This same parallax can be measured to determine differences in elevation from a stereoscopic pair of pictures, and these geometric qualities are the foundation of all photogrammetric systems.

Mosaic Maps Have Arrived

The mosaic map of the early days was the work of an artist dedicated to the principle that no error was serious unless it could be seen. Thus, these early mosaicists were in reality a breed of prestidigitators—so clever that they made things look all right even when they were all wrong.

Today, cleansed by the blessing of a scientific approach, the mosaic has been accepted in practically all circles as a good map for many purposes, and good maps have been needed, especially by the U. S. Army. Only about 25% of the United States was adequately mapped prior to World War II. Since that time the precise mosaic, accurate and quickly made, has become a map acceptable for many military purposes.

Mosaics are built on a graphical triangulation net known as radial control, which depends upon the fact that all displacements due to difference in ground elevation are radial, and consequently the angle subtended at the center of the picture by any two points is approximately a true angle. Thus, by using merely the direction of various image points from the center of the picture, rather than the images themselves, as a triangulation net, it is possible from the pictures themselves to establish the true position of any number of points in each picture. An occasional point of known position establishes the desired scale and the necessary geodetic ties. The true position of various points in a picture can now be compared with the image position, and a computation made by which the picture may be brought to scale. It is enlarged or reduced, and, in case of sloping ground, tilted so that the scale of one side is corrected more than the other. In mountainous terrain, pyramiding is utilized, whereby the picture is broken up into small sections, each of which can be brought to a uniform scale so that the picture can be laid accurately to the radial control.

Slotted Templet Method of Control.—One widely used method of radial control is known as the "slotted templet method." By this method a templet, usually cardboard, is made from each individual photograph. The principal point, or center point, of the photograph is pricked through from the picture to the cardboard, and at least eight radial points around the edges are similarly designated. A hole is punched in the center of the picture, and through each of the outlying points a radial slot is cut. Due to the overlap of the pictures, each of these points is common to several pictures—sometimes as many as six. The templets are then "buttoned up" with connecting studs through the slots, so that all the slots representing a given common point from the various pictures are assembled over a stud. Thus an entire map can be "buttoned up" and, due to the freedom of the slots, can be enlarged or reduced accordion-like, so that the entire area automatically conforms to a given scale. A very few control points, established by conventional surveying, serve to establish the map scale, and the map stretches or shrinks with movement in its radial slots to conform to these pre-established points.

When the entire area is covered by the templets, each one of which represents one of the pictures, the studs that hold the templets together represent the true position of an infinite number of myriad points. These studs have a hole through which a pin can be applied to the board beneath the templets. The points are pricked through each stud and properly designated, thus becoming the field notes, as it were, from which the ultimate map can be compiled. Typical mosaic specifications may require that 90% of all points shall be within 0.05 in. of their true location.

Planimetric Maps

Planimetric maps are compiled in much the same way as mosaics. Ordinarily, the first step in the compilation of a planimetric map is to make the radial control, or slotted templet, base. This base is traced on to tracing cloth or acetate sheets, and the planimetric detail is added by projecting the photographs on to the sheets in such a way that the control points on the photographs conform with their map positions as established on the radial control plot. Government agencies, such as the U. S. Geological Survey and the Soil Conservation Service, have done considerable planimetric mapping since about 1936.

Mechanized Contour Maps

Whereas the main problem in constructing mosaic and planimetric maps is to eliminate the effects of parallax, the contour map, on the other hand, is made by measuring this parallax. The aerial photograph, when properly taken, is a perfect triangulation record, made as though a transit were set at the same point in space as the lens of the camera. Two such pictures taken from the extremities of the air-base line afford the means by which all of their common points may be located by photogrammetry which, in common language, is triangulation by photography. To be sure, the modern photogrammetric machine does its work automatically, so that it is unnecessary to solve triangles for each individual point. However, were one inclined to this tedious process, a perfectly satisfactory answer could be derived in this way.

Two-Dimensional Processes

The Stereocomparagraph.—Contour drawing machines are essentially of two types. The first is the parallax measuring stereoscope, which is a two-dimensional machine, drawing its contours to the same conic projection as the picture itself. The most widely known machine of this type is the stereocomparagraph (not to be confused with the stereoplanigraph), a device by means of which a pair of stereoscopic prints is pinned to a board, and by the simplest of processes viewed stereoscopically with a couple of measuring dots in contact with the picture. These dots merge into a single dot appearing to float at a given distance. Their separation may be varied with a micrometer screw, and this micrometer is set according to the parallax of the pictures, for any desired elevation. Work done with the stereocomparagraph can be quite accurate, providing the pictures have been first restituted accurately; that is that the pictures have been corrected for the tilt of the camera at the instant of exposure. Even without restitution, a reasonably accurate job can be done by parallax methods, providing a great abundance of control has been established. However, it is generally accepted that the stereocomparagraph is inefficient for precision work.

The Brock Process.—The parallax principle is also incorporated into a very precise system known as the Brock process. This system involves the substitution of glass plates which have been carefully restituted for the simple paper prints of the stereocomparagraph method. The stereoscope by which the image is viewed is an extremely precise affair. With the multiplex and the stereoplanigraph methods, the entire map is drawn from beginning to end in one operation and in one machine, whereas the Brock system breaks this process down into a number of operations. This has the advantage of simplicity in that less highly skilled personnel is required, and it also affords opportunity for more operators to work simultaneously on different steps of the operation for each plate pair. The first major step of the Brock process is to determine the tilt of the aerial photograph, and to make a restituted aerial photograph, on glass. Next, this pair of stereoscopic glass plates is placed in the stereoscopes, and the same floating mark is made to follow the contours by an operator turning two wheels. The operator actually causes the floating mark to traverse perhaps one-half inch of the picture, and then with pen and ink draws this half inch of contour on a piece of overlaid tracing paper or acetate sheeting. Then he proceeds to cause the floating mark to indicate another half inch of contour, and draws this with pen and ink. After all the contours from one plate pair have thus been drawn, the result is put in a projecting machine, and each contour is projected at the proper ratio to transform it from the conic to the orthographic projection. The contour is thus traced off, and at the same time the planimetric detail, such as roads and streams, are plotted off, the detail falling between a given pair of contours being drawn at the setting of each contour. Thus, if a 20-ft contour is being traced, the planimetric detail falling within half a contour interval each side of the contour would be drawn at this same setting.

THREE-DIMENSIONAL PROCESSES

The stereocomparagraph and the Brock processes are two-dimensional methods which measure the parallax of the conic projection all in the horizontal plane and subsequently convert to the orthographic projection by differential enlargement.

Multiplex System.—Perhaps the most widely known of the three-dimensional processes is the multiplex system, which has been adopted by the U. S. Geological Survey, and by the U. S. Corps of Engineers. The photographs are reduced to a small glass diapositive about 2.25 in. square. This miniature, which is one fourth the original size, is then placed in a small projector. Its stereoscopic counterpart is placed in a second adjacent projector, and the two images are projected simultaneously on to a screen of adjustable height. On this screen the two images merge into a three-dimensional miniature of the ground. The two projections are made through complimentary colored filters, and the operator wears spectacles with a red and a blue glass, whereby each eye sees its separate image. Thus a stereoscopic effect is obtained. On the screen is a tiny illuminated dot, called the "floating mark," which rises and falls with the screen. The screen is carried by a stand, which can be pushed around over the table on which the map paper is placed. In the center of this stand is a pencil. The screen is elevated by a thumbscrew to a height representing a desired contour at the scale of the map (1:10,000, for example) and then the stand is pushed around under the direction of the observer, who causes it to move in such a fashion that the floating dot follows the undulating surface of the ground. This floating dot, moving parallel to the horizontal surface of the table, can only move in this one contour level. Thus the operator, by moving the dot continuously over the miniature surface of the ground, getting neither below it nor above it, causes the pencil to draw that particular contour. When this contour is completely drawn, the thumbscrew is turned and the screen and floating dot are raised to the next contour elevation. For example, if the contours are 20 ft apart, the screen and floating dot will be raised by the amount 20 ft ÷ 10,000 or 0.002 ft, which is 0.024 in. Planimetric details, such as roads and streams, are drawn by causing the floating dot to follow the stereoscopic image of these features, rising and falling with the feature by manipulating the thumbscrew.

There are several hundred multiplex units in the various mapping agencies of the United States government, and this system, by reason of its general familiarity, is unquestionably the most popular in the United States.

Stereoplanigraph.—Another three-dimensional method for stereoscopic contour mapping is the stereoplanigraph method,[2] which involves the same general type of stereoscopic pictures as all the other methods. The machine is by far the most elaborate of any of those described, and the most difficult to operate. Its compensating features are great speed and accuracy.

The stereoplanigraph uses a pair of glass diapositives the same size as the original photographs. As in the case of the multiplex, these two diapositives are placed in projectors, although the projectors are four times as large, and consequently four times as far apart. The image from these photographs is projected down on to mirrors, on which is the ubiquitous floating mark. Image and floating mark are observed through an enlarging optical system, by means of which the image appears from five to ten times its original size.

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[2] "Stereo-Topographic Mapping," by C. H. Birdseye, *Transactions*, Am. Soc. C. E., Vol. 98 (1933), p. 771.

Contours are traced by an operator turning a pair of hand wheels—the left-hand wheel controlling the "X" motion, and the right-hand wheel controlling the "Y" motion. Elevation is changed by a foot wheel. Drawing is done on a separate table connected with the stereoplanigraph. This table has an X and a Y slide actuating a pencil, under the surveillance of an assistant. Thus the stereoplanigraph is a two-man machine, whereas all the others heretofore described are operated by only one person.

The stereoplanigraph method is similar, in function, to the multiplex, in that the complete operation performed on one plate pair consists of first making the stereoscopic model; that is, bringing the two pictures into perfect correspondence by tipping them and tilting them, and ultimately, by trial and error, returning them to the exact relative position they occupied in the air. In the Brock process this same thing is accomplished by the first step of making the diapositive, the position of which has been determined by calculation. Then, in the multiplex and stereoplanigraph operations, the pair is "horizontalized." This means that the two pictures which have been placed into perfect relative position are now set in their correct relationship to the horizontal reference plane. Then the next step in both the multiplex and stereoplanigraph operations is to draw all the detail on the map. The stereoplanigraph differs most widely from the multiplex at this stage of the proceedings. Whereas the multiplex operator is pushing his screen stand, or pencil stand, around over the map, causing it to draw, the stereoplanigraph operator is doing his drawing by turning a pair of hand wheels. It is the speed of this hand-wheel operation, the fine accuracy, and the optics of the stereoplanigraph, which are responsible for the much greater production per man-hour than can be achieved with any other type of machine. Production, per man-hour, with the stereoplanigraph, on comparative types of mapping, is from five to ten times faster than on the multiplex.

Comparison of Operating Costs

A comparison of the merits of the various systems described briefly herein involves, first, an understanding of some of the fundamental criteria used, in evaluating the work.

The most efficient system of mapping is that by which a given map made to a given degree of accuracy can be produced cheapest and fastest. The principal comparative items of cost in compiling a contour map by photogrammetric processes may be classified roughly as follows: (1) Amortization of the capital investment; (2) flying cost; (3) control cost; and (4) plotting cost.

Other items of cost, such as drafting and the routine editing, are more or less similar for all systems of mapping, and consequently do not constitute one of the bases of comparison.

(*1*) *Amortization of Cost.*—The stereocomparagraph costs about $300. The Brock equipment is not listed, only one unit of this equipment being in existence. The writer estimates that it might cost $25,000 or $30,000 to duplicate one of the Brock units. The multiplex costs about $10,000 a unit; and the stereoplanigraph costs about $45,000 per unit. Assuming that the equipment will be used for ten years, there is no amortization to be worried about on the stereocomparagraph; the multiplex must be written off at the rate of $1,000 per yr; the Brock equipment at the rate of about $2,500 per yr; and the stereoplanigraph at the rate of about $4,500 per yr. On the assumption that the equipment will be used only one shift (say, 2,000 hr per yr) the multiplex has an amortization charge of 50¢ per hr; the Brock equipment has an amortization charge of $1.25 per hr; and the stereoplanigraph an amortization charge of $2.25 per hr.

(*2*) *Flying.*—Since the stereocomparagraph is rarely, if ever, used for precision mapping, the remaining comparison will be confined to the other processes and instruments. For multiplex work, flying is generally undertaken at an elevation of from 700 to 1,100 times the permissible tolerance in the contours, as compared with 2,500 times for the stereoplanigraph. The net area covered per photograph can be compared on the following basis: The multiplex method covers the least area per photograph. Thus, if the multiplex covers 810 units of area per stereoscopic model to a given degree of accuracy, the stereoplanigraph method will cover 1,620 units of area. These figures can be clarified somewhat by stating that the multiplex, flying 1,000 times the permissible tolerance, uses a 6-in. lens covering a 9-in. by 9-in. photograph. The stereoplanigraph flies at 2,500 times the permissible tolerance, using a 6-in. lens on a 9-in. by 9-in. negative. The different systems also use a different overlap which is taken into consideration in the foregoing figures. The number of pictures required and the degree of control required will be in the following ratio: Stereoplanigraph 0.64 and multiplex 2.00.

The relative separation between the photographic strips will be 7.2 for the multiplex, as compared with 14.4 for the stereoplanigraph. The actual amount of flying to be done will be inversely proportional to these figures; in other words, the most flying must be done for the multiplex work.

(*3*) *Control.*—Although there is some difference of opinion among experts as to the most economical method to control a photogrammetric survey, the cost of doing the work for the different methods, according to the standard of any expert, will be substantially in the same proportion as the area covered per photograph. Control generally involves about four known points per stereoscopic pair. Since most of these points are common to a number of stereoscopic "models," the net by almost all systems will prove to be approximately one and one-half control points per picture. So, by almost any conceivable method of control, the relative cost will be approximately twice as much per unit of area for the multiplex method as for the stereoplanigraph method. (Note: It should be kept in mind that the writer is most familiar with the stereoplanigraph process. Much has been published on the multiplex process, and persons familiar with the multiplex system will have an opportunity to discuss this paper. Little has been published as to the cost of the Brock process, and it is hoped that someone thoroughly familiar with the Brock process will take this opportunity to present its case in the form of a discussion.)

(*4*) *Plotting the Map.*—The fact that a map can be plotted by stereoplanigraph from five to ten times as fast as by multiplex is due to many factors. First and foremost is the relative speed of the hand-wheel control as compared with the clumsy job of pushing the multiplex stand around by hand. Second, the time is affected by the greater area per stereoscopic model which is available to the stereoplanigraph, and consequently less time is needed in the preparatory setting up work, as it takes about as long to set up one model as another, regardless of scale or flying altitude. Another advantage of the stereoplanigraph is that it affords a means of enlargement between the picture scale and the plotting scale. Usually the map is drawn directly by the stereoplanigraph at a scale about six times larger than the original picture scale. The multiplex, on the other hand, is at a distinct disadvantage in that it first reduces the pictures to one-fourth size, and then has a limited range of enlargement due to the shallow depth of focus of the multiplex projectors. Therefore, it will be seen that, in plotting the same map, the multiplex operator actually will have to draw from two to three times the number of linear inches of contour that the stereoplanigraph operator will have to draw.

Although no data are available on the actual rate of drawing in the Brock stereoscope, the writer believes that the drawing is probably the slowest of any of the three methods. The Brock process does not permit the continuous drawing of a contour line, as do the other two processes. In the Brock process, the contour line must be drawn by short increments, the operator turning the hand wheels of the stereoscope and observing where the contour line goes for a short distance such as one-half inch, and then drawing that contour line by hand. Then he observes another half inch and draws that by hand.

There is another factor which enters the cost of the drawing by the three machines, however: Whereas the stereoplanigraph permits much faster drawing, it also can be operated only by a much higher type of personnel, the training period for efficient operators being about two years. One of the great advantages of the multiplex is that the method is simple and very apparent, and that reasonably alert operators can be trained in probably one quarter of the time that it would take to train them for the stereoplanigraph. In this respect the Brock process has an advantage over both other methods in that an ordinary topographical draftsman with good eyes can be taught to become a Brock operator in a comparatively short period of time—probably in one tenth the time that it would take to train a stereoplanigraph operator. The reason for this is that the Brock plates come to the operator with the tilt entirely eliminated, so that all the operator has to do is to set them in the machine and start to draw. On the other hand, both the stereoplanigraph and the multiplex operators must learn to set up the stereoscopic model and to horizontalize it—processes which take a considerable period of time to master. The stereoplanigraph must be set up with the greatest precision of any machine, because the pictures are taken at such very high altitudes and are observed under such high magnification. The optical qualities of the multiplex image are not nearly so good as those of the stereoplanigraph, and consequently the stereoscopic model cannot be established with the same degree of precision. Therefore the instrument can be set up more quickly and with less experience.

In summary, all three methods will produce an excellent map. The multiplex appears to be the least efficient from the standpoint of the flying and control required, and second in the time required to plot. The stereoplanigraph will actually permit plotting the map much faster per man-hour than the Brock process; but in so doing it utilizes a machine that has a high rate of amortization. It involves an operator and an assistant to run the machine, in comparison to a single lower-salaried operator on the Brock process. If the stereoplanigraph can be kept in use continually, so that the amortization charge is not a major factor, it is the most efficient method now known, by a considerable margin. However, if the machine is only to be used occasionally, it is conceivable that the Brock process, or even the multiplex, if the period of use compared with the total available period is small enough, might be more efficient, due to the amortization charges.

Comparison with Ground Methods

So far, this paper has considered only a comparison of the different photogrammetric methods. The subject could not be considered thoroughly covered without making some comparison between photogrammetric methods and the older and more conventional ground method—that is, the plane-table method—of drawing a contour map. There are undoubtedly many situations in which the plane table is more efficient than any photogrammetric method. Any very small area can be surveyed more economically by a plane table. The job is done completely and finished by the man in the field. With the photogrammetric method, it is necessary first to send an airplane to the area to be photographed. Subsequently control parties are sent out to establish ground control, and finally the data are put through a machine to produce the map. Just where this line of demarcation is between the efficiencies of the two, according to scale and number of acres, is difficult to determine. The smaller and farther away an area is from the flying base, the more relative efficiency favors the plane-table method.

As an hypothetical illustration, consider an area of four hundred acres right beside the flying field, to be done either by plane-table methods or by photogrammetric methods for a map scale of 1 in. = 200 ft, and 5-ft contours. If this area were rolling to mountainous, the photogrammetric method probably would only cost half as much as the plane-table method. On the other hand, if it were reasonably flat to rolling, and open, the costs would probably be about the same. Remove this same four hundred acres two hundred miles away from the flying field, and the costs by the two methods might be about the same. The cost of using the airplane will be perhaps $50 an hr, and five hours may be required to complete the job—providing the photographers record all the data correctly on the first attempt. Perhaps, also, an allowance of 50% should be made for possible re-takes, which would be 7½ hr, at $50 per hr. Thus, there is a charge of about $1 an acre for the flying, really before the work is started.

It may also be said that, in areas which are very flat with contours far apart, plane-table methods probably are more efficient than any photogrammetric method.

The plane-table method may be more efficient in the case of a road location which had already been substantially staked out; for example, for a strip of topography only 100 ft wide and 20 miles long. In this instance, the photogrammetric survey would cost almost as much as if the width of the strip to be mapped were the entire width that could be covered by a picture, which might be 2,000 ft or 3,000 ft. The plane-table operator can confine his cost to the actual width of 100 ft. Consequently, there are many cases where, in locational surveys, it will be found more economical to run a plane-table survey, or to cross section the location, than to make a contour map by photogrammetric methods. This is not universally true, however, as many circumstances, such as accessibility to the ground, etc., will often favor the photogrammetric method.

It can be stated definitely that for final determination of canal grades, where the fall of the canal must be confined to a few inches per mile, photogrammetric methods are of no avail except in the preliminary stages of the survey. The final location must be established by level.

SUMMARY

Today in the United States it is probable that nearly all topographic mapping is being done by photogrammetric methods. The U. S. Geological Survey is the only organization among many now engaged upon the federal mapping program that still utilizes the plane-table method to any extent. This organization operates what is probably the largest multiplex unit in the world, so that even within the Geological Survey a large percentage of the total work is being done by photogrammetric methods. Other federal organizations and private organizations under contract to the government use the photogrammetric methods almost exclusively.

During the cantonment construction program for World War II, when detailed contour maps were required in a great hurry, the photogrammetric method was not found wanting. The speed of photogrammetric mapping has proved a surprise to people unfamiliar with the methods, and the uniform accuracy of the results has made design from photogrammetric maps a pleasure to the engineer. Photogrammetric maps are being made with a guarantee that 90% of all points checked shall be within one-half contour interval of their true position. Photogrammetric maps have been made at scales ranging from 1 in. = 20 ft, with 1-ft and 2-ft contours, to scales of 1:62,500, with 100-ft contours.

Photogrammetry has finally arrived at the age of full maturity. Although the methods will unquestionably continue to improve from year to year, they are already thoroughly practicable, and afford a tool which will find continuing and increased acceptance at the hands of thoughtful engineers.

DISCUSSION

Raymond A. Hill,[3] M. Am. Soc. C. E.—Aerial mapping was a "child" of World War I; and, as described by the author, it has since grown to full stature even though most engineers have no appreciation of that fact. Those engaged in the photogrammetric field have suffered many disappointments because of the lack of interest shown by most engineers and because of the lack of understanding on the part of the few interested engineers. These disappointments should have been expected, however, because most engineers are not concerned with maps, and even those who use maps in their work often have no part in the preparation of maps. Hence, only a few engineers have had more than an academic interest in aerial surveys and photogrammetry, and it was not natural that all of them would turn from conventional methods and accept something which was new and untried. Furthermore, it must be confessed that even the small number of engineers who today rely greatly upon photogrammetry do so more because of faith in the result rather than because of understanding of a method that depends upon an optical illusion.

If a poll were taken of all members of the Society, the writer believes that a substantial number would be found to have no acquaintance with photogrammetry, that half of the membership would not distinguish between modern photogrammetry and aerial photography, and that half of the remainder would state that the same information could be obtained more cheaply and equally well by conventional ground methods. The author is thus to be commended for presenting, clearly and fairly, the distinction between the "child," aerial mapping, and the "adult," photogrammetry.

It is not likely that the majority of engineers will ever be interested more than academically in photogrammetry, any more than most engineers are concerned with developments in hydraulics, or in other specialties foreign to their own fields of endeavor. On the other hand, every engineer who uses maps should acquire an appreciation and understanding of both the advantages and the limitations of modern developments.

It should not be presumed from the statements of the author that aerial photography has been supplanted by photogrammetry. On the contrary, aerial photography probably has more applications to engineering problems than do the contour maps produced with a photogrammetric instrument. This is natural, because the relative positions of physical features and types of culture are generally more important than absolute elevations. Many illustrations may be cited: Changes along a shore line due to erosion or deposition; general routes for railroads, highways, and transmission lines; city planning, as distinguished from site planning; and general surveys, such as land and soil classifications, agricultural censuses, etc.

In most of the foregoing instances, the contact print from an aerial photograph presents, directly, more information than could be obtained by ground methods without excessive cost. When successive photographs are examined stereoscopically, the land form is clearly disclosed. The observer then has before him an accurate relief map on which is located every natural feature and all culture in the proper relationship.

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[3] (Leeds, Hill, Barnard & Jewett), Los Angeles, Calif.

Too little attention has been given to the interpretation of aerial photographs. Those in the industry, by reason of their own familiarity, took it for granted that any one could read an aerial photograph, not appreciating the fact that familiar objects viewed horizontally become strange when seen vertically. It should have been recognized that people must be taught how to read maps, even though maps have been in use for thousands of years. It is only natural that most people should look upon aerial photographs as interesting pictures and not as maps, because they have not been taught to interpret the shades and shadows which take the place of conventional symbols on a map.

Assembly of contact prints of aerial photographs into a mosaic produces a map that is frequently of great value in spite of inherent distortions. The accuracy of a mosaic, of course, depends upon the degree of control and the extent to which each part is brought to a common scale. A rough mosaic is little more than a rephotograph of overlapping contact prints placed in approximate relation one to another. A matched mosaic presents a better picture in that, to the eye, it has the appearance of a single photograph. For many purposes such a mosaic is ample, because the engineer who uses the map, as distinguished from the engineer who prepares the map, is generally most concerned with the relative position of natural features and cultures. Such mosaics are particularly valuable in general planning. They have been used extensively in flood control studies, in rerouting highways, in locating industries, and for many other purposes where a bird's-eye view of large areas was helpful.

The development of the controlled mosaic and the precise mosaic was in response to the demands of those who made maps rather than to satisfy the needs of those who used maps. Most of the former think in terms of the accuracy of the survey for which they are responsible; and, hence, they are prone to demand comparable accuracy when aerial methods are used. Actually, the comparison should be on the basis of the finished and reproduced map. A ground survey may be made at an accuracy of 1:5,000, but no man can plot an observed distance on a map with one tenth of that accuracy. Furthermore, when a draftsman completes a map on tracing cloth, it is of little value until reproductions are made. Changes in humidity and temperature cause some distortion of the tracing even before prints are taken. Unless these prints are made entirely by a dry process, large distortions result; and, even under the best conditions, the map that is available to the user is neither true to scale nor free from distortion. It has been the writer's experience that a reproduction of a carefully made, matched mosaic can be used with confidence for almost all the purposes, excepting only elevations, for which any reproduction of any map at the same scale would be used.

Whenever either the absolute elevation or the relative elevation of features must be obtained from maps, the engineer responsible for the preparation of the map should ascertain whether photogrammetric methods should be used. As stated by the author, if the area is small, the answer is probably "no." Likewise, if the terrain to be mapped is open and relatively flat or if only a narrow strip of topography is needed, the answer is again "no." On the other hand, if a substantial area of broken terrain is to be mapped, the answer is probably "yes." In some cases, the nature of the cover will determine whether ground methods would be cheaper and faster or whether photogrammetric methods would be better. The writer could cite the case of one Army cantonment where the required contour map was prepared photogrammetrically because of the presence of a mantle of brush but where, otherwise, plane-table methods would have been more suitable.

In general, ground surveys, supplementing aerial photographs and matched mosaics, will usually be found most advantageous in the solution of specific engineering problems. On the other hand, photogrammetry will probably supplant ground surveys of large areas, and the future of photogrammetry is limited only by the extent of all the continents.

RAYMOND L. MOORE,[4] ESQ.—The multiplex "aero-projector" method is probably the most widely used of the three types of precision equipment described in the paper. The method is also the most recent addition to the field of photogrammetry although the spatial model was first constructed by dichromatic projection in 1858. Practical equipment for utilizing this principle in the production of topographic maps did not become available in the United States, however, until about 1935.

In general, the equipment of 1944 is similar to that of 1935, but vast improvements, both optical and mechanical, have been incorporated in later models. The most important change has been the result of improved lens design, not only in the projector unit but in the aerial camera itself, by increasing the effective field of coverage.

Early multiplex equipment was designed for aerial photography obtained with the multi-lens camera, of which the 9-lens tandem T-3A camera is probably the best known in the United States. Photography of this type necessitated two general projector designs—one for use with the vertical center chamber, and the other for use with oblique wing prints. Since distortion, displacement of images due to relief, and generally unsatisfactory quality were present in the oblique photographs, accurate mapping with these projectors was extremely difficult. The multi-lens camera is considered obsolete for general mapping purposes and it has been replaced by the so-called wide-angle, single-lens camera. This camera provides an effective field of coverage between 90° and 100°, depending upon the camera type; and the field of coverage for a single vertical photograph is about equal to the usable field of the older multi-lens cameras. Although most of the older normal-angle equipment is still being used, the newer wide-angle equipment is found in most multiplex mapping laboratories. This equipment makes possible aerial photography with most favorable base-altitude ratios—which results in greater contour accuracy and materially reduces the number of photographs required.

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[4] Lt.-Col., Corps of Engrs., U. S. Army, Commanding Officer, 29th Engrs., Portland, Ore.

In comparing the relative merits of the different systems of photogrammetric plotting devices on a cost basis, it is necessary to examine the fields in which each system is used. Both the Brock equipment and the stereoplanigraph are suitable for the production of large-scale contour maps—a field in which the multiplex has not been used extensively. Sufficient mapping by both the Brock process and the stereoplanigraph has been published and tested to prove conclusively that exceedingly high standards of accuracy can be obtained. In this field the multiplex has not yet been used extensively enough to compete on a cost basis with the other methods.

Within the continental limits of the United States, more medium-scale mapping (that is, mapping on scales of 1:24,000 to 1:62,500) has been completed by multiplex than by either of the other two processes. Many factors have influenced the widespread acceptance of the multiplex method for this type of mapping. Multiplex maps have met rigid accuracy standards, and the comparatively low initial cost of equipment undoubtedly has been considered; but these factors alone do not entirely account for the popularity of the method. Availability of the equipment on the domestic market is a primary asset. Neither additional stereoplanigraphs, which are of foreign manufacture, nor Brock equipment, which was custom-built and not a standard commercial product, is available. Before the priorities of World War II made acquisition difficult, multiplex equipment was available to those organizations that had occasion to do topographic mapping and is the only precision plotting equipment that has been in actual production during the war.

The availability of photogrammetric operators has been a major problem of all agencies utilizing stereoscopic plotting equipment, since trained operators are not available and must be educated within the organization. As in all types of precision work, there are certain basic requisites of knowledge and aptitude that potential operators must possess. Only part of the people with this knowledge and aptitude will develop the acuity of vision necessary to become capable "photogrammetrists." Nevertheless, a larger percentage capable of operating the multiplex will be developed than would be developed if the training were on other types of equipment. If candidates are selected carefully, the average operator will be able to draw contours on oriented models after about three months; he will become a reasonably skilled "phototopographer" in six months; and he may be expected to do satisfactory extension work in nine months or one year.

Inherent in the multiplex method are operational features which are not provided (or are difficult to execute) by the other types of equipment, the most important of which is the capability to "phototriangulate" or "bridge" field control both vertically or horizontally. Secondly, and of considerable importance in military mapping, contour plotting may progress simultaneously on a large number of stereoscopic models for later assembly into the finished map.

The cost of field control is a primary consideration in the planning of any mapping operation and may well be the deciding factor in determining which method, or methods, should be utilized. In general, the amount of field control is decreased as the scale of the map is reduced and the contour interval increased. Thousands of square miles of mapping have been produced by the multiplex method since 1936 with a publication scale of 1:24,000 and an average contour interval of 20 ft. For this type of mapping horizontal control is generally placed along the 7.5-min quadrangle with boundaries normal to the direction of the flight lines. Horizontal control, for the orientation of interior models, is obtained by "bridging" this control without the necessity for additional horizontal field control, although for this scale and contour interval it is still accepted practice to obtain a limited amount of vertical control on each interior photographic pair. Much work has been completed during this same period on standard quadrangle maps published on a scale of 1:62,500, with contour intervals averaging about 50 ft. Control lines are placed along the 15-min quadrangle borders crossing the flight lines; and, because of the greater contour interval, little or no vertical field control is required on the intermediate models. In planning projects of this type, both vertical and horizontal control bands are placed about 15 miles apart. Between these control bands the multiplex bridge, or "phototriangulation," produces adjusted positions and elevations upon which the intermediate models for drawing the topography are oriented.

Military and civilian practice differs somewhat inasmuch as time is frequently the paramount consideration in any military mapping project; and, for this reason, methods are frequently used which do not necessarily produce maps in a minimum number of man-hours. More frequently, the problem is the production of a map in the absolute minimum lapsed time even though the number of man-hours may be excessive. To produce a maximum amount of work in the minimum over-all length of time, control bridge work is conducted independently of contour mapping by an organized bridging section. Only the highest types of operators are assigned to this section. In general, the same type of equipment is used as that used in the plotting section, except that the multiplex bars are longer. In an emergency, projectors normally used in bridging work may be diverted to contour plotting, provided a sufficient number of additional operators are available.

The multiplex has been used to a limited extent in the field of reconnaissance mapping with highly satisfactory results. A brief description of a project recently completed will illustrate the potentialities of the method in this field. The topography of an area approximately 500 miles long and 10 miles wide was required for publication on a scale of 1:250,000, with a contour interval of 100 ft. The need for this work was urgent; the area was inaccessible; and part of the route was over terrain probably never before traversed by white men. There were available, within the area, seven control points of known geographic position and a few elevations of somewhat doubtful quality. Some additional elevations were secured by prorating known or assumed river grades. The longest single multiplex bridge extended 285 miles, and a single horizontal control point was available on each of the terminal models. This bridge did not run in a straight line but traversed river valleys and high mountain passes. Aerial photographs for the project were taken in the middle of the winter with the sun never more than 5° above the horizon. Although the multiplex work was completed in a remarkably short time, subsequent field work has indicated that this map is surprisingly accurate.

Although it was designed for the production of topographic maps, numerous other uses have been found for the multiplex. By a rather simple method it has been used in the production of mosaics. With the multiplex bridge adjusted for horizontal scale, sufficient planimetry to fix the orientation of each photograph is drawn for transfer to the mosaic base. With dianegatives substituted for diapositives, projection prints can be made directly below the projectors. These prints are both adjusted as to scale ratios and horizontalized; and they may be pasted down in the normal manner.

Unfortunately, many topographic maps are actually obsolete before they are published; and, to preserve their maximum usefulness, frequent revision of cultural detail may be necessary. This revision work will be a major mapping problem in some localities. For example, in California, several areas were mapped very accurately as recently as 1925. Because of development, cultural details in some of these areas have changed to such an extent that, without revision, an entirely erroneous picture of the terrain is presented. The work in some of these areas has been revised easily, and with utmost accuracy, by the multiplex method. After the war a large amount of map revision will be required where wartime activities have caused major cultural changes.

With large areas of the United States still unmapped, there is ample work to keep all types of plotting equipment in use for some years. It is obvious that, in particular instances, certain types of equipment are preferable to others. For some time after the war the popularity of the multiplex method will probably continue, if for no other reason than the availability of equipment and operators. In the field of large-scale mapping the method remains to be proved. It may be found that large-scale work by the multiplex method will be less economical in the long run and that this particular type of work should be left for instruments of greater precision. In the intermediate-scale field—which includes most of the mapping to be done in the continental United States—there is ample proof that the multiplex method can compete with other methods, both in economy and accuracy. Economy in field costs is possible because, unlike the other methods, the multiplex does not require survey control on every stereoscopic model. When contour intervals of 50 ft or more are used, costs of aerial photography will be the same for the multiplex as for the stereoplanigraph, inasmuch as photographs for both methods will be taken close to the service ceiling of the aircraft employed, and this ceiling will be the deciding factor in determining the number of models and the unit area covered by each model.

A discussion of existing photogrammetric plotting devices should not be concluded without some mention of the K.E.K. Plotter (named for J. E. King, J. W. Elliott, and P. B. Kail) being used by the United States Forestry Service in the production of standard quadrangle maps in some parts of California. This plotter was developed by personnel of the Forestry Service and might be considered a much-refined and enlarged stereocomparagraph in which certain of the most undesirable characteristics have been eliminated. The principal improvements incorporated in the K.E.K. Plotter are: (1) An arrangement by which contours are drawn to correct orthographic scale, rather than on the conic projection (which is one of the principal disadvantages of the stereocomparagraph); and (2) a system of restitution by an appropriate inclination of the contact prints with respect to the operator's point of view. As with other devices, except the multiplex, vertical control is required on each model. Little information is available as to the amount of horizontal control necessary, but it is probably about the same as that required for the other processes. Comparatively low initial cost and portability may make the K.E.K. Plotter a most valuable instrument for some types of work.

Ralph S. Pearse,[5] Jun. Am. Soc. C. E.—It is gratifying to find a paper on photogrammetry in the publications of the Society, since engineers are relying more and more on accurate, complete maps. Two additional methods should be included in this discussion. For example, the K.E.K. Plotter is an improved form of the stereocomparagraph incorporating the third dimension, together with an accurate means of correcting the photographs for tilt. A pair of contact prints are oriented and corrected for any tilt, the resulting model then being viewed through a stereoscope. A lens with a small dot in the center is suspended under each eyepiece of the stereoscope between it and the image. These dots fuse to form a floating mark with which the operator traces planimetric detail and contours. The pair of photo-tables on which the photos are fastened are raised or lowered, according to a varying scale, to compensate for the conic projection, thus allowing the floating mark to trace the contours directly to scale. The floating mark is connected to a pencil or pantograph which permits drawing the detail on to the map. A considerable amount of control is required for this method, at least two horizontal and four vertical points being needed for each pair of photographs. An accurate map may be obtained and the instrument may be moved easily from place to place. The cost of the K.E.K. Plotter is several hundred dollars—considerably less than that of either the multiplex or the stereoplanigraph.

The aerocartograph is an instrument quite similar to the stereoplanigraph, incorporating a similar optical-mechanical system for obtaining map detail from glass diapositives. There are three in the United States, but they are not much used. The advantages and disadvantages of this method over the other methods are similar to those of the stereoplanigraph.

Several items concerning the comparative economy of the multiplex and stereoplanigraph methods should be mentioned. The cost of $10,000 for one multiplex unit is extremely high, $5,000 or $6,000 being more nearly correct. In many instances, it has become the practice to equip one unit for bridging (described more fully in a later paragraph) and to streamline four or more units, thus reducing the cost of an average unit to $3,000 or $4,000.

The flying costs will be greater for the multiplex than for the stereoplanigraph because of the lower flying height, which necessitates more pictures and more flying to cover a given area. However, the cost of the flying, especially for extensive areas, is a minor item in the total cost of the finished map. With an increase in flying height, some detail is very likely to be sacrificed regardless of the magnification of the pictures in the plotting machine. In many instances, the detail thus lost is of minor importance or of no value at all. However, at other times this detail is of major importance, outweighing the cost of extra control and flying.

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[5] Photogrammetric Engr., U. S. Geological Survey, Arlington Va.

The multiplex adapts itself to bridging over an area of several consecutive models, since a number of stereoscopic models can be set up simultaneously. Thus, considerable horizontal control is eliminated. The stereoplanigraph can accomplish the same result if an adjacent model is set up on the established control, which is a system of "pass points." However, as the stereoplanigraph can accommodate only one stereoscopic pair at a time, a number of single models, requiring considerable time and repetition, must be set up and oriented.

The multiplex tracing table, or stand, is far from clumsy. Sliding on quartz feet, it moves smoothly and easily, being limited in speed only by the skill of the operator. Although the multiplex may require more models, the time of operation represents only from about 5% to 8% of the total time consumed. The increased flying height at which the pictures for the stereoplanigraph are taken necessitates a much more precise setup of the model to obtain the desired accuracy. With the multiplex, the average operator can place the floating mark in the model on the ground to an accuracy of ±0.05 mm which, at a scale of 1:10,000, represents about 3 ft. Assuming the contour interval to be 20 ft, this is well within the accuracy of 90% of the contours in one half of an interval. In setting up a model in the stereoplanigraph, the floating mark would have to be set accurately to ±0.025 mm—a very limited tolerance.

One distinct advantage of the multiplex is that the operator is working directly over the map and with a flip of a switch can see what he has drawn, whereas the stereoplanigraph requires an assistant whose job is to observe the actual drawing, the operator not being in a position to view his work readily. Although the multiplex system can be used by somewhat less skilled personnel with less training, it has proved highly desirable to utilize only the higher type of personnel—preferably operators with a working knowledge of topographic features. The stereoplanigraph has an advantage over the multiplex in that it can accommodate a wide range of scales from a given pair of photographs, whereas the range of the multiplex is very limited, the flight height depending on the scale and contour interval desired.

Two factors that have a tremendous bearing on the comparison of the two methods—photogrammetric versus ground—are the extent of the area and the scale. The multiplex is adaptable to scales from 1:2,500 to 1:24,000; the stereoplanigraph has a slightly wider range; and neither of them is as economical for small areas as for large areas—say, 60 sq miles or greater. Flying, sending control parties into the field, etc., to map only a few square miles by the multiplex or other photogrammetric method are not economical procedures under present conditions as compared with field or ground methods. In numerous instances, a planimetric base map has been prepared by the multiplex method and assigned to a field party for addition of the contours, especially in flat areas.

Another factor that is important in comparing the economies of the multiplex with those of the stereoplanigraph is the photographic laboratory and its skilled personnel. In connection with a large or medium sized multiplex organization, a complete photographic laboratory is maintained under the supervision of a highly trained laboratory photographer with several assistants. Since there is only one stereoplanigraph in the United States and since there is little likelihood of ever having many assembled in one place, it would appear that either economy or the advantage of a highly skilled photographer would have to be sacrificed unless photographic work of a similar nature were available to occupy part of the photographer's time.

To summarize: For small areas up to several square miles the ground method should be far more economical. In flat areas either the ground method or a combination of ground and photogrammetric methods should prove most satisfactory. Over extended areas, especially where the terrain does not permit easy access by foot, the photogrammetric method should be superior and more economical. As to which is the most suitable photogrammetric method, one must take into consideration the conditions pertaining to each particular assignment. The tremendous first cost of obtaining equipment, plus the training of personnel, must be considered along with the type and extent of mapping to be done before the most economical method can be ascertained. For example, the U. S. Geological Survey is engaged in a continuous and extensive mapping program where expensive equipment and trained personnel will be utilized over a long period of time; thus, the multiplex method combined with ground methods has proved the most economical. On the other hand, for a private company interested in a small local area on a job of short duration, the ground method, the stereocomparagraph, or the K.E.K. Plotter should prove more economical.

George D. Whitmore,[6] M. Am. Soc. C. E.—This discussion of Mr. Eliel's excellent and comprehensive paper is limited to that part which deals with comparisons and relations between contour mapping by photogrammetric methods and by ground-survey methods.

Mr. Eliel's explanations may leave an impression, with those who are not professional mappers, that a modern contour map may be completely and finally compiled by stereoscopic plotting instruments, without recourse to any follow-up ground surveys. The topographic mapping program of the Tennessee Valley Authority, directed by its Maps and Surveys Division, has included standard contour maps prepared by all three of the principal processes Mr. Eliel describes—that is, (1) multiples, (2) stereoplanigraph, and (3) Brock and Weymouth. This experience makes it seem virtually mandatory that all contour maps compiled by stereophotogrammetric processes be field inspected and field checked. Otherwise, there is no positive assurance that the map as drawn in the photogrammetric laboratory does correctly represent the terrain, and unless certain field checks are made as to contour accuracy, there can never be any real assurance that the stereoscopically compiled contours are accurate within specifications. Thus, it would seem proper to state that, except in emergencies or other rare occasions, the photogrammetric mapping procedures should always provide for the follow-up field inspection and field checking. Accuracy checks are often obtained by random transit and vertical-angle profiles, by numerous side "shots" from a few well-selected plane-table setups, and by other means.

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[6] Prin. Civ. Engr., Maps and Surveys Div., TVA, Chattanooga, Tenn.

Incidentally, the field engineers who are responsible for field inspecting and field checking operations, commonly called field completion, require as much training and skill as do plane-table topographers. Hence, the skilled topographers, who at one time may have thought that photogrammetry would force them "out of business," instead will probably be more in demand, and for work that should be more interesting.

Another matter not mentioned by Mr. Eliel, but obvious to most experienced mappers, is that probably in most cases a combination, in some degree or other, of photogrammetric contouring and ground-survey contouring will be best. For example, most map sheets will include certain flat or relatively flat areas. It is well known to experienced photogrammetrists that, with present equipment, these flat areas quite often present a difficult problem—that is, it is relatively more difficult to obtain satisfactory contours in flat areas than in rugged terrain by stereophotogrammetric instruments (unless special lower-altitude photographs are provided for those areas). On the other hand, such areas are usually most important economically and, therefore, extra precautions should be taken to make sure that the contours in such areas are relatively more exact. Fortunately, in this case, what is difficult for the stereoscopic plotting instruments is easy for the plane-table topographer. Therefore, many experienced mappers today choose this combination method—that is, the "rolling to steep" terrain to be contoured by the stereoscopic plotting instruments; the flat terrain to be contoured by plane table, usually during the later field inspection and field checking processes.

Another well-known combination of photogrammetric and ground-survey methods that has proved efficient on large projects is that of first preparing a detailed planimetric manuscript base sheet by stereophotogrammetry, on which all contours and elevations are later added in the field by plane-table topographers. After the planimetric base sheet is completed in the laboratory, it is usually reduced photographically to the field scale, the topographer being furnished a blue-line print on drawing paper mounted on aluminum or other scale-holding material. This eliminates the need for long plane-table traverses, and usually results in more accurate planimetry with respect to horizontal position. Of course, it is especially advantageous and economical in flat or gently rolling terrain, and where contour intervals of 10 ft or less are required.

Mr. Eliel did not mention the difficulty of obtaining accurate contours through heavily wooded areas by photogrammetric processes. Densely wooded terrain represents one of the unsolved problems of photogrammetric contour mapping. If the pictures are taken during summer, with the trees tall and closely spaced, and the ground surface irregular beneath the trees, it is extremely difficult for the photogrammetrist to compile standard accuracy contours. He can do better if the pictures are taken during the winter, when the leaves are off the trees, and when snow is not too deep on the ground. Obviously, however, it will be a difficult matter to schedule extensive flying in northern latitudes for satisfactory winter pictures, because of the predominance of cloudy weather during the winter season, because the flying days are short and the shadows long, and because of risk of deep snow. Also, regardless of latitude, an evergreen forest photographs no better in winter than in summer. Unfortunately, ground-survey plane-table methods are also at a disadvantage in mapping contours in wooded areas. It is true that any desired degree of accuracy can be obtained by ground-survey methods, but the costs accelerate rapidly. The development of a procedure for obtaining accurate contours in densely wooded terrain is the real challenge to photogrammetry today.

BENJAMIN A. WASIL,[7] JUN. AM. SOC. C. E.—The problems of present-day photogrammetric economics are presented admirably in this paper, which affords an opportunity to look into the future—into postwar mapping—and to appraise the war mapping program as it applies to the future possibilities of making a cheaper map having the same standards of accuracy.

The cost of field control is a major portion of the total cost of a map. Any reduction in field control necessarily would mean a reduction in the accuracy of the map. However, if an office method can be substituted in establishing a part of the control and still maintain the same standards of accuracy, the cost of the map would be lowered. Two office methods now used to establish secondary control are "multiplex bridging" and "radial plotting." Operators of the multiplex do not try to bridge more than twelve photographs. By radial plotting one can do better if perfectly vertical photographs are available. The basic principle to multiplex bridging and radial plotting is the same. The multiplex is an optical method involving three dimensions. Radial plotting is a mechanical method involving two dimensions; the third dimension of relief is projected orthographically on to the plane of the photograph. An advantage of the multiplex is that, in bridging forward, the entire overlap can be seen at once and it is as if an infinite number of pass points were used. However, the greatest advantage of multiplex bridging, as against radial plotting, is that it permits the elimination of tilt. Disadvantages of multiplex bridging are: (1) The final images as viewed have passed through three sets of lenses—the aerial camera lens, the diapositive camera lens, and the multiplex projector lens; and (2) only one flight can be scanned at a time.

In radial plotting, as many adjoining flights can be used as the operator can handle. Adjoining flights help to tighten up the network of secondary control. However, the always present accidental tilt of vertical photographs limits the work of radial plotting.

If the accidental tilt of the photograph were known, it would be possible to use a rectified photograph in the radial plot with the radial center as the nadir point. This would increase the accuracy and span of the radial plot manyfold. The usual methods of determining the tilt of a nearly vertical photograph require at least three ground-control points to each photograph. The necessity for this ground control defeats the purpose of expanding secondary control. However, there is a method of determining the tilt of a nearly vertical photograph without the aid of ground control. The answer is "tri-metrogon photography."

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[7] Asst. Civ. Engr., TVA, Chattanooga, Tenn.

Tri-metrogon photography presently will be used for reconnaissance mapping. It is used in construction of planimetric maps of scales of from 1 : 250,000 to 1 : 1,000,000. Tri-metrogon system consists of exposures made with three cameras, one camera facing vertically downward and two cameras turned away from the vertical, facing perpendicular to the line of flight. The oblique cameras are faced so that the horizon and a small portion of the vertical are photographed. All three cameras are exposed simultaneously, photographing a strip of ground from horizon to horizon perpendicular to the line of flight.

Inasmuch as all three pictures of a tri-metrogon series are taken from the same photo station they have the same nadir point. The tilt and thus the nadir point can be determined from the oblique photographs because the horizon is visible. By a simple raying-in process the nadir point can be determined on the vertical photograph. A check is possible inasmuch as two independent determinations of the nadir point can be made from the two obliques. The nadir point of the central photograph having been determined, the tilt can be found easily and the photograph rectified.

Therefore, by taking oblique photographs, together with the verticals just for the purpose of determining the tilt of the vertical, the operator can determine the degree of tilt without ground control. The flying should be done in the usual manner, with the vertical flights overlapping 40%. The additional cost of photography is negligible. Flying cost, in a mapping project, is a small part of the total cost, and it is just as easy to take three pictures at one time as it is to take one. The additional cost of films, processing, and amortization of two added cameras will be slight. Of course, there is the added cost of constructing the radial plot, but this will be greatly overbalanced by the saving in the cost of the horizontal field control that can be eliminated. Another saving would be in the elimination of scaling in the multiplex work, for the radial plot will establish many secondary control points to each model. It may also mean a greater use of the K.E.K. Plotter, a cheaper apparatus that requires control on each model.

Charles H. Davey,[8] Esq.—Mr. Eliel's interesting technical paper is timely and should serve to focus attention on a development in the engineering and scientific field which, from day to day, is finding wider application. In general, the presentation can be accepted without debate, although in detail it is believed that the author's extended knowledge of the stereoplanigraph method has resulted in conclusions that cannot be fully accepted by those who are thoroughly familiar with other methods.

The stereoplanigraph is acknowledged widely among photogrammatrists to be a versatile and probably the most accurate plotting instrument yet devised. It can be accepted as a standard of comparison, if the comparisons are restricted to demonstrated values. Some of the author's statements bear the implication of demonstrated facts, whereas, actually, they partake more of the nature of personal opinions or theoretical considerations that differ materially from practical applications. For instance, under "Comparison of Operation Costs," item (2), citing the relative flying altitude required for each method, in much of the country that is best adapted for mapping by photogrammetric methods and for contour intervals of 50 ft or more, both methods require photographs taken at altitudes that exceed the practical operating ceiling of commercially available planes and here the theoretical advantage of the stereoplanigraph over the multiplex method disappears at once. It is here that one outstanding advantage of the multiplex method comes into play as it is possible to set up, at one time, a complete stereoscopic model of a strip exceeding 30 miles in length, at a scale of 4 in. equals 1 mile, and adjust this model to horizontal control as a unit. This is an advantage possessed by no other method and, in practical application, it upsets much of the theoretical advantage as to control required, cited by Mr. Eliel, for the stereoplanigraph.

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[8] Chf., Section of Photo-Mapping, Geological Survey, U. S. Dept. of the Interior, Washington, D. C.

In item (4), "Plotting the Map," Mr. Eliel states as a fact that a map can be plotted by stereoplanigraph from five to ten times as fast as by multiplex. He has not demonstrated this statement by any logic and it is extremely doubtful if any impartial test has been made under comparable conditions from which a factual comparison could be made. It is most debatable if impartial observers would concede the argument that it is easier and faster to perform the drawing of a map by controlling one's motions through the use of hand wheels operating in the X-direction and Y-direction than to control them directly by moving the plotting pencil at will. It is further stated in the same paragraph that the multiplex operator will have to draw from two to three times the number of linear inches of contour that the stereoplanigraph operator will have to draw. The writer is not too familiar with operations of the stereoplanigraph, but using Mr. Eliel's data the comparison is about as follows under the comparable condition of mapping with a 20-ft contour interval: The scale of the photographs would be approximately 1 : 27,000 for the multiplex and 1 : 50,000 for the stereoplanigraph; the stereoscopic model would be viewed at a scale of 1 : 10,000 for the multiplex and 1 : 5,000 or 1 : 10,000 for the stereoplanigraph; and the map would be drawn at a scale of 1 : 10,000 and 1 : 8,330, respectively. Thus it would appear that the drawing by the multiplex is less than by the stereoplanigraph. The critical point to consider is that the stereoscopic model is scanned by the operator at the same scale in both instances and for work performed with the same care and attention to detail there appears to be little advantage of one method over the other aside from the admitted advantage of the better optical model which is present in the stereoplanigraph.

It is regretted that definite costs are not available for a careful analysis and comparison of the two methods, for it is the opinion of the writer that each method would be found to have its own particular sphere, where it would operate to best advantage, with perhaps a broad middle ground where there would be little choice between the two. It follows, therefore, that in any extensive project it would be most advantageous to have both types of equipment available.

Mr. Eliel may convey a slightly misleading picture in his statement in the "Summary": "Today in the United States it is probable that nearly all topographic mapping is being done by photogrammetric methods." It is true that photogrammetric methods have a part in nearly all mapping operations at present and in most cases play a major rôle; but the plane table is by no means obsolete. The Geological Survey has maintained an open mind regarding the relative merits of the plane table and photogrammetric methods and it is believed that, for most economical operation, the best features of both methods should be utilized as conditions dictate, either singly or in combination. It has been demonstrated amply that there are extensive areas in the United States where at the present stage of the art photogrammetry alone cannot compete successfully with the plane table used in combination with photogrammetry.

ROBERT H. RANDALL,[9] M. AM. SOC. C. E.—Mr. Eliel's paper fills a very real need. His clear description of the principal photogrammetric methods used in United States mapping operations is of value not only to the engineering profession as map makers—for most of the practicing photogrammetrists are engineers—but also to those in the profession who are users of maps.

Although Mr. Eliel's paper is valuable as a clear description of photogrammetric processes, its principal contribution is in the comparative analysis it affords of the economy of these different procedures. The author's conclusions are presented as the result of analyses based upon comparison of the innate characteristics of the various types of equipment, supplemented and reinforced by a wide acquaintance with the various methods in actual practice. For those who, like him, are concerned with the progress of photogrammetry, it may be interesting to note that there is a large and growing volume of cost information by which the economy of the various procedures used, now or in the future, can also be gaged.

All topographic quadrangle mapping now done by or for the United States Government is done to standard specifications. (Mr. Eliel summarizes these specifications, in respect to vertical accuracy, when he states that 90% of the elevations tested must be correct within one half the contour interval.) Further, federal topographic mapping is tested for compliance with specifications, and any federal agency producing and testing such a map is entitled to certify its accuracy by the following statement printed in the margin of the reproduced copy: "This map complies with the national standard map accuracy requirements."

Federal map-making agencies keep cost and performance records of the principal operations involved in topographic mapping, according to a uniform system which they devised jointly at the suggestion of the Bureau of the

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[9] Chf. Examiner, U. S. Bureau of the Budget, Washington, D. C.

Budget. Records of work done by contract for federal agencies (as, for instance, the so-called strategic mapping program financed by the War Department but actually performed by five federal mapping agencies and by commercial contractors as well) are also available.

From records such as the foregoing the effectiveness of various photogrammetric methods and combinations of methods may be appraised. In addition to information on work done by the stereoplanigraph, multiplex, and Brock systems, information is accumulating in respect to work done by other methods. Among these may be mentioned the Forest Service method, which employs the K.E.K. Plotter; the stereoscopic plotting equipment of the Coast and Geodetic Survey, which employs photographs taken by a 9-lens camera; and, of course, combination methods involving the use of aerial photographs and plane-table operations.

Mr. Eliel's closing remarks, to the effect that photogrammetry has reached the age of full maturity, are very much to the point. Perhaps it could also be said that photogrammetry, in its present stage, is like a man in the full possession of his powers who is fortunate enough to be situated in virgin territory, with all the opportunities of pioneer enterprise before him; for, although photogrammetry has unquestionably "arrived," further developments are sure to increase its potentialities. New opportunities for its use are constantly arising, and new and improved equipment is being tested. In the matter of control, for example, radar, or some procedure related to it, may greatly improve and cheapen the process of locating the plane at the instant a photograph is being taken; or, to some degree, radar may replace present methods of locating ground points.

W. J. Ryan,[10] M. Am. Soc. C. E.—The difficulties involved in mapping heavily wooded areas are worthy of note in any discussion of photogrammetry. These difficulties are much more serious in the Pacific Northwest than in other parts of the United States. The Corps of Engineers, U. S. Army, has mapped a large part of the Pacific Coast area on the scale of 1 : 62,500 with 100-ft contours. Although these maps have great value, they are not entirely dependable because the actual position of the ground cannot be determined through a cover of dense coniferous forest. The method most generally used is: First, mapping the tree tops and then correcting for the height of the timber (which may vary from 25 ft to 250 ft, resulting in large errors).

Photogrammetric mapping is also under a heavy handicap because of the small number of suitable flying days. Forests are frequently covered with smoke during the late summer and fall, and the days or hours of bright weather during the spring are limited. In the latitude of Oregon and Washington the winter months do not provide satisfactory light.

Heavy forest areas also make suitable ground control difficult. In many cases, it is impossible to identify picture points without artificial marking, which sometimes involves climbing and topping trees. Some controls have been established by building towers of trees topped 100 ft or more from the ground. These factors make aerial mapping less satisfactory, because they may delay a project a full year beyond the planned time for completion.

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[10] Chf. Engr., Weyerhaeuser Timber Co., Tacoma, Wash.

If a map is to be of practical value to the logging industry, it must necessarily have the section corner positions as established by the General Land Office (GLO). The proper location of these corners on the map is one of the most difficult factors in mapping. The original surveys were frequently made with little care, and the accepted plat often does not represent the position of the corners on the ground. For commercial use, a map with a scale of anything less than 1 in. equals 400 ft is more valuable for representing the positions of the section corners with relation to geographical points than for providing extreme accuracy in the actual distances on the map. It has been found possible to place section corners on these aerial maps on the basis of the information available from ground surveys and from stream crossings shown on the GLO plat, thus preparing maps of value for reconnaissance work. Such maps must give the drainage and the shape of the terrain, but extreme accuracy in elevation or position are not necessary. The specifications for such work could be much less rigid than those adopted as a standard by mapping agencies of the U. S. Government, if methods were offered for doing the work more rapidly and economically. The U. S. Geological Survey is using a combination of aerial and ground methods that seems to be the most practical answer to the problem if federal requirements for map accuracy are to be maintained. This process is slow and expensive.

If the vast area still unmapped could be covered by another system, and if maps could be made available for this generation instead of the next, it would be a genuine economic accomplishment.

In forest surveys, the preparation of mosaics seldom seems worth the effort, as individual pictures, properly indexed on a planimetric map, are usually more serviceable. For any larger scale than that used for a purely reconnaissance map, the study of the individual prints through a stereoscope is frequently more satisfactory than the contour map that can be made in a heavily timbered area.

In the case of actual working maps for locating roads, railroads, logging plans, and forestry developments, it is still necessary to use ground methods. The writer has found that contour maps made by the strip method, using a slope level and chain, are more dependable and more economical than maps made by the use of plane table. On a scale of 400 ft to the inch with 10-ft to 25-ft contours, such maps cost as much as one dollar per acre. When photogrammetry can develop a method for making such maps, it should be economically advantageous.

In spite of difficulties, there is still a large field for aerial photography in the logging industry. Possibilities of distinguishing the type and conditions of the forest through photographs are unlimited and the further development of color and filters will add to the value of these studies.

The author has stressed the relative value of the various types of equipment, thus clarifying the possibilities that are available to the engineer. Consideration of the subject from the viewpoint of cost is commendable, and further reports are needed to define the relationship between cost and accuracy before any economic procedure for a project can be established.

Oliver S. Reading,[11] M. Am. Soc. C.E.—An excellent description of present photogrammetric practices and of their varied applications and limitations is presented in this paper. Comparisons of the relative economy or efficiency of the different instruments and combinations of methods emphasize the great flexibility of photogrammetry. As Mr. Eliel has shown, excellent work may be done by almost any combination of methods.

Many factors in addition to the methods described in the paper cause wide variations in efficiency. The character of the terrain, size and location of project, skill of personnel, quality of photographs—any of these factors are capable of causing a greater range in cost than the variation in efficiency of the methods used. Although many of these factors will average out as cost data are accumulated for a very large number of projects, fully adequate data for determining costs are not yet available. To the present time most photogrammetric work has been so sporadic and subject to such fluctuation of funds and to such urgency for production regardless of cost, training, or skill when funds have been available, that present cost data probably are misleading.

Arguments based on theoretical considerations are likely to err both from lack of consideration of factors which may have been unimportant on projects with which the individual is familiar, and from differences in evaluation resulting from special conditions that may prevail in certain organizations. Thus, the factor of smaller scale and larger area in air photographs is limited by the ceiling of the aircraft and the resolution of the lenses and photographic materials available. The ratio of control cost to number of photographs is important, but this argument favors extrawide-angle, or multi-lens photographs requiring different instruments. Also, where many roads and adequate breakdown of geodetic control exist, the cost of supplemental control may become a minor consideration. Again, if it were easier and faster to trace, accurately, the lines of a map by the manipulation of wheels moving the tool in the X-direction with one hand and in the Y-direction with the other hand than to guide the tool directly by hand, probably the wheel drives would be widely used for engraving. However, more information on this question will be available soon from a direct comparison of two plotting machines under construction in the photogrammetric office of the United States Coast and Geodetic Survey. One of the machines will use direct motion of the hand, accompanied by motion of the head in tracing the lines of the map; and the other will drive the tracing point by an X-handwheel and a Y-handwheel, while the operator's head remains stationary as with the stereoplanigraph.

Until recently there has been but one stereoplanigraph in the United States, and it has been operated much of the time by a man who admittedly has exceptionally acute stereoscopic vision and skill as an operator. The capture and importation of several stereoplanigraphs from Germany to the United States will allow a direct comparison in the same office, on the same projects, of the different machines and methods. Thus, much more reliable data should be obtained on the most economical field of application for each instrument or combination. Such direct comparisons of the stereoplanigraph, wide-angle multiplex, and 9-lens stereoplotters are contemplated in the Division of Photogrammetry of the U. S. Coast and Geodetic Survey during 1946 and 1947. Other organizations also will doubtless make direct comparisons.

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[11] Chf., Research Section, Div. of Photogrammetry, U. S. Coast and Geodetic Survey, Washington, D.C.

Incidentally, recent surveys indicate that multi-lens cameras and equipment are more efficient than the 95° single-lens equipment for many projects. Multi-lens photographs are far from obsolete, although possibly further development of extrawide-angle lenses with admitted distortion in the air photograph corrected in subsequent printing, or of lenses using spherical focal surfaces may lessen the advantages of multi-lens cameras.

Certainly, photogrammetry has arrived at maturity in that no sizable mapping project can be executed with maximum efficiency unless some use is made of photographic surveys; yet those who are concerned with improving the efficiency and usefulness of photogrammetry are eagerly trying so many promising ideas (new lenses, camera stabilization, simpler plotters, color film, and radar control) that the economics of photogrammetry will surely require frequent discussion in the future.

Leon T. Eliel,[12] Esq.—It is very gratifying to the writer that his paper provoked so much excellent and constructive discussion. He is especially grateful that all the discussion on such a controversial subject has been presented with great temperance and restraint.

The twelve stereoplanigraphs captured from Germany should yield some fine comparative data between the different photogrammetric systems, as suggested by Mr. Reading. It is to be hoped that they will actually produce very efficiently—which objective certainly transcends, in importance, all academic considerations.

Among the discussions were six by map makers and two by map users. The latter, by Messrs. Hill and Ryan, have been especially constructive. Mr. Hill's remarks are of special importance to users who are less interested in how aerial photography is done, than they are in what it can do.

Mr. Ryan has given a thoughtful and thorough analysis of the application of photogrammetric methods to the problems of the forest engineer. The difficulties of control, inability to see the ground through the trees, and the problems of weather apply almost universally, as well as to forest problems.

Frequent reference has been made to the advantage the multiplex enjoys in "bridging" and thus minimizing control. Reports presented to the American Society of Photogrammetry show that the Germans (who invented and use both the stereoplanigraph and the multiplex) when under the pressure of war preferred the stereoplanigraph to the multiplex for bridging, using the multiplex to fill in the details.

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[12] Vice-Pres., Fairchild Aerial Surveys, Inc., Los Angeles, Calif.

The multiplex gives about half the vertical accuracy obtainable from the stereoplanigraph when the flight altitude is the same. For comparable accuracy, in other words, multiplex pictures are taken from half the height required for stereoplanigraph flying. In the bridging method, the accuracy is found to be somewhat less than if control had been available for each model. Assuming that a "bridge" of a certain length gives half the accuracy obtainable from control in every model, the altitude for stereoplanigraph flying and a "bridged" multiplex series would be 4 to 1. Thus, the stereoplanigraph area per model, compared to the "bridged" multiplex area per model, at comparable vertical accuracy, might be 16 to 1.

In this hypothetical case, bridging causes such a sacrifice in flight altitude that the "bridge" would have to be sixteen models long to have the same control economy as the stereoplanigraph with control in each model. Whether the bridge involves sixteen models, or some greater or smaller number, it is apparent that there is a diminishing return that may actually prove that the stereoplanigraph requires less control. Add the inefficiency of handling all the additional models required with the lower flight altitude and the "bridging" argument can readily "boomerang" in favor of the stereoplanigraph.

Probably W. Schermerhorn of the Dutch University at Delft, Holland, and Prime Minister of Holland, has conducted more research on bridging than any one. He concluded that, if bridging must be done, the stereoplanigraph was better for the purpose than the multiplex. He operated both instruments, as well as many others, very extensively. Of course, his experiments were with the Zeiss narrow-angle multiplex which Americans who have used both consider inferior to the Bausch and Lomb's wide-angle instrument.

The writer sees little justification for bridging as an economy measure. Only under war conditions or when the terrain is inaccessible because of natural conditions does there seem to be a clean-cut case for bridging; and, if bridging must be done, many authorities prefer the stereoplanigraph to the multiplex.

Little information was available on the K.E.K. Plotter or the Reading 9-lens plotter, when this paper was first written. Fortunately, these new instruments have been described in the discussion.

The writer recommends a "decent burial" for one plotting instrument—the aerocartograph, which was mentioned in one discussion as comparable to the stereoplanigraph. Having had the two instruments side by side for twelve years, the writer considers that they are alike only in complexity of appearance and nomenclature. In fact, the aerocartograph is so completely obsolete that sometime soon the writer will be glad to offer a good one to a museum.

Mr. Davey made it very clear that, for small-scale mapping, the airplane cannot fly high enough to exceed the efficient limits of any of the methods. Thus, for 50-ft contours the multiplex flying could be from 30,000 ft aboveground, whereas the stereoplanigraph flying should theoretically be done at 62,500 ft. The latter is obviously impossible as 30,000 ft is substantially the limit of presently available aircraft. Thus, the contour interval must be less than 50 ft before the efficiency of the stereoplanigraph can be realized.

In summary, no photogrammetric method is a cure-all for mapping problems. Photogrammetry is a new and useful tool, and it richly supplements all the other well-known methods. Under many conditions, photogrammetric methods are superior. Sometimes the methods are not practicable, and yet again, a combination of methods is best. In discussing photogrammetry, engineers should not lose sight of the many valuable applications of aerial photography. A great many more square miles are studied usefully from aerial photographs and mosaics than from contour maps.

Each of these new aerial tools should be placed carefully in its proper category of usefulness.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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Paper No. 2278

FLOW FROM DRAINAGE BASINS DETERMINED BY SHORT-TERM RECORDS

BY LLOYD L. HARROLD,[1] ASSOC. M. AM. SOC. C. E.

WITH DISCUSSION BY MESSRS. SAMUEL ROBERTS, AND LLOYD L. HARROLD.

SYNOPSIS

A small number of short-term records of runoff of less than 10 years' duration are becoming available from the agricultural watersheds of Soil Conservation Experiment Stations, U. S. Department of Agriculture, throughout the various climatic regions of the United States. Data from one of these stations, at LaCrosse, Wis., are presented utilizing these relatively short records to derive values of maximum rates of flow for various recurrence intervals.

INTRODUCTION

Long-term records of peak rates of runoff, and flood-crest elevations for large drainage areas, have been published for years. These data frequently have been used as the basis of the hydraulic design of large flood-control and power projects as well as large culvert designs for highways and bridges. They have also been the subject of numerous scientific papers. Where actual runoff records were not available it has been common practice to derive peak runoff values for various recurrence intervals by using the rational method[2] with rainfall-intensity values published by the late David L. Yarnell,[3] M. Am. Soc. C. E. More recently the design of extremely costly flood-control projects in some cases has warranted the more detailed analysis as applied by W. W. Horner, M. Am. Soc. C. E., to storm sewers[4] and to large agricultural watersheds.[5]

Unlike the records for large drainage areas, those for small areas have not been published widely; nor have they been used extensively in designs for small hydraulic structures. The difference is due primarily to the fact that, until recently, data of this type were collected for only a few localities and for short periods. The need for runoff data from small urban areas has long been

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NOTE.—Published in April, 1945, *Proceedings*. Positions and titles given are those in effect when paper or discussion was received for publication.

[1] Project Supervisor, SCS, North Appalachian Experimental Watershed, Coshocton, Ohio.

[2] "Runoff—Rational Run-Off Formulas," by R. L. Gregory and C. E. Arnold, *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 1038.

[3] "Rainfall Intensity—Frequency Data," by David L. Yarnell, *Miscellaneous Bulletin No. 204*, U.S.D.A., 1935.

[4] "Surface Runoff Determination from Rainfall Without Using Coefficients," by W. W. Horner and S. W. Jens, *Transactions*, Am. Soc. C. E., Vol. 107 (1942), p. 1039.

[5] "Rôle of the Land During Flood Periods," by W. W. Horner, *ibid.*, Vol. 109 (1944), p. 1269.

recognized in connection with the design of storm sewers, but the need for runoff data from small rural areas has not been so apparent in connection with the design, construction, and maintenance of highways, railroads, and drainage systems. This latter need became more urgent when programs for soil and water conservation came into front-page prominence in the early 1930's. Works by C. E. Ramser,[6,7] M. Am. Soc. C. E., based on rainfall and runoff data from six watersheds ranging in size from 1.25 to 112 acres on the Lexington silt loam soil of the Murchison farm near Jackson, Miss., contributed much toward meeting the need in this field of soil and water conservation. These data formed the basis for the first runoff values used widely throughout the United States in the design of gully control structures, terrace outlet ditches, road culverts, and other hydraulic construction on small drainage areas.

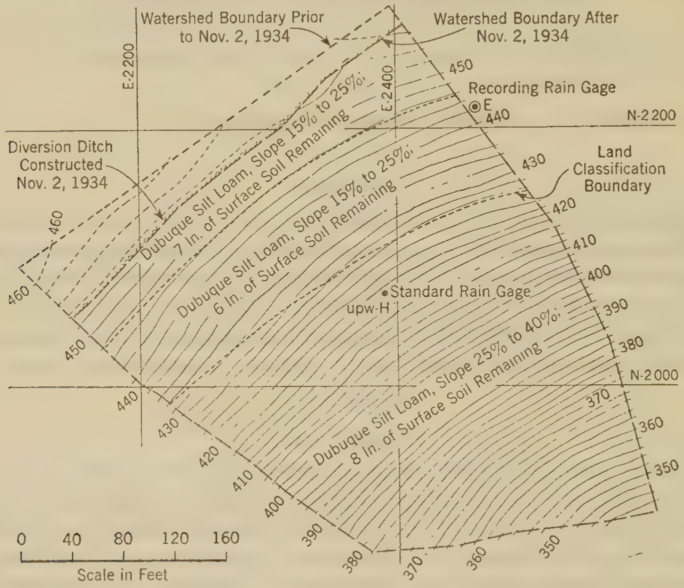
Watershed Boundary Prior to Nov. 2, 1934
Watershed Boundary After Nov. 2, 1934
E-2200
E-2400
450
Recording Rain Gage
E
440
N-2200
Diversion Ditch Constructed Nov. 2, 1934
Dubuque Silt Loam, Slope 15% to 25%, 7 In. of Surface Soil Remaining
430
Land Classification Boundary
Dubuque Silt Loam, Slope 15% to 25%, 6 In. of Surface Soil Remaining
420
410
400
460
460
Standard Rain Gage
upw-H
390
450
380
Dubuque Silt Loam, Slope 25% to 40%, 8 In. of Surface Soil Remaining
N-2000
440
370
430
360
420
350
410
400
0 40 80 120 160
Scale in Feet
390
380
370
360
350

Fig. 1.—Unterraced Pasture Watershed No. 1

As it was realized that similar data should be obtained on other soil types and in other climatic and agricultural regions, ten soil and water conservation stations were established in cooperation with the various state agricultural experiment stations, and hydrologic observations were begun on small agricultural watersheds at each station, under the Bureau of Chemistry and Soils and the Bureau of Agricultural Engineering, U. S. Department of Agriculture. In 1935 these stations were transferred to the Soil Conservation Service. Records from the LaCrosse station, one of the original ten stations, are used herein to present a procedure for deriving values of peak runoff for various recurrence intervals.

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[6] "Runoff from Small Agricultural Areas," by C. E. Ramser, *Journal of Agricultural Research*, Vol. 39, May, 1927, p. 797.

[7] "Brief Instructions on Methods of Gully Control," by C. E. Ramser, Bureau of Agricultural Engineering, U.S.D.A., August, 1933 (mimeographed).

Description of Watersheds

Watershed No. 1 (Fig. 1) was established in 1932. The records used in this example are for the 10-yr period, January 1, 1933, to December 31, 1942. Prior to November 2, 1934, the area comprised 2.713 acres, reducing to 2.412 acres after that date. It was pastured during 1933 and from 1935 to 1942. Pasturing was discontinued and hay was cut periodically from January 1, 1934, to July 17, 1935. Traces of an old road across the watershed were removed in June, 1936. The average land slope is 24%. Dubuque silt loam predominates.

Watershed No. 2 was established in 1932. Records used for this area shown in Fig. 2 are for the 10-yr period, 1933 to 1942. From March 22, 1934, to April 13, 1938, the area comprised 2.335 acres, reducing to 2.245 acres thereafter. This drainage area, on an average land slope of 15%, was entirely cultivated on the contour from 1933 to 1937. By the fall of 1937 the lower third of this area had been so severely eroded that it was seeded to meadow. The meadow was cut periodically for hay after 1938, and the upper two thirds remained in contour cultivation with a 3-yr rotation of corn, small grain, and hay. During the 4-yr period, 1934 to 1937, the 7-in. depth of topsoil on the lower third of the area was reduced by erosion to 3.5 in. This does not mean that land of 15% slope or more cannot be cultivated. Conservation measures, such as stripcropping and terracing, are proving quite effective on steep slopes in this region. As shown in Fig. 2, the soil of the upper portion of the area is Clinton silt loam and that of the lower is Dubuque silt loam.

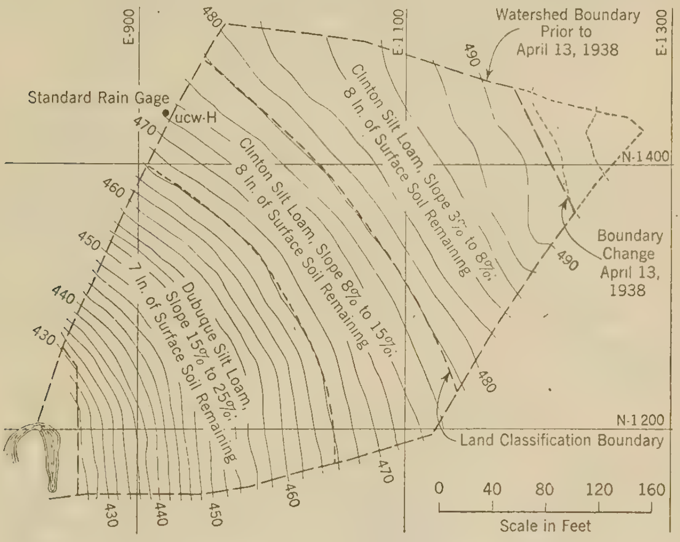
E-900
480
E-1100
Watershed Boundary Prior to April 13, 1938
E-1300
490
Clinton Silt Loam, 8 In. of Surface Soil Remaining, Slope 3% to 8%
Standard Rain Gage
ucw-H
470
Clinton Silt Loam, 8 In. of Surface Soil Remaining, Slope 8% to 15%
N-1400
460
Boundary Change April 13, 1938
450
490
7 In. of Surface Soil Remaining Dubuque Silt Loam, Slope 15% to 25%
440
430
480
N-1200
Land Classification Boundary
0 40 80 120 160
Scale in Feet
430
440
450
460
470

Fig. 2.—Unterraced Cultivated Watershed No. 2

Watershed No. 3 was established on January 1, 1937. Records used for this area shown in Fig. 3 are for the 6-yr period, 1937 to 1942. The drainage area of 2.705 acres on an average land slope of 17% lies on Dubuque and Clinton silt loam soils. The lower third of the watershed was continuously in alfalfa

TABLE 1.—PEAK RUNOFF AND LAND USE, 1933–1942

Date	WATERSHED NO. 1 (PASTURE)		WATERSHED NO. 2 (CONTOUR CULTIVATED)[a]		WATERSHED NO. 3 (STRIP-CROPPED)		TERRACE A-5 (CULTIVATED)	
	Rate[b]	Land cover	Rate[b]	Land cover or treatment	Rate[b]	Land cover or treatment	Rate[b]	Land cover or treatment
1933								
July 1	0.10	Grass; good cover	1.23	Barley; 15-in. to 20-in. cover	....[c]	....	0.164	....
July 1	0.11		0.60		....[c]	....	0.254	....
1934								
June 23	0.19	Grass; fair cover; not pastured	1.05	Mixed hay; poor stand	....[c]	....	....[c]	....
June 25	0.13		1.10		....[c]	....	....[c]	....
July 5	2.81		3.75		....[c]	....	0.80	Hay
1935								
July 5	0.12	Hay; cut June 11	4.63	Corn	....[c]	....	1.45	Corn
July 27	0.23	Grazing	1.67		....[c]	....	0.17	
Aug. 1	0.79		2.17		....[c]	....	1.18	
Aug. 3	0.54		1.91		....[c]	....	1.10	
Aug. 5	2.12		2.66		....[c]	....	2.08	
Oct. 30	0.12		2.83		....[c]	....	0.21	
1936								
Apr. 30	....[c]	Grass; 6 in.; dense cover	5.74	Planted to barley and clover Apr. 28	....[c]	....	0.98	Apr. 28, prepared seed bed, planted barley[d]
May 22	0.57		5.71	Barley, 3 in.; thin stand	....[c]	....	2.16	Barley, 3 in.; thin stand
Aug. 27	0.71		2.29	Clover reseeded Aug. 24[e]	....[c]	....	0.44	New seeding; weak cover
Sept. 6	0.07	Grass; 1 in. to 3 in.; good cover	1.21	New seeding; weak cover	....[c]	....	0.20	
Sept. 15	0.46		1.51		....[c]	....	0.42	
Sept. 20	0.07		1.43		....[c]	....	0.22	
Oct. 3	1.17		2.38		....[c]	....	0.60	
1937								
June 20	2.28	Grass; 4 in. to 9 in.	4.17	Sparse cover	2.96	Alfalfa and barley, 36 in.[f]	1.19	Clover cut June 7[g]
Aug. 3	0.17		1.38		0.38	Barley cut	0.21	Fair growth of clover
Aug. 18	0.16		1.34		0.48	Alfalfa cut	0.19	
1938								
July 2	0.87	Grass; 2 in. high	1.12	Meadow, 12 in.; corn, 24 in.	1.50	Alfalfa cut; corn, 24 in.; barley, 30 in.	0.66	June 20 Corn cultivated
July 5	1.54		1.98		2.01		1.45	15 in. to 18 in.
July 6	1.50		2.34		1.92	Barley cut	0.95	....
July 21	1.87		5.80		5.56		0.54	July 13[h]
Aug. 5	....[c]		2.54		1.78		0.47	....
Sept. 9	2.06	Grass; 4 in. to 6 in.	1.76	Hay, 12 in.; corn, 108 in.[i]	2.09	Hay, 12 in.; corn, 108 in.[i]	0.66	....
1940								
Aug. 16	1.73	Grass	2.17	Hay cut Aug. 5	1.92	Hay cut Aug. 5[j]	0.94	Hay cut Aug. 5
1941								
Apr. 17	0.19		1.60	Hay	0.77	Seeded barley Apr. 17	0.13	May 2 Corn cultivated
May 15	0.23		1.17		1.86	Hay cultivated May 5	2.05	May 12
Sept. 15	2.24	Grass	1.77	Hay cut June 23 and Aug. 6	2.59	Hay cut Aug. 6	0.51	Corn cut and shocked Sept. 18
Sept. 16								
1942								
Aug. 8	1.74	Grass	0.25	Hay cut Aug. 19	0.94	Cut hay strips Aug. 19	0.14	Barley stubble
Sept. 17	1.22		1.87		1.79		0.27	
Sept. 19								

[a] Entire area was cultivated from 1933 to 1937. The lower one third of the area was meadow from 1938 to 1942 and the remaining two thirds of the area was cultivated. [b] Peak runoff rate, in inches per hour. [c] No record available. [d] Also seeded red clover. [e] In barley stubble. [f] Also corn, 3 in. [g] Young clover 4 in. to 8 in. high does not offer much protection. [h] Corn, 60 in. to 72 in. [i] Also stubble and clover, 9 in. [j] Barley cut July 23.

which was cut periodically for hay, and the upper two thirds was strip-cropped with a 3-yr rotation of corn, barley, and hay.

Terrace A-5 is 1,385 ft long, has a drainage area of 1.9 acres and a variable grade of from 1 in. to 3 in. per 100 ft of terrace, and lies on Clinton silt loam having a surface slope of 12%. The vertical spacing between terraces is 7 ft. The drainage area is cultivated parallel to the terrace ridge.

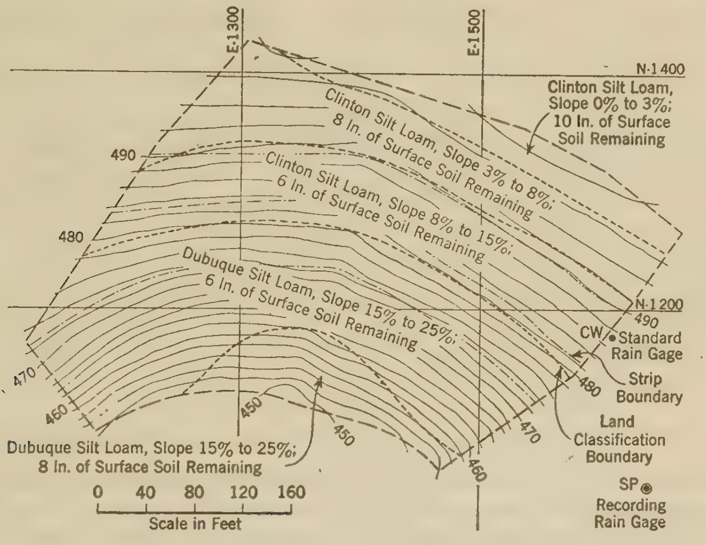
E-1300
E-1500
N-1400
Clinton Silt Loam, Slope 0% to 3%; 10 In. of Surface Soil Remaining
Clinton Silt Loam, Slope 3% to 8%; 8 In. of Surface Soil Remaining
490
Clinton Silt Loam, Slope 8% to 15%; 6 In. of Surface Soil Remaining
480
Dubuque Silt Loam, Slope 15% to 25%; 6 In. of Surface Soil Remaining
N-1200
490
CW • Standard Rain Gage
Strip Boundary
470
480
460
450
450
470
460
Land Classification Boundary
Dubuque Silt Loam, Slope 15% to 25%; 8 In. of Surface Soil Remaining
SP ◉ Recording Rain Gage
0 40 80 120 160
Scale in Feet

FIG. 3.—CONTROLLED (STRIP-CROPPED) WATERSHED NO. 3

Rainfall records were obtained from automatic weighing and recording rain gages near the watersheds. Runoff was measured in Parshall flumes and trapezoidal flumes. Hydrographs and summary tabulations of rainfall and runoff for watersheds Nos. 1, 2, and 3 for the period, 1932 to 1938, have been published.[8]

PEAK FLOWS

The purpose of this paper is to present a method for the derivation of runoff values for various recurrence intervals for cultivated, strip-cropped, and pastured areas of different sizes, and for cultivated terraces, using short-term records of different lengths from small watersheds. All of the peak runoff rates greater than 1 in. per hr in each year for each watershed and terrace were tabulated in chronological order (see Table 1). These data were then arranged in descending order of magnitude (see Table 2) for either the period of record or the period during which there was no material change in land use. If the data given in Table 2 are used for purposes other than the method presented in this paper, careful attention should be given to the description of land cover. It is likely that a cultivated watershed, at times when small grain or hay is on the area, may have a better hydrologic cover than the permanent meadow watershed. Only the number of highest values corresponding to the number of years of record were used, as additional values would not appear in the plotting. From this duration type of tabulation the data were plotted on logarithmic probability paper (Fig. 4), as described by the late Allen Hazen,[9] M. Am. Soc. C. E., and as illustrated in an army manual.[10] Plotting points were derived from the equation—

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[8] "Hydrologic Studies: Compilation of Rainfall and Runoff from the Watersheds of the Upper Mississippi Valley Conservation Experiment Station, LaCrosse, Wisc. 1932–38," SCS–TP–29, U.S.D.A., November, 1939 (mimeographed).

TABLE 2.—Peak Runoff in Order of Magnitude and Plotting Position (Eq. 1)

Order	Watershed No. 1 (Pasture)		Watershed No. 2 (Contour Cultivated)				Watershed No. 3 (Strip-Cropped)		Terrace A-5 (Cultivated)	
	1933–1942		1933–1937		1938–1942[a]		1937–1942		1933–1942	
	Peak rate (in. per hr)	Plotting position (%)	Peak rate (in. per hr)	Plotting position (%)	Peak rate (in. per hr)	Plotting position (%)	Peak rate (in. per hr)	Plotting position (%)	Peak rate (in. per hr)	Plotting position (%)
1	2.81	5	5.74	10	5.80	10	5.56	8	2.16	5
2	2.28	15	5.71	30	2.54	30	2.96	25	2.08	15
3	2.24	25	4.63	50	2.34	50	2.59	42	2.05	25
4	2.12	35	4.17	70	2.17	70	2.09	58	1.45	35
5	2.06	45	3.75	90	1.98	90	2.01	75	1.45	45
6	1.87	55	....	....	....	....	1.92	92	1.19	55
7	1.74	65	....	....	....	....	....	....	1.18	65
8	1.73	75	....	....	....	....	....	....	1.10	75
9	1.54	85	....	....	....	....	....	....	0.98	85
10	1.50	95	....	....	....	....	....	....	0.95	95

[a] Lower third in meadow and upper two thirds in contour cultivation.

$$F = \frac{N - 0.5}{t} = \frac{2\,N - 1}{2\,t} \qquad (1)$$

in which: F = the plotting point; N = the order number of the peak (highest value = 1, next highest = 2, etc.); and t = the number of years of record. This formula for the plotting position of peak runoff values is based on the assumptions that the 10-yr record is one of ten such periods in 100 years, that the highest value in the actual 10-yr record is the median of highest values in each of ten 10-yr periods in 100 years, and that in a period of 100 years the actual recorded peak value for the 10-yr record will be equaled or exceeded about five times (recurrence interval 20 years).

The records for cultivated watershed No. 2 show that the entire area was cultivated for the first 5 years, 1933 to 1937, and that only the upper two thirds of the area was cultivated in the last 5 years, 1938 to 1942. Curve A (Fig. 4) resulted from using the 10-yr (1933 to 1942) record for watershed No. 2 as one period and, as it represents two widely different watershed conditions, it should not be used. Curves B (1933 to 1937) and C (1938 to 1942) are those for the separate periods and represent therefore the specific watershed conditions for the separate periods. Curve C represents all points based on watershed No. 3, which was strip-cropped for its entire period of record, and on watershed No. 2 during the period when its lower third was placed in permanent meadow and the upper two thirds was devoted to contour crop rotation (which is somewhat similar to strip-cropping).

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[9] "Flood Flows; A Study of Frequencies and Magnitudes," by Allen Hazen, John Wiley & Sons, Inc., London (1930), p. 106.

[10] "Engineering Construction, Flood Control," *Publication X-156*, Engr. School, Fort Belvoir, Va., p. 106.

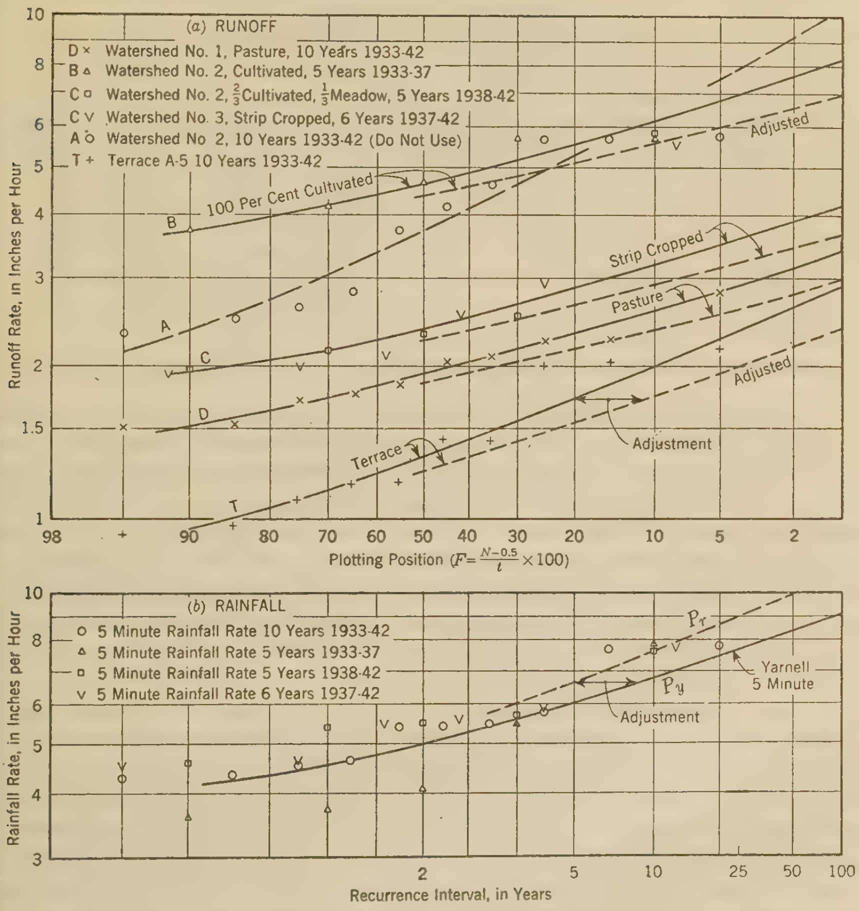

Fig. 4.—Values of Peak Runoff for Various Recurrence Intervals, Eq. 1

Values of runoff for various recurrence intervals may be derived directly from the curves in Fig. 4. However, these values may be too high or too low depending on whether the period of record was normal or of high or low rainfall intensity. To determine this factor, the rainfall intensity data for all the maximum 5 min (approximate time of concentration) for each of the periods of years used in the runoff plotting were arranged in descending order of magnitude (Table 3) and plotted on logarithmic probability paper, as shown by the points near curve P_r on the lower part of Fig. 4. It is assumed that curve P_r represents the rainfall potentiality of the period of record to produce high runoff rates. A comparison of curve P_r with curve P_y which represents Yarnell's[3] 5-min rainfall values for the LaCrosse area shows whether the rainfall potentiality of the period was above or below normal.

TABLE 3.—MAXIMUM RAINFALL RATES FOR FIVE MINUTES, IN DESCENDING ORDER OF MAGNITUDE

Order	1933–1942		1933–1937		1938–1942		1937–1942	
	Rate (in. per hr)	Plotting position (%)	Rate (in. per hr)	Plotting position (%)	Rate (in. per hr)	Plotting position (%)	Rate (in. per hr)	Plotting position (%)
1	7.92	5	7.92	10	7.68	10	7.68	8
2	7.68	15	5.42	30	5.72	30	5.72	25
3	5.72	25	4.08	50	5.52	50	5.52	42
4	5.52	35	3.72	70	5.43	70	5.43	58
5	5.43	45	3.60	90	4.64	90	4.64	75
6	5.42	55	....	....	....	....	4.56	92
7	4.64	65	....	....	....	....	....	....
8	4.56	75	....	....	....	....	....	....
9	4.35	85	....	....	....	....	....	....
10	4.32	95	....	....	....	....	....	....

It should be noted that the rainfall intensities used for curve P_r do not necessarily belong to the same storms as the peak runoff values plotted in Fig. 4. In other words, the highest rainfall value recorded in the 10 years may belong to a 1938 storm, whereas the highest peak runoff value recorded in 10 years may belong to a 1936 storm. Although 5-min rainfall values recorded in the various periods reveal the potentiality of that period (as far as rainfall is concerned) to produce high runoff peaks on these small areas, additional factors (such as land cover, soil moisture, and degree of aggregation, and rainfall amount and intensities preceding the maximum 5 min) play a large part in the determination of the magnitude of the peak flow.

In Fig. 4, inasmuch as the rainfall-intensity values (curve P_r) greater than (to the right of) a 2-yr recurrence are higher than the Yarnell values (curve P_y), it was assumed that the rainfall recorded in the 10-yr period was abnormally high. It was concluded, therefore, that the runoff curves were also abnormally high and should be adjusted to represent a normal period. As the recurrence interval of rainfall-intensity values for this period of record are considerably lower than (to the left of) the Yarnell curve (P_y), the runoff curves were raised (moved to the right) by recurrence-interval amounts equal to the horizontal difference between the rainfall curves P_y and P_r at various points. For example (Fig. 4), the recurrence interval for a rainfall intensity of 6.6 in. per hr for the period of record is 5 years, whereas the Yarnell recurrence interval for this same intensity is about 10 years; therefore, all 5-yr runoff values were raised to about 10 years. Similar adjustments were derived for other points on the rainfall curve and applied to corresponding points on each runoff curve. The unadjusted and adjusted runoff curves are shown on arithmetic coordinates in Fig. 5, which is the form most commonly used in field application.

It was considered better to make adjustments to normal in the recurrence interval values than to adjust the runoff values (curves in Fig. 4) by the same amount as the vertical differences between curves P_r and P_y at corresponding recurrence intervals. The latter method appears illogical as it assumes that, if the rainfall experience (for example) was 1 in. per hr greater than normal, the normal runoff value should be 1 in. per hr less than that on the observed curve.

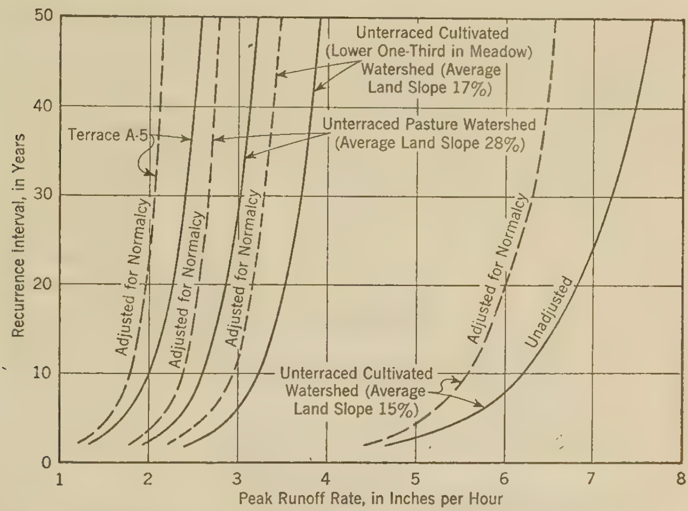

FIG. 5.—PEAK RUNOFF VALUES FOR VARIOUS RECURRENCE INTERVALS DERIVED FROM PLOTTING EQ. 1

Curves of recorded peak flows and maximum 5-min rainfall for various recurrence intervals derived from plotting values in the equation—

$$F = \frac{N}{t} \qquad (2)$$

along with the Yarnell 5-min curve are shown in Fig. 6. Although the Yarnell rainfall curve in this figure is the same as that in Fig. 4, the rainfall curve P_y and the runoff curves differ from the corresponding curves in Fig. 4. The data plotted in Fig. 6 are based on the assumption that the highest value in a 10-yr record has a recurrence interval of 10 years (will be equaled or exceeded once every 10 years on the average) and that in 100 years this runoff value will be equaled or exceeded ten times.

As in Fig. 4, the runoff values in Fig. 6 for various recurrence intervals must be adjusted for normalcy. Inasmuch as the rainfall rates recorded in the period of record were abnormally high when compared with the Yarnell curve (P_y), it was concluded that the runoff values were also higher than those to be expected on the average. Rather than to adjust the runoff value, the values of the recurrence interval for the various runoff rates were adjusted as explained for Figs. 4 and 5. Unadjusted and adjusted runoff curves are shown in Fig. 7.

The procedure discussed hereinbefore has shown how peak runoff values for various recurrence intervals may be derived for drainage areas of nearly the same size as those on which the basic data were collected; namely, about 2 or 4 acres. Further "stretching" of the original data may be required to furnish peak runoff values for drainage areas of different sizes. Some data have been gathered on watersheds from 2 acres to 300 acres in size, but the periods of record are too short to provide a reliable method of extending the results on small areas to larger ones. It is expected that a few more years of record on these watersheds will provide sufficient data to establish such a method.

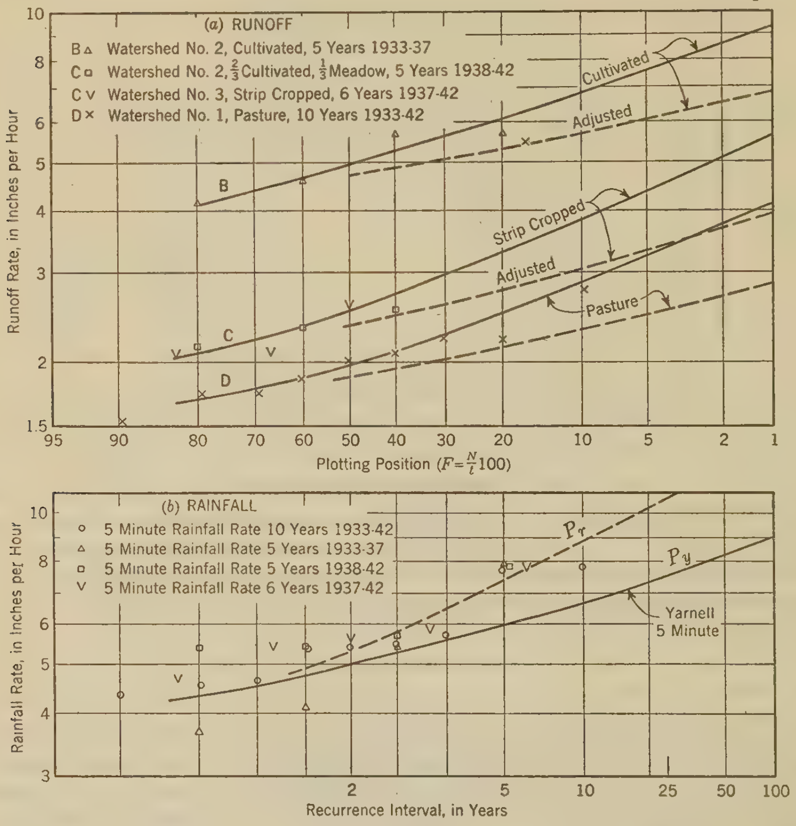

FIG. 6.—PEAK RUNOFF VALUES FOR VARIOUS RECURRENCE INTERVALS, EQ. 2

It is recognized that Mr. Yarnell's values for rainfall intensity and recurrence interval may not serve as a satisfactory basis for the determination of normalcy of the period of observation. However, it is used herein and probably will continue to be used until a more reliable basis is established. Where a long record of rainfall intensity data is available at near-by weather stations, it should probably be used in preference to the Yarnell values.

The position of the adjusted curves (Figs. 4, 5, 6, and 7), derived from the plot of the recorded values according to Eq. 1 and those according to Eq. 2, reveals the fact that the method of adjustment eliminates the differences due to plotting methods. The final results are essentially the same regardless of the plotting method, and whenever small differences occur they are caused by the fact that the plotted points do not define the position of the curves adequately.

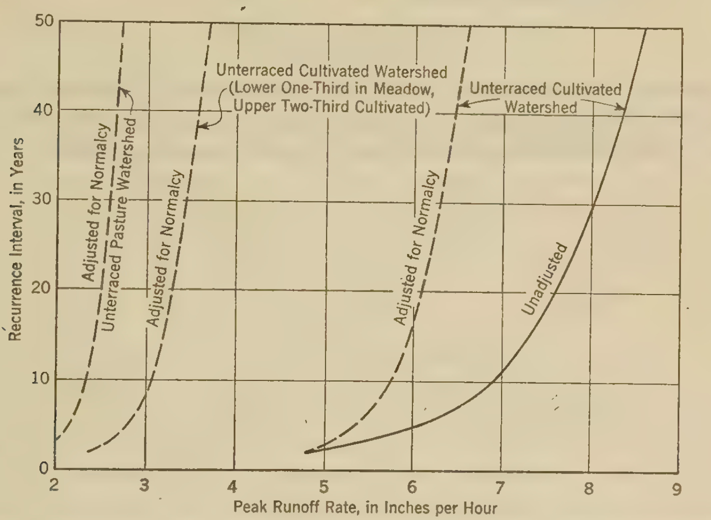

FIG. 7.—PEAK RUNOFF VALUES FOR VARIOUS RECURRENCE INTERVALS DERIVED FROM PLOTTING EQ. 2

This procedure is presented in the expectation that constructive criticism and discussion will follow which will provide an adequate procedure or method for the treatment of similar data.

CONCLUSION

In estimating a peak runoff value for a particular drainage area it has been a common practice in the past (and still is in some hydrologic and hydraulic fields) to apply a selected coefficient to a Yarnell rainfall value for the time of concentration of the area and a given recurrence interval. Usually the selected coefficient has been rather high, as it was assumed that the occurrence of that condition of soil and cover which would yield most runoff would be coincident with that of maximum rainfall intensity for the given recurrence interval. Actually this does not always happen, and the runoff values thus determined are probably too high when compared with those based on a long-term record of runoff.

It is recognized that nothing can take the place of long-term runoff records for this type of runoff-frequency study. Additional years of record on the drainage areas used herein will either verify or revise the results shown in Figs. 4 and 5 or 6 and 7. In such 5-yr or 10-yr records as those used in this paper, a 25-yr rain may fall on the area when its soil and cover are most conducive to runoff, and the resulting peak may have a recurrence interval of 50, 100, or even a greater number of years. Such may be the case of the storm of July 21, 1938, on watersheds Nos. 2 and 3, the points for which (5.8 and 5.6 in. per hr) lie considerably higher than their respective curves (Fig. 4). About 6.5 in. of rain fell on the area during the 30 days preceding this storm. Pasture watershed No. 1 recorded a peak of 1.87 in. per hr which is more consistent with the other values for this area.

ACKNOWLEDGMENTS

The work of O. E. Hays, H. B. Atkinson, and J. C. Trieloff, of the Soil Conservation Experiment Station at LaCrosse, in supplying watershed records for use in the preparation of this paper is gratefully acknowledged.

DISCUSSION

SAMUEL ROBERTS,[11] JUN. AM. SOC. C. E.—The extension of short-term records for design purposes is a problem of frequent occurrence in flood control and drainage practice. Where long-term records are available, a straightforward procedure may be used to present the data in frequency form. The flood peaks are plotted in accordance with a suitable formula, such as Eq. 1, and a curve is drawn to fit all the plotted points, as in Fig. 4. For short-term records this procedure must be modified by an examination of the data to determine whether or not the record is a truly representative sample of the long-term record and, if not, what correction should be applied. The frequency curve finally drawn need not be the average of all the plotted points. If there is a sufficient scatter of points in a short record, such as ten years, it may be expected that one or more of the high peaks should have greater recurrence intervals than ten years, and that the other peaks will define the lower portion of the frequency curve. The final curve drawn with this premise in mind, therefore, may deviate considerably from the plotted positions of the higher peaks.

For plotting purposes, the writer prefers log-log paper instead of the probability paper used by the author and he prefers the plotting of the data by frequency in years, the reciprocal of F in Eq. 1. Peak discharge frequency curves plotted by this method usually are parabolic in shape, making extrapolation comparatively simple. Deviation from this shape occurs only in a watershed displaying material differences in characteristics between moderate and severe floods, such as would be caused by storage or by regulation.

The data given in Tables 2 and 3 have been replotted on log-log paper in Fig. 8, in accordance with the formula:

$$f = \frac{t}{N - 0.5} \qquad (3)$$

in which f equals the frequency of recurrence in years; t equals the number of years of record; and N equals the order number of the peak (highest equals 1, etc.). The Yarnell frequency curve, as given in Fig. 4, has also been replotted in Fig. 8. It may be noted that the plotted rainfall points agree reasonably well with the Yarnell curve and no adjustment in frequency is warranted. Furthermore, the plotted peak runoff points for the short records show a good scatter in magnitude, indicating a representative sample of the long record in each case.

It is singular to note that the curves thus drawn seem to support the common practice of using the rational method and the Yarnell rainfall frequecy curve. Judicious choice of the runoff coefficient based on such information as that given by the author and the frequency curves in Fig. 8 would result in a more balanced design. The coefficient of runoff for area A-5 varies from 0.25 to 0.3. For area 1 it is about 0.4; and for area 3 and area 2 (between 1938 and 1942), it varies from 0.45 to 0.6. The coefficient for area 2 (between

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[11] Engr. (Hydrologic), U. S. Engr. Office, New York, N. Y.

1933 and 1937) is extraordinarily high and seems to indicate an inconsistency in the data, possibly caused by the changes in cultivation or plowing methods that may have reduced the concentration time; or perhaps the introduction of ground-water flow from other areas increased the flow measured at the gage. If the concentration time of area 2 had remained 5 min, it is improbable that the runoff rate would have exceeded the maximum 5-min rainfall rate for four out of five of the plotted values for the same period (see Fig. 8).

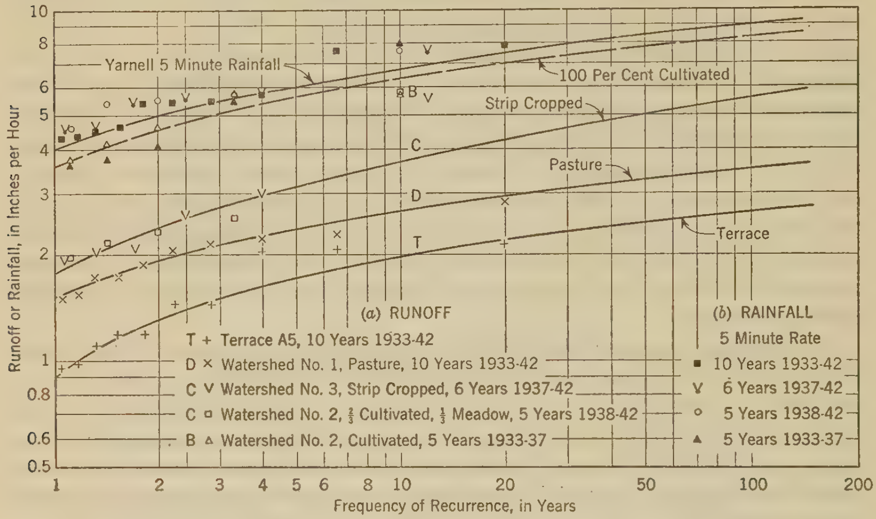

FIG. 8.—VALUES OF PEAK RUNOFF FOR VARIOUS RECURRENCE INTERVALS, EQ. 3

The areas studied by the author are so small that special cultivation methods may have an appreciable effect on the concentration time. From the descriptions given, the areas do not appear as dissimilar as the range in derived runoff coefficient indicates, and the assumption of a 5-min concentration time for each of the areas may not be valid. The data on hand are too inconsistent and variable to use for direct extrapolation to other watersheds. However, the paper can serve as a useful basis for further study, particularly if more information is published later for the same areas.

LLOYD L. HARROLD,[12] ASSOC. M. AM. SOC. C. E.—In view of the limited discussion of this paper, the writer could possibly infer that the procedures used therein have the sanction of the profession; but he will not. Although certain parts of the method received no comment—either confirmatory or refutatory—some phases of the procedure have been used for analyzing short-term watershed runoff for a number of locations. The writer has been reluctant, however, to adjust any of the runoff curves to normalcy. This hesitancy was in full view of the fact that almost without exception the observed rainfall rates for the periods of record have exceeded the normal expectancy. Long-term records at near-by U. S. Weather Bureau Stations, or Yarnell values, were used as normal in this comparison. The adjustment, if used, would lengthen the period of recurrence in each case; or, for the same recurrence interval, would reduce the flood peak. The unadjusted curves give flood-peak values that may be high. It appears better to have this factor of safety in the design flood peak rather than to apply an adjustment that may be questionable.

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[12] Project Supervisor, SCS, North Appalachian Experimental Watershed, Coshocton, Ohio.

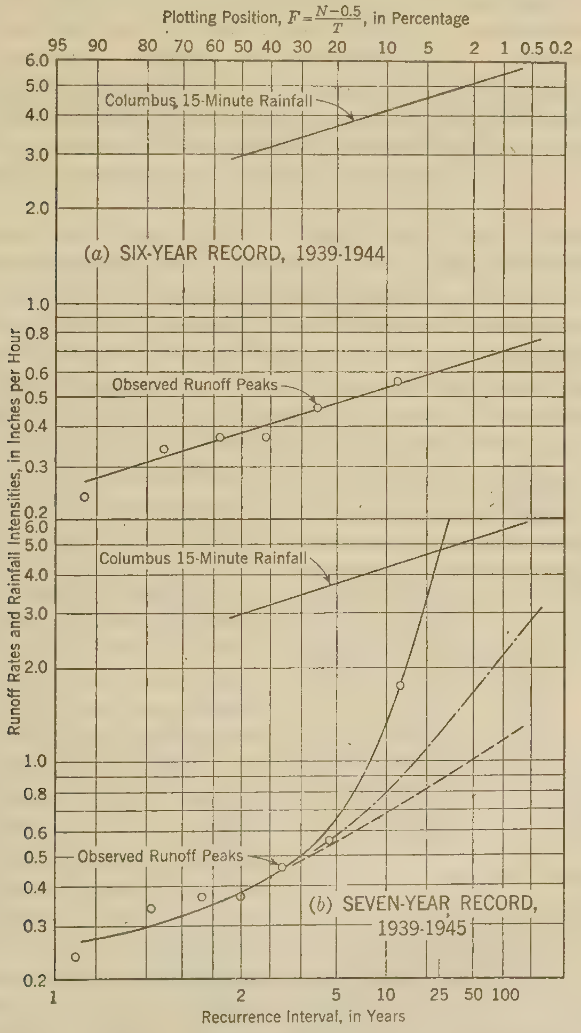

Fig. 9.—Record of Runoff on Watershed No. 10 (122 Acres) Coshocton, Ohio

Mr. Roberts' preference for log-log paper and the use of Eq. 3 (the reciprocal of Eq. 1) does not introduce any material change in the flood values for 25, 30, or 100 years. Data taken from the unadjusted curves in either Fig. 4 or Fig. 8 would be almost identical. Even the plotted rainfall points in both figures compare similarly with the Yarnell 5-min curves—Mr. Roberts' conclusion to the contrary. The plotting method does not eliminate the adjustment to normalcy, if such an adjustment is warranted in the first place.

Both the writer and Mr. Roberts feel the inadequancy of defining flood-frequency curves from short records. This has been brought out rather forcibly. A 6-yr record, 1939–1944, for a 122-acre watershed had been analyzed at the end of 1944 (Fig. 9(*a*)). The rainfall intensities and the soil moisture conditions recorded in this 6-yr period appeared to establish the fact that a normal period had been sampled. Press of other duties prevented the completion and publication of the report on this analysis. In view of the 1945 records this delay proved to be a blessing. It is necessary, frequently, to prepare preliminary reports which may be used until sufficient amount of additional data are obtained to warrant a revision on preparation of a final report. One cannot continue, indefinitely, to delay the analyses and publication of such reports.

In September, 1945, a flood peak of 1.72 in. per hr was recorded—three times as great as the highest flood in 6 years. The maximum 15-min rainfall for the storm was not exceedingly high—3.56 in. per hr. This is about a 5-yr storm according to the late David L. Yarnell, M. Am. Soc. C. E. Antecedent moisture, however, was very high, but not more than that occurring in 1941 (date of highest peak prior to 1945). Six inches of rain fell in the 3-week period prior to the date of the 1945 storm. There was no material change in the land use over the watershed throughout the entire 7-yr period. From an extension of the curve in Fig. 9, the recurrence interval of this 1945 peak would be somewhere above 1,000 years. This is unreasonable.

TABLE 4.—FLOOD PEAKS, WATERSHED NO. 10

Order	SIX-YEAR RECORD (1939–1944)		SEVEN-YEAR RECORD (1939–1945)	
	Runoff (in. per hr)	Plotting position	Runoff (in. per hr)	Plotting position
1	0.56	8.3	1.76	7.2
2	0.46	25	0.56	21
3	0.37	42	0.46	36
4	0.37	58	0.37	50
5	0.34	75	0.37	64
6	0.24	92	0.34	79
7	0.23	...	0.24	93
8	0.207	...	0.23	...
9	0.198	...	0.207	...
10	0.192	...	0.198	...
11	0.146	...	0.192	...

The peak values for the 7-yr period, 1939–1945, were replotted as shown in Fig. 9(*b*). The recorded peak values for the 6-yr and 7-yr periods are listed in Table 4. A line through the plotted points defines curve A, Fig. 9(*b*)). An extension of this curve crosses the rainfall curve. At a 50-yr recurrence the flood peak would be about twice as large as the rainfall rate. This is most unreasonable. Mr. Roberts' statement, that the frequency curve need not be the average of all the plotted points, certainly applies to this condition. All the points should not be used in defining this curve.

A curve (line C, Fig. 9(*b*)) through all but this highest value gives the 1945 peak as one occurring once in a little less than 1,000 years. This seems to be too great. Obviously, the probable recurrence of this peak value is somewhere between curves A and C—perhaps along curve B. Only a longer period of record will help to establish the true position of this flood-frequency curve.

Mr. Roberts' interest in this paper and his discussion are appreciated. The writer is subjecting the procedures to further study and may at some future date submit his findings to the profession.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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TRANSACTIONS

Paper No. 2279

ELIMINATION OF CROSS CONNECTIONS IN LOS ANGELES, CALIF.

By Harry Hayes,[1] Assoc. M. Am. Soc. C. E.

With Discussion by Messrs. Roy O. Van Meter, Robert P. Lowe, and Harry Hayes.

Synopsis

The City of Los Angeles, Calif., has undertaken what is probably one of the most carefully planned and executed programs for the control and elimination of cross connections of any city in the United States. This is the outcome of development work started by the City Department of Water and Power, at first acting alone but gradually acquiring the cooperation of all governmental units having jurisdiction, which act in unison through a joint committee. The purpose of this paper is to describe briefly some of the factors which enter into the development of this program, the formation of the joint committee, and results accomplished.

History

When potable water delivered to a consumer from a source approved for domestic consumption may be polluted or contaminated by admission of polluting material through a connection to the consumers' water piping system, such connection is known as a cross connection. (By the term "cross connection" as used in these regulations is meant any connection between any part of a water system used or intended to supply water for drinking purposes, and any other source or kind of water that is not or cannot be approved as reasonably safe and potable for human consumption whereby water from said unapproved source may be forced or drawn into the drinking water system.[2]) The existence of dangerous cross connections inimical to the public health has been recognized for many years. Publications are replete with reports of epidemics of water-borne diseases resulting in sickness and death which have been attributed directly to dangerous cross connections. Investigations indicate that the number of instances of sickness caused by contaminated drinking

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Note.—Published in March, 1945, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

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[2] "Cross Connection Regulations," adopted August 28, 1943, *Bulletin No. 64*, Dept. of Public Health, State of California.

water through backflow connections has been many times greater than the number actually reported in print. It is further apparent that, instead of diminishing, the number of instances where pollution has occurred is on the increase, due largely, it is believed, to the rapid increase and expansion of war industries. This is attested to by the reported incidents of salt-water pollution of drinking water lines in shipyards alone.

Many states have regulations governing cross connections, administered through public health authorities, some of which forbid the existence of cross connections even with secondary supplies that may be considered potable, whereas others allow such cross connections if the primary supply is protected by an approved double check-valve installation. As yet, no guaranteed procedure for backflow protection has been developed. Although some consider parallel piping systems with air-gap or broken connections as ideal, systems originally protected by such means have become cross connected by operators, which nullifies the supposed protection entirely. Others place reliance upon mechanical protective devices although such devices have not been fully developed to afford complete protection. It becomes a question of whether present mechanical deficiencies are more serious than the human element in making new cross connections. Current conditions in regard to cross-connection control by various state and local governmental bodies have been reported quite thoroughly by R. F. Goudey[3] and Robert E. Moore, Jr.[4]

The City of Los Angeles began conducting an increasingly vigorous campaign of cross-connection control and elimination in 1930, mostly through the Sanitary Engineering Division of the Department of Water and Power. Since that time most buildings in the downtown area, and many buildings in outlying areas, have been surveyed. Every new water-service sale is checked for use and all properties to which the services are sold, with the exception of dwellings with service connections less than 1.5 in. in diameter, are inspected for cross connections. In many cases properties or building plans are investigated before the service is sold, and, where it is found that backflow protection is required at the meter, the service is withheld until such protection is provided.

Until 1942, the control of cross connections on water piping systems connected to the municipal water supply mains was solely for the purpose of preventing backflow of polluted water into the street mains. That contamination of the water within a private water piping system might occur, with resulting danger to the health of occupants of the property, was not the primary concern of the Water Bureau so long as that contamination did not flow through the meter into the street main. Where contamination was known to have occurred, or was likely to occur, in a private water piping system, the matter was reported to the local health authorities for further action.

Right of entry on to private property for the purpose of investigating the water use, and for inspection for the existence of cross connections, was established by rules and regulations passed by the Board of Water and Power

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[3] "Practical Aspects of Cross-Connection, Inter-Connection and Back-Flow Protection," by R. F. Goudey, *Journal*, A.W.W.A., March, 1941, p. 391.

[4] "Legal Aspects of Cross-Connection, Inter-Connection and Back-Flow Protection," by Robert E. Moore, Jr., *ibid.*, p. 495.

Commissioners[5] which have all the force and effect of general laws.[6] Other rules and regulations provide for the prohibition of cross connections on the consumers' property or protection of the water supply and the right to discontinue water service for violations.[7]

On a routine investigation of a water service for cross connections the inspector began at the meter and followed the water supply piping from the point of entry to the property, through all branches to the last foot of pipe, inspecting every connection, outlet, and water-using device and fixture found on the service. Every cross connection was listed and located, using sketches and drawings where required. When the survey was completed, the cross connections were listed (except cross connections to plumbing fixtures which were referred to the Department of Building and Safety) and presented to the owner or person in charge of the property with a legal order requiring their elimination from the potable water piping system.

Two alternates were allowed: The first was the complete severance of each individual cross connection on the system; and the second was the installation of a backflow protective device at the meter. By the use of the first method, the public water supply system was protected so long as no new cross connections were made. By the use of the second method, the mains were reasonably well protected so long as the checks were maintained in good condition. In either case, direct connections to the sewer were ordered severed.

ORGANIZATION AND FUNCTIONS OF THE COMMITTEE FOR CROSS-CONNECTION CONTROL

In 1942 the Cross-Connection Control Advisory Committee was founded to take over cross-connection control and elimination for the entire city. It was formed by a joint resolution passed by the Boards of Commissioners of the Building and Safety Department, the Health Department, the Water and Power Department, and the Board of Mechanical Engineers of the City of Los Angeles. Its membership is comprised of one representative from each of the departments and one attorney from the city attorney's office, plus certain advisory counselors who may be appointed from time to time. The sanitary engineer of the Department of Water and Power is the member from that department and is the permanent chairman of the committee.

The committee is a quasi-legal organization and is invested with all the police powers of the signatories to the joint resolution, in so far as they pertain to cross-connection control. The policy of the committee is to accomplish its objectives through education and cooperation rather than through enforcement, using its authority and the laws back of it only as a last resort. The committee has sponsored evening classes in Los Angeles at the University of Southern California and the Frank Wiggins Trade School for the purpose of educating the public on what constitute cross connections, the dangers of allowing their existence, and the methods for correcting or eliminating them, thus protecting the water supply system. These classes have been well attended by plant engineers, master plumbers and journeyman plumbers, plumbing and health inspectors, representatives of manufacturers of plumbing equipment, and many others who are concerned with the maintenance of the purity of the water supply system.

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[5] *Rule and Regulation No. 58*, Board of Water and Power Commrs., City of Los Angeles, Calif.

[6] *City Charter*, City of Los Angeles, Calif., Chapter I, Sections 82 and 220.

[7] *Rule and Regulation No. 59 to No. 62*, inclusive, Board of Water and Power Commrs., City of Los Angeles, Calif.

Field inspection forces were obtained by assigning to the committee inspectors from the various departments represented. A chief inspector and two assistants, with previous experience in this type of work, were selected from the engineering staff of the Sanitary Engineering Division of the Department of Water and Power to take charge of the inspection work. Most of the inspectors who have worked under the jurisdiction of the committee were obtained from the Plumbing Inspection Division of the Department of Building and Safety because the type of work involved is akin to plumbing inspection.

The procedure in making a survey under the authority of the committee is identical to that used by the Department of Water and Power before the committee was conceived. Great care is taken to see that each pipe line is traced to its end and every cross connection located. When the survey of a property is completed, the inspector writes a legal order requiring the elimination of all cross connections found having listed them in an attached appendix. In making up the list each cross connection is numbered consecutively, its location given, and the method of elimination stated. Thereafter it is referred to by number only. Wherever possible, cross connections of a like nature that can be eliminated by similar methods, such as the raising of low inlets into tanks, or installation of vacuum breakers on flush valves, are grouped together so that the order for elimination covers the entire group.

For many items on the list there is but one method of elimination. If such is the case, it is so ordered, but for others where the elimination may be accomplished by any of a number of methods, involving, perhaps, complete changes in piping systems, the owner is given the option of using the method that serves his purpose best.

In order to further unify the methods of elimination for all properties so that none will be discriminated against or given special advantages, a code of standard wording was devised for each type of cross connection, and in writing up his orders each inspector must follow this standard explicitly. The only time when deviation from the standard is allowed is when a new type of cross connection or an unusual condition is found that is not covered by the standard wording, and in that case it is referred to the engineers for codifying.

The legal order is dated as of the day it is served; and the person to whom it is served, or his agent or representative, is required to sign the order to acknowledge receipt of it. The order is then dated as to the time allowed for completion of the work ordered or provision made for its completion. The time limit given is usually two weeks. If the required work cannot be completed in this time, the owner is allowed to file biweekly progress reports and the limit placed on the time of completion is extended as long as evidence of good faith is shown. However, no order is allowed to be pigeonholed and forgotten.

Each inspector working on cross-connection control work, regardless of the department from which he is obtained, is deputized to act for each of the member departments in cross-connection control work. His duty is not only to insure the purity of the water supply in the city mains, but he is also interested in protecting the public health by the elimination of all cross connections and illegal plumbing installations on private premises that might adversely affect the purity of the water in the consumers' water piping system and endanger the health of any individual drinking from that system.

The three engineers from the Department of Water and Power and two plumbing inspectors from the Building and Safety Department who had had some experience in cross-connection inspection work formed the nucleus for the inspection. From time to time inspectors from other member departments were added. Each new man, when assigned to the field inspection force, was paired with an experienced man for a period of training until he was able to do the field work alone and perhaps train new men. The purpose of the committee was to allow every inspector working for a member department sufficient time on cross-connection inspections to become thoroughly familiar with the work so that he could recognize and order the elimination of any cross connections he might find when conducting his regular department inspection work. The time required to train regular plumbing inspectors in this work depends on the ability of the men assigned. Exceptional men will require from six to nine months; men who are slower to learn will require a year or more; and some will never be able to make a satisfactory survey. Overconfidence on the part of the trainee is the greatest stumbling block to overcome. It takes at least thirty days on the job before a conscientious inspector discovers how little he knows.

Inspections of water piping systems in buildings in the business district, food processing plants, hotels, hospitals and sanitariums, manufacturing plants, etc., that had previously been conducted by the Water Department engineers were undertaken by the committee's inspection force. With the outbreak of World War II and the necessity for a continuous flow of war materials, the committee confined its inspections more and more to plants and industries connected with the war effort. This safeguarded large concentrations of war workers and it was also found that preferential ratings could be used for obtaining the necessary critical materials. The inspection of residential, business, and other properties not connected with the war effort was abandoned for the duration, with the exception of special cases involving sickness.

Evidences of Pollution

Early in 1942, a series of incidents occurred which focused the attention of the committee on the Los Angeles Harbor area. The first of these was a report from the Sanitary Engineering Division of the Water Department that analyses of water samples collected at certain locations from the water distribution system in the harbor area showed evidences that pollution was getting into the mains from some source in the area. A sanitary survey indicated the possible sources of pollution as a shipbuilding yard and an army fort. The bacterial pollution appeared to be coincident with periods of heavy water use in the shipyard which caused low water-pressure conditions in the street mains and distribution systems serving both the yard and the fort. A quick check up showed both places to be replete with cross connections. An order was issued immediately that backflow protection be installed at the meters on all services entering the suspected properties. Thereafter there appeared to be less evidence of pollution in water samples collected from the city water mains in that area, but the water in the mains in the fort continued to show evidence of contamination and was quarantined for drinking purposes.

Systematic and complete cross-connection surveys of each of the properties revealed the existence of more than 350 cross connections in the shipyard and nearly a thousand at the fort.

Second in the series of incidents was the actual pollution of the fresh drinking water lines within the aforementioned shipyard. A ship's officer on board a ship tied to a wharf connected the fresh-water system in the yard to the salt-water system on board the ship, started the ship's pumps, and pumped sewage-polluted salt water throughout the yard's drinking water system. This was the first of five occurrences within six months where polluted water was forced back into drinking water systems in the harbor area, and although there are no official records of sickness resulting from these occurrences, it is known unofficially that there were many cases of illness due to these causes, and the loss of man-hours in the plants involved was high.

Such occurrences have not been confined to plants in Los Angeles or in California. Reports have been published of similar cases occurring in shipyards and industrial plants in various parts of the United States, indicating that hazardous conditions exist in all hastily built war production plants because, in most cases, it is impossible to get designers and engineers who have a practical as well as a theoretical knowledge of cross-connection control.

One of the most hazardous conditions found by the committee was the existence of many cross connections between the fresh-water domestic and fire-fighting systems and secondary piping systems carrying sewage-polluted salt water. Most of these were newly made and were intended to augment the supply of fresh water for fire-fighting purposes and many involved new installations of auxiliary salt-water pumps.

The committee notified the State Board of Public Health of the conditions found in the harbor area, with the request that action be taken to assist in removing the dangers. As a result, that Board called a meeting on June 22, 1942, of all city and county health officials of Southern California and other interested parties, for the purpose of considering this problem. As an outcome of this meeting the State Board of Health, on June 27, 1942, passed a resolution[8] condemning the use of cross connections and requiring their elimination or control. This same resolution was adopted by reference by the Los Angeles City Board of Health and became a law in the city on October 8, 1942.

THE SEVEN-POINT PROGRAM

In August a meeting was called by the committee and was attended by representatives of various federal, state, and local governmental agencies to discuss plans and procedures for cleaning up the cross-connection situation in war industry plants in Los Angeles, in general, and in the harbor area in particular. It was agreed that the committee should undertake this job and that ten inspectors from the Building and Safety Department would be assigned permanently to the committee for this work.

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[8] *Resolution Relative to the Protection of Water Supplies Against Pollution Due to Cross-Connections with Auxiliary or Emergency or Firefighting Supplies*, State Board of Health, State of California, adopted June 27, 1942.

On September 15, 1942, a meeting of representatives from numerous shipyards and industrial plants, the U. S. Army, Navy, Coast Guard, Maritime Commission, the U. S. Public Health Service, State Department of Health, City Council, and other governmental departments concerned was called by the committee at the U. S. Naval Operating Base at Terminal Island (California) for the purpose of acquainting the representatives with the committee's proposed plan of survey and the policies to be followed by the committee in requiring cross-connection control and elimination and to discuss backflow protective installations.

The following seven-point program was presented to and adopted by the group:

1. Backflow protection at meter of all services to premises fronting on the harbor and to adjacent industrial and commercial plants.
2. Backflow protection at the wharf for all ship supplies.
3. Segregation of domestic and fire lines as far as possible so that no unprotected cross connections will endanger the domestic supply from pierhead, Siamese, or salt-water pump intakes.
4. Elimination of all dangerous plumbing cross connections.
5. Provision for the isolation of all pipe lines carrying processing water.
6. Adoption of color and sign scheme for designation of safe and unsafe water lines.
7. Organization of water foremen on each premise to supervise a unified plan.

These seven points represent the uniform policy established at the meeting of September 15 and followed by the committee.

On September 10, 1942, an office was opened in Wilmington, Calif., with ten inspectors and three engineers assigned to the survey. Work was started immediately on the waterfront properties. Preliminary contacts with plant managers were made by the engineers to arrange passes and entry for the inspectors, to obtain maps and drawings of the water piping system on the property if available, and to enlist the cooperation of the plant engineer in making the survey. In general, the cooperation of the management of the plants has been very satisfactory, but, as with individuals in a group, there are contrary ones who make it difficult for the inspectors. Although the inspectors carry sufficient legal authority to enter any of the plants for inspection purposes, it is the policy of the committee never to make use of such authority or to make any show of force in dealing with the plants unless compelled by a refusal of cooperation. That holds true also in the issuance of orders for the elimination of cross connections or any other required work for the protection of the drinking water system.

The survey proceeded with diligence, and the report to the committee (dated January 1, 1945) shows that the following had been accomplished as of that date:

Premises surveyed and legal orders issued	267
Cross connections found	12,500
Work completed on orders issued	99%
Survey of harbor area completed (industrial area only)	100%
Approximate cost of work ordered	$600,000

As the report shows, the survey of the harbor area is completed. Part of the inspector's duty is to be available on call to consult with plant managers in perfecting methods for the required elimination. As the work in the harbor is completed the inspectors are moved north into defense plants in the "shoestring" strip connecting the harbor area and the city and into the city area.

During 1943, the District Security Officers for the Army, Navy, Air Force, and other branches of the armed services were charged with the responsibility of securing the safety of the drinking water systems in all plants on war material production under their jurisdiction. As the Cross-Connection Control Advisory Committee was at the time proceeding with its program of cross-connection elimination in war production plants, it was requested by the District Security Officers to expand its activities to include water safety security.

The committee was supplied with master responsibility lists of war production plants, and surveys were made at these plants in accordance with their priority on the lists.

When the survey of a plant was completed, and an order for corrections was issued, a copy of the order was forwarded to the District Security Officer, who thereupon assumed responsibility for its enforcement. The committee, however, maintained a continuing inspection at the plant to see that the terms of the order were fulfilled.

(*1*) *Protection at the Meter*.—The requirement of backflow protection at the meter (point 1) is not confined to harbor-front properties alone, but to any plant or property having access to auxiliary water supplies and pressure producing equipment for utilizing such supplies. Fig. 1 comprises two reproductions of prints issued to property owners with orders for cross-connection elimination involving such necessary protection, showing various methods which have been approved by the committee, for installation at the meter, to protect against the pollution of the water supply in the street main.

The fundamental principle of backflow protection is the air-gap or broken connection. There is as yet no completely satisfactory substitute for a broken connection or a complete severance of a cross connection for permanent protection to a drinking water system. However, modern backflow protective equipment has been developed that will give a reasonable degree of protection if properly installed and maintained.

The first method of protection shown in Drawing No. A-1, Fig. 1, makes use of the air-gap principle by requiring the water supply to be discharged into a receiving tank through an overhead inlet which is sufficiently high above the overflow rim of the tank to prevent any siphoning back of water from the tank should a vacuum occur in the supply main. Furthermore, it is obvious that any pressure producing device in the consumer's water piping system could not force water back across this gap (as would be possible through a direct connection), either against the pressure in the main or into a vacuum that might occur in the main.

In general, the least air-gap distance required is twice the diameter of the inlet pipe, plus additional distances in special cases. As shown in Drawing No. A-1, the tank may be placed in an elevated position so as to provide gravity pressure in the consumer's water piping system, or the pressure may be produced by a pressure pump taking the suction out of the receiving tank. A device not shown in the drawings, which is the nearest equivalent to an air gap in protection, and which allows a solid connection to the water supply so that the main pressure can be utilized, is the siphon loop. This is a loop in the supply pipe which rises to a height of not less than 35 ft above the highest point from which back-siphonage of polluting material might occur. It can be used only on premises where there are no steam boilers, auxiliary water supply, or any pressure producing device connected into the water piping system except where the loop is raised to a height sufficient to compensate for the maximum pressure that may occur.

It is seldom practicable to use such a loop, as it requires a support that may extend a considerable distance above the roof of the building housing the piping system, although it has been recommended at times where conditions seem to warrant its use.

Where it is impossible or impracticable to require the use of the foregoing methods of protection on a water service, the committee allows the use of backflow protective devices of approved types, provided they are installed in an approved manner and maintained in good operative condition at all times. Such approved devices are shown in Drawing No. A-1, alternate method (Fig. 1) for services less than 6 in. in diameter, and Drawing No. A-2, Fig. 1, a typical double check-valve installation for 6-in. services and larger. Any device for such use must be submitted to the laboratory of the Building and Safety Department for tests and approval before its use is allowed in Los Angeles.

Although it is recognized that no mechanical device is the equivalent of a broken connection for backflow protection, great progress has been made in the development and the design of modern backflow protective devices. The trend is toward the perfection of the operating mechanism together with the use of devices which operate to destroy the negative and positive forces that cause backflow.

In general, the forces that tend to cause backflow through any cross connection are: (1) Reduced pressure or a vacuum in the water supply mains caused by main breaks, shutdowns for repairs, excessive water use, pumping connections for fire fighting, etc.; (2) gravity head of water in the consumer's piping system; (3) pressure producing equipment connected into the consumer's water piping system, or connections to a secondary water piping system having a higher (or occasionally higher) pressure than the primary system.

The fundamental principle of all backflow protective devices for installation at the meter has been the use of double check valves. Until recently they did

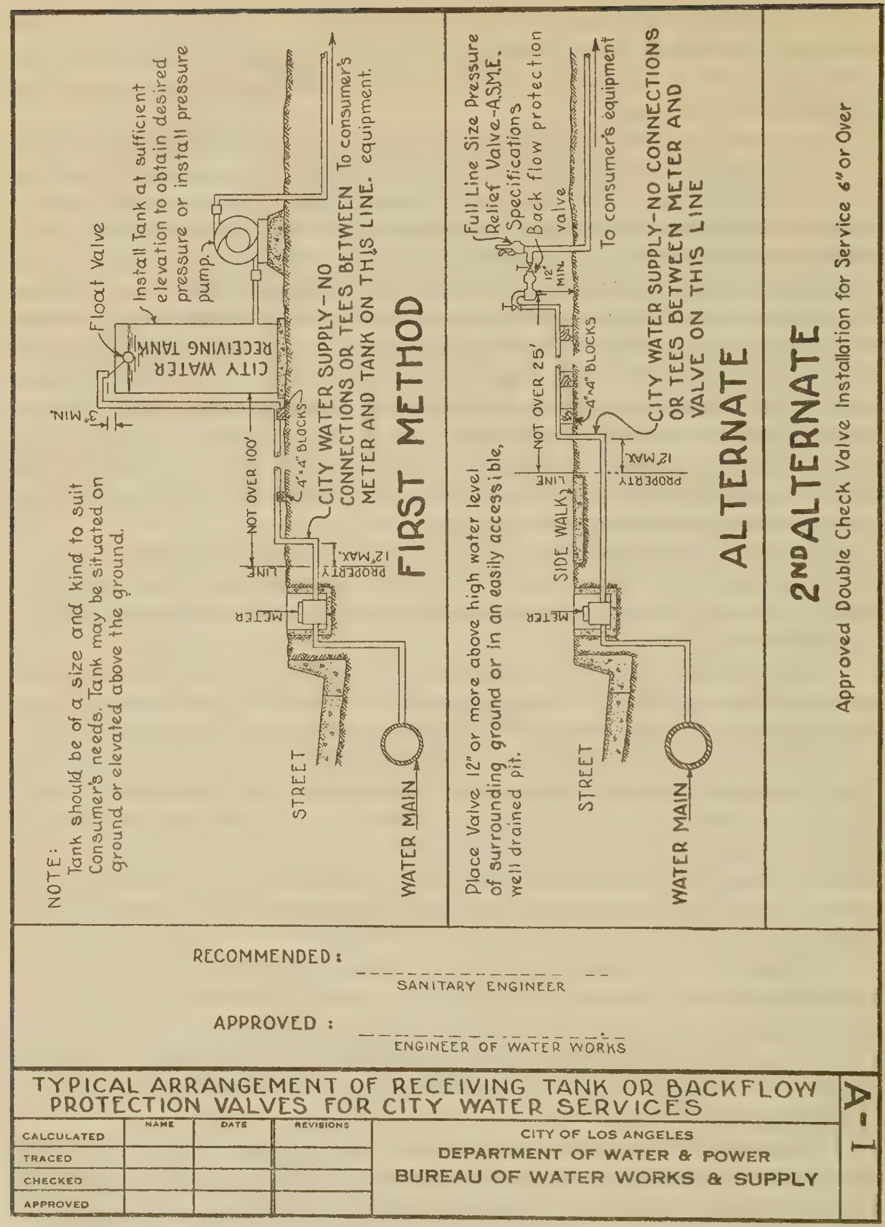
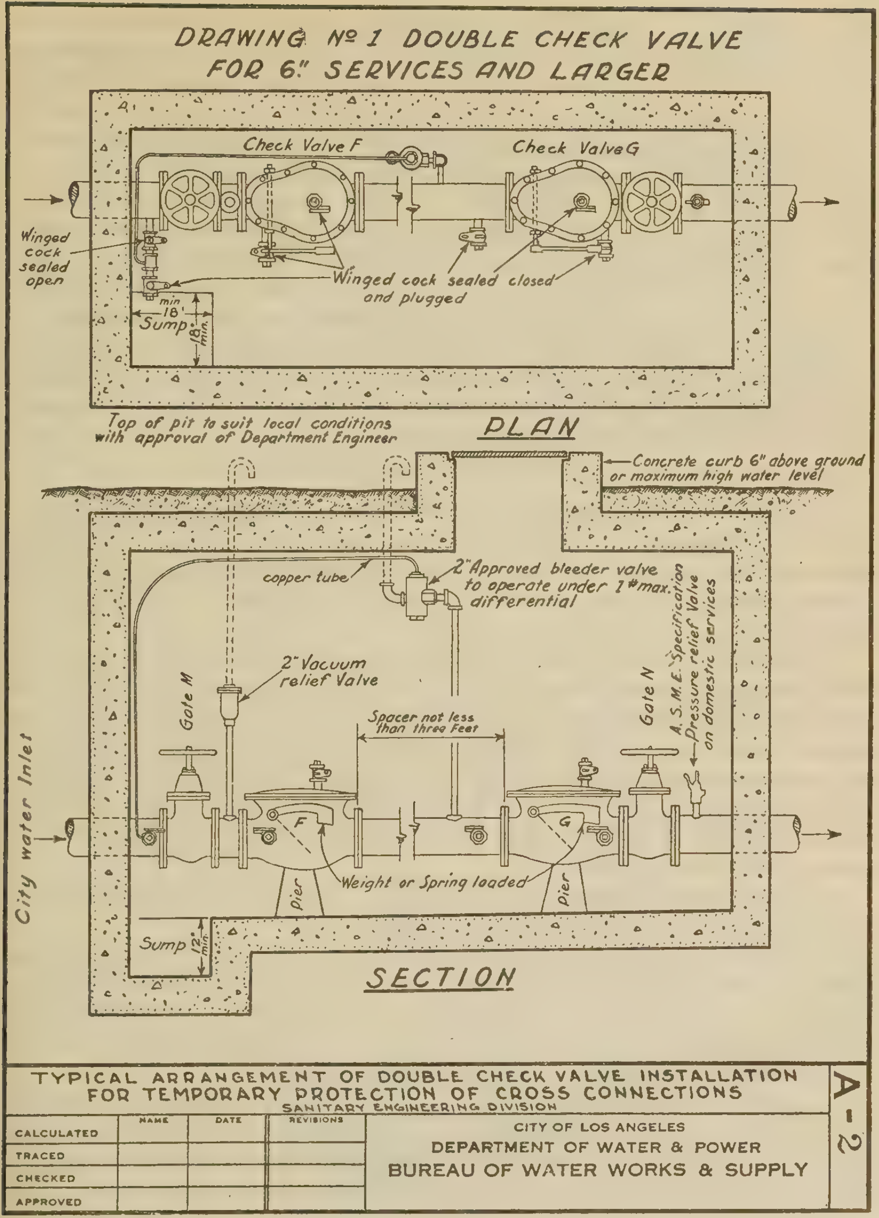

NOTE:
Tank should be of a size and kind to suit Consumer's needs. Tank may be situated on ground or elevated above the ground.
Float Valve
Install Tank at sufficient elevation to obtain desired pressure or install pressure pump.
3" MIN.
CITY WATER RECEIVING TANK
NOT OVER 100'
4"x4" BLOCKS
LINE
PROPERTY
12" MAX.
METER
STREET
WATER MAIN
CITY WATER SUPPLY - NO CONNECTIONS OR TEES BETWEEN METER AND TANK ON THIS LINE.
To consumer's equipment.
FIRST METHOD
Place Valve 12" or more above high water level of surrounding ground or in an easily accessible, well drained pit.
Full Line Size Pressure Relief Valve-A.S.M.E. Specifications
Back flow protection valve
12" MIN.
NOT OVER 25'
4"x4" BLOCKS
SIDE WALK
LINE
PROPERTY
12" MAX.
METER
STREET
WATER MAIN
To consumer's equipment
CITY WATER SUPPLY - NO CONNECTIONS OR TEES BETWEEN METER AND VALVE ON THIS LINE
ALTERNATE
2ND ALTERNATE
Approved Double Check Valve Installation for Service 6" or Over
RECOMMENDED:
SANITARY ENGINEER
APPROVED:
ENGINEER OF WATER WORKS
TYPICAL ARRANGEMENT OF RECEIVING TANK OR BACKFLOW PROTECTION VALVES FOR CITY WATER SERVICES
A-1
NAME
DATE
REVISIONS
CALCULATED
TRACED
CHECKED
APPROVED
CITY OF LOS ANGELES
DEPARTMENT OF WATER & POWER
BUREAU OF WATER WORKS & SUPPLY

Fig. 1.—Examples of Prints Issued to Property Owners

not include vacuum breakers and pressure-differential relief valves which have been included to destroy the forces creating backflow. Use of the double check valve was first recommended by the New England Water Works Association, and has been adopted without question by many state and local health


DRAWING Nº 1 DOUBLE CHECK VALVE
FOR 6" SERVICES AND LARGER
Check Valve F
Check Valve G
Winged cock sealed open
Winged cock sealed closed
and plugged
min 18"
Sump
18" min.
Top of pit to suit local conditions with approval of Department Engineer
PLAN
Concrete curb 6" above ground or maximum high water level
copper tube
2" Approved bleeder valve to operate under 1# max. differential
2" Vacuum relief Valve
Gate M
Gate N
A.S.M.E. Specification Pressure relief Valve on domestic services
Spacer not less than three Feet
City water Inlet
F
G
Weight or Spring loaded
Pier
Pier
Sump
12" min.
SECTION
TYPICAL ARRANGEMENT OF DOUBLE CHECK VALVE INSTALLATION
FOR TEMPORARY PROTECTION OF CROSS CONNECTIONS
SANITARY ENGINEERING DIVISION
A-2
NAME
DATE
REVISIONS
CALCULATED
TRACED
CHECKED
APPROVED
CITY OF LOS ANGELES
DEPARTMENT OF WATER & POWER
BUREAU OF WATER WORKS & SUPPLY

WITH ORDERS FOR THE ELIMINATION OF CROSS CONNECTIONS

authorities. The original recommendations were for check valves made entirely of bronze, but, as the cost is high for the larger sizes, the Department of Water and Power, in some of the early installations, allowed the use of a certain type of checks made of cast iron with galvanized body which has proved quite satisfactory. These checks were originally developed as detector checks on fire-sprinkler services and for compound meters where it was found desirable to check the main flow-through channel in order to force small flows through the by-pass meter. To accomplish this purpose the clapper of the check is weighted internally to cause positive closure of the valve under static or low-flow conditions and to require a substantial decrease in pressure on the property side to make it open. In addition, the valve contains other desirable features, such as bronze seat rings, hinge and hinge pins, rubber or composition bushed hinge-pin bearings, and rubber-faced clapper disk. This type of check valve is the first choice of the committee for use in double check-valve installations.

With the increased scarcity of strategic materials due to war conditions, it has become increasingly difficult to obtain valves meeting previous specifications and the committee has allowed the use of other types of valves, provided they are rebuilt to meet certain requirements. Practically any check valve that has a bronze seat ring and hinge-pin bearings in the body of the valve can be rebuilt to meet the requirements, which are as follows:

(*a*) The face of the clapper disk must be machined flat and fitted with a rubber face composed of fabric-reinforced medium soft rubber. Any valve already on the market which has a rubber face in the seat ring may be used without change; but the clapper disk must be bronze, or must have a bronze face.

(*b*) The bearing for the bolt that fastens the disk to the hinge arm must be rubber bushed and must be provided with rubber washers for each side of the hinge arm. These washers and bushing must provide a tight fit between the disk and hinge arm and yet be resilient enough to allow sufficient movement for proper seating of the disk when the valve closes. If the valve to be used is double-hinged with rubber bushings for the hinge pins, or, if it has an otherwise solid but resilient connection between the disk and hinge arm, the aforementioned change is not required.

(*c*) The hinge-pin bearing in the body of the valve must be drilled out and fitted with rubber or reinforced plastic bushings.

(*d*) The bearing on one side must be fitted with a packing gland and an extended hinge pin of bronze, stainless steel, or other rust-resisting material. The hinge pin must be keyed or pinned to the hinge arm. An outside lever must be fastened to the hinge pin with a key, or a substantial pin, in such a position that it is horizontal when the valve is closed.

(*e*) A weight must be attached to the arm lever and adjusted in a position that will require a differential pressure of approximately 1 lb per sq in. to open the valve. Check valves are available which are already fitted with a packing gland and outside weighted lever arms as required in items (*d*) and (*e*).

The committee made a study of double check-valve installations and recognized the fact that double check valves alone cannot be relied upon to provide adequate protection against backflow. Therefore, the installation of other devices was specified in conjunction with the check valves for the purpose of dissipating the forces that cause backflow. Referring to Drawing No. A-2, Fig. 1, it will be seen that there is a vacuum-relief valve inserted in the water supply line ahead of the first check valve. Whenever a vacuum occurs in the city's water supply line, this valve will open to supply air to satisfy the vacuum, thereby reducing the negative pressure on the inlet side of the check valve.

Again referring to Drawing No. A-2, it will be seen that a differential pressure-relief bleeder valve is connected into the spacer between the two check valves with a control line connected into the main supply line ahead of the first shutoff gate. This valve (which is power operated by water pressure) opens to atmosphere when the water pressure in the main drops below the pressure in the spacer between the checks, thus allowing the excess pressure to bleed off. This valve is set to operate on approximately a 1-lb differential but its action must be retarded so as not to be affected by water hammer.

The purpose of these two devices is to maintain an equalized pressure on both sides of the first check valve so that there will be no force to cause backflow through that valve if it leaks or is defective. Backflow protective devices for smaller services must incorporate these principles and in addition must have three checks, or their equivalents.

The latest development, in equipment for protection against backflow, is a device incorporating a zone of reduced pressure in the line of flow through the device. The chamber between the main barriers to backflow is maintained at a pressure lower than the inlet pressure for all conditions of flow, by means of weighted, spring-loaded, or power-operated check valves, and the differential pressure thus created is utilized to control the operation of a port opening to atmosphere from this central chamber.

The operation of this port has a dual purpose. It will operate to maintain a reduced pressure in the central chamber if leakage occurs into the chamber through failure of first barriers under normal conditions, or through the second barrier under backflow conditions. A small amount of bleeding from the central chamber occurring at intervals during extremes of fluctuation of pressures in the supply main is normal to its operation, and serves to maintain its automatic features in good operating condition. Excessive bleeding, however, indicates failure of the main barriers and the need for servicing.

(2) *Protection at the Wharf.*—All pierhead outlets, on wharves, for supplying fresh water to ships must be protected against backflow (point 2). In every case this means that a protective device must be installed on each individual outlet as it is possible for a ship, tied to a wharf with fresh-water lines from the pierhead outlet aboard, to pump sewage-polluted salt water not only into the supply mains but into the fresh-water tanks of all other ships taking on water at that wharf or into their own. The city is legally responsible for the delivery of fresh water to all ships taking their supplies from its mains and cannot tolerate the pollution of the water supply of one ship by another through water lines being fed from the city mains. These indicants are of common occurrence and have been the cause of much sickness, inconvenience, and loss of man-hours in shipyards all over the United States.

(3) *Pollution Through Fire Lines.*—Domestic water lines and fire-protection lines must be separated as far as possible where there are auxiliary pumps or intakes, or cross connections to a polluted secondary water supply (point 3).

In a large percentage of the plants in Los Angeles the domestic water lines and the fire-sprinkler lines are entirely separate systems using the same source of supply and are required to be maintained separately by the Rules and Regulations of the Department of Water and Power. Backflow protection is required on these fire-fighting lines only where there is equipment or conditions that may cause backflow of polluted water into the city water mains.

In many plants a single system is used for domestic and fire-fighting purposes and is supplied from the city water mains. Such arrangements have been called combined systems. Backflow protection on these systems is required only where local conditions show the necessity for it. In many large plants using a combined water system, and particularly in the Harbor area, auxiliary fire-fighting water pumps with polluted sources of supply (such as salt water) have been connected into the combined systems. These pumps in most cases are inoperative at all times except in case of fire and a dire need for water. In these plants the committee allows the cross connections to be maintained provided a permanent automatic chlorinator, delivering chlorine gas, is connected into the pump discharge or into the lines delivering the auxiliary water supply. The chlorinator must be maintained in a continuously operative condition so that, whenever the auxiliary pumps begin to operate and polluted water starts to flow into the combined water lines, it will open up full force and thoroughly chlorinate all water going into the system. At the end of the emergency operating period the combined system must be thoroughly flushed out with heavily chlorinated fresh water. A fire alarm must be in operation when the polluted water pump is operating; and a water foreman must be in constant attendance to report each instance.

(*4*) *Plumbing Cross Connections.*—The requirements that all plumbing cross connections must be eliminated (point 4) are covered by plumbing ordinances and include such specifications as low inlets to tanks and basins, toilet flush valves without vacuum breakers, and other items which can, and have, caused extremely hazardous conditions where the water piping system has been overloaded. The need for proper design of water piping systems and the elimination of plumbing hazards have been largely ignored by war plant contractors, under the erroneous idea that they were constructing a federal government operated or contractual plant and therefore were above all local ordinances and regulations. These items have been cleared up in the survey.

(*5*) *Pollution Through Processing Water.*—In many plants the drinking water lines are used for supplying water for industrial or processing purposes and are usually connected directly into the processing equipment so as to utilize the main pressure (point 5). In ordinary times when material is available the committee would require a separate system for carrying all industrial or processing water. With the present scarcity of material, however, it is the policy of the committee to allow the use of the domestic water for supplying industrial processes, provided the lines running to the processing equipment are "industrialized" or isolated from drinking water systems by the installation of backflow protective devices at the point of take off, so that, under all conditions of use, water cannot be forced or siphoned back into the drinking water lines.

Such corrections may require the running of a small-size line to supply drinking fountains, lavatories, etc., but, in general, such changes will call for the use of the minimum amount of material that will give the maximum amount of protection.

(6) Color Warnings.—Wherever there are two water systems in a plant, one carrying a safe, potable water used for drinking and other domestic purposes, and the other carrying a polluted, unsafe water (or water which is not protected and might at any time become polluted and unsafe), the committee requires that both systems be painted a distinctive color (point 6) so that plumbers or pipe fitters will not by accident make a cross connection between the two lines. In addition the committee requires that all outlets on the secondary system be posted with a reasonably large placard signifying that the water from that outlet is dangerous and should not be used for drinking purposes. The colors recommended by the committee for this use are solid blue, or blue stripes, for the drinking water lines, and red for the polluted lines. On the waterfront, maritime or naval standards may be used and are also recommended.

Proctorship Supervision.—An organization of water foremen for each premise, to supervise a unified plan of water protection, was started early in the program (point 7). The plan was to select, or have selected, from each plant or premise surveyed, one man or several men who would be thoroughly familiar with, or who would make themselves familiar with, the entire water piping system on the premises where they were employed. The plant engineer, or one of his pipe foremen, would be the logical man for the post. These men were to be made responsible for the purity of the water on the premises and were to see that no new cross connections were made on the premises, that all new installations of water piping systems were in accordance with existing laws and regulations, and that in those plants governed by point 3, where emergency fire-fighting connections are allowed to exist, all the precautions for maintaining the purity of the drinking water were observed. This organization has proved quite satisfactory, and has since been made a requirement in all plants by the State Board of Health.[2]

SUMMARY

Approximately 50,000 cross connections had been eliminated in Los Angeles by December 1, 1944, representing sizes from 1 in. to 12 in. in diameter at a total cost well over $3,000,000 to the consumer. Since the war this program has embraced practically all the defense plants and military establishments in the Harbor area and many such installations in other parts of the city. It is gratifying to note that all local Army, Navy, Coast Guard, and defense plant organizations are supporting this program. The War Production Board has approved the use of critical materials as required.

The committee is cognizant of the fact that the situation in Los Angeles is not unique but that conditions are duplicated in every war industry center. By adhering to its cross-connection control and elimination program, the committee hopes to make the city the safest place for industrial and other workers in the United States.

DISCUSSION

Roy O. Van Meter,[9] Esq.—The cross-connection control and elimination program in Los Angeles has been described by Mr. Hayes in a comprehensive manner. The subject is so broad that many interesting problems could be included. The City of Los Angeles did not embark on a survey to enumerate the cross connections but rather to eliminate all cross connections that were found. The surveys were started on a piecemeal basis. A certain critical area was laid out for surveys, a single property in the area was inspected, orders to eliminate the cross connections were issued, and the inspectors diligently prosecuted the case until every cross connection was eliminated or corrected. This procedure sounds easy, but in practice it resulted in many painful experiences.

Mr. Hayes has described the formation of the Los Angeles Cross-Connection Control Advisory Committee and also the problems confronting the committee prior to and after the outbreak of World War II. The permanent chairman of the committee had at his command just three trained engineers on cross-connection control work at the time the Los Angeles Harbor survey was started. The Department of Building and Safety provided six plumbing inspectors untrained in cross-connection control work and later increased the number to eleven, after the first group had become familiar with the work.

To complete the survey as soon as possible and to utilize the trained men to the fullest extent, the work, legal orders, and other features of the survey were standardized as much as possible. In general, the survey of a given area is conducted by the inspector in nine steps as follows: (1) A surface map is prepared; (2) file records are made on service cards; (3) the owner of the area is informed of the problem; (4) an underground location survey is made; (5) a legal order is prepared; (6) the legal order is presented to the owner; (7) any necessary steps are taken to enforce the legal order; (8) corrections having been made, the area is reinspected; and (9) final inspection having been approved, the case is closed.

(*1*) *The Map.*—A tracing of the area to be surveyed is made from a district sheet provided by the Department of Water and Power. The scale is 100 ft to the inch, and shows the location and size of the street mains; the locations, numbers, sizes, and serial numbers of the water services; the property lines; the street locations; and street names and numbers.

(*2*) *Service Cards.*—Information from service cards is then obtained. Each card (on which are listed meter number and size, owner's name and address, collection name and address, when the meter was set and turned on or off, and other pertinent data) is checked against information obtained in the field.

(*3*) *Informative Visit.*—Contact is made with the owner or manager of the premises and a full and complete detailed explanation is given of the historical background; the prevalence of cross connections, water-borne diseases, water-borne disease epidemics; the formation of the Cross-Connection Control

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[9] Chf. San. Inspector, Bureau of Water Works and Supply, Los Angeles, Calif.

Advisory Committee and its powers and duties; federal, state, and local laws; rules and regulations; enforcement policies; the seven-point program of cross-connection control and elimination; types of cross connections normally found; methods of control and elimination; duties and liabilities of the owners or managers; the type of educational and cooperative campaign the committee is conducting; and the need for making an inspection of premises. After the introductory talk, permission to make the survey is requested; the manager is also asked to furnish a competent engineer or foreman who is familiar with the water system to help on the survey. Blueprints are sometimes found helpful; but, in every case, the information on them must be proved.

(4) *Underground Location Survey.*—A physical survey is made of every foot of exposed piping and the underground system is checked as accurately as possible. All lines are sketched on the tracing in the field. A water-use survey is then made to determine whether there are any cross connections or conditions that might tend to pollute the drinking water system.

(5) *Preparation of the Legal Order.*—In writing the legal order to eliminate the cross connection, the inspector must present to the assistant engineer in charge of the field work a summary of his findings together with sketches, blueprints, and piping layout. At this conference each cross connection is reviewed and classified as thoroughly as possible, and special cases are treated as independent problems. After deciding on the proper procedure to be followed in each case, the inspector writes his order using a standard wording form that has already been determined. Special wording may be used when existing wording forms do not apply. After the order is written, it is again checked by the assistant engineer. Then it is reviewed by the engineer in charge of field work; and finally the inspector must demonstrate the soundness of the order to the engineer while the two are reviewing the work together on the premises. If the order displays a good sound sanitary engineering basis, it will be approved for service.

(6) *Presenting the Legal Order.*—The legal order is then issued to the owner or manager instructing him to eliminate or correct the insanitary conditions. The legal order lists the cross connection or condition; location; proposed method of elimination; length of time for compliance; instructions as to what to do in case an extension of time is necessary; the laws, rules, or regulations that have been violated; complainants or enforcement agencies involved; and defendants and their duties and obligations during period of compliance.

(7) *Enforcing the Legal Order.*—Pressure is then brought to bear on the owner or operator to gain compliance as soon as possible. This may take the form of a recapitulation of the explanation given at the original conference (step (3)); the presentation of copies of the laws involved; conferences with keymen; letters from the city attorney regarding legal status; letters from the city health officer certifying the need for immediate compliance to help the management in efforts to gain priorities; orders from the Industrial Accident Commission, Naval and Army Security Officers, and other interested agencies; reinspections; etc.

(8) *Final Inspection.*—A final reinspection is made after all the work has been completed.

(9) *Closing the Case.*—The completed orders together with the maps, sketches, field notes, etc., are filed for future use as the War Department requires a continuing inspection of all defense plants.

The cross-connection problem was recognized as a health menace by public health officers several years prior to 1919. In that year, New Hampshire passed a law prohibiting the maintenance of such connections. Between 1920 and 1930, many similar laws were enacted. The problem received official national recognition in 1930, when the surgeon general of the United States issued a directive to all health officers, in which he ordered the outright elimination of cross connections or the installation of "* * * modern protective devices approved by the State Department of Health and installed and operated under the direct control of that Department." Cross connections are not effectively eliminated by the mere passage of laws or by surveys such as the $1,000,000 survey in Chicago, Ill., after the amebic dysentery outbreak in 1933; the $1,081,830 survey conducted in Cleveland, Ohio, in 1942; or the cross-connection survey made by the U. S. Public Health Service and the Works Progress Administration of all federal buildings located in Detroit, Mich., and New York, N. Y., in 1936 and 1937—unless such surveys are followed up by the issuance of individual orders to make necessary corrections; and, what is far more important, these orders must be enforced.

To those who have been endeavoring to control the cross-connection problem, it has become quite obvious that the situation has been growing worse instead of better. This is true for at least seven reasons, as follows: (*a*) Failure to pass control laws, (*b*) failure to enforce laws, (*c*) failure to publicize the problem, (*d*) failure to control the design of water supply systems, (*e*) failure to standardize and codify good practice, (*f*) failure of management to recognize the problem, and (*g*) failure of the engineer to promote good practice.

(*a*) *Control Laws.*—The failure of federal, state, and local lawmakers to pass cross-connection control laws—or rather cross-connection control laws that are uniform, specific, and at the same time understandable—is a contributing cause of the present situation. In most cases, engineers make the preliminary draft of the laws, after which the draft is referred to the attorneys for a redraft. When the engineers again see the draft, they often cannot recognize it as the product of their efforts. Therefore, they abandon the effort in disgust; the draft is approved by the lawmakers and codified. When the enforcement officer tries to enforce the provisions of the law, he is told by the attorneys that nothing can be done because the law contains too many loopholes and that a conviction could not be secured. In other words, the attorneys would have used the word, "may," where they should have used the word, "shall." Instead of being specific and describing the cross connection involved, the attorneys are vague to the point of ambiguity. Again, the attorneys fail to realize that the proper control of each cross connection is just as important in the problem of health and sanitation as is the determination of an individual stress in the design of a bridge truss.

(*b*) *Law Enforcement.*—The failure on the part of federal, state, and local law enforcement agencies to enforce such laws as are on the books—either because of negligence, ignorance, or fear of political pressure groups—is another contributing cause of present conditions.

(*c*) *Publicity.*—A third cause is the failure on the part of all concerned to publicize the cross-connection control problem. This results from the fear of enforcement officers that they will be criticized because such a condition was allowed to exist, the fear of political reprisals by financially interested pressure groups, and the fear of criminal and civil actions which might be filed against enforcement officers, cities, or utilities (both public and private) for past failure to perform duties and obligations properly. The cross-connection control problem is just as widespread and just as important as the syphilis problem, for example, and the sooner the public recognizes that fact and publicizes the cause and the cure, the sooner the situation can be controlled.

(*d*) *Design Controls.*—Federal, state, and local health authorities fail to recognize the need for specific regulations governing the design of water piping systems, the use of water in all its phases, and the continuing inspection of water systems and water uses. Little, if any, effort has been made to prevent either the improper design of water systems or the overloading of existing systems. The overloading of existing water systems presents one of the greatests hazards in cross-connection control. In one plant a sustained vacuum of 16 in. of mercury was recorded for a period of 5 min in the water system on the second floor. The operating head during the daytime was 5 lb per sq in. in the system on the first floor.

(*e*) *Good Standard Practice.*—Federal and state public health authorities fail to set up a "uniform practice code" for the elimination of cross connections, the testing of backflow protective devices, and the certifying of modern backflow protective devices. Everyone, without regard to authority or fitness, seems to have a voice in making decisions, and local health enforcement agencies are criticized and crucified if they propose any advanced ideas. In most cases the only approved method is the obsolete double check-valve assembly proposed about 1925. The principle of installing vacuum breakers and pressure differential releases on every pressure-operated backflow protective device should be universally adopted. Inventors, designers, manufacturers, and others are afraid to spend their money to develop new ideas for modern backflow protective devices because they know they will not be able to recover their money or because they fear that they will be accused of promoting a device on which they have a monopoly. If the standards were made high enough, and were enforced, many devices would be developed. There is a growing demand for dependable equipment.

(*f*) *Responsibility of Management.*—Management fails to give due consideration to the cross-connection control problem, and is often willing to expend large sums of money for the installation of water systems without consulting local authorities regarding sanitary engineering requirements. The fault for this condition lies with the enforcement authorities rather than with the plant management. The authorities have assumed that the management has as much knowledge of the subject as experts. This is especially evident in the large industrial plants where extensive changes have been ordered. After the reasons for the required changes have been explained, the management has usually concurred and has cheerfully ordered the work completed. In several cases the work order has involved the expenditure of more than $50,000 for a single area.

(*g*) *Responsibility of Engineers.*—Engineers, individually and collectively in their associations, fail to give more than lip service toward solving the problem. The individual plant engineer tries to establish a record for economy. Either consulting and designing engineers are not familiar with water piping and cross-connection problems, or they deliberately fail to incorporate in designs the necessary safety factors required to insure a safe and potable water supply. The associations have made studies and recommendations; but, as yet, they have not made insistent efforts to see that the results of their studies are incorporated in a "Uniform Water Piping and Cross-Connection Control Code."

Summary.—So far as cross-connection control is concerned, enforcement officers have conspicuously failed in carrying out their sworn duty—that is, the safeguarding of public health; the formulation of equitable laws; and, what is far more important, the enforcement of all laws in a uniform manner without fear of political pressure groups and without favor to minority groups.

ROBERT P. LOWE,[10] ASSOC. M. AM. SOC. C. E.—The difficulties that must be surmounted to produce an effective program for elimination of cross connections are given in this paper. The formation of the Cross-Connection Control Advisory Committee is one solution to the diversity of interested jurisdictions which have hindered vigorous action against this recognized danger to the public health. Therefore, the ability of the City of Los Angeles to combine the jurisdictions of the Building and Safety Department, the Health Department, the Water and Power Department, and the Board of Mechanical Engineers for this specific purpose is commendable. A number of cities have made extensive surveys, but nearly all have resulted in very little actual elimination on a large scale, because of complications in authority which were not entirely overcome. This condition will continue to exist unless more authority can be granted under one jurisdiction, as the Los Angeles solution is not always possible.

It is obvious that considerable educational work is necessary to indicate the urgency for the elimination of the cross-connection menace which is often not realized by the layman nor even by the average civil engineer who is not associated with the sanitation field. This was quite evident on a cross-connection elimination survey prosecuted by the War Relocation Authority at its Centers in 1943 and 1944. In many cases plumbers were found to be most aware of the conditions, but they could not control the allocation of time and funds for the changes. Action by the sanitary engineer in the Health Section finally overcame most of this difficulty. Removal was obtained in practically all cases by persistent check ups with renewed emphasis of importance to every one in direct authority.

The condition at the Army fort in the Los Angeles Harbor area was no doubt an extreme case for army installations. As the Centers for the War Relocation

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[10] San Engr., War Relocation Authority, Washington, D. C.

Authority are built on army camp style, they can be compared very easily with the usual cantonment type of installation for troops. This consists of central block latrines and mess halls that centralize and simplify plumbing installations. Very little trouble was found with these installations. In contrast, the Center hospitals were found to be the source of about 90% of cross connections of all types. A few were found at sewage treatment plants and in repair shops. Strict control of hospital plumbing can eliminate the largest source of possible water supply contamination at most army installations of the cantonment type.

The Public Health Service Drinking Water Standards that were adopted on September 25, 1942, raised the requirements for drinking (and culinary) water provided by common carriers for the use of passengers carried in interstate traffic. These standards recognize the seriousness of the cross-connection menace and emphasize responsibility for potable water throughout the distribution system. In the past, great stress has been placed on the source of supply and purification plant control; therefore it appears that modern water purification has become inconsistent with distribution system control. This fault does not lie with any one group. Instead, it is a jurisdictional problem that is clearly indicated by the formation of the Los Angeles Cross-Connection Control Advisory Committee.

Conclusion.—All indications point to the fact that improved distribution control is necessary before present water-purification standards will be consistently beneficial. War, with its necessity for speed and temporary installations, has focused more importance on the seriousness of the cross-connection problem, and there seems to be little doubt that the postwar period will see action on a larger scale. However, it appears that legislation and educational methods on a state-wide basis will be necessary for uniform control, as the jurisdictional difficulty has proved to be slow and almost insurmountable in most instances.

Harry Hayes,[11] Assoc. M. Am. Soc. C. E.—Mr. Van Meter's indictment of the apathy of public officials in prosecuting cross-connection control is no longer applicable to the City of Los Angeles. There has been such apathy in the past, however; but, after a change in personnel and an intensive program of education, an active spirit of cooperation has become evident among all city officials concerned with the elimination of cross connections. Several of these officials have undertaken aggressive programs in their respective spheres of influence.

The Army post mentioned by Mr. Lowe is not a new installation, but a permanent post constructed many years ago. Almost all the plumbing fixtures were old in style and constituted dangerous cross connections. The water system was heavily overloaded and partial vacuums were a common occurrence. These conditions, together with many cross connections of long standing, created a hazardous health problem.

Since 1945, there has been a notable addition to the cross-connection control program in the City of Los Angeles—the establishment of a $50,000

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[11] Asst. San. Engr., Dept. of Water & Power, Los Angeles, Calif

Foundation for Cross-Connection Control and Research by the University of Southern California, conducted under the direction of the dean of engineering. A separate laboratory building has been constructed and equipped under the auspices of this foundation for the complete testing of all backflow-prevention devices, from the smallest size, to and including 12-in. pipe sizes. This Foundation began its work by conducting rigorous research into all conditions of cross-connection control and backflow-prevention devices, and it is anticipated that the findings of this body will constitute a source of impartial and authoritative information on all such matters. In addition, a plumbing laboratory has been established to determine standards for backflow devices used in connection with standard plumbing.

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COMPUTING COAGULANT REQUIREMENTS IN SLUDGE CONDITIONING

BY A. L. GENTER,[1] M. AM. SOC. C. E.

WITH DISCUSSION BY MESSRS. PAUL D. MCNAMEE, D. L. MCLEAN, K. FRASCHINA, WALTER H. BROWN, JR., AND A. L. GENTER.

SYNOPSIS

The practice of partly dewatering sewage sludges in continuous vacuum filters is relatively recent. The procedure for estimating the initial cost of such an installation for different types of sludges is comparatively simple. It involves estimating the tons of sludge solids to be disposed of during each operating day and the capacity and dimensions of the component mechanical elements of the filtration equipment. The mathematical basis for the latter factors has been supplied by rather extensive experience with such equipment.

However, when it comes to the more important item of predicting the annual operating charges involved in using this equipment, a logical mathematical basis is lacking. This is due to the fact that experience with such filters has demonstrated the necessity of using chemical reagents to render all typical sludges sufficiently curdled or coagulated to permit their rapid draining under suction. This experience has shown that the sludge demand for such reagents varies greatly with various types of sludges in different plants; and, even in the same plant, filter yields are largely a function of the amount of coagulant added to the sludge, providing the drum speed of the continuous filter is adapted to certain ranges of coagulant dosage.

Empirical knowledge of these items relating to chemical requirements has been gained by methods of trial and error and involves assumptions that need clarification by mathematical analysis. At times these assumptions have been disappointing from the standpoint of annual operating costs. In quite a number of plants the cost of chemicals per ton of sludge solids treated is an extremely expensive item. Instances are also known in which such filters have been installed and then either abandoned or partly substituted by less expensive methods of sludge lagooning, sand-bed dewatering, or trucking of liquid sludge

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[1] Cons. Engr., Baltimore, Md.

to neighboring farm areas because the annual operating charges for chemicals and incidental filter upkeep were unexpectedly high.

The objective of this paper is to present some evidence of a mathematical relationship between the quantities of chemical reagents demanded by various sludges and certain basic sludge assays. Such a relationship should accomplish several useful purposes:

(*a*) It should supply an explanation of where chemical coagulants go in dosing typical sludges;
(*b*) It should enable a designing engineer to select the most economical method of sludge processing and disposal for given local conditions;
(*c*) It should enable the plant superintendent to determine what may be done to improve the economics of sludge conditioning and filtration in his plant; and
(*d*) It should stimulate further investigation of the factor presented herein relating to the influence of volatile matter on coagulant demand.

As ferric chloride remains the most effective coagulant—that is, produces the best sludge floc with the least amount of chemical—the following studies relate particularly to this compound. Other coagulating aids and substitutes, such as lime, chlorinated copperas, and ferric and aluminum sulfate, are also considered.

Review of Established Pertinent Facts

Experience has already taught the following pertinent facts about sludges and their liquid requirements for chemical coagulants:

(*a*) The major fraction of all sludges is the water that must be strained from the solids by coagulation and filtration;

(*b*) The minor fraction of sludge solids consists of organic and inorganic matter, which are expressed approximately in terms of volatile matter and ash by the combustion of the dried sludge;

(*c*) The volume of the major fraction of water depends primarily on the specific gravity of the solids, which in turn depends on the ratio of organic to mineral matter in the sludge solids;

(*d*) The organic fraction of sludge solids is destroyed partly by bacterial action during storage of freshly collected raw solids, and it is destroyed materially in controlled sludge digestion. This action not only reduces the ratio of organic to mineral matter, but it also increases both the water fraction and the solute concentration of such decomposition products as the bicarbonates of ammonium and calcium, in the water fraction. Instances are known in which the bicarbonates present in fresh raw sludges increase almost one hundred times through digestion. Measured in terms of percentage on solids present, an increase of ten to twenty times is not uncommon.

SLUDGE-WATER DEMAND FOR FERRIC CHLORIDE

Studies of the effects of elutriating decomposing sludges by the writer,[2] C. E. Keefer, M. Am. Soc. C. E., and Herman Kratz, Jr.,[3] at Baltimore, Md., and P. D. McNamee[4] at Washington, D. C., have disclosed and confirmed the fact that the water associated with the solids in a sludge has a specific chemical demand of its own which may be regarded as distinct from that of the solids. This water or "liquid demand" is largely a function of items (*c*) and (*d*).

The addition of ferric and aluminum salt solutions to solutions of ammonium and calcium bicarbonates causes definite chemical reactions; that is, the liberation of carbon dioxide gas and the formation of precipitates of ferric and aluminum hydroxide. According to the combining weight relations of this reaction for ferric chloride:

$$\underset{(162.21)}{FeCl_3} + \underset{(3 \times 79.06)}{3NH_4HCO_3} \rightleftharpoons \underset{(106.86)}{Fe(OH)_3} + \underset{(3 \times 53.5)}{3NH_4Cl} + \underset{(3 \times 44.01)}{3CO_2} \quad \ldots(A)$$

and

$$\underset{(2 \times 162.21)}{2FeCl_3} + \underset{(3 \times 162.12)}{3Ca(HCO_3)_2} \rightleftharpoons \underset{(2 \times 106.86)}{2Fe(OH)_3} + \underset{(3 \times 111)}{3CaCl_2} + \underset{(6 \times 44.01)}{6CO_2} \quad \ldots(B)$$

In these reactions one need be concerned only with the numerical weight relation of the reaction between the ferric chloride and bicarbonate ion. As the standard method of measuring bicarbonate alkalinity (HCO_3^-) is in parts per million of calcium carbonate, $CaCO_3$, the weight relationship between $FeCl_3$ and HCO_3^- shifts to that between parts per million of calcium carbonate, and parts per million of ferric chloride. In reactions *A* and *B*, $2FeCl_3 = 6HCO_3^- = 3CaCO_3$. Therefore, 1 ppm $CaCO_3 = \frac{2}{3} \times \frac{162.21}{100}$ $FeCl_3$ $= 1.08$ ppm $FeCl_3$.

As it is common practice in sludge conditioning to measure the chemicals used in terms of sludge solids—namely, as the percentage of sludge solids—the alkalinity should be measured in the same manner. The percentage alkalinity should then be multiplied by the foregoing constant 1.08 to arrive at the liquid demand for ferric chloride in terms of the solids.

Let Y_1 = alkalinity expressed as percentage of solids, then

$$Y_1 = \frac{\text{Percentage of sludge water} \times \text{ppm alkalinity} \times 0.0001}{\text{Percentage of sludge solids}} \quad \ldots\ldots(1)$$

Let X_1 = percentage of ferric chloride required on sludge solids to complete the chemical reactions with this alkalinity. Then,

$$X_1 = 1.08\ Y_1 \quad \ldots\ldots(2a)$$

or

$$Y_1 = 0.926\ X_1 \quad \ldots\ldots(2b)$$

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[2] "Adsorption and Flocculation as Applied to Sewage Sludges," by A. L. Genter, *Sewage Works Journal*, July, 1934, p. 689.

[3] "The Vacuum Filtration of Elutriated Sludge," by C. E. Keefer and Herman Kratz, Jr., *ibid.*, September, 1934, p. 845.

[4] "Filtration Tests with Elutriated and Unelutriated Digested Solids," by Paul D. McNamee, *ibid.*, September, 1939, p. 764.

Eqs. 2 are the simplest possible formulas of a straight line, expressing the fact that the sludge liquid demand for ferric chloride is a straight-line function of the percentage of alkalinity present on sludge solids.

Solids Demand for Ferric Chloride

As some of the bicarbonates and other solute reagents are undoubtedly trapped within the spongy or capillary porous structure of the wetted sludge solids it is logical to assume that this solid water complex likewise has a chemical affinity or demand for ferric chloride.[5] This is conveniently termed the "solids demand" in order to distinguish it from that of the large fraction of free water in which the solids are suspended. The latter liquid demand must be satisfied largely before additional ferric chloride can reach the solid complexes.

In view of what has already been stated about the influence of the organic-to-mineral ratio of the sludge solids on sludge moisture, the solids demand for ferric chloride should be subject, in some manner, to the same influence. If the solids were entirely mineral, they would require little or no coagulation for vacuum filtration.

Basic Data Used in Determining Solids Demand

If the foregoing statements are fundamentally reliable, it should be possible to estimate approximately the combined liquid and solids demands for ferric chloride with such simple plant data as percentages of solids, water, and volatile matter in a sludge and the parts per million of alkalinity in the sludge water. In plant practice this involves not only reliable statistical averages in sampling and analytical methods but also careful vacuum filter operation. Careful filter operation infers: (*a*) Regulating the ferric chloride dosage so that this chemical is not wasted; (*b*) operating with a filter speed and drum submergence as required by the conditioned sludge; and (*c*) operating with a minimum of filtrate resistance in the filter cloth and in all drainage channels under this cloth. Items (*a*) and (*b*) are quite important in avoiding errors due to frequent faulty addition of any coagulant to a sludge, as well as overflowing sludge that has been conditioned either from a conditioning tank or from the filter itself. LeRoy W. Van Kleeck[6] has emphasized tersely the importance of item (*c*).

With these objectives and items in mind all data presented in Table 1 are purposely selected in order to establish a fairly reliable starting point for evolving a factor for the sludge solids demand. The other incidental objective was to choose data representing geographic variations in plant location, and variations in types of sludges, sludge moistures, alkalinities, volatile content, and ferric doses. The number of plants collecting sludge alkalinity data is not very large and in some plants the tabulation of such data does not extend over several years.

The largest raw sludge filtration plant publishing complete data in the United States is the Minneapolis-St. Paul (Minn.) plant (line 1, Table 1). It is also outstanding in careful filter operation. The essential assay figures for this plant are taken from analytical results for 1941 by George J. Schroepfer,[7] Assoc. M. Am. Soc. C. E. As both lime and ferric chloride are used at this plant, the equivalent ferric dose (Col. 9, Table 1) is determined by the method developed in this paper under the heading, "Other Coagulants: Lime and Ferric Chloride."

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[5] "Principles and Factors Influencing Vacuum Filtration of Sludge," by A. L. Genter, *Sewage Works Journal*, November, 1941, p. 1182.

[6] *Ibid.*, p. 1207.

TABLE 1.—TYPES AND ASSAYS OF SLUDGES CHOSEN TO DETERMINE A FACTOR FOR SLUDGE SOLIDS DEMAND

Line	Type of sludge (1)	Alkalinity (ppm) (2)	SLUDGE ASSAYS (%)				Alkalinity[a] / Solids (7)	Alkalinity on solids[b] (8)	$FeCl_3$ (%)			
			Water (3)	Solids (4)	Volatile (5)	Ash (6)			Total dose (9)	Used for alkalinity (10)	Left for coagulation (11)	Total dose[c] (12)
1	Primary thickened[d]	260	90.65	9.35	62.1	37.9	0.00252	0.252	2.5	0.27	2.23	2.9
	Digested Elutriated—											
2	Hartford, Conn.	150	88.6	11.4	41.1	58.9	0.00117	0.117	1.26	0.13	1.13	1.25
3	Springfield, Mass.	235	88.6	11.4	48.3	51.7	0.00183	0.183	1.63	0.2	1.43	1.69
4	Winnipeg, Man., Canada	1,000	90.0	10.0	50.5	49.5	0.009	0.90	2.6	0.97	1.63	2.63
5	San Francisco, Calif.	470	96.85	3.15	63.5	36.5	0.0145	1.45	4.3	1.6	2.7	4.3
	Digested Waste—											
6	Activated	3,500	96.5	3.5	60.0	40.0	0.0965	9.65	13.0	10.42	2.58	12.8
7	Elutriated	500	94.2	5.8	57.0	43.0	0.0081	0.81	3.0	0.87	2.13	3.0
8	Waste activated thickened	140	98.0	2.0	80.0	20.0	0.007	0.69	7.3	0.74	6.56	7.14

[a] Ratio by weight. [b] Percentage of alkalinity on solids. [c] Computed. [d] Minneapolis-St. Paul plant.

Whereas the Minneapolis-St. Paul data are outstanding in effecting coagulant economies resulting from careful operations in the filtration of primary raw sludge, the writer regards Springfield, Mass. (line 3, Table 1) as the pacemaker in formulating similar methods of filtering elutriated digested primary sludge for populations exceeding 100,000. Line 3, Table 1, gives data for the entire year of 1941 (the first year of operation at Springfield).

The data of line 2, Table 1, were selected primarily for comparison with the data in line 3 and do not represent the entire corresponding average for the year at Hartford, Conn. A particular time was selected during which the averages showed identical solids in the case of Hartford and the year's average at Springfield (Col. 4, Table 1), but showed lower alkalinity, volatile content, and ferric dose (Cols. 2, 5, and 9, Table 1) at Hartford. It was assumed that the lower values for alkalinity and volatile percentage with identical solids content at Hartford accounted for the lower ferric dose during the particular time and should, therefore, provide extreme low points for such values in plotting the graphic results for Fig. 1.

The averages in lines 4 and 5, Table 1, like lines 1 and 3, are the averages for the entire year of 1941 at Winnipeg, Manitoba, Canada, and the Richmond-Sunset treatment plant in San Francisco, Calif. Particular care is used in conditioning the sludge at Winnipeg, and K. Fraschina[8] has emphasized studies undertaken to improve careful operation at the Richmond-Sunset plant.

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[7] *Transactions*, Am. Soc. C. E., Vol. 110 (1945), p. 194 (and private correspondence).

In a way the sludge filtered at Richmond-Sunset is remarkably different from the others. Operation of this plant involves mechanical preflocculation of the solids suspended in the sewage, single-stage digestion with about 62% destruction of 82% raw volatile matter, and elutriation of both supernatant and bottom solids from the digestion tank.

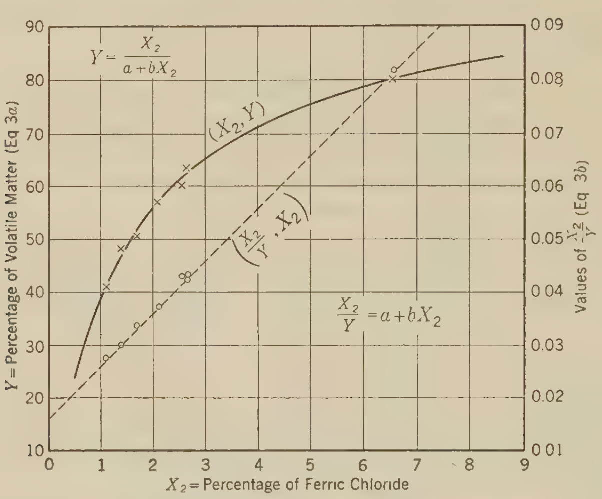

FIG. 1.—VOLATILE-FERRIC CHLORIDE DOSING CURVES

Line 6, Table 1, is chosen to represent about the most difficult type of sludge to coagulate and filter, either with ferric alone or ferric and lime combined—namely, digested mixed primary and waste activated sludge. Such sludges are usually low in solids and high in alkalinity. The sludge shown in line 6 was conditioned with lime and ferric chloride at Ann Arbor, Mich., in 1940 just prior to making some plant-scale elutriation tests. The combined price of the lime and ferric used amounted to the equivalent of about 13% ferric chloride alone, which sum is entered as Col. 9 for this sludge. A large quantity of this sludge was elutriated, conditioned, and filtered with the results shown in line 7, Table 1. Although the data shown are experimental, the writer regards them as characteristic.

Ferric chloride is used alone in conditioning waste activated sludges. However, in larger plants which condition and filter such sludges, alkalinity data are either entirely or largely missing because the ferric dose is primarily regulated by the hydrogen-ion concentration of the conditioned sludge. This is generally about pH 4.3. As such sludges are notably high in water content, considerably more ferric may be used in conditioning them to a pH 4.3 than to a pH 4.9 or pH 5, especially with automatic control of the dosing. Therefore, purely experimental data had to be used for such a sludge in line 8, Table 1. A sludge of high volatile content and low alkalinity was purposely selected to provide an extreme high point for graphic plotting of the influence of volatile matter on conditioning. The sludge was conditioned to about pH 4.9. Like waste activated sludge, the elutriated digested sludges in lines 2, 3, 4, 5, and 7, Table 1, are all conditioned with ferric chloride alone.

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[8] "Laboratory Control of the Operation of the Richmond-Sunset Sewage Treatment Plant," by K. Fraschina, *Sewage Works Journal*, January, 1940, pp. 95–96.

In Table 1 it will be noted that the variations in volatile contents of the listed sludges are greater than 100%, the variations in solids and alkalinity are 600% and 2,400%, respectively, and the ferric doses vary from about 1.3 to 10.0 times as much—namely, 13%.

FACTOR FOR SOLIDS DEMAND EVOLVED

From the selected data in Table 1 the percentage of ferric chloride used for coagulation of the wetted sludge solids and incidental reactions should be determined with fair practical accuracy by deducting the percentage used for the alkalinity reaction, determined by Eq. 2*a* from the total ferric chloride used. In Table 1 this has been done in Cols. 9, 10, and 11. Col. 11 then shows the percentage ferric chloride on solids used for the volatile-solid complex, which may also be termed the ferric demand of the wetted solids themselves.

When the values of this demand in ferric chloride are plotted on the abscissa axis X_2 (see Fig. 1) against the percentages of volatile matter in the respective sludges listed as ordinate (Y-axis) values, a remarkably regular curve results. As shown in Fig. 1, this is a hyperbolic curve of the type

$$Y = \frac{X_2}{a + b\,X_2} \qquad (3a)$$

which converts to a straight-line equation

$$\frac{X_2}{Y} = a + b\,X_2 \qquad (3b)$$

Plotting the values for X_2 (Col. 11, Table 1) against these values divided by the respective volatile percentages Y (Col. 5, Table 1), as shown in the right-hand side of Fig. 1, results in a straight line. Using the simple method of averages on the six sludges dosed only with ferric chloride, the two constants calculate closely to $b = 0.01$ and $a = 0.016$.

Therefore the straight-line equation for solids demand is

$$\frac{X_2}{Y} = 0.016 + 0.01\,X_2 \qquad (4a)$$

which, multiplied by 100 to eliminate the constant 0.01, converts to $X_2 = \frac{1.6\,Y}{100 - Y}$. As Y is the percentage of volatile matter in the sludge, then $100 - Y$ is nothing but the percentage of ash, which simplifies Eq. 4*a* to

$$X_2 = 1.6\,\frac{\text{Percentage volatile}}{\text{Percentage ash}} \qquad (4b)$$

Eq. 4*b* is another straight-line equation for the solids demand.

The total ferric chloride dose, X, as percentage on solids is therefore the sum of the two straight-line equations (Eqs. 2*a* and 4*b*); that is,

$$X = 1.08\ Y_1 + 1.6\ Y_2 \qquad (5)$$

in which Y_1 is the percentage of alkalinity on sludge solids according to Eq. 1 and Y_2 is the ratio of volatile matter to ash in the sludge.

Col. 12, Table 1, shows the doses calculated according to this equation for the various sludges listed.

FACTS REVEALED BY EQ. 5

As a result of the reagents involved, Eq. 5 reveals a definite chemical influence in sludge conditioning. As it is primarily based on alkalinity assays and water-to-solids ratios this formula measures a definite reaction with ferric chloride and the alkalinity of the total water—namely, the free water and the residual within the sludge solid capillary structure. Such chemical reactions obviously involve basic and experimental constants.

In order to relate the ferric dose required for alkalinity to the sludge solids present in the first part of Eq. 5, it is necessary to use the ratio of water to solids. This ratio is evidently determined by the ratio of volatile matter to ash in the sludge solids in the second part of Eq. 5. This is thoroughly logical and may be regarded as a confirmation of the empirical law relating to the Mohlman sludge index, used for measuring the concentration of solids from mixed liquors, which states that this index correlates closely and varies inversely as the ash content of the sludge.[9] In reality all sludge solids complexes suspended in water may be said to have a sludge index.

Therefore it appears that, at present, the constant 1.6 is the only empirical factor or constant in the equation. It should be emphasized that the value of this constant may increase beyond 1.6 with dosing increments purposely used to increase filter yields or to produce minimum cake moistures for the heat drying of some sludges. Instances may be cited in which the constant is practically double the value herein evolved. Conversely there are instances of very low filter speeds (8 min per revolution), heavy sludge, and low dosage, for which this constant becomes practically the same as the first constant, 1.08.

For general mathematical purposes, Eq. 5, when ferric chloride and alkalinity assays are used, evidently becomes

$$X = 1.08\ Y_1 + K\ Y_2 \qquad (6)$$

in which K is an empirical constant which probably varies from the value of the first constant (1.08) to approximately three times this value. From the selected data for careful dosing it is about 1.5 times the minimum value—namely, 1.6.

The range of values for this constant, below 1.6, may be demonstrated to be representative of partial or incomplete ferric dosing of heavy sludges of relatively low alkalinity. The less complete the alkalinity and coagulation reactions are the slower will be the resulting filtration rates. This is quite evident from examination of the numerous filter yield graphs that have been published since 1933. Mr. Schroepfer[10] clearly shows this effect on raw sludge dosed with progressive increments of lime and ferric. From the data in Table 1, Item 1 (Cols. 5, 6, and 11), this constant for the 1941 Minneapolis-St. Paul data is computed to be about 1.35 when using the price equivalent of ferric alone. This certainly indicates partial, and therefore most careful, dosing.

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[9] "Problems and Trends in Activated Sludge Practice," by Robert T. Regester, *Transactions*, Am. Soc. C. E., Vol. 106 (1941), p. 176.

Particle Size.—In revealing reactions largely of a chemical nature, Eqs. 5 and 6 ignore particle size—that is, the presence of finely dispersed colloids in any sludge. This is entirely logical. It would be unreasonable to assume that the various sludges listed in Table 1 contained particles or sludge aggregates of the same size.

Very finely divided solids in sewage will settle to a thin sludge having a high water-solid ratio. With bicarbonates present in the water this high ratio automatically increases ferric demand in terms of the solids. An outstanding example of this statement will be found in Mr. McNamee's filtration tests on the liquor associated with digested sludge.[4] Such supernatant liquor is rich in finely divided solids. In Mr. McNamee's paper[4] the total solid content was reported as 0.4% dispersed in a very high water-to-solid ratio (249 to 1) with a total alkalinity of 3,630 ppm in the water, and a volatile-ash ratio of 1.26. On the basis of Eq. 5, the liquid alkalinity demand for ferric chloride was 98% and the solids demand only about 2%, making a total of 100% ferric dose on solids present. This is remarkably close to the total optimum requirements (105%) for completed alkalinity and coagulation reactions established by properly analyzing the McNamee filtration data. With 0.38% actual sludge solids (which is practically the 0.4% total solids listed by Mr. McNamee), the calculated values total 105% ferric chloride. This is regarded as a rigid test for the validity of Eq. 5.

Floc Size.—This is an important factor in obtaining good filter yields at careful dosing rates and will be briefly dealt with hereinafter.

Eq. 5 Applied to Other Experimental Results Not Tabulated

Usually the best operating control for conditioning sludges is made by frequent laboratory checks with Buechner funnel filters and about 200 g sludge samples of the dosed sludge going to the filter. The laboratory-tested samples should dry to a vacuum break in less time than that required to complete a normal filter drum revolution. More informative, and therefore more reliable, checks are made on sludge samples dosed in the laboratory over various ranges of ferric chloride additions, and the Buechner filter rates in grams of dry solids per minute plotted against the percentages of ferric added are as shown in Fig. 2.

Sludges dosed for actual filter practice require more coagulant to produce sufficiently dry filter cake in a given time than do sludges tested in the laboratory. What may be a dose optimum in laboratory testing will be less than optimum in field practice. There are numerous logical reasons for this: Laboratory testing is essentially small batch filtration, usually done more carefully and at a higher vacuum than in field practice; sludge drainage is directly downward instead of upward against a revolving drum surface and therefore without cake scouring; and the dosed sample is not as long in reaching the filter with the consequence that there is no destruction of the coarsened drainable floc structure produced by either partial or complete coagulation. Therefore, laboratory filtration rates for 4 or 5 min of drying time may be pushed to considerably lower partial ferric doses than in the field, where coarse floc may be initially produced but cannot always be maintained for many reasons.[8,11,12,13]

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[10] *Transactions*, Am. Soc. C. E., Vol. 110 (1945), Table 12, p. 201.

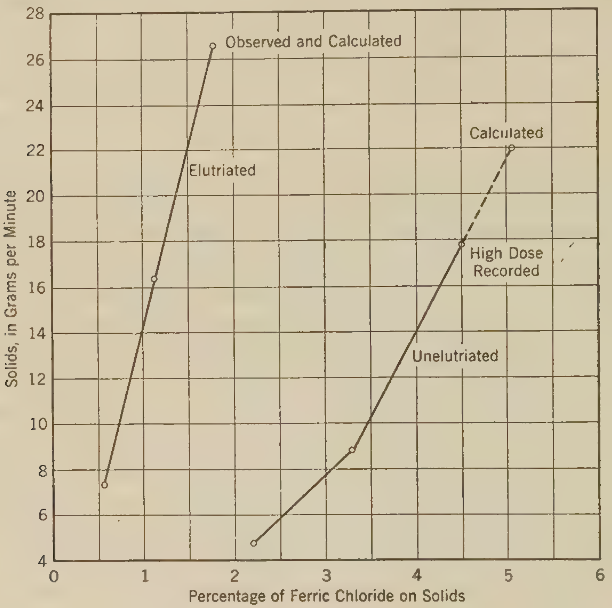

FIG. 2.—FILTER YIELD, FERRIC CHLORIDE DOSING CURVES (MCNAMEE TESTS)

As Eq. 5 accounts for complete alkalinity reaction and probably complete volatile coagulation, the confirmation of this equation by Buechner filter tests has two requirements—the calculated dose should be somewhere near the upper end of the plotted graphs (that is, near the optimum dry solid yield rate), and it should fall directly on such graphs. Obviously reliable sampling, sludge assays, testing technique, and the average of a number of tests are best for this purpose.

Mr. McNamee's laboratory studies of digested supernatant were adopted as the initial test of the validity of Eq. 5. In his published work,[4] the course of alkalinity removal from digested sludge through elutriation and the effects of this removal on sludge conditioning were followed for the first time. His tables showing seventy-two coagulation and filtration tests on unelutriated and elutriated sludges were next used in preparing graphical data for Fig. 2 and as a test of the validity of Eq. 5. Here the graph points of average low, medium,

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[11] "Principles and Factors Influencing Vacuum Filtration of Sludge," by A. L. Genter, *Sewage Works Journal*, November, 1941, pp. 1187, 1189, and 1192.

[12] "Effect of Stirring of Sludge on Vacuum Filtration," by C. E. Keefer and Herman Kratz, Jr., *ibid.*, September, 1937, p. 743.

[13] "The Richmond-Sunset Sewage Treatment Plant," by B. Benas, *ibid.*, January, 1940, p. 93.

and high doses used are plotted against average solids yields for the thirty-six filtration tests on straight digested sludge and the thirty-six tests on the elutriated sludges. At practically completed alkalinity and coagulation reactions the equation was found valid, and the dose calculated from the average assays of the unelutriated and elutriated sludges fell on the respective graphs near the maximum recorded yield points.

The essential analyses for the digested sludge at Cranston, R. I., before and after elutriation and comparisons of the computed doses are presented in Table 2. This new plant[14,15] produces digested mixed primary and waste activated sludge, which for obvious reasons has a high ferric chloride demand. In Fig. 3 and Table 2 the optimum graphed doses are remarkably close to the calculated doses.

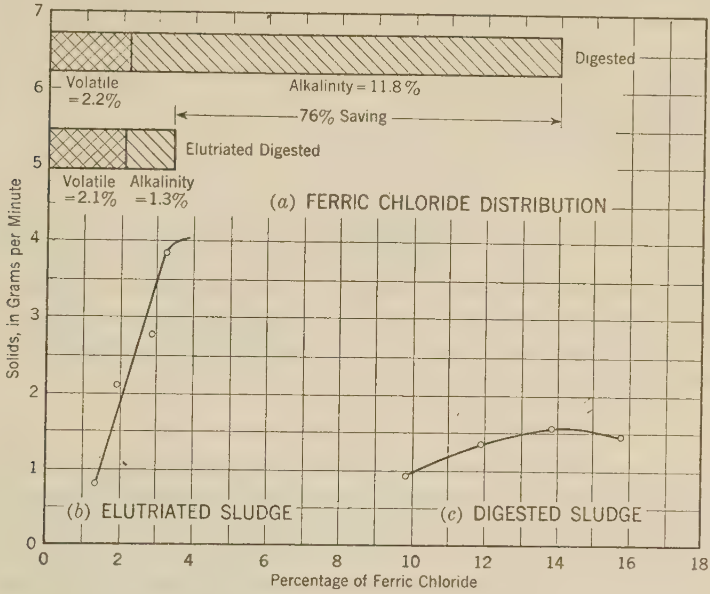

FIG. 3.—FILTER YIELD, FERRIC CHLORIDE DOSING CURVES, CRANSTON, R. I.

Instances of laboratory tests on other sludges may be cited. In some of these the calculated dose is somewhat less than the recorded optimum, indicating a somewhat higher optimum constant than 1.6. However, in all cases thus far recorded the calculated doses fall on the recorded graphs and are therefore regarded as distinct point indexes of near optimal doses in laboratory testing and a reasonable pacemaking low dose in actual practice.

Similarly, in the exhaustive field tests made and recorded by Messrs. Keefer and Kratz,[3] Eq. 5 demonstrates clearly, through point indexes, the relative positions of the various graphs (similar to Figs. 2 and 3) for all sludges dosed

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[14] "Up-to-the-Minute Cranston, R. I., Sewage Disposal Plant and Sewerage System," by Ralph W. Horne, *Public Works*, May, 1943, p. 20.

[15] "Compact Sewage Treatment Works Units at Cranston, R. I.," by Ralph W. Horne, *Civil Engineering*, September, 1943, p. 417.

with ferric chloride. These field tests, made in 1933 and 1934, were run in a manner very similar to laboratory tests—that is, on different batches of large samples of various sludges within various dosing ranges. Here the effects of various concentrations of dissolved ammoniacal nitrogen in parts per million on coagulant requirements were followed. Through the known ratio of ammonium to total bicarbonates the recorded parts per million of ammoniacal nitrogen produced values for Eq. 5. The various graphs were then located at points near the dose optima recorded.

TABLE 2.—COMPUTATION OF FERRIC CHLORIDE DISTRIBUTION IN ELUTRIATED AND DIGESTED SLUDGE; CRANSTON, R. I.

Description	Elutriated (Fig. 3(*b*))	Digested (Fig. 3(*c*))
Alkalinity (parts per million)	300	2,700
Analysis (Percentage by Weight[a])—		
Volatile matter	57	58
Mineral matter	43	42
Computed Values of $FeCl_3$—		
X_1 (by Eq. 2*a*)	1.3	11.8
X_2 (ratio of volatile to mineral, multiplied by 1.6) (Eq. 4*b*)	2.1	2.2
Total, by computation	3.4	14.0
Total, recorded	3.3	13.9

[a] Water 97.6% and solids 2.4% in both sludges.

EQS. 5 AND 6 AS AN OPERATOR'S GUIDE

From the foregoing observations it is evident that Eq. 5 establishes more of a point than a line on such ferric dosing and yield graphs. Other K-values in Eq. 6 may be established from proper laboratory and field testing. In practice, if the actual average ferric dose is materially in excess of values calculated by Eq. 5, careful Buechner filter tests should be run in the laboratory. Here this equation will be an indicating guide to the approximate dose optimum necessary.

If such laboratory tests indicate the approximate validity of the equation and if plant practice on the same sludge does not, the operator has good evidence that chemical is being wasted somewhere in processing the conditioned sludge. The chief sources of this trouble have been cited.[5,8,12,13]

If laboratory tests denote that much higher doses are required than are indicated by Eq. 5, this should necessitate the graphical plotting of the average of several laboratory tests to determine if the constant K in Eq. 6 should be changed to a value materially higher than 1.6.

Exceptions to Eq. 5.—Such a change in the value of K may be due to the logical exceptions found in sludges containing undue amounts of ferric reducing agents or other coagulant-consuming compounds resulting from industrial wastes. In some cases such waste products may be beneficial and in others distinctly inimical to good coagulant economy. In sludges containing mineral oil wastes, cutting oil with soaps, etc., various emulsions and ferric consuming smears (ferric soaps) can result, thus increasing the ferric demand while adversely influencing dry cake yields. Such atypical conditions do not invalidate

Eq. 5. Some of these complexes may be accounted for through proper laboratory tests and the constant K changed to fit local conditions; or the complexes can be largely eliminated by other methods.

DISTRIBUTION OF COAGULANT REQUIREMENTS IN VARIOUS TYPES OF SLUDGES

In Fig. 3(*a*) the distribution of the portions of the total ferric dose for liquid and solids demand is shown graphically for the sludges tested in the laboratory at Cranston. With the unelutriated sludge, 84.3% of the total dose is used to offset alkalinity alone, leaving only about 16% for the actual coagulation intended. As elutriation displaced most of the alkalinity factor and a small portion of the volatile matter, most of the ferric dose remained for solids demand.

A similar graphical method could be used on the various sludges listed in Table 1. However, such a procedure would not depict the quantity of ferric chloride related to the various quantities of sludge collected and processed in different types of plants.

A more illuminating comparison results when the data are limited to various types of sludges collected from average domestic sewage only. For this purpose the writer has used the amounts of average fresh sludge solids removed by the different treatment methods. Tables of data presented by Karl Imhoff and Gordon M. Fair,[16] Members, Am. Soc. C. E., were extended to coagulation and vacuum filtration and the results are given in Table 3, the ferric chloride distribution for the liquid and solids demand being shown in pounds per 1,000 persons daily.

In calculating the pounds of digested solids on this basis, the writer has assumed in all cases a reduction of 65% of the fresh solid volatile in digestion, and 66% in digestion followed by elutriation, as the latter process removes some volatile and further concentrates the sludge. There is always a loss of some solids through digester supernatant withdrawals, elutriation, and vacuum filtration. Consequently, in Table 3 the average digested and elutriated solids reaching the filters will be found to be somewhat higher than in practice. A good example of this statement is to be found in the 1942 results for Springfield. About 130 lb per 1,000 persons daily of primary solids, of 74% volatile matter, were collected. The percentages of volatile matter reduction in digestion, and through digestion and elutriation, were 65% and 66.3%. The removal efficiency of suspended solids was about 60%. The digested sludge contained 8.5%, and the elutriated sludge 12%, solids compared to 8% and 10% shown in Table 3. The percentage of ferric chloride consumption was identical with that computed in Table 3. However, due to loss of solids in processing digester supernatant, elutriate, and filtrate, the actual ferric consumption was about 0.7 lb per 1,000 persons daily instead of 1 lb shown in Table 3.

The percentage solids (Col. 6, Table 3) and alkalinities (Col. 5, Table 3) in the various sludges are taken from conservative averages collected by the writer. In storing and concentrating primary sludge comprising 72% volatile

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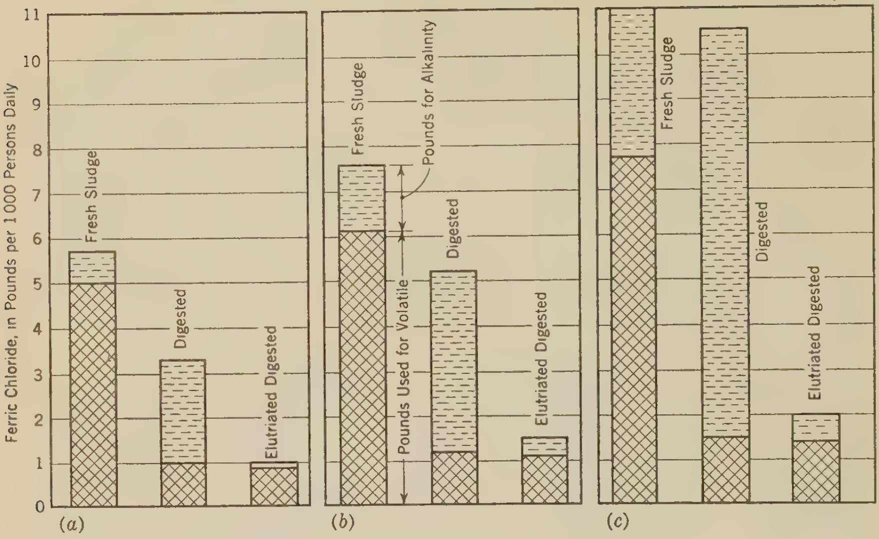
[16] "Sewage Treatment," by Karl Imhoff and Gordon M. Fair, John Wiley & Sons, Inc., New York, N. Y., 1940, Tables 11 and 12, pp. 185–189.

TABLE 3.—COMPUTATIONS OF THE QUANTITY AND NATURE OF DOMESTIC SLUDGE AND FERRIC CHLORIDE REQUIRED FOR VACUUM FILTRATION



No.	Treatment process (1)	Solids[a] (2)	PERCENTAGES Volatile (3)	PERCENTAGES Ash (4)	Alkalinity (ppm) (5)	Percentage of solids (6)	FERRIC CHLORIDE USED[a] For alkalinity (7)	FERRIC CHLORIDE USED[a] For volatile (8)	FERRIC CHLORIDE USED[a] Total (9)
	(a) PLAIN SEDIMENTATION (SEE FIG. (a), ABOVE)								
1	Solids removed..	119	72	28	400	6	0.7	5.0	5.7
	Remainder:								
2	Digested......	64	47.5	52.5	3,000	8	2.3	1.0	3.3
3	Elutriated[b]....	62	46.5	53.5	220	10	0.1	0.9	1.0
	(b) TRICKLING FILTERS AND SEDIMENTATION (SEE FIG. (b) ABOVE)								
4	Solids removed..	148	72	28	400	6	1.5	6.1	7.6
	Remainder:								
5	Digested......	79	47.5	52.5	3,000	6	4.0	1.2	5.2
6	Elutriated[c]....	77	46.5	53.5	330	7	0.4	1.1	1.5
	(c) ACTIVATED AND SEDIMENTATION (SEE FIG. (c) ABOVE)								
7	Solids removed..	187	72	28	500	3	3.3	7.7	11.0
	Remainder:								
8	Digested......	100	47.5	52.5	3,000	3.5	9.0	1.5	10.5
9	Elutriated[c]....	97	46.5	53.5	330	4.5	0.7	1.3	2.0

[a] In pounds per thousand persons daily. [b] Elutriated with water of 100 ppm alkalinity at an elutriation ratio R of 4 to 1 (countercurrent). [c] Elutriated with plant effluent of 200 ppm alkalinity at an elutriation ratio R of 4 to 1 (countercurrent).

matter and its mixtures with secondary sludges for filtration, the alkalinity (Col. 5, Table 3) can easily exceed the listed values, especially in warm weather. Obviously such sludges of high alkalinity can be benefited by proper elutriation. However, such treatment has not been included in Table 3, since it is not typical practice in the United States.

Mechanically flocculated sludges like those involved at Richmond-Sunset and the new San Diego, Calif., plant are quite light and will probably be found to be somewhat like the graph for Table 3(*c*), with activated sludge when properly digested and thoroughly elutriated.

Although sludges collected from combined sewerage systems may double in quantity the amounts shown in Col. 2, Table 3, the total coagulant and its distribution will not double. In fact for fresh plain sedimentation the Col. 9, Item 1 value may be but little more. This is due to the increase of mineral and decrease in volatile matter in such sludges. This is quite evident at Minneapolis-St. Paul where the percentage of volatile matter in the concentrated raw sludge is actually less than that of the digested sludge at Richmond-Sunset (see Table 1) with the result that the primary sludge at the former plant is very heavy and very easy to condition. The 1941 Minneapolis-St. Paul figure for Col. 2, Item 1 of Table 3, was about double the listed value, that is, 246 lb. As a result of the low dose requirements (see Table 1) the value in Col. 9, Item 1 of Table 3, is 6.15 lb, which is only 8% more than the 5.7 lb listed. If this particular sludge were digested and elutriated the values in Col. 9 for Items 2 and 3 would increase about 60% and become more like the Col. 9 values for Items 5 and 6.

The bar graphs in Table 3 supply illuminating evidence of where ferric chloride goes for the various types of sludges. Where the percentage of alkalinity on solids is relatively low as in the primary sludge, its mixtures, and the elutriated sludges, most of the ferric is used for the solid demand. The reverse is the case with the unelutriated digested sludges in which, despite good mineralization of the solids and lowered volatile-to-ash ratio, the high percentage of alkalinity makes this factor predominant in coagulant requirements.

The graph in Table 3(*c*) certainly indicates that there is little if anything gained in using vacuum filters on digested mixed primary and waste activated sludges of relatively high volatile content without the benefits of elutriation. Due to the light solid content of such sludges it can be demonstrated in several instances known to the writer that more ferric chloride or equivalent ferric chloride and lime per 1,000 persons is used on the digested mixture than would be used on the fresh undigested mixture. For example, by changing the percentage of solids in this digested sludge (Col. 6, Item 8, Table 3) to 2.5, the alkalinity factor (Col. 7) becomes 12.6 lb, which, together with the same volatile factor, totals more than 14 lb of ferric chloride (Col. 9) per 1,000 persons daily. This is 27% greater than the chemical demand shown for the fresh sludge mixture. In this example of digested sludge, the alkalinity demand is almost 90% of the total requirements. It is certainly faulty practice to coagulate such sludges and dewater them in vacuum filters without elutriation. If elutriation is not to be used, it is far better engineering to provide sand-bed filtration.

Volatile Matter Destruction as a Measure of Chemical Saving

The percentage of volatile matter destroyed is identical with the percentage reduction in the volatile-ash ratio through the digestion of any sludge. Although the true significance of this statement has been overlooked, Eqs. 5 and 6 now lend it importance. As the reduction in the volatile-ash ratio of a sludge means a corresponding reduction in ferric chloride demand for this ratio, the reduction in volatile matter through digestion is a direct measure of ferric chloride saved per pound of original volatile matter, providing the resulting alkalinity gain is removed by elutriation. However, as the reduction in ferric demand through this procedure also leaves less solids to be conditioned, the final ferric chloride demand is used on the percentage of original raw solids left. This makes the over-all percentage of ferric reduction, for the remaining volatile-ash ratio, equal to

$$R = 100 - \frac{1}{100} \times \text{Percentage of original raw solids remaining} \\ \times \text{percentage of original volatile content remaining} \quad \text{(7)}$$

For example, in the case of Springfield, the 1941 averages (in either pounds or percentages) showed:

No.	Description	Quantity
1	Raw volatile content (69.5% of which is removed in digestion and 30.5% of which remains)	73.6
2	Ash	26.4
3	Total	100.0
4	Remainder of original volatile content (30.5% × 73.6)	22.4
5	Plus original ash	26.4
6	Total original solids remaining	48.8

Therefore, $R = 100 - \frac{48.8 \times 30.5}{100} = 85\%$ saving in ferric chloride through digestion and elutriation. This can be checked readily through methods previously shown in Eq. 4*b*. Thus, neglecting the original and final alkalinity factors, the volatile-ash, ferric chloride requirements are:

$$\text{Original, } \frac{73.6}{26.4} \times 1.6 = 4.46\%$$

$$\text{Final, } \frac{22.4}{26.4} \times 1.6 = 1.36\%$$

The original ferric chloride requirements are intended for 100 lb of raw solids, whereas the final 1.36% is used on 48.8 lb of solids remaining; namely, 0.66 lb of ferric chloride. This is 3.8 lb less than the original 4.46 lb of ferric chloride required, or 85% saving. This saving is based on keeping the alkalinity factor constant in both instances. As a matter of fact proper elutriation may add to this saving by further reducing this factor.

One particularly interesting feature to be noted about Eq. 7 is the fact that, regardless of the ultimate value assigned to the constant, K (1.6 in this example), Eq. 7 will not be changed.

Calculations for Sludge Elutriation.—Eqs. 5, 6, and 7 reveal the genuine advantages gained by reducing the volatile-ash ratio through digesting the sludge as much as economically possible and then removing the inimical alkalinity factor by sludge elutriation. These two procedures combine to reduce the amount of sludge solids to be handled, size of equipment required for filtration and drying or incineration, annual operating charges for chemicals, power for filter hours, and maintenance, at the same time increasing filter yields and the uniformity of sludge characteristics, and eliminating odor nuisance.

The alkalinity (in parts per million) remaining in the sludge moisture after single-stage, multi-stage, and countercurrent elutriation in two tanks may be computed from the following equations, wherein D is the alkalinity in the digested or stale raw sludge water before elutriation, E is the alkalinity of the elutriated sludge, W is the alkalinity of the elutriating water, and R is the ratio of volumes of elutriating water to the volume of moisture in the sludge: For single-stage elutriation—

$$E = \frac{D + R\,W}{R + 1} \qquad (8a)$$

for n-stage elutriation—that is, repeating the elutriation with fresh water n times at the same ratio R—

$$E = \frac{D + W\,[(R + 1)^n - 1]}{(R + 1)^n} \qquad (8b)$$

and for countercurrent elutriation in two mixing and settling tanks—

$$E = \frac{D + (R^2 + R)\,W}{R^2 + R + 1} \qquad (8c)$$

If the water and sludge are both metered, to obtain the true value of R (the ratio of water to sludge moisture by volume), the metered ratio is divided by the fraction of water present in the sludge.

With the alkalinities D, E, and W determined before and after elutriation, the elutriation ratios, R, used to obtain the alkalinities, E, may be computed by solving for R in Eqs. 8: For single-stage elutriation—

$$R = \frac{D - E}{E - W} \qquad (9a)$$

for n-stage elutriation (each wash)—

$$R = \sqrt[n]{\frac{D - W}{E - W}} - 1 \qquad (9b)$$

and for countercurrent (two tanks)—

$$R = \sqrt{\frac{D - E}{E - W} + 0.25} - 0.5 \qquad (9c)$$

In Eqs. 9, to convert R to the ratio of water to sludge, multiply the estimated values of R by the fraction of water present in the sludge.

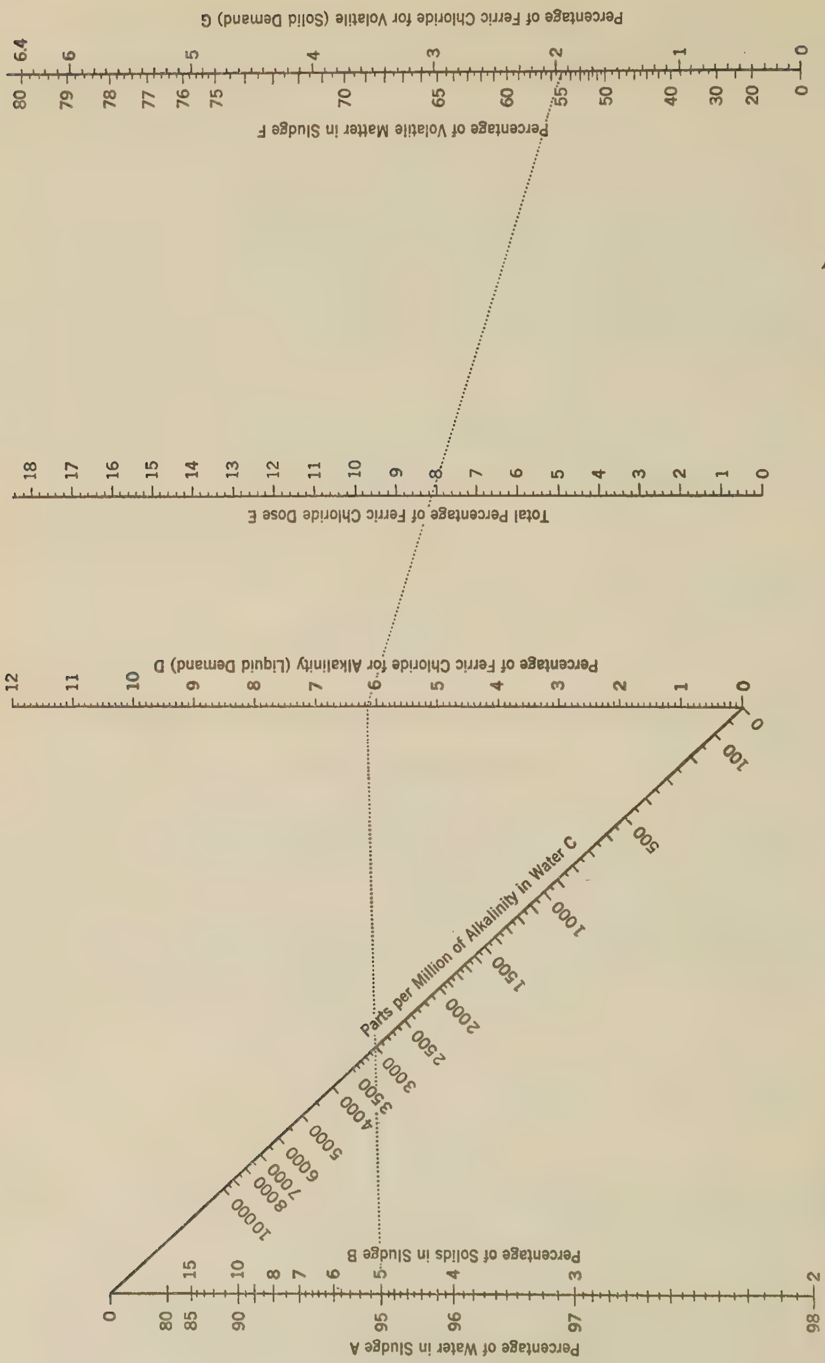


Fig. 4.—Nomograph for Sludge Dosing; Eq. 5

Nomograph.—Fig. 4 presents a useful nomograph of Eq. 5. The N-shaped part of this graph for the alkalinity factor is scientific and therefore relatively exact, with proper sludge sampling and basic analyses.

As an example, consider a digested sludge with 5% solids, 95% water, 3,000 ppm of alkalinity, and 55% volatile content. Connect 95% on scale A, through 3,000 ppm on scale C to 6.15% on scale D. Next connect 6.15% on scale D with 55% volatile on scale F. The total ferric chloride dose (8.1%) is read at scale E. Thus, the actual alkalinity demand in this typical example is more than 6% $FeCl_3$, which is easily 75% of the total requirements. By elutriating this sludge to 300 ppm alkalinity and thickening to 8% solids the alkalinity factor becomes negligible. Fig. 4 also shows that the alkalinity factor in waste activated sludges can play a definite rôle in consuming ferric chloride; for example, at 2% solids and 400 ppm alkalinity the liquid demand is more than 2% $FeCl_3$.

OTHER COAGULANTS

Chlorinated copperas, ferric sulfate, and aluminum sulfate are being used and have been tried extensively instead of ferric chloride. More of these chemicals than ferric chloride must be used for various reasons—heavier molecular weight relations in reacting with total bicarbonates, difference in the compressible hydroxide floc formed in the presence of the sulfate radical, less purity in case of the sulfates, and greater difficulty in effectively dissolving the ferric sulfate. Therefore, these substitutes for ferric chloride must be materially cheaper than the latter in order to find successful use. They also appear to be only of value on well mineralized sludges of low alkalinity.

Chlorinated Copperas (Chemical Symbol, $FeSO_4Cl$; and Molecular Weight, 187.36).—Theoretically it takes 1.16 times more of this pure coagulant than ferric chloride to complete the reactions of Eq. 5. This chemical has been used extensively at Baltimore. The indications are that the dosage approximates this ratio of about 1.2 of chlorinated copperas to 1 of ferric chloride.

Ferric Sulfate (Chemical Symbol, $Fe_2(SO_4)_3$; and Molecular Weight, 399.87).—Theoretically 1 mol of this compound reacts the same as 2 mols of ferric chloride with bicarbonates. This means that 1.233 times more of the pure sulfate salt than ferric chloride is required to complete the reactions of Eq. 5. Although this chemical is delivered in an anhydrous state and is easy to handle, as marketed commercially it is far from being as pure as ferric chloride and is much less soluble in the cold. Consequently the quantity of the commercial salt used will exceed the theoretical weight ratio to ferric chloride.

This salt as ferri-floc is being used successfully on elutriated digested sludge at Annapolis, Md., due to its price of $36.10 per ton in 1941 compared with about $99.00 per ton for anhydrous ferric chloride. Since 1941 about 2.7% ferri-floc on sludge solids was used. This is about 1.6 times as much ferri-floc as ferric chloride previously used.

Aluminum Sulfate.—The chemical symbol of pure crystal sulfate of aluminum is $Al_2(SO_4)_3 \cdot 18H_2O$; and its molecular weight is 666.42. Most commercial sulfate, however, has less water of crystallization, and a molecular weight of about 595. Theoretically it takes about 1.84 times more of this commercial salt than of ferric chloride to complete the reactions of Eq. 5. In some parts of eastern United States this product, known as "paper maker's alum," is cheaper than any of the foregoing reagents. Its use on partly-elutriated and well-elutriated digested sludges presents some interesting disclosures relative to Eq. 5.

It was given extensive trials on elutriated digested sludges at Hartford in 1939, and in Winnipeg in 1940, with rather discouraging results. At Winnipeg the trials disclosed that this relatively pure alum was required in quantities 2.8 times larger than ferric chloride. Furthermore, the cake moistures were higher and the filter yields lower. These findings were confirmed at Hartford where the mineralization of the sludge was better than at Winnipeg. This was undoubtedly due to the precipitation of too much aluminum hydroxide by the bicarbonates left in the sludge. This hydroxide precipitate is exceedingly difficult to filter especially when present in excessive amounts in any sludge. What constitutes a beneficial amount of this precipitate was demonstrated in 1941 and 1942 at Springfield, where its use proved encouraging.

The same grade of alum as that used at Hartford and Winnipeg was tried at Springfield. However, in the Springfield tests the sludge was more thoroughly elutriated than at the other two plants. Therefore, the sludge alkalinity was lower. As a consequence these trials demonstrated the validity of Eq. 5 by the use of a chemical other than ferric chloride.

In 1942, during July and August, 8,023 lb of this chemical were used on 284,176 lb of sludge solids producing an average cake solid yield of 7.4 lb per sq ft hourly. The average sludge assays were 14.2% solids, 45% volatile content, and 212 ppm alkalinity. The average dose was about 2.8% alum on sludge solids. On this basis the calculated ferric chloride dose is 0.14% for alkalinity and 1.31% for the volatile factor, or a total of 1.45%. For aluminum sulfate this calculated dose would be about $1.84 \times 1.5 = 2.8\%$.

Aluminum hydroxide floc is evidently more compressible and fragile than the sludge floc produced when using ferric chloride. J. D. McDonald, engineer in charge of sewage treatment at Springfield, informs the writer that when alum was used, flash coagulation was necessary—that is, running the alum solution into the sludge stream being discharged from the bucket elevator and through baffles in the mixing chamber and then directly to the filter without any further stirring. Ferric sulfate solution is added in a similar manner at Annapolis.

Evidently, under proper conditions, crystal alum can compete with ferric salts when its pound price is less than half that of anhydrous ferric chloride, the sludge is well mineralized through digestion and then elutriated to a low alkalinity. A similar statement applies to the use of chlorinated copperas and ferric sulfate.

Lime and Ferric Chloride.—This combination is used extensively on raw and digested sludges but not on waste activated and elutriated sludges.

Eq. 5 and its nomograph, Fig. 4, are of material help in using lime and ferric chloride on those sludges where the combination may be advantageous. Lime is used with the intention of saving ferric chloride and reducing the total chemical cost per ton of sludge solids.

The combined lime and ferric requirements are just as much a function of the liquid and solids demands of any sludge as are the requirements for ferric chloride, or any of the other ferric or aluminum salts used alone. In this combination lime functions as calcium hydroxide and cannot entirely replace ferric chloride except with some sludges containing relatively large amounts of certain industrial wastes.

In a strict sense, calcium hydrate is not a sludge coagulant. Added to sludges relatively rich in bicarbonate alkalinity before ferric chloride is added, calcium hydrate precipitates the bicarbonates as calcium carbonate. The latter is an inert filter aid which prevents the liberation of carbon dioxide gas as bothersome foam when ferric chloride is added. As calcium hydroxide is only slightly soluble in water, an excess is always needed. On fairly heavy sludges, lime as CaO displaces about one half the ferric chloride dose that would be necessary if no lime were used.

Ratio of Lime to Ferric Chloride.—The importance of this ratio depends largely on the local market price of the two chemicals. This fact seems to have been overlooked in a number of installations.

The saving in the amount and cost of ferric chloride by the addition of lime depends on four essential factors: (1) The ratio of the price of ferric chloride to lime or available CaO per pound; (2) the actual ratio of the two chemicals used; (3) the minimum percentage of ferric chloride permissible with lime; and (4) the actual ferric chloride demand of the sludge. The amount of ferric chloride displaced by using lime equals the percentage of lime (L) used on sludge solids divided by the ratio (R) of the cost of ferric chloride to the cost of available CaO per pound; that is, equal to L/R. Furthermore, if the total percentage of ferric chloride used alone to produce an economic filter yield is equal to C, and the minimum percentage of ferric chloride used with lime to produce the same yield is equal to F, the economic limit to which the engineer may go in using lime is

$$C = F + \frac{L}{R} \qquad (10a)$$

$$R = \frac{L}{C - F} \qquad (10b)$$

Obviously $(C - F)$ is the weight or percentage, on sludge solids, of ferric chloride displaced by using lime. Therefore, the moment the ratio of percentage of lime used to maximum percentage of ferric displaced exceeds the price ratio (R), money is definitely being wasted. Nothing is gained when the two ratios are equal. In fact, it may be argued that money is being lost due to the added expense of removing lime incrustation. Consequently money can be saved only when $R > \dfrac{L}{C - F}$.

Increasing the value of R in Eq. 10b increases the range within which lime (L) can be of value in displacing ferric chloride $(C - F)$. Thus two plants in different localities may produce digested sludges having about the same total chemical demand but with ferric chloride and available lime costing $30 and

$10 per ton respectively in one plant ($R = 3$), and $72 and $12 respectively in the other plant ($R = 6$). With experimentally determined values of $C = 6$ and $F = 3$ in both plants, the lime dosage (L) in the former plant will have to be less than 9% to save money, whereas in the latter plant money may be saved at all lime doses less than 18%.

As a result of these observations, Table 4 has been compiled from the records of four of the largest and best known plants using lime and ferric chloride. The data for all plants are averages from 1941 records. Table 4 indicates that the first plant (Item 1) is the only one that definitely saved ferric chloride and money by using ferric and lime. Aside from the emphasis placed on filtration economies at the former plant, it must be remembered that the low ferric demand of the sludge and the higher price ratio (R) combine to produce most of this result.

TABLE 4.—TYPICAL FERRIC CHLORIDE EQUIVALENTS TO FERRIC CHLORIDE AND LIME COMBINATIONS (PERCENTAGES)

No.	Plant	Price ratio R	CaO used L	$FeCl_3$ used F	$\frac{L}{F}$	TOTAL $FeCl_3$		$FeCl_3$ on cake	CaO on cake	TOTAL $FeCl_3$	
						C[a]	$R = 4.1$[b]			Observed	Computed
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
1	Minneapolis-St. Paul, Minn...	4.1	3.77	1.53	2.46	2.5	2.5	1.4	3.5	2.3	3
2	Buffalo, N. Y...............	2.82	11.2	2.9	3.8	6.9	5.6	2.4	9.7	5.8	5
	Cleveland, Ohio										
3	Westerly................	3.24	14.0	3.0	4.67	7.3	6.4	2.4	11.1	5.8	6
4	Southerly...............	3.24	11.2	4.0	2.8	7.5	6.7	3.5	9.8	6.5	5
5	Averages..................	3.3	10.0	2.9		6.0	5.3			5.0	5.0

[a] Total percentage of ferric chloride, equivalent (symbol C). [b] Total percentage of ferric chloride equivalent with R (Col. 2) the same in all plants as in plant No. 1.

The use of lime and ferric chloride on the digested sludges in the remaining plants, with their higher alkalinity and lower cost ratios, is further complicated by both higher ratios of lime to ferric chloride necessary to meet the higher liquid demand and lower price ratios. Consequently the economy of this procedure becomes more difficult and somewhat questionable. The low lime to ferric chloride ratio (Col. 5, Table 4) for the Southerly plant at Cleveland (Item 4) is not quite typical. On a few occasions during 1941 it was necessary to increase the ferric dose to make up for the absence and scarcity of lime supply. In 1940 the average was 3.5 as compared with 2.8 in Col. 5, Table 4.

The values in Col. 6 are determined by Eq. 10*a*; those in Col. 10 are determined in the same manner when computing lime and ferric chloride percentages on dry cake weight. The values in Col. 7 are computed by the same equation when R is 4.1 in all cases. This places all plants on the same price ratio basis as Minneapolis-St. Paul and removes this particular advantage this plant has over the others in determining Col. 6 values.

Eq. 5 was used on the yearly average sludge assays for the values in Col. 11 and decimals were eliminated in the tabulation; for example, the Westerly and

Southerly plants were computed as 5.7% and 4.8% of ferric chloride, respectively. On this basis the average calculated doses show a rather close approximation to the average of Col. 7 and a closer agreement with the average values in Col. 10, Table 4.

In a way this should be expected. In most cases of Table 1, where ferric chloride alone was used in developing Eq. 5, the ferric chloride doses have been determined on dry cake solids. This chemical increases the weight of cake solids very little. In using lime and ferric chloride this is not the case. In this practice the cake solids are increased sufficiently to account for the differences between Cols. 6, 10, and 11, Table 4. Furthermore, in such installations using ferric chloride and lime, the actual sludge solids handled are estimated both from weighed cake and gallons of sludge sent to the filters. In some instances this procedure may result in an error. The error should be less when calculations are made on cake solids actually weighed and determined through accurate sampling and moisture determinations despite the fact that integrated weightometer readings are frequently light.

In this case, Cols. 10 and 11, Table 4, indicate that Eq. 5 is approximately valid for sludges dosed with lime and ferric chloride when the combined percentages of these chemicals on cake solids are converted to the equivalent percentage of ferric chloride according to Eq. 10*b*. Furthermore, at price ratios of 5 and 6, common to smaller plants using crystal ferric chloride, Eq. 5 will be found relatively accurate.

Therefore, if Eq. 5 or its nomograph, Fig. 4, is to be used as a guide for careful dosing of lime and ferric chloride with essential assays known, Eq. 10*b* also will be of value, for then the percentage addition of lime (L) will be:

$$L = R\,(C - F) \qquad \text{(10c)}$$

with C determined either by the nomograph or careful tests, and R from local prices.

SUMMARY

Eq. 5 has been developed with the purpose of increasing current knowledge of the basic factors that influence coagulant requirements in sludge conditioning. Research indicates that these factors are broadly two in number: (*a*) The liquid demand of the chemically and biochemically contaminated water associated with the sludge solids; and (*b*) the demand of the wetted sludge solids themselves. In this paper, the former is based on the scientific weight relationship of bicarbonate alkalinity to ferric chloride, whereas the latter is based on an empirical relationship of ferric chloride to the volatile or organic portion of the sludge solids.

In determining both of these demands experimentally, the writer has purposely chosen only those plant records and experimental results that are pacemaking in their efforts to save conditioning chemical. Such a choice of statistical averages does not involve dosing sludges to produce the best filter yields and driest cakes in actual practice. With these facts in mind it is believed that the second part of Eq. 5 clearly indicates that the solid demand for ferric chloride and its chemical substitutes is a function of a constant times the ratio of volatile matter to ash in the sludge solids. Undoubtedly the ratio of water to sludge solids in the first part of the equation is likewise determined by this same ratio. As the volatile-ash ratio only approximates the true ratio of organic to mineral matter the true ratio may disclose more meaning to the constant evolved. For general mathematical purposes the formula for total ferric chloride requirements (X) when using alkalinity assays is as expressed by Eq. 6, in which K is the only empirical constant. According to the degree to which the liquid and solid demand of a sludge is combined with ferric chloride prior to filtration, constant K probably varies from the value of the first constant (1.08) to some multiple of this value. For careful vacuum filter practice it is approximately 1.6 (see Eq. 5).

Such careful practice primarily depends on constancy of sludge characteristics and operating averages. To designers of vacuum filter installations either Eq. 5 or 6 should be of definite aid in choosing the best type of sludge processing to produce the lowest values for both liquid and solids demands and to gain constancy of operating averages. However, the operator's chief aid in taking economic advantage of the constancy of sludge characteristics through careful filter operation is the installation of coagulation and filtration equipment that does not tend to destroy the coagulated sludge floc and keep the conditioned sludge unduly long in process before reaching the filter cloth.

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DISCUSSION

PAUL D. McNAMEE,[17] Esq.—The question, "How much coagulant is necessary to condition a sewage sludge for filtration?" would rate $64 on any quiz program. As a rule, fresh solids require less coagulant than do digested solids, whereas activated sludges require the highest dose. Without running actual filter tests, it is hazardous even to venture a guess as to the actual amount of coagulant required by a particular sludge. The common procedure is to add varying amounts of coagulant and then to compare filter yields or filtration times. Mr. Genter has presented a simple method for calculating coagulant requirements which should prove helpful to all persons engaged in the filtration of sludge. Eq. 6 appears to hold for all types of sludge; but, as Mr. Genter has stated, it may have to be modified somewhat when more results are available for comparison. To date, Eq. 6 has had limited application and it will be necessary to apply it to many other sludges before its true worth can be evaluated.

An equation of this type establishes the optimum dose required and gives the operator a definite value at which to aim. In actual practice it is often difficult to attain theoretical values; but, as Mr. Genter has stated, the use of such an equation does show when something is wrong. Mr. Genter's basic data showed that the Washington, D. C., plant used more coagulant than is indicated by Eq. 6. Laboratory tests at the plant verify this conclusion. At the Washington, D. C., plant, there has been some difficulty in maintaining a constant flow of elutriated solids to the sludge meter, which in turn controls the ferric chloride doser. In other words, the ferric chloride is discharged every time the sludge meter makes a certain number of revolutions, and the sludge meter will revolve with a full or a partial load. If the operator regulates the discharge for a partial load and then receives a full load, the sludge will be underconditioned, which causes the filters to become clogged with slime. This necessitates shutting down the filter and washing the cloth. To keep the filters running, it is necessary for the operator to set the dose for a full load, with the result that the sludge is overconditioned when a partial load is received. The trouble is caused by small pieces of wood and trash that find their way to the plant. The plant itself has no screens; the sewage is screened at the main sewage pumping station. The bottleneck in the plant for this material occurs at the sludge pumps that handle digested and elutriated sludge. A single stick will become lodged in a pump and a mat of fine material is built up which eventually clogs the pump. An extremely small amount of trash can cause considerable trouble. For example, if just one sizable stick escapes in every 4,000,000 gal of sewage treated, sufficient interference has been created to require thirty pump cleanings per day. Various steps have been taken to remedy this condition, such as increasing the size of the pumps and lines, but the condition, although relieved, still exists. A different type of coagulant doser would eliminate the overdosing, but additional screening facilities are needed for final solution of the problem.

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With the present setup, it is necessary to overcondition the sludge by a definite percentage even if straight digested sludge is being filtered. Thus, the saving in coagulant by elutriation under present conditions is higher than the theoretical saving since the same percentage increase is three times more for the digested, than for the elutriated, sludge in terms of ferric chloride.

Eq. 6 is divided into two parts: One part determines the ferric chloride required to satisfy the alkalinity of the solution, and the other part determines the amount of coagulant actually needed to coagulate the sludge. The first part is based on well-known and established facts. Although there are many theories in chemistry, there are but a few fundamental laws. The first part of Eq. 6 is based on the law of combining weights, which is well established. If any fault is found with Eq. 6, it will probably involve the second part where Mr. Genter was limited to eight sewage sludges for the derivation. The ferric chloride requirements are shown to be a function of the ratio of volatile matter to mineral matter in the sludge. This condition is somewhat difficult to explain since the determination of volatile matter complicates the picture. The volatile matter is generally taken as a measure of organic matter, although this assumption is only approximately true. Considerable inorganic material is changed by ignition into other forms—with corresponding changes in respective weights. Compounds such as sulfates break down with the loss of sulfur dioxide; carbonates lose carbon dioxide; and other compounds may sublime. The true organic content of a sample, therefore, would probably be lower than the volatile content. This fact does not invalidate Eq. 6, but it does make the use of Eq. 6 more difficult since the operator does not know just how much true organic matter is measured or to what extent the volatile loss may have included inorganic substances capable of reacting with coagulant. The fact that Mr. Genter's ratio of volatile matter to mineral matter does show such remarkable agreement with coagulant requirements must mean that the mineral-matter conversion factor measured by the volatile test is fairly constant for the different sewage sludges.

Breaking down the coagulant requirements of a sludge into two components as Mr. Genter has done affords a clear-cut picture of the value of elutriation. Not only can it be seen at a glance which sludges lend themselves to elutriation but also it is possible to calculate the exact savings of coagulant that can be realized. Eq. 6 should prove helpful to any person estimating the value of elutriation.

In the case of partial dosing to produce partial coagulation, the writer believes that it is a question either of the mixing or of the contact of the ferric chloride itself with the sludge, rather than a question of the precipitation of ferric hydroxide by partial alkalinity neutralization. In running batch samples, the stirring is usually fairly constant. When a strong solution of ferric chloride is added to a sample, there will be a zone where all the alkalinity is satisfied and the local sludge particles are in contact with more ferric chloride than is necessary to condition them. The resulting sample contains conditioned and unconditioned sludge. In the case of sludges with high alkalinity, more ferric chloride is precipitated before it can come in contact with a sludge particle and the percentage of sludge conditioned is not directly proportional to the percentage increase in dosing.

Mr. Genter's paper certainly supplies "food for thought." It is probably the first attempt to correlate the coagulant requirements of all types of sewage sludges with certain sludge characteristics. It was known that, with well-digested sludges, where the volatiles are low and more or less constant, the coagulant requirements vary almost directly with the alkalinity; but this relationship did not hold when other types of sludge were considered. By introducing the ratio of volatile matter to mineral matter, Mr. Genter has correlated the other sludges. A study of Eq. 6 will prove worth while. For instance, if the operator is interested in discovering how the thickness of the sludge will change coagulant requirements, it is only necessary for him to select a given alkalinity value and plot the X-values for different sludge concentrations. About ten times as much coagulant is required for a 1% sludge as for a 10% sludge to satisfy the first or X-part of the equation. If for no other reason than curiosity, all operators who filter sludge should try to apply Eq. 6 to the particular sludge with which they are dealing.

D. L. McLEAN,[18] Esq.—The liquid wastes from a modern community vary from day to day and from season to season to such an extent that any mathematical analysis of the amount of coagulant required to condition the sludge from these wastes for partial dewatering by vacuum filters would appear to be a most difficult, if not impossible, task. However, Mr. Genter simply divided the constituents of the sewage sludge into its liquid and solid parts and used the alkalinity of the liquid portion and the volatile matter of the solid portion as his fundamentals. In this most ingenious manner he has reduced the whole intricate problem to the simple mathematical relationship of Eq. 6, which can be applied to all types of sludge and adapted to meet any set of conditions.

Eq. 5, and particularly the nomograph of this equation (Fig. 4), are of particular value and use in the operation of a treatment plant where vacuum filters are used for the partial dewatering of sludge. To check the application of the use of this nomograph and equation, and to show the relationship between the amounts of coagulant determined from them as compared with the actual amount used in practice, the writer offers some data from the treatment plant records of the Greater Winnipeg Sanitary District at Winnipeg. In this plant vacuum filters are used to partly dewater the elutriated digested sludge with ferric chloride as the coagulant. First, a check will be made on the yearly quantities of ferric chloride used for conditioning elutriated sludge; next, a special run will be checked where digested sludge was filtered without elutriation; and finally the filtering runs for a month will be compared, run by run, as well as the monthly average. If the actual results and the nomograph check within reasonable limits, then the value of the nomograph for checking this plant's coagulant requirements will be demonstrated.

The results of applying Eq. 5 can be compared with the actual average values for 1944. In that year, 1,906,255 lb of dry filter cake solids were conditioned with 53,739 lb of ferric chloride. This gave an average of 2.8% of ferric chloride used. Properties of the elutriated sludge were as follows:

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[18] Superintendent, Greater Winnipeg San. Dist., Winnipeg, Man., Canada.

Average alkalinity, 940 ppm; sludge solids, 9%; sludge moisture, 91%; volatile matter, 52%; and ash, 48%. With these values, Eq. 5 yields the following values for the percentage of ferric chloride required: For the alkalinity demand, $\frac{91}{9} \times 0.094 \times 1.08 = 1.03\%$; and, for the solids demand, $\frac{52}{48} \times 1.6 = 1.73\%$. The ferric chloride required for the total dose is the sum of the two, or $1.03 + 1.73 = 2.76\%$. For all practical purposes, this calculated dose is the same as the 2.8% actually used.

Using 940 ppm for alkalinity, as the average for the past three years of operations and combining this with the tabulated averages of 90.5% water, 9.5% solids, 50.7% volatile matter, and 49.3% ash, the calculated dose of ferric chloride is found to be 2.7%, which checks with the 2.7% actually used.

To compare the results by Eq. 5 with those obtained from a special filtering of digested sludge that was not elutriated, the following data may be used:

On an isolated special plant run of digested sludge which was not elutriated, more than 9% of ferric chloride was required to render the sludge filterable. This sludge had an alkalinity of 4,430 ppm, a water content of 93.8%, a solid content of 6.2%, and a volatile content of 57%, with 43% ash. The ferric chloride requirement calculated by Eq. 5 is: For liquid demand, $\frac{93.8 \times 0.443 \times 1.08}{6.2} = 7.2\%$; and, for the solid demand, $\frac{57}{43} \times 1.6 = 2.1\%$. The ferric chloride total requirement is $7.2 + 2.1 = 9.3\%$, which checks with that actually found by the test.

TABLE 5.—COMPARISON OF ACTUAL AND PREDICTED FERRIC CHLORIDE REQUIREMENTS

September, 1944	Alkalinity (ppm)	Water (%)	Volatile (%)	FERRIC CHLORIDE (%)	
				Actual	Fig. 4
2	920	88.1	51.7	2.3	2.4
5	730	90.9	52.7	2.7	2.8
9	830	90.8	53.8	2.7	2.8
11	860	88.1	52.9	2.5	2.5
13	850	88.5	49.3	2.5	2.3
16	940	88.4	48.2	2.1	2.2
18	850	88.3	51.5	2.6	2.4
20	860	90.8	51.5	2.8	2.6
23	930	89.9	51.7	2.9	2.6
26	890	90.9	57.1	2.8	3.0
29	920	91.7	56.2	2.9	3.1
Average	....	....	....	....	2.59

The results of filter runs for one month, compared run by run with the quantities of ferric chloride taken from Fig. 4, are given in Table 5. The nomograph average for the month (2.59%) compares with the actual percentage (2.54%) computed from the 5,101 lb of ferric chloride used in conditioning 200,912 lb of dry filter cake solids.

These examples of the use of Eq. 5 or Fig. 4 show that for a wide range of conditions it is a most valuable and rapid method to give a check of the ferric chloride used in conditioning sludge in treatment plant operation. A wide divergence between the value given by Fig. 4 and the ferric chloride actually used indicates the probability of inefficiency in some part of the filtering process and should lead to a remedy for any such condition.

As an aid to plant operation Mr. Genter's work is of great value. It should be of equal value to the designing engineer in the selection of the most economical method of sludge disposal. In research it suggests economies to be effected in sewage treatment by concentrating sludges and selecting sources of wash water for efficient elutriation, and emphasizes the value of thorough digestion.

K. FRASCHINA,[19] ESQ.—The problem of calculating coagulant requirements in sludge conditioning has been reduced by Mr. Genter to a simple mathematical process of apparently wide application which should prove to be very useful to all persons engaged in sludge filtration. In applying his equations to plant practice, however, Mr. Genter warns of the necessity of reliable statistical averages in sampling and analytical methods, and of careful vacuum filter operation.

This discussion will be confined to some theoretical aspects of the equations formulated, and to subsequent operational data for the Richmond-Sunset plant, with tentative explanations of conformity to, and departure from, Mr. Genter's equations.

In the first half of Eqs. 5 and 6 the amount of ferric chloride required to react with the chemically and biochemically contaminated water forming the major portion of the liquid sludge is determined by the reaction of the coagulant with dissolved bicarbonates, as shown in Eqs. (*A*) and (*B*) (see heading, "Sludge-Water Demand for Ferric Chloride"). Any possible effects of phosphates, borates, or silicates, if present, are not considered. It is entirely logical that the reactions given should go practically to completion because the solubility of hydrous ferric oxide is extremely small in water, the reaction proceeding quantitatively even in weakly acid solution.[20] The solubility product of hydrous ferric oxide is not exactly known; it depends somewhat upon the manner of precipitation but is of the order of 10^{-36}. It follows, therefore, that the amount of iron left in solution at a pH-value of about 4 will be in the order of 0.05 mg per liter. Since alkalinity is also determined, by titration with acid, to a definite pH-value of about 4, it can be inferred, if the foregoing is correct, that the pH-value of the conditioned sludge should be about 4 at the completion of the reactions expressed by Eqs. (*A*) and (*B*), and before any coagulant is available for the "solids demand" of the second part of Eqs. 5 and 6. This is not necessarily the case, at least with elutriated digested sludge. At the Richmond-Sunset plant, for instance, some residual titratable alkalinity remains, for sludge need not be conditioned with ferric chloride to a lower pH-value than about 6 for successful filtration. The effect of this on Eq. 6 would be to decrease the coefficient of Y_1 and increase the value of K.

The second part of Eq. 6 measures the coagulant required for the "solids demand." The fact that the solids-water complex does have a ferric chloride demand can be demonstrated by the presence of considerable amounts of reduced iron in the ferrous state in the filtrate. This "solids demand" is measured in the equation by an empirical constant K and the ratio of percentage volatile matter to percentage ash. Mr. Genter has emphasized the fundamental importance of this ratio in estimating the loss of volatile matter during the digestion of any sludge and the economies that can be realized in digestion and elutriation. These relationships are neatly expressed in Eq. 7. In sludge ignition, organic matter is destroyed and considerable inorganic matter is converted to a variable extent into other forms depending on the temperature of ignition and the particular salts present. The volatile content determined by ignition probably exceeds, and at best only approximates, the actual organic content. The latter presumably determines the proportion of coagulant required; little or no coagulant is required for the mineral matter, as stated by Mr. Genter.

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[19] Supt., Richmond-Sunset Sewage Treatment Plant, San Francisco, Calif.

[20] "Textbook of Quantitative Analysis," by I. M. Kolthoff and E. B. Sandell, The Macmillan Co., New York, N. Y., 1936, p. 294.

The determination of volatile matter and ash is further complicated by the fact that both are based on the weight of dried solids in the sludge; volatile matter is the percentage loss of solids on ignition, and ash is the percentage remaining. The weight of dry solids is not susceptible of absolute determination. In the standard method,[21] the sample is evaporated overnight in a 103° C oven, cooled in a desiccator, and weighed; or evaporated on a water bath, dried at 103° C, cooled, and weighed to constant weight. At the Richmond-Sunset plant it has been found that a continuous apparent decrease in solids percentage occurs on continued heating at 103° C, probably due to loss of the more volatile organic fractions or to the decomposition of the more unstable organic materials. A limiting or constant weight is approached only after prolonged heating. Furthermore, the dried solids are very hygroscopic; they absorb water very rapidly. Unless weighings are made under such conditions that absorption of water is reduced to negligible proportions, solids and calculated volatile percentages in general will be too high, and computed ash percentage too low. The net effect on the volatile-to-ash ratio depends on the relative loss in weight on heating and gain in weight from moisture absorption, and may either increase or decrease the apparent ratio with corresponding changes in the value of K in Eq. 6. For example, if ash weight remains constant and solids are reported too high by 5%, the apparent volatile-to-ash ratio of a 3% solids sludge of 65% volatile matter will be increased 7.7%; with the same error in a 3% sludge of 50% volatile, the increase will be 10%.

Another factor which may affect the percentage of coagulant required is the rate of flocculation. Hydrous ferric oxide, when first formed, is a positively charged colloid in the acid region to a pH-value of about 6 to 7.5, negatively charged in the alkaline region.[22] In the first case, hydrogen ion and probably some ferric ion are adsorbed from solution; in the second case, hydroxide ion and probably some ferric ion are adsorbed. Flocculation is promoted in the acid region by anions and in the alkaline region by cations, the most effective in either case being those of higher valence. The rate of flocculation and, therefore, the amount of coagulant required for any given sludge may depend to a certain extent, then, on the particular dissolved salts present. The rate of flocculation may also be influenced by the presence of certain impurities in the ferric chloride itself, such as calcium, zinc, copper, ferrous iron, hydrogen, sulfate, and phosphate ions.

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[21] "Standard Methods for Examination of Water and Sewage," published jointly by the A.P.H.A. and the A.W.W.A., New York, N. Y., 8th Ed., 1936, p. 173.

[22] "Handbook of Applied Hydraulics," by C. V. Davis, Editor-in-Chf., McGraw-Hill Book Co., Inc., New York, N. Y., 1942, p. 802.

None of the foregoing invalidates the use of Eq. 6 but some factors have been presented that may serve to explain anomalous behavior in applying Eq. 6 and to indicate modifications in sampling and analytical methods that may be required.

TABLE 6.—COAGULATION REQUIREMENTS; RICHMOND-SUNSET PLANT

Line	Period	Alkalinity (ppm)	SLUDGE ASSAYS (%)			Ratio[a] Col. 4 / Col. 5	$FeCl_3$ (%)		Constant K by Eq. 6
			Solids	Volatile matter	Ash		Actual	Eq. 5	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
				(a) BY FISCAL YEARS					
1	1941–1942....	470	3.04	65.3	34.7	1.88	4.17	4.63	1.35
2	1942–1943....	...[b]	2.78	68.0	32.0	2.12	5.35	5.17	1.73
3	1943–1944....	465	3.00	68.4	31.6	2.16	6.05	5.08	2.04
4	1944–1945....	450	3.28	69.1	30.9	2.23	5.02	5.01	1.61
5	Weighted avgs.	465	3.03	67.7	32.3	2.10	5.13	4.97	1.68
				(b) BY MONTHS, IN THE FISCAL YEAR 1944–1945					
1	July.........	455	3.01	69.8	30.2	2.32	5.64	5.30	1.75
2	August.......	485	3.07	69.0	31.0	2.22	5.58	5.20	1.77
3	September....	505	3.44	66.8	33.2	2.02	5.95	4.76	2.18
4	October......	465	3.24	68.0	32.0	2.12	4.84	4.90	1.58
5	November....	390	3.30	68.0	32.0	2.12	4.06	4.63	1.33
6	December....	425	3.63	71.2	28.8	2.47	2.94	5.18	0.70
7	January......	445	3.57	70.6	29.4	2.40	3.98	5.14	1.12
8	February.....	460	3.72	68.8	31.2	2.20	5.86	4.80	2.08
9	March.......	420	3.77	68.8	31.2	2.21	5.18	4.69	1.82
10	April........	420	3.46	69.6	30.4	2.28	4.68	4.92	1.50
11	May.........	445	3.25	69.8	30.2	2.31	5.18	5.13	1.62
12	June........	465	3.09	69.9	30.1	2.32	5.64	5.29	1.75

[a] In Table 6(*b*) the values in Col. 6 were computed for each month from the actual accumulated pounds of solids, volatile matter, and ash. The percentages in Cols. 4 and 5 are rounded values computed from these same monthly totals. In some months, therefore, the values in Col. 6 will not coincide with the ratio, Col. 4 : Col. 5. [b] No data available; the value 470 was assumed for purposes of computation.

In Mr. Genter's paper the data presented for the Richmond-Sunset plant are for 1941. Subsequent data computed as weighted averages by fiscal years are given in Table 6(*a*). The data for 1944–1945 are the most reliable statistically because improvements have been made in the measurement of volumes of sludge filtered and in methods of sludge sampling and analyses. In Table 6(*b*), monthly weighted averages during 1944–1945 are given to show the deviations not apparent when weighted averages over the year are considered as in Table 6(*a*).

The data in Table 6 can be considered as valid tests of Eqs. 5 and 6 with the following qualifications: Wartime dislocations in personnel were felt since early in 1942 but were most acute during the fiscal year 1943–1944. The technique of sludge filtration has conformed in most respects with Mr. Genter's criteria of careful operation, although it has been necessary to operate the single filter at practically maximum speed at all times to treat the sludge produced. If more filter area were available, filter speed could be reduced and somewhat greater economy in the use of ferric chloride would be attained.

In Table 6(*a*), except in 1943–1944, fairly close agreement is shown between computed ferric chloride doses by Eq. 5 and actual doses (Cols. 7 and 8); and computed values of K by Eq. 6 (Col. 9) are approximately 1.6, the tentative value set for careful filter operation. The higher values for 1943–1944 can be attributed in part to new and inexperienced operators. This confirms the necessity for competent and experienced personnel if most effective filter operation and maximum operating economy are to be attained.

When the data for 1944–1945 (which, in Table 6(*a*), line 4, show excellent agreement when compared with computed values by Eqs. 5 and 6 on a weighted yearly average basis) are compared by months, rather wide discrepancies appear. In Table 6(*b*), K-values computed from Eq. 6 vary from 0.70 to 2.18. The high K-value in September may be attributed in part again to changes in personnel, and the high value in February to errors in measurement arising from the small quantity of sludge filtered. The low doses in November, December, and January (Col. 7, Table 6(*b*)) with respective K-values of 1.33, 0.70, and 1.12 (Col. 9) are more difficult to explain. At this plant single-stage digestion is employed followed by elutriation of both supernatant and bottom solids from the digester. It is not possible to obtain a supernatant of much less than 1.5% solids with the limited capacity of this single digester. The major portion of elutriated sludge filtered usually consists of solids from the supernatant. From the middle of November to the middle of January the digester was in an abnormal state of activity with most of the sludge accumulated in an actively digesting mass at the top and with only a small amount of partly digested sludge at the bottom. The solids content of the intervening layer decreased to less than 1%. Relatively greater proportions of bottom sludge were withdrawn to elutriation during this period than usual and at the same time there occurred the greatest dilution of elutriating water with storm water. Several possible explanations of the apparently anomalous behavior of this sludge on filtration—that is, less coagulant required than computed—suggest themselves. First, digestion had not proceeded as far as usual and, in the sludge solids, intermediate digestion products including more than usual quantities of scum solids containing such excellent aids as disintegrated hair, paper, etc., were present whose coagulant demand no longer could be evaluated properly by the volatile-to-ash ratio; and, second, the flocculating effect of dissolved salts in the associated water phase was changed by the presence of unusual soluble decomposition products.

The plant was shut down for mechanical repairs from the middle of January to the end of February; the digester then reverted to normal activity. The aforementioned atypical behavior in sludge filtration has not recurred since.

Walter H. Brown, Jr.,[23] Esq.—The purpose of this discussion is to show that, through the use of the equation recommended by Mr. Genter, material savings were made in the use of ferric chloride at the Cranston sewage treatment works. The writer is not a chemist and will therefore limit his remarks to the mechanical results obtained.

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[23] Supt. Sewage Treatment, Dept. of Public Works, City of Cranston, Cranston, R. I.

By substituting Eqs. 2*a* and 4*b* into Eq. 5 in place of Y_1 and Y_2, an equation for the total dose of ferric chloride for elutriated sludge is evolved, as follows—

$$\text{Total dose } FeCl_3\ (\%) = \frac{H_2O\ (\%)}{\text{Solids}\ (\%)} \times 0.0001 \times \text{alkalinity (ppm)} \times 1.08 + \frac{\text{Volatile matter}\ (\%)}{\text{Ash}\ (\%)} \times 1.6 \quad \text{(11)}$$

By using Eq. 11, Buechner funnel tests were performed in the laboratory at Cranston, the results of which checked closely the dosage as determined theoretically. However, when such dosage was applied to the elutriated sludge in actual practice, the sludge was absolutely unmanageable. It was found that almost an 80% increase above the theoretical was required. This fact led to a suspicion that the ferric feeders or mixing tank, or both, were not functioning as efficiently as possible.

TABLE 7.—CHECK TESTS ON FERRIC FEEDERS, SEWAGE TREATMENT WORKS AT CRANSTON, R. I.

(Units Are Percentages Except as Indicated)

Month	PRIMARY SLUDGE		DIGESTED SLUDGE			ELUTRIATED SLUDGE			FILTER CAKE		$FeCl_3$ (lb)	$FeCl_3$ (PERCENTAGE OF DRY SOLIDS)		
	Solids (1)	Volatile (2)	Solids (3)	Volatile (4)	Alkalinity[a] (5)	Solids (6)	Volatile (7)	Alkalinity[a] (8)	H_2O (9)	Dry solids[b] (10)	(11)	Actual (12)	Theoretical (13)	Excess[c] (14)
							(a) ORIGINAL FEEDERS							
1944:														
July	4.60	77.8	4.58	61.1	4,745	4.12	60.5	415	78.2	58,930	3,771	6.40	3.4	88
August	4.61	78.4	4.49	60.1	4,714	4.36	59.8	416	77.3	56,930	3,043	5.34	3.4	56
September	4.92	78.3	5.21	59.7	5,675	4.52	59.2	431	77.4	51,590	3,395	6.58	3.3	100
October	4.75	80.3	5.62	59.0	6,125	4.03	58.9	413	78.2	40,100	2,323	5.80	3.4	71
November	5.11	76.3	...	...	...	3.89	59.3	374	78.8	39,360	2,050	5.21	3.3	58
December	4.58	75.8	7.38	44.1	5,360	4.08	51.1	553	76.7	45,364	2,325	5.12	3.1	64
1945:														
January	4.51	80.8	5.50	53.6	5,312	4.37	54.4	613	78.3	31,508	1,909	6.06	3.4	79
February	4.80	78.3	6.42	52.9	6,350	4.24	57.7	760	79.7	5,420	317	5.84	4.1	41
							(b) NEW FEEDER							
1945:														
March	4.77	80.3	6.74	53.4	6,425	4.37	55.7	586	78.6	37,980	1,844	4.86	3.4	44
April	5.43	80.1	5.57	54.2	5,815	4.45	56.1	553	78.1	25,800	1,100	4.26	3.3	30
May	4.83	76.3	6.08	55.1	5,400	4.47	56.2	362	77.1	31,770	1,092	3.43	3.0	13
June	5.80	74.9	6.00	54.0	6,083	4.83	54.5	411	76.8	44,580	1,506	3.38	2.8	21
July	6.02	71.8	7.01	52.9	6,250	5.63	51.7	559	76.0	50,450	1,628	3.23	2.8	14
August	5.48	76.9	7.44	53.1	6,500	5.12	51.4	583	76.0	59,220	1,684	2.85	2.8	4
September	5.60	79.2	6.47	52.1	6,080	4.85	54.4	609	76.3	44,220	1,259	2.84	3.2	−12

[a] Parts per million. [b] Total dry solids, in pounds. [c] Percentage excess (Col. 12 − Col. 13)/Col. 13.

Examination proved that the ferric feeders were very erratic in their operation. They were subsequently discarded and a volumetric type feeder was designed by the writer. After a period of adjustment and refinement, the actual dose was found to be approaching the theoretical. Further investigation showed that the mixing tank at Cranston was short-circuited; and, after that was remedied, Eq. 11 yielded the actual as well as the theoretical dosage.

Table 7 contains the data that led to a check of the equipment at the Cranston works. In the period reported a fairly wet filter cake (77.8% H_2O) was being produced with an average excess ferric dose of approximately 74%. For the months of May through September, 1945, the overdose was only about 8% and the filter cake had been reduced to about 76.5% H_2O. This resulted in a substantial saving in the use of ferric chloride. Furthermore, the digested sludge at the Cranston works now contains, on the average, less volatile matter and much more alkalinity than shown in Table 2. Due to the relatively large quantities of brewery waste discharged into the sewers and treated at the Cranston plant, sludge digestion is particularly turbulent (gas production, 1.7 cu ft per capita per day) and the resulting alkalinity of the digested sludge is notably high. As will be seen from comparison of Cols. 2, 4, and 7, showing the volatile contents of the digested and elutriated sludges, the latter is frequently higher than the former. This is due to the accumulation of top, or less completely digested, solids sent to elutriation.

In view of the effects of such industrial wastes on the treatment at Cranston, for some time the writer doubted the applicability of Mr. Genter's equation to the sludge treated at Cranston; but subsequent tests and investigations have demonstrated that this basic equation undoubtedly holds.

A. L. GENTER,[24] M. AM. SOC. C. E.—The discussions of this paper on the mathematical analysis of a practical and theoretical problem have originated in widely separated localities. In all efforts to understand natural phenomena, and to control them, both the practical and the theoretical are equally important. From the standpoints of presenting more statistical evidence of the validity and usefulness of Eqs. 5 and 6 to the designing engineer and operator and of critically examining the component parts of these equations—the discussions have been valuable contributions to the subject and are gratefully acknowledged.

Reply to them will be confined chiefly to the practical and theoretical phases of that part of Eqs. 5 and 6 which cannot be understood immediately in an intellectual manipulation of either equation. This difficulty refers to Eq. 4*b* which states mathematically that in addition to including the liquid alkalinity requirements of a sludge, measured in terms of water per unit weight of dry solids, the chemical requirement is a function of a constant multiplied by the weight of volatile matter per unit weight of ash present in the sludge.

Mr. McNamee agrees that the first part of Eqs. 5 and 6 (that part determined by Eq. 4*a*) is scientifically valid. He is logically cautious, however, about deriving equivalent validity from Eq. 4*b* and from the second part of Eqs. 5 and 6 without further evidence of supporting data. This caution arises chiefly from the difficulty of explaining the ratio of volatile to mineral matter in terms of the ratio of true organic matter to mineral matter. Mr. Fraschina agrees with this observation. As the writer has previously indicated (see heading, "Facts Revealed by Eq. 5") and will presently attempt to demonstrate, the volatile-ash ratio in these equations takes in considerably more territory in sewage treatment in general, and in sludge disposal in particular, than has been generally contemplated in the examination of the effect of this ratio on coagulant consumption.

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[24] Cons. Engr., Baltimore, Md.

The mathematical language involved in the use of this ratio in Eqs. 4*b*, 5, and 6 does not necessarily imply measurement of organic matter itself. It does, however, measure the amount of sludge water present in a sludge per unit weight of sludge solids, and this is most important. The amount of volatile matter in a sludge evidently contains mostly organic and possibly some inorganic matter essential to the organic capillary structures—both of which ingredients are responsible for the polar-surface activities of the solid and its water-imbibing properties acting as capillary structures and colloidal aggregates. The ratio of volatile to ash, times some constant, therefore, measures these water-imbibing capacities instead of presenting any picture of the amount of true organic matter present in a sludge.

There is nothing novel in the idea that the amount of water in a sludge is some function of the volatile matter present. There is something new, however, in evolving mathematical tools for measuring this function. In discovering such tools, it seems to the writer that it is not so essential that the mineral-matter conversion factor in the volatile test be fairly constant, as it is that the analytical methods and data be kept on an "even keel" by closely following "standard-method" procedures. In fact, the data presented from all parts of North America, and the equations evolved therefrom, are a composite tribute to the reliable uniformity of standard scientific analytical procedure and the competence of analysts and observers.

Such equations cannot be derived without an ample supply of supporting data. The writer feels that this condition was met in the data presented in Table 1. He also feels that it is not so important that data be collected from seven or eight sludges as it is that they cover a wide range of treatment practices, reliable essential assays, and careful dosing operations. Nevertheless, the greater the number of data collected and correlated, the more reliable will be the average data used in deriving such equations. If a scientific hypothesis is sufficiently important, additional evidence of its truth should be sought. This is undoubtedly the meaning of Mr. McNamee's comment on this point and the writer thoroughly agrees with it. Therefore, he will attempt presently to supply ample additional evidence.

In addition to these observations, it should be noted that, unless the hypothesis geared into the mathematical machinery of such pioneering investigations is capable of confirming experimental averages already recorded and unless it becomes a guide in the investigation of new data and consequently becomes equally capable of predictions which observations later verify, the hypothesis is a museum piece instead of a useful scientific instrument.

At the time the data presented in Table 1 were subjected to the impartial machinery of mathematical correlation, the writer offered Eq. 5 and its more generalized expression Eq. 6 as instruments of prediction. The statistical data involved approximately 7,000,000 lb of sludge solids, 191,000 lb of ferric chloride, and 8,000 lb of alum.

In the succeeding five years, notwithstanding the influence of war emergencies on the nature of sewage treated, plant overloading, operation, changes in volatile and alkalinity contents of the sludges, and additional recorded data, together with data from other plants where careful dosing is practiced, expanded the statistical data to considerably more than ten times the sludge solids and ferric chloride involved in Table 1. These additional data demonstrate that Eqs. 5 and 6 have served certain plants as reliable instruments of prediction and promise to continue to do so. This is made evident not only by the new data generously contributed and discussed by Messrs. McLean, Fraschina, and Brown but by considerable additional data collected from plants in Springfield, Mass., Hartford, Conn., and other places not tabulated and discussed.

All these data could be tabulated and summarized to indicate the validity of that part of the equations involving the so-called solids demand—that is, the constant 1.6 times the pounds of volatile matter per pound of ash—notwithstanding the great differences in the volatile and alkalinity contents listed in Tables 1, 2, 5, 6, and 7 and in the recent unpublished records from other plants. What could be demonstrated by such a comprehensive comparative tabulation can be made equally evident from the data relative to the Richmond-Sunset plant in San Francisco, Calif., in Tables 1 and 6.

Since the publication of the 1940–1941 data for that plant in Table 1 (line 5, Cols. 5 and 6), the volatile-ash ratio of the elutriated sludge has gradually increased from 1.74 to the 2.23 average of Table 6 (line 4, Col. 6). This material increase of more than 28% has taken place without any significant change in the yearly average value of the constant 1.6. Notwithstanding the causes of the increase in the volatile-ash ratio, incidental to the use of an overloaded single-stage digestion tank, occasional variations in the K-values of Table 6, and Mr. Fraschina's logical explanation of them, the 1940–1941 and 1944–1945 statistical averages for this plant have the same average value of 1.6 for K as do those listed for Winnipeg, Man., Canada, Hartford, Conn., Springfield, Mass., etc., and ultimately for Cranston, R. I. Since the publication of Table 7 relative to the Cranston plant, a private communication from Mr. Brown regarding the improved ferric feeder shows that the average K-value for the months of from June to December, 1945, also settled down to 1.6. That the value remained near this average all over North America can scarcely be thought mere coincidence.

Volatile-Ash Ratio and Sewage Treatment.—The writer has elsewhere emphasized[25] that sewage treatment, from plain sedimentation to final dewatering, is a system of correlated, progressive steps in removing undesired suspended and settleable solids from a large quantity of water. Everything in the system is connected in some definite manner with the amount of volatile matter in the solids being dewatered. This definite correlation may be traced from the effects of the volatile matter on the specific gravity of the solids in plain sedimentation (where gravity alone frequently accomplishes 99.96% of solids dewatering) to all the ever-increasing economic difficulties incidental to further dewatering.

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[25] "Principles and Factors Influencing Vacuum Filtration of Sludge," by A. L. Genter, *Sewage Works Journal*, November, 1941, pp. 1164 and 1165.

In one of these final dewatering steps, investigations have disclosed the fact that the amount of solute-carrying water per pound of sludge solids is a function of a constant times the amount of volatile matter per pound of ash in the sludge—the inevitable result of conditions associated with this ratio in previous steps of the dewatering system. Therefore, the validity of the statement can be extended to: "Every step in this system is mathematically related to the volatile-ash ratio and some constant value."

The causes associated with this ratio will be found in the surface energy or polar nature of water related to polar groups and ions in the wetted solids. Water is the best solvent and ionizing agent and constitutes 99.9+% of sewage and from 90% to 98% of most sludges. Its solvent and hydration properties are due to the free surface energy of water which results from the polar character of its molecules (H – OH), "* * * because of which water acts as though it contains positive and negative electrical charges slightly separated from each other."[26] The polar nature of most solids submerged in water manifests itself by the presence of polar groups of molecules in the wetted solid surfaces (interfaces) having a strong solution tension or attraction for the polar-water molecules, as a result of which the molecules orient themselves toward the water:

> "The theory that molecules are oriented in surfaces and interfaces has a profound effect upon the development of surface chemistry, of colloid science and upon the theories of the internal structure of liquids,* * *."[26]

This theory also accounts for the surface charges of the wetted solids, for coagulation phenomena, and for all the water-imbibing properties of capillary gelatinous structures containing volatile matter and ash—namely, starches, agars, albumin, gelatin, pectins, zoogloeal masses, activated sludge floc, and all organic solids found in sewage which are subject to bacterial decomposition. Even hydrated mineral structures of a gelatinous nature that show volatile and ash contents by decompsoing at red heat owe their powers of hydration to the same polar forces—silicic acid gels and the flocculent precipitates of iron and aluminum hydroxide. Furthermore, most of the finely divided mineral solids found in sewage are attracted to water by polar groups such as silica, clays, oxides, and sulfides.

In capillary-active solid and semisolid structures common to most suspended and settleable sewage solids, the volatile and mineral ingredients are primarily responsible for the polar activities, hydration properties, and surface chemistry pertinent to the subject. In these structures the ingredients are related in some definite manner to other definite properties, such as:

(1) Specific gravity of sludge solids;
(2) Hydraulic subsiding rates or settleable solids;
(3) Capacity of sludge solids for imbibing and retaining large amounts of enmeshed water;
(4) Concentration of polar solutes in the imbibed and enmeshed water which necessarily has an influence on item (5);

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[26] "Surfaces of Solids in Science and Industry," by William Draper Harkins, *Science*, Vol. 102, September 21, 1945, pp. 293–294.

(5) The chemical coagulant requirements; and
(6) Elastic properties of solids aggregates (compressibility of solids).

Item (6) concerns deformation under filtration pressure and has been fully discussed elsewhere.[27] Like solids, aqueous gels are characterized by definite elastic properties.[28] Although the influence of volatile matter on sludge compressibility in filtration is quite pronounced and remains to be mathematically investigated, it is altered advantageously by chemical coagulation, which, in items (4) and (5), has been mathematically related to the volatile-ash ratio.

In pushing the causes associated with this ratio in items (4) and (5) back a step to item (3), one encounters some fresh evidence of a most convincing nature on the general validity of this basic ratio. This evidence primarily involves referring to another field of organic colloids in order to relate the water-carrying power of solid gels to the amounts of volatile matter and ash in such gels.

Most gelatinous or amorphous aggregates of sludge solids are similar to gels, zoogloeal and otherwise. This fact becomes quite evident in relating hydration properties to the volatile matter and ash present. C. K. Tseng,[29] in his article on seaweed colloids (phycocolloids), shows that the various types of agars, in natural conditions, are calcium, magnesium, potassium, and sodium salts of various agar acids. The gel-forming properties of the various agars, and their allied phycocolloids, are functions of the ash contents, which is equivalent to stating that they are functions of the volatile and ash contents. The dried amorphous agar which scientists use (the standard agar of the bacteriologist) is prepared from seaweed species Gelidium, has an ash content of less than 5%, and contains 95% or more volatile matter. Such agars are:

> "* * * insoluble in cold but soluble in hot water, a one per cent neutral solution of which sets at 35° to 50° C to a firm gel melting at 80° to 100° C."

In other words, the solid gel of 19 to 1 volatile-ash ratio imbibes ninety-nine times its own weight in water to set to a solid gel that has a melting point near the boiling point of water.

Another seaweed colloid is carrageenin, which resembles agar in a general way, but has a much higher ash content (about 20%, or a volatile-ash ratio of 4 to 1) and requires concentrations of 3% or more to form solid gels containing about thirty times its own weight in imbibed water. These gels also melt at lower temperatures.

In any case, the Tseng article[29] clearly demonstrates that the power to imbibe water and to form firm gels, at ordinary temperatures, by different seaweed colloids is definitely a function of the volatile and ash contents of the different types listed. These colloids are believed to be chiefly cell-wall constituents of various seaweeds, comparable to the pectins of the land plants.

These pertinent facts were emphasized because the hydration and water-imbibing properties of such gels are remarkably similar to the disclosures relative to the water-imbibing and enmeshing properties of activated sludges made by Gail P. Edwards,[30] M. Am. Soc. C. E., as will become more evident through mathematical analysis of Mr. Edwards' disclosures. His pioneer studies relating to the effects of volatile matter on solids concentration in sedimentation tanks have deserved much more attention from the research engineer and scientific investigator than has heretofore been evidenced. They have been particularly valuable to the writer in the present investigation, not only because of the ample supply of statistical evidence on the influence of volatile matter on the water content of the sludges investigated, but also because of the wide range of volatile contents in the sludges. The percentages of volatile matter in the sludges investigated at the sewage treatment plants at Wards Island and Tallmans Island in New York, N. Y., ranged from 38 to 85.5, thus the volatile-to-ash ratios varied from 0.61 to 5.9.

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[27] "Principles and Factors Influencing Vacuum Filtration of Sludge," by A. L. Genter, *Sewage Works Journal*, November, 1941, pp. 1171, 1174, and 1177 to 1179, inclusive.

[28] "Colloid and Capillary Chemistry," by Herbert Freundlich, E. P. Dutton & Co., Inc., New York, N. Y., 1922, pp. 716–719.

[29] "The Terminology of Seaweed Colloids," by C. K. Tseng, *Science*, Vol. 101, June 15, 1945, pp. 600 and 601.

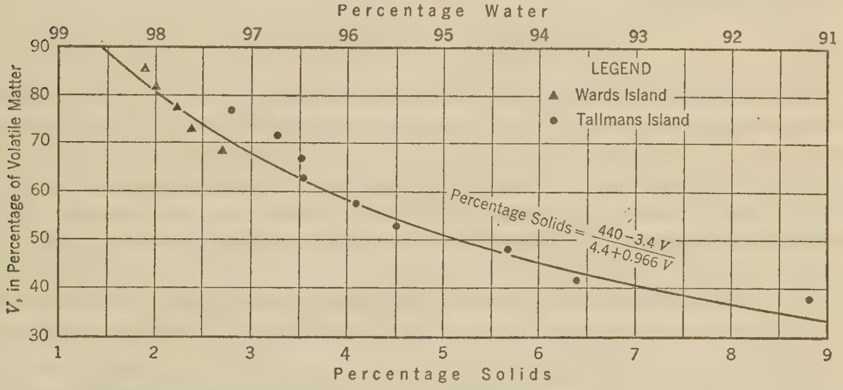

FIG. 5.—RELATION BETWEEN PERCENTAGES OF VOLATILE MATTER AND SOLIDS (ILLUSTRATING EQ. 13)

Fig. 5 illustrates the graph of the mathematical equation evolved by the writer from the original data published by Mr. Edwards.[31] Separate graphs are shown by Mr. Edwards for Wards Island and Tallmans Island sludges.[32] In Fig. 5, the full line is plotted from the writer's equation and accounts for both sludges. The various data-point averages for the two New York plants are indicated in Fig. 5 by black circles for Tallmans Island and by black triangles for Wards Island. The percentages of volatile matter are plotted as ordinate (y-axis) values, and the percentages of solids are plotted in ascending order, with the corresponding percentages of water in descending order as abscissa (x-axis) values.

Mr. Edwards' published data and graph averages for Tallmans Island indicate a curve of a hyperbolic type which (when the different y-axis and x-axis values are symbolized as V equals percentage of volatile matter, W equals percentage of water, and S equals percentage of solids in sludge) the writer evolves as:

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[30] "New Developments in Activated Sludge Plant Operation," by Gail P. Edwards, *Sewage Works Journal*, November, 1940, p. 1080.

[31] *Ibid.*, p. 1080, Tables I and II.

[32] *Ibid.*, p. 1081, Fig. 3.

$$W = \frac{100\ V}{4.4 + 0.966\ V} \quad \text{.......................(12)}$$

and

$$S = \frac{440 - 3.4\ V}{4.4 + 0.966\ V} \quad \text{.......................(13)}$$

Dividing Eq. 12 by Eq. 13 to determine the quantity of water per unit weight of solids (Eq. 1), a fact of major importance to this discussion is developed:

$$\frac{W}{S} = \frac{29.4\ V}{29.4 + 100 - V} \quad \text{.......................(14)}$$

Because the value 100 − V, as formerly stated in developing Eq. 4*b*, is nothing but the percentage of ash in the sludge, Eq. 14 clearly states that in these particular sludges:

$$\frac{\text{Water (\%)}}{\text{Solids (\%)}} = K' \frac{\text{Volatile (\%)}}{K' + \text{Ash (\%)}} \quad \text{................(15)}$$

Eq. 15 is undoubtedly the general law expressing the amount of evaporable water per unit weight of dry solids in sludges containing volatile matter. It establishes the measurable influence of the volatile-ash ratio and some constant value on the water-carrying power per unit weight of sludge solids in the first stage of solids dewatering in sewage treatment (that is, in sedimentation) and confirms the validity of this influence in sludge dewatering by vacuum filtration.

Eq. 15 also clarifies the subject of the capacity of sludges to imbibe water. In the presence of the total water included in this equation, the ratio of volatile percentage to a constant plus the ash percentage is somewhat greater than the volatile-to-ash ratio itself. Since the ratios in the equation were determined from the sedimentation of wetted solids and include enmeshed water freely associated with the settled solid flocs, this additional constant amount indicates a water-imbibing or enmeshing factor (see sludge property item (3)). This factor will vary greatly with the polar or water-binding properties of the sludge solid structures—it will be highest with the water-enclosing, gelatinous structures of activated sludge floc, before and after digestion (also with other hydrated gel structures) and lowest with denser and less hydrated sludge solids. Processing suspended solids through activation and primary solids through digestion increases their value.

The average of several of the most prominent treatment plants shows K'-values of about 7 for primary raw sludges, 16 for digested primary, and 27 for waste activated and digested waste activated in plants having primary sedimentation with sludge activation. The actual observed K'-values for Tallmans Island average 26.2; and, for Wards Island, 31.5. The average values at the Southerly and Easterly plants at Cleveland, Ohio, for concentrated waste activated sludge fed to digestion and after digestion agree fairly well with the Tallmans Island values.

As the K'-value includes enmeshed water freely associated with the flocculent or hydrated structures of various sludge solids, the value will obviously vary somewhat with the detention time in sedimentation and with sedimentation tank design. Without standards for these factors, the K'-value is best determined by evaporation. In any case it indicates the existence of water-imbibing and enmeshing properties resulting from the sludge-solid structure associated with the amount of volatile matter per unit weight of mineral matter present.

It must be noted that Eqs. 5 and 6 really express the sum of two conveniently separated coagulant-consuming reactions occurring in the same total amount of water per unit weight of solids as shown by Eq. 15. The quantity $\frac{K' V}{K' + 100 - V}$ in Eq. 15 could be used to represent the water-solids ratio in Eq. 6 and would then be related to the quantity $K \frac{V}{100 - V}$ of the solids demand in Eq. 6. When multiplied by the ferric chloride constant 1.08, the constant K' in Eq. 15 would be some multiple ($n\ K$) of the basic K in the solids demand. Notwithstanding the magnitude assigned to the values K and $n\ K$ by practice and theory, the volatile-to-ash ratio in the sludge remains the essential factor in determining, in terms of sludge solids, the total water containing the solutes reacting with the added coagulants in free solution and in the wetted solid surfaces.

There is ample theoretical background for the foregoing disclosures provided by the buoyant effect of the lighter volatile fraction on the specific gravity of the sludge solids. The general statement found in the literature that the volume of a sludge (that is, the ratio of water to solids) depends mainly on water content and only slightly on the character of the solid matter can be quite misleading. The simple truth lies in reversing this statement; thus:

> "It is the capillary and polar characters of the wetted sludge particles and gelatinous aggregates of the sludge submerged in water that determine both the specific gravity of the sludge solids and the water content of a sludge."

Sufficient evidence has been supplied to demonstrate that the amount of water per unit weight of solids in a sludge is a function of the amount of volatile matter per unit weight of ash.

The influence of this important ratio on the specific gravity of sludge solids and their consequent gain or loss in hydraulic subsiding values are of prime importance to the general subjects of sedimentation and further sludge dewatering.

Among the factors controlling free settling rates of solids in water—particle size and structure and the frictional or viscous resistance of water to the falling solid—the gravitational pull on the solid is of prime importance. This is measured in terms of the gravity constant times the difference in the specific gravities of the solid and water. In centimeter-gram units, the difference indicates the apparent weight of the submerged solid. The apparent weight loss in water occurs because of the buoyant force of the water on the solid; and, in the plain sedimentation of sewage solids, the apparent weight loss depends on the volatile-ash ratio. This loss increases as the specific gravity of the sludge solids (G_s) decreases. Since the specific gravity of the volatile matter (G_v) is so near that of water, it has a material influence on this loss or on the resulting buoyant force of the water on the sludge solids.

The weight loss of sludge solids in water is the sum of the losses of the volatile and mineral fractions or percentages in the solids—namely, with 100 grams of solids in water the sum of the cubic centimeters of water displaced and the apparent total weight loss in grams is $\frac{V}{G_v} + \frac{100 - V}{G_m}$, in which V is the percentage or grams of volatile matter; $100 - V$, the percentage or grams of ash present; and G_v and G_m, the respective specific gravities of the volatile and mineral matter.

In this sum of apparent losses that of the lighter volatile part is always $\frac{G_m V}{G_v (100 - V)}$ times greater than that of the mineral part; that is: $\frac{V}{G_v} \div \frac{100 - V}{G_m} = \frac{G_m V}{G_v (100 - V)}$. As the average values for G_m and G_v will be fairly constant in most plants, this ratio (R) of the weight loss of the volatile part to that of the mineral part of sludge solids may be expressed as:

$$R = K_2 \frac{\text{Volatile (\%)}}{\text{Ash (\%)}} \qquad \text{(16)}$$

in which K_2 is the ratio of the specific gravity of the mineral part of the solids to that of the volatile part.

With this constant in mind Eq. 16 simply measures the gain or loss in the buoyant power of water on the submerged sludge solids through the increase or decrease in the volatile-ash ratio of the sludge.

The ratio R of the buoyant force of water on the volatile part to that of the mineral part of a sludge is a straight-line function of the specific gravity constant K_2 times the volatile-ash ratio of the sludge. For example, with $G_m = 2.5$ and $G_v = 1$, K_2 equals 2.5 and the buoyant power of water on the volatile part of a sludge averaging 50% volatile matter (volatile-ash ratio 1 : 1) is 2.5 times greater than that on the mineral part. The buoyancy ratio doubles in a sludge having a 2 : 1 volatile-ash ratio (66.7% volatile : 33.3% ash) and quadruples in a sludge having a 4 : 1 volatile-ash ratio (80% volatile : 20% ash).

The buoyancy ratio therefore is one of the fundamental causes of the gain or loss in hydraulic subsiding rates of settleable solids and of the consequent gain or loss in the quantity of water with its solute per unit weight of sludge solids in the progressive dewatering steps involving Eqs. 5, 6, and 15.

Adding the imbibed and adsorbed water, previously associated with the polar nature of the volatile solids, will change the effective displacement volume of the volatile and mineral parts of a sludge and the buoyancy ratios expressed by Eq. 16. Thus, both the free and impeded settling rates and the final water content of the sludge will be changed. The final effect has already been accounted for in Eqs. 13, 14, and 15 and the discussion concerning them.

The plot of percentages of volatile matter against the specific gravity of sludge solids (G_s) determined by the usual equation $G_s = \frac{100}{\frac{V}{G_v} + \frac{100 - V}{G_m}}$ results in a graph similar to Fig. 5 for the same volatile and solids range. However, the latter experimental graph shows the buoyant effects of imbibed and enclosed water in the settled sludge floc. Furthermore, experimental Eq. 13, when extended to a sludge containing solids of a hypothetical 100% volatile content, shows 1% solids and 99% water content. Although such an extension may lead to erroneous conclusions, the graphed values taken within the experimental range do indicate that at Tallmans Island the specific gravity of the volatile matter is slightly greater than that of water—probably 1.02 or 1.04—which denotes that some hydrated mineral matter later appearing in the ash is probably a part of the wetted volatile structure.

Although the analytical determination of volatile matter and ash in sewage treatment is not as scientifically exact as chemical determination of solutes in the sludge water, it is quite as exact and reliable as the current method of determining the amount of water and dry solids by evaporation. The measurements involving weighted average percentages of moisture and solids, volatile matter, and ash in all the dewatering steps are sufficiently accurate and informative when computed to the first decimal place. Therefore, further precision is not warranted in measuring weighted average percentages of solute and coagulant on solids that have been determined by evaporation. With this in mind, measurements of amounts of water per unit weight of sludge solids and of volatile matter per unit weight of ash are just as reliable yardsticks in Eqs. 5, 6, 15, and 16 as is the law of combining weights in Eqs. 5 and 6; and they have quite a reliable scientific background.

In these equations and those from which they are derived, the separation of a sludge into volatile and ash components is analytically convenient in relating and measuring the various properties of polar water and wetted polar volatile solids to successive solids dewatering steps. As noted before in discussing Eq. 15, the separation of sludge requirements for coagulating chemicals into an alkalinity demand, and added solids demand, is also expedient. The solids demand is altered by digestion and the alkalinity demand may be altered by elutriation previous to chemical conditioning. Nevertheless, both reactions involve the same solute ions in the same sludge water. In addition to the alkalinity reactions, there are the rather widely known oxidation-reduction reactions associated with the use of ferric chloride, which are conveniently associated with the carbon compounds, sulfides, etc., in the wetted solids demand. Probably other minor reactions are associated with this demand. However, because of the presence of calcium and magnesium in sludges, it is doubtful whether borates, silicates, and phosphates will be found in sufficient quantities as true solute ions to affect the coagulant consumption in average sludges significantly. The chief polar ions involved in the surface chemistry of coagulation will be the various bicarbonates.

Aside from the polar compounds found in the chemically and biochemically fouled water associated with wetted sludge solids, the surface chemistry of the polar solids holding this water and its solutes is active in coagulation. As described elsewhere,[33] the process of coagulating water-logged sludge solids is similar, in principle, to the curdling of milk and depends on the chemical transformation of polar groups in the water-imbibing structures, with the result that these structures clump into solid aggregates having far less attraction for water.

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[33] "Principles and Factors Influencing Vacuum Filtration of Sludge," by A. L. Genter, *Sewage Works Journal*, November, 1941, p. 1179.

Such clumping may also be termed a coarse flocculation process, but the rate of obtaining properly clumped solid aggregates in coagulating sewage sludges has only a limited likeness to the rate of flocculation common to the colloid chemical practice of coagulating rather pure sols dispersed in relatively pure water. The writer, therefore, doubts if the clumping rate in current vacuum filtration practice has any measurable effect on ferric chloride consumption. He does agree, however, that both the method of mixing coagulant with a sludge, and the duration of the mixing, are of material importance to coagulant economy. This has been emphasized elsewhere,[8,34] including the last paragraph of the "Summary."

In any case, the research involved in this paper and in its discussions should leave little doubt that the coagulant requirements of various sludges are functions of the solutes in the sludge water; and, what is most important, the amount of this water per unit weight of sludge solids is a function of the amount of volatile matter per unit weight of ash in the sludge. In presenting the foregoing investigations of the causes and consequences of this relationship from the initial stages of solids concentration in plain sedimentation to the final stages of the disposal of the solids, it is felt that the amount of evidence has left the problem less baffling.

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[34] "Principles and Factors Influencing Vacuum Filtration of Sludge," by A. L. Genter, *Sewage Works Journal*, November, 1941, pp. 1187, 1188, and 1189.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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Paper No. 2281

DETERMINATION OF PRESSURES WITHIN A SIPHON SPILLWAY

By I. M. Nelidov,[1] M. Am. Soc. C. E.

With Discussion by Messrs. A. Shukry, and I. M. Nelidov.

Synopsis

A correct determination of the internal pressures in closed conduits is an important factor in their design. In ordinary pipes with a straight alinement the computation of internal pressures is rather simple. It is limited to determination of the height of the piezometric line above the center line of the pipe after which a multiplication of this piezometer height by the unit weight of water gives the average value of the internal pressure within the pipe. This pressure is normally called "positive"—that is, in regard to the pressures exerted by air it is greater than the latter and their difference is positive. However, there are cases in which pressures become "negative," or smaller than the pressure exerted by air. Such cases occur (*a*) in closed conduits when the vertical alinement is above the piezometric line, (*b*) in conduits behind valves or gates; and (*c*) in siphons in which the effect of the curvature of the flow filaments is revealed by a distribution of internal pressures different from hydrostatic pressures. In the siphons the pressures may become either positive or negative. Expressions for the pressures within a siphon are derived in this paper.

Introduction

In designing a true siphon the strength of the walls should be based on pressures within the siphon, because they may become smaller or greater than atmospheric pressure. The unbalanced pressures resulting from the curvilinear flow will produce forces which will be especially noticeable at the summit and at the outlet, provided the latter has the usual reverse curves.

Notation.—The letter symbols used in this paper are defined where they first appear in the paper, and are assembled for reference in the Appendix.

Note.—Published in March, 1945, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

[1] With U. S. Engr. Dept., Sacramento, Calif.

THEORY

The theory upon which the derivations were based is that of the free vortex or irrotational flow with a velocity potential.[2] With this assumption the energy content E for each filament is the same, independent of the location of the filament within the flow. By inspection in Fig. 1, the atmospheric vacuum head h_a is equal to

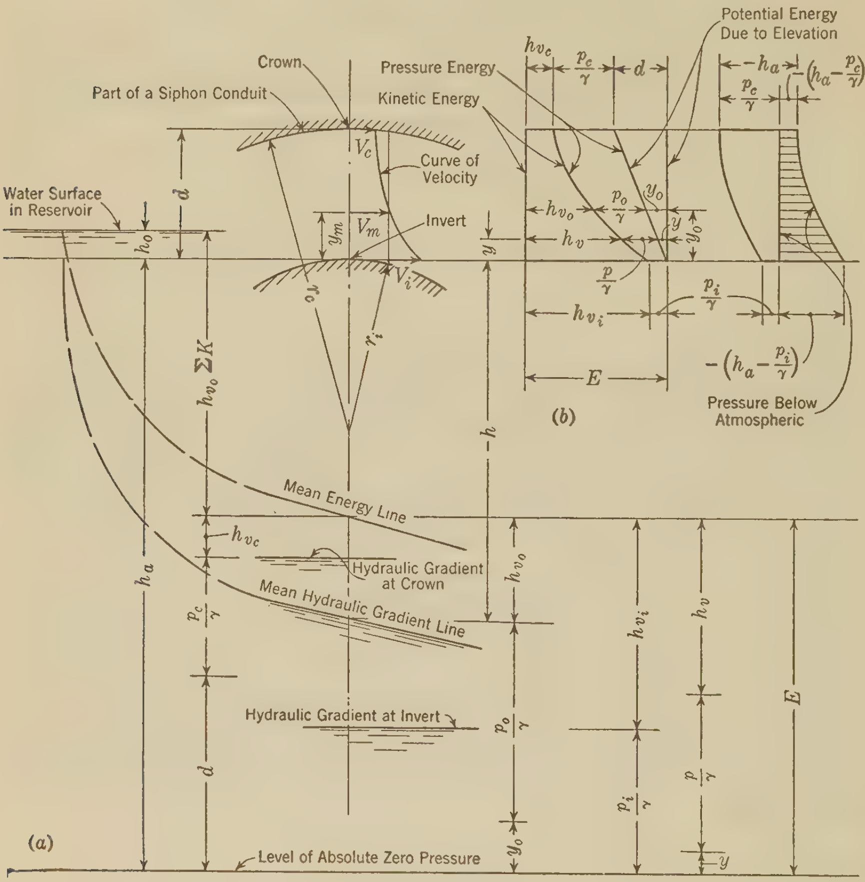
Potential Energy Due to Elevation
Crown
Pressure Energy
Kinetic Energy
Part of a Siphon Conduit
h_{v_c}
$\frac{p_c}{\gamma}$
d
h_a
$(h_a-\frac{p_c}{\gamma})$
V_c
Curve of Velocity
Water Surface in Reservoir
h_o
y_m
V_m
Invert
V_i
r_c
r_i
h_{v_o}
$\frac{p_o}{\gamma}$
y_o
y
h_v
$\frac{p}{\gamma}$
$\frac{p_i}{\gamma}$
h_{v_i}
$-(h_a-\frac{p_i}{\gamma})$
E
(b)
Pressure Below Atmospheric
$h_{v_o}\Sigma K$
$-h$
Mean Energy Line
h_{v_c}
Hydraulic Gradient at Crown
Mean Hydraulic Gradient Line
$\frac{p_c}{\gamma}$
Hydraulic Gradient at Invert
$\frac{p_i}{\gamma}$
(a)
Level of Absolute Zero Pressure

FIG. 1.

$$h_a = -h + y + \frac{p}{\gamma} + h_v - h_{vo} \ldots\ldots\ldots\ldots\ldots\ldots (1a)$$

or

$$\frac{p}{\gamma} = h_a + h - y - h_v + h_{vo} \ldots\ldots\ldots\ldots\ldots\ldots (1b)$$

in which h = the distance from the invert to the mean water gradient (positive when the mean water gradient is above the invert); y = the height above the invert to a filament where a value of pressure is to be determined; p = pressure intensity at the filament, distant y above the invert; h_v = velocity head at the filament distant y above the invert; h_{vo} = mean velocity head corresponding to pressure energy $\frac{p_o}{\gamma}$ and distance above the invert y_o; and γ = the unit weight of water = 62.5 lb per cu ft.

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[2] Discussion by B. A. Bakhmeteff of "Tests of Broad-Crested Weirs," by J. G. Woodburn, *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 430.

Likewise, by inspection in Fig. 1,

$$\frac{p'}{\gamma} = h_{vi} + \frac{p_i}{\gamma} - h_v - y \qquad (2)$$

in which h_{vi} = velocity head at the filament along the invert. Combining Eqs. 1b and 2:

$$\frac{p_i}{\gamma} = h_a + h - h_{vi} + h_{vo} \qquad (3)$$

Once $\frac{p_i}{\gamma}$ is found, $\frac{p_c}{\gamma}$, $\frac{p_o}{\gamma}$, $\frac{p_i}{\gamma}$ and $\frac{p}{\gamma}$ can be determined from the following expression (see Fig. 1):

$$h_{vc} + \frac{p_c}{\gamma} + d = h_{vo} + \frac{p_o}{\gamma} + y_o = h_v + \frac{p}{\gamma} + y = h_{vi} + \frac{p_i}{\gamma} = E \qquad (4)$$

from which

$$\frac{p_c}{\gamma} = E - h_{vc} - d \qquad (5a)$$

$$\frac{p_o}{\gamma} = E - h_{vo} - y_o \qquad (5b)$$

$$\frac{p_i}{\gamma} = E - h_{vi} \qquad (5c)$$

and

$$\frac{p}{\gamma} = E - h_v - y \qquad (5d)$$

In Eqs. 3, 4, and 5, p_c (at the crown), p_o (at y_o), and p (at any point y) are absolute pressures; that is, the atmospheric pressure has been added in each case. The pressures relative to the atmospheric pressure p_r can be obtained by deducting h_a from the absolute pressures p_a as shown in Fig. 1(b), or:

$$p_r = p_a - h_a\gamma \qquad (6)$$

These relative pressures (p_r) are the ones to be used in structural computations. (In this paper, a minus sign indicates that the resulting pressure is directed toward the inside of the conduit, and a plus sign means the reverse.)

For a complete solution it is necessary to determine h_a, h, h_{vi}, h_{vc} (velocity head at the crown), h_{vo}, and the dimensions of the siphon cross sections—such as the width b, the depth or height d of a cross section normal to the direction of flow, and one of the radii. Let h_a = 24 ft to 28 ft as reduced from its theoretical value of 33.9 ft due to vapor pressure. Other values of head —h, h_{vi}, h_{vc}, and h_{vo}—are determined from flow computations.

Assume that the mean velocity V_m (equal to the velocity at distance y_m

above the invert) and the cross section area A are given. First, the flow will be referred to mean hydraulic gradient and mean energy gradient, thus:

$$h = h_o - (1 + \sum K)\, h_{vo} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

in which h_o = the head necessary for priming the siphon or in general the head above the invert of the summit of a siphon; and K = the coefficient of loss from the inlet to the point under consideration. Usually the value of h or the position of the mean hydraulic gradient is fixed by the conditions at the outlet, whether free or submerged, and with the discharge defined by these conditions the mean velocity V_m can be found. Then the mean velocity head entering Eq. 7 must be expressed in terms of the mean velocity.

A coefficient $n = \dfrac{h_{vo}}{h_{vm}}$, representing the ratio of mean velocity head to that due to mean velocity, can be determined from the expression:

$$h_{vo} = n\, h_{vm} = \frac{\displaystyle\int_0^d \frac{V^2}{2\,g}\, dq}{\displaystyle\int_0^d V\, dy} = \frac{\displaystyle\int_0^d \frac{V^2}{2\,g}\, V\, dy}{\displaystyle\int_0^d V\, dy} = \frac{\displaystyle\int_0^d \frac{V^3}{2\,g}\, dy}{\displaystyle\int_0^d V\, dy} \ldots\ldots\ldots (8)$$

in which h_{vm} = the velocity head due to a mean velocity V_m; V = the velocity at any filament distant y from the invert; g = the gravity constant; and q = the discharge per unit width. For a flow convex upward, the velocity at any filament y is:

$$V = V_c \frac{r_i + d}{r_i + y} = V_c \frac{r_c}{r_i + y} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

in which V_c = the velocity at the filament at the crown; r_i = the radius of the siphon at the invert (Fig. 1); and r_c = the radius of the siphon at the crown. Substituting Eq. 9 in Eq. 8 and writing the solution for n:

$$n = \frac{h_{vo}}{h_{vm}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10a)$$

that is:

$$n = \frac{\displaystyle\int_0^d \frac{V^3_c}{2\,g}\left(\frac{r_c}{r_i + y}\right)^3 dy}{\displaystyle\frac{V^2_m}{2\,g}\int_0^d V_c\left(\frac{r_c}{r_i + y}\right) dy} = \left(\frac{V_c}{V_m}\right)^2 \frac{\displaystyle\int_0^d \left(\frac{r_c}{r_i + y}\right)^3 dy}{\displaystyle\int_0^d \left(\frac{r_c}{r_i + y}\right) dy} \ldots (10b)$$

Let $\dfrac{V_c}{V_m} = \beta$ and $\dfrac{r_c}{r_i} = m > 1$; then Eq. 10b becomes

$$n = \beta^2\, r^2_c \frac{\displaystyle\int_0^d \frac{dy}{(r_i + y)^3}}{\displaystyle\int_0^d \frac{dy}{r_i + y}} = \beta^2\, r^2_c \frac{-\,0.5\,(r_i + y)^{-2}\Big|_0^d}{\log_e (r_i + y)\Big|_0^d}$$

$$= -\frac{\beta^2\, r_c^2}{2}\, \frac{r^{-2}_c - r^{-2}_i}{\log_e r_c - \log_e r_i} = \frac{\beta^2\,(m^2 - 1)}{2 \log_e m} \ldots\ldots\ldots\ldots (11)$$

For a flow concave upward, the velocity at any filament distant y above the invert is:

$$V = V_e \frac{r_i - d}{r_i - y} = V_e \frac{r_e}{r_i - y} \qquad (12)$$

Substituting Eq. 12 in Eq. 8 and again writing the solution for n:

$$n = \frac{h_{vo}}{h_{vm}} = \frac{\int_0^d \frac{V^3_e}{2\,g}\left(\frac{r_e}{r_i - y}\right)^3 dy}{\frac{V^2_m}{2\,g}\int_0^d V_e \frac{r_e}{r_i - y}\,dy} = \left(\frac{V_e}{V_m}\right)^2 \frac{\int_0^d \left(\frac{r_e}{r_i - y}\right)^3 dy}{\int_0^d \frac{r_e}{r_i - y}\,dy}$$

$$= \beta^2\, r^2_e \frac{\int_0^d \frac{dy}{(r_i - y)^3}}{\int_0^d \frac{dy}{r_i - y}} \qquad (13)$$

As with Eq. 10, when Eq. 13 is written in terms of β and m, the algebraic result is identical with Eq. 11. In applying this formula, however, a different numerical result is obtained for the case of flow convex upward (in which m is greater than unity) than for the case of flow concave upward (in which m is less than unity). Now to determine the expression for β: Since

$$V_m = \frac{q}{d} = \frac{\int V\,dy}{d} \qquad (14)$$

the value of $\beta = \frac{V_e}{V_m}$, for the case of flow convex upward, is determined from the following expression:

$$\beta = \frac{V_e}{V_m} = \frac{V_e\,d}{\int_0^d V\,dy} = \frac{V_e\,d}{\int_0^d V_e \frac{r_i + d}{r_i + y}\,dy}$$

$$= \frac{d}{r_e}\,\frac{1}{\log_e r_e - \log_e r_i} = \frac{d}{r_e \log_e \frac{r_e}{r_i}} = \frac{1}{\frac{r_e}{d}\log_e m} \qquad (15)$$

in which $m > 1$. For the case of flow concave upward it is:

$$\beta = \frac{V_e\,d}{\int_0^d V_e \frac{r_i - d}{r_i - y}\,dy} = -\,\frac{d}{(r_i - d)\log_e (r_i - y)\Big|_0^d}$$

$$= \frac{-\,d}{r_e \log_e \frac{r_i}{r_e}} = \frac{1}{\frac{r_e}{d}\log_e \frac{1}{m}} \qquad (16)$$

in which $m < 1$. Eqs. 15 and 16 are not identical, and they do not render identical numerical results.

The relationships between the various velocity heads are as follows. By definition (see Eq. 10a) $h_{vo} = n\, h_{vm}$. The corresponding relationship between h_{vc} and h_{vm} is defined as:

$$h_{vm} = \frac{V^2{}_m}{2\,g} \times \frac{V^2{}_c}{V^2{}_c} = \frac{V^2{}_c}{2\,g}\left(\frac{V_m}{V_c}\right)^2 = \frac{1}{\beta^2}\, h_{vc} \ldots\ldots\ldots\ldots \tag{17a}$$

Likewise,

$$h_{vc} = \frac{V^2{}_c}{2\,g} \times \frac{V^2{}_m}{V^2{}_m} = \frac{V^2{}_m}{2\,g}\left(\frac{V_c}{V_m}\right)^2 = \beta^2\, h_{vm} \ldots\ldots\ldots\ldots \tag{17b}$$

The relationship between h_{vi} and h_{vm} is defined as follows, since $\dfrac{V_i}{V_c} = \dfrac{r_c}{r_i} = m$,

$$h_{vi} = \frac{V^2{}_i}{2\,g} \times \frac{V^2{}_c}{V^2{}_c} = \frac{V^2{}_c}{2\,g}\left(\frac{V_i}{V_c}\right)^2 = h_{vc}\, m^2 = \beta^2\, m^2\, h_{vm} \ldots\ldots\ldots \tag{18}$$

The relationship between h_v, h_{vc}, h_{vm}, and h_{vi} for the flow convex upward is as follows: $\dfrac{V}{V_c} = \dfrac{r_c}{r_i + y}$ or

$$h_v = h_{vc}\left(\frac{r_c}{r_i + y}\right)^2 = \beta^2\, h_{vm}\left(\frac{r_c}{r_i + y}\right)^2 = \frac{h_{vi}}{\left(1 + \dfrac{y}{r_i}\right)^2} \ldots\ldots \tag{19a}$$

In the same way, for the case of flow concave upward:

$$h_v = h_{vc}\left(\frac{r_c}{r_i - y}\right)^2 = \beta^2\, h_{vm}\left(\frac{r_c}{r_i - y}\right)^2 = \frac{h_{vi}}{\left(1 - \dfrac{y}{r_i}\right)^2} \ldots\ldots \tag{19b}$$

The position of the filament at which the mean velocity head h_{vo} occurs is determined as follows: For the flow convex upward, since $h_{vo} = n\, h_{vm}$ (see Eq. 10a),

$$h_{vo} = n\, h_{vm} = h_{vc}\left(\frac{r_i + d}{r_i + y_o}\right)^2 \ldots\ldots\ldots\ldots\ldots\ldots \tag{20a}$$

or, since $h_{vc} = \beta^2\, h_{vm}$,

$$n = \frac{\beta^2\, r^2{}_c}{(r_i + y_o)^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \tag{20b}$$

Therefore,

$$y_o = \frac{\beta\, r_c}{\sqrt{n}} - r_i = r_i\left(m\sqrt{\frac{2\log_e m}{m^2 - 1}} - 1\right) \ldots\ldots\ldots\ldots \tag{21}$$

For the flow concave upward:

$$h_{vo} = n\, h_{vm} = h_{vi}\,\frac{r^2{}_c}{(r_i - y_o)^2} \ldots\ldots\ldots\ldots\ldots\ldots \tag{22a}$$

or

$$n = \beta^2\left(\frac{r_i - d}{r_i - y_o}\right)^2 = \frac{\beta^2\, r^2{}_c}{(r_i - y_o)^2} \ldots\ldots\ldots\ldots\ldots \tag{22b}$$

and

$$y_o = -\left(\frac{r_c}{\sqrt{n}} - r_i\right) = r_i\left(1 - m\sqrt{\frac{2\log_e m}{m^2 - 1}}\right) \ldots\ldots\ldots\ldots \tag{23}$$

Example 1. Pressures at the Crown and Invert of the Summit Section (Convex Upward).—Consider a siphon for which the general conditions are as listed in Table 1(*a*). As shown in Table 1(*b*), other basic quantities are: $r_c = 20$ ft

TABLE 1.—COMPUTATIONS FOR PRESSURES AT THE CROWN AND INVERT OF A SIPHON

(*a*) GENERAL DATA

$Q = 100$ cu ft per sec, $A = 10$ sq ft, $b = 2.5$ ft, $d = 4$ ft, $r_i = 16$ ft, $q = 100/2.5 = 40$ cu ft per sec, $V_m = 40/4 = 10$ ft per sec, $h_{vm} = 1.55$ ft, and $h_a = 24$ ft

Equation	Symbol	Computation	Quantity	Unit
(*b*) EXAMPLE 1.—SUMMIT SECTION (CONVEX UPWARD, $h = -10$ FT)				
. . .	r_c	$16 + 4$	20	Ft
. . .	m	$r_c/r_i = 20/16$	1.25	. . .
15	β	$1/(5 \times 0.223)$	0.90	. . .
17	h_{vc}	$0.90^2 \times 1.55$	1.26	Ft
18	h_{vi}	$1.25^2 \times 1.26$	1.96	Ft
11	n	$\dfrac{0.90^2\,(1.25^2 - 1)}{2\,(\log_e 1.25)}$	1.025	. . .
10*a*	h_{vo}	1.025×1.55	1.59	Ft
3	p_i/γ	$24 - 10 - 1.96 + 1.59$	13.63	Ft
. . .	p_i	$13.63 \times 62.5 \times 1/144$	5.93	Lb per sq in.
5*c*	E	$1.96 + 13.63$	15.59	Ft
5*a*	p_c/γ	$15.59 - 1.26 - 4$	10.33	Ft
. . .	p_c	$10.33 \times 62.5 \times 1/144$	4.48	Lb per sq in.
21	y_o	$16\left(1.25\sqrt{\dfrac{2 \log_e 1.25}{1.25^2 - 1}} - 1\right)$	1.81	Ft
5*b*	p_o/γ	$15.59 - 1.59 - 1.81$	12.19	Ft
. . .	p_o	$12.19 \times 62.5 \times 1/144$	5.33	Lb per sq in.
. . .	p_a	$24 \times 62.5 \times 1/144$	10.42	Lb per sq in.
(*c*) EXAMPLE 2.—SECTION NEAR THE OUTLET (CONCAVE UPWARD, $h = +10$ FT)				
. . .	r_c	$16 - 4$	12	Ft
. . .	m	$r_c/r_i = 12/16$	0.75	. . .
16	β	$1/(3 \times 0.288)$	1.15	. . .
17	h_{vc}	$1.15^2 \times 1.55$	2.06	Ft
18	h_{vi}	$0.75^2 \times 2.06$	1.16	Ft
11	n	$\dfrac{1.15^2\,(0.75^2 - 1)}{2\,(\log_e 0.75)}$	1.01	. . .
10*a*	h_{vo}	1.01×1.55	1.58	Ft
3	p_i/γ	$24 + 10 - 1.16 + 1.58$	34.42	Ft
. . .	p_i	$34.42 \times 62.5 \times 1/144$	14.90	Lb per sq in.
5*c*	E	$1.16 + 34.42$	35.58	Ft
5*a*	p_c/γ	$35.58 - 2.06 - 4$	29.52	Ft
. . .	p_c	$29.52 \times 62.5 \times 1/144$	12.78	Lb per sq in.
23	y_o	$16\left(1 - 0.75\sqrt{\dfrac{2 \log_e 0.75}{0.75^2 - 1}}\right)$	2.19	Ft
5*b*	p_o/γ	$35.58 - 1.58 - 2.19$	31.81	Ft
. . .	p_o	$31.81 \times 62.5 \times 1/144$	13.78	Lb per sq in.
. . .	p_a	$24 \times 62.5 \times 1/144$	10.42	Lb per sq in.

and $m = 1.25$. The remaining steps of the solution follow in the sequence of steps in Table 1(*b*).

By Eq. 6 and Table 1(*b*) the pressures relative to atmospheric pressure, in pounds per square inch, are:

$$p_{cr} = 4.48 - 10.42 = -5.94$$
$$p_{or} = 5.33 - 10.42 = -5.09$$
$$p_{ir} = 5.93 - 10.42 = -4.49$$

The direction of action of these pressures is inward—that is, toward the inside of the conduit. The diagram of distribution of the absolute and relative pressures can be obtained from Eqs. 5*d* and 19*a*, thus:

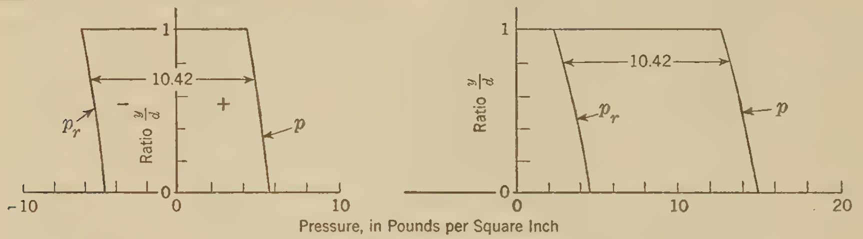
$$p = (E - h_v - y)\,\gamma = \frac{\gamma}{144}\left[E - \frac{h_{vi}}{\left(1 + \frac{y}{r_i}\right)^2} - y\right] \quad \text{........(24)}$$

or, substituting appropriate values,

$$p = \frac{62.5}{144}\left[15.59 - \frac{1.96}{\left(1 + \frac{y}{16}\right)^2} - y\right] = 1.74\left[3.90 - \frac{0.49}{\left(1 + \frac{y}{4d}\right)^2} - \frac{y}{d}\right] . \text{(25)}$$

in which $\frac{y}{d} = \frac{y}{4}$. The values of p are given in Table 2(*a*).

TABLE 2.—PRESSURES IN A SIPHON, IN POUNDS PER SQUARE INCH; ILLUSTRATIVE EXAMPLES



(*a*) EXAMPLE 1 (*b*) EXAMPLE 2

Item	Pressure	Ratios y/d					
		1.00	0.80	0.60	0.40	0.20	0.00
(*a*) Example 1.—Flow Through the Summit; Convex Upward							
1	p	4.48	4.80	5.10	5.40	5.66	5.93
2	p_r	−5.94	−5.62	−5.32	−5.02	−4.76	−4.49
(*b*) Example 2.—Flow Through the Section Near the Outlet; Concave Upward							
3	p	12.78	13.30	13.75	14.17	14.60	14.90
4	p_r	2.36	2.88	3.33	3.75	4.18	4.48

Example 2. Pressures at the Crown and Invert of a Siphon Section Near the Outlet.—The basic quantities, Q, A, b, k, r_i, q, V_m, h_m, and h_a, are the same as in Example 1. Other basic quantities (see Table 1(*c*)) are: $r_c = 12$ ft and $m = 0.75$. The remaining steps of the solution are listed in Table 1(*c*). By Eq. 6 the pressures relative to atmospheric pressure, in pounds per square inch, are:

$$p_{cr} = 12.78 - 10.42 = 2.36$$
$$p_{or} = 13.78 - 10.42 = 3.36$$
$$p_{ir} = 14.90 - 10.42 = 4.48$$

The direction of these pressures is outward—that is, toward the outside of the conduit. Similar to Example 1, the diagram of absolute and relative pressures is obtained from Eqs. 5*d* and 19*b*; thus (compare Eq. 24):

$$p = (E - h_v - y) = \frac{\gamma}{144}\left[E - \frac{h_{vi}}{\left(1 - \frac{y}{r_i}\right)^2} - y\right] \ldots\ldots\ldots (26)$$

Again, substituting appropriate values,

$$p = \frac{62.5}{144}\left[35.58 - \frac{1.16}{\left(1 - \frac{y}{16}\right)^2} - y\right] = 1.74\left[8.90 - \frac{0.29}{\left(1 - \frac{y}{4d}\right)^2} - \frac{y}{d}\right] . (27)$$

in which $\frac{y}{4} = \frac{y}{d}$. The values of p for Example 2 are given in Table 2(*b*).

SUMMARY

The derivations in this paper, which are based on the formulas for an idealized vortex flow in a siphon, and the subsequent numerical examples, illustrate that the pressures both at the summit and at the outlet of a siphon become unbalanced relative to the atmospheric pressures. Therefore, structural design should provide for these unbalanced pressures with due amount of steel reinforcement in the concrete if the siphon is a reinforced concrete structure or with an additional quantity of steel if it is of steel. Actual failures of siphons indicate the necessity for the provisions recommended.

The unbalanced pressures in the examples cited herein are about 5 lb per sq in. maximum, which is about 0.4 ton per sq ft or about 720 lb per sq ft. If it is remembered that a live load created by a crowd of people is usually assumed to be 100 lb per sq ft and in warehouses as much as 400 lb per sq ft, then it will be realized what magnitude of pressure is imposed on siphons and why some of them fail. It is therefore considered that a siphon (or as a matter of fact any closed conduit) with a curvilinear flow should be designed for the unbalanced pressures resulting from such flow.

APPENDIX

NOTATION

The following letter symbols, used in the paper, conform essentially to American Standard Letter Symbols for Hydraulics (ASA—Z10.2—1942), prepared by a Committee of the American Standards Association, with Society representation, and approved by the Association in 1942.

Without subscripts, the symbols refer to the summit, outlet, or anywhere along the line of the siphon. Subscripts c and i, respectively denote the crown point and invert at any section.

A = area of siphon cross section;
b = width of the siphon;
c = subscript denoting "crown";
d = height of the cross section under consideration, normal to the direction of flow;
E = energy content as shown in Fig. 1 (see Eq. 4);
g = gravity constant;
h = head = distance from the invert to the mean water gradient (positive when the mean water gradient is above the invert);
h_a = atmospheric vacuum head;
$h_{\blacksquare}$ = distance from the invert at the summit of a siphon to the water level in the reservoir;
h_v = velocity head at a filament distant y from the invert:
  h_{vc} = velocity head at the crown;
  h_{vi} = velocity head at the invert;
  h_{vm} = velocity head due to mean velocity;
  h_{vo} = mean velocity head;
i = subscript denoting "invert";
K = coefficient of energy loss from the inlet to the section under consideration;
m = ratio of the radius at the crown r_c to the radius at the invert r_i;
n = ratio of the mean velocity head h_{vo} to the velocity head h_{vm} created by the mean velocity V_m;
p = pressure intensity at a filament distant y above the invert:
  p_a = absolute pressure intensity;
  p_c = pressure at the crown;
  p_i = pressure at the invert;
  p_o = pressure intensity at a filament with mean kinetic energy, distant y_o above the invert;
  p_r = pressure relative to atmospheric pressure;
Q = rate of flow in a siphon;
q = rate of flow per unit width of siphon;
r = radius, with appropriate subscripts;
V = velocity of flow, with appropriate subscripts:
  V_m = velocity at a filament equal to the mean value of V, distant y_m from the invert;
y = distance to any filament measured upward from the invert:
  y_m = distance above the invert where the velocity of a filament is the same as the mean velocity of the section;
  y_o = distance above the inert where the kinetic energy of a filament is the same as the mean kinetic energy of the section;
β = ratio of the velocity of flow at the crown, V_c, to the mean velocity, V_m;
γ = unit weight of water, assumed equal to 62.5 lb per cu ft.

DISCUSSION

A. Shukry,[3] Esq.—The main assumptions on which the author has based the formulas for pressures within the sections at the summit and near the outlet of a siphon spillway are: (*a*) The flow around the siphon bends obeys the free vortex law; and (*b*) the elevation of the mean hydraulic and energy gradients at any point can be determined by adding the head losses of the individual elements from the inlet to the point under consideration.

As given by Eq. 7, the second assumption really was made because the given formulas must depend on a foreknowledge of the elevation of the mean water gradient at the point concerned.

The writer would like to discuss the foregoing two assumptions in the light of experiments conducted in 1942[4] on siphon models in the irrigation research laboratory of Fouad I University, Gîza, Egypt. Under the direction of Prof. H. Hefny, the writer measured the pressure and the velocity distribution across different sections of siphon models to study the behavior of the siphonic flow and to determine the extent to which the flow obeys the laws of pipe flow. Fig. 2(*a*) illustrates a view of one of the models; and Fig. 2(*b*), the arrangement of the laboratory. The identity of the various points marked by letters in Fig. 2(*b*) is as follows:

Point	Description
A	Adjustable copper gate shown in Fig. 2(*a*)
B	Side scales for the air vent
C	Lip of the siphon
D	Glass side for observing the action of priming
E	Pitometer tube for measuring velocity distribution
F	Differential gage for measuring velocity heads
G	Suction pump
H	Rubber tube connections with the pressure gages

Regarding assumption (*a*), the theoretical velocity distribution based on a free vortex flow at the summit and outlet bends of a siphon was derived by integrating a discharge element of depth dr. If the working head on the siphon is designated by H, then, according to Fig. 3,

$$Q = \int_{r_i}^{r_c} dQ = b \int_{r_i}^{r_c} V\,dr = b \int_{r_i}^{r_c} \frac{V_i\,r_i}{r}\,dr = b\,V_i\,r_i \int_{r_i}^{r_c} \frac{dr}{r}$$

$$= b\sqrt{2\,g\,H}\;r_i \log_e \frac{r_c}{r_i} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (28)$$

The velocity at any point distant r from the center would then be:

$$V = \frac{V_i\,r_i}{r} = \frac{Q}{b\,r \log_e \frac{r_c}{r_i}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (29)$$

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[3] Lecturer, Irrig. Dept., Faculty of Eng., Fouad I Univ., Gîza, Egypt.

[4] "The Flow of Water Through Siphon Spillways," by A. Shukry, thesis presented to the Faculty of Engineering, Fouad I Univ., Gîza, Egypt, in 1942, in partial fulfilment of the requirements for the degree of Master of Science in Civil Engineering.

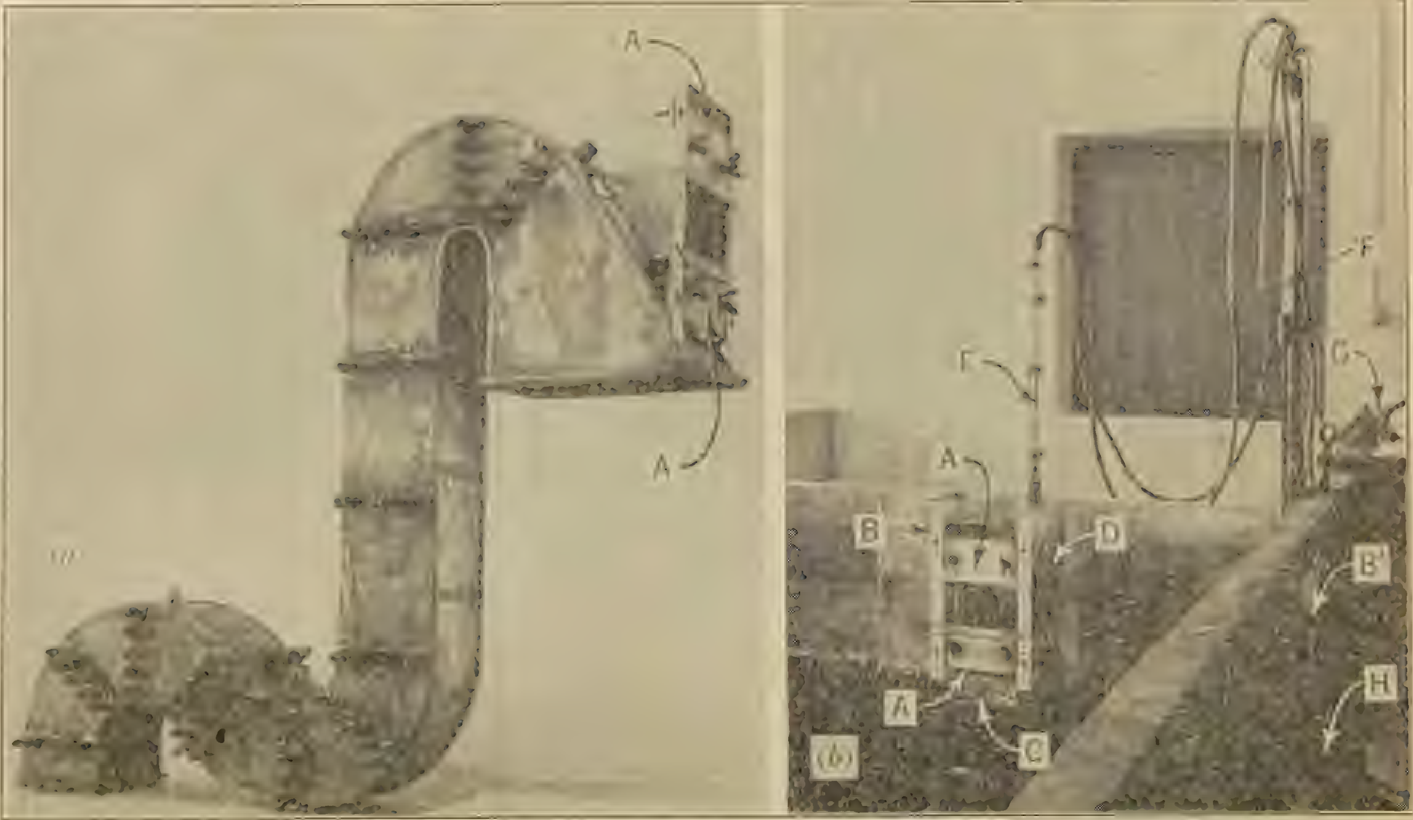
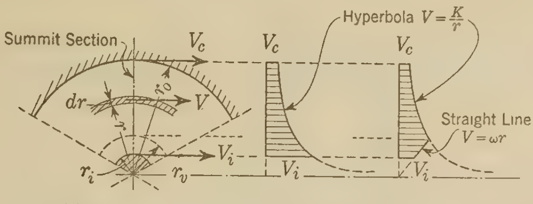

A
A
(a)
F
A
B
D
F
C
B'
A
C
H
(b)

FIG. 2.—MODEL OF SIPHON, AND LABORATORY ARRANGEMENT

The working head H considered in Eq. 28 should not exceed 24.0 ft of water, which is the practical limit of partial vacuum, however high the working head may be.[5] The mean percentage difference (ΔV) between the theoretical and the actual velocity distribution curves was calculated at any section by the formula:



(a) (b) FREE VORTEX POTENTIAL (c) COMBINED VORTEX

FIG. 3.—VELOCITY DISTRIBUTION AT THE SUMMIT SECTION OF A SIPHON SPILLWAY

$$\Delta V = \frac{\int_{r_i}^{r_e} [V \text{ (theoretical)} - V \text{ (actual)}]\, dr}{\int_{r_i}^{r_e} V \text{ (actual)}\, dr} \times 100 \quad \ldots\ldots\ldots (30)$$

The following three types of sections were considered: (1) All sections at the bends of the siphon when operated under a constant working head (Fig. 4); (2) the summit section of the siphon when operated under different working heads (Table 3); and (3) the summit section of the siphon for various shapes of outlet, operated under the same working head (Fig. 5).

The results showed that similarity between the actual and the theoretical flow distribution was closer for smaller than for larger ratios of $r_c : r_i$, and that the velocity distribution at the summit bend did not appreciably change within the possible range of the working head or with different shapes of outlets.

To study the applicability of the free vortex law in predetermining the discharge passing through any siphon spillway, the theoretical and the actual discharges of seven different types of siphon spillways were compared. As seen in Table 4, the theoretical discharges are always less than the actual discharges. The percentage difference can thus be considered as a factor of safety; but, beyond a ratio of $r_c/r_i = 3.0$, the design, according to a free vortex assumption, may become uneconomic.

The reason for a departure of the actual flow from the ideal free vortex is explained by the fact that the free vortex law, when applied to bends of closed or open conduits, should be limited with respect to the pressure induced at the inner wall of the bend. According to this law the velocity at any point in the bend is inversely proportional to its radius. Consequently, as r_i approaches the value zero in very sharp curves, V_i approaches infinity and P_i/γ

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[5] "Design of a High-Head Siphon Spillway," by Elmer Rock, *Transactions*, Am. Soc. C. E., Vol. 105 (1940), p. 1050.

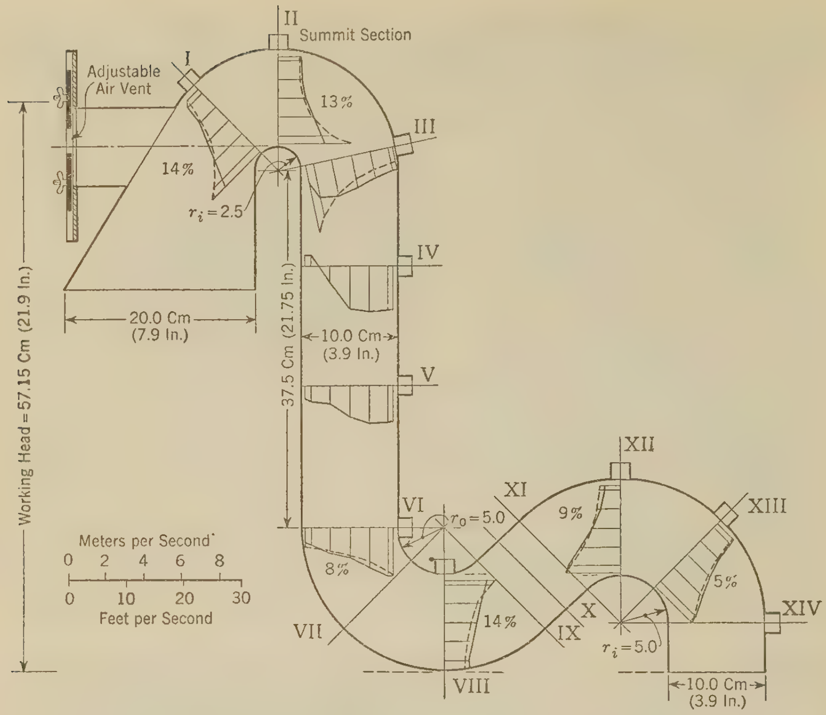

II
Summit Section
I
Adjustable Air Vent
13%
III
14%
$r_i = 2.5$
IV
Working Head = 57.15 Cm (21.9 In.)
20.0 Cm (7.9 In.)
37.5 Cm (21.75 In.)
10.0 Cm (3.9 In.)
V
XII
XI
9%
XIII
VI
$r_o = 5.0$
Meters per Second
0 2 4 6 8
0 10 20 30
Feet per Second
8%
5%
XIV
X
14%
VII
IX
$r_i = 5.0$
VIII
10.0 Cm (3.9 In.)

FIG. 4.—THEORETICAL AND ACTUAL VELOCITY DISTRIBUTION ACROSS A SIPHON WITH AN OUTLET A AS IN FIG. 2 (Q = 16.41 LITERS PER SEC OR 260 GAL PER MIN)

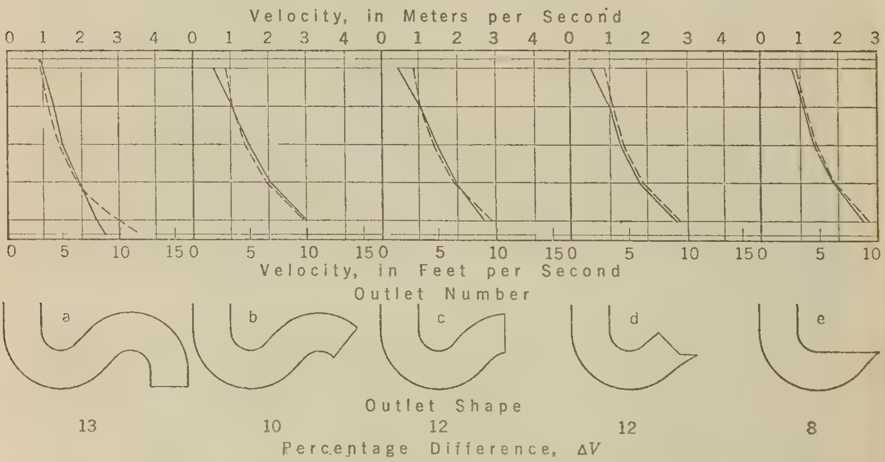
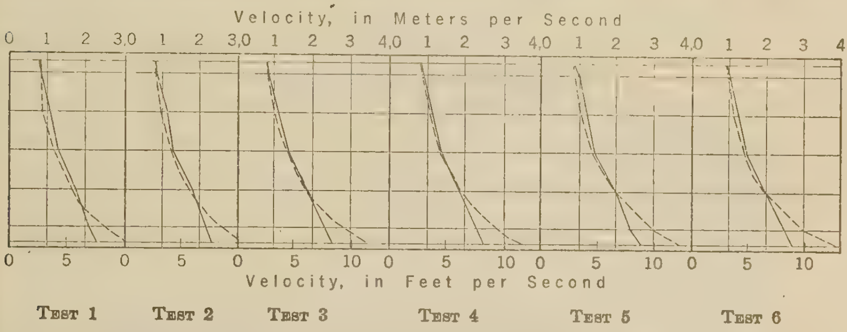


FIG. 5.—THEORETICAL AND ACTUAL VELOCITY DISTRIBUTION AT THE SUMMIT SECTION FOR VARIOUS OUTLETS AND CONSTANT DISCHARGE (Q = 17 LITERS PER SEC OR 269 GAL PER MIN)

approaches a negative infinity. Since the negative pressure in closed conduits cannot exceed 24.0 ft of water, this pressure condition actually limits the radius at which the flow follows the theoretical free vortex law. Calling this

TABLE 3.—THEORETICAL AND ACTUAL VELOCITY DISTRIBUTION AT THE SUMMIT SECTION FOR VARIABLE WORKING HEADS ON A SIPHON WITH AN OUTLET A AS IN FIG. 2



Description	Test 1	Test 2	Test 3	Test 4	Test 5	Test 6
Discharge:						
Liters per second	14.69	15.38	15.94	16.41	17.06	17.82
Gallons per minute	232.8	243.8	252.7	260.1	270.4	282.5
Working Head:						
Centimeters	43.50	48.00	51.70	53.70	57.15	60.92
Inches	17.1	18.9	20.4	21.2	22.5	24.0
Percentage difference[a]	....	14	14	13	14	13

[a] Percentage difference between the theoretical and the actual distribution.

radius r_v, the inner portion of the fluid behaves as a separate vortex filament, all particles of which have a constant rate of rotation ω, similar to conditions in a forced vortex where $V = \omega r$. The velocity distribution in this combined potential, generally known as the Rankine vortex, is shown diagrammatically in Fig. 3. It is obvious that the energy of flow in this vortex filament cannot be considered a constant because of its rotational motion. If r_i is greater than r_v, the flow in the bend will obey the free vortex law. On the other hand, if r_i is smaller than r_v the flow will obey the combined vortex potential. The value of r_v for any bend can be determined in terms of the dimensions and the flow conditions of the bend.

As regards the second assumption made by Mr. Nelidov, it should be mentioned that there has been considerable discussion as to whether the siphonic flow obeys the laws of flow in pipes—in other words, whether it is possible to divide the total length of the siphon into a number of bends and straight sections and to apply the existing coefficients and formulas for the individual parts so as to obtain the progressive net losses in friction and eddies. Most of this discussion[6,7,8,9,10] was based on pressure measurements at some sections of siphon models. The pressure head at any section was considered as the mean reading of the pressure gages, and the total head was calculated by adding to the pressure head the velocity head h_{vm}. Through the omission of the head-correction factors resulting from the nonuniform distribution of velocities and pressures, serious discrepancies in the energy lines were encountered, especially at the *vena contracta* downstream from the summit bend. Investi-

TABLE 4.—COMPARISON BETWEEN THEORETICAL AND ACTUAL RATES OF DISCHARGES FOR SOME OF THE EXISTING SIPHON SPILLWAYS[a]

Item	Description	LEABURG[b]		LAGGAN[c]		INGLIS[d]		ALPINE[e]		OCOEE[f]		WALTERVILLE[g]		FERRO[h]	
		Q_{th} (2)	(%) (3)	Q_{th} (4)	(%) (5)	Q_{th} (6)	(%) (7)	Q_{th} (8)	(%) (9)	Q_{th} (10)	(%) (11)	Q_{th} (12)	(%) (13)	Q_{th} (14)	(%) (15)
	(1)														
1	Ratio r_e/r_i for the summit section	2.375		1.500		2.670		2.750		2.330		2.835		5.500	
2	Working head (H) on the siphon, in feet	28.4		66.0		23.0		31.0		27.2		45.9		13.1	
	Flow in the Siphon:														
3	Actual, Q_a, in cu ft per sec	470	...	650	...	501	...	667	...	220	...	700	...	353	...
	Theoretical,[i] with Head Losses as Computed by the Methods of:														
4	W. P. Creager and J. D. Justin	468	0	590	−9	494	−1	667	+1	202	−8	700	0	299	−16
5	J. Williamson	630	+34	670	+3	546	+9	770	+15	251	+14	890	+27	357	+ 1
6	Theoretical; free vortex[j]	407	−13	650	0	453	−10	595	−11	200	− 9	587	−16	192	−46

[a] The headings of this table are abbreviated considerably for compactness, the full significance of each being described in footnotes ([b]) to ([h]). The heading Q_{th} denotes the theoretical rate of flow in the siphon, in cubic feet per second, as applied to values in lines 4, 5, and 6; and (%) denotes the percentage difference, $\frac{Q_a - Q_{th}}{Q_a} \times 100$, for these three lines.

[b] The behavior of the siphons of the Leaburg Hydro-Electric Plant, Eugene, Ore.; as reported by J. C. Stevens, President, Am. Soc. C. E. (*Transactions*, Am. Soc. C. E., Vol. 99 (1934), p. 986).

[c] Siphon, Battery No. 2, at the Laggan Dam in England, as reported by A. H. Naylor ("Siphon Spillways," Edward Arnold & Co., 1935, London).

[d] The Inglis Siphon reported by A. H. Gibson, Thomas H. Aspey, and Fred Tattersall (*Minutes of Proceedings*, Inst. C. E., London, England, Vol. 231, 1931, p. 203).

[e] The Siphon at Alpine Dam in Marin County, California ("Construction of Dams," by Edward Wegmann, 8th Ed., p. 430).

[f] See "Design and Tests of Wasteway of Large Flume Connected with Tennessee Power Company's Second Ocoee River Development," by W. P. Creager, *Engineering Record*, Vol. 69, 1914, p. 567.

[g] The behavior of the siphons of the Walterville Hydro-Electric Plant, Eugene, Ore., as reported by Mr. Stevens (*Transactions*, Am. Soc. C. E., Vol. 104, 1939, p. 1787).

[h] The siphon type developed by Cavicini and Masini, as reported by G. Ferro (*Engineering*, London, Vol. 121, 1926, p. 5).

[i] Theoretical rate of flow (Q_{th}) in the siphon, based on the total sum of head losses, in cubic feet per second.

[j] Theoretical rate of flow (Q_{th}) in the siphon, based on a theoretical free vortex at the summit section.

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[6] "Experiments on Siphon Spillways," by A. H. Gibson, Thomas H. Aspey, and Fred Tattersall, *Minutes of Proceedings*, Inst. C. E., London, England, Vol. 231, 1931, p. 203.

[7] "The Laws of Siphon Flow," by P. Davies, *ibid.*, Vol. 235, 1932–1933, p. 352.

[8] "The Flow of Water through Rectangular-Pipe Bends," by P. Davies and S. V. Puranik, *Journal*, Inst. C. E., London, England, Vol. 2, 1935–1936, p. 83.

[9] "Experiments on Pipe Bends," by H. Addison, *Selected Paper No. 5084*, Inst. C. E., London, England, 1936.

[10] "Considerations on Flow in Large Pipes, Conduits, Tunnels, Bends, and Siphons," by James Williamson, *Journal*, Inst. C. E., London, England, Vol. 11, 1938–1939, p. 451.

gators found the same phenomena when analyzing the energy losses from the recorded pressures in some existing siphon spillways.[11,12]

For the purpose of determining the correct analysis, the energy method was followed in deriving the mean values of the pressure, velocity, position, and total heads at any section. This was accomplished by integrating the energy curves (instead of the head curves) with respect to the area of the section, as given by the following: Mean pressure head,

$$h_{pm} = \frac{1}{V_m A} \int_0^A \frac{p}{\gamma} V \, dA \qquad \text{(31}a\text{)}$$

mean velocity head,

$$h_{vm} = \frac{1}{V_m A} \int_0^A \frac{V^3}{2 g} dA \qquad \text{(31}b\text{)}$$

mean position head,

$$h_{sm} = \frac{1}{V_m A} \int_0^A y \, V \, dA \qquad \text{(31}c\text{)}$$

and mean total head,

$$h_{tm} = \frac{1}{V_m A} \int_0^A \left(\frac{p}{\gamma} + \frac{V^2}{2 g} + y \right) V \, dA \qquad \text{(31}d\text{)}$$

In the writer's investigation, the aforementioned energy process eliminated the apparent discrepancies in the energy line of a siphon. The siphon was then divided into three parts—the summit bend, the straight leg, and the outlet bend. The coefficient K expressing the loss of head in terms of h_{vm} was obtained for each part and compared with the value given in the existing published data. The loss of head due to wall friction was calculated according to the modified Chézy formula (in foot-second units):

$$h_l = \frac{V^2{}_m \, n^2}{R^{1.333}} \qquad \text{(32)}$$

in which h_l is loss of head per unit length; n is Kutter's coefficient of roughness equal to 0.009 for the model walls; V_m is mean velocity in the section, and R is the hydraulic radius. The loss of head due to a bend, which is the head in excess of that which would occur in a straight conduit of equal length, has been obtained by at least two investigators: (*a*) William P. Creager and Joel D. Justin, Members, Am. Soc. C. E., determined coefficients based on the mean results of most of the tests which had been conducted on pipe bends;[13] and (*b*) James Williamson derived coefficients[10] in which the effect of the wall material of the bend was taken into consideration.

Comparison of the two revealed that the actual losses in the siphon elements were totally different from the losses of these elements when tested separately, as may be noted in Table 5. It was found, however, that the various elements of the siphon seemed to have a counterbalancing effect on

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[11] "On the Behavior of Siphons," by J. C. Stevens, *Transactions*, Am. Soc. C. E., Vol. 99 (1934), p. 986.

[12] "Siphons as Water-Level Regulators," by J. C. Stevens, *ibid.*, Vol. 104 (1939), p. 1787.

[13] "Hydro-Electric Handbook," by W. P. Creager and J. D. Justin, John Wiley & Sons, Inc., New York, N. Y., and Chapman Hall, London, England, 1927, p. 118.

each other. A reduction of the loss at one part was offset by an increase of the loss at another, so that the total sum of the energy losses appeared to be practically unchanged. This may help to compute the rate of discharge passing through any type of siphon spillway with a reasonable degree of accuracy, as shown in Tables 4 and 5, but fails to give any correct indication for the pressures that may be induced at some sections of a siphon or the elevation of the energy line.

TABLE 5.—COMPARISON BETWEEN EXPERIMENTAL (K_a) AND COMPUTED (K_c) VALUES OF THE ENERGY LOSS COEFFICIENTS (BASED ON THE MEAN VELOCITY AT THE SUMMIT SECTION) AND THE RATE OF FLOW IN A SIPHON MODEL

Item	Sources of energy loss	Experimental values K_a	COMPUTED VALUES, K_c	
			Creager and Justin[a]	J. Williamson[a]
1	Entry plus friction between inlet and outlet	1.448	0.416	0.416
2	Summit bend	0.400	1.308	1.122
3	Outlet bend	1.175	2.030	1.020
4	Outlet	1.150	1.150	1.150
5	Total	4.173	4.904	3.708
6	Percentage difference $(\Sigma K_a - \Sigma K_c)100/\Sigma K_a$	....	+17.5	−11.1
	Working Head H:			
7	In meters	0.5500	0.5500	0.5500
8	In feet	1.805	1.805	1.805
	Mean Velocity V_m at the Summit Section ($\sqrt{2\,g\,H/\Sigma K}$):			
9	In meters per second	1.605	1.485	1.700
10	In feet per second	5.266	4.872	5.577
	Experimental Rate of Discharge Q_a:			
11	In liters per second	16.22	....	....
12	In gallons per minute	259	....	....
	Computed Rate of Discharge Q_c:			
13	In liters per second	....	15.00	17.15
14	In gallons per minute	....	238	272
15	Percentage difference $(Q_c - Q_a)100/Q_a$	....	−8	+6

[a] Computed value of the energy loss coefficient K_c, with bend losses as determined by the methods of W. P. Creager and J. D. Justin, and by J. Williamson, respectively.

Mr. Nelidov has shown clearly the relatively high pressures within the summit and near the outlet bends of a siphon, and the importance of providing good reinforcement for these unbalanced pressures, due to which failure of the siphon may occur. It may be added that a good factor of safety in the structural design should also be provided to withstand the intermittent violent shocks and serious vibrations during the actions of priming and de-priming, especially if the siphon is to be operated under high heads.

I. M. NELIDOV,[14] M. AM. SOC. C. E.—The discussion by Mr. Shukry is constructive and positive in its conclusions. On the basis of experiments, it is demonstrated that a similarity exists between the actual and the theoretical velocity, with variations on the safe side—that is, theoretical velocities are always smaller than actual velocities by various percentages ranging from 5% to 14% (see Table 3 and Figs. 4 and 5). Mr. Shukry makes a valuable comparison between the theoretical and actual discharges for seven existing siphons, which indicates that theoretical discharges are always smaller by various percentages, ranging from zero to 16%. On one occasion (for the Ferro siphon) the percentage variation was as high as 46%, which leads Mr. Shukry to conclude that for large ratios of $\frac{r_c}{r_i}$ (greater than 3.0) the discrepancy increases.

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[14] Civ. and Structural Engr., El Cerrito, Calif.

With decreasing values of r_i a state may be reached such that the velocity V_i at the summit will be the limiting value based on the available atmospheric vacuum, after which the discharge will not be appreciably increased. The writer wishes to refer to his discussion of the article by Elmer Rock,[15] Assoc. M. Am. Soc. C. E., in which he indicated the possibility of a "limiting" discharge and a corresponding "limiting" head with the inference that further increase of the head will not produce increase in discharge. Similar results may be obtained with a constant head by decreasing the radius r_i. Referring to Example 1 and Table 1 of the paper, it may be found that by making $\frac{p_i}{\gamma} = 0$ in Eq. 3 and with $h_a = 24$ ft, $h = -10$ ft, and $h_{vm} = 1.55$ ft:

$$h_{vi} = h_a + h + n\, h_{vm} = 14 + n\, h_{vm} \ldots\ldots\ldots\ldots\ldots (33)$$

in which (see Eq. 11) $n = \frac{\beta^2 (m^2 - 1)}{2 \log_e m}$.

On the other hand, by Eq. 18, $h_{vi} = \beta^2 m^2 h_{vm}$. Equating Eqs. 18 and 33 and eliminating h_{vi}: $14 + n\, h_{vm} = \beta^2 m^2 h_{vm}$; or $-n + \beta^2 m^2 = \frac{14}{h_{vm}} = \frac{14}{1.55} = 9$.

By Eqs. 11 and 15 and with $m = \frac{r_c}{r_i}$:

$$\beta^2 \left[-\frac{m^2 - 1}{2 \log_e m} + m^2 \right] = 9 \ldots\ldots\ldots\ldots\ldots\ldots (34a)$$

and

$$\left(\frac{d}{r_c}\right)^2 \frac{1}{\log^2_e \frac{r_c}{r_i}} \left[-\frac{\left(\frac{r_c}{r_i}\right)^2 - 1}{2 \log_e \left(\frac{r_c}{r_i}\right)} + \left(\frac{r_c}{r_i}\right)^2 \right] = 9 \ldots\ldots\ldots (34b)$$

With $d = 4$ ft and $r_c = 20$ ft, Eq. 34b becomes

$$\frac{0.04}{\log^2_e \frac{20}{r_i}} \left[-\frac{\left(\frac{20}{r_i}\right)^2 - 1}{2 \log_e \left(\frac{20}{r_i}\right)} + \left(\frac{20}{r_i}\right)^2 \right] = 9 \ldots\ldots\ldots\ldots (34c)$$

By trial it is found that a value of $r_i = 0.3$ ft will be the limiting radius of the invert. It is true that the value of the ratio $\frac{r_c}{r_i} = \frac{20}{0.3} = 67$ is so great that the theoretical results may be questioned, but this example was used only as a numerical illustration of the process of obtaining r_i. At the same time the small value of r_i demonstrates that in this example the flow will agree with the assumed theory of irrotational vortex.

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[15] Discussion by I. M. Nelidov of "Design of High-Head Siphon Spillway," by Elmer Rock, *Transactions*, Am. Soc. C. E., Vol. 105 (1940), p. 1069.

Another point of considerable interest was treated by Mr. Shukry in his discussion of the mean pressure, velocity, and position heads corresponding to the nonuniform distribution of velocities. As shown by Fig. 1, the writer has related his energy and hydraulic gradient lines to their mean values and not to those corresponding to the mean velocity.

The use of losses in the siphons computed with the coefficients selected from nonrelated tables should give quite divergent results. However, the main fault is the scarcity of information pertaining specifically to the siphon flow rather than defective experiments.

The writer wishes to thank Mr. Shukry for his exhaustive study of the problem.

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Paper No. 2282

DESIGN OF ROADBEDS

BY IRA B. MULLIS,[1] ESQ.

WITH DISCUSSION BY MESSRS. CHARLES W. BRITZIUS, J. T. L. McNEW, GUSTAVO PÉREZ GUERRA, JACOB FELD, JASPER L. STUCKEY, DELBERT L. LACEY, RENÉ S. PULIDO Y MORALES, K. B. WOODS, AND IRA B. MULLIS.

SYNOPSIS

The purpose of this paper is fourfold:

1. To review the fundamental properties of the three classes or phases of matter and to show that a roadbed material in particular is either strong or weak according to the phase in which it is found;

2. To offer experimental evidence showing the influence of gases, liquids, and solids on the stability of roadbeds;

3. To cite examples of roads and certain experience supporting the theory advanced; and

4. To outline a standard of procedure under which roads may be designed and built so they will remain stable under given conditions of traffic.

INTRODUCTION

The materials of roadbeds are of infinite variety. In size, the constituents range from boulders to finite particles of matter; in quality, from rigid and durable rocks to fluids. That all these materials are of equal structural value, no one would claim; but, unless the several relationships are controlled adequately, the essential balance cannot be established and maintained. However, before the relationships of these materials can be controlled wisely, their natures must be understood.

As in other matter, the constituents of roadbeds may be classified profitably into three distinct states or phases—gases, liquids, and solids. This approach is followed because it is the accepted method of classifying matter in general and provides an orderly approach to a clear understanding of roadbeds in particular.

Since matter may exist in any one of three states, no one state can be considered as being characteristic of a given substance at all times. Its state, therefore, depends upon conditions, such as temperature and pressure. Water,

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NOTE.—Published in April, 1945, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

[1] Cons. Engr., Raleigh, N. C.

for example, at atmospheric pressure is a solid below 0° C and a gas above 100° C. Unqualified statements, such as "air is a gas," "water is a liquid," and "quartz is a solid," are inaccurate unless atmospheric conditions are specified, as each of these substances may occur in any one of these states.

Obviously, roadbeds should be constructed and maintained as rigid bodies and, to attain this condition, the properties of the component materials must be understood. Moreover, the proper balance of the components must be maintained at all times; otherwise, structural failure is certain. The ability to resist loads is a property of solid or rigid bodies; but, whenever fluids are present in critical amounts, the property of rigidity is lost. The several states of matter and their effects on the structural properties of roadbeds have been discussed by the writer elsewhere.[2] Although the effects of heat, pressure, and solvents on the structural properties of certain rigid bodies are commonly recognized, most specifications and professional papers indicate a lack of appreciation of how earthy substances may be protected against the weakening effect of air and water.

Of course, the temperature of roadbed materials never approaches the point of liquefaction, although under freezing temperatures the beds do become very rigid. These freezing conditions, however, do not have an adverse effect on roadbeds unless they are very wet. Therefore, the only condition remaining that can lower the measure of their rigidity to that of a plastic or fluid condition is the presence of critical amounts of fluid, such as air or water. The method of approach is a rational one, the principles of which have been long understood, although habitually overlooked.

States of Matter

Gases.—A gas consists of an aggregation of atoms or molecules in which the force of attraction is so overbalanced by the movement of the particles that it cannot act. In this condition there is no cohesion between the particles, therefore; they fly in all directions. If the attractive force between the particles is normally weak, the substance exists as a gas at ordinary temperatures; but any substance, whether normally a liquid or a solid, may be turned into a gas by heating it sufficiently. Temperature is a relative comparison of molecular or atomic vibration; therefore, if atoms and molecules are made to vibrate fast enough, the attractive forces between them become ineffective. The result is a gas.

Due to the foregoing properties, gases, unlike other forms of matter, have no constant volume but fill all parts of the container regardless of its size. Therefore, gases are highly compressible and, under sufficient pressure and loss of heat, may be converted into liquids and in certain cases directly into solids.

Liquids.—A liquid consists of a mass of molecules in which motion and the relative distance between the molecules are considerably less than in a gas; hence, the attractive forces are greater. In a liquid the molecules are free to move about but they cannot move very far apart. The result is that the liquid maintains a relatively constant volume, assumes the shape of its container, and unlike gases maintains a definite upper surface.

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[2] "Principles Applying to Highway Road-Beds," by Ira B. Mullis, *Transactions*, Am. Soc. C. E., Vol. 104 (1939), p. 1381.

In a liquid, the attractive forces are great enough to hold the molecules together; however, the attractive forces do not have absolute control. Occasionally, a molecule at the surface of the liquid moves fast enough to break away and fly off into space. This phenomenon is known as evaporation. Hence, liquids left in the open air long enough disappear by evaporation. A liquid may be turned into either a gas or a solid under appropriate changes of heat and pressure.

Substances in the liquid state have greater densities, greater internal friction, larger cohesive forces, and much smaller compressibility than in the gaseous state. However, loads can be supported by liquids only when confined or under conditions that permit the utilization of buoyancy and velocity.

Solids.—In contrast to gases and liquids, the attractive forces in solids have the upper hand. The molecules are no longer free to move as they do in fluids, each molecule being tied to a definite position in the solid. These ties, therefore, have varying strengths in different substances, and they are one reason, at least, why different solids have different melting points. If the forces holding the solid together are weak, the solid will have a low melting point; but, if the forces are strong, the solid will have a high melting point. The reason for this is apparent if one remembers that temperature is a measure of molecular vibration. When the attractive forces are weak, only a little vibration is required to separate the molecules. A weak solid, such as butter, begins to melt on a warm day; but a solid, such as tungsten, one of the strongest metals, requires a temperature of more than 3,000° C to melt it. In a similar manner, one may say that, since gumbo clay is one of the strongest sediments, much water is required to soften it; but kaolin being among the weakest, little water is required for softening. As solids are only slightly compressible, this is an indication that the atoms or molecules already are under very high compressive forces so that the application of more pressure does not largely increase the total volume change.

By dissolving a foreign substance in a metal, the resistance to plastic deformation may be changed greatly. Thus, the mechanical properties of steel in the iron-carbon system are determined by very small amounts of carbon; and minute quantities of gases such as hydrogen, nitrogen, or oxygen which may be dissolved in a steel have an influence on the yield point and also on the relation between stress and plastic deformation. Thus, it will be seen that substances are weakened by excessive heat and by admixtures tending to accelerate molecular vibration. Bodies of clay, for example, are weakened by absorbed water that accelerates the molecular vibration of the mixture. In a like manner, solid bitumen is weakened by the liquefying action of either heat or naptha.

INFLUENCE OF FLUIDS ON SOLIDS

Under normal conditions of the atmosphere, the component fluids and solids in a roadbed may be confined to air, water, and rock matter. Since the physical properties of each of these substances have been established, the effect of one on the other should be the better understood, through stress-strain measurements of an adequate number of combinations of these substances.

Air and Rock Matter.—The effect of air on the structural properties of masses of rock matter, coarser than fine sand, is so insignificant as to require little attention either during or following construction; but, in material of a smaller particle size, the effect of air is one of major importance. During construction, air becomes entrapped in the pore spaces of the material, tends to hold the particles apart, and thus prevents consolidation of the mass. This condition may be exemplified by the use of a recently sifted Portland cement in a partly filled bag. If pressure is applied, the mass of air and cement tends to flow away from the area under pressure and, upon removal of the pressure, recovery is not noticeable. In mixtures of this sort, solid particles aided by intervening films of air are free to slip; therefore, the body may be said to be a dry plastic mass. Even if it were possible to reduce the thickness of the air films to the minimum, and thereby obtain fair contact, there can be no consolidation when films of air intervene. (The word "consolidate" connotes more than compaction and includes a bonding together of the particles. It is here used only in this sense.)

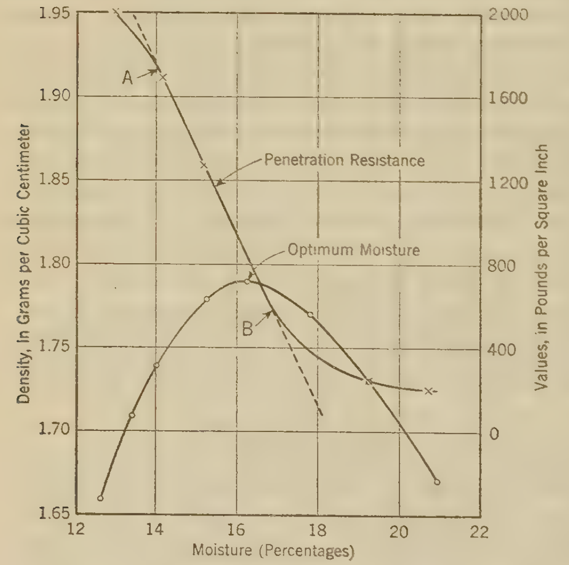

Fig. 1.—Relations of Density and Water Content to Penetration Resistance

The effect of air and water between particles of solid matter is exemplified in Fig. 1. This figure is based on data obtained through the use of an A.S.T.M. (American Society for Testing Materials) standard 5.5-lb hammer dropped 18 in., here referred to as the Modified Proctor method. On the dry side of the moisture-density curve, slipping begins under a moisture content of 14%. This manifestation is indicated by the break in the moisture-resistance curve at point A, Fig. 1. Corresponding to this point, the density approximates 1.74 g per cu cm and the moisture content 14%. Assuming an absolute density of 2.65, the voids occupy 34% of the volume. Of this volume 24% is water and the remaining 10% is air. Since slip is not indicated at the apex of the curve, corresponding to a density of 1.79 and a water content of 17%, slip starting at point A must be due to air content.

Although detailed information on the dry side of the moisture-density and stress-strain relations of the curve is meager, it is apparent that the presence of a critical amount of air in the pores of a solid produces a marked effect on the slip. For this and other reasons, the air permitted in a roadbed should be restricted to small quantities.

A good example of the adverse effect of occluded air in dry loessial sediments is found often during droughts on road construction projects on the Great Plains of the United States. Adequate compaction is impossible and slippage of the earthy material makes even moderate speed hazardous. Another well-known example is the slip experienced in attempting to travel over fine, dry, air-filled deposits of sand. Both of these examples are manifestations of voluminous pore space filled almost entirely with air.

Water and Rock Matter.—The effect of water on the structural properties of rock matter, coarser than fine sand, is of minor importance with the exception of the erosive action in streams during flood stages; therefore, no discussion will be given to this phase of the subject. However, the effect of water on the structural properties of sediments finer than coarse sand is one of major importance.

To assist in the elimination of air and to aid in the consolidation of sediments, the presence of water is essential. Without water, even clay particles cannot be made to stick together; however, when the compactive force is large, less water is necessary. This statement has been verified by J. W. Mellor[3] who found that, by increasing the pressure from 1 to 200 kg per sq cm, the water content for stickiness was reduced from 26.4% to 5.6%. Moreover, face brick is quite generally formed with moisture contents of 7% to 8%.

As water tends to weaken all hydrous materials, its use in fine sediments utilized in making roadbeds should be held to the minimum. Tough clays, such as the gumbos, can be consolidated more adequately by increasing the weight of the roller rather than by the addition of water. When excessive quantities of water are used, the body is thereby weakened and upon drying its shrinkage produces large cracks that aid in the perpetuation of cycles of excessive moisture change and cause much reduction in capacity to support loads.

If less water had been applied with a greater amount of compacting force, the density of the mass exemplified in Fig. 1 could have been increased greatly. This would have produced a consequent decrease in pore space and also would have increased the resistance to loads and the absorption of water. If the water content had been reduced to 14%, the density could have been increased to about 1.90; moreover, if the water content had been set at 10%, with an adequate method of compaction, the density could have been increased to 2.05. Since it is one of the purposes of this paper to develop a basis for an adequate standard of compaction for roadbeds, no specific numerical value of compaction will be proposed at this time.

By referring again to Fig. 1, it will be seen that slip due to water content starts at or near point B, representing a value very close to that of the optimum moisture as determined by the Modified Proctor method. With this moisture content, the mass is a plastic solid. Solid particles are at least partly surrounded by water and touch other particles only at certain points. As a result of this condition, the mass partakes of the properties of both solid and liquid.

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[3] "On the Plasticity of Clays," by J. W. Mellor, *Transactions*, Ceramic Soc. (England), Vol. 21, 1921–1922, p. 95.

In this condition, solid particles under pressure slip over one another and, as the water itself is under pressure, the wet mass tends to flow in its characteristic manner. As the thickness of the films of water between solid particles is increased, the mass becomes more and more like a liquid; but, as the films are reduced in thickness, the mass becomes more and more viscous until the critical point is reached when the mass is a rigid body, a state essential in roadbeds at all times.

Although the late Mansfield Merriman,[4] M. Am. Soc. C. E., defined a material as plastic when it has no elasticity, so that the smallest forces cause permanent deformation, he was undoubtedly referring to a perfectly plastic material that does not actually exist. As in the case of viscous bodies, a certain amount of pressure must be applied before a plastic clay body can be made to flow through an orifice. In this statement research workers including W. D. Bancroft,[5] E. C. Bingham,[6] and Henry Green[7] are in agreement, although different clays in a plastic condition have different yield values.

The ideal state of a roadbed is that in which the elastic limit is high even when the pore spaces are saturated. To attain this state the moisture-density curve (Fig. 1) should be required to indicate a much higher density with an attendant decrease in water content. This change would leave point B, Fig. 1, on the penetration resistance curve little changed but point A would be raised to a higher value of resistance, provided the water content was not excessive.

Air-Water Relationship.—Obviously pore spaces will be filled with either air or water at all times. During dry weather, evaporation losses are to be expected. As water is lost, air takes its place; and in a similar manner, the pore spaces receive water and discharge air during wet weather.

The volume relationship of these fluids is of vast importance but detailed information in widespread areas is meager. From data developed in the State of Iowa by Bert Myers and the writer,[2] it would seem that in consolidated mixtures one should expect from 45% to 80% pore saturation during wet weather. In ordinary roadbeds, an average of from 70% to 80% is not unusual; in poorly compacted clay, 80% or more is found; and in silt, from 80% to 100%.

Since it has been shown that excessive volumes of either air or water are detrimental to roadbeds, the necessity of restricting these fluids is obvious. This can be done only through the restriction of pore space. If the pore space is small, the volume of air or water must likewise be small. Moreover, the smaller the pore space the more constant the water content—a fact explained by the enormous increase in molecular attraction of water to solids when the films are quite thin. In this condition, evaporation losses are indeed small and the entrance of additional water is made difficult by frictional resistance and the presence of occluded air.

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[4] "Mechanics of Materials," by Mansfield Merriman, John Wiley & Sons, Inc., New York, N. Y., 10th Ed., 1912, p. 44.

[5] "Applied Colloid Chemistry," by W. D. Bancroft, McGraw-Hill Book Co., Inc., New York, N. Y., 2d Ed., 1926, p. 183.

[6] "Plasticity as a Means for Control of Properties," by Eugene C. Bingham, *Journal*, Am. Ceramic Soc., Vol. 7, 1924, p. 430.

[7] "Further Development of the Plastometer and Its Practical Application to Research and Routine Problems," by Henry Green, *Proceedings*, A.S.T.M., Vol. XX, Pt. II, 1920, p. 451.

Although certain writers[2,8,9] have stressed heretofore the necessity of restricting pore space, little heed has been given to this essential phase of construction. As a consequence, numerous roadbeds fail even where the Modified Proctor standard of compaction has been followed.

Charles H. Lee,[9] M. Am. Soc. C. E., sets a tentative limiting value for air at 5% by volume or 3.5% expressed as equivalent water in terms of dry weight. This limit appears to be practicable, provided the limit of total pore space is adequately restricted.

CHANGE OF STATE

Many substances have definite melting and freezing points. Furthermore, the strength of metals is greatly affected by heat and the presence of carbon and certain other elements. As bituminous pavements are weakened by heat or the presence of solvents such as naptha, so are road surfaces composed of mixtures of aggregates and clay or clay alone weakened by the solvent action of water. As long as these materials remain rigid, they are strong; but, when they are overcome by solvents, a plastic and therefore weak condition prevails. The knowledge of these facts led Bennett H. Levenson,[10] in the fall of 1926, to make an investigation of clays and other sediments to determine the exact manner in which they are affected by absorbed water. He wished to determine, by experimental methods, whether sediments soften gradually by the absorption of water or whether the change from the rigid to the plastic state occurs suddenly.

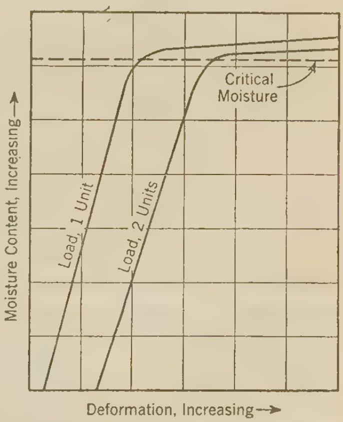

FIG. 2.—GENERALIZED CURVES SHOWING THE EFFECT OF CRITICAL MOISTURE ON LOAD AND DEFORMATION

To begin, forty-two samples of roadbed materials were obtained from various parts of the United States. These were selected so as to yield a wide range of plasticity and strength and all were tested in duplicate to determine their lower plastic and lower liquid limits. The effect of moisture content on uniformly prepared specimens was also determined in duplicate under various loads, all being measured at a given deformation.

After preliminary tests had indicated that the curves would be of the general form of the two shown in Fig. 2, the procedure was modified to include only two pairs of test values at moisture contents above and also below what seemed to be the critical moisture. This procedure was planned to test the validity of an indication that the lower limit of plasticity might be determined by the stress-strain method as truly as by the well-known Atterberg method.

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[8] "Fundamental Principles of Soil Compaction," by R. R. Proctor, *Engineering News-Record*, August 31, 1933, p. 246; September 21, 1933, p. 350; September 28, 1933, p. 374.

[9] "Selection of Materials for Rolled-Fill Earth Dams," by Charles H. Lee, *Transactions*, Am. Soc. C. E., Vol. 103 (1938), p. 57.

[10] "Simplified Soil Tests for Subgrades and Their Physical Significance," by Charles Terzaghi, *Public Roads*, Bureau of Public Roads, U.S.D.A., Vol. 7, No. 8, pp. 153–162.

The detailed data obtained from tests on twenty-two plastic or coherent sediments are shown in Table 1(*a*). Unconfined tests were performed on small cylinders whose walls had no outside support and the bearing block covered the

TABLE 1.—PERCENTAGE RELATION BETWEEN CRITICAL MOISTURE AND LOWER LIQUID LIMIT

(Averages Are Shown in Parentheses)

CRITICAL MOISTURE DETERMINED UNDER THE CONDITION AND THE LOAD SHOWN					Lower limits,[a] plastic and liquid	Ratio, Col. 5 / Col. 6
Not Confined		Confined		Mean critical moisture		
5.6-lb load (1)	12-lb load (2)	16-lb load (3)	32-lb load (4)	(5)	(6)	(7)
(*a*) PLASTIC SEDIMENTS						
....	....	41.5	41.0	41.2	42.6	0.967
36.5	35.5	....	....	36.0	36.2	0.994
33.5	33.5	....	....	33.5	34.4	0.974
31.5	33.0	....	....	32.2	31.9	1.009
30.5	27.5	....	....	29.0	32.7	0.886
28.0	30.0	....	....	29.0	28.1	1.031
28.5	27.5	....	....	28.0	29.9	0.936
27.5	27.5	....	....	27.5	29.4	0.935
25.0	25.0	24.0	23.5	24.4	25.4	0.960
23.5	23.5	....	....	23.5	25.1	0.936
24.0	22.5	....	....	23.2	24.6	0.943
23.0	21.0	23.5	21.0	22.1	23.0	0.960
21.5	21.0	....	....	21.2	23.2	0.914
19.0	20.0	23.0	21.0	20.8	24.5	0.848
20.5	19.5	20.5	20.5	20.2	23.2	0.871
19.5	17.5	....	....	18.5	22.0	0.841
17.5	18.0	18.5	18.0	18.0	18.7	0.962
18.5	17.5	17.5	17.5	17.8	20.0	0.890
17.5	17.5	....	....	17.5	19.2	0.912
....	....	17.5	16.0	16.8	17.7	0.948
15.0	15.0	15.0	14.5	14.9	17.3	0.861
....	....	14.5	13.5	14.0	18.3	0.765
(24.2)	(23.8)	(21.5)	(20.6)	(24.0)	(25.8)	(0.924)
(*b*) NONPLASTIC SEDIMENTS						
38.5	32.5	....	....	35.5	47.5	0.748
....	....	31.0	30.0	30.5	37.0	0.824
....	....	29.0	29.0	29.0	39.0	0.743
....	....	29.0	28.5	28.8	36.0	0.800
....	....	27.5	27.5	27.5	45.0	0.611
....	....	26.5	27.5	27.0	43.4	0.622
....	....	25.5	24.5	25.0	34.0	0.735
....	....	21.0	20.5	20.8	29.1	0.715
....	....	21.0	20.0	20.5	28.0	0.732
19.0	19.0	....	....	19.0	24.5	0.775
19.0	18.5	....	....	18.8	20.9	0.900
18.0	17.5	....	....	17.8	20.9	0.852
....	....	17.5	17.5	17.5	24.2	0.723
15.0	15.0	....	....	15.0	21.8	0.688
....	....	13.0	12.5	12.8	16.0	0.800
12.0	12.0	12.0	12.0	12.0	17.8	0.674
....	....	10.5	10.5	10.5	15.3	0.686
....	....	10.0	9.5	9.8	15.8	0.620
....	....	9.5	9.0	9.2	13.0	0.707
....	....	9.0	8.5	8.8	12.2	0.721
(20.2)	(19.1)	(19.5)	(19.1)	(19.8)	(27.1)	(0.734)

[a] In Table (*a*) "lower plastic limits," and in Table (*b*) "lower liquid limits."

entire end of the cylinder. Confined tests had the walls of the cylinders supported by tight-fitting brass cups, but to allow bulging at the upper surface the bearing block did not cover the entire end of the cylinder.

Thus, the unconfined cylinders under load were free to deform by bulging at the circumference, but in the confined cylinders deformation took the form of a bulge at the top end of the cylinder in the open area around the bearing block. That freedom to deform or to bulge was equal in both types of cylinder is indicated by the close agreement of critical moistures under different loads shown in Table 1(*a*).

Table 1(*a*) also shows that, whether the load was doubled or increased as much as sixfold, the change in the moisture content at which slip occurs is little affected. This was found to be true also in the confined series of tests. Detailed examination discloses that the ratio of mean critical moisture (moisture at which critical slip occurs) to Atterberg's lower plastic limit averages 0.924 with a maximum deviation of 0.266.

A comparison of average critical moistures for the several loads as compared to the mean lower plastic limit is rated as shown in Table 2(*a*). Although the numerical differences in the average critical moistures at the several loads are not large, the tendency of the percentage to be decreased under increasing load is apparent. However, since clays and other earthy sediments are known to undergo certain changes of state due to the absorption of water, the facts disclosed by Table 1(*a*) are to expected. One important fact, however, must be kept in mind: The change from the plastic to the rigid state is always marked by rupture. It matters not whether the force producing rupture is the weight of the hand, as by rolling out under the Atterberg method, or the force exerted by the largest testing machine; the principle involved remains identical. However, the speed of rolling out like the speed of a testing machine must have an influence on the results obtained.

TABLE 2.—COMPARISON OF AVERAGE CRITICAL MOISTURE

Description	(*a*) With Mean Lower Plastic Limit					(*b*) With Average Lower Liquid Limit				
	Plastic limit	5.6-lb load	12-lb load	16-lb load	32-lb load	Liquid limit	5.6-lb load	12-lb load	16-lb load	32-lb load
Percentage.........	25.8	24.2	23.8	21.5	20.6	27.1	20.2	19.1	19.5	19.8
Ratio..............	1.000	0.938	0.922	0.833	0.798	1.000	0.746	0.705	0.720	0.730

The second series of tests by Mr. Levenson is represented by twenty incoherent or nonplastic sediments obtained and tested in a manner similar to that described. However, owing to the nature of these sediments, obviously no lower plastic limit could be determined; hence, the lower liquid limit as defined by the Atterberg method was substituted as the reference point. In all other respects, determinations were made in accordance with the method followed in the plastic group.

The results obtained from this group are shown in Table 1(*b*). The mean critical moisture or the moisture at which slip was started, under conditions indicated, is expressed in Col. 7, Table 1(*b*), as a ratio of the lower liquid limit.

Although only one set of tests (Table 1(*b*), line 16) in both confined and unconfined conditions is shown, the critical moisture content is identical under all loads. However, additional tests are needed.

For unconfined loads of 5.6 and 12.0 lb, Table 1(*b*) shows average critical moistures of 20.2% and 19.1%, respectively, and for confined loads of 16 lb and 32 lb average critical moistures of 19.5% and 19.1%, respectively. The maximum difference in critical moistures obtained in confined tests is 6% with no other differences greater than 0.5%. The maximum difference obtained in the confined tests is 1.0%. Col. 7 shows the ratio of mean critical moisture to the lower liquid limit as 0.734 with a maximum deviation of 0.289.

A comparison of average critical moistures obtained under the several loads is shown in Table 2. Although the effect of load on lowering critical moisture is not always shown in individual cases, this tendency is indicated. For example, the average critical moisture in the plastic sediments tested under a 32-lb load was found to be 0.798 of the plastic limit and that in the nonplastic sediments 0.730 of the lower liquid limit. Thus, it will be seen that slip or plastic flow is affected by both fluids and loads.

Inasmuch as Col. 7, Table 1, shows variations between the critical moisture as measured by the Atterberg method and that of stress and strain, the need of a factor of safety is apparent. Moreover, if pore space is reduced to 70% of that required for a critical moisture (lower plastic sediments or 75% of the lower liquid limit in nonplastic sediments) as recommended by the writer[2] in another paper, roadbed failure is avoided.

The occurrence of slip at a definite water content has been observed by H. Thiemecke,[11] and also by Fred Burggraf,[12] who emphasizes the importance of keeping the moisture content in coherent or plastic types of roadbeds below that of their respective lower plastic limits.

TABLE 3.—EFFECT OF POROSITY-MOISTURE CONDITIONS ON STABILITY STRESS IN ROADBEDS

Sample	DENSITY		Pore space (%)	MOISTURE		Lower plastic limit (%)	Pore saturation (%)	Ratio, Col. 6/Col. 7	Stability index	Stability stress (lb per sq in.)
	Absolute	Bulk		Content (%)	Capacity (%)					
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
B-2	2.66	1.70	36.1	13.0	21.2	21	61.3	1.02	1.62	20
A	2.60	1.70	34.6	16.3	20.4	18	79.9	1.13	1.10	14
E	2.63	1.48	43.6	23.6	29.5	24	80.0	1.23	1.02	5
B	2.66	1.47	44.7	24.6	30.4	24	80.9	1.26	0.98	3

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[11] "Plastic Flow of a Nearly Dry Clay Body," by H. Thiemecke, *Journal*, Am. Ceramic Soc., Vol. 19, No. 3, 1936, p. 67.

[12] "Field Tests and Their Application to the Design of Stabilized Soil Roads," by Fred Burggraf, *Bulletin No. 50*, Am. Road Builders Assn., 1940.

In another paper, Mr. Burggraf[13] shows the adverse effect of water in excess of the lower plastic limit on the stability of roadbeds and corroborates his and other conclusions relative to the abrupt loss of bearing capacity of clays containing water equivalent to the lower plastic limit. The basic values obtained by him have been arranged by the writer in Table 3, where, for convenience, the original bulk density values given in pounds per cubic foot have been converted into grams per cubic centimeter.

Samples B-2 and A, Table 3, show stability stresses of 20 lb per sq in. and 14 lb per sq in. corresponding to moisture contents of 62% and 90%, respectively, of the lower plastic limit with pore saturations of 61% and 80%, respectively. A comparison of values in Cols. 10 and 11, Table 3, shows that when the stability index is high, so is the stability stress; and, in a similar manner, the values in Col. 9 indicate the downward trend of stability to be expected when the pore space becomes saturated. (The "stability index" is the ratio of the lower plastic limit to the moisture content.)

TABLE 4.—EFFECT OF WATER CONTENT ON BEARING CAPACITY

Sample No.	Lower plastic limit	MOISTURE CONTENT AT TIME OF TEST (%)		Actual moisture difference at test (%)	BEARING CAPACITY[a] CORRESPONDING TO MOISTURES I AND II		Bearing capacity increase (%)
		Test I	Test II		I	II	
1	30.7	33.0	30.0	3.0	400	700	75
2	26.3	26.7	24.8	1.9	373	628	68
3	24.7	24.2	22.6	1.6	410	642	57
4	21.6	21.3	19.8	1.5	462	719	55
5	19.5	18.5	17.5	1.0	537	769	43
6	18.4	16.2	15.4	0.8	650	887	36
7	16.7	14.0	13.4	0.6	698	928	33
8	15.4	12.1	11.7	0.4	737	1,035	40
9	14.6	10.4	10.2	0.2	945	1,106	17
10	15.4	9.9	9.8	0.1	1,433	1,533	7

[a] Pounds per square foot.

Additional data tending to show that the bearing capacity of earthy sediments becomes critical at or near the lower plastic limit are shown in Table 4. Data in this table are from the work[14] of K. B. Woods and R. R. Litehiser, Members, Am. Soc. C. E. (Instead of showing all the values given by Messrs. Woods and Litehiser, under the heading, "Summary of Test Averages for 467 Ohio Soil Samples,"[15] the writer shows only the moisture and bearing capacity relation of their data.)

Attention is called to the increase of 75% in bearing capacity due to a 3% decrease in moisture of sample 1, Table 4. Test II shows the moisture to be only 0.7% less than that of the lower plastic limit and that of test I, 2.3% more than this limit. In sample 2, test I is 0.4% wetter than the lower plastic limit; and test II, 1.5% drier with a consequent increase of 68% in bearing capacity.

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[13] "Field Tests on Shearing Resistance," by Fred Burggraf, *Proceedings*, Highway Research Board, December, 1939, pp. 484–490.

[14] "Soil Mechanics Applied to Highway Engineering in Ohio," by K. B. Woods and R. R. Litehiser, *Bulletin No. 99*, Ohio State Eng. Experiment Station, 1938.

[15] *Ibid.*, p. 62.

In samples 3 to 10, Table 4, all moistures are less than the lower plastic limit and the increase in bearing capacity due to decrease in moisture content ranges from 7% to 57%. In none of the latter tests is the break near the lower plastic limit as sharp as is found in the Levenson tests. However, when test values are measured above and also below the lower plastic limit, the change is marked. Moreover, inasmuch as the bearing capacity apparently was measured by means of the Proctor needle, an instrument which does not permit a steady rate of application of load, very sharp breaks or knees should not be expected. This belief is corroborated by many needle resistance curves shown as a part of the report by Messrs. Woods and Litehiser[14] and also by others, all of which are of the characteristic shape of the curves in Fig. 1.

Examination of many published data shows that the so-called optimum moisture (A.S.T.M. D698–42T) is commonly near that of the lower plastic limit and it is not unusual for the two moistures to differ as little as 1% to 5%. Furthermore, the record of density-moisture-penetration resistance values indicates that Fig. 1 is typical of many. Point B, Fig. 1, occurs entirely too close to the maximum density obtained to provide adequate bearing capacity. The plastic or near plastic condition of the mass at the maximum density obtained (not necessarily the ultimate density obtainable) is shown not only by the change beginning in the vicinity of point B, but this condition has been observed also many times on earthwork construction jobs where rolling operation showed a plastic condition, although the moisture content of the material was at "the optimum" as indicated by the laboratory test mentioned.

Evaluation of Roadbeds.—Table 5, based on data by Messrs. Woods and Litehiser,[14] is offered to demonstrate how the structural properties of roadbeds may be evaluated correctly. The average of maximum and minimum density values determined by these writers corresponding to "good," "fair," "poor," "very poor," or "unsatisfactory" roads is shown in Cols. 1 and 9. Data in Cols. 2, 3, 4, 7, and 8 were computed from basic data observed by Messrs.

TABLE 5.—COMPARISON OF MODIFIED PROCTOR STANDARD AS USED IN OHIO WITH CRITICAL MOISTURE STANDARD

PROCTOR MAXIMUM DENSITY		Pore space (%)	Moisture capacity of pores (%)	Water content of pores (%)	Lower plastic limit (%)	METHOD OF CLASSIFICATION		
Lb per cu ft (1)	g per cu cm (2)	(3)	(4)	(5)	(6)	Ratio, $\frac{\text{Col. 4}}{\text{Col. 6}}$ (7)	Pore capacity[a] (8)	Grade (9)
131.1	2.10	20.8	9.9	9.8	15.4	0.64	10.8	Good
126.7	2.03	23.4	11.5	10.2	14.6	0.79	10.4	Good
122.2	1.96	26.1	13.3	11.7	15.4	0.86	11.6	Good
117.5	1.88	29.1	15.5	13.4	16.7	0.93	11.7	Fair
112.3	1.80	32.1	17.8	15.4	18.4	0.97	12.8	Fair
107.8	1.73	34.8	20.1	17.5	19.5	1.03	13.6	Poor
103.0	1.65	37.8	22.9	19.8	21.6	1.06	15.1	Poor
97.5	1.56	41.2	26.4	22.6	24.7	1.07	17.3	Poor[b]
93.2	1.49	43.8	29.4	24.8	26.3	1.12	18.4	Poor[b]
85.0	1.36	48.7	35.8	30.0	30.7	1.16	21.5	Poor[b]

[a] Pore capacity reduced to 70% of the lower plastic limit. [b] Unsatisfactory or exceptionally poor.

Woods and Litehiser, with an assumed absolute density or apparent specific gravity of 2.65.

A comparison of Cols. 7 and 9, Table 5, shows that the order of numerical values is in agreement with the values assigned by Messrs. Woods and Litehiser. Table 3 shows a similar agreement with work by Mr. Burggraf.[13] Moreover, if pore capacity can be reduced to 70% of the lower plastic limit as shown in Col. 8, Table 5, all the materials represented can be rendered satisfactory for use in roadbeds, provided they are compressed to the densities required.

In further support of the writer's claim that roadbed stability under given moisture conditions is predictable by the ratio of moisture content to the lower plastic limit, reference is made to Table 3, Cols. 10 and 11; but, for the most adverse moisture conditions to be expected, Col. 9, Table 3, should be used instead of Col. 10. Without a single exception, Cols. 9 and 10 are in accord with the stability stresses obtained.

Whether the reader is willing to accept the factor of safety proposed is of minor consequence; but, with a knowledge of the well-known properties of the three constituents—air, water, and solids—of roadbeds, it would seem that all must agree that the mixing of fluids with solids must be controlled rigidly if stability is to be developed. As fluids weaken the structure of rocks and other less consolidated masses of matter only by entering pore spaces, the necessity of restricting the volume of the spaces is obvious. Although air is less detrimental to the structural properties of hydrous materials than water, it is indeed essential that the air content be restricted also if adequate support is to be attained. Moreover, if the air content is not restricted, a part of it is likely to be replaced by water during rainy seasons, resulting in a weakened roadbed.

Since water weakens roadbeds in a greater measure than air, it seems desirable that pore space be restricted so that if it does become entirely filled with water the roadbed will not be critically soft. Obviously the percentage of water required to produce critical softening must be determined before the plans are made. Also, a suitable factor of safety must be set and maintained during construction. Of course, the water content of the roadbed material during construction must not be in excess of the capacity of the designed pore space; but for facilitating consolidation it should be 80% to 90% of the pore capacity, a value close to that proposed by Mr. Lee[9] as 5% by volume of the pores.

Much effort has been given to the determination of the measure of consolidation required to produce stable roadbeds. The methods in general use consist of applying compactive force to laboratory samples by means of static or impact loads of given magnitude and applying this as a standard for roadbed compaction or consolidation. A report by James H. Stratton,[16] M. Am. Soc. C. E., giving the experience of the U. S. Corps of Engineers on the usefulness of these standards (A.S.T.M. D698–42T) seems to indicate that they have not been entirely satisfactory.

Inasmuch as work cannot be measured in terms of energy applied to different materials, the findings of the Corps of Engineers are to be expected. For example, a well-graded mixture of sand and clay, such as that used for sand-clay road surfaces, can be consolidated to a density of, say, 2.15 to 2.25 without great effort, although a much greater amount of energy is required to compact a gumbo clay to a density of, say, 2.00. Indeed, certain materials cannot be consolidated by ordinary mechanical processes. Of such materials, one carrying an abundance of flake mica is an example. The elasticity of the flakes causes the mass to expand or recover upon removal of the pressure. Therefore, the mere application of a compacting force cannot be utilized successfully as an index of consolidation of roadbed materials; if this were not true, tooled steel could be machined to the same shape as soft steel in an equal period of time.

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[16] "Military Airfields": A Symposium, *Transactions*, Am. Soc. C. E., Vol. 110 (1945), pp. 670–696.

Consolidated Roadbeds.—In the case of two approach fills made of black gumbo clay, the following contrast found at the time of paving between rains in the month of December is recited: The east approach, graded and water jetted for compaction several years previously, had been subjected to traffic until the time of paving. The west end was graded and consolidated a few weeks previous to the time of being paved so that the pore space was less than that required for water equivalent to the lower plastic limit. The east end had to be "planked" throughout for the paving operations but the west end did not rut at all. Although the subgrade was quite slippery on the west end, a few shovelfuls of sand applied to wheel tracks eliminated this slipperiness. Weather conditions were considered nearly identical during the time that both approaches were paved.

On another black gumbo clay road, a section of brick pavement had to be removed for an overhead bridge approach. The bricks were laid on a thin layer of sand and the surface of the pavement was reasonably smooth. The gumbo clay under the pavement was found to be moist but firm. The manner of consolidation is not known; but, whatever the method used, it appeared from the smoothness of the old brick pavement to be adequate for heavy loads such as those needed on truck-line highways in a region engaged in the production of mineral oil.

A concrete pavement built in 1917 in California between Patterson and Newman by Lloyd Aldrich,[17] M. Am. Soc. C. E., then county engineer of Stanislaus County, California, was found three years later to be in fine condition over all types of soil (including Yolo adobe) except at the extreme north end through a village where the pavement had been laid on a roadbed not prepared by Mr. Aldrich. In contrast to other roads, no shrinkage beneath the road slab was observed.

The method followed by Mr. Aldrich in constructing the roadbed requires light lifts of pulverized and moistened material compacted by means of heavy rollers. Before laying the pavement the subgrade is allowed to dry five or more days, during which time shrinkage cracks do not develop. The failure of shrinkage cracks to develop is a positive indication of the use of a very limited amount of water and of a high measure of compaction of the roadbed—two conditions essential to the longevity and smoothness of all types of pavement.

The Pittsburg, Calif., test highway[18] also built under the supervision of Mr.

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[17] "Report on California State Highways," by J. B. Lippincott, Waller C. Howe, C. Derleth, Jr., and H. J. Brunnier, copyrighted 1921 by Automobile Club of Southern California, Los Angeles, Calif., and Calif. State Automobile Assn., San Francisco, Calif., pp. 112–114.

[18] "Report of Highway Research at Pittsburg, California, 1921 and 1922," by Lloyd Aldrich and John B. Leonard, California State Printing Office, Sacramento, Calif., 1923.

Aldrich (1921) was constructed in a manner similar to that of the Patterson-Newman road; and, of the seven conclusions following the tests, the first one is significant. It reads: "The method of constructing the subgrade on adobe soil, as described, so reduced the objectional features of this material that a reliable foundation for the pavement resulted."

Numerous other instances of the stability of naturally occurring or artificially produced beds of consolidated clay or sand-clay could be recited. Frost action on these consolidated beds has been studied by the writer in regions where the ground freezes to depths of 6 ft and not a single instance is recalled where freezing appeared to be detrimental. The reason for this is simple—pore space being limited, little water finds its way into these masses of material and enough air space is left to allow for the expansion of freezing water.

Unconsolidated Roadbeds.—The sight of a curb in a southern city is recalled where plastic movement had distorted a curve horizontally and vertically, all in a length of 100 ft. The material in which the curb was placed was a silty clay.

Following the inundation of the Omaha (Nebr.) Municipal Airport by the Missouri River in April, 1942, W. H. Campen, Assoc. M. Am. Soc. C. E., and J. R. Smith[19] made a survey to determine the effect of water on the density and water content of the compacted earth subbase and also the gravel-clay base of the airport runways. The inundation covered the airport grounds to depths of from 3 ft to 7 ft for a period of from 10 to 17 days. The depth of water and the duration of the inundation created conditions for saturating the runways which were unusually favorable. The compacted earth subbase was from 6 in. to 12 in. thick and the gravel-clay base approximately 6 in. thick.

The survey revealed at least one fact that is almost always overlooked. It is the effect of pore capacity on the maximum water content, the uniformity of saturation to be expected for given types of materials, and their measures of compaction. The values given by Messrs. Campen and Smith indicate skill and careful work. In only a few cases is the computed pore capacity less than the actual water content, a condition probably due to the writer's use of an erroneous value of specific gravity of fine solid particles—hence, the confidence that is warranted in the values given in Table 6. The table justifies the conclusion that water content can be controlled by the restriction of pore capacity. This evidence and numerous other data further confirm the writer's belief expressed elsewhere[2] that the pore spaces are rarely completely filled with water even after long periods of inundation. In only two cases did the tests show complete saturation of the compacted earth subbase and only one case of saturation as low as 76%. In the base, 99% saturation was found at two points and at only one point was saturation found as low as 63%.

The data on bases indicate a much lower percentage of saturation than do the data on compacted subbases. This tendency has been observed often by the writer when comparing highly compacted bodies with less compact ones. A case is recalled in which a very stable federal-aid sand-clay road surface in the State of Nebraska showed an average saturation of only 31% at the time of a spring thaw following a winter of considerable snow.[20] Such projects demonstrate the value of density and water resistance of minute pore spaces.

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[19] "The Behavior of Densified Soil-Water Mixtures Under Very Adverse Water Conditions," by W. H. Campen and J. R. Smith, *Proceedings*, Highway Research Board, Vol. 23, 1943, pp. 450–458.

The writer's tentative conclusion is that, as pore spaces are restricted, the penetration of water and the escape of air become increasingly difficult, thus accounting for the greater abundance of air in the minute and sealed pore spaces. On the other hand, it appears that deposits of silt nearly free from clay are more susceptible to high or complete saturation, due probably to the ease with which these fluids enter and escape from sediments low in cementing or claylike matter. This condition has been observed many times in eastern Iowa.

TABLE 6.—MEAN MOISTURE AND CAPACITY OF THE FLIGHT STRIPS OF THE OMAHA (NEBR.) AIRPORT FOLLOWING ITS INUNDATION IN 1942

(Percentages)

Item No.[a]	Moisture		Pore saturation	Year built
	Capacity	Content		
(*a*) SUBBASE				
1	21.4	19.8	92.5	....
2	20.8	19.7	94.6	....
3	18.4	18.0	97.8	....
4	17.8	17.0	94.9	....
Mean	....	....	(94.9)	....
(*b*) BASE				
6	8.1	7.0	86.4	1936
6	7.1	5.5	77.5	1938
7	6.4	5.2	81.2	1941
7	6.2	5.1	82.2	1942
Mean	....	....	(88.2)	....

[a] These item numbers correspond with the table numbers in the work by W. H. Campen and J. R. Smith (*Proceedings*, Highway Research Board, Vol. 23, 1943, pp. 450–458).

Many instances of "flow ridges" or "cat-step" topography are recalled in the "Great Plains" of the United States and in belts along the Ohio River. In these areas landslides are common. Investigations show that these sediments are very porous and hence become so filled with water that they tend to slip downhill. Often the slip is so pronounced that trees become bent in an effort to right themselves. These areas contain many "soft spots" and in regions of intense freezing "frost boils" are numerous.

Characteristics of certain beds of material in their natural states, and also in compact states, were presented by the writer in 1938.[20]

In contrast to the statement made relative to the stability of consolidated deposits of sediment, the following statement concerning unconsolidated sediments can be made: In an area extending from the Canadian border to the Gulf of Mexico and from the Atlantic Ocean to the Rocky Mountains, the writer has found roadbed failures only in unconsolidated sediments. Many failures of "stabilized bases" and bituminous mats have been observed, laid on roadbeds that had not been compacted to extents that precluded the entrance of water into the pore spaces in amounts equal to or in excess of that of critical moisture. Numerous failures have been observed also when the roadbed foundations have been compacted to so-called "maximum density at optimum moisture content." Frequently these "compacted roadbeds" have become so saturated with water from windrows of wet aggregate on the roadway that mixing and laying of bituminous mats or other surfacing material had to be delayed to allow "soft spots" to dry. Although the soft spots may have been allowed to dry before stabilized surface courses or bituminous mats were laid, many observations of later failures on these areas furnish clues that future failures are to be expected. Pore spaces for absorbing water being present, only water and loads are needed to complete the ever-recurring cycle of wetting, instability, failing, and drying. Truly it may be said that the patterns of the soft spots of today are to be those of failure tomorrow.

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[20] "Principles Applying to Highway Road-Beds," by Ira B. Mullis, *Transactions*, Am. Soc. C. E., Vol. 104 (1939), Table 1, p. 1393.

Adequate Consolidation.—What constitutes adequate consolidations? Ideally two types of material are essential—a hard durable aggregate, well-graded from coarse to fine, and a glue-like material such as bitumen or a high-strength clay to coat the aggregate and bind it into a dense and highly resistant mass. The proportion of binder to aggregate must be such that the load will be carried only by the aggregate. Hence, the volume of the binder must be slightly less than that of the voids. The fluidity of the binder at the time of mixing should be such that all solids are evenly though thinly coated. This is a condition to be sought when applying any binder, whether it be bitumen or clay. However, if the volume of the binder plus the volume of its solvent is in excess of the volume of voids, the solvent must be allowed to volatilize until the binder and solvent can be contained in the voids as designed; otherwise, a plastic and porous mass will be obtained.

In case the binder is clay, consolidation must be such that water cannot find room to enter in a quantity sufficient to cause critical softening of the mass. However, since small quantities of water are essential to toughen or temper the clay binder, the presence of limited amounts of both air and water is essential, as these work in harmony under properly guarded conditions to preserve the stability of the structure.

Since it has been proved that clays and other fine-grain sediments show yield values at definite moisture contents, it seems obvious that the moisture content at which slip or yield starts, even under light loads, should furnish a sound basis for the design of roadbeds. The points at which materials yield under high loads or high temperatures and the laws that attend these critical changes must not be violated. Engineers recognize with profit the action of solvents on paint, wax, etc., and they must also recognize the weakening effect of water on clay.

Water does not weaken any form of rock matter until it penetrates the pores. Moreover, rock matter is weakened in direct proportion to the water absorbed into the pore spaces until the critical point is reached. At this point, the additional loss of strength far exceeds the final increase in water content.

Two methods—the Atterberg method and the so-called stress-strain method—have been mentioned. The Atterberg method can be used only on fine-grain sediments, but the stress-strain method can be applied to all coherent bodies.

Following the practice of designing with reference to the yield point of structural materials, the engineer must learn to follow a similar procedure in designing roadbeds. As with other engineering materials, a factor of safety is essential. To determine such a factor, the writer[2] proposed in 1938 that the capacity of the pore space be restricted to 70% of the water content at the lower plastic limit. This means that a material with a lower plastic limit of, say, 20 must be consolidated so that its pore space could contain only 14% of water, the equivalent to a density of 1.93. For such consolidation, 11.5% to 12.5% of water and a heavy sheepsfoot roller or some other equally efficient means would be required.

However, the recommendation that pore space be restricted to a water capacity of not more than 70% of the lower plastic limit is not to be regarded as all that is desired. Whenever economically possible to do so, the reduction to smaller percentages will result in greater safety against water softening and also will increase the bearing capacity. Moreover, in incoherent materials, such as sand and silt, the mixing of clay is necessary to consolidation and its attendant stability.

Inasmuch as roadbeds are designed to resist both traffic and weather, it would seem that a measure of resistance to impact and a measure of resistance to the disintegrating effect of water should be of value. For this reason the plasticity index was compared to the corresponding cementation and slaking values.

Cementation or resistance to impact was measured by means of the well-known Page machine, once used as a standard measure of cementing value of rock dust used in macadam construction. The earth specimens to be tested were molded in the standard manner, dried to constant weight and cooled. The number of blows required for breaking a standard briquet represents its bond strength. The bond strength of individual specimens was plotted against its plasticity index. Fig. 3 shows the relationship of the values obtained.

Time of slaking was determined on specimens molded in the same manner and at the time that briquets were molded for the cementation tests. The slaking value obtained for each specimen was plotted against its plasticity index value as shown in Fig. 4.

The results obtained from the two series of tests show values of the scatter chart type without apparent relationship. From these values one may compute the mean average number of occurrences for the several plasticity indexes and thereby derive a statistical relationship similar to that found in life expectancy tables used by life insurance companies. Of course, the average death rate and expectation of life are quite accurate for large groups but for a

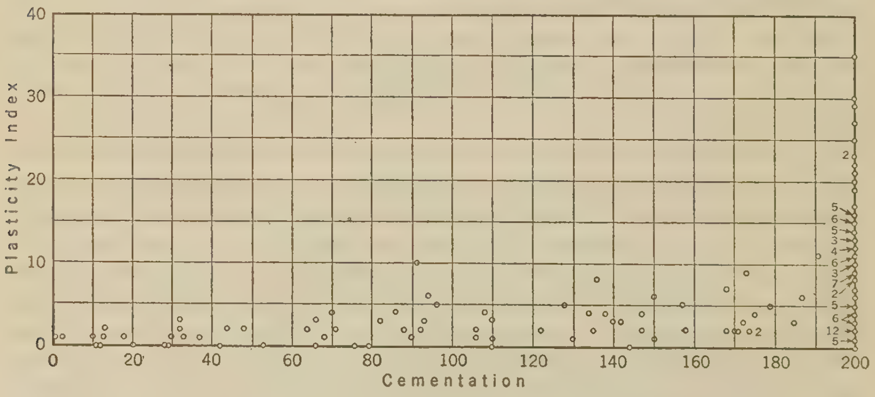

Fig. 3.—Relation Between Plasticity Index and Cementation

single person the data do not apply. For example, a large group of persons twenty-one years of age are expected to die at the rate of 7.85 per 1,000 and have an expectation of life of 41.53 years, although any one of a group may not live another day. Although age does have a definite effect on death rate and expectation of life, there are too many chance causes, such as disease and accident, to permit tables of mortality to be of any value whatsoever in predicting the duration of life of a particular person. Similar relationship is found in many engineering data and to be safe one should consider the probability of occurrence of the least favorable condition in applying such data. It would seem, therefore, that the average plasticity index curve is not only worthless for individual applications, but that it may be dangerous to apply. Numerous road failures on projects constructed on the basis of plasticity indexes support this statement.

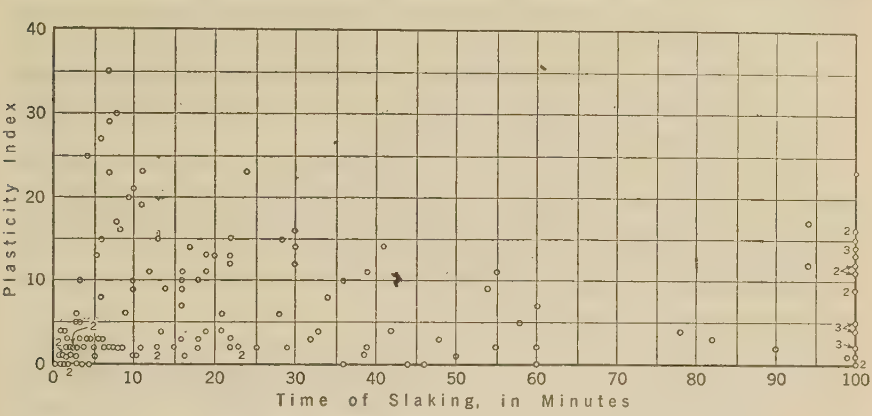

FIG. 4.—RELATION BETWEEN PLASTICITY INDEX AND TIME OF SLAKING

Once a roadbed is consolidated as described herein, it must be protected against weather by a weather-resistant and traffic-resistant surface such as sand-clay, bitumen, etc.; otherwise, these forces tend to destroy the stability of the crust thus obtained. The principles discussed in this paper are well known. Some must have been apparent to prehistoric man and others are as old as the science of physics itself. Geologists have observed them many years and have recognized their effects on resistance to topographic change. Applied to highway work, these principles are equally indicative of roadbed stability. Indeed, it would seem that, since it is known that hydrous materials require certain proportions of water to produce critical softening, the necessity of restricting pore space to prevent it should be a self-evident conclusion.

SUMMARY

The writer invites a full and free discussion of this method of design and construction in comparison with other methods now in use for the purpose of obtaining roadbeds of higher measures of stability for postwar roads. Entirely too many roadbed failures are occurring, even on projects built under the supervision of those following approved methods; hence it would seem that a thorough review of basic principles is imperative.

At least fourteen topics are offered as a guide to such discussion:

1. Three essentials—air, water, and solids—constitute all roadbeds. Each constituent has its own function but the volume of fluids must be restricted rigidly.

2. The presence of excessive volumes of fluid produces a change of state that destroys roadbed stability. This change of state is a critical point.

3. The change of state in roadbeds, as discussed in this paper, is characterized by the loss of rigidity, and failure under load occurs through plastic distortion rather than through planes of rupture.

4. High resistance to loads and the penetration of water can be developed best when pore spaces are caused to be minute and of the isolate type. In this condition the movement of water into or out of pore spaces is restricted by molecular forces of considerable magnitude. Moreover, pore spaces must be so restricted that the presence of either air or water will not be sufficient to destroy the rigidity of the roadbed.

5. Although high-strength clays are difficult to shape and consolidate, they offer great resistance to loads and weather when consolidated at moisture contents that preclude cracking upon loss of water. More thought and experimentation on clays of the gumbo type are urgently needed.

6. The use of tenacious clays is recommended for the bonding of aggregates. These clays can be best ground in water and applied to aggregates in the form of a slurry. The elimination of air by puddling is important but consolidation must be delayed for adequate drying.

7. Clays of the kaolin type are shaped easily, require little water for compaction, but are too weak to offer much resistance to traffic and weather.

8. Consolidation of tenacious clays should be attempted only by means of sheepsfoot rollers exerting pressures of from 300 lb per sq in. to 675 lb per sq in. or more. The water content should be the optimum for the force or pressure utilized.

9. The shrinkage of clays is not an inherent property of the solid matter but a measure of water lost. Therefore, clays should be consolidated under high pressures with water contents equaling from 80% to 90% of the designed pore space.

10. The use of stabilizing materials such as gravel, slag, and stone aggregates may be profitable, but not until local material has been utilized and compacted to its economic limit.

11. As applied energy is not equivalent to work required, standards of compaction or consolidation cannot be measured successfully in terms of static or impact loads applied as compactive effort.

12. The standard of consolidation proposed for roadbeds is based on critical moisture with an adequate factor of safety. Obviously, the higher the factor of safety, the more resistant the roadbed will be to traffic and weather.

13. Measurement of resistance to loads and weather should be made by means of stress-strain methods on laboratory specimens or on roadbeds artificially saturated with water.

14. For further protection against traffic and weather, fine-grain sediments must be surfaced with protective coatings, such as sand-clay, bitumen, etc.

DISCUSSION

Charles W. Britzius,[21] Assoc. M. Am. Soc. C. E.—The paper seems somewhat academic. Admittedly, to provide a stable subgrade, it is necessary to keep the moisture content low, and this can be done best by compacting the subgrade soil to a low void content. It is impracticable, however, to compact the heavier soils to such a density and low void content that the critical moisture content would not be exceeded. A compacted soil mass does not absorb water in its air-void spaces without expanding, and thus increasing in volume.

TABLE 7.—Factors that Reduce Expansion and Water Absorption

Material	Percentage Retained:				Liquid limit (%)	Plastic limit (%)	Plasticity index	Optimum moisture (%)	Maximum density[a]	Air Voids at 95% Proctor Density:			
	No. 10	No. 270	0.005 mm	<0.005 mm						Total voids (%)	Moisture content[b] (%)	Plastic limit (%)	$\frac{\text{Col. 11}}{\text{Col. 12}}$
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
Sandy loam...	0	63	27	10	23	14	9	10.0	127.5	26	13.4	14	96
Clay loam....	0	38	42	20	35.5	16.5	19	15.5	111.4	35	20.4	16.5	124
Clay.........	0	10	35	55	39.5	20.5	19	19.5	99	43	28.7	20.5	140

[a] Modified Proctor density, in pounds per cubic foot. [b] Moisture content when voids are filled.

The factors that will reduce the amount of expansion and water absorption are the weight of the overlying soil, the moisture available, and the moisture content and density at the time of compaction. The writer wishes to present some data on the latter factor. Consider three soils, a sandy loam, a clay loam, and a clay. Their properties are as given in Cols. 1 to 9, Table 7.

A reasonable, practical, and workable requirement for compaction of these soils is 95% of the Modified Proctor maximum density. At this density the air voids in the soil mass will be as shown in Cols. 10 to 13, Table 7. The air-void values, for these soils, do not conform to the suggested requirement that the pore space be restricted to such a value that, when the space is filled, the moisture content does not exceed 70% of the plastic limit. Because extreme densification is not practicable, thicker and more carefully graded base courses must be used to carry the loads.

Compacted soil masses swell and absorb water to some extent after compaction, until a so-called equilibrium is reached. Soils compacted drier than the optimum swell and absorb water to such a great extent that they finally reach a high moisture content—the same condition that would have resulted if the soils had been compacted beyond the optimum originally. The new residual density curve for sandy and silty soils lies just below the Proctor moisture density curve; but, as the soil becomes heavier, the top of the curve moves to a higher moisture content. The practical implication is that a soil should be compacted to the specified density at such a moisture content that the lowest final moisture content and greatest density will exist. For most soils this moisture content is just greater than the Proctor optimum, and for expansive and gumbo clays it may be considerably greater.

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[21] Twin City Testing & Eng. Laboratory, Inc., St. Paul, Minn.

In field soil compaction, many times it is expedient to compact soils drier than the optimum. To prevent greater swell and moisture absorption than would have occurred if the compaction had been done at the optimum, a greater density should be required.

The observation made by Mr. Mullis (see heading, "Change of State: Consolidated Roadbeds") to the effect that soils compacted to low air voids are resistant to frost action is misleading. Soils are adversely affected by frost action not because of any change in the soil, water, and air constituents of a unit soil mass due to freezing, but because of an increase in moisture content due to the movement of water toward the source of the freezing. If the soil is silty and thus of a type that has a favorable porosity and capillarity, the heaving due to moisture accumulation will be as much as 1 ft. When the frost heave has melted, a frost boil remains.

J. T. L. McNew,[22] M. Am. Soc. C. E.—One solution to the problem involved in the design of stable earthwork is proposed by the author. Although not specifically identified as such, the author has upheld vigorously the fundamental engineering principle that diagnosis must be thorough and complete before any proposed solution may have better than an average opportunity for success.

Design of earthwork structures involves, from the very beginning, a frank and straightforward admission that soils are materials far different from steel, concrete, and stone. These latter materials are almost always such that they may be assigned certain average characteristics, and the designs resulting therefrom may be expected to have average behaviors. On the other hand, soils are subject to almost all the "peculiar" behaviors and are influenced in their permanent behavior by many more natural occurrences than is steel.

Probably the most serious oversight now common in the design of soil structures is the failure to modify a "theory" as dependent upon the variation between the climate at the site of the structure and the "climate" of the laboratory. Certainly the roadbed that is successful in arid regions of the southwestern United States might not be successful in the Gulf Coast section, nor in regions subject to hard freezes and thaws. The addition of water beyond a small proportion weakens plastic soils so far as load-carrying ability is concerned, and the water that enters a dry, compacted soil first enters through pore space. Just what happens after pore space in a soil is filled with water is quite another matter. Unless such things are considered before construction is started, and unless provisions are made for them, new failures may, and often do, result.

As a principle of design to be adopted, the author's concept, that consolidation should be such as to attain a chosen percentage of pore space in terms of the lower plastic limit, is unquestionably sound. Such a procedure may provide the best design for a given soil; and, as the author states, such densities may not be attainable with reasonable construction effort. The requirement of high density may properly lead to the use of the author's suggested procedure in the selection of better raw materials, or even to the use of other expedients to exclude water from the irreducible pore spaces. Perhaps the soil structure may be planned for a locality where the intrusion of water seldom occurs.

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[22] Vice-Pres. for Eng., Agri. and Mech. College of Texas, College Station, Tex.

There seems to be no substitute yet for competent and careful engineering diagnosis of soil characteristics and for an accurate evaluation of the "loads" to be carried, both as a result of climate and traffic, in the engineering design of roadbeds. "Rules of thumb" probably do not exist that will replace the engineer's work in a diagnosis of all the problems prior to the prescription of a remedial treatment or cure. It is stimulating, therefore, that Mr. Mullis has insisted that the fundamental behaviors of soils be studied thoroughly before a soil structure is erected. There are reasons for the inability of a soil to do some things; there are ways in which almost all types of soil can do certain jobs; and there are many ways that these types of soil can be misused. Therefore, the engineer must discover these ways with the aid of his "on the job" investigations of the fundamental soil behaviors and, in the light of these findings, propose the design most likely to succeed. Truly, as the author states (see "Introduction"): "The method of approach is a rational one, the principles of which have been long understood, although habitually overlooked."

Gustavo Pérez Guerra,[23] Jun. Am. Soc. C. E.—Considerable difficulty has been encountered, in several cases, in attempting to compact soils that were at their optimum moisture content. In most such cases the difficulty is attributed to imperfect control of embankment operations. Disregarding that simple explanation, many engineers have thought that there was a missing link in the practical application of the compaction principle.

The method of design proposed by the author is founded on well-established facts and sound reasoning; it should be considered with interest by soil engineers. To the writer it seems that the case presented is so clear and evident that this paper should lead to a complete revision of compaction standards.

When it was developed in 1933, the Proctor method was a valuable scientific tool which eliminated rule-of-thumb methods. It was later standardized and the mechanics of its routine application obscured several well-known and important factors.

It is a known fact that, more often than otherwise, the optimum moisture content for standard compaction is only a few points below the lower plastic limit. It also is known that there is a "slip" when that optimum content is exceeded. The basic process of compaction was not misunderstood, only that the standard compaction was the best that could be obtained using roller pressures of about 200 lb per sq in., as was the case in the early thirties, while at present it is common to use pressures as high as 290 lb per sq in. for regular work and much higher pressures for special compaction problems. Several manufacturers list, as standard, rollers producing a ground pressure of 390 lb per sq in. The tendency for rolling to create higher ground pressures has emphasized the deficiencies of the standard method and at the same time has made possible the development of the new method of design proposed by the author.

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[23] Civ. Engr., Parsons, Brinckerhoff, Hogan & MacDonald, Caracas, Venezuela.

In general, the writer agrees with the conclusions of the paper and draws the following practical conclusions:

(1) The curves of moisture-density relation obtained by the use of different compacting forces can be considered as theoretically tangent to the zero air-voids curve with the ascending maxima located on a curve roughly parallel to that curve. The standard curve determined with a specified number of blows is one of the lowest of the range and thus this test furnishes a minimum of compacted density with a safety factor equal to or slightly greater than unity. It is obvious that the designer should try to avoid that minimum.

(2) Rollers of different sizes are available in the market, and their indiscriminate use in conjunction with the standard compaction test defeats the advantage gained by the use of higher pressures.

(3) Extensive correlated laboratory-field tests are urgently needed to set up new compaction standards. There appears to be no need to discard the popular standard apparatus for routine determinations, provided enough correlation is secured. Nevertheless, the basic determinations of critical moisture for the new approach, and the need for load and weather tests, require the use of the stress-strain method and the testing of full-size sections.

Jacob Feld,[24] M. Am. Soc. C. E.—The paper analyzes the stability of the soil as a mixture of gas (air), liquid (water), and soil solids (rock fragments), and attempts to fix limiting percentages of each state of matter as a criterion for the absence of undesired physical characteristics. Actually, the argument presented is applicable to all soil stability problems, not only to roadbeds.

Complete logical solutions of the problem can be expected only when all the factors are considered. A soil is not merely a mixture of solid rock fragments, liquid water, and gaseous air. The air fraction may be partly gas, partly liquid, and even partly solid (occluded gas held as solid constituents by certain minerals and metals). The water in the soil may be (and usually is) partly gas, partly liquid, and partly solid. The rock fragments are sometimes entirely solid (clean granular soils), but also sometimes entirely liquid, and even partly gaseous (dust storm and soot additions). The state in which the soil constituents occur depends largely upon the absence or presence of colloid admixtures or impurities. The colloids may result from the mechanical or chemical disintegration of the rock fragments, or from animal and vegetable refuse leachings. Minute colloid admixtures, if chemically and electrostatically correct, have major control of the physical action of soil mixtures.

When the influence of colloids is fully appreciated, and when a soil can be properly identified and analyzed for all its physical and chemical constituents, not only can the physical behavior of the soil be prophesied, but the desired physical characteristics can be produced by the addition of colloidal admixtures.

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[24] Cons. Engr., New York, N. Y.

In the 1921, 1922, and 1923 progress reports of the Special Committee to Codify Present Practice on the Bearing Value of Soils for Foundations,[25] there is included an exhaustive treatment of the colloidal properties of clay as a soil constituent. In a discussion of these reports, the writer extended that treatment to include the other colloids often found in soils—silicic acid, silicates, aluminum and iron hydroxides, humus acid, as well as colloidal clays of various types.[26] The effect of various colloids on different soil types is there described not only on the physical property but also on the fertility of soil. Much useful information on this subject can be found in the reports of soil studies by the agronomists.

That air, water, and rock fragments are not always gas, liquid, and solid, respectively, at normal temperatures and pressures, is not a new idea. For instance, Galileo, in 1640, understood that there was a difference between liquid water and pore water. He states that the smallest parts into which water can be divided must be smaller than the smallest consistency of water; for, no matter how small or finely a powder is divided, the particles can still be wetted. That water is different from the smallest divisible quantities of liquid water.[27]

In 1920, Karl Terzaghi, M. Am. Soc. C. E., stated that water films less than two millionths of an inch in thickness are semisolid in action. Such water does not boil or freeze at normal temperatures. C. A. Hogentogler,[28] Assoc. M. Am. Soc. C. E., has shown that, although the division between the solid state and the plastic state depends on the percentage of moisture, the percentage decreases as the soil is put under pressure; and the pressure can be applied either externally or internally, the latter by reduction of the water film thicknesses. H. F. Winterkorn,[29] Assoc. M. Am. Soc. C. E., has added, to this study, the proof that the entire relation between moisture percentage and plastic state can be changed by adding chemicals to the water.

When it is kept in mind that, as W. D. Bancroft[30] states, all solids tend to adsorb or condense upon their surfaces any gases or liquids with which they come into contact, the importance of colloid-sized particles must be evident; for colloidal particles have very great ratios of surface area to weight.

Many practical applications of soil control by colloids and of clay soil control by chemicals can be found in recent reports. One application developed by experience, and not as a result of laboratory or analytical study, may be of interest. Seneca County, in New York State, is almost entirely covered by soft shale and extremely fine clay. When the surface is disturbed, there are only two conditions—dust clouds when dry and a slick slippery surface when wet. Nevertheless, local farmers have used a layer of crushed local shale rock, taken from the exposures where thin limestone seams occur, to build the secondary roads. The rock, in pieces up to 50 lb in size, are spread along the desired location, and preferably they are spread directly upon the ground surface. Traffic and weather form a fairly smooth road surface in a few months. These surfaces, locally known as "shuck roads," cannot be made from shale that lacks the requisite limestone content.

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[25] *Proceedings*, Am. Soc. C. E., February, 1921, p. 9; March, 1922, p. 523; and October, 1923, p. 1732.

[26] *Ibid.*, January, 1924, p. 114.

[27] "Opere di G. Galilei," by Galileo Galilei, 1808 Ed., Vol. 3, p. 25.

[28] *Proceedings*, Highway Research Board, Vol. 18, Pt. II, 1938, p. 9.

[29] *Ibid.*, Vol. 21, 1941, p. 415; also Vol. 16, 1936, p. 293.

[30] "Colloid Chemistry," by W. D. Bancroft, McGraw-Hill Book Co., Inc., New York, N. Y., 1932.

When the writer was required to produce large areas of serviceable parking fields and bus terminal pavements, with a minimum of labor and no outside materials, as part of the construction facilities for a war project in Seneca County, he applied the "shuck road" idea, with very good results. A local fine sand deposit was found to contain a large limestone content, from the leaching of limestone bedrock. The sand was unfit for construction purposes. By disking about 1 in. of this sand into the clay surface, adding a slight quantity of water, and rolling with a 10-ton roller, a hard impervious surface was obtained. The top crust, about 4 in. thick, hardened in a few days and was used as a parking field, with practically no maintenance, for a year. A sprinkling of limestone chips was applied in the bus terminal areas, and these were used for three years with little maintenance.

The paper by Mr. Mullis is of great value in pointing to a logical approach to soil compaction studies. The method can be made of far greater value by adding the necessary factors of colloid action.

Jasper L. Stuckey,[31] Esq.—Strong arguments are presented by Mr. Mullis for the rigid control of liquids and solids in the development of stable roadbeds; also excellent reasons are offered for a better knowledge of geological materials and the fundamentals of geology on the part of the highway engineer. A better appreciation of both of these should be helpful to all highway engineers, and especially to those concerned with the problem of constructing permanent roadbeds for modern traffic conditions.

Long before the development of modern roadbeds it was known that too much water on the roadway caused softness and ruts. As a result, roads generally followed the ridges and divides in order to take advantage of natural drainage. It is but natural that, with changed demands for roads and highly specialized facilities for research, methods should be developed for producing permanent roadbeds where they are needed. Mr. Mullis should be especially commended for the contribution he is making to this end.

It is interesting to note that of fourteen topics offered as a guide to any discussion of the paper, seven deal with the physical properties of the geological materials necessary in roadbed construction. It is also interesting that these materials are properly designated and not erroneously called "soil" as is so often done.

Since most of the materials used in roadbed construction are obtained along the line of the road or from local sources of supply, the engineer should consider two factors: (1) The geological conditions that may affect the permanence and stability of the roadbed; and (2) the kind, character, and proper utilization of the materials available for construction. Failure to do this has caused, and is causing, entirely too many roadbeds to prove unsatisfactory.

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[31] Prof. of Geology, Univ. of North Carolina, Raleigh, N. C.

Dense rocks such as granites, quartzites, and many limestones, in undisturbed masses, might be considered ideal materials for roadbed foundations. They have low porosity and are known to be highly resistant under most conditions. In the majority of cases, however, fresh, unaltered rock of any type is seldom found in adequate quantities at or near the surface. In many areas, weathering and rock decay have produced a mantle, of variable thickness, of decayed rock material which overlies the hard and fresh rock and grades gradually into it. If, as in the Atlantic Coastal Plain, sediments which are not deeply weathered are found at or near the surface, they are often of variable character and poorly consolidated. Since this is true, the highway engineer is forced in most cases to use materials that have high porosity and low resistance and require special treatment to be made into a stable mass such as is required in a modern roadbed.

Building stone usually is tested dry because its strength often decreases when wet. T. L. Watson, F. B. Laney, and G. P. Merrill[32] have shown that sandstone, with an absorption of 3.71% and 4.2%, tested only slightly more than one half as strong when saturated as when dry. This evidence strongly supports the argument for the controlled mixing of fluids with solids if roadbeds are to be stabilized.

The crushed stone, gravel, and sand used in the production of concrete for road paving and the preparation of the concrete have been standardized and closely controlled for many years. This paper definitely indicates the need for higher standards and better controls in the building of roadbeds.

Delbert L. Lacey,[33] Esq.—Among highway engineers there are two schools of thought regarding the proper density to which soils in roadbeds should be compacted. One group contends that an extremely high soil density with a resultant low void space is necessary to provide adequate permanent support for traffic loads. The other group contends that each soil has some "natural" density or range of "natural" density, and although placed outside the density range, either above or below it, when exposed to cycles of freezing and thawing and moisture changes, the soil will, return to its "natural" density state. Therefore, it is the contention of the second group that it is useless and a waste of effort to compact fine-grained plastic soils to a density greater than the "natural" density as such soils will eventually return to that density.

The contention of the second group of engineers is largely based on observations made on plastic soils that were compacted at low moisture contents and fairly high densities. Pavements placed on such soils become distorted as the soil beneath the edges of the pavement and beneath expansion joints absorbs moisture from the surface—causing the soil to undergo a considerable volume increase and to exert sufficient force to lift sections of the pavement above the original position. Since not all the subgrade soil has access to this moisture increase, the part of the soil under the center of the pavement and for some distance back of the joints does not undergo any volume change, and the remaining pavement stays in its original position. The result is a very disagreeable riding surface.

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[32] *Bulletin No. 2*, N. C. Geological Survey, 1906, pp. 235–236.

[33] Field Soils Engr., State Highway Comm. of Kansas, Topeka, Kans.

However, laboratory research conducted by the Kansas Highway Department[34] to determine the expansive properties of soils indicates that the moisture content at which a soil is compacted has more influence on volume increase (while the soil absorbs moisture) than does the density to which it was compacted. During this research, specimens of a soil were compacted at a number of different moisture contents and to various degrees of compaction. A load equivalent to the weight of a concrete pavement was placed on each specimen, and each specimen was permitted to absorb water until all vertical movement of the soil ceased. The vertical movement was measured and the volume increase of the specimen, based on original volume, was calculated. The results of a series of these tests conducted on a silty clay loam soil to determine percentage of volume increase are shown in Table 8. The volume change of a soil definitely decreases as the initial moisture content at which the soil was compacted is increased. At any given moisture content, on the other hand, there is very little decrease in the soil swell if the density to which the soil was compacted is increased.

TABLE 8.—THE PERCENTAGE INCREASE IN THE VOLUME OF SOIL AS AFFECTED BY THE COMPACTED DENSITY AND THE PERCENTAGE MOISTURE CONTENT AT WHICH THE SOIL WAS COMPACTED

Compacted density (lb per cu ft)	PERCENTAGE MOISTURE WHEN COMPACTED				
	10	12	14	16	18
94	1.0	3.2	2.6	1.6	....
100	5.5	3.6	2.6	1.8	1.2
106	5.7	4.4	2.6	1.6	0.7
112	5.6	4.1	2.5	1.4	0.0
115	5.5	3.6	1.5	0.8	....

Therefore, had the plastic soils that caused pavement distortion been compacted to the same high densities, but at higher moisture contents, the volume change would not have been large enough to cause any appreciable change in the pavement elevation. This conclusion is logical when it is considered that, under an impervious surface that retards evaporation, soils tend to absorb moisture until their void space is nearly filled. Therefore, if sufficient moisture is present in the soil during compaction nearly to fill the void space and to satisfy the colloidal requirements, there will not be space for enough additional moisture to cause detrimental soil swell. In the light of the foregoing information and line of reasoning, there seems to be very little danger of overcompacting a soil if it contains sufficient moisture to fill the voids when compaction is completed.

In further support of the author's contention that high soil densities are essential to a durable roadbed, it should be emphasized that the deflection of the surface of a soil caused by a given wheel load is directly dependent upon the existing density of the soil. Thus, a loose, poorly compacted soil might deflect enough under a wheel load to rupture the wearing surface. This deflection would be due to an increase in the soil density; whereas if the soil had been compacted to a high density before the surface was constructed, this surface deflection due to density change would have been eliminated and all deflection would have been due to the plastic flow of the soil. Since resistance to plastic flow is largely dependent upon friction between the soil particles and since these particles are in more intimate contact when the soil exists at a high density, the resistance of the soil to plastic flow is increased, and consequently a stronger roadbed results.

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[34] "The Results of Tests to Determine the Expansive Properties of Soils," by Harold Allen and A. W. Johnson, *Proceedings*, Highway Research Board, National Research Council, 1936, p. 220.

Additional evidence to substantiate the author's contention that a high state of compaction provides additional soil strength or bearing power is presented in Table 9. These data are the results of triaxial shear tests conducted on a number of soils, largely from the central and western semiarid areas of Kansas. The values in Cols. 9 and 12, Table 9, are the moduli of deformation of the soil tested, for a depth below the point of load application equal to the thickness of flexible surface required. Thus, a soil having a high modulus would require a thinner base course to support a given load than would a soil having a low modulus.

TABLE 9.—Subgrade Modulus for Soils Tested at Various Densities (Triaxial Shear Tests)

Sample No.	Class	Plastic index	Sand (%)	Clay (%)	Specific gravity	Test 1, Density Lower			Test 2, Density Higher		
						Density[a]	Moisture[b]	Modulus[c]	Density[a]	Moisture[b]	Modulus[c]
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
216	SiL	20	20	19	2.55	100	22	905	108	20	2,345
9	SiL	11	36	8	2.55	81	16	645	89	17	1,285
12	SiL	11	26	14	2.60	100	20	810	112	18	1,155
231	SiL	13	32	14	2.55	93	20	540	106	20	4,600
212	SiL	13	31	13	2.59	88	16	550	100	16	2,300
18	SiL	10	20	15	2.54	82	10	335	103	16	1,025
21[d]	SiL	4	23	6	2.55	96	17	470	108	4	4,755
44	SiL	15	18	18	2.51	98	23	660	113	18	1,135
44[d]	SiL	15	18	18	2.51	105	18	445	115	14	1,657
64	SiL	13	25	17	2.51	109	18	1,400	117	15	2,480
64[d]	SiL	13	25	17	2.51	105	17	1,990	121	11	3,195
72	SiL	16	22	13	2.45	91	16	600	101	20	2,655
84	SiL	9	20	8	2.55	83	19	611	97	21	2,680
2A[d]	SiC	14	13	34	2.51	94	21	820	99	23	1,530
130	SiC	18	16	21	2.58	97	19	1,095	104	19	1,385
138	SiCL	25	14	27	2.58	103	23	745	112	18	4,310
146	SiL	14	17	17	2.55	88	20	336	101	20	2,300
153	SiL	11	32	15	2.54	101	20	940	110	19	1,570
160	SiCL	13	18	21	2.56	94	17	1,280	98	13	3,720
164	SiL	14	19	18	2.55	100	19	2,415	109	19	6,040

[a] Initial density (before saturation) in pounds per cubic foot. [b] Initial moisture content (before saturation), expressed as percentage of dry weight. [c] Modulus of deformation, in pounds per square inch, for a depth below the point of load application equal to the thickness of the flexible surface required. [d] The four samples indicated in Col. 1 were remolded in the laboratory. The remaining sixteen were obtained in an undisturbed condition from the existing roadbed.

The results of triaxial shear tests conducted on two types of samples are summarized in Table 9. One type is obtained by cutting out an undisturbed section of an existing roadbed and trimming it to proper size for testing, with a minimum of particle disturbance. The second type of sample is prepared for testing by compacting loose, moist soil into a mold of proper size. After the samples are either cut or molded to proper size, they are saturated by admitting a supply of water to the lower part of the sample and applying a vacuum to the upper part until free water is drawn through. Therefore, the two specimens prepared from each sample were treated in the same manner, the only difference being that one specimen existed at a lower density prior to saturation than the other. In each case, regardless of whether undisturbed or molded samples were tested, an increase in density resulted in an appreciable increase in the subgrade modulus value.

One specimen of each undisturbed sample was obtained from the extreme upper section of the roadway, representing a high-density stratum condition developed by the action of traffic. The density of such a sample is usually higher than standard density for the soil, and is always considerably higher than the density of the material below that stratum. Some of these crust-density values were so high that it was not possible to duplicate them in the laboratory using a large static load applied by a hydraulic compression machine.

A large proportion of the undisturbed samples was obtained from beneath flexible bituminous wearing surfaces that had been in place for several years. However, the high density condition in the crust, developed by traffic before the surface was constructed, still existed. Thus, soils similar to those in Table 9 do not undergo any appreciable loss of density with the passage of time if they have reasonable protection from surface moisture.

The data presented by Mr. Mullis and the limited quantity of data included in this discussion indicate definitely that high compaction of roadbeds is essential if the best possible use is to be made of materials that exist within the right-of-way limits. Although most of the data presented in this discussion have been within the present working range of densities (that is, those commonly obtained during construction), they indicate the need for research to determine the increase in durability and bearing power obtained by compacting soils to much greater densities than can be obtained with present compaction equipment. Such a research program might show that it would be more economical to spend more effort and money during construction to compact the soils that comprise the upper section of the road structure to an extremely high density than to spend the same amount of effort and money on expensive road surfaces.

René S. Pulido y Morales,[35] Esq.—Soil is the most important of the infinite varieties of material in a roadbed. For this reason and may others, engineers are developing the new science of soil mechanics and pedology in order to explore the properties of the different constituents of soils. Mr. Mullis' work is another valuable contribution to the science of pedology.

In the middle of the nineteenth century research investigators, in the fields of physics and chemistry, first approached a study, and thus gained an understanding, of the properties of gases and liquids in the soil, and of their influence on mechanical structures. In 1930, N. F. Lebedeff stated[36] that water occurs in soil as:

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[35] Civ. Engr., Havana, Cuba.

[36] "Soil Water," by N. F. Lebedeff, Moscow, 1930.

(1) Vapor water in the voids, fissures, underground channels, and cavities;

(2) Hygroscopic water absorbed by soil particles from the vapor of the air (according to R. Rodewald[37] this water forms a monomolecular film on the particles;

(3) Film water, that covers the soil particles and is retained by gravitational molecular forces (this process is a continuation of the hygroscopic absorption in item 2);

(4) Gravity water, further subdivided as—

  (*a*) Capillary water that ascends from the phreatic water,
  (*b*) Suspended water that fills the pores of the soil but without communication with the phreatic water, and
  (*c*) Gravity water or free water that flows downward by gravity force;

(5) Solid water;
(6) Crystallized water; and
(7) Water containing chemicals.

The complexity of the system, soil-water-air, is apparent from the many forms that water can assume. This complexity and the variety of forms, sizes, and types of soil particles explain why no theory is valid for all soils, and why a liberal engineering judgment is necessary to produce a successful and stable road design.

In all soil mixtures the maximum stability is obtained at the highest density. The free water acts like a lubricant for the individual particles. The density varies according to four moisture phases; that is: (*a*) Hydration,[38] (*b*) lubrication,[39] (*c*) excess water and air, and (*d*) total saturation.

The optimum moisture occurs in phase (*b*); in phase (*c*) the volumes of free water and free air exceed the volumes of the pores, and the density diminishes because the specific gravity of the water is less than the soil particles; and, in phase (*d*), the water begins to occupy the air space, thus completing the total saturation of the soil.

In soils with a great quantity of large particles, the maximum density is obtained easily; probably because the large particles form a structure that permits transmitting the exterior forces deeper into the subsoil. M. M. Filatoff[40] states that the optimum grading of materials to secure stability in a roadbed soil is obtained by studying their cohesive qualities. His experience indicates that stability is dependent on the clay fraction although it is evident that under different conditions this clay fraction can possess different properties. The maximum and minimum hygroscopicity for clay soils is sometimes very useful. When the maximum is greater than the plastic limit, the soil is not suitable for flexible pavements. Factors such as these, that affect the capacity of a soil to support external forces, are very numerous.

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[37] "Uber Quellungs und Benetzungserscheinungen," by R. Rodewald, 1900.

[38] "Essentials of Soil Compaction," by C. A. Hogentogler, *Proceedings*, Highway Research Board, National Research Council, Vol. 16, 1936, p. 309.

[39] "Fundamental Principles of Soil Compaction," *Engineering News-Record*, August 31, 1933, p. 245; September 21, 1933, p. 348; September 28, 1933, p. 372.

[40] "Principles of Pedology Applied to Road Construction," by M. M. Filatoff, Moscow, 1936.

The complexity of the system, soil-water-air, is the cause of the many failures in roadway design. The subject, so well presented by Mr. Mullis, deserves coordinated and widespread research by everyone interested in it.

K. B. Woods,[41] M. Am. Soc. C. E.—The idea used by the author in establishing a degree of compaction in roadbed materials sufficient to exclude water in detrimental amounts may be accepted generally as a satisfactory approach to the problem. This same thesis was developed independently by the writer and L. E. Gregg, Assoc. M. Am. Soc. C. E.[42]

The author proposes an arbitrary value of seven tenths of the plastic limit as the limiting value of moisture content for roadbeds. However, it is impractical, if not impossible, to achieve this value in plastic, low-weight (Proctor peak) soils. In contrast, soil-aggregate mixtures with high Proctor weights would be compacted inadequately under this requirement. This discussion is directed to a consideration of these two points.

Considering first the last line of Table 5, the soil with a Proctor maximum of 85.0 lb per cu ft has a saturation moisture content of 35.8% and a plastic limit of 30.7%. If the soil were to be compacted so that the saturation moisture content would be only seven tenths of the plastic limit or 21.5%, the soil would need be compacted to a weight of 105.3 lb per cu ft or to a percentage compaction of 123.9. This degree of compaction is impossible with present-day equipment. Therefore, if it is necessary to construct a roadbed upon such a subgrade, it is generally more economical to utilize base courses constructed of granular material (from local sources of supply).

Referring again to Table 5, if the first line is considered, it will be noted that, at 100% compaction, a soil with a Proctor maximum density of 131.1 lb per cu ft has a saturation moisture content of 9.9%, and a plastic limit of 15.4; however, if the degree of compaction is changed such that the saturation moisture content is seven tenths of the plastic limit or 10.8%, then the material would have a dry weight of 128.6 lb per cu ft and the degree of compaction would be only 98.1%. In other words, it is more advantageous, in this instance, to specify 100% of the Proctor dry-weight peak than to limit the saturation moisture content to seven tenths of the plastic limit. Furthermore, assuming that this material consists of 80% aggregate, that would be retained on a No. 200 sieve and 20% soil binder that would pass a No. 200 sieve, the saturation moisture content of 10.8% would be contained in the soil binder and in the voids of individual aggregate grains themselves. In the latter instance, the soil binder or material passing the No. 200 sieve is likewise contained in the voids between the aggregate grains, and, as a result, the moisture filling the voids is 100% effective in lubricating the soil fines. In other words, by using the foregoing illustration, it can be readily shown that the moisture content of these soil fines amounts to 34.0% (that is, $\frac{(128.5 \times 0.108) - (102.8 \times 0.05)}{(128.5 \times 0.2)} = 34.0$)

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[41] Associate Director, Joint Highway Research Project, and Associate Prof., Highway Eng., Purdue Univ., West Lafayette, Ind.

[42] "Pavement Performance Related to Soil Texture and Compaction," by K. B. Woods and L. E. Gregg, *Proceedings*, Highway Research Board, Vol. 24, 1944, pp. 426–435.

when the entire soil-aggregate mass has a moisture content of 10.8% and assuming that the absorption of moisture within the aggregate is 5.0%. In most instances, this moisture content will be several points above the liquid limit of the material passing the No. 200 mesh sieve. Thus, it can be seen that the limiting value for moisture content of seven tenths the plastic limit for semigranular mixes used in base courses is not necessarily adequate.

In conclusion, the writer feels that the over-all perspective of obtaining a degree of compaction sufficient to minimize the entrance of detrimental amounts of water is sound; however, it is often impractical to apply these principles to their ultimate with very plastic soils. In contrast, the use of the seven-tenths plastic limit as a limiting moisture content value for semigranular materials used as base courses may be entirely inadequate.

IRA B. MULLIS,[43] ESQ.—The science of roadbeds is being retarded by the use of too many empiricisms. Too many engineers have become engulfed in channels of procedure so deep as to obscure many of the simple laws of physics, thus beclouding the essentials of roadbed design. It was this situation that caused the writer to begin the paper with a review of the properties of gases, liquids, and solids which one discusser referred to as "academic." Therefore, for the sake of directing attention to these hidden causes of roadbed failure, the writer gladly accepts the criticism.

Attention is again directed to the effect of air at point A, and that of water at point B, in Fig. 1. The effect of these fluids on penetration resistance (load) is so apparent that no one familiar with stress-strain curves should overlook it. But they do! Common experience should tell one that for maximum resistance to weather and traffic the apex of the density-moisture curve should intersect the penetration resistance curve midway between points A and B.

Rock matter of which roadbeds are formed consists of minerals having usually a definite chemical composition and, if formed under favorable conditions, a certain characteristic atomic structure is expressed in its crystalline form and in other physical properties. Even clay, the most weathered and changed residue of the original minerals, consists largely of crystalline matter and hence has definite properties. Of these properties, specific gravity is an important one. The specific gravity of kaolin, according to E. S. Dana and W. E. Ford,[44] is from 2.60 to 2.63, a value not far removed from that of the feldspars. It should be stated, however, that certain of the less common members of the kaolin group may have a specific gravity as low as from 2.00 to 2.20, although the high silica content of most clays utilized in engineering work causes the value to range near 2.65. In the absence of definite information, one should remember that a ±0.05 change produces a difference in pore space in most cases less than 4%. Another important property, particularly involving fine sediments such as silt and clay, is the critical point at which they become plastic. Other critical points of crystalline materials need not be discussed because of their lack of application to roadbeds.

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[43] Cons. Engr., Raleigh, N. C.

[44] "Textbook of Mineralogy," by E. S. Dana and W. E. Ford, John Wiley & Sons, Inc., New York N. Y., 1932.

Critical Moisture.—A very important critical point of roadbed material is the one at which a definite quantity of water causes a rigid body to change to a plastic one. Apparently no discussers were willing to contradict the value of this critical point although two denied its practicability. This phase will be discussed in a subsequent paragraph.

The "stability index" as defined in the paper (see heading, "Change of State" and the discussion of Table 3) is the ratio of the lower plastic limit to the moisture content. Since the date of first publication (April, 1945) research has been continued and the writer now (1946) desires to invert the ratio to read: The stability index is the ratio of the moisture content to the lower plastic limit. The latter definition is the one originally set up, but it was revised before publication to avoid an inverse relationship between the stability index and the stability stress (Cols. 10 and 11, Table 3). With this revision it is possible to combine the ratio of the moisture capacity to the critical moisture (lower plastic limit) with the stability index by multiplying the one by the other to give the "voids-moisture function" (Cols. 6, 7, 8, Table 10). By this combination it will be seen that an inverse relationship is found to exist between the voids-moisture function and the compressive strength of the cylinders of earth tested (Cols. 8 and 9). This relationship is to be expected as it is apparent that the actual water content and the pore volume also should affect strength in an inverse measure. The values in Cols. 8 and 9, Table 10 (from basic data presented by Henry C. Porter, M. Am. Soc. C. E.[45]), are shown graphically in Fig. 5. The characteristic break in the curve is due, no doubt, to the effect of critical moisture that occurs between 0.90 and 1.00. Similar data are shown in Table 11 with like relationships shown between Cols. 12 and 13. In this table no stability index is shown. The explanation for this may be found in Col. 10 with values slightly higher than those in Col. 9 obtained at estimated sat-

TABLE 10.—SERIES A; RELATIONSHIP BETWEEN MOISTURE, MOISTURE CAPACITY, CRITICAL MOISTURE, AND CAPACITY FOR LOAD

Specimen No. (1)	Density (g per cm^3) (2)	PERCENTAGE MOISTURE			Ratio, $\frac{\text{Moisture capacity}}{\text{Critical moisture}}$[b] (6)	Stability index (7)	Voids-moisture function (8)	Compressed strength (lb per $in.^2$) (9)
		Capacity[a] (3)	Content (4)	Saturation (5)				
1	1.40	33.6	28.4	84.4	1.33	1.13	1.50	40
2	1.46	30.7	28.4	92.5	1.22	1.13	1.38	45
3	1.43	32.2	25.6	79.5	1.28	1.02	1.31	60
4	1.52	28.0	25.6	91.4	1.11	1.02	1.13	75
5	1.54	27.2	25.6	94.0	1.08	1.02	1.10	90
6	1.51	28.5	24.3	85.2	1.13	0.96	1.08	95
7	1.57	26.0	24.3	93.4	1.03	0.96	0.99	100
8	1.51	28.5	19.5	68.4	1.13	0.77	0.87	142
9	1.56	26.4	19.5	73.7	1.05	0.77	0.81	170
10	1.60	24.7	19.5	78.9	0.98	0.77	0.75	200

[a] The specific gravity assumed by the writer for computing moisture capacity is 2.65. [b] The recorded critical moisture (lower plastic limit) is 25.2%; the optimum is 27.7%.

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[45] "Effect of Moisture on Compacted Soils Revealed by Laboratory Tests," by Henry C. Porter, *Engineering News-Record*, August 23, 1945, p. 101.

uration. This shows that actual moisture content (Col. 10, Table 11) is also practical saturation; hence, the voids-moisture function is equal to Col. 11 squared. Again the voids-moisture function follows the compressive strength values in inverse order without exception. These values (from basic data

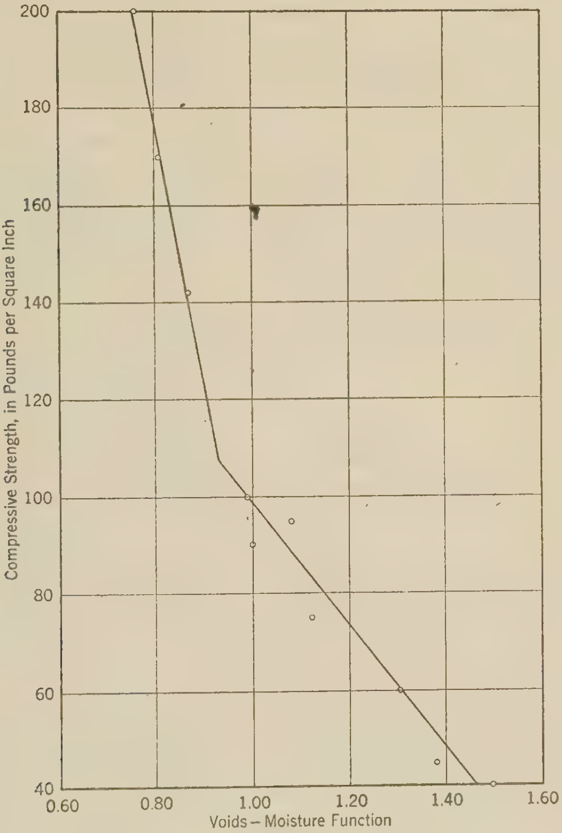

FIG. 5

presented by Mr. Porter[45]) are shown graphically in Fig. 6, the break in the curve being somewhat in excess of the voids-moisture function 1.00. The extent to which this deviation may occur is not known.

Values in Table 12 show certain physical conditions found in a survey of roadbed conditions in the State of Iowa. The table is considered as being representative of roadbeds in that state that did, and those that did not, have frost boils. Col. 11, Table 12, shows a mean ratio of pore capacity to the plastic limit of 1.49 in frost-boil roadbeds and of 1.23 in those that did not have frost boils. The mean thickness of gravel on frost-boil roadbeds was found to be 0.4 ft and in those without frost boils only 0.2 ft. At the time the writer computed the

TABLE 11.—SERIES B; RELATIONSHIP BETWEEN MOISTURE, MOISTURE CAPACITY, CRITICAL MOISTURE, AND CAPACITY FOR LOAD

Specimen No.	CONDITION AT START				CONDITION ON 63d DAY			Moisture at estimated saturation (%)	CONDITION AT FINAL TEST			Compressive strength (lb per sq in.)
	Density (g per cm³)	Percentage Moisture			Density (g per cm³)	Moisture (%)			Actual moisture content	Ratio[b]	Voids-moisture function	
		Capacity[a]	Content	Saturation		Content	Increased capacity					
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
1	1.51	28.5	19.5	67.4	1.41	33.2	16.4	29.7	29.7	1.18	1.39	15
2	1.59	25.2	18.6	73.7	1.48	29.9	18.7	27.2	27.6	1.09	1.19	19
3	1.61	24.3	19.3	79.0	1.50	28.9	19.0	26.1	27.0	1.07	1.15	23
4	1.63	23.6	18.6	78.8	1.51	28.5	20.9	26.1	26.8	1.06	1.12	23
5	1.62	24.0	18.7	77.9	1.50	28.9	20.5	26.3	26.9	1.07	1.15	23
6	1.65	22.8	17.6	77.1	1.54	27.2	19.3	24.3	25.2	1.00	1.00	29
7	1.63	23.6	13.9	58.9	1.44	31.7	34.3	29.1	30.0	1.19	1.42	11
8	1.61	24.3	11.7	48.1	1.35	36.4	49.9	32.8	33.6	1.33	1.77	5

[a] The specific gravity assumed by the writer for computing moisture capacity is 2.65. [b] In the ratio, $\frac{\text{moisture capacity}}{\text{critical moisture}}$, the recorded critical moisture (lower plastic limit) is 25.2%; the optimum moisture is 27.7%.

TABLE 12.—SUMMARY OF MEAN VALUE OF PROJECTS IN THE STATE OF IOWA

Project	Absolute density	Cut density	Cut moisture	Pore capacity	Pore saturation	Liquid limit	Plastic limit	Ratio[a]	Surface thickness	County	No. of tests
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
(a) FROST-BOIL AREAS											
FA358	2.69	1.61	21.9	25.2	86	33	19	1.30	0.6	Delaware	18
FA173	2.68	1.48	25.2	30.4	83	47	22	1.37	0.2	Adair	24
FA60AB	2.71	1.58	21.4	26.5	80	44	19	1.38	0.2	Van Buren	7
FA134	2.66	1.55	23.4	27.4	87	40	20	1.35	0.7	Worth	6
FA851B1[b]	2.68	1.52	20.6	28.7	72	36	21	1.42	0.3	Webster	6
FA822	2.60	1.34	28.4	36.0	79	43	19	1.89	0.2	Tama	6
FA791A[b]	2.69	1.48	25.5	30.4	84	35	20	1.51	0.3	Marshall	6
FA7	2.68	1.51	25.0	28.9	87	33	19	1.52	0.2	Marion	1
FN544	2.66	1.49	26.1	29.7	87	37	19	1.56	1.1	Mahaska	5
FA873	2.68	1.51	26.6	29.0	92	39	21	1.38	0.3	Wapello	3
FA835C1	2.69	1.33	27.9	37.8	74	37	23	1.64	0.1	Lyon	6
P128B	2.70	1.56	25.2	27.4	92	47	21	1.28	0.2	Keokuk	6
FN234[b]	2.68	1.52	23.1	29.7	81	29	19	1.50	0.5	Jones	9
FA765[b]	2.68	1.51	23.1	29.3	79	32	18	1.63	0.5	Johnson	6
FA764B	2.69	1.50	26.9	29.7	91	39	22	1.36	0.2	Jasper	8
Means[c]	(2.68)	(1.50)	(24.7)	(29.7)	(84)	(38)	(20)	(1.49)	(0.4)		(117)
(b) NON-FROST-BOIL AREAS											
FA134	2.70	1.78	16.8	19.3	88	38	18	1.07	0.8	Worth	2
FA851B1[b]	2.68	1.51	21.9	28.9	76	36	20	1.44	0.2	Webster	1
FA791A[b]	2.71	1.85	15.2	17.0	89	32	15	1.16	0.2	Marshall	6
FA7[b]	2.71	1.60	21.8	25.6	85	50	19	1.32	0.2	Marion	3
FA765[b]	2.69	1.65	16.7	24.1	73	35	18	1.34	0.2	Johnson	8
FA764B[b]	2.72	1.76	15.7	19.5	79	43	19	1.05	0.2	Jasper	6
Means[c]	(2.70)	(1.69)	(18.0)	(22.4)	(82)	(39)	(18)	(1.23)	(0.2)		(26)

[a] Ratio of Col. 5 to Col. 8. [b] Part of project. [c] Grand average means; in Table 12(*a*), of fifteen items, and in Table 12(*b*), of six items.

data in Col. 9, he had not developed the term "voids-moisture function" although he has since computed the mean voids-moisture function of the frost-boil roadbeds in this survey and finds they have a value of 1.82. Those without frost boils have a value of 1.24. Therefore, by the use of the voids-moisture function, the relative strengths of the two conditions are probably more truly indicated.

Because of the lack of adequate experience in the use of the voids-moisture function, the writer hesitates to compare the values obtained from one material with those of another. He will suggest, however, that the frost-boil voids-moisture function of 1.82 be compared with specimen No. 8, Table 11, having a voids-moisture function of 1.77 and a strength of 5 lb per sq in. In a like manner, the non-frost-boil strength of 1.24 may be compared with specimen No. 2, Table 11, having a voids-moisture function of 1.19 and a strength of 19 lb per sq in. Again it may be compared with specimen No. 3 or specimen No. 4, Table 10, with a voids-moisture function of 1.31 and a strength of about 65 lb per sq in. This problem is one that seems to offer promise of success to the research engineer although additional research is needed.

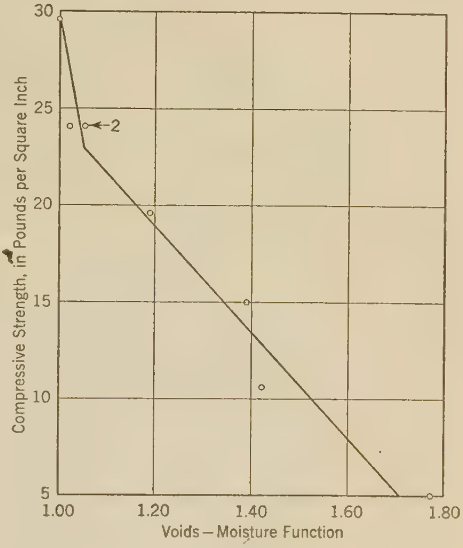

FIG. 6

Density.—The most striking manifestation of the density-strength relationship can best be shown by reference to Fig. 7, the rock cycle. This figure is a modification of one by Carey Croneis and William C. Krumbein[46] (see Richard M. Field,[47] and C. K. Leith and W. J. Mead,[48] Affiliate, Am. Soc. C.E.) but is modified by the insertion of the point at which the identity of the immediately preceding rock or sediment is lost. This loss of identity is the method used by the Committee on Sedimentation, Division of Geology and Geography, National Research Council,[49] as well as by William H. Twenhofel,[50] for differentiating the several forms of rock matter. Several states or phases of rock formation are shown in Fig. 7. Fig. 7 includes all the major stages of the cycle, including magmas, igneous rocks, unconsolidated sediments, consolidated sediments, and metamorphic rocks. In each stage certain types of rock representing important members belonging to it are shown. The several stages are connected by arrows showing the rock trend under given conditions of environment. Attention is directed to the term, "loss of identity," designated as the practical point at which one stage of the rock cycle passes into the next one.

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[46] "Down to Earth," by Carey Croneis and William C. Krumbein, The Univ. of Chicago Press, Chicago, Ill., 1936, p. 212.

[47] "An Outline of Principles of Geology," by Richard M. Field, Barnes and Noble, Inc., New York, N. Y., 1938, p. 12.

[48] "Metamorphic Geology," by C. K. Leith and W. J. Mead, Henry Holt and Co., New York, N. Y., 15, Plate I.

[49] "Treatise on Sedimentation," by William H. Twenhofel, The Williams and Wilkins Co., Baltimore, Md., 1932, p. xxvii.

[50] "Principles of Sedimentation," by William H. Twenhofel, McGraw-Hill Book Co., Inc., New York, N. Y., 1939, p. 2.

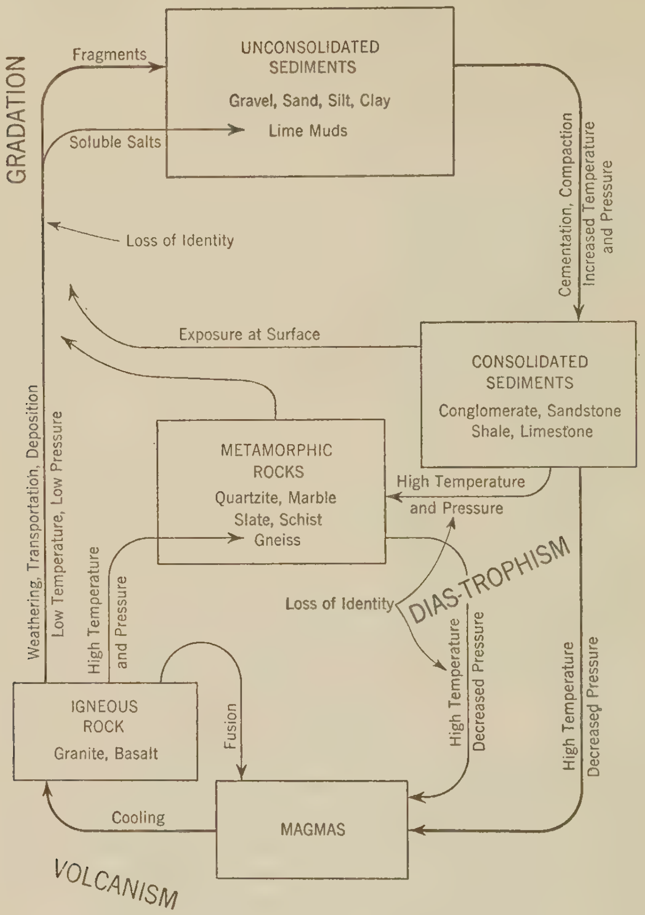
GRADATION
Fragments
UNCONSOLIDATED SEDIMENTS
Gravel, Sand, Silt, Clay
Lime Muds
Soluble Salts
Loss of Identity
Cementation, Compaction
Increased Temperature and Pressure
Exposure at Surface
CONSOLIDATED SEDIMENTS
Conglomerate, Sandstone
Shale, Limestone
METAMORPHIC ROCKS
Quartzite, Marble
Slate, Schist
Gneiss
High Temperature and Pressure
Weathering, Transportation, Deposition
Low Temperature, Low Pressure
High Temperature and Pressure
Loss of Identity
DIAS-TROPHISM
High Temperature
Decreased Pressure
High Temperature
Decreased Pressure
IGNEOUS ROCK
Granite, Basalt
Fusion
Cooling
MAGMAS
VOLCANISM

Fig. 7.—Rock Cycle

The strongest and most weather-resistant rocks, such as granite, have very small pore spaces and hence absorb very little water (Fig. 7). Of course weathering, including temperature changes, eventually enlarges the pore spaces and at the same time causes them to lose part of their capacity to resist loads and weather. In the case of granites, for example, when weathering has caused the feldspar minerals to change to clay, the resultant material is a mixture of clay and sand (quartz). According to the authorities cited, the granite is said to have lost its identity and therefore becomes unconsolidated sediment, Fig. 7. If the parent granite is fine-grained, the resultant sediment will consist of fine sand, silt, mica, and clay. This unconsolidated sediment, whether wet or dry, will have much more pore space and hence can offer little resistance to loads and weather. The specific gravity of the parent granite was near 2.65 and that of the sediment is nearly identical. The outstanding change observed by the road engineer is the increase in pore space and attendant loss of resistance to loads and weather.

If a road is proposed across a belt of sediment derived from granite, the major problem of the engineer is the consolidation of the sediment to a measure that will not allow it to become plastic when fully saturated. The ideal toward which he should strive is to reduce the pore spaces to the measure of those in the parent rock, although the ideal is unattainable. Whether the proposed road is to be paved or used as an earth road, means must be found to seal the pores against the entrance of water in critical amounts or failure will follow sooner or later. How can this be accomplished? Returning to Fig. 7 one finds that nature converts unconsolidated sediments into more resistant consolidated sediments through cementation and compaction. Of course, the first step that should be made is to determine the plastic limit, the natural boundary between the rigid and plastic states. The second step should determine the possibility of consolidating the sediment to a measure that pore spaces will not receive water in critical amounts. To be successful, the engineer must determine the best method of converting certain unconsolidated sediments into consolidated equivalents. He must also know how nature converts gravel aggregate into gravel conglomerate, sand into sandstone, silt into siltstone, and clay particles into claystone. This is imperative if he is to build resistant roads. Moreover, this natural law applies to the making of Portland cement concrete, bituminous concrete, sheet asphalt, and sand-clay roads as well.

The application of data such as those shown in Table 10 is recommended as a means for analyzing roadbed problems and pointing the way to successful solution.

The values obtained on a homogeneous sample of clay in Table 10 computed from basic data by Mr. Porter[45] show the effect of density and water content on compressive strength. Table 10 also shows why the density-strength relationship is not always constant. For example, compare specimens Nos. 2 and 3, 5 and 6, also 7 and 8, in Table 10. Note the constancy of relationship between Cols. 8 and 9. Similar comments are also applicable to values in Table 11 computed from another series of tests by Mr. Porter and described in the same paper.

Table 12 shows the effect of density (Col. 3) and hence pore capacity (Col. 5) of roadbeds having frost boils and those without them. Although the mean percentage of pore saturation (Col. 6) in frost-boil roadbeds and the mean percentage of pore saturation in non-frost-boil roadbeds are nearly identical, one may observe the effect of low porosity (Col. 5) in the prevention of failure.

The part of Table 9 on this phase of the subject, in Mr. Lacey's discussion, shows striking contrasts between the modulus of deformation of like specimens of the higher densities (less wet) and that of those with the lower densities (wetter). In an effort to make the data as concise and conclusive as possible, the writer has recomputed the pore capacity in each case as a ratio of the lower plastic limit and presents it as Table 13. Without a single exception the reduction of the ratio (Cols. 2 and 4) causes a considerable increase in the modulus value (Cols. 3 and 5). In the case of sample No. 21 a reduction of about 70% of the modulus increases the strength about ten times. No doubt it would be very enlightening if one might have similar test data on identical samples compacted to produce pore capacities ranging from small to large values. Table 9 shows several cases in which an increase in density has produced a higher modulus even when the water content remains the same or is higher.

Messrs. McNew, Guerra, Feld, and Stuckey also support this method of approach in obtaining stability of roadbeds. Although Messrs. Britzius and Woods recognize the need of compaction, both seem to regard the standard proposed as impracticable although neither one offers any data in support of his belief. Apparently both base their opinions on the so-called Proctor density method (A.S.T.M. D698–42T and A.A.S.H.O. T99-38) as their standard of compaction used in conjunction with light rollers of comparable effectiveness.

TABLE 13.—TRIAXIAL SHEAR TESTS (SUMMARY OF PART OF TABLE 9)

Sample No.	TEST 1; DENSITY LOWER		TEST 2; DENSITY HIGHER		$\frac{\text{Col. 4}}{\text{Col. 2}}$	$\frac{\text{Col. 5}}{\text{Col. 3}}$
	$\frac{\text{Pore capacity}}{\text{Lower plastic limit}}$	Modulus value	$\frac{\text{Pore capacity}}{\text{Lower plastic limit}}$	Modulus value		
(1)	(2)	(3)	(4)	(5)	(6)	(7)
212	1.61	550	1.20	2,300	0.745	4.17
18	1.68	335	0.96	1,025	0.570	3.06
21[a]	1.12	470	0.80	4,755	0.715	10.12
44[a]	1.03	445	0.76	1,657	0.738	3.72
64[a]	1.03	1,990	0.61	3,195	0.593	1.60
72	1.53	600	1.16	2,655	0.759	4.41
84	1.80	611	1.26	2,680	0.700	4.37
130	1.17	1,095	0.96	1,385	0.821	1.26
146	1.37	336	0.98	2,300	0.716	6.84
160	1.24	1,280	1.12	3,720	0.914	2.89
Mean	....	....	....	....	(0.727)	(4.24)

[a] The four samples indicated in Col. 1 were remolded in the laboratory. The remaining sixteen were obtained in an undisturbed condition from the existing roadbed.

Mr. Britzius states that the writer's claim, that sediments compacted to low air voids are resistant to frost action, is misleading. Table 12 offers specific evidence in support of the value of low voids; and Tables 10 and 11 contain evidence showing that, since water content decreases with pore space, less damage through frost action should follow.

Mr. Woods states that it is impracticable, if not impossible, to compact certain clays to a water-capacity value equal to 70% of their plastic limit; he is no doubt thinking in terms of lightweight rollers and not of ones with weights suggested by the writer. Although the writer does not claim that certain clays, such as China clay and certain so-called pipe clays, can be compacted to prevent them from becoming plastic, he does insist that the use of excessive quantities of water, required to soften tough clays to bring them within the compaction range of light rollers, is a fallacy. This statement is proved when wide shrinkage cracks develop through drying after such light roller operation. Certainly one should expect roadbeds to be poor with pore spaces indicated in the last five tests in Table 5.

Through oversight, the writer failed to account for approximately 35% of coarse granular material in the first item in Table 5, referred to by Mr. Woods. This fact, of course, lowers the critical moisture in the mixture to perhaps 10%. On the basis proposed by the writer, the mixture should have a capacity of about 7%, although it is essential that the critical moisture point of mixtures be tested by the stress-strain method. Certain clays have been found with a density of 2.10 and experience teaches the writer that they are satisfactory when surfaced. It was this type of material the writer had in mind.

Compaction and Moisture Standards.—In 1933 R. R. Proctor[8] published a standard of compaction for earth dams which was intended to cause a body of earth to offer a needle resistance of not less than 300 lb per sq in. when saturated, and to restrict the percolation rate of water to 0.3 ft per yr. The quality of the work was measured by the capacity of the earth to support wheel loads and to resist the penetration of water. The weight of the roller and the number of its passes were such as to produce the required resistance to load and water. Density was intended only as a means for obtaining adequate resistance.

Some time after the introduction of this method, certain engineers undertook to improve on the Proctor method by applying a certain number of foot-pounds of energy to earth using the mold and rammer designed by Mr. Proctor. Whatever density this procedure produced was considered satisfactory by them. The method has been standardized by the American Association of State Highway Officials (T99-38). It has also been adopted by the American Society for Testing Materials as a tentative standard (D698-42T).

Like other materials, clays and other roadbed materials offer varying resistance to compaction or shaping. China clay or kaolin, being weak, offers little resistance, although gumbo and adobe offer much resistance to change of shape unless quite wet. Of course, all clays are brittle when dry but tend to become tough when the ideal proportion of water is absorbed. This being true, the tougher the clay the greater will be the quantity of water required to deform a clay under a small force. Since the deforming force in A.S.T.M. D698-42T is only 5½ ft-lb applied seventy-five times, it follows that the tougher the clay the greater will be the proportion of water required to soften it to the point where 5½ ft-lb will eliminate the maximum percentage of air space.

Table 14 substantiates this point.[51] Col. 3 shows that the material represented by curve No. 1 is the toughest as it requires the largest percentage of water to soften it within the range of the rammer. Optimum moisture, of course, decreases constantly throughout the series. The density value follows in inverse order; but the percentage of saturation of pore spaces remains near 90% in every test, with a maximum deviation of 8.2%.

According to the facts presented and the experience cited in the paper, roadbeds constructed with densities such as those in the upper third of Table 14 should be expected to offer little resistance to loads or the penetration of water.

With adequate means for compacting the upper third of the table, it probably could have been compacted to densities and moisture contents shown in the lower third of the table.

To expect to produce adequate consolidation in a mass of an unknown clay by means of applying a given amount of energy is like supplying a given quantity of fuel to an unknown steam engine or gas motor and assuming the delivery of a definite amount of power from the shaft. Of course, this is a grave fallacy that cannot be condoned. The effectiveness of consolidation can be measured only in terms of the resistance it offers to loads and weather.

In an effort to uncover as much fact as possible from Table 8, the writer has recomputed the data with results as shown in Table 15. For these computations the specific gravity of the sediment is 2.62 and the lower plastic limit is 19. With a density of 1.51 the water capacity is nearly 1.5 times the critical moisture (19%) and hence is of low bearing capacity. The pore space is so great that little swell occurred when compacted with only 12% moisture (42.8% saturation). Obviously, air content would be displaced if water entered and little swell followed. With a density of 1.85 and a moisture content of 14% (88% saturation) swell amounted to only 1.5% because the pore spaces were already so nearly saturated that little additional moisture was absorbed. The writer's interpretation of these and other similar data seems to indicate that for minimum volume change pore spaces should be reduced to a minimum, should be from 85% to 90% filled with moisture, and evaporation should be restricted by means of an adequate surface.

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[51] "Compaction of Subgrades and Embankments," Highway Research Board, National Research Council, August, 1945, p. 10.

TABLE 14.—MOISTURE-DENSITY RELATIONS

Curve No.	DENSITY		Optimum moisture (%)	Moisture capacity[a] (%)	$\frac{\text{Col. 3}}{\text{Col. 4}}$	Col. 4 less Col. 3
	Lb per ft^3	g per cm^3				
	(1)	(2)	(3)	(4)	(5)	(6)
1	87.2	1.40	30.0	33.7	89.0	3.7
2	90.7	1.45	26.6	31.2	85.2	4.6
3	92.1	1.48	26.0	29.8	87.2	3.8
4	94.5	1.51	24.8	28.4	87.2	3.6
5	97.5	1.56	23.3	26.4	88.2	3.1
6	99.5	1.60	22.2	24.8	89.5	2.6
7	102.4	1.64	20.5	23.2	88.3	2.7
8	104.4	1.68	19.7	21.7	90.6	2.0
9	106.9	1.71	18.4	20.1	91.5	1.7
10	109.4	1.76	17.2	19.1	90.0	1.9
11	111.7	1.79	15.9	18.1	87.8	2.2
12	114.2	1.83	14.8	16.9	87.5	2.1
13	117.2	1.88	13.6	15.4	88.2	1.8
14	119.3	1.92	13.0	14.3	90.8	1.3
15	121.5	1.95	12.0	13.5	88.8	1.5
16	124.7	2.00	11.4	12.2	93.4	0.8
17	126.8	2.03	10.6	11.5	92.1	0.9
18	128.9	2.06	9.3	10.8	86.1	1.5
19	131.0	2.10	9.0	9.9	90.8	0.9
Mean	....	....	....	....	(89.1)	(2.2)
Maximum	....	....	....	....	(93.4)	(4.6)
Minimum	....	....	....	....	(85.2)	(0.8)

[a] For computing moisture capacity a specific gravity of 2.65 is assumed.

Economy of Consolidation.—Objections to raising the standard of consolidation seem to be based on the belief that it will be impracticable or too expensive, although no cost estimate has been submitted by the discussers. The writer found that prior to 1940 many materials could be consolidated to adequate densities at a cost of from 4¢ to 5¢ per cu yd. Cost data published under the auspices of the Tennessee Valley Authority[52] show a cost usually less than 5¢ per cu yd for sheepsfoot rollers exerting from 300 to 400 lb per sq in.; 4¢ per cu yd is equivalent to a cost of $1\frac{1}{3}$¢ for rolling 1 sq yd 1 ft thick. A cost of 10¢ per cu yd is only $3\frac{1}{3}$¢ per sq yd 1 ft thick.

In lieu of high consolidation two discussers would stabilize with aggregates; but it should be remembered that aggregates must be placed on firm roadbeds to avoid potholes or other types of failure. Moreover, it is not economical to import selected material until local roadbed material has been rendered as efficient as may be economically possible. For example, it would not be economical to haul pit-run gravel costing 60¢ per cu yd to be placed 4 in. thick at 4.4¢ per sq yd when a square yard could be rendered stable for surfacing by rolling it 1 ft thick at a cost of $1\frac{1}{3}$¢ per sq yd. Also, if one consolidated a road to a thickness of 1 ft at a cost of $1\frac{1}{3}$¢ and then placed only 2 in. of gravel on it at a cost of 2.2¢, the total cost would be only $3\frac{1}{2}$¢ and this of course would be much more valuable than the entire 4 in. of gravel placed on a less compact roadbed.

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[52] "Granular Stabilized Base Construction of Access and Relocation Roads, by Tennessee Valley Authority," by F. W. Webster and F. H. Kellogg, *Proceedings*, Highway Research Board, Vol. 24, 1944, p. 483.

TABLE 15.—Effect, on the Total Swell, of Density and the Percentage of Saturation at the Beginning of the Swell Test (Recomputation of Table 8; "Sat." Denotes "Saturation" and "Sw." Denotes "Swell")

Compacted density (g per cm³)	Water capacity (%)	Percentage Moisture When Compacted									
		10		12		14		16		18	
		Sat.	Sw.	Sat.	Sw.	Sat.	Sw.	Sat.	Sw.	Sat.	Sw.
1.51	28.0	35.7	1.0	42.8	3.2	50.0	2.6	57.1	1.6	64.2	....
1.60	24.3	41.1	5.5	49.3	3.6	57.6	2.6	65.8	1.8	74.0	1.2
1.70	20.6	48.5	5.7	58.2	4.4	67.9	2.6	77.6	1.6	87.2	0.7
1.80	17.4	57.5	5.6	68.8	4.1	80.4	2.5	91.9	1.4	....	0.0
1.85	15.9	62.8	5.5	75.5	3.6	88.0	1.5	....	0.8	....	....

When aggregates cost considerably more than a dollar per ton (as they usually do), the economy of roadbed consolidation produces a striking contrast to usual procedure.

The reader is again directed to Table 12 showing the effect of roadbed compaction on the possible reduction of gravel thickness. Col. 5 shows a mean pore capacity of 29.7% in the frost-boil roadbeds with a mean gravel thickness of 0.4 ft (Col. 10). In contrast, the pore capacity in non-frost-boil roadbeds was 22.4% and did not fail with only 0.2 ft of gravel surface. As far as could be determined, no difference other than porosity existed. This survey seems to indicate that non-frost-boil roadbeds with a mean pore capacity of about 75% of that of frost-boil roadbeds remained stable with only one-half the gravel thickness, whereas frost-boil roadbeds failed. It is apparent, therefore, that a 25% reduction in pore capacity was of greater value than a 0.2-ft increase in thickness of gravel surfacing. Indeed if the cost of rolling should be as much as 15¢ per cu yd (5¢ per sq yd 1 ft thick) and the cost of crushed stone $1.40 per cu yd (7.8¢ per sq yd 4 in. thick)—a total of 12.8¢ per sq yd—this procedure would be more economical than spending the entire 12.8¢ for $6\frac{1}{2}$-in. stone alone. On the basis of past experience, including the survey shown by Table 12, if the writer had only 8.9¢ per sq yd available, he would appropriate 5¢ for consolidation and the remaining 3.9¢ for a 2-in. crushed stone surface.

Summary.—As a summary the following conclusions are offered:

1. The change from the plastic to the rigid state is indicated by rupture.

2. The point of rupture in fine sediments, as determined by the Atterberg and stress-strain methods, are nearly identical (Cols. 5 and 6, Table 1).

3. The point of rupture is affected more by moisture than by load (Cols. 1, 2, 3, and 4, Table 1).

4. The apex of the ideal density curve of a roadbed should intersect the penetration resistance curve midway between points A and B, Fig. 1.

5. Masses of earth compacted, as described in the paper under the heading, "Change of State: Consolidated Roadbeds," and in Table 6, absorb little additional water; hence, they remain stable.

6. The plasticity index is of very limited use in predicting resistance to impact or weather (Figs. 3 and 4).

7. The "voids-moisture function" appears to offer splendid possibility in predicting the capability of roadbeds to resist load and weather (Tables 10 and 11).

8. Since the most resistant rocks have little pore space and the least resistant rocks (unconsolidated sediments) have much pore space, it is evident that the least resistant ones can be made resistant only by the process of consolidation (Fig. 7 and Tables 10, 11, 12, and 14).

9. The two outstanding characteristics of the data in Table 14 are the constancy of saturation (Col. 5) at the time of compaction and the lack of uniformity of density (Cols. 1 and 2) upon completion of the compaction effort.

10. Obviously, sediments of the highest density and least capacity for moisture are the most resistant to traffic and water. It is also obvious that those having the least density and greatest capacity for water are the least resistant (Table 14). By increasing the compactive effort to a magnitude that would reduce optimum moisture to a nearly constant value, the density and moisture capacity in most cases, if not all cases, could have been reduced to a nearly constant value. However, micaceous and one-size sediments would continue to have large capacity for moisture.

11. To render sediments—especially clays—the most resistant to swell, they must be made very compact with pore spaces from 80% to 90% saturated (Tables 14 and 15).

12. To prevent sediments, especially clays, from losing moisture by evaporation, they must be surfaced with a covering that prevents undue evaporation.

13. Excessive evaporation losses from clay roadbeds can be prevented by the application of a thin blanket of loose sand, although a rigid surface (sand-clay, sand-asphalt, etc.) is superior.

14. Money spent in adequate roadbed compaction becomes an asset to the future road surface or pavement. Indeed, it may warrant the use of a less thick pavement at a considerable reduction in cost (Table 12).

In July, 1945, the U. S. Waterways Experiment Station,[53] Vicksburg, Miss., reported certain stress-strain data to which the present writer compared the voids-moisture function as in Figs. 5 and 6 and Table 11. The Vicksburg data, plotted as in Figs. 5 and 6, showed a similar relationship. As in Figs. 5 and 6, the data conform more closely to the curve when the voids-moisture or strength function is low. Also it should be stated that, since the Vicksburg materials were tested at moisture contents drier than that of the lower plastic limit, no break occurred in the straight-line curve as in Figs. 5 and 6. It is now obvious that only identical materials produce like voids-moisture functions, because each material may not possess a strength identical to that of another.

In conclusion, the writer expresses his deep appreciation for the several discussions submitted.

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[53] "The California Bearing Ratio Test as Applied to the Design of Flexible Pavements for Airports," *Technical Memorandum No. 213-1*, U. S. Waterways Experiment Station, Vicksburg, Miss., July 1, 1945, plates 124 and 127.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DETERIORATION OF CONCRETE DAMS DUE TO ALKALI-AGGREGATE REACTION

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WITH DISCUSSION BY MESSRS. RALPH R. PROCTOR, MILTON D. BURRIS, H. A. KAMMER, THOMAS E. STANTON, R. W. SPENCER, W. L. CHADWICK, J. C. WITT, DUFF A. ABRAMS, AND R. F. BLANKS AND H. S. MEISSNER.

SYNOPSIS

For many years engineers have been concerned over a peculiar and characteristic type of concrete deterioration, manifested by large-scale, random-pattern cracking. Such cracking is not unusual or new; in fact it is difficult to find massive concrete structures that do not display random cracking in some degree. Most instances of pronounced pattern or map cracking, as distinguished from ordinary small-scale crazing, have been attributed to excessive internal expansion, aggravated by varying degrees of surface drying shrinkage. The mechanisms producing internal expansion have been explained in the past on the basis of unsound aggregate or unsound cement, the latter resulting from high magnesia content or underburning and high free-lime content. Although some instances of expansive deterioration appear to be explained adequately by unsoundness, most cases, until recently, have remained enigmas and subjects for speculation only. Many concrete dams in all sections of the United States have developed evidence of this type of degeneration. Several of these cases have been caused by adverse chemical reactions between the alkalies in Portland cement and certain siliceous mineral constituents in the aggregates.

Identification of alkali-aggregate reactions as the cause of many cases of concrete deterioration has been recorded in contemporary technical literature. A brief selected bibliography is included as an Appendix of this paper. The destructive effects of the chemical instability of some aggregate constituents in the presence of alkalies contained in Portland cements have been definitely established. The problem is extremely complicated, however, and many questions, practical and technical, remain to be answered satisfactorily.

NOTE.—Published in January, 1945, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

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Symptoms and Diagnosis of Alkali-Aggregate Deterioration in Concrete Dams

The random-pattern cracking is very conspicuous, as may be observed from the photograph of a typical affected surface reproduced in Fig. 1. However, positive association of such surface manifestations with adverse alkali-aggregate reactions usually requires detailed investigation. Although crack openings of as much as 1.5 in. have been observed, they extend into the concrete only a comparatively short distance. The superficial nature of cracks caused by alkali-aggregate expansion has been established by drill cores, as illustrated in Fig. 2. The shallow depth to which such cracks penetrate has also been indicated by attempts to fill the openings by grouting. In one instance a hole was drilled parallel to a concrete face at a distance of 2 ft to intercept large, open cracks in evidence at the surface. Water under a pressure of 50 lb per sq in. failed to develop any connection between the surface cracks and the drill hole 2 ft inside the mass. Water tests from one surface to another, both displaying large, open cracks which from superficial evidence appeared to be connected, have been similarly unsuccessful.

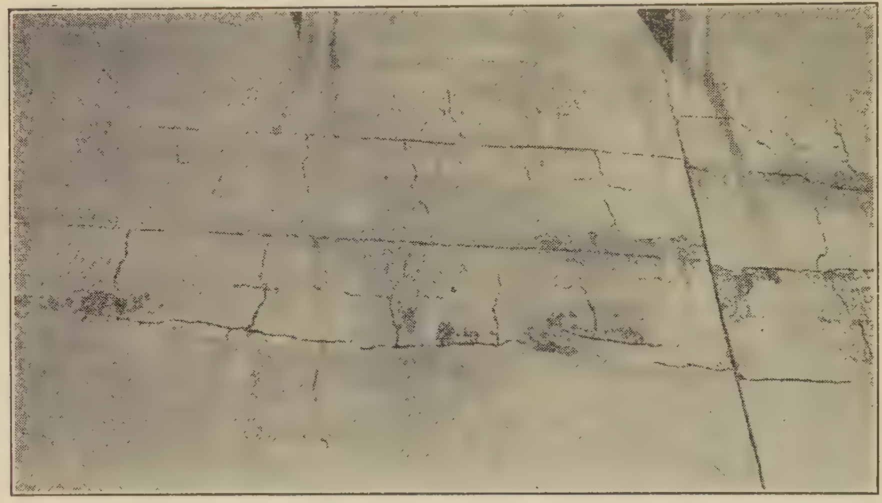

Fig. 1.—Expansive Reaction Between Aggregate and High-Alkali Cement Produces Typical Pattern Cracking on a Dam

Outward evidences of excessive internal expansion in concrete dams affected by adverse combinations of high-alkali cement and reactive aggregates are usually obscured by the massive proportions of the structure and the difficulty of establishing reference points for observing expansive movements. It is suspected that, under some conditions, restraint is sufficient to prevent most of the expansion that would normally occur if the concrete was not restrained. It is also suspected that such restraint of expansion tendencies in no way lessens the destructive action, except to reduce the surface crack openings.

In fact, deterioration may even be aggravated by internal stresses set up by the expansion forces. Some indication of the latter concept is given by the extremely low modulus of elasticity obtained in tests of cores drilled from affected structures.

Excessive expansion in massive concrete structures can sometimes be detected by the abnormal closing of expansion joints, particularly joints in a superstructure on a massive concrete foundation. Misalinement or displacement of hydraulic machinery such as the binding of gates, the cracking, bending, or buckling of penstocks, pit-liner plates, guide vanes, and speed rings, and the shifting or tilting of turbine-generator shafts are other signs of abnormal expansive movements in concrete which accompany alkali-aggregate reactions. Offsets at cracks, particularly in tunnels and galleries, frequently occur in affected concrete. Precise surveys and other forms of accurate measurements will sometimes indicate both the extent and direction of movement.

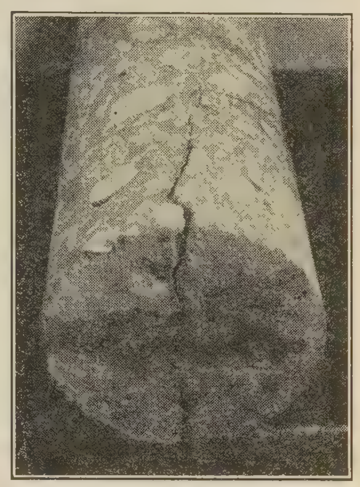

Fig. 2.—Cores Reveal That Wide Cracks Are of Shallow Depth

Positive identification of the alkali-aggregate disease in concrete dams usually requires laboratory study and tests of cores drilled from the structure, and of specimens made with the particular aggregate used in the dam. Invariably, cores from affected concrete, when stored in the presence of moisture, will develop gelatinous exudations and whitish amorphous deposits on their surfaces. These gel deposits are also found in pores in the concrete mass, particularly adjacent to pieces of reactive aggregate. Chemical analyses of the gels show them to be composed essentially of sodium and potassium silicates. The gels may be transparent, closely resembling ordinary water glass as illustrated in Fig. 3, or they may be white or gray with a porcelaneous luster as shown in Fig. 4. Table 1 shows the results of chemical analyses of typical gel deposits from alkali-aggregate reactive concrete.

TABLE 1.—Analyses of Gel Deposits Show Them To Be Composed Essentially of Alkali Silicates (Values Are Percentages)

Type of gel sample	Loss at ignition	Silica SiO_2	R_2O_3	Lime CaO	Magnesia MgO	Sulfate SO_3	Soda Na_2O	Potassa K_2O
Bead	....	51.2	...	...	...	0	16.8	
Fig. 4	10.5	81.9	0.7	1.1	...	...	4.0	2.0
Fig. 3	20.5	53.4	1.5	4.7	0	0	12.7	4.5

Hand specimen and microscopic study of core samples from affected concrete also invariably reveal what appear to be zones of chemical reactivity, alteration, or infiltration around certain aggregate particles, as shown in Fig. 5. Aggregate pebbles which appear to have experienced reaction in concrete are

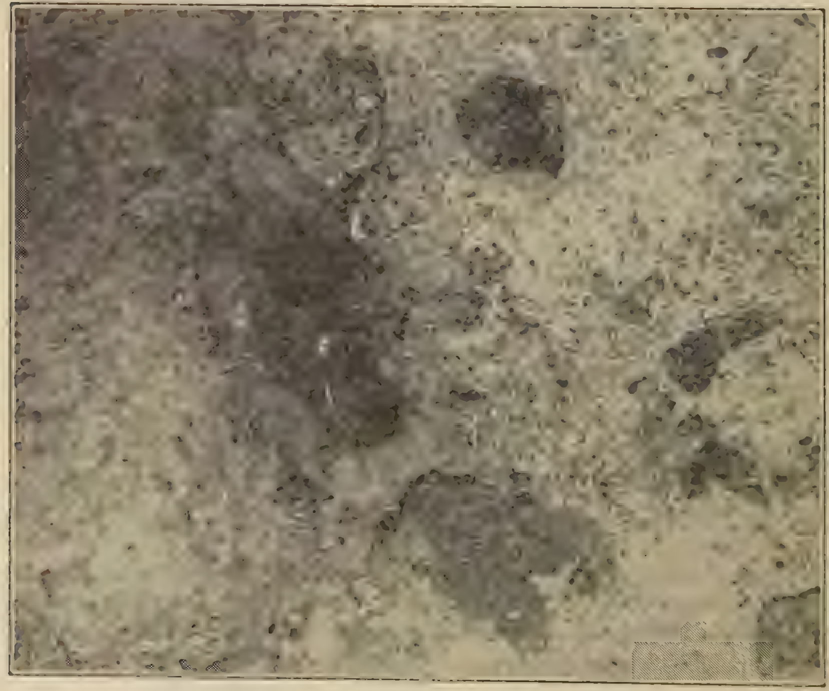

FIG. 3.—TRANSLUCENT GEL ON THE INTERIOR FRACTURED SURFACE OF A CORE

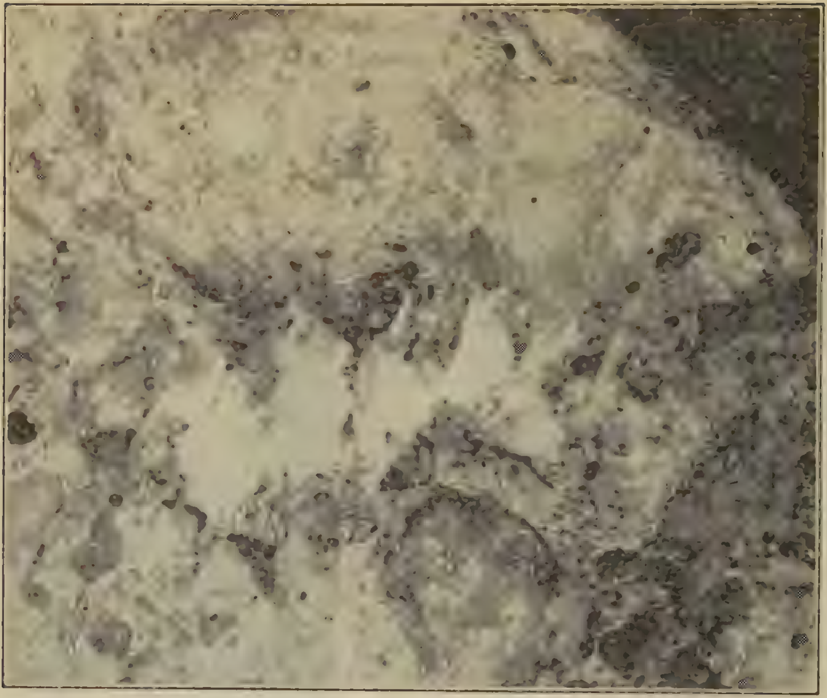

FIG. 4.—CARBONATED OPAQUE GEL ON A CONCRETE CORE

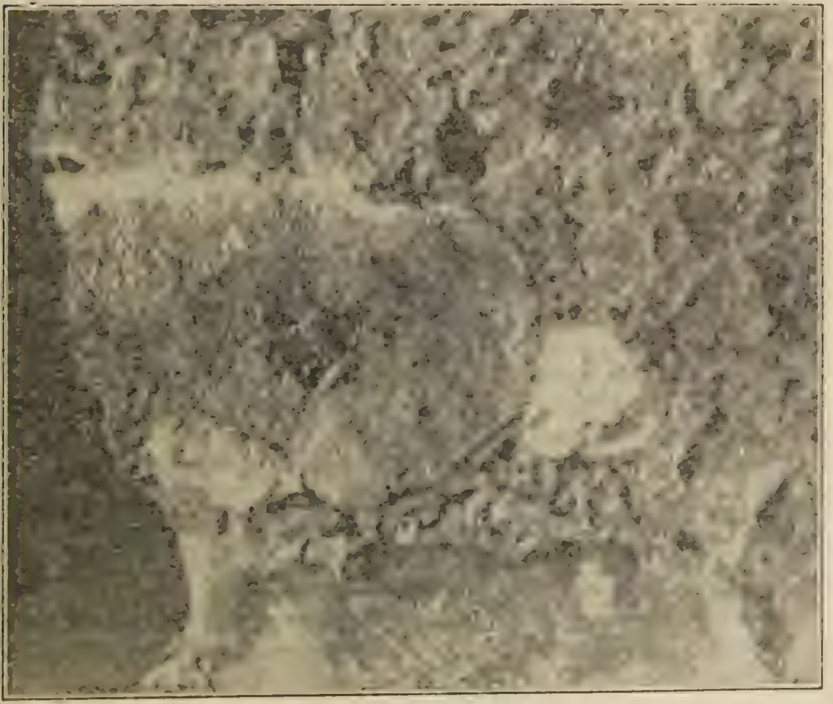

FIG. 5.—RIMS AND GEL POCKETS OFTEN SHOW ON REACTIVE AGGREGATE PARTICLES IN FRACTURED SECTIONS OF EXPANDING CONCRETE

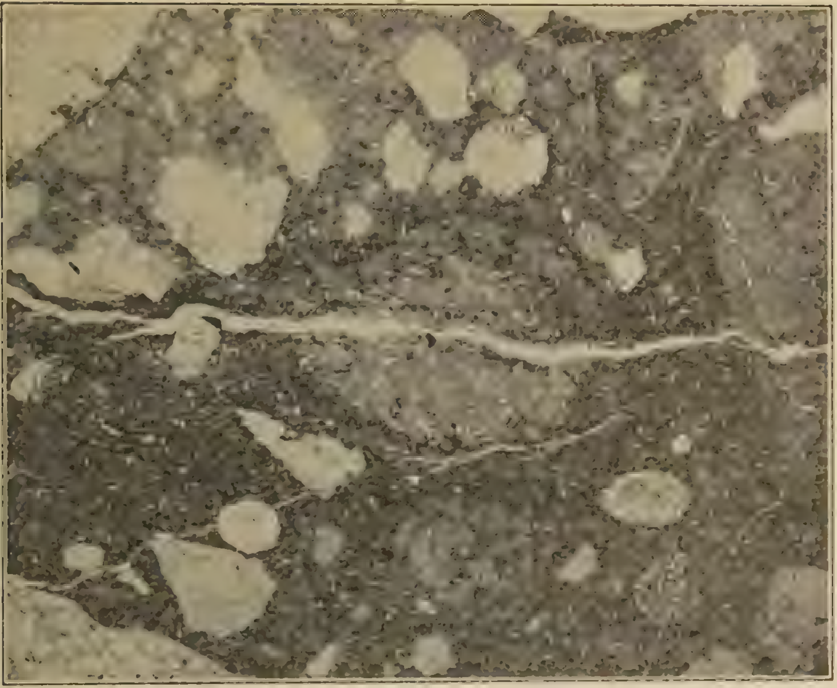

FIG. 6.—CRACKS IN AFFECTED AGGREGATE PARTICLES EXTEND INTO THE PASTE AND ARE FILLED WITH GEL DEPOSITS

frequently softened and fractured. The cracks extend outward into the surrounding matrix and are usually wholly or partly filled with gel substances as illustrated in Fig. 6. The zones around reactive particles are characterized by discolored borders and loose sockets filled with gel material or other abnormal products.

Far too little is known about the specific mineral constituents of various rock types which contain silica in forms susceptible to reactivity with alkalies in cement to permit positive predetermination of troublesome combinations by petrographic analyses. Some of the more conspicuously reactive aggregate materials include: Opaline and other siliceous rocks such as some cherts, some siliceous limestones, some chalcedonies and intimate opal-chalcedony admixtures; certain acid to intermediate volcanic rocks such as some andesites and rhyolites, including tuffs and glasses of similar composition; and miscellaneous rock types such as phyllite. Eminent authorities have stated that all rocks, with the possible exception of pure limestone, are potentially reactive in concrete. Small amounts of adversely reactive substances are known to occur in a large number of aggregate materials; but it is not known, as yet, what is a tolerable limit below which amount such substances can be ignored and above which they must be considered as dangerous in combination with high-alkali cements.

In studying the various dams that are deteriorating because of alkali-aggregate reactions, it has been impossible in most cases to establish the exact alkali content of the cements used, because determinations for Na_2O and K_2O were normally not included in routine chemical analyses of Portland cements until quite recently. However, from information disclosed by occasional analyses made by the manufacturers, it has been proved with reasonable certainty that high alkali ($1.0 \pm \%$ $Na_2O + K_2O$) cements were used in all structures that have exhibited serious deterioration because of alkali-aggregate reaction. It has been possible to obtain samples of the aggregate materials actually used in constructing the various dams, which have been subjected to accelerated expansion tests in mortar or concrete bars. Again invariably, these aggregate materials have developed high expansions in comparatively short periods of time in combination with high-alkali cements and no abnormal expansion when used with low-alkali cements.

When correctly interpreted, results from laboratory expansion tests are in agreement with the service histories of a number of cases where reactive aggregates from a given source have been used in construction, with both high-alkali and low-alkali cements. The experiences from such cases have repeatedly demonstrated that concrete made with reactive aggregates and high-alkali cements develops expansive deterioration but the same aggregates in combination with cement of less than 0.6% alkali content cause little or no trouble. Typical examples of such service records are provided by the Parker, Gene Wash, and Copper Basin dams in comparison with the Gene and Intake pumping plants, all in southeastern California.

Parker, Gene Wash, and Copper Basin dams were completed late in 1937 or early in 1938. These structures were all built with the same aggregates (a natural gravel deposit at the confluence of the Bill Williams and Colorado rivers in Arizona), and cement from the same mill containing 1.44% total alkalies. All three of these structures evidence profuse random-pattern cracking and expansive deterioration, which started from two to three years after completion. The Gene and Intake pumping plants were built almost coincidentally with the dams using aggregates from Colorado River deposits a few miles below the mouth of the Bill Williams River. Investigations have shown that the two aggregate sources (1) are of the same geologic age, (2) that they contain the same mineral elements in about the same proportions, and (3) that they show about the same degree of reactivity in the mortar-bar expansion test. The cement used in the pumping plants contained 0.51% total alkalies. After five or six years of service the concrete in these plants

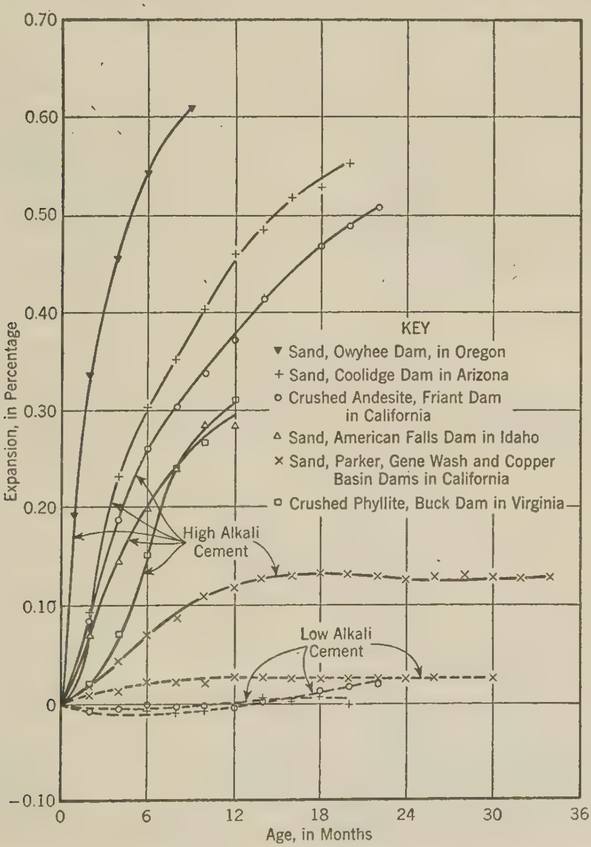

FIG. 7.—AGGREGATES USED IN SEVERAL DAMS EXPAND WHEN COMBINED WITH HIGH-ALKALI CEMENT

is as nearly perfect as any concrete could be. The same mill that supplied cement for the dams also later furnished cement for the Parker powerhouse but with a specification provision which limited the alkali content to 0.60% of ($Na_2O + 0.658\ K_2O$). The concrete in this latter structure, some of which was poured the latter part of 1940, was still in excellent shape at the end of 1944, although the aggregate is known to give expansion with a high-alkali cement. Expansion-test data obtained with aggregates from various dams in combination with high-alkali and low-alkali cements are shown in Fig. 7. The mortar bars tested were first stored with moisture at 100° F, and sealed.

Although excellent correlations have been obtained between laboratory results and service performance, the mortar-bar expansion test has not been developed so that it can be relied upon as an infallible indication of troublesome combinations of cements and aggregates. Neither can it be considered as a satisfactory acceptance test for either cements or aggregates. The test is useful only as an indication of possible troublesome combinations of reactive aggregates and high-alkali cements. If high expansions are obtained in mortar bars within a period of three or four months, it may be concluded safely that the materials contained in the bars are reactive. However, slowly reactive aggregates may require much longer periods of time to develop excessive expansions in the bar test. In such cases it is not always easy to determine whether the expansion is due to inherent alkali-aggregate reaction tendencies or to extraneous causes. Furthermore, because of the highly accelerated nature of the test, excessive expansions may ultimately be indicated with a highly reactive aggregate and relatively low-alkali cements, although field records do not indicate that such combinations will necessarily cause trouble in actual service.

Progressive Development of Alkali-Aggregate Deterioration in Concrete Dams

The rate at which expansive reaction develops does not appear to follow any predictable schedule. Perhaps the variable nature of the progression is due to the fact that the action is influenced by so many factors, including the alkali content of the cement, the thickness of section, the quality of the concrete, and the conditions of exposure, particularly with respect to moisture conditions within and surrounding the concrete.

At Parker, Gene Wash, and Copper Basin dams, severe open-surface cracking developed with the appearance of warm weather two years after the structures were completed. The cracking extended rapidly until all exposed surfaces were covered by the random pattern. Measurements across typical cracks in these structures indicate that the cracks are increasing in width very slowly, if at all. Embedded instruments appear to record some slight expansive growth.

At Buck Dam in Virginia excessive expansion was apparently not observed until ten to fifteen years after construction. Surface cracking and expansion progressed until extensive repairs of the dam and power plant were required after some thirty years of service, as reported (1)[3] by Philip Sporn, M. Am. Soc. C. E., and H. A. Kammer, Assoc. M. Am. Soc. C. E.

Although the natural aggregate used in building Owyhee Dam in Oregon is very reactive according to expansion-bar tests, only two or three small isolated areas of random-pattern cracking were in evidence in 1937, five years after its completion. The deterioration in these areas had progressed noticeably after another six years (1943), as shown in Fig. 8, and several other affected areas had appeared.

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[3] Numerals in parentheses, thus: (1), refer to corresponding items in the Bibliography (see Appendix).

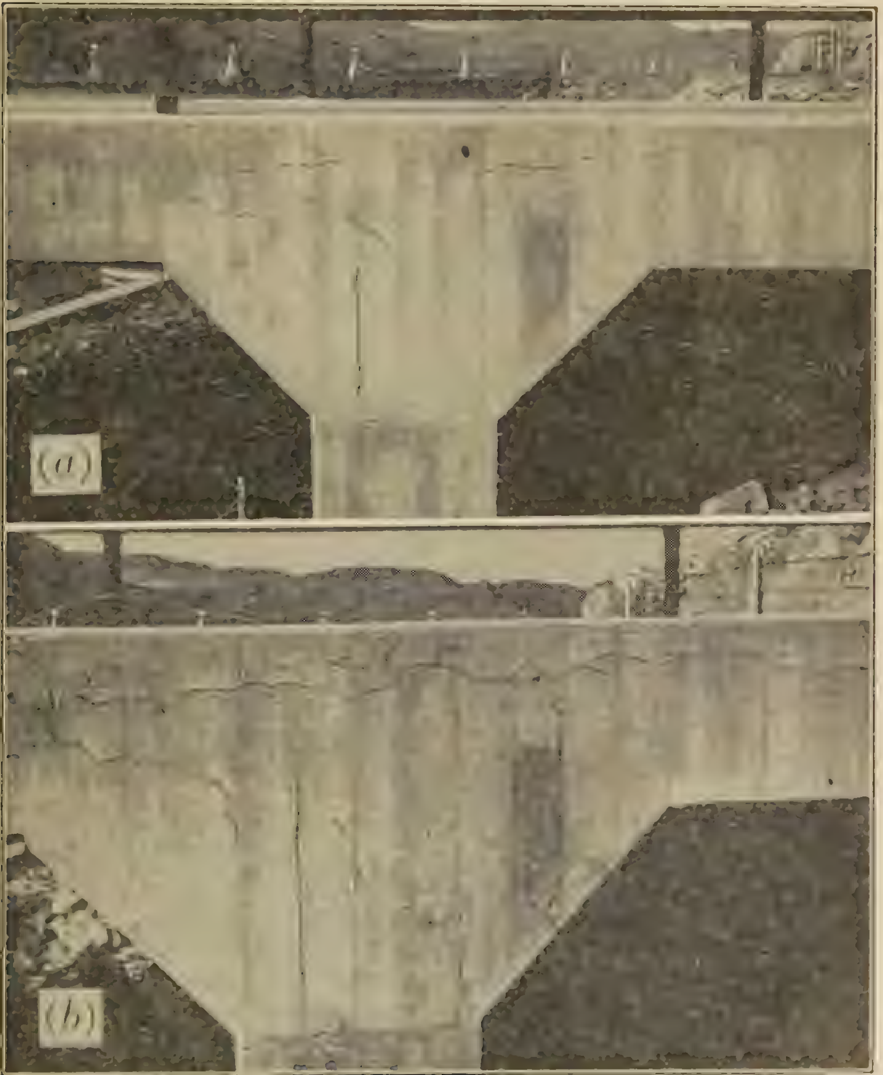

FIG. 8.—EXPANSIVE CRACKING HAS DEVELOPED PROGRESSIVELY IN THIS CONCRETE MEMBER, BUILT IN 1932

At Coolidge Dam in Arizona expansive deterioration has developed only in the parts where high-alkali cement was used, the greatest part of the structure (where low-alkali cement was used) being unaffected. During the period of fifteen years since its completion, severe cracking has necessitated minor repairs to the parapets and maintenance work to correct binding of the reinforced concrete spillway gates.

American Falls Dam in Idaho was completed in 1925. In 1929 there was no evidence of random-pattern cracking. There was noted at that time an efflorescence of sodium carbonate on the walls of the gate-operating gallery. By 1940 all of the previously described symptoms of alkali-aggregate deterioration were in sharp evidence. From information available, it can only be concluded that there has been a marked progression of cracking, spalling, and disintegration since that time.

Although some constituents in the Boulder Dam (Arizona-Nevada) aggregate have been found to be reactive, there is only minor development of random-pattern cracking in the structure, some of which must be ascribed to extreme drying exposure in the hot, arid location. The only place where alkali-aggregate reaction had been definitely identified at Boulder Dam, after eight years of service, was in the lining of a drainage tunnel through some wet rock.

From information collected on various types of structures, it appears that, in those cases where early, rapid expansion occurs, the action generally seems to slow down or even stop after a few years. The slower cases seem to be more insidious and deterioration progresses, comparatively slowly, for thirty years or longer, or until rehabilitation is required. Apparently almost any combination of conditions may be anticipated.

Significant Effects of Alkali-Aggregate Deterioration on the Service Life of Concrete Dams and Appurtenant Works

The preceding evidence has been given in considerable detail in order to present as clearly as practicable the symptoms by which alkali-aggregate degeneration is recognized, how it may be diagnosed positively, and a brief review of some known case histories. The next question of more immediate concern to engineers is the manner in which the "disease" affects structures in service.

Since the cracking that accompanies alkali-aggregate reaction is only a superficial surface condition, its effect on the structural stability of massive concrete structures would appear to be of minor importance. The esthetic and psychological effects of such cracking may require particular attention. Concrete dams subjected to severe conditions of exposure, such as freezing and thawing, corrosive waters, extreme temperature ranges, and extreme drying shrinkage, of course, will be weakened materially in their resistance to the disintegrating elements by the surface cracking which accompanies alkali-aggregate reaction.

Concrete cores drilled from affected structures show that the interior concrete is characterized by a chalky, lifeless appearance. Cores break easily during drilling and it is difficult to obtain representative samples sufficiently intact for strength and elasticity tests. Therefore, strength tests with cores from alkali-aggregate concrete must be considered as representing the best of such concrete. In general, such tests reflect some retrogression in strength as compared with that which might be expected from sound concrete under comparable conditions. In most cases, however, core samples have developed higher strengths than might be expected from their dead appearance.

Cores taken from Parker Dam, three years after construction, averaged 3,480 lb per sq. in. in compressive strength, with an average modulus of elasticity of 2,200,000. Previous cores, six months old, gave an average strength of 4,180 lb per sq in. and an elastic modulus of 3,800,000. In contrast to these results, 28-day control cylinders made during construction had an average strength of 4,180 lb per sq in., and 1-yr laboratory tests on Parker Dam concrete yielded 4,620 lb per sq in. strength and showed an elastic modulus of 4,200,000.

Similarly, although 28-day control cylinders showed the concrete in Gene Wash and Copper Basin dams to produce 3,450 lb per sq in. compressive strength, cores taken two years later showed the strength of this concrete to be 3,295 lb per sq. in., and the elastic modulus to be 2,420,000. Two-year-old cores, taken from pumping-plant structures which were built with the same aggregate as that used on Gene Wash and Copper Basin dams, but with low-alkali cement, were found to give 5,775 lb per sq in. compressive strength and to have a modulus of 4,800,000.

Mr. Kammer, with R. W. Carlson, Assoc. M. Am. Soc. C. E., reports (2) the strength of cores from the 26-yr-old concrete in Buck Dam as 2,100 lb per sq in.; and cores from the 15-yr-old affected concrete in American Falls Dam gave 2,840 lb per sq in. The latter concrete showed a low elastic modulus of 1,880,000 which, as may be observed from the other moduli listed in the foregoing paragraphs, appears to be a characteristic of concrete afflicted with expansive reaction between aggregate and high-alkali cement. The change or decline in elastic modulus has been used by many investigators as a measure of durability, or of the effect produced by some treatment of the concrete and may be considered as reflecting some deterioration. A marked reduction in modulus of elasticity, therefore, is interpreted as a sign of disintegration. Such changes in elasticity will also affect the distribution of stresses in a structure and will result in deflections or strains differing from those assumed in the design.

Measurements of the crown deflections of Copper Basin Dam, made regularly since completion of the structure, by the Metropolitan Water District of Southern California, show a progressive and steady upstream movement of 0.5 in. per year, amounting to 3.75 in. of total displacement in the latter part of 1944 (see Fig. 9). Superimposed upon the record are seasonal fluctuations due to temperature variations and load changes resulting from filling and drawdown of the reservoir; however, the data are positive in regard to a continuous movement of the structure. These deflection measurements have been corroborated by several independent observations from undisturbed control points on the solid-rock abutments and by plumb lines on the downstream face of the dam. Accurate measurements of several chord lengths across the top of the dam likewise reveal enlargement of the concrete, amounting to

0.018% linearly during the period of one full year. Other substantiating evidence of growth in the concrete exists in precise level measurements over points on the top of the dam, which show the crest of the dam to be rising over a 2-yr period. The increase in elevation at various points is roughly proportional to the height of the dam and with reference to this distance indicates a rate of expansion of 0.018% per year, corresponding to the data gathered from the chord measurements. Although these movements are indicative of the general enlargement of the concrete mass, the upstream deflection, which is already large for a thin arch structure, is significant because of the effect that it may have in changing the stress distribution from that originally contemplated.

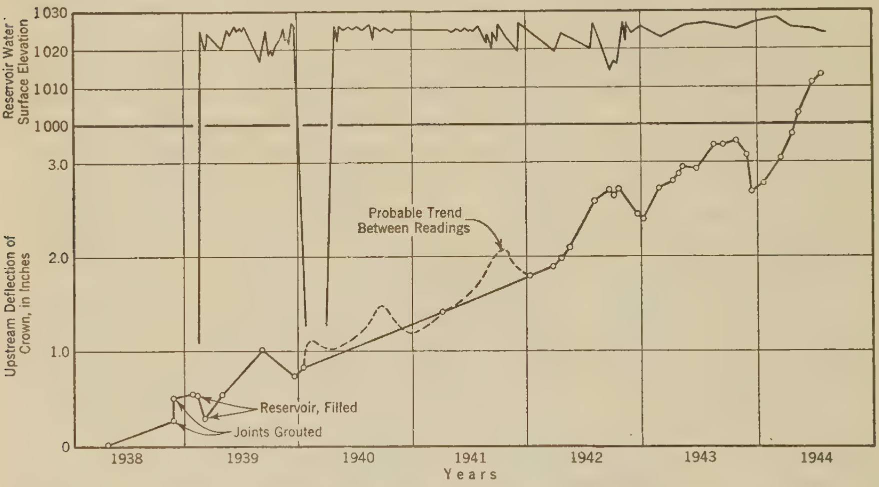

Fig. 9.—The Crown of Copper Basin Dam Has Shown Progressive Upstream Movement

Positive evidence of expansion in the concrete of Parker Dam exists in the record of measurements obtained from Invar rods mounted in holes drilled in the concrete. The rods are anchored in the bottom of four adjacent vertical holes (12, 30, 60, and 120 in. deep) and have dial gages attached to their upper ends so that any expansion or contraction over the span of the instruments is registered. As may be seen in Fig. 10, a general progressive expansion is recorded with seasonal temperature variation impressed upon the data from each hole. It is further indicated that expansion of Parker Dam concrete is greater within the interior than near the surface, the shallow holes showing the least expansion.

Aside from surface cracking and general deterioration, reactive expansion in concrete has been most significantly felt through its effect on embedded machinery. The alinement of generator and turbine shafts has been disturbed to the extent that further adjustments required extensive resetting and repairs. The strain in speed rings has been great enough to break the guide vanes, necessitating costly shutdowns and difficult repairs.

CORRECTIVE AND PREVENTIVE MEASURES

In mild climates expansive cracking may not present the serious aspect that it does in northern regions where it will be aggravated by exposure to the elements. In the latter cases, surface cracks, although superficial, serve as points of entrance for precipitation and assist the destructive processes of freezing and thawing, necessitating repairs much earlier than concrete in more moderate temperatures would require. Such repairs generally would comprise chipping away the affected concrete and incasing the new, taking pains to secure good bond and to provide the new work with joints which would allow additional growth in the underlying mass without producing random cracks in the new work. The paper by Messrs. Sporn and Kammer (1) describes restorations of this nature.

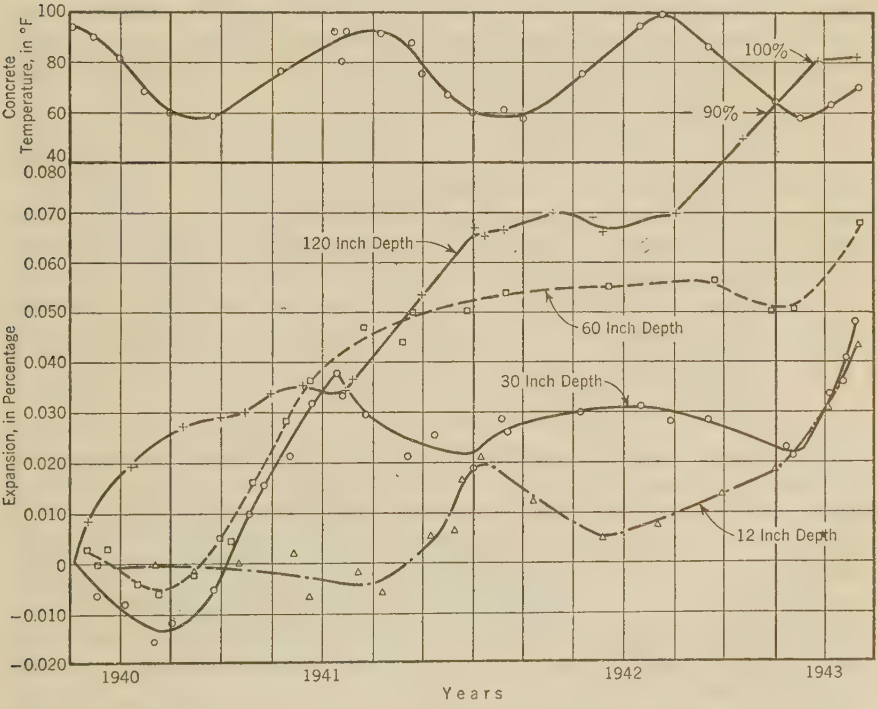

FIG. 10.—INVAR ROD METERS, MOUNTED IN DRILL HOLES, INDICATE PROGRESSIVE EXPANSION IN THE CONCRETE OF PARKER DAM

Where expansion has affected the alinement of embedded machinery to the extent that its operation can no longer be continued, satisfactory repairs can be effected only by removing the defective concrete in immediate contact with the parts and resetting the machinery in new, sound concrete. Precaution requires that the new concrete be free of the old, with joints or articulations that will insure no disturbance by further movement in the old concrete. Where watertightness in the entire structure must be preserved, the joints should be provided with flexible metal seals or plastic filler, or both. Continued frequent maintenance, alteration, and adjustment of the embedded machinery itself is the only alternate to any such major restoration work.

New concrete construction in areas where expansive reaction may occur dictates that precautionary measures be taken. The quantities of aggregates required in dam building usually necessitate the utilization of near-by sources for economic reasons. Such sources may have established service records from previous use but more than likely they have not. Therefore, it is usually not feasible to avoid the use of aggregates that may be known to be, or are suspected of being, reactive in combination with high-alkali cements. Furthermore, in most cases, reactive aggregates have been found to possess excellent physical and structural qualities, making them first-class materials for use in concrete in all respects, except in their tendency toward adverse chemical activity in the presence of inordinately high-alkali cements. Experience to date indicates that reactive aggregates can be used without danger of destructive reaction, provided that low-alkali cement (less than 0.60% $Na_2O + K_2O$) is employed (see Fig. 7).

Specification of low-alkali cement for all important hydraulic structures is strongly recommended regardless of what may or may not be known about the reactive tendencies of the aggregates to be used. Such a precautionary measure not only provides insurance against possible eventual deterioration due to alkali-aggregate reactions, but limits undesirable constituents of Portland cements and usually improves the quality in several other ways.

The Bureau of Reclamation, U. S. Department of the Interior, has purchased several million barrels of cement under specifications limiting the alkali content. This cement has been produced by a number of mills, and there is unanimous agreement that the result has been a much improved product, based on presently accepted criteria for judging the quality of Portland cements. In most cases the reduction in alkali content has been accompanied by marked reductions in C_4AF (tetracalcium alumino-ferrite), which is a fluxing material having little cementing value in itself, and lower C_3A (tricalcium aluminate), which is generally considered to be an undesirable compound. Coincidentally the principal cementing compounds, C_3S (tricalcium silicate) and C_2S (dicalcium silicate), have been increased as much as 9%. It seems evident that the additional cost of low-alkali cement, which may range from nothing to about 20¢ per bbl, is more than repaid by the improvement in quality alone.

Tests have indicated that some Portland pozzolanic cements may be safely used with reactive aggregate. It has also been demonstrated that additions of fine pozzolanic materials to reactive aggregate and high-alkali cement combinations or a replacement of some of the high-alkali cement by the pozzolanic material will reduce the expansion. In the construction of Friant Dam, on the San Joaquin River in California, 20% of naturally fine pumicite (by weight of the cement) was used as one of the concrete ingredients. Both high-alkali and low-alkali cements were used in the concrete since wartime production conditions made it impossible to secure low-alkali cement for all the work. The aggregate used in this dam was found to be reactive with high-alkali cement and it is therefore felt that pumicite has furnished some protection against expansion where the low-alkali cement was not provided.

No specific preventive for alkali-aggregate reaction has as yet been discovered although the problem is under intensive investigation. Specification of low-alkali cement, while admittedly inadequate, is the only recourse available to engineers at the present time.

SUMMARY

Expansive reaction between aggregate and high-alkali cement has been found responsible for the random cracking and deterioration of the concrete of a large number of dams. The binding of gates and of generator rotors, the misalinement of generator and turbine shafts, and the straining and breakage of guide vanes and other machinery embedded in concrete have been traced to the excessive expansion which accompanies such reaction. Aggregates that have been found to react with high-alkali cement include opaline silica, highly siliceous rocks (such as siliceous limestones, chalcedony, and some cherts), and acid to intermediate volcanic rocks. These same materials have caused no trouble in service when combined with cement low in alkali content.

Aggregate which will give this trouble in concrete may be recognized if it produces expansion within a short time in test specimens made with high-alkali cement. Absence of such expansion in a short period is no guarantee, however, against a latent type of reaction, for which no reliable accelerated test has appeared. Prudence dictates that aggregates known or suspected of adverse activity be used only with cement of low-alkali content or not at all. Low-alkali cement is recommended generally for all concrete in hydraulic structures as its use provides security against the uncertainty of encountering expansive reaction, and otherwise improves the general quality of the concrete.

ACKNOWLEDGMENTS

Acknowledgment is due Julian Hinds, M. Am. Soc. C. E., general manager and chief engineer, Metropolitan Water district of Southern California, for the data on deflection measurements at Copper Basin Dam; and Raymond E. Davis, M. Am. Soc. C. E., director, Engineering Materials Laboratory, University of California, for encouragement and advice. Activities of the Bureau of Reclamation are supervised by Harry W. Bashore, M. Am. Soc. C. E., commissioner; and all engineering and construction are under the direction of S. O. Harper, M. Am. Soc. C. E., chief engineer. Research and design work are directed by J. L. Savage, Hon. M. Am. Soc. C. E., chief designing engineer.

APPENDIX

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DISCUSSION

Ralph R. Proctor,[4] M. Am. Soc. C. E.—The cracking of concrete from internal expansion has been under investigation by the writer since about 1930, together with the general subject of expansion and shrinkage of concrete in connection with various structures that should be watertight. There seems to have been much more of this type of deterioration in the concrete used in construction since 1930 or 1935 than in older structures. As concrete aggregate probably has not changed much, it would appear that the first place to look for the trouble would be in all the new types of cement that have been produced during recent years; particularly as a considerable part of this trouble has occurred where new types of cement have been used.

Effect of Carbon Dioxide on Various Cements and Its Relationship to Alkali Aggregate Reaction.—During the driving of a long tunnel, large amounts of carbon dioxide were encountered. Tests were made to determine what the effect of this carbon dioxide would be on the concrete tunnel lining and what types of cement should be used to resist this action. As the contract for the cement to line the tunnel had already been entered into, five separate types of cement produced by the manufacturer who was to supply the cement were secured for testing. Small test beams for use in a miniature modulus-of-rupture test were made from sand mixed with each of the five types of cement, two of which were new type cements, and subjected to immersion in water under one atmosphere of carbon-dioxide pressure for eighteen months. At the end of that time it was found that all of the small beams, 1 in. wide, $\frac{1}{4}$ in. deep, and 3 in. long, had expanded and broken into bits, except those made from one type of cement not considered suitable for use in dam construction. The latter beams were unaffected, having the same strength when tested as similar beams that had been kept immersed in water in the concrete curing room of the laboratory. Thin sections from beams made from each type of cement were prepared and examined under the petrographic microscope. All those that had failed exhibited a white chalky substance between most of the sand grains, but the specimens that had resisted carbon dioxide showed a glassy substance, light amber in color, between the sand grains with only an occasional small speck of chalky substance embedded in the glassy material. Unfortunately, the sample of cement from which these carbon-dioxide resistant beams were made was lost so it could not be tested further.

The conclusion to be drawn from this experiment, in connection with this discussion, appears to be that, if a cement could be manufactured that, when set, would exhibit this glassy carbon-dioxide proof structure, there would need to be no worry about any kind of reaction between aggregate and cement. There would be no travel of chemical compounds, with resulting contact and crystallization, through such a glassy material, as it must have been completely watertight or the carbon dioxide would have broken down the structure as it did in all of the other specimens. Therefore, it appears to the writer that the afore-mentioned carbon-dioxide test of cement should be investigated as a possible means for preventing the kind of concrete deterioration under discussion.

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Relationship of Carbon-Dioxide Test to Merriman Cement Test.—Further testing of various cements (about eighty separate samples being tested) for resistance to carbon dioxide disclosed a wide variety of test results which could not be correlated with chemical compounds in the cement or any other cement designations such as sulfate resistant, low alkali, low heat, high early strength, etc.; and only two samples from the group showed reasonable resistance to carbon dioxide. All attempts to correlate test results with any means of testing or designating cement failed, except the comparisons made with the tests proposed by the late Thaddeus Merriman, M. Am. Soc. C. E. (9). The two cements that resisted carbon dioxide to a reasonable extent were not proposed by any one as carbon-dioxide resistant cements, but had simply been added to the test group because they were cements actually available even though made supposedly for other purposes.

When thin sections of these two best specimens were examined under the petrographic microscope, they exhibited similar glassy characteristics between the sand grains as has been described previously, although there were more small areas of chalky material visible. As far as can be determined, the main difference between the cements that resisted carbon dioxide and those that did not was in the temperature at which they had been burned during manufacture, as indicated by the Merriman sugar solubility index, rather than in chemical compounds. It is of special interest to note that the writer visited a cement mill that was producing a "low-alkali" cement for one large organization which does extensive research in cement; a "sulfate-resistant" cement for another large organization equipped similarly; and a "Merriman" type cement for a third large organization; and the exact same clinker was burned and subsequently ground and sold to each of the three separate organizations under their own separate specifications for cement. Furthermore, the cement thus manufactured met exactly all three specifications, according to statements of the manufacturer's chemist.

As far as could be determined, the essential characteristic of this clinker was the temperature in the kilns during manufacture, which appeared exactly in accord with several conversations that the writer had with Mr. Merriman on this subject at the time the tests were being made.

Evidence of Excessive Shrinkage in Expanding Concrete.—Fig. 1 is apparently the upstream or downstream face of a dam. It will be noted that over the distance shown in the picture (possibly 25 ft) there are at least eight vertical cracks that might be $\frac{1}{4}$ in. wide each. If the concrete in the dam had expanded enough to cause such cracking, it would mean that a 25-ft length of the dam had expanded as much as 2 in. (certainly not less than 1 in.) which raises the question of what happened to the dam when it elongated to this extent. Such expansion could occur vertically, of course, but it is difficult to visualize how it could occur horizontally, except in the case of a thin arch dam. Therefore, it would appear that the concrete must have excessive shrinkage properties when dried. This is further evidenced by referring to Fig. 5 to describe a condition noted by the writer on a specimen of concrete from the Parker Dam which had been freshly broken. To illustrate this point, it will be assumed that the piece of aggregate shown in the upper center of Fig. 5 represents the surface of a piece of nonreactive aggregate and that the white spots shown in the picture represent crystal growth on the surface of the nonreactive aggregate and between the aggregate and the remainder of the concrete.

The question arises then as to what circumstances permitted the formation of these crystals between the piece of nonreactive aggregate and the remainder of the concrete, for if the aggregate had been coated originally with cement paste there would be no reason for large quantities of these crystals to form next to the nonreactive aggregate rather than at any other location in the mass of the concrete. Therefore, it appears reasonable to assume that the aggregate probably had a higher coefficient of shrinkage when dried than did the concrete mass, with the result that it pulled away from the surrounding concrete and left a thin void that became filled with solutions that, upon subsequent drying caused the crystallization. Then, upon subsequent wetting, the aggregate expanded, crushing the crystals against the concrete and producing pressure. The formation and shape of the crystals bore out this conclusion. There has been other evidence found by the writer that this type of expansion in concrete can be associated with aggregate having a high shrinkage rate, or, to express it differently, an aggregate which expands considerably if immersed in water when in a dried condition. Therefore, it appears that the shrinkage or expansion properties of aggregates should also be investigated in connection with important structures.

Effect of Adding Admixtures to Concrete.—The free-alkali content of admixtures should also be investigated. It came to the attention of the writer in connection with one large dam where great care was being used in the selection of low-alkali cement, and where the aggregate was being scrutinized by all known methods, that the admixture used contained enough of the same free alkalies customarily found in cements to more than overcome the benefits secured from using low-alkali cement.

Tests for Reactive Cements.—Test bars similar to those used in autoclave tests were made in an attempt to determine the reactive properties of several types of Portland cement. These were immersed in water and dried through several cycles and the net result secured from the tests was shrinkage rather than expansion, even though the specimens were made with at least 50% of the aggregate and sand comprising supposedly reactive aggregate; however, the cements were not high in the Merriman sugar solubility index.

Conclusion.—It would be most interesting to secure test results similar to those shown in Fig. 7 wherein the expansion of concrete made from the various aggregates would be expressed in terms of the Merriman sugar solubility index of the cements used.

MILTON D. BURRIS,[5] ESQ.—For several years the writer has used petrographic methods to determine the reactive properties of aggregates with the alkalies contained in Portland cements. Thin sections of proposed aggregates are prepared in the usual manner: A small piece of rock is selected and polished on one side, after which it is cemented with Canada balsam to a glass microscope slide; and the other side of the rock is ground and polished until a thickness of 0.03 mm is secured. The rock section is then nearly transparent and the various minerals contained therein may be identified by polarized or plain light transmitted through the specimen and by the use of other standard petrographic methods for mineral determination.

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[5] Petrographer, Los Angeles, Calif. (formerly with Los Angeles Dept. of Water and Power).

In the section on "Symptoms and Diagnosis of Alkali-Aggregate Deterioration in Concrete Dams," the authors state that

> "Far too little is known about the specific mineral constituents of various rock types which contain silica in forms susceptible to reactivity with alkalies in cement to permit positive predetermination of troublesome combinations by petrographic analyses."

The structure of the thin section of rock and the texture of the individual minerals contained therein are usually among the first of the many properties determined by the petrographer. The texture of each specific mineral readily reveals the state of weathering undergone. Since practically all rock specimens show alteration or weathering to some extent, a perfect, unweathered or unaltered thin section of rock is seldom encountered.

This is well illustrated in samples of "andesite" aggregate from the Parker Dam analyzed by the writer. Under the microscope the ground mass (a glassy material filling the space between the larger crystals) in thin sections of these rocks proved to be quite dense and virtually unaltered; however, the andesine crystals (soda-lime-alumina silicates) were almost invariably affected. Numerous andesine crystals were poorly formed; others had been altered in varying degrees by chemical action. Many of these crystals showed replacement of portions of the andesine crystals by calcium carbonate to a considerable extent. This is not unusual and is quite common in many rocks where hydrothermal and other forms of metamorphism have taken place. Furthermore, the partial weathering of the aggregate renders it more susceptible to attack by the free alkalies in cement, particularly as some of the silica (SiO_2) had been released in the form of hydrated or opaline silica SiO_2 (n) H_2O—the amount of hydration expressed by n in the chemical formula varying from 1% to 21% and averaging between 9% and 13%.

Reactive Minerals Found in Parker Dam Aggregate.—Although the reactive aggregate in the Parker Dam is generally referred to as an andesite rock, this is partly disproved by the presence in some thin sections of albite and in others of oligoclase as the prevailing feldspar. Albite and oligoclase were also found more or less altered, and when combined with andesine they form the three feldspars that chemically contain the most sodium in the group of six soda-lime feldspars. Thus, it is logical to assume that the further release of sodium from the feldspars as well as from the cement would tend—through chemical reaction with the silica in feldspars and cements—to promote expansion even in a low-alkali cement but at a slower rate than in the case of a high-alkali cement.

Types of Siliceous Rocks that May Be Reactive with Cements.—In the "Summary," the authors state that

> "Aggregates that have been found to react with high-alkali cement include opaline silica, highly siliceous rocks (such as siliceous limestones, chalcedony, and some cherts), and acid to intermediate volcanic rocks."

Wollastonite ($CaSiO_3$) is formed only from a limestone deposit by contact or metamorphism; it is highly siliceous (51.7%) and has a calcium content of 48.3%. This rock has a specific gravity of 2.85 and a hardness of 5, and has been used, when available, with much success as a concrete aggregate. The siliceous limestones to be avoided are those that contain nodules of silica derived from organic matter such as Radiolaria and diatoms. They are readily distinguishable under the petrographic microscope because of the different optical properties of this type of silica. Opaline silica becomes dark under crossed "nicols" (the light passing through the thin section being passed through polarizers placed below and above the thin section and set at right angles to each other), and remains dark as the stage is rotated whereas pure silica, SiO_2, alternates between dark and white for every 90° the stage is rotated.

Additional Precautions To Be Taken in Cement Manufacture.—The authors state (see heading, "Corrective and Preventive Measures") that

> "In most cases the reduction in alkali content has been accompanied by marked reductions in C_4AF (tetracalcium alumino-ferrite), which is a fluxing material having little cementing value in itself, and lower C_3A (tricalcium aluminate), which is generally considered to be an undesirable compound."

These two compounds are dependent wholly upon the clay (alumina) and iron content of the materials used in the manufacture of cement. Most of these two compounds are derived from the waste material found in fault zones and from fractures that are always present to some degree in practically all quarries of consequence. These two compounds constitute 20% and more of most cements and range in value from "practically useless" to "quite detrimental." Furthermore, rather than depend upon a coincidence for a reduction of these compounds in the finished product, it is suggested that closer attention be paid to the elimination of waste material from the quarry.

Aggregate Reaction Shown by "Popouts."—Fig. 11 is a view of the underside of a "popout" from the surface of a concrete slab showing a piece of aggregate of unknown composition that became gelatinous through chemical reactions of alkalies and silicates. This chemical change increases the volume of the aggregate particle; and, when the compressive stress thus formed within the aggregate exceeds the tensile strength of the concrete, "popouts" are the result. Popouts occur but an inch or two below the surface of the concrete and over a 3-in. or 4-in. diameter, but the chemical reactions are identical with those which occur at greater depths in mass concrete structures. The shrinkage cracks shown in the aggregate particle near the center of the photograph are the result of dehydration of the gel after exposure to the sun for three or four hours. The dehydrated piece of aggregate shown in Fig. 11 is one of a number of mineral substances that do not yield definite chemical formulas and furthermore show no signs of crystallinity. These substances have been called "gel minerals" or "mineraloids." The power of these minerals to absorb other substances accounts for their often wide variation in chemical composition although silica predominates greatly in all analyses.

Fig. 1 shows a typical view of concrete where expansion within the mass has occurred. However, if the reaction was entirely between coarse aggregate and cement, there should be evidence of this by popouts caused by pieces of aggregate lying close to the surface. The writer has observed considerable expanded concrete where there were no popouts visible, leading to the conclusion that at least the coarse aggregate was not involved. Accordingly, two thin sections were prepared from a small piece of Parker Dam concrete available from an area adjacent to some expansion cracks having pronounced expansion cracks within one section and slight expansion cracks in the other. There was no positive evidence of reaction between cement and coarse aggregate or between cement and sand, leading to the conclusion that the expansion possibly might be caused by reaction within the cement. It is suggested that the possibility of opaline silica from the limestone remaining in the cement, because of too low kiln temperatures, and uniting with free alkalies remaining in the cement for the same reason, should be investigated thoroughly as a possible source of this expansion.

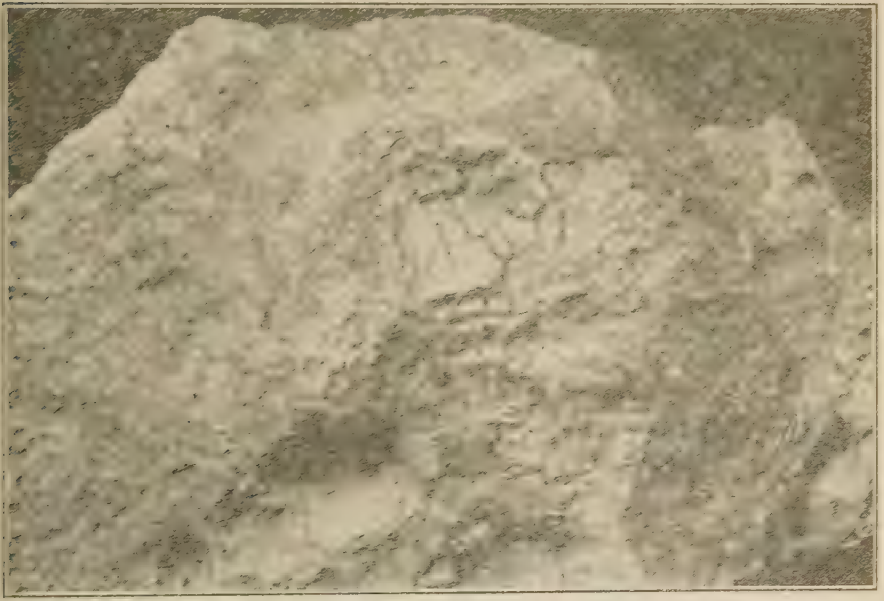
FIG. 11.—UNDERSIDE OF A "POPOUT" AFTER SUBSEQUENT AIR DRYING
(Approximate Magnification, 3 X)

Conclusions.—All rocks, with the possible exception of pure limestone, are potentially reactive in concrete. This is especially true of any aggregate that shows the least sign of weathering. Incipient alteration in any aggregate chosen should be observed and studied with a polarizing microscope.

Radiolaria and diatoms are microscopic animals and plants, respectively, whose tests or shells are composed of hydrated silica which contains from 1% to 21% of H_2O. Hydrated silicas react with the sodium and potassium hydroxides referred to as free alkalies in Portland cement to form sodium and potassium silicates, thus causing expansion by the formation of a gel.

Far too few data have been made available to engineers concerning reactive aggregates, such as the mineral and chemical composition and the state of weathering, whether incipient or otherwise. For instance, during the process of weathering, an albite feldspar produces a sodium silicate gel and free lime which eventually carbonates causing further expansion. Secondary crystal growth and carbonation of freed calcium, as well as the formation of a highly siliceous ooze or gel, have been observed under the microscope, by the writer, in all expanded concretes examined.

The large amount of concrete manufactured without trouble from expansion appears to indicate that ordinarily both cement and aggregate must contain an excess of available free alkalies to cause expansion and that when either cement or aggregate is practically free of alkalies no serious difficulty has been encountered. However, for an important permanent structure, a full investigation of the free-alkali properties of both cement and aggregate should be made, with special emphasis on the analysis of the aggregate by petrographic methods as outlined herein in order that all known precautions may be taken to prevent concrete expansion.

H. A. Kammer,[6] Assoc. M. Am. Soc. C. E.—The deterioration that develops in concrete structures as a result of the reaction that sometimes occurs between certain concrete aggregates and the alkalies in some Portland cements has been well described in this paper. The authors also have recommended certain corrective and preventive measures that might be used to limit, or possibly to eliminate, the destructive effects of this phenomenon.

In 1941 it was established definitely that the serious deterioration of the concrete structures at the Buck Hydroelectric Plant in Virginia was caused by a reaction that was taking place between the alkalies in the cement and the phyllite aggregate used in the concrete. The basis for this conclusion was reported by R. W. Carlson, Assoc. M. Am. Soc. C. E. (2), and the writer at that time.

In the summer of 1942 work was started on rebuilding and rehabilitating some sections of the substructure of the powerhouse and the exposed surfaces of the dam. In the powerhouse the old concrete structure supporting the turbine and generator was removed to a level a few feet below the bottom of the scroll case and the foundations were rebuilt using cement with an alkali content of approximately 0.5% and an aggregate which the engineers were reasonably certain was not reactive with the cement. The details of this work were reported by Philip Sporn, M. Am. Soc. C. E., and the writer in 1944 (1). Reference points were established in pairs, one of each pair being established in the old concrete which was left in place and the other in the new concrete. The joint between the old and new concrete was treated to break bond.

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[6] Constr. Engr., Am. Gas & Elec. Co., New York, N. Y.

Rebuilding of the first unit was completed early in 1943 and periodic measurements made since that time definitely indicate that the old concrete adjacent to the machine foundations is still growing. The measurements, made at the four corners of the foundation, indicate some difference in the amount of movement at the several corners which may be due to the variation in the depth of the old concrete still in place below the reference points. Inaccuracies in measurement may also be a contributing factor in these variations. In May, 1945, the surfaces of the old concrete structures were an average of 0.05 in. higher than the new.

The replacement of the foundations for the second and third units has been completed but, although a trend in the same direction as unit No. 1 is noticeable, sufficient time has not elapsed to permit accurate and conclusive information to be obtained relative to the behavior of the structures at these locations. Further growth of the old concrete was expected and allowed for in the reconstruction of the machine foundations. Therefore, no trouble is expected to develop as a result of the continued growth of the old structure.

The downstream face of the north abutment section of the dam was repaired by cutting off the affected face and replacing it with new concrete provided with regularly spaced control joints which were expected to open as the underlying mass continued to grow and expand. After two years this section of the work was behaving as expected and cracks developed in all of the joints. No random map cracking of the type previously experienced had developed by May, 1945.

Messrs. Blanks and Meissner point out that experience thus far demonstrates that reactive aggregates can be used without danger of developing destructive reactions in the concrete mass if the alkali content of the cement is less than 0.6% $Na_2O + K_2O$.

At one of the hydroelectric developments erected on the American Gas and Electric System in 1938 a dolomitic limestone was used for the manufacture of both the coarse and fine concrete aggregates. At that time nothing was known about any destructive reactions developing between some aggregates and the alkalies in some Portland cements. The cement was purchased under rather rigid specifications but no limit was placed on the alkali content.

When the cause of the trouble at the Buck Hydroelectric Plant was discovered, tests were made to ascertain if the dolomitic aggregate used in the new development was reactive. These tests showed that the particular dolomitic limestone that was used was more reactive than the phyllite aggregate used at the Buck development.

Fortunately, samples had been taken of all cements used in the new structure and they had been carefully stored. Tests were made to determine the alkali content ($Na_2O + K_2O$) which was found to be:

Cement No.	Alkali content (%)
1	0.75
2	0.62
3	0.46
4	0.36

Reference points were established at a number of locations on the concrete structures at the new plant and periodic measurements and observations have been made since 1942. Thus far (1945), no unusual expansions have been measured nor has any random-pattern cracking, which is so typical of alkali-aggregate deterioration, been observed.

THOMAS E. STANTON,[7] M. AM. SOC. C. E.—The material presented by Messrs. Blanks and Meissner is a valuable and interesting contribution to the exceedingly important topic of the "Deterioration of Concrete Dams Due to Alkali-Aggregate Reaction."

Reactive Minerals.—The authors have well stated:

> "Far too little is known about the specific mineral constituents of various rock types which contain silica in forms susceptible to reactivity with alkalies in cement * * *."

This is a fertile field for research.

Opal, Chalcedony, Limonite, Glauconite, Dolomite, and Calcite.—In the early reported California experience, it was definitely determined that opal was the reactive ingredient (4). Test data with 1:2 mortar and high-alkali cement (that is, 1.14% Na_2O), covering four years of observation, are given in Table 2. Because of the high lime-magnesia content, rock No. 28039 (Col. 2, Table 2) was classified as a siliceous magnesian limestone (4*a*). The expansion results in Col. 4, Table 2, however, indicate that opal is undoubtedly the reactive constituent of this rock. The long-time (forty-eight-month) tests confirm the original conclusions based on tests of specimens less than one year old, thereby indicating conclusively that pure chalcedony, limonite, glauconite, dolomite, and calcite have no abnormal reactive properties—at least in the percentages used in this test series.

TABLE 2.—REACTION CHARACTERISTICS OF MINERALS SIMILAR TO THE CONSTITUENTS OF SILICEOUS MAGNESIAN LIMESTONE (ROCK NO. 28039)

Mineral (1)	PERCENTAGE DISTRIBUTION: In rock No. 28039 (2)	PERCENTAGE DISTRIBUTION: In each test specimen[a] (3)	Percentage expansion after 48 months (4)
Chalcedony	15.0	10.0	0.019
Opal No. 1	12.0	2.5	1.574
Opaline chert	1.0	10.0	2.837
Limonite	1.0	5.0	0.012
Glauconite	1.0	5.0	0.010
Dolomite	10.0	10.0	0.007
Calcite	60.0	10.0	0.046
Combined	100.0	10.0	0.776

[a] These are the values substituted for equivalent percentages of the nonreactive sand used in each sand-mortar specimen. Separate bars were fabricated for each of the minerals in Col. 1.

Other tests have been made with higher percentages of chalcedony without the development of an expansive reaction, whereas tests on opal-bearing chalcedonic cherts and other cherts have developed expansive results proportional to the percentage of opal. An excellent discussion on this phase was contributed by Roger Rhoades in 1942 (10). Mr. Rhoades stresses the fact that:

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[7] Materials and Research Engr., State Div. of Highways, Sacramento, Calif.

> "* * * authoritative conjecture surrounds the possibility that some chalcedony verges toward an opaline character in becoming hydrous, and the question naturally arises as to the possibility of such types also assuming other similarities to opaline silica, perhaps becoming potentially reactive in alkaline environments."

He cites the possibility that:

> "* * * opaline material minutely disseminated in chalcedonic aggregates may escape observation and yet impart opaline properties which appear to pertain to the enclosing chalcedony."

The studies of the rocks found in the coastal region of California, where the opal-bearing chalcedonic cherts have caused so much trouble, check Mr. Rhoades' observations.

Andesites, Rhyolites, Tuffs, and Volcanic Glasses.—The Bureau of Reclamation has definitely traced the cause of the trouble at Parker, Gene Wash, and Copper Basin dams to an andesitic rock. As a result, all andesitic rocks have justly been under suspicion as potentially reactive. This rock type has been observed in the Cowlitz (Washington) gravels and is present in the aggregates from the Friant and other Fresno County (California) areas along the San Joaquin River. The andesites from these areas have been examined petrographically; and, although there are wide variations as to type, texture, and other characteristics, they all have one feature in common—a glassy groundmass. Specimens of concrete from Friant Dam, the Belmont Traffic Circle in Fresno, Calif., the Summers Creek and Courtright Creek bridges in Washington, and the Parker Dam have been examined to observe which aggregates were reacting. Numerous thin sections were made of various particles that appeared to be participating in gel formation. The phenocrysts of plagioclase, hypersthene, hornblende, olivine, augite, rutile, and a few other less common minerals were, in most cases, unaffected by contact with the cement mortar. In various instances, however, the glassy groundmass showed signs of reaction.

It was decided, therefore, to study the glassy groundmass of various basic or intermediate volcanic rocks further. To insure sufficient uniform material for study, and for fabrication into expansion bars, it became necessary to synthesize a quantity of glass having a chemical composition equivalent to that of an average andesite (11)(12). A small furnace was constructed of firebrick. The finely pulverized charge was first moistened with sufficient water to mold it, and thus to prevent the flame from blowing the powder away. The charge was made of the straight oxides of aluminum, iron, silicon, and magnesium with equivalent amounts of the carbonates, calcium, sodium, and potassium. (Compounds of the rarer elements that are usually found in andesites, such as titanium, zirconium, phosphorus, manganese, etc., were omitted from the charge.) Chemical analyses were made of the charge before fusion, and of the fused glass. These analyses are shown in Table 3. Reaction characteristics of this glass are given in Table 4.

As soon as melted, the glass was ladled from the furnace; and, after cooling at room temperature, it was crushed and sieved into various sizes for fabrication into expansion bars with both low-alkali and high-alkali cements. The percentage of silica dissolved after twenty-one days in 10% NaOH (using the Bureau of Reclamation method) was 0.9%. Solubility in NaOH by the foregoing method does not appear to be significant, however, except possibly in the case of the highly soluble rocks, in which case the high solubility may be indicative of opal.

TABLE 3.—PERCENTAGE COMPOSITION OF SYNTHETIC ANDESITIC GLASS

Constituents	Unfused (original mixture)	Fused glass
Silica as SiO_2	54.3	61.8
Alumina as Al_2O_3	17.4	18.8
Iron as Fe_2O_3	3.3	2.5
Lime as CaO	12.8	7.6
Magnesia as MgO	3.9	4.0
Sodium as Na_2O	6.3	3.0
Potassium as K_2O	2.0	1.2
Undetermined	0.0	1.1
Total	100.0	100.0

TABLE 4.—PERCENTAGE EXPANSION OF 1:2 MORTARS OF A HIGH-ALKALI CEMENT (1.07%) AND NEUTRAL RUSSIAN RIVER SAND COMBINED WITH CRUSHED SYNTHETIC ANDESITIC GLASS

Age (month)	PERCENTAGE OF ANDESITIC GLASS							
	At 70° F				At 110° F			
	5	10	20	30	5	10	20	30
	(a) UNWEATHERED MATERIAL							
24	0.020	0.019	0.031	0.055	0.061	0.217	0.351	0.491
36	0.028	0.040	0.080	0.148	0.072	0.241	0.368	0.512
	(b) WEATHERED MATERIAL							
3	0.012	0.143	0.063	0.008	0.026	0.046	0.123	0.055
12	0.024	0.218	0.461	0.022	0.054	0.271	0.486	0.418
24	0.026	0.218	0.470	0.391	0.049	0.299	0.549	0.627
27	0.025	0.218	0.472	0.549	0.061	0.334	0.616	0.726

Chemical Alteration as an Activator.—Observations of many andesite specimens known to be reactive indicated that the specimens were moderately or partly altered and weathered. This condition was existent prior to fabrication into concrete. Work done by R. D. Evans and Howel Williams (13) in Lassen Volcanic National Park (California) shows a few rocks (with opal as a principal constituent) in the vicinity of hot springs to be highly altered. To simulate the Lassen Park condition, some of the synthetic glass was altered in the presence of SO_2 and water, for comparison of expansive reaction with the fresh glass.

A portion of the untreated synthetic andesite glass, crushed to $-20+50$ size was placed in a long glass tube, moistened with water and subjected to daily evolution of SO_2 from a cylinder. A petrographic study of the (SO_2-H_2O)-treated glass indicated a definite change not only in the appearance of the material, but also in its composition and refractive index. Tests at various ages on the unweathered and weathered material (Table 4) indicate a definite increase in reactivity following weathering. This work should be checked against, or compared with, glasses of other composition. A fertile field is indicated for the investigator who has the time.

Experience with the San Joaquin River aggregates from the vicinity of the Friant Dam and several miles lower down at Herndon, Calif., indicated that some of the andesitic rocks in these deposits are highly reactive and some only slightly so (Table 5).

TABLE 5.—PERCENTAGE EXPANSION, IN THIRTY-SIX MONTHS, OF ROCK SAMPLES FROM STOCK PILES AT A COMMERCIAL PLANT NEAR FRIANT, SAN JOAQUIN COUNTY, CALIF. (1:2 MORTAR)

Type[a] No.	(a) HIGH-ALKALI CEMENT "GS" (1.07% ALKALI)				(b) LOW-ALKALI CEMENT AS "AS" (0.48% ALKALI)			
	PERCENTAGE OF ANDESITE ADDED TO NEUTRAL SAND							
	10%		25%		10%		25%	
	70° F	110° F	70° F	110° F	70° F	110° F	70° F	110° F
1	0.027	0.102	0.025	0.066	−0.006	0.017	−0.003	0.016
2	0.030	0.506	0.365	0.861	0.000	0.153	−0.015	0.345
3	0.018	0.067	0.019	0.086	−0.004	0.011	−0.004	0.041
4	0.013	0.029	0.014	0.042	−0.006	0.001	−0.005	0.005
5	0.023	0.330	0.029	0.519	−0.005	0.005	0.001	0.011

[a] A mineralogical description of each type of rock is listed in the text.

The test data from which Fig. 7 was compiled indicate a definitely reactive andesite. California highway experience at the Belmont Traffic Circle, and on the concrete pavement between Fresno and Herndon is a further demonstration of the fact that at least portions of the San Joaquin River aggregates are highly reactive on occasion. However, because experience indicated that aggregates from this source are not consistently reactive and in some cases not reactive at all, a special series of laboratory tests on samples of rock from the San Joaquin River, which, on superficial inspection appeared to be andesitic in character, was begun several years ago with the results shown in Table 5.

The expansion observations in Table 5 indicate that only one (type No. 2) of the five andesites tested is highly reactive at 70° F but that one other (type No. 5), although not excessively reactive at 70° F, is materially activated at 110° F—thereby indicating potential ultimate excessive reactivity at 70° F. Type No. 2 is reactive at 110° even with the low-alkali cement. That the high results with type No. 2 are not accidental is indicated by the trend of all specimens containing this material in combination with a high-alkali cement at either 70° F or 110° F.

The rock samples in Table 5 were selected from stock piles of coarse aggregate at a commercial plant near Friant. There were no marked visible distinctions between the different samples; and it is difficult, therefore, to estimate the probable percentage of each in the combined product of the plant. That these relative percentages are not constant, however, can be inferred by the difference in performance in different projects using the same cement or cements of similar composition. Obviously, the only safe procedure with such variable quality aggregates is the use of a low-alkali cement as was done as far as practicable by the Bureau of Reclamation in the construction of Friant Dam.

In an effort to determine the reactive ingredients in the San Joaquin River aggregates, a petrographic analysis was made of each of the five rock types tested. The results of these analyses are as follows:

1. *Megascopically, a Dense, Dark-Colored Andesite with a Few Relatively Large Scattered Crystals (Phenocrysts) of Feldspar.*—The rounded pebbles of this type show little or no weathering on the outside surfaces. Under a microscope, the dense groundmass is found to be a fine felt of little rods of plagioclase feldspar arranged in more or less flowing lines with their long dimensions parallel with one another (trachytic texture). Included in the felty groundmass are ferro-magnesium minerals (principally augite), magnetite, and amorphous basic glass.

2. *A Dense, Mottled, Grayish-Green Andesite Which Shows Flow Structure.*—Along numerous fine cracks and planes of weakness parallel to the flow lines the rock has been altered to a reddish-brown material. Microscopic examination shows a felty, locally trachytic groundmass composed of plagioclase microlites (embryonic crystals) with interstitial pyroxene, finely divided magnetite, and basic glass. Portions of the groundmass (from 10% to 20%), particularly along seams and fracture planes, have been altered to a brownish, amorphous, hydrated glass which is probably the mineraloid palagonite.

3. *A Dark-Colored, Dense, Porphyritic Rock Which Shows Flow Structure and Is Streaked with Fine Light-Colored Veinlets.*—This rock consists of large phenocrysts of feldspar, quartz, and hornblende in a once glassy-flowing groundmass which has devitrified, or broken up into aggregates of quartz and feldspar in excessively minute crystals. Numerous quartz veinlets are scattered throughout the rock. This metamorphosed volcanic rock may be classified as an aporhyolite.

4. *A Weathered Andesite Whose Light, Grayish-Brown, Rounded Pebbles Have Fresh Dense Unweathered Cores.*—The amount of this weathering varies; some of the pebbles are completely altered, whereas others have only a rim of altered material on the outside. The unaltered central core of the rock consists of lathlike phenocrysts of plagioclase feldspar (andesine) in a felty, occasionally trachytic groundmass of plagioclase microlites set in a random mesh, the interstices of which are filled with augite grains, magnetite, and amorphous glass. The altered portion of the rock consists of corroded phenocrysts of plagioclase in a groundmass which has been almost completely altered to palagonite.

5. *A Grayish Andesitic Rock with Prominent Flow Lines and Embedded Eruptive Breccia.*—Microscopic examination shows that the embedded fragmental breccia and the matrix are very similar. Both are composed of a finely crystalline net of plagioclase feldspar needles with interstitial pyroxene, magnetite, and glass. Well-developed flow banding is common and alteration to palagonite has occurred along seams in some sections.

Although the cause of the high reactivity of type No. 2 can be traced to the glassy groundmass, the analyses indicate that type No. 4 should have been equally if not more reactive—judging by the petrographic analysis, which classifies it as a weathered andesite in which the weathered or altered portions consist of corroded phenocrysts of plagioclase in a groundmass that has been almost completely altered to palagonite. Since palagonite has been determined to be highly reactive, it was felt that type No. 4 should have shown high, instead of apparently negligible, reactive properties.

Because of the possibility that this was another case in which too great a percentage of a highly reactive ingredient had resulted in no measurable reaction, a new series of tests was begun in which only 2½% and 5% of type No. 4 were incorporated with a neutral sand instead of 10% and 25% as in the original series. Although preliminary 28-day test results indicate some greater reactions than do the original tests, they are not sufficient to justify any definite conclusions.

Phyllite.—The phyllite referred to by the authors, the reactivity of which is well illustrated in Fig. 7, was discussed by H. A. Kammer and R. W. Carlson, Assoc. Members, Am. Soc. C. E., in 1941 in connection with a report on the Buck Hydroelectric Plant in Virginia (2). The reaction at the Buck Plant was slow, and the expansion did not become sufficiently extensive to be noticeable until 1922. However, the expansion continued until, by 1941, the dimensions were reported to have increased about 0.5% and to be still growing. A possible explanation of this slow development of the reaction, if the phyllite is at fault, is the fact that this rock was used only in the coarse aggregate portion of the concrete mix. Crushed to sand size, reactivity becomes apparent at a much earlier period, as demonstrated by the Bureau of Reclamation tests which are checked by some work done by the California Division of Highways.

The authors declare (see heading, "Symptoms and Diagnosis of Alkali-Aggregate Deterioration in Concrete Dams") that "Eminent authorities have stated that all rocks, with the possible exception of pure limestone, are potentially reactive in concrete." This conclusion might, with accuracy, have been expanded to state that any such potential reactivity increases with temperature and that most tests indicate a higher expansion at 110° F and 130° F than at 70° F, thereby indicating the effect of temperature as an accelerator and, even on occasion, as an activator. Concrete temperatures exceeding 100° F are not unusual, particularly in pavements and thin walls subject to the direct rays of the sun in the hot summer temperatures that prevail in many areas in California as well as elsewhere throughout the United States. The effect of temperature has been demonstrated by the writer elsewhere (14).

The authors contend (see paragraph preceding the heading, "Progressive Development of Alkali-Aggregate Deterioration in Concrete Dams") that

> "* * * the mortar-bar expansion test has not been developed so that it can be relied upon as an infallible indication of troublesome combinations of cements and aggregates."

This may be true as far as mildly reactive combinations are concerned. However, the mass of evidence accumulated since 1939 has convinced the writer that not only is the mortar-bar test an excellent indication of reactivity but that it is the only dependable test developed to date.

If the authors had qualified this conclusion by a time limit clause such as "within 28, 60, or 90 days when no excessive expansion develops at earlier ages," the conclusion would have greater validity, as undoubtedly in certain slowly reacting combinations, such as the Buck Dam phyllite, a somewhat longer than normal time is required. However, any excessive expansion at early periods can be considered as a definite indication of reactivity, roughly proportional to the magnitude of the expansion.

It is conceded, however, that absence of early expansion cannot be considered as proof of the nonreactivity of the aggregate from a given source when used with a high-alkali cement. The aggregate, if reactive, reacts very slowly; or, if no reaction is observed, the particular sample under test just happens to contain none of the reactive mineral or minerals normally present. The truth of this last statement is well illustrated by experience in California where the mineral causing the reaction (opal) was sometimes present in such small percentages (less than 1.0% of the aggregate) that reactive particles might be entirely missing from a small-sized test sample secured from a commercial supply.

If (as the authors state) the mortar-bar test cannot be considered a satisfactory acceptance test for either cements or aggregates, the only alternative in the absence of any more suitable test is to limit the alkali content of all cements as a standard requirement if engineers are to insure against destructive combinations.

The writer agrees with most, if not all, of the authors' comments on the deficiencies in the mortar-bar test but would have no hesitation in being guided by the results thereof in the absence of evidence to the contrary. On the other hand, the authors are certainly justified in their conclusion that

> "* * * because of the highly accelerated nature of the test, excessive expansions may ultimately be indicated with a highly reactive aggregate and relatively low-alkali cements, although field records do not indicate that such combinations will necessarily cause trouble in actual service."

An illustration of the validity of this conclusion is the California low-alkali cement AS (Table 5) which apparently never develops excessive reaction at normal temperatures but is peculiarly sensitive and frequently becomes highly reactive even at temperatures as low as 110° F. It would appear logical, however, to follow the lead of the mortar-bar tests in the absence of any contrary and reliable job experience.

The authors cite the case of the Owyhee Dam aggregate which, although highly reactive in the mortar-bar tests, failed to show much reaction in service during the first five years. Neither the alkali content of the cement nor the amount of cement per cubic yard used in the Owyhee Dam construction is given. Is it not possible that an intermediate alkali cement, slow in developing sufficient reaction to be objectionable, was used at Owyhee Dam? The accelerated laboratory tests were made with a high-alkali cement and, therefore, are truly indicative of the ultimate potential reactivity of the aggregate. They constitute a warning that a low-alkali cement should be used under similar conditions.

Furthermore, the nature of exposure is undoubtedly an important factor in the development and progress of the reaction. Many structures in which reactive combinations are used show early distress in portions exposed to adverse weathering conditions, whereas protected portions of the same structure fail to develop appreciable distress, if any. Therefore, although the structural stability may not be seriously lowered, the evidences of the reaction occur where they are most readily observed or where there is the greatest likelihood of subsequent failure through either alterations of freezing and thawing or attack by alkalies from soils or sea water.

Corrective and Preventative Measures.—The authors' comments and suggestions with regard to the necessity for, and desirable procedure to be followed in, making repairs to damaged structures are excellent, as are their comments regarding precautions to be taken in the original construction in so far as the use of low-alkali cement (0.60% or less) is concerned. In addition, the authors state:

> "Tests have indicated that some Portland pozzolanic cements may be safely used with reactive aggregate. It has also been demonstrated that additions of fine pozzolanic materials to reactive aggregate and high-alkali cement combinations or a replacement of some of the high-alkali cement by the pozzolanic material will reduce the expansion."

With regard to the Fresno pumicite, however, there is some evidence indicating possible objectionable rather than beneficial qualities.

The California Division of Highways has tested a number of finely divided minerals for use as correctives but so far has found only two types of materials which appear to be always truly corrective when ground to −200 mesh and used in practicable amounts. These two materials are the opaline cherts and the calcined Monterey shale of the type used in the pozzolanic type of cement (HP) (4*b*)(4*c*). Cement HP had a relatively high-alkali content (0.78%) but has been entirely nonreactive over a period of six years although the standard cement of the same brand and alkali content is decidedly reactive. Checks have been made by adding this same pozzolana (calcined Monterey shale) to the high-alkali cement GS with uniformly successful results. All other local siliceous powders with the exception of the ground opaline rocks, however, have been found relatively ineffective.

TABLE 6.—EFFECT OF TWO TYPES OF POZZOLANIC ADMIXTURES ON SEVERAL REACTIVE CEMENT-AGGREGATE COMBINATIONS

Cement No.	Alkali (%)	PERCENTAGE EXPANSION IN FORTY WEEKS AT 70° F		
		No Admix-	Fresno pumicite	Calcined Monterey shale
1	0.47	0.067	0.162	0.014
2	0.47	0.168	0.183	0.015
3	0.51	0.118	0.108	0.016
4	0.62	0.255	0.123	0.011
5	1.17	0.230	0.221	0.086

Table 6 shows the relative effect of two pozzolanic admixtures, ground to pass a 200-mesh screen. The admixtures were Friant pumicite and calcined Monterey shale. Pozzolanic admixtures equal to about 20% by weight of the cement were substituted for an equivalent quantity of the neutral sand portion of the fine aggregate combination. The mortar mix—cement by weight to fine aggregate combination—was 1:2.25; and the fine aggregate was Perkins sand plus 10% of rock No. 28039. Specimens were cured in sealed containers at 70° F. The cements were from lots reported to have been used in the Friant Dam.

The answer to the erratic effect of the Friant pumicite may lie in the alkali content of this material. Chemical analyses show the following percentages of alkali in the Perkins sand, reactive rock No. 28039, Friant pumicite, and calcined Monterey shale:

Material	$Na_2O + K_2O$
Perkins sand	3.53%
Reactive rock No. 28039	0.19%
Friant pumicite	7.04%
Calcined Monterey shale	1.94%

R. W. Spencer,[8] Assoc. M. Am. Soc. C. E.—Damaging expansion of the concrete is known with certainty to occur at only a few points in major structures of the Southern California Edison system although expansion effects may exist, masked by surface deterioration from weathering. One of the examples of excessive expansion occurs in the drainage gallery in the base of a dam where a pit-run tunnel muck was used in placing a small section of concrete. The expansion is made evident by the cracking along exposed corners of the concrete. The concrete was placed in 1926 and the cracking first appeared in 1940. As of June, 1945, however, tests had not shown the presence of silica gel deposits typical of alkali-aggregate reaction. The remainder of the dam was constructed of quarried crushed granodiorite and shows no signs of excessive expansion.

A second instance of damaging expansion has appeared in the mass concrete footings of one of several flumes serving one of the Company's Kern River hydro plants. This concrete was placed in 1920 and surface cracking was noted in 1942, although it is probable that it began several years earlier. The footing concrete was made with crushed phyllite obtained from a near-by tunnel dump. The concrete flume box was made from a portion of the tunnel dump which is low in phyllite and high in granitic material, and no excessive expansion has occurred. Tests by the Bureau of Reclamation on a sample of the cracked footings showed the presence of silica gel typical of alkali-aggregate reaction.

A third case of excessive concrete expansion is known to occur in the walls of a gallery through a heavy tunnel plug at the Company's Big Creek No. 3 plant. The concrete was placed in 1923 and surface cracking appeared in 1940. The aggregate was chiefly granodiorite obtained from a near-by tunnel dump. The concrete where the expansion occurred was badly segregated and the expansion is confined to portions of the structure that are high in mortar and low in coarse aggregate.

In none of the three aforementioned cases has the quantity of concrete involved been large enough nor the effects of the expansion been important enough to require repair, but the fact that the expansion did occur made it desirable to test as many aggregates as possible in the region for reactive tendencies. The results of tests on thirty-one aggregates taken from various sources on the Edison system, including the three discussed herein, are given in Table 7.

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[8] Civ. Engr., Southern California Edison Co., Los Angeles, Calif.

TABLE 7.—TESTS FOR REACTIVE AGGREGATES, SOUTHERN CALIFORNIA EDISON SYSTEM

(Expansion[a] in Millionths of an Inch Per Inch After Approximately 880 Days)

Designation	Aggregate	High alkali[b]	Low alkali[c]	Difference
	U. S. Bureau of Reclamation:			
I	Crushed quartz	250	90	160
	Bear Creek Tunnel; Dump:			
B	Fresh crushed granite	210	110	100
	Florence Lake; Stock Pile:			
FN	Fresh crushed granite	250	150	100
FO	Old crushed granite	170	150	20
	Florence Lake; Jackass Meadow:			
FJ	Natural sand	330	180	150
	Camp 60 Tunnel; Dump:			
	Fresh crushed granite—			
WN	Normal	230	50	180
WA	Aplitic	280	160	120
WG	Gneissic	220	40	180
WD	Partly decomposed	120	0	145
	Huntington Lake Dam; Spillway Stock Pile:			
HO	Old crushed granite	170	60	110
HN	Fresh crushed granite	250	130	120
	Huntington Lake Dam; Old Quarry:			
	Fresh crushed granite—			
HQN	Normal	30	−40	70
HQA	Altered	140	20	120
	Huntington Lake Dam; Line Creek:			
HS	Natural sand	313	93	220
	Tunnel 2, Adit 4; Dump:			
2N	Fresh crushed granite; normal	90	10	80
2A	Selected granite; altered	170	30	140
2O	Old crushed sand	170	130	140
	Italian Creek:			
IL	Crushed lava; probably andesite	630	20	610
	Italian Bar:			
IS	Natural sand	287	13	274
	Tunnel 3, Adit 3; Dump:			
3O	Old crushed granite	230	10	220
3F	High ferro-magnesian material; granitic	130	−30	160
3N	Fresh crushed granite	430	170	260
	Tunnel 5, Camp 19; Dump:			
5O	Old crushed sand	200	−90	290
5N	Fresh crushed granite	160	20	140
	Shaver Lake; Reservoir:			
SS	Natural sand	180	133	47
	Shaver Lake; Quarry:			
SQ	Fresh crushed granite	200	20	180
	Last Hope Bar:			
LS	Natural sand	220	120	100
	Kerckhoff Reservoir; Project No. 4:			
4S	Natural sand	187	60	127
4B	Freshly crushed, slightly altered, granite	270	−15	285
	Sanger:			
S	Natural sand	320	87	233
	Wishon Marble Claim:			
M	Crushed marble	120	90	30
	Kern River, Adit No. 3; Tunnel Dump:			
KP	Freshly crushed phyllite	790	190	600
	Standards for Comparison:			
R	Known reactive aggregate[d]	8,450	170	8,280
RX	100% siliceous magnesian limestone	327	27	300

[a] Three 1 in. by 1 in. by 6 in. aggregate-cement mortar bars (mix, 2 : 1 by weight) for each condition tested; sealed and stored over water at 100° F, water-cement ratio, 0.57 by weight. [b] Specially manufactured, containing 0.89% sodium oxide and 0.43% potassium oxide. [c] Commercial cement containing 0.18% total alkali as Na_2O. [d] 20% siliceous magnesian limestone and 80% Bureau of Reclamation quartz.

In no case was the expansion measured in the mortar-bar test high in comparison with the expansion caused by a known reactive aggregate such as the siliceous magnesian limestone supplied by T. E. Stanton, M. Am. Soc. C. E., of the California State Highway Department. However, the Kern River phyllite (KP) used in the footings previously mentioned and the crushed andesite from Italian Creek (IL) showed expansion much greater than normal for non-reactive aggregate. The Italian Creek andesite has never been used in concrete but, because its geological age and vicinity are similar, its action in concrete probably resembles the reactive Friant andesite for which the authors show an expansion curve in Fig. 7.

The expansion of the Kern River phyllite test bars checked the field expansion observed in concrete made with this same material.

The test bars made with the crushed granodiorite (3N) used in the tunnel No. 3 plug, although showing greater than average expansion, did not show enough to put the aggregate definitely in the dangerous class if used with a high-alkali cement. Cored samples of the tunnel plug concrete when examined by the Bureau of Reclamation and the Portland Cement Association did not show deposits of silica gel characteristic of the alkali-aggregate reaction.

Test bars made from a tunnel dump aggregate similar to that used in the expanding block of the dam noted have shown an expansion greater than the average, but not enough to indicate that the aggregate would be dangerous if used with a high-alkali cement. It is possible that the test bars made with the granodiorite aggregates used for this work and that used for the tunnel plug fail to show excessive expansion because the action differs from the typical alkali-aggregate reaction. On the other hand, it may well be, as the authors suggest in the section on "Symptoms and Diagnosis," that the present accepted test procedure for alkali-aggregate reaction is not sensitive enough to predict expansions that may take place in structures twenty years or more after they are completed.

W. L. Chadwick,[9] M. Am. Soc. C. E.—Engineers concerned with the use of concrete are indebted to Messrs. Blanks and Meissner and to others in the Bureau of Reclamation for their straightforward presentation of troubles with concrete as a result of alkali-aggregate reaction. Because the activities of the Bureau cover many projects scattered over the entire western part of the United States, the experience gained is particularly valuable in determining the relatively few undesirable or dangerous cement-aggregate combinations.

The Southern California Edison Company has a number of dams and major concrete structures in the granitic region of the middle Sierra Nevada where natural sand and gravel for concrete aggregates are scarce. Consequently, it has been necessary for most of the concrete in these structures to be made from crushed granodiorite taken either from quarries or from tunnel excavation dumps. Some concrete in these Sierra structures has stood up well but some shows troublesome surface deterioration. Over a period of twenty years continuing efforts have been made to determine the reason for the differences. These efforts have included many tests and petrographic examinations of aggregates and concretes without positive results. When the alkali-aggregate reaction first became known, these structures were re-examined for evidences of such action and a few probable examples were found. The possibility that some particular mineral in the granodiorite was responsible for such trouble and also for other deterioration in the presence of high alkali was considered, as a result of which a fellowship was established at the California Institute of Technology, at Pasadena, in 1942 to test the susceptibility of these individual minerals to reaction with cement-borne alkalies.

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[9] Mgr., Eng. Dept., Southern California Edison Co., Ltd., Los Angeles, Calif.

Under the direction of John P. Buwalda and Ian Campbell of the Institute, Lloyd Pray undertook the research including the difficult task of separating enough pure minerals from a 500-lb sample of granodiorite to permit the manufacture of a number of 1 in. by 1 in. by 6 in. mortar bars with various concentrations of each mineral. The average composition of the fresh granodiorite was:

Mineral	Percentage
Plagioclase (andesine)	38
Quartz	24
Potash feldspar (chiefly microcline)	18
Biotite	10
Hornblende	8
Magnetite	1
Sphene	0.5
Zircon	Trace
Apatite	Trace

Mr. Pray was able to obtain all the minerals listed with the exception of the plagioclase feldspar, zircon, and apatite in purities of 96% or better. The plagioclase was obtained as a concentrate containing 43% plagioclase and 53% quartz with the remainder largely potash feldspar. The minerals tested were used as aggregates in different percentages ranging from 2% to 100%. The remainder of the aggregate consisted of a crushed crystalline quartz furnished by the Bureau of Reclamation. The selected percentages were based on consideration of the natural proportion of the minerals in the rock and also of the amount of mineral concentrate readily available. The test bars were made up in triplicate, using a 1 : 2 mix by weight and high-alkali and low-alkali cements of known analyses. The low-alkali cement contained 0.18% of total alkali measured as Na_2O. The high-alkali cement contained 0.87% Na_2O and 0.47% K_2O. After curing for seven days at 70° F and 100% humidity, the bars were placed in a cabinet, over water, with a controlled temperature of 100° F, and length measurements were made periodically over a period of two years.

The volume changes for the principal series of test bars at an average age of 750 days are compared in Table 8, including the results from the use of the same cements with an aggregate made from altered or weathered granodiorite and also with siliceous magnesium limestone of proved reactivity obtained from T. E. Stanton, M. Am. Soc. C. E., of the California Highway Commission.

While making the mineral separations it was found that occasional pieces of slightly altered granodiorite contained a zeolitic mineral known as stellerite. Since a zeolite reacts with the ordinary tap water used in washing and may be eliminated by the washing operations, specimens were hand picked for high stellerite content and crushed and concentrated in the dry to obtain a sand containing 33% stellerite. Two series of test bars were made, one containing

2% stellerite and the other 33% stellerite, each mixed with the required complement of crushed granodiorite or quartz (see item 10, Table 8).

To date (1945) the test results are largely negative but they are of value nevertheless. These results may be summarized as follows:

(1) None of the bars showed clearly harmful expansion and presumably, therefore, the aggregate minerals are nonreactive with high-alkali cement;

(2) All the mortars made with high-alkali cement expanded more than those made with low-alkali cement; and

(3) Specimens containing large percentages of biotite mica show rather large expansions for mortar bars, but the rate of change over the last year has been so slow that these specimens cannot be placed definitely in the dangerous class.

TABLE 8.—EFFECT OF HIGH-ALKALI CEMENT (H) AND LOW-ALKALI CEMENT (L) ON THE VARIOUS MINERALS IN FLORENCE LAKE GRANODIORITE

(Expansion[a] in Millionths of an Inch Per Inch)

No.	Mineral	EXPANSION[b] FOR THE FOLLOWING PERCENTAGES OF MINERAL CONCENTRATE IN AGGREGATE											
		2		5		10		25		50		100	
		H	L	H	L	H	L	H	L	H	L	H	L
1	Quartz plagioclase	...	...	357	171	318	171	338	168	362	189	425	236
2	Potash feldspar	388	160	362	212	331	231	369[c]	150	92	−192	95	−75
3	Biotite	370	129	294	176	325	126	265	210	480	344	506	368
4	Hornblende	355	213	370	323[d]	399	170	370	189	352	124	...	...
5	Magnetite	331	74	248	163	457	226	349	247	...	...	...	...
6	Sphene	415	291	488	312	425	268	...	...	...	...	...	...
7	Fresh "granodiorite"	...	...	...	...	...	...	...	...	...	...	331	10
8	Altered "granodiorite"	...	...	...	...	...	...	...	...	286	87	320	165
9	Siliceous magnesian limestone	...	...	...	...	2,430[e]	152[e]	...	...	...	...	...	...
10	Stellerite[f]	356	185	...	...	...	...	305[g]	237[g]	...	...	...	...

[a] Volume-change observations were made, after two years, on three cement-aggregate test bars for each condition. The mortar bars (1 in. by 1 in. by 6 in.; mix, 1 : 2 by weight; and water-cement ratio, 0.57 by weight) were sealed and stored over water at 100° F. [b] The abbreviations H and L denote, respectively, high-alkali cement, specially manufactured, containing 0.89% sodium oxide and 0.43% potassium oxide; and a commercial, low-alkali cement containing 0.15% total alkali as Na_2O. [c] Average of six bars. [d] Average of two bars only. [e] 20% of siliceous magnesian limestone in each case. [f] Age 270 days. [g] 33% stellerite.

The authors have assisted the writer materially in the work done under the fellowship and in searching for the causes of other concrete troubles, and this opportunity is taken to express appreciation for material assistance, advice, and free exchange of progress information.

J. C. WITT,[10] M. AM. SOC. C. E.—For convenience in discussion, the writer has divided the title of this paper into three sections, each of which refers to a section of the paper:

$$\overbrace{\text{"Deterioration of Concrete Dams}}^{1}\quad\overbrace{\text{Due to}}^{2}\quad\overbrace{\text{Alkali-Aggregate Reaction."}}^{3}$$

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[10] Tech. Director, Marquette Cement Mfg. Co., Chicago, Ill.

The first section of the paper consists largely of a rather general account of deteriorated concrete, together with descriptions and illustrations of some examples. Many papers have appeared in which the deterioration of various types of building materials has been discussed, and examples almost without limit could be listed. In the third section, chemical reactions between compounds of alkalies and compounds of silicon are mentioned. These reactions have been known for a long time.

The tying together of the two groups of phenomena by the second section of the title suggests that the relation of the two is to be demonstrated in the paper. This relation is not demonstrated. In fact, if, by the reading of previous papers, one had been convinced that an increase of a few tenths of one per cent of alkali compounds in a cement would cause concrete to deteriorate, some statements in this paper might weaken his belief materially. Some of the statements that suggest this conclusion are:

(*a*) "The problem is extremely complicated, however, and many questions, practical and technical, remain to be answered satisfactorily" (see heading, "Synopsis").

(*b*) "It is suspected that, under some conditions, restraint is sufficient to prevent most of the expansion that would normally occur if the concrete was not restrained" (see heading, "Symptoms and Diagnosis of Alkali-Aggregate Deterioration in Concrete Dams"). Statement (*b*) implies that even in concrete free from cracks reactions tending to cause deterioration may be in progress.

(*c*) "Chemical analyses of the gels show them to be composed essentially of sodium and potassium silicates" (see paragraph that introduces Table 1). The analyses in Table 1 indicate that the gels contain sodium, potassium, and silicon; but they do not indicate how these elements are combined.

(*d*) "Hand specimen and microscopic study of core samples from affected concrete also invariably reveal what appear to be zones of chemical reactivity, alteration, or infiltration around certain aggregate particles, as shown in Fig. 5."

The authors have cited a number of references in support of their statements, and they could have cited additional ones. Many statements in opposition could also be made. For example, in discussing the expansion of mortar bars made with 106 cements varying in alkali content, W. C. Hanna (15) states:

> "At the age of 2 months the expansions of these bars, with few exceptions, were observed to be definitely related to the alkali content of the cements. At 6 months many exceptions were observed in this relation, and at the age of 18 months nearly all of the bars had expanded excessively. In many instances bars of low alkali content expanded to a greater extent than those of higher alkali content. * * * Some bars to which a deliberate addition of sodium hydroxide was made showed lower expansions than companion bars made without addition."

No one should accept the relation between the deterioration of concrete and the alkali content of cement expressed, and implied, in the paper without familiarizing himself with the great mass of information on this subject that is available in literature.

Under the heading, "Corrective and Preventive Measures," the following statements are made:

(*e*) "Experience to date indicates that reactive aggregates can be used without danger of destructive reaction, provided that low-alkali cement (less than 0.60% $Na_2O + K_2O$) is employed (see Fig. 7)."

(*f*) "Specification of low-alkali cement for all important hydraulic structures is strongly recommended regardless of what may or may not be known about the reactive tendencies of the aggregates to be used." If the deterioration of concrete may be caused by internal alkali compounds, and if there is a preventive measure, surely this preventive measure (statements (*e*) and (*f*)) is not so simple as to be merely a matter of reducing the percentage of alkali compounds in the cement. Many factors are involved, some of which are the total sodium and potassium in the concrete (not in the cement only), and the nature of their compounds.

(*g*) "Such a precautionary measure [reduction in alkali content] not only provides insurance against possible eventual deterioration due to alkali-aggregate reactions, but limits undesirable constituents of Portland cements and usually improves the quality in several other ways." Some examples are mentioned in which, "the reduction in alkali content has been accompanied by marked reductions in C_4AF (tetracalcium alumino-ferrite), which is a fluxing material having little cementing value in itself, and lower C_3A (tricalcium aluminate), which is generally considered to be an undesirable compound. Coincidentally the principal cementing compounds, C_3S (tricalcium silicate) and C_2S (dicalcium silicate), have been increased as much as 9%." There is no fixed relation between the alkali content of a cement and the percentages of the principal clinker minerals. This applies to the succeeding paragraph also.

(*h*) "It seems evident that the additional cost of low-alkali cement, which may range from nothing to about 20¢ per bbl, is more than repaid by the improvement in quality alone." If the authors mean the manufacturing cost, such a general statement is not warranted.

The lag between research accomplishments and their application is one of the most serious retarders of industry. Also serious, although far less common, is the lag between application and research accomplishments.

DUFF A. ABRAMS,[11] M. AM. SOC. C. E.—A notable feature of the paper by Messrs. Blanks and Meissner is its many contradictory statements. A few of the more obvious contradictions are listed here as an introduction to the arguments to follow:

Cracks in Boulder Dam (see heading, "Progressive Development of Alkali-Aggregate Deterioration in Concrete Dams") "must be ascribed to extreme drying exposure in the hot, arid location," but in Parker Dam, made of the same cements, a similar concrete mix, built by the same contractor, using the same equipment, in a hotter and more arid location, the cracks were ascribed (see heading, "Significant Effects of Alkali-Aggregate Deterioration on the Service Life of Concrete Dams and Appurtenant Works") to alkali-aggregate reaction. Surface cracks in concrete dams are attributed to internal expansion due to alkali-aggregate reaction, but the photographs that illustrate this phenomenon clearly show contraction cracks.

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[11] Cons. Engr., New York, N. Y.

The mortar-bar expansion test on sand from Owyhee Dam shows the highest expansion of any sand reported. Photographs of "affected areas" (Fig. 8) do not show Owyhee Dam, but only part of a small viaduct.

Nearly every positive assertion by the authors is modified or nullified by contrary statements elsewhere in the paper.

Failure of Parker Dam.—A number of concrete dams were mentioned; but, to the writer, the paper seems primarily an attempt to explain the failure of Parker Dam on the Colorado River 155 miles south of Boulder Dam. This is an arch dam, 322 ft high, 100 ft thick at the base, and 39 ft thick at the roadway level, containing 276,000 cu yd of concrete. It was completed in 1938 by the U.S. Bureau of Reclamation (U.S.B.R.) for the Metropolitan Water District of Southern California, at a cost of $9,000,000. Two years later it was found to be in an advanced stage of disintegration.

It may be thought that "failure" is not the correct term, since, at last report, the dam was still in service. The following statements were made by the chief engineer, U.S.B.R., early in 1941 (16):

> "In the construction of Parker Dam full advantage was taken of every known advancement and modern improvement in the selection and utilization of materials, production control, construction methods and equipment; the concrete was cooled and no precaution was overlooked to insure the very highest quality of construction in that dam. Nevertheless, we have a dam there whose permanence must be very seriously questioned.***
>
> "Of course, Parker Dam is not going to fall down next year or the year after, but it is going to require a lot of maintenance work, and there is the possibility that at some time in the distant future it may have to be largely replaced."

When the chief engineer, two and one-quarter years after completion, sees a large concrete dam "whose premanence must be seriously questioned," and a dam that "may have to be largely replaced," the only accurate term is "failure." After five years of "intensive investigation," the authors attributed the failure to reactions between the alkalies in the cement and the aggregates.

This discussion examines the authors' evidences of alkali-aggregate reaction in concrete. It will be shown that the failure of Parker Dam was due to an entirely different cause, and that it was exactly what should have been expected from properties and behavior of concrete that had long been well known to the profession.

High-Alkali Cements.—It has been known for nearly a century that Portland cements contain alkalies. The first complete analysis of Portland cement was reported, in 1849, by A. Hopfgartner (17), an Austrian chemist, from tests made on English cement that gave 2.7% alkalies. Every subsequent complete analysis of Portland cement has shown alkalies. It is doubtful whether a commercial cement was ever made that did not contain alkalies.

Prior to the construction of Boulder Dam, the U.S.B.R. spent a large sum of money in studies of cement and concrete (18)(19). As a result of these studies they prepared a specification for what is now known as a low-heat cement. The principal characteristic of this cement is the high ratio of dicalcium silicate. The most direct method of producing such a cement is to reduce kiln temperature. It has long been known that low kiln temperatures increase the alkalies in the cement. This was brought to the attention of American engineers by W. F. Hillebrand (20) in 1903, and by A. V. Bleininger (21) in 1904.

The low-heat cements used in Parker Dam were manufactured by the same four mills that furnished cement to Boulder Dam.

"Gel Deposits."—The incomplete chemical analyses in Table 1 are the only data that have been released on the composition of the "gel deposits" from cores taken from Parker Dam. These analyses were first made public in February, 1941 (6). The entire case of "gel deposits" collapses in the light of evidence that the cores were packed in damp sawdust for shipment to Denver, Colo. (22). Many woods contain organic acids that readily attack compounds in cements and aggregates. Evidences of acid action would be destroyed by exposing the samples to a high temperature. The nature of the material that was driven off in the "loss-at-ignition" test (Table 1) was not disclosed; nor was the temperature given. These losses seem to represent free water, although it may have been carbon dioxide, water of crystallization, alkalies, or many other compounds.

The material that the authors identified as water glass was probably formed in the crucible during the "loss-at-ignition" test. Water glass is formed only under the most favorable conditions and at a temperature high enough to fuse alkalies and silica; it certainly was not formed in this concrete.

Mortar-Bar Tests.—The writer made a serious effort to ascertain what the facts were with reference to the mortar-bar expansion tests reported by the authors, but was soon lost in conflicting tests and in "yes-no-maybe" language. Typical statements were:

Statement A (See Heading, "Summary").—

> "Aggregate which will give this trouble in concrete may be recognized if it produces expansion within a short time in test specimens made with high-alkali cement."

Statement B (Last Paragraph Under "Symptoms and Diagnosis of Alkali-Aggregate Deterioration in Concrete Dams").—

> "The test is useful only as an indication of possible troublesome combinations of reactive aggregates and high-alkali cements."

Statement A is positive and unequivocal, but it is completely nullified by Statement B. The key words in Statement B are "indication" and "possible." Used in science or engineering, "indication" is a "strong" word; Webster's

Dictionary gives the synonyms "manifestation, mark, evidence, signal, and proof." The word "possible" is "weak" and takes a position at the opposite end of the scale. Something that is "useful only as an indication" of a possibility is no indication at all. The net result of the two statements is a sort of double vacuum.

Another example is afforded by the sentences preceding Statement B:

Statement C.—

> "Although excellent correlations have been obtained between laboratory results and service performance, the mortar-bar expansion test has not been developed so that it can be relied upon as an infallible indication of troublesome combinations of cements and aggregates. Neither can it be considered as a satisfactory acceptance test for either cements or aggregates."

What were these "excellent correlations"? In no instance was the cement used in the tests, the same as that used in the dams which showed "deterioration due to alkali-aggregate reaction." In the case of "Crushed Phyllite, Buck Dam in Virginia" (Fig. 7), the test showed a very rapid expansion; in three months it was 0.4%, but, for the dam, expansion was not observed until the concrete was ten years old, and it required eighteen years to reach 0.5% (2). "Excellent correlations" would of themselves establish the test as an "infallible indication of troublesome combinations" of cement and aggregates. The positive assertion ahead of the comma in Statement C was promptly canceled by the contrary statement after the comma.

Statements A, B, and C are fair examples of what the authors have to say on the mortar-bar expansion test after using it for five years; but the tests in Fig. 7 cannot be accepted as evidence of alkali-aggregate reaction. A few of the reasons are: (*a*) The reader is not informed as to the composition or any other properties of the sands; (*b*) he is not told the alkali content or any other properties of the cements; and (*c*) he is not informed whether all high-alkali tests were made with the same cement. Perhaps the excessive expansion found with certain sands and high-alkali cement was due to high free lime that generally accompanies low burning temperatures. Furthermore: (*d*) With high-alkali cement, crushed andesite from Friant Dam showed, at three months, seven times the expansion of Parker sand, but in an earlier report Mr. Blanks (4*d*) showed that Parker andesite gave no appreciable expansion at three months; (e) at six months, with high-alkali cement, Owyhee sand showed eight times as much expansion as Parker sand, but there was no evidence to show that Owyhee Dam expanded several times as much as Parker Dam; and (*f*) no values were given for Owyhee and American Falls sands with low-alkali cement. Perhaps these tests showed more expansion than the high-alkali cement.

The writer finds no foundation for the claim of "excellent correlation" between the mortar-bar test and service performance. The mortar-bar tests are inconclusive and misleading, and bear no relation to what occurs in a dam. The conflicting treatment of the paper supports the writer's deductions.

Reactive Aggregates.—The entire burden of the paper is that alkali-aggregate reaction was the cause of the deterioration of concrete dams; but, when the reader inquires into the scientific basis for this hypothesis, and attempts to identify the "reactive aggregates," he encounters insurmountable dificulties.

Opaline Silica.—This material heads the list of rocks that "have been found to react with high-alkali cement." What is "opaline silica"? It is not defined by any dictionary in the New York Public Library; it does not appear in the index of the Encyclopædia Britannica; and it is not found in the index of Chemical Abstracts, started in 1907. A commercial house that has supplied mineralogical specimens from all parts of the world for eighty-two years could not fill an order for a sample of "opaline silica." Dana's "Manual of Minerology" has been a standard reference work for nearly one hundred years; "opaline silica" did not appear in any of the fifteen editions of this book published between 1848 and 1941. Opal is a form of hydrous silica; hence "opaline silica" is a redundancy which being translated becomes: "(hydrous-silica)—like silica."

Rhyolite.—"Some rhyolites" were found by the authors among the "more conspicuously reactive" aggregates; but a rhyolite (pumicite) was used to replace 20% of the cement in Friant Dam, and the authors "felt that pumicite has furnished some protection against expansion where the low-alkali cement was not provided" (see heading, "Corrective and Preventive Measures"). In other words, "some rhyolites" are reactive with high-alkali cement; but, when the rhyolite is very finely divided, contains 9% alkalies, and is called "pumicite," it is not harmful, but has distinct merits when used with a high-alkali cement.

Andesite.—The writer was struck by the use of the word "some," which occurred seven times as a modifier of various rock types that the authors claim to be reactive with high-alkali cement. Fortunately, there is evidence by which to determine what was included in "some andesites." Early papers by the authors (6)(22)(4d) claimed that andesite was the principal reactive aggregate in Parker sand. In 1941, however, Mr Blanks (4d) published the results of tests on mortar bars with a high-alkali cement and 10% of Parker andesite crushed to four different finenesses. All tests showed negligible expansion at one, two, and three months. The authors now give, in Fig. 7, a curve for "crushed andesite, Friant Dam," which shows a very high expansion. The native rock at Boulder Dam is described as "andesite breccia," but, according to the authors (see heading, "Progressive Development of Alkali-Aggregate Deterioration in Concrete Dams"):

> Statement D.—
>
> "The only place where alkali-aggregate reaction had been definitely identified at Boulder Dam, after eight years of service, was in the lining of a drainage tunnel through some wet rock."

Obviously, this rules out Boulder Dam andesite, although the same high-alkali cements were used as at Parker Dam. In two out of three instances andesite has been found nonreactive. This reduces "some" to less than one third. It is more logical to conclude that andesite is nonreactive, and that repeat tests on the same material gave conflicting results.

Other Rock Types.—In view of what has been written herein under "andesite," little weight can be given to the authors' claims (see heading, "Symptoms and Diagnosis of Alkali-Aggregate Deterioration in Concrete Dams") that:

Statement E.—

> "Some of the more conspicuously reactive aggregate materials include: Opaline and other siliceous rocks such as some cherts, some siliceous limestones, some chalcedonies and intimate opal-chalcedony admixtures; certain acid to intermediate volcanic rocks such as some andesites and rhyolites, including tuffs and glasses of similar composition; and miscellaneous rock types such as phyllite."

The authors' term "other siliceous rocks" (a meaningless classification in regions where all rocks are siliceous) just about exhausts the rock types that are claimed to be reactive. The only conclusion that the writer can draw from the data and the statements of the paper is that the authors failed to name a single recognizable rock type that is reactive with high-alkali cement.

Alkali-Silica Reaction.—The paper encourages readers to believe that an alkali-silica reaction had been identified as the cause of the "alkali-aggregate disease in concrete dams," but the expression generally used was "alkali-aggregate reaction." The following is typical of the misleading inferences of the paper (see heading, "Symptoms and Diagnosis of Alkali-Aggregate Deterioration in Concrete Dams"):

Statement F.—

> "Far too little is known about the specific mineral constituents of various rock types which contain silica in forms susceptible to reactivity with alkalies in cement to permit positive predetermination of troublesome combinations by petrographic analyses."

To make such a statement regarding "silica in forms susceptible to reactivity with alkalies" begs the question; it merely asserts something that has not been proved. Dozens of scientists have devoted their lives to the study of silica. The properties of the various forms of silica that occur in nature are fairly well understood. There is no evidence to show that the authors made any attempt to determine whether or not these forms are reactive with alkalies. Alkalies are among the most widely distributed compounds in nature. The fact that silica in sand and gravel has not been destroyed by alkalies is of itself strong evidence that it is not reactive.

The only parts of the paper that bear on this question are the high silica content of one of the "gel deposits" in Table 1, and the statement that "The gels may be transparent, closely resembling ordinary water glass***." The writer has shown that: (*a*) No credence can be given to the inexpert and incomplete chemical analyses of the "gels"; (*b*) water glass cannot possibly be formed in the concrete; and (*c*) not a single recognizable rock type was identified as reactive with high-alkali cements. In other works, the paper contains no evidence of alkali-silica reaction.

Before the hypothesis of alkali-silica reaction becomes acceptable, the authors must explain:

(1) If certain constituents of Parker sand were attacked by cement alkalies in the dam, why were they not long ago destroyed by the same alkalies in Williams River water (a tributary to the Colorado River)?

(2) If 5 lb of cement alkalies is a menace to a cubic yard of concrete, why was the aggregate not long ago destroyed by 225 lb of the same alkalies that they contain?

(3) Parker Dam cements contained 16 to 47 times as much silica as alkalies. Why did not this silica combine with the alkalies during hydration of the cement?

(4) Parker Dam cement had 1.2% alkalies; the mortar contained 0.3% of cement alkalies. The "gel deposits" had 17% alkalies, or 55 times as much as the mortar. How can the cement alkalies move about so freely within the hardened concrete? Why did this high concentration of alkalies occur in the hardened concrete and not in the cement paste alone, or in the aggregate alone?

The mere statement of the foregoing considerations discredits the hypothesis of alkali-silica reaction in concrete. In the case of Parker Dam, the authors gave this hypothesis a slight semblance of credibility by: (*a*) Incomplete, inconsequential tests, (*b*) foggy language, and (*c*) playing down or failing to present important information on concrete materials and test methods.

Solution of an Enigma.—In the foregoing paragraphs the writer has shown that: (*a*) The mere assertion that there are "rock types which contain silica in forms susceptible to reactivity with alkalies in cement," does not constitute a demonstration; (*b*) incomplete chemical analyses cannot be accepted as a basis of a hypothesis of alkali-silica reaction; (*c*) contradictory statements on the results of mortar-bar tests make those tests valueless; and (*d*) such expansion as was found was probably due to causes other than alkali-aggregate reaction. The remainder of this discussion is constructive in that it solves the "enigma" of the failure of Parker Dam.

Parker Dam Climate.—The school geography used by the writer had printed across the map of the southwestern part of the United States: "The Great American Desert." At Parker Dam the sun shines nearly 100% of the possible hours, shade temperatures reach 125° F, and concrete surface temperatures 145° F. The air is almost devoid of moisture; relative humidities of 6%, 5%, and 4% are frequently recorded. In parts of this desert 10-yr periods have passed without any measureable rainfall. It is doubtful whether a large concrete structure has been built any place in the world where more severe combinations of high temperature and low humidity were encountered. The U.S.B.R. has used the terms "blistering desert," and "almost unbearable heat," to describe the conditions at Boulder Dam, which is 155 miles north of Parker Dam.

Concreting Parker Dam.—In many important features of the design and construction of Parker Dam, there was failure to take account of the severe climate, and well-known properties and behavior of concrete were ignored. Three examples are given:

(1) Grinding of Aggregates in Mixer.—Concrete was mixed in 4-yd batches. The specifications required concrete to be mixed $2\frac{1}{2}$ min. Since 1870, leading

authorities have repeatedly warned against overmixing to such an extent as to produce excessive breaking or grinding of aggregates. Three important series ot tests were conducted before concreting was begun on Parker Dam in 1937. The late W. A. Slater, M. Am. Soc. C. E. (23), made such tests in 1930; S. C. Hollister, M. Am. Soc. C. E. (24) in 1931; and W. A. Blanchette (25) in 1934. All studies showed that long mixing, using gravel up to 1¼ in., resulted in grinding large quantities of aggregates to the fineness of cement. Little argument will be required to show that even with modest times of mixing, excessive grinding occurs with gravel up to the size of a football. An ideal grinding machine is created by 17,000 lb of water, cement, sand, and gravel as large as 6 in. The writer has shown (26) that at Grand Coulee Dam (on the Columbia River, Washington), 4-yd mixers produced rock dust equivalent to 26% of the weight of the cement in 2½ min, and 95% in 15 min. There are several reasons for the belief that there was more mixer grinding at Parker Dam than at Grand Coulee Dam.

(2) Rate of Concreting.—The rate of placing concrete should be restricted to permit the dissipation of part of the heat of hydration of the cement. Parker Dam Specifications provided (27):

> "The rate of placing concrete in any panel or block of the dam shall be such that not more than 5 ft. shall be placed in 72 hours, and not more than 30 ft. in depth shall be placed in 30 days unless specifically authorized by the contracting officer."

Construction records show that, in the hottest months of the year, neither of the foregoing provisions was enforced. In many instances 10 ft or 15 ft of concrete was placed in 72 hours. Instead of limiting the depth to 30 ft in 30 days, 45 ft to 70 ft was placed in a month. In three instances, 60 ft was placed in a month; in seven instances, 65 ft; and in six instances, 70 ft.

(3) Curing Concrete.—The importance of adequate curing of concrete has been well understood since 1920. How was the challenge of this "blistering desert" met at Parker Dam? The Specifications required (27):

> "For concrete made with low-heat or moderate heat cement the period of moist curing shall be extended to not less than three weeks to correspond to the minimum curing period of two weeks for concrete made with standard portland cement. The method of keeping the concrete moist shall be by continuous sprinkling, spraying, or other methods approved by the contracting officer."

A concession of one week (which was not enforced) was made on account of the low-heat cement, but no concession whatever was made on account of sun temperatures up to 145° F or the near-zero humidity. The upper, thin, reinforced concrete sections of the dam were not cured by "continuous sprinkling, or spraying," but by three coats of a patented curing compound that was poorly adapted to the conditions.

Cracking of Parker Dam.—On this job every effort should have been made to restrict mixing water to a minimum. By a better grading of aggregates, and a method of mixing that minimized mixer grinding, the mixing water in a cubic yard of concrete could have been reduced from 250 lb to 175 lb. All concrete should have been effectively water cured for the entire summer after the main portion of the dam was finished early in 1938. Instead, the concrete contained a great excess of mixing water, and the curing was not up to the standard practice in regions with a favorable climate. Surface cracking at Parker Dam was due to too much mixing water and to inadequate curing. The effects of these factors were well understood long before this dam was begun. It is not necessary to search for a mysterious alkali-aggregate reaction to explain these cracks.

The first twelve pages of the paper give the impression that serious deterioration was shown by the internal expansion that caused surface cracks in concrete dams; in the remainder of the paper the authors' attempt to demonstrate that such cracks "would appear to be of minor importance." The work "crack" (cracks, cracking, crazing) was used about forty times, but the authors concentrated the reader's attention on surface "random-pattern cracking." Not once did they refer to structural cracks in either the dam or the superstructure. The structural cracks contradict the alkali-aggregate hypotheses of the paper.

Expansion Measurements at Parker Dam.—To the writer, the curves in Fig. 10, presented to "indicate progressive expansion in the concrete of Parker Dam," have a meaning entirely different from that suggested by the paper. There is a well-known cause of concrete expansion that has nothing to do with alkali-aggregate reaction, which certainly was present in this dam—namely, temperature rise due to the hydration of the cement. This effect of the heat of hydration has long been recognized, and has generally been partly counteracted by an embedded-pipe system of cooling the concrete. Cooling was not applied to the abutments, however, and it is probable that the cooling of the other portions of the dam was ineffective, since the cooling medium was river water and cooling was delayed until serious damage had occurred.

The trend of the curves in Fig. 10 shows a net expansion, in the 5-ft to 10-ft range, of about 0.00013 per yr. Assuming a coefficient of thermal expansion of 0.0000055, this expansion would be accomplished by a temperature rise of 24° F per yr. If this continued for five years, from the time the concrete was placed until the end of 1942, it means a total rise of 120° F.

Parker Dam has five 50-ft by 50-ft gates. The two abutments, outside of the gate system, are about 375 ft long. The total horizontal movement into the gate openings in five years would then be: $0.00013 \times 5 \times 375 \times 12 = 2.9$ in. In the same way there would be a "hump" of about $2\frac{1}{2}$ in. in the middle of the dam due to vertical expansion. This placed the gates, the gate-operating structure, and the roadway bridges in a gigantic vise where they were crushed like eggshells. It is not surprising that there was considerable "binding of gates" which the authors mention three times.

The expansion, in which the authors (see heading, "Symptoms and Diagnosis of Alkali-Aggregate Deterioration in Concrete Dams") mistakenly saw "signs of abnormal expansive movements in concrete which accompany alkali-aggregate reactions," was due to hydration of cement. The measured expansion (or computed temperature rise) enables one to estimate the excess stress developed in the restrained concrete in the dam proper. Assuming a modulus of elasticity of 4,500,000 lb per sq in., the stress would be: 0.00013 × 5 × 4,500,000 = 3,250 lb per sq in. Part of this excess stress near the top of the dam was relieved, no doubt, by pushing the crown of the arch upstream. In an earlier discussion (28) the writer gave some evidences of similar high stresses in Boulder Dam, due to unconsidered temperature rise, as a result of continuing hydration of cement.

That the writer's analysis is not mere theory is shown by two other facts: (*a*) During construction, a wedge of concrete about 100 ft long broke off where the downstream corner of one of the abutments joined the rock; (*b*) the reinforced concrete girders that carry the highway over the 50-ft gate openings showed, within a few months after completion, that they had been squeezed and crushed like so many eggshells. The foregoing phenomena were clearly not due to alkali-aggregate reaction; they can be accounted for readily by temperature rise of the concrete; they were not mentioned by the authors.

Missing Data.—A large number of extensometers and thermometers were embedded in Parker Dam. Instead of giving these data which have been available from the beginning, the authors base their conclusions on expansion and temperatures as measured in four shallow holes that were drilled in an abutment several months after the dam was known to be in a precarious condition. The deepest of these holes was 3% of the maximum depth of the dam.

It seems obvious that the "concrete temperature" curve at the top of Fig. 10 does not represent the interior of the dam; the dam temperature could not possibly show seasonal variations of 40° F. A study of the curves suggests that the temperatures for the 30-in. hole were plotted; the points do not, in general, coincide with the dates of expansion readings in the 12-in., 60-in., or 120-in. holes.

The authors are requested to give for Parker Dam: (*a*) Concrete temperatures for, say, April, 1938, 1939, 1940, 1942, and 1945—in an abutment at mid-depth of the gate opening and in the middle of the dam 50 ft below tailwater level; (*b*) the horizontal movement of each abutment into the gate opening; (*c*) the rise of the top of the dam; (*d*) the upstream movement of the crown of the arch; and (*e*) all temperature readings taken in the four holes drilled in an abutment.

Conclusions.—This discussion is a condensation of a more voluminous study of the paper. The space available makes it necessary to omit many significant features, and merely to state some of the conclusions without supporting data or arguments.

General.—

(1) The paper was notable for its many contradictory statements; practically every positive assertion of the authors was modified or nullified by contradictory statements elsewhere in the paper.

(2) To the writer it appears that the authors did not avail themselves of evidence that contradicts the hypothesis of the alkali-aggregate reaction.

(3) The photographs that were given to illustrate expansive cracking of dams clearly show cracks to be due to contraction of concrete.

(4) The only general conclusion that can be drawn logically from the paper is that dams should not be built of concrete. The writer does not subscribe to that conclusion.

Alkali-Aggregate Reaction.—

(5) The "gel deposits" from Parker Dam concrete probably resulted from attack by organic acids; this accounts also for the "chalky, lifeless appearance" of the concrete. These phenomena were not due to alkali-aggregate reaction.

(6) The resemblance of the "gel deposits" to water glass probably resulted from a manufacture of a glass in the crucible at the high temperature of the ignition test. It is certain that water glass was not formed in the concrete. There was no evidence to show that the "gel deposits" were harmful to concrete.

(7) Five years of "intensive investigation" failed to identify a single rock type that reacted with high-alkali Portland cement. The hypothesis of alkali-silica reaction has no basis except foggy language, unsupported assertions, and trivial circumstances.

(8) The "yes-no-maybe" language of the paper makes it impossible to determine what the authors' conclusions were with reference to the mortar-bar expansion tests. The tests that have been reported are contradictory and misleading.

(9) The laboratory tests were in general meaningless, due to limited scope, lack of scientific controls, and faulty technique. In most instances the material tested did not remotely resemble the concrete used in dams.

(10) The authors' blanket indictment of all types of concrete aggregate "with the possible exception of pure limestone" (that does not occur in Nature) is not supported by data in the paper; it is contradicted by one hundred years of experience by the concrete industry.

Failure of Parker Dam.—

(11) Since January, 1940, it has been fashionable to attribute concrete failures to alkali-aggregate reactions. The evidence shows that Parker Dam was in a precarious condition early in 1940, less than two years after completion. Failure was at once attributed by U.S.B.R. to an excess of 0.6% of alkalies in the cement that they had designed especially for this work.

(12) Concrete specifications were inadequate for this "blistering desert" climate; the few ineffective requirements were ignored.

(13) Abnormal surface cracking was due to excessive volume shrinkage of concrete as a result of rapid evaporation of mixing water. The excessive quantity of mixing water resulted from poorly graded and contaminated aggregates and from excessive mixer grinding. Surface cracking could have been reduced to negligible proportions by exercise of well-recognized precautions with reference to materials, proportions, and curing.

(14) The damage to Parker Dam is much more serious, and from a cause entirely different from that suggested by the paper. Unconsidered temperature rise due to hydration of the cement resulted in a destructive expansion that the authors mistakenly attributed to alkali-aggregate reaction. This placed

five large gates, the gate-operating structure, and the roadway bridges, in a gigantic vise, where they were crushed like eggshells.

(15) Too rapid placing of concrete, although prohibited by the Specifications, contributed to this destructive expansion.

(16) There is no evidence to show that high-alkali cement contributed in any way to the cracking, expansion, or failure of the dam. There is no reason to think the results would have been different if a low-alkali cement had been used.

(17) The measured expansion, given in the paper, corresponds to a computed compressive stress in the concrete of 3,250 lb per sq in. in excess of that for which the dam was designed.

(18) The trouble at Parker Dam was inherent in the proportions of concrete mixes and the concreting methods. If information that had long been available to the profession had been used, there is every reason to believe that this dam would be in perfect condition today—a constant reminder that (as stated by the U.S.B.R. in 1942) concrete "is in reality a simple and practical art."

R. F. Blanks,[12] and H. S. Meissner,[13] Members, Am. Soc. C. E.—Since the paper was written, another dam, afflicted with expansion and accompanying cracking, has come under observation. Stewart Mountain Dam was built in 1929 and 1930 on the Salt River in Arizona, with cement from two mills that later supplied low-heat cement of high-alkali content (1.25% and 1.13% $Na_2O + K_2O$) for Parker Dam. The same two mills also furnished cement for the upper part of Coolidge Dam, built in 1927–1928 on the Gila River in Arizona, the upper part of which structure was described in the paper as suffering from expansive cracking. Fig. 12, illustrating the deterioration of concrete in Stewart Mountain Dam, is typical of all concrete in which reaction between aggregate and high-alkali cement has been found. Mortar-bar expansion tests have been made with sand, taken from the river near the original aggregate deposit, using the high-alkali and low-alkali cements of Fig. 7. After 18 months the specimens containing high-alkali cement show 0.33% expansion, whereas those containing low-alkali cement show only 0.02% expansion. Further evidence of reaction is to be noted in Fig. 13, which shows gel oozing from a concrete core recently drilled from the structure, after the core had been stored in a fog room for 22 days.

Stewart Mountain Dam is mentioned because Mr. Proctor implies that more of this type of deterioration has appeared in structures erected since 1930 or 1935 than in older concrete work, and he considers that much of the trouble has occurred where "new types" of cement have been used. Of the several affected hydraulic structures discussed in the paper—American Falls Dam, Buck Dam, Coolidge Dam, and Owyhee Dam, together with Stewart Mountain Dam—were all built prior to the period mentioned and did not contain "new-type" cement.

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Mr. Proctor advances the theory that cements passing the Merriman tests (which are incorporated in cement specifications of the New York City (N. Y.) Board of Water Supply—B.W.S.), particularly the sugar solubility test, would not have caused expansion in concrete. Results of the Merriman tests are available for the cements used in Parker Dam and, for convenience, these have been listed in Table 9, together with the B.W.S. specification limits for the tests and for the cement composition. None of these cements meet the B.W.S. specifications completely, although all meet the sugar solubility test, which is considered the criterion of complete burning. However, the allowable composition range permitted by B.W.S. is extremely narrow—much more so than any range that has ever been used by the Bureau of Reclamation (U.S.B.R.). Mr. Proctor believes that the expansion trouble has appeared with the introduction of the new-type cements, but it should be apparent that few of the older cements would have passed these rigid specifications entirely. Therefore, it cannot be presumed that older cements have good service records because they are similar to present-day B.W.S. cements.

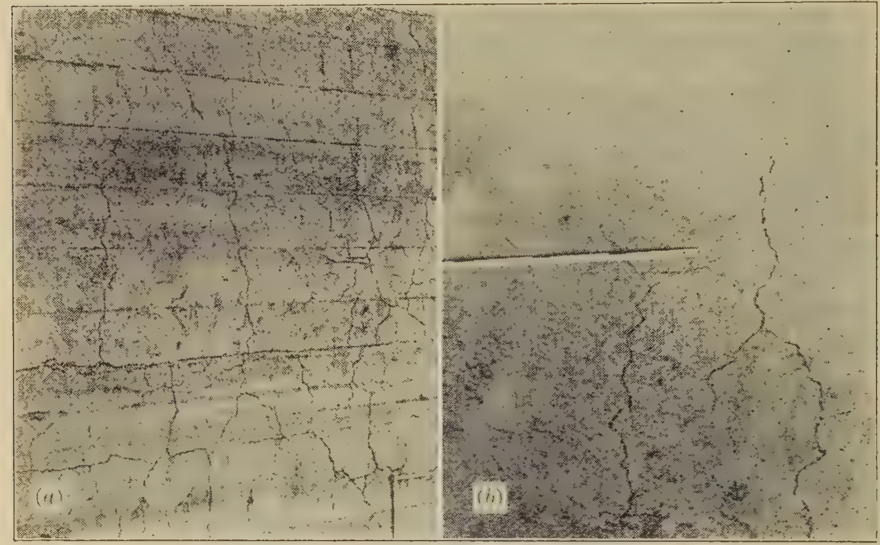

Fig. 12.—Deterioration of Concrete in Stewart Mountain Dam

In response to Mr. Proctor's inquiry, the values of the sugar solubility index for the cements used in the expansion tests shown in Fig. 7 are as follows:

Cement	Cloud point	Clear point
Low alkali	18.6	25.5
High alkali	2.9	2.9

They are not in the order that he evidently suspects since the low-alkali cement, with which the reactive aggregates did not expand, fails to meet the B.W.S. limits. The high-alkali cement which produces expansion with the reactive aggregates, on the other hand, is an acceptable product by the standard of the sugar test. Fig. 7 and Table 9 supply good evidence that a low sugar solubility does not guarantee low expansion, when the aggregates are reactive.

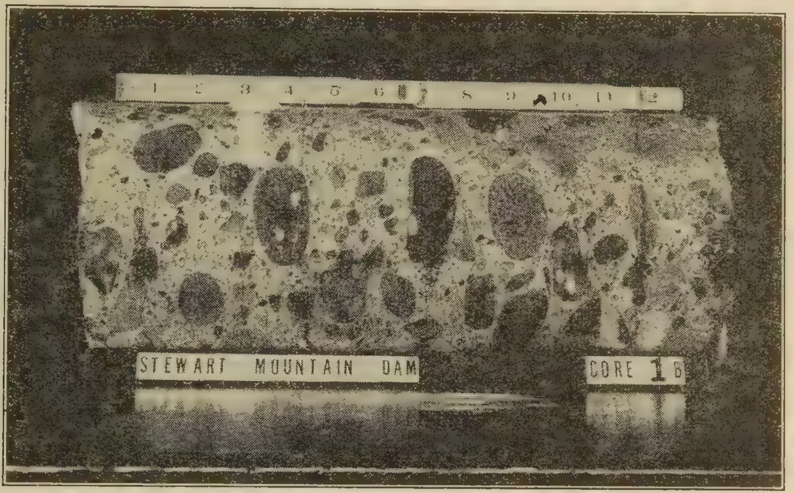

FIG. 13.—GEL CAN BE SEEN OOZING FROM THIS CONCRETE CORE, AT THE 2-IN. AND 3-IN. MARKS OF THE RULER

TABLE 9.—COMPARISON OF CEMENTS USED IN PARKER DAM

Description	B.W.S. limits[a]	Cement PV	Cement PC	Cement PM	Cement PR
Percentages:					
Silica (SiO_2)	21.50[b]	26.60	22.71	22.72	22.11
Alumina (Al_2O_3)	5.60	3.14	5.17	5.71	5.27
Magnesia (MgO)	4.00	2.12	3.06	1.16	4.27
Sulfurtrioxide (SO_3)	1.75	1.76	2.04	2.07	1.68
Loss	0.90	1.12	1.43	1.68	1.55
Insoluble	0.30	0.12	0.06	0.13	0.08
Ratios:					
Alumina/iron oxide (A/F)	1.60 1.20[b]	2.05	1.02	0.99	0.97
Lime/silica (C/S)	2.90	2.40	2.64	2.63	2.67
Molecular ratio	2.60	2.32	2.26	2.20	2.24
Alkalinity	3.8	2.2	4.4	4.5	3.7
Free alkali	3.5	3.3	8.2	4.9	3.1
Sugar Test—					
Clouded	8.0	2.5	2.4	2.3	2.0
Clear	10.0	2.5	2.7	2.3	2.0

[a] Limits specified by the New York, N. Y., Board of Water Supply (B.W.S.); maximum except as indicated by [b]. [b] Minimum limits specified by B.W.S.

Mr. Proctor has suggested that some kind of carbon dioxide test be investigated. Inasmuch as he tested eighty samples and found that only two cements showed reasonable resistance to carbon dioxide, it does not appear that the test is discriminatory. The only apparent connection between this test and the present discussion is that a watertight cement should prevent migration of alkalies. Watertightness, as indicated by absorption, is a function of several variables, such as aggregate grading, water-cement ratios, and curing—all large as opposed to the cement variable. Although it may be possible to rate cements in the laboratory by holding other variables constant, practically the results would mean nothing because variability in field conditions would have so much larger an effect on mass concrete.

Like Mr. Abrams, Mr. Proctor appears to assign the cause of cracks in Fig. 1 to drying shrinkage and has made estimates of the amount of over-all expansion (that would have to take place to ascribe the cracks to this cause) by summing up the crack openings. Fig. 1 was intended to portray typical expansive cracking. Admittedly, shrinkage of the concrete surface, caused by drying, aids in producing such cracks; however, it is not the underlying cause in the cases discussed. The photograph from which Fig. 1 was made was retouched slightly for reproduction, in order to reveal more clearly the detail that was intended to be shown, and Mr. Proctor was thus mislead in attempting to estimate the size of cracks shown and from them to postulate the accompanying amount of over-all expansion. Messrs. Proctor and Abrams (and perhaps also many other readers) are to be excused for diagnosing Figs. 1, 8, and 12 as examples of cracking due to drying shrinkage. This was also the impression of many U.S.B.R. engineers on first viewing the cracks at Parker Dam. Such a simple explanation, however, does not fit the conditions, which include evidence of increasing internal volume. Other concrete in the vicinity of the afflicted structures does not suffer from drying, and the concrete in question cannot be shown to possess any but ordinary sensibility to drying shrinkage. The cracks exhibited in Figs. 1, 8, and 12 are open and are not filled with deposits; they are fairly wide and form a pattern larger than that resulting from the crazing usually associated with drying shrinkage. The concrete between the cracks has all appearances of soundness, although the microscope will sometimes reveal additional minute cracks.

Mr. Burris' discussion calls attention to the methods employed by petrographers in identifying rocks and minerals. In this connection, his comments may be of interest to those not sufficiently familiar with the apparatus and techniques to comprehend, fully, the integration of the various procedures. The interpretations given in the paper are supported by detailed petrographic studies of this same nature conducted in the U.S.B.R. laboratories in Denver, Colo.

Mr. Burris states:

> "* * * it is logical to assume that the further release of sodium from the feldspars as well as from the cement would tend—through chemical reaction with the silica in feldspars and cements—to promote expansion even in a low-alkali cement but at a slower rate than in the case of a high-alkali cement."

In summarizing his conclusions he states, further, that:

> "* * * both cement and aggregate must contain an excess of available free alkalies to cause expansion and that when either cement or aggregate is practically free of alkalies no serious difficulty has been encountered."

If it is correctly understood from these statements that Mr. Burris believes that sodic feldspars react in a deleterious manner with high-alkali cement; and, furthermore, that expansive reactions take place only when the aggregate contains alkali-bearing constituents, he is mistaken on both counts. Mr. Stanton (4*e*) made autoclave tests on several feldspars and other materials immersed in sodium carbonate and sodium hydroxide solutions, but found only opal and parts of specimens of highly reactive siliceous magnesian limestone to be affected. Expansions obtained for laboratory mortar bars fabricated with high-alkali cement and several types of aggregates are shown in Table 10, from which it is evident that the more sodic plagioclase feldspars do not expand significantly with high-alkali cement, although several aggregates that do not contain more than traces of soda expand markedly. The data shown in Table 10 for a given aggregate represent the greatest expansion obtained with that aggregate when mixed with crushed pegmatite quartz in various percentages—the test series for each aggregate comprising seven groups.

TABLE 10.—PERCENTAGE OF EXPANSION OF LABORATORY MORTAR BARS FABRICATED WITH HIGH-ALKALI CEMENT

Quartz, mixed with:	6 months	12 months
Microcline	0.017	0.021
Albite	0.018	0.041
Oligoclase	0.002	0.003
Andesine	0.022	0.051
Opal	1.723	2.218
Novaculite	0.499	0.537
Chalcedony	0.227	0.264

Mr. Burris raises the question as to how positively and how completely some of the mineral constituents of fine-grained rocks can be identified with the microscope when he states:

> "The structure of the thin section of rock and the texture of the individual minerals contained therein are usually among the first of the many properties determined by the petrographer."

There are some minerals that cannot be determined uniquely with the microscope, and under these circumstances it is often necessary to resort to microchemical tests and other means of identification. Aside from rarer instances of this kind most rock-forming minerals are readily identifiable by a skilled petrographer; but, after a mineral name such as "lussatite" or even "chalcedony" has been applied, there are still numerous questions that remain unanswered. Chalcedony produces an X-ray diffraction pattern virtually indistinguishable from that of quartz; but there are differences in the optical properties of quartz and chalcedony since these two varieties of silica behave differently as concrete aggregates. Some investigators have concluded that chalcedony probably is not a single-phase system but may be a mixture of two or three components. It is probable that some of these fundamental questions concerning silica minerals will not be further resolved with the microscope but will require X-ray or electron diffraction for ultimate resolution. When knowledge on this subject has become sufficiently complete to indicate what is present in some of these so-called minerals, it may be possible to decide what is reacting with the cement alkalies, and thus to explain why some cherts are reactive whereas others are not deleteriously reactive. Petrographers have progressed in discovering many of the constituents of rocks that are capable of interaction with cement alkalies, but this work is not finished.

In his discussion concerning "popouts" such as that shown in Fig. 11, Mr. Burris appears to conclude that where such "popouts" are not observed on the surface of expanding concrete, particularly as at Parker Dam, the expansion might be attributed to reaction within the cement. He further suggests:

> "* * * that the possibility of opaline silica from the limestone remaining in the cement, because of too low kiln temperatures, and uniting with free alkalies remaining in the cement for the same reason, should be investigated thoroughly as a possible source of this expansion."

However, data obtained in the U.S.B.R. laboratories indicate that neat cement bars fabricated with high-alkali cement, particularly the ones used at Parker Dam and the one used in the tests of Fig. 7, usually do not expand and that the exceptional cases where expansion did occur can probably be accounted for in terms of other characteristics of the cement.

Mr. Burris discusses Parker andesites briefly and alludes to the feldspars being too sodic for these rocks to be classified as andesites. However, Mr. Burris apparently has noticed also that the andesites are somewhat altered by hydrothermal processes because he comments that one of the alteration products is calcium carbonate. He does not state just what term he would apply to these rocks containing "* * * albite and * * * oligoclase as the prevailing feldspar," but presumably he is aware of the hydrothermal process called "albitization" in which more calcic feldspars are altered to more sodic varieties. Thus, it is probable that Mr. Burris would speak of these as "albitized andesites," although the additional adjective contributes comparatively little to the general scope of the problem inasmuch as andesites do not have to be albitized to be reactive. Actually, some of the andesites are only insignificantly altered whereas others are greatly altered; some are weathered and others are comparatively unweathered. Many intermediate and acid types are present in addition to andesites, including rhyolites, dacites, and trachytes, and these show various amounts of hydrothermal alteration and weathering.

Mr. Burris uses several adjectives whose meanings are not entirely self-evident. For example, reference is made to "free alkalies" and "pure silica." The adjective "free" in referring to chemical oxides usually means uncombined with other oxides present, but probably Mr. Burris does not believe that the alkalies of cement clinker are largely in the uncombined state—most investigators do not concur in such an opinion. However, it is apparent that such alkalies are readily released by hydration and hydrolysis when the cement is wetted. "Pure silica" apparently refers to some form of silica other than opal or silicates, but it is not clear whether it means quartz, tridymite, or cristobalite. Also, the writers do not understand why Mr. Burris discusses a contact metamorphic rock, such as one containing wollastonite, in a manner implying a relationship to the siliceous limestones mentioned by the writers. There is no relationship; the term siliceous limestones was not intended to include marbles or contact metamorphic rocks and there is nothing inherent in the connotation of this term to suggest including these rocks. Mr. Burris is perfectly correct, however, in stating that the siliceous limestones to be avoided are those that contain silica derived from organic matter such as Radiolaria and diatoms which are detected, by the aid of the polarizing microscope, to be noncrystalline or amorphous.

The discussions of Messrs. Kammer, Stanton, Spencer, and Chadwick supply valuable additional information which should aid toward a better understanding of this type of deterioration in concrete. These investigators have provided factual data and useful information from their experiences.

Mr. Stanton adds to the list of materials reactive with high-alkali cement and states that, of the active andesites, all have a glassy groundmass as a common characteristic. The increased activity observed in a synthetic andesitic glass when it is artificially weathered is of great interest because so many of the suspect andesites possess unmistakable signs of preconcrete alteration.

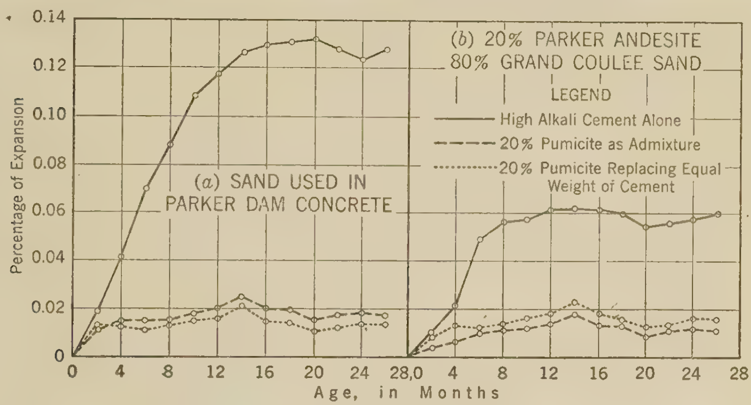

FIG. 14.—EFFECT OF PUMICITE ON EXPANSION

Since Fresno pumicite was used in Friant Dam, and there previously has been assurance that such use afforded protection against expansion from the reactive aggregate in this dam, the data in Table 6 are disturbing. However, in previous work Mr. Stanton, and O. J. Porter, M. Am. Soc. C. E., L. C. Meder, and Allen Nicol (8*a*) have shown pumicite (not necessarily Fresno pumicite) to be almost as effective as Monterey shale in reducing expansive reaction. Another statement by Mr. Stanton (4*f*) is also comforting:

> "The addition of a pozzolanic material seems to be effective, as 25% pumicite substituted for an equivalent weight of high-alkali cement reduced the expansion to a negligible amount at early periods."

The expansion of mortar bars made with two types of reactive aggregates (Fig. 14) was reduced to an insignificant amount when 20% of Fresno pumicite, by weight of cement, was either added to the mix or used as a cement replacement. Mortar bars 1 in. by 1 in. by 10 in. were made with reactive aggregate and cement (mix, 1:3) containing 1.44% of total alkalies ($Na_2O + 0.658\ K_2O = 1.21\%$). They were cured in sealed containers in the presence of moisture at 100° F.

Concrete cores taken from Friant Dam show no evidence of gel formation where pumicite was used in the mix, although some slight amount of gel was found in cores containing high-alkali cement alone. There is substantial evidence, therefore, that Fresno pumicite has aided in preventing expansion in Friant Dam, where an aggregate now known to be reactive was used.

Mr. Witt makes it clear that he has not been convinced, by the paper or any of the many articles (8*b*) on the subject, that concrete can deteriorate because alkalies in the cement react with some aggregates. Many cement manufacturers, however, do not share this skepticism; they agree that a considerable volume of concrete is suffering because some cements and aggregates do not "get along" agreeably with each other. One producer (29) has made this statement,

> "* * * this does not permit the manufacturer of portland cement to put the matter aside; there are probably cases in which low-alkali cement should be used, and the problem of making it economically is therefore of considerable importance."

The discussion by Mr. Kammer is significant in this connection. He considers a case where reactive aggregates were used successfully with low-alkali cement in contrast to the failures noted with cement of high-alkali content. There are a growing number of cement consumers who have been convinced of the necessity for controlling the alkali content of cement and who are demanding that it be limited when aggregates known or suspected of activity must be used. Also (in Mr. Witt's words), "* * * serious, although far less common, is the lag between application and research accomplishments."

Mr. Abrams presents one major theme—that the cracking in Parker Dam is not due to the cause ascribed by the writers but to the extreme temperature and drying conditions at the site, coupled with defective technique. He maintains that (conclusion (2)) "* * * the authors did not avail themselves of evidence that contradicts the hypothesis of the alkali-aggregate reaction"; and (conclusion (3)) "The photographs that were given to illustrate expansive cracking of dams clearly show cracks to be due to contraction of concrete." The possibility of extreme drying conditions being responsible for the cracking was once explored and found untenable under most of the evidence. The writers will not deny that the climate at Parker Dam is hot and dry; but official data of the U. S. Weather Bureau show that, although extremely low humidities are frequently encountered in this region, the average relative humidity is far from the low values that readers might infer from Mr. Abrams' discussion. For instance, the average relative humidity recorded at Yuma, Ariz. (the closest station to Parker Dam where such records are kept), over the period from 1940 to 1945 is 44%. Whereas Mr. Abrams infers that no rain might fall at Parker Dam in a ten-year period, the average yearly precipitation is about 5.5 in.—only 0.5 in. less than at Boulder Dam. Whereas Mr. Abrams quotes shade temperatures of 125° F, the mean annual temperature at Parker, Ariz., is about 71° F, which is not greatly different from that at Boulder Dam.

Regardless of how severe the climate may or may not be, drying is not responsible for the cracks in Parker Dam. Two pumping plants, built by the Metropolitan Water District coincidently with Parker Dam and in the same vicinity, have not developed these cracks, although they have been subject to all the rigors described by Mr. Abrams—neither has the powerhouse at Parker Dam. Abundant evidence that the concrete in Parker Dam is expanding is presented in the paper. More such evidence has been presented elsewhere (5) (6).

Another important point is that, although the aggregates used in the pumping plants and powerhouse have been proved reactive when used with high-alkali cement, they did not cause expansion in those structures because they were built with low-alkali cement. In contrast, Gene Wash and Copper Basin dams, built during the same period as Parker Dam with the identical aggregate and high-alkali cement, are expanding and have suffered similarly to Parker Dam. Mr. Abrams further attributes the cracking at Parker Dam to (*a*) the use of large (4-yd) mixers and (*b*) an inordinately long mixing time of 2½ min. The concrete for Copper Basin Dam was mixed for 1½ min in a 2-yd mixer and that for Gene Wash Dam was mixed for the same period in a 1-yd mixer and yet both dams have developed irrefutable expansive cracking. Except for the size of batches, the concrete was identical in the three dams; the materials were all batched in the same plant.

Mr. Abrams states that the U.S.B.R. requirements for curing Parker Dam concrete were (*A*) inadequate for the "blistering desert" conditions and (*B*) poorly enforced. It is left to the reader to decide if three weeks of continuous sprinkling or spraying with water is not as much as has been demanded for the curing of concrete in any region. At least, it is one week more than was given the concrete in the pumping plants just referred to, which did not crack under the "blistering desert" conditions. Mr. Abrams is incorrect in stating:

> "The upper, thin, reinforced concrete sections of the dam were not cured by 'continuous sprinkling, or spraying,' but by three coats of a patented curing compound that was poorly adapted to the conditions."

The fact is that all exposed concrete on the dam itself was thoroughly cured for three weeks as directed. The curing compound mentioned was applied to the inside surfaces of the hollow piers in which a high humidity would naturally obtain and where it is believed that even better curing was achieved than elsewhere.

Mr. Abrams states also that:

> "By a better grading of aggregates, and a method of mixing that minimized mixer grinding, the mixing water in a cubic yard of concrete could have been reduced from 250 lb to 175 lb. * * * Surface cracking at Parker Dam was due to too much mixing water * * *."

The writers know of no mass concrete, with maximum size of aggregate restricted to 6 in., which could be made satisfactory and workable with as little as 175 lb of water. The concrete-making properties of the Parker Dam aggregates were thoroughly investigated with trial mixes of various gradings, and with no combination could a water content approaching 175 lb per cu yd be used to produce a satisfactory mix, even in a small mixer. Again, however, Mr. Abrams' argument that "cracking at Parker Dam was due to too much mixing water" fails when attention is directed to the perfect concrete in the near-by Gene Wash and Intake pumping plants which contained 100 lb more water per cubic yard than was present in the Parker Dam concrete. The subject of grinding action in large mixers is treated in some detail by Mr. Abrams, who supplements his comment by reference to his previous discussion of a paper by S. P. Wing, M. Am. Soc. C. E., Valens Jones, and R. E. Kennedy, M. Am. Soc. C. E. (26). Those who are interested in the details of this subject should also read the authors' reply to the discussion (26*a*). To describe a mixer as an "ideal" grinding machine, as Mr. Abrams does, is certainly an exaggeration.

In referring to Fig. 10, Mr. Abrams states,

> "It seems obvious that the 'concrete temperature' curve * * * does not represent the interior of the dam; the dam temperature could not possibly show seasonal variations of 40° F."

The temperatures shown are in fact the averages at the bottom of the 12-in., 30-in., 60-in., and 120-in. holes, and it was expected that the reader would so regard them. In spite of the foregoing statement, the concrete does show seasonal variations of more than 30° F at a depth of 10 ft below the roadway. Mr. Abrams has attempted to relate the expansion reported by the writers to volume change that would accompany "unconsidered temperature rise, as a result of continuing hydration of cement." He visualizes: "* * * the gates, the gate-operating structure, and the roadway bridges in a gigantic vise where they were crushed like eggshells." To some extent he was led to this distorted picture by the statements concerning "binding of gates" which "the authors mention three times." Nevertheless, nothing was written in the paper to suggest that the gates at Parker Dam were binding. The gates at Coolidge Dam were binding, as stated by the writers, and data were cited in the paper (1)(2) to support the other statements. The conditions reported necessitated the repairs described by Mr. Kammer in his discussion of this paper.

After going through some calculations which appear highly speculative to the writers, Mr. Abrams comes to the conclusion that by 1942 the internal temperature of Parker Dam had risen some 120° F. Similar assertions were made concerning Boulder Dam (28) which have been adequately refuted by showing that no temperatures faintly approximating that value are to be found in the dam. As requested, the writers report the temperature in the middle of Parker Dam 50 ft below tailwater, at El. 315, to be 65° F minimum and 69.8° F maximum as of 1944. In the abutment at middepth of the gate opening, during 1943, the temperature was 70.6° F minimum and 77.6° F maximum.

Mr. Abrams refers to "inexpert and incomplete chemical analyses," "inconsequential tests," foggy language," and then proceeds as follows:

> "Many woods contain organic acids that readily attack compounds in cements and aggregates. Evidences of acid action would be destroyed by exposing the samples to a high temperature. The nature of the material that was driven off in the 'loss-at-ignition' test (Table 1) was not disclosed; nor was the temperature given. These losses seem to represent free water, although it may have been carbon dioxide, water of crystallization, alkalies, or many other compounds."
>
> "The material that the authors identified as water glass was probably formed in the crucible during the 'loss-at-ignition' test. Water glass is formed only under the most favorable conditions and at a temperature high enough to fuse alkalies and silica; it certainly was not formed in this concrete."

The paper states that the gels may resemble ordinary water glass, not that they are ordinary water glass. The paper clearly identifies the location of this gel (Fig. 3) as being on the interior (broken) surface of a core—not on the outside surface in contact with sawdust. Certainly the gel did not form in the crucible. The remarks on the formation of water glass, therefore, are completely irrelevant as well as entirely erroneous. An authoritative description of both wet and dry methods for producing silicate solutions from silica and alkalies has been published by J. G. Vail (30). The production of sodic silicas does not necessarily depend on the fusion temperature of alkalies and silica, but can be achieved at temperatures below 100° C. In a study of the quaternary system soda-lime-silica-water at 25° C, George L. Kalousek (31) reports gel analyses similar to those given in this paper. He also reports analyses of gels synthesized in the laboratory.

To clarify this issue on gel, Fig. 15 is presented, showing a gel body recently picked from the downstream surface of Copper Basin Dam. This material is shown just as found, oozing from a crack, and by analyses consists of:

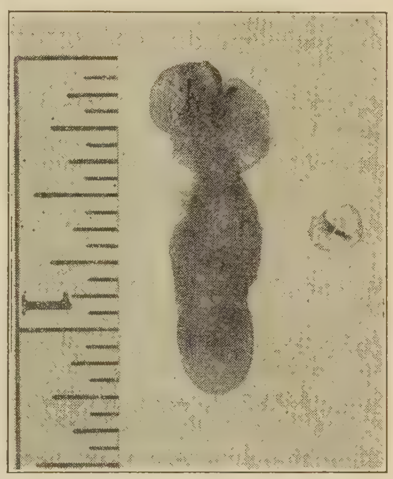

Fig. 15.—Gel, Taken from the Downstream Face of Copper Basin Dam

Chemical	Proportion (%)
Loss on ignition	26.8
Sulfur trioxide, SO_3	0.9
Silica, SiO_2	45.5
R_2O_3	0.7
Lime, CaO	0.0
Magnesia, MgO	0.0
Sodium oxide, Na_2O	20.0
Potassium oxide, K_2O	4.1
Total	98.0

The smallness of the sample (0.6 g), made it impossible to separate the part represented in "loss on ignition" into its components, or to make further analyses to determine constituents other than those listed.

The relationship between the damp sawdust and the formation of a sodic silica gel is somewhat obscure in spite of Mr. Abrams' allusion to organic acids and their attack on cements and aggregates. These gels can be, and have been, produced by the interaction between high-alkali cements and reactive aggregates in the laboratory by the U.S.B.R., the California State Division of Highways, the Public Roads Administration, and by other reputable agencies. The formation of these gels has nothing to do with sawdust or organic acids.

These gels have been produced from aggregates used at Parker Dam in laboratory bars and cylinders in the complete absence of sawdust.

Mr. Abrams asks:

> "If certain constituents of Parker sand were attacked by cement alkalies in the dam, why were they not long ago destroyed by the same alkalies in Williams River water (a tributary to the Colorado River)?"

This question is not pertinent to the problem of the paper. The water of the Bill Williams River ultimately will destroy the sand through chemical attack and, in a geologic era or two, all the sand in the river bed will be completely destroyed by chemical weathering processes—that is, if man can devise a method of preventing its being carried away by the natural currents of the river. Simultaneously, the sand thus destroyed will be replaced by other sand which forms farther upstream.

The "alkalies" in the Bill Williams River water are balanced chemically by such ions as carbonates, chlorides, sulfates, etc. The water of the river is principally saline although slightly basic, in contradistinction to the solutions that exist in hydrating cement, which are strongly basic. Thus, the relationship between the behavior of the aggregates in the river water and their behavior in concrete does not seem pertinent to the discussion.

Mr. Abrams disposes of alkali reaction at Parker Dam by a series of illogical processes and erroneous assumptions but he disregards a large number of other structures that have similarly suffered this type of disintegration. He questions the very existence of alkali reaction when he states,

> "Five years of 'intensive investigation' failed to identify a single rock type that reacted with high-alkali Portland cement. The hypothesis of alkali-silica reaction has no basis except foggy language, unsupported assertions, and trivial circumstances."

This sweeping statement is not in any way consistent with the facts. Experience gained from laboratory bars and from concrete structures indicate that there are several rock types which have proved reactive with high-alkali cement. The writers have stated what some of these rock types are, and others have been described by Messrs. Stanton, Kammer, Spencer, and Chadwick in their discussions of the paper.

The writers feel that neither Mr. Abrams' approach nor his deductions give any material help in solving the problem presented for discussion in this paper. Considerable of his discussion involves trivial matter, concerning which they are loath to burden the reader. As an example, exception is taken to the term "opaline silica," in which redundancy is recognized, but which has enjoyed use by prominent authorities (32)(33) as well as by Mr. Burris in his discussion of this paper.

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MOVING LOADS ON RESTRAINED BEAMS AND FRAMES

BY R. C. BRUMFIELD, M. AM. SOC. C. E.

WITH DISCUSSION BY MESSRS. R. W. STEWART, NATHAN D. BRODKIN, T. F. HICKERSON, MAX W. STRAUSS, D. A. MACKENZIE, CHARLES E. SCHAFFNER, J. KALB, G. ROLSMA AND F. W. GIESEKING, STANLEY S. SCHURE, ALEXANDER DODGE, FREDERICK S. MERRITT, ALFRED B. HEISER. T. B. RIGHTS-PHIL M. FERGUSON, LEON BESKIN, BRUCE JAMEYSON, CHARLES W. DOHN, CLIFFORD PETERSEN, A. A. EREMIN, AND R. C. BRUMFIELD.

SYNOPSIS

Engineers have long been familiar with the criterion for placing a load on a simple beam, for maximum moment under one of its concentrations, which requires that the center of the span bisect the distance from the center of gravity of the load to the concentration in question. There is also that other criterion for placing the load for maximum moment at a given point, which requires that the average load in each segment of the span shall be the same as that throughout the span. When the ends of a beam are restrained these criteria cannot be applied; and the problem is further complicated when the object is to find the loading position for maximum moment at the restrained ends. In the past the problem of placing moving loads on fixed spans has usually been solved, rather laboriously, by the use of influence lines.

In this paper the concept of the transmission coefficient will be introduced and equations expressing its value will be derived. From the idea of the transmission coefficient will be developed a method for computing the restraining bending moments at the supports of restrained beams, due to any combination load placed on the span. The resulting equations will be maximized and criteria developed for placing the load in position for maximum bending moment at the support and out in the span. A new concept will be introduced for the sign or sense of the bending moment in rigid frames; and methods will be developed for distributing the moment to the frame in a single operation.

Although the methods contained in this paper may be rendered applicable to beams of all types, the present scope will be limited to straight beams of uniform cross section. Other applications will be outlined but not discussed.

NOTE.—Published in May, 1945, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

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1. Definitions and the Nomenclature of Unusual Terms

As far as practicable, the effort is to conform with standard letter symbols in common use for structural analysis, and these symbols are defined where they first appear in the paper. Inherent in the usefulness of the paper, however, are certain symbolisms peculiar to the method itself, and the nomenclature needs to be understood clearly to grasp the full import of the paper. Such special symbols are as follows:

C = transmission coefficient, a dimensionless fraction by which a bending moment, introduced into one end of a span by forces outside the span, is multiplied in order to obtain the induced bending moment at the opposite end of the same span. This coefficient becomes identical with commonly used "carry-over factor" when its direction of transmission is toward a fixed support.

U = moment of inertia of the loading group, which is equal to the sum of the products of the separate concentrated loads of the group and the squares of their respective distances from the center of gravity of the group.

r = radius of gyration of a series of loads with fixed spacing, found by taking the square root of the quotient that results from dividing the moment of inertia of the group of loads by the total load.

λ = span distribution ratio, $\frac{r}{L}$, or the radius of gyration of the loading group divided by the length of the span on which the load is placed.

S = modulus of dissymmetry, or the algebraic sum of the products of the individual concentrated loads of a group and the cubes of their respective distances from the center of gravity, distances on the right being considered positive and those on the left negative.

e = radius of dissymmetry, or the cube root of the quotient found by dividing the modulus of dissymmetry by the total load.

ϵ = dissymmetry span ratio, $\frac{e}{L}$, the quotient found by dividing the radius of dissymmetry of the loading group by the length of the span in which the load is placed.

i = stiffness index, the product of the span ratio $Q \left(= \frac{L}{I} \right)$ of a member and the quantity $(2 - C)$.

R = restraint index which, for a particular member at a joint, is equal to the reciprocal of the sum of the reciprocals of the stiffness indexes of all other members radiating from the same joint.

m = moment split. For a particular member, moment split is that number by which the moment delivered to the joint by a second member of the same joint is multiplied to obtain the induced moment in the first member.

t = torque split. For a particular member of a joint the torque split is that number by which artificial torque, applied to a joint by some means such as a crank, is multiplied to obtain the induced moment in the particular member to which the split applies.

Q = the quotient found by dividing the span length of a member by the moment of inertia of the span. (It will be noted that this is the reciprocal of the "K-value" often used.)

2. THE TRANSMISSION COEFFICIENT

The theorem of three moments is familiar to engineers as a tool of computation. It can be derived simply from the use of the second principle of moment areas. When any number of spans radiate from a common rigid joint, the theorem of three moments may be applied to any pair of them, if one member of the pair actuates rotation at the common joint and the other opposes it. Let the two spans being considered be given the numbers 1 and 2. Let the quotients found by dividing the lengths of the spans by their respective moments of inertia $\left(\frac{L}{I} = \frac{1}{K}\right)$ be Q_1 and Q_2. If there are single loads W_1 and W_2 in each of the spans, placed at distances $k_1 L_1$ and $k_2 L_2$, respectively, from the left supports, the theorem of three moments is stated as follows:

$$M_{1L} Q_1 + 2\,(M_{1R} Q_1 + M_{2L} Q_2) + M_{2R} Q_2 = -\,W_1 Q_1 L_1 (k_1 - k^3{}_1)$$
$$-\,W_2 Q_2 L_2 (2\,k_2 - 3\,k^2{}_2 + k^3{}_2) \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

The subscripts L and R refer to the left and right of the spans. When no loads exist in either of the two spans, the right-hand side of the equation becomes zero.

The three contiguous spans, AB, BC, and BD, in Fig. 1, radiate from a common, rigid joint B. The supports at A, B, C, and D are free supports.

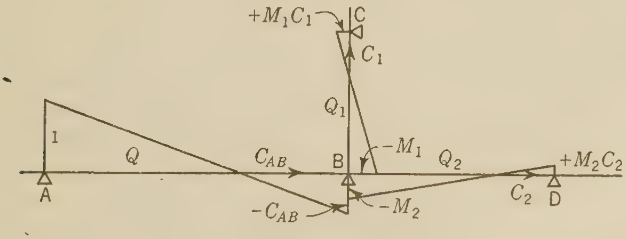


FIG. 1

The spans, as indicated, may continue indefinitely away from the joint B, at supports A, C, and D. Their connections beyond these points are wholly immaterial to this discussion, except as expressed by the transmission coefficient. There are two transmission coefficients C for any span, one in either of the two directions. The letters used to designate the span are used as subscripts to denote the direction of transmission in the span. Thus C_{AB} denotes that coefficient which transmits moment from A to B; C_{BA} that coefficient transmitting moment from B to A.

In Fig. 1, a unit positive couple is assumed to have been introduced into span AB, at end A, from the left. By definition of transmission coefficient, this induces C_{AB} at the opposite end of span AB. The bending moment C_{AB} is negative, according to the ordinary mechanics of materials concept of bending moment, since the two couples cause a reverse curvature in member AB. The moment $-C_{AB}$ divides at joint B, and induces two unknown moments $-M_1$ and $-M_2$, at joint B, in spans BC and BD. These in turn induce moments of $+M_1 C_1$ and $M_2 C_2$ at the outer ends of their respective spans. Remembering that there are no loads in either of the three spans, the theorem of Eq. 1 may be applied to span AB, in combination with either of the other two; thus:

$$Q - 2\,Q\,C_{AB} - 2\,Q_1\,M_1 + M_1\,Q_1\,C_1 = 0 \quad \text{(2)}$$

from which:

$$-M_1 = \frac{2\,Q\,C_{AB} - Q}{Q_1\,(2 - C_1)} = \frac{2\,Q\,C_{AB} - Q}{i_1} \quad \text{(3a)}$$

and, by analogy,

$$-M_2 = \frac{2\,Q\,C_{AB} - Q}{Q_2\,(2 - C_2)} = \frac{2\,Q\,C_{AB} - Q}{i_2} \quad \text{(3b)}$$

Now the sum of M_1 and M_2 must be equal to C_{AB} in order to have equilibrium at joint B. Adding the values of M_1 and M_2, in Eqs. 3, in terms of C_{AB}, and equating them to C_{AB}:

$$C_{AB} = \frac{Q}{2\,Q + \dfrac{1}{\dfrac{1}{i_1} + \dfrac{1}{i_2}}} \quad \text{(4)}$$

The value of C that appears in the stiffness index i is that one which transmits moment away from the joint of the span, toward which C_{AB} transmits moment.

As defined in Section 1 the reciprocal of the sum of the reciprocals of the stiffness indexes of the receiving members of a joint is known as the restraint index, R, of the transmitting member. If one member of a joint transmits bending moment to that joint, the other members all resist the rotation of the joint due to this moment. The restraint index for this transmitted moment, or rather for the transmitting member, is made up of the reciprocal of the sum of the reciprocals of the stiffness indexes of all the resisting members of the joint. These include all members except the transmitting member. The transmission coefficient C of any particular resisting member, which appears in the stiffness index of that member, will be that one which transmits moment away from the joint under consideration. With these concepts in mind, Eq. 4 can be written:

$$C = \frac{1}{2 + \dfrac{R}{Q}} \quad \text{(5)}$$

The Q-value in Eq. 5 is that of the member whose C-value is being computed. The restraint index R contains the entire effect, on C, of all the restraining members, including those directly and remotely connected. When R becomes zero, the value of C becomes 0.5. This is true for a beam with fixed ends. This value of 0.5 is the greatest possible value of C for straight beams of uniform section. Greater values than this may be had when the beam is of non-uniform section. When R becomes infinite in value, the computed value of C

becomes zero. This occurs in the case of a free support. Negative values of the coefficient occur for curved beams.

3. Values of Moment Split

Eq. 5 gives the value of the transmission coefficient for a beam. It is also desirable to know how the moment, which arrives at a joint, such as joint B, Fig. 1, is divided between each of the two receiving members 1 and 2. The multiplier that gives this division is termed the moment split for the receiving member. Such values are readily deduced from the former computation.

Dividing Eq. 3*a* by Eq. 3*b*:

$$M_1 = M_2 \frac{i_2}{i_1} \tag{6}$$

Then:

$$M_1 + M_2 = M_2 i_2 \left(\frac{1}{i_1} + \frac{1}{i_2} \right) = M_2 \frac{i_2}{R} \tag{7}$$

In the case under consideration, C is equal to $M_1 + M_2$, and

$$M_2 = C_{AB} \frac{R}{i_2} \tag{8a}$$

and

$$M_1 = C_{AB} \frac{R}{i_1} \tag{8b}$$

in which R is the restraint index for the member, which delivered the moment to the joint, and i is the stiffness index of the (receiving) member whose part of the moment division is being sought. The moment split is the ratio of the two, or:

$$m = \frac{R}{i} \tag{9}$$

The sum of these m-values for all the receiving members to which moment divides, from a particular actuating member at the joint, must be equal to unity.

4. Restraining Moments in a Loaded Beam

When the transmission coefficients of a span are known, it is possible to compute, accurately and without the concept of fixed-end moments, the induced restraining moments due to loads placed on the span.

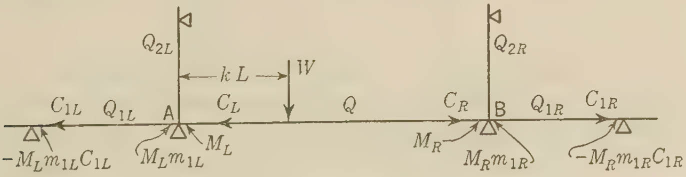


Fig. 2

In Fig. 2, span AB is loaded with a single concentrated load W, placed at a distance $k L$ from the left support. The span AB has the two transmission coefficients C_R and C_L, toward the right and left, respectively. Its Q-value is designated without subscript. The span is shown restrained, at the left and right ends, by two spans at either end. These are designated, in all respects by number-letter subscripts, as 1R—that is, No. 1 span on the right, etc.

Let it be assumed that the induced bending moment due to W, in span AB, at the left end, is M_L, and at its right end is M_R. The induced bending moment in span 1L at joint A will then be $M_L\, m_{1L}$, and at its outer end will be $-M_L\, m_{1L}\, C_{1L}$. A similar set of moments exists in span 1R as shown.

The theorem of three moments, shown in Eq. 1, may now be applied first to the pair of spans AB and 1L, and second to span AB and span 1R. Thus:

$$\begin{aligned} &- M_L\, m_{1L}\, C_{1L}\, Q_{1L} + 2\,(M_L\, m_{1L}\, Q_{1L} + M_L\, Q) \\ &\qquad + M_R\, Q = -\, W\, Q\, L\,(2\,k - 3\,k^2 + k^3) \qquad (10a) \end{aligned}$$

and

$$\begin{aligned} &M_L\, Q + 2\,(M_R\, Q + M_R\, Q_{1R}\, m_{1R}) \\ &\qquad - M_R\, m_{1R}\, C_{1R}\, Q_{1R} = -\, W\, Q\, L\,(k - k^3) \qquad (10b) \end{aligned}$$

Segregating the M_L-terms and the M_R-terms in each of these equations:

$$M_L \left[\frac{2\,Q + m_{1L}\, Q_{1L}\,(2 - C_{1L})}{Q}\right] + M_R = -\, W\, L\,(2\,k - 3\,k^2 + k^3) \qquad (11a)$$

and

$$M_L + M_R \left[\frac{2\,Q + m_{1R}\, Q_{1R}\,(2 - C_{1R})}{Q}\right] = -\, W\, L\,(k - k^3) \qquad (11b)$$

By definition: $Q_{1L}\,(2 - C_{1L}) = i_{1L}$; $Q_{1R}\,(2 - C_{1R}) = i_{1R}$; $m_{1L} = \frac{R_L}{i_{1L}}$; and $m_{1R} = \frac{R_R}{i_{1R}}$. Therefore, Eqs. 11 become:

$$M_L \left(2 + \frac{R_L}{Q}\right) + M_R = -\, W\, L\,(2\,k - 3\,k^2 + k^3) \qquad (12a)$$

and

$$M_L + M_R \left(2 + \frac{R_R}{Q}\right) = -\, W\, L\,(k - k^3) \qquad (12b)$$

An inspection of Eq. 5 will show that Eqs. 12 may finally be written:

$$\frac{M_L}{C_L} + M_R = -\, W\, L\,(2\,k - 3\,k^2 + k^3) \qquad (13a)$$

and

$$M_L + \frac{M_R}{C_R} = -\, W\, L\,(k - k^3) \qquad (13b)$$

Eqs. 13 now may be solved simultaneously for the two unknown quantities M_L and M_R, with the following results:

$$M_L = -\, W\, L\, \frac{C_L}{1 - C_R\, C_L}\, [(2\,k - 3\,k^2 + k^3) - C_R\,(k - k^3)] \qquad (14a)$$

and

$$M_R = -\, W\, L\, \frac{C_R}{1 - C_R\, C_L}\, [(k - k^3) - C_L\,(2\,k - 3\,k^2 + k^3)] \qquad (14b)$$

Eqs. 14 give values of the induced bending moments, at the supports of any restrained beam, when a single load W is applied at any point in the span. The negative sign of the bending moment indicates the usual concept of positive and negative bending moment—that is, that the radius of curvature extends from the beam in the same direction as the positive inducing force presses against the beam.

The accuracy of Eqs. 14 may be verified readily. Let the case of a single span be considered, which is fixed at the right end, freely supported at the left end, and is loaded with a single load W at the center. In this case, C_R is 0.5, C_L is 0.0, and k has the value of 0.5. Substituting in Eq. 14b:

$$M_R = -W\,L\,\frac{0.5}{1-0}\left[(0.5 - 0.5^3) - 0\right] = -\frac{3}{16}\,W\,L$$

Eqs. 14 may be transformed readily into suitable equations for computing the restraining effects of uniform loads. In Fig. 3 is shown a single span with a uniform load of w pounds per foot throughout its length. A single differential portion of this load is indicated at the distance $k\,L$ from the left. Its value is obviously $w\,L\,dk$. This differential load may be considered in the equations as a concentrated load, of which an infinite number make up the total uniform load. Their complete effect may be had by integration, thus:

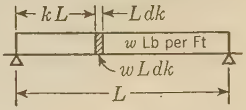


Fig. 3

$$M_L = -w\,\frac{C_L}{1 - C_R\,C_L}\,L^2 \int_0^1 \left[(2\,k - 3\,k^2 + k^3) - C_R\,(k - k^3)\right] dk \quad \text{..(15)}$$

Integrating and substituting the limits, the expression for beams uniformly loaded throughout is:

$$M_L = -\frac{w\,L^2}{4}\,\frac{C_L}{1 - C_R\,C_L}\,(1 - C_R) \quad \text{...............(16)}$$

Similarly:

$$M_R = -\frac{w\,L^2}{4}\,\frac{C_R}{1 - C_R\,C_L}\,(1 - C_L) \quad \text{...............(17)}$$

In verification of Eqs. 16 and 17, let the beam previously considered, with a fixed right support and a free support at the left, be loaded with w pounds per foot. Then: $M_R = -\frac{w\,L^2}{4}\,0.5\,(1 - 0) = -\frac{w\,L^2}{8}$.

5. Restraining Moments Due to Combination Loads

Eqs. 14 are entirely serviceable for the case of a span with restrained ends, loaded with a single load. They may be made to serve also for a span loaded with a series of concentrations, by computing the separate effects and adding them algebraically. This is tedious, and therefore it is desirable to modify the equations to apply to the case of a span loaded with a series of loads with fixed spacing such as a truck load, a locomotive load, etc. When so modified the net effect of all loads in the series may be computed at one operation.

In Fig. 4 a single restained span carries two concentrations, W_1 and W_2. The sum of these two loads is W. The center of gravity of the load lies at a distance $\bar{k}\,L$ from the left support. Each of the loads in the series lies at a distance $a_n\,L$ from the center of gravity of the entire group. Thus W_1 is distant $a_1\,L$ from the center of gravity. By definition, the fraction a has the peculiar property of being negative when the load which it measures lies to the left of the center of gravity, and positive when to the right. All values of a will then reverse sign, when the load is turned around.

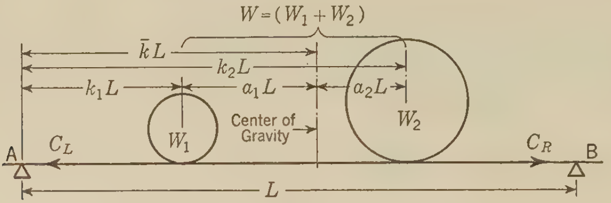


FIG. 4

For the purposes of substitution in Eqs. 14, it is obvious from Fig. 4 that:

$$k_1 = \bar{k} + a_1 \qquad (18a)$$

and that

$$k_2 = \bar{k} + a_2 \qquad (18b)$$

also,

$$A_R = \frac{C_R}{1 - C_R\,C_L} \qquad (19)$$

Substituting these values of k in Eq. 14b and expanding:

$$M_{R1} = -L\,A_R\,W_1\,\{\bar{k} + a_1 - \bar{k}^3 - 3\,\bar{k}^2\,a_1 - 3\,\bar{k}\,a^2_1 - a^3_1$$
$$-\,C_L\,[2\,(\bar{k} + a_1) - 3\,(\bar{k}^2 + 2\,\bar{k}\,a_1 + a^2_1) + \bar{k}^3 + 3\,\bar{k}^2\,a_1 + 3\,\bar{k}\,a^2_1 + a^3_1]\} \qquad (20a)$$

plus

$$M_{R2} = -L\,A_R\,W_2\,\{\bar{k} + a_2 - \bar{k}^3 - 3\,\bar{k}^2\,a_2 - 3\,\bar{k}\,a^2_2 - a^3_2$$
$$-\,C_L\,[2\,(\bar{k} + a_2) - 3\,(\bar{k}^2 + 2\,\bar{k}\,a_2 + a^2_2) + \bar{k}^3 + 3\,\bar{k}^2\,a_2 + 3\,\bar{k}\,a^2_2 + a^3_2]\} \qquad (20b)$$

In Eqs. 20, let $W_1 + W_2$ be equal to W. Adding vertically the like terms of the right-hand side of Eq. 20:

$$\begin{aligned} M_R = -L\,A_R\,\{&W\,\bar{k} + (W_1\,a_1 + W_2\,a_2) - W\,\bar{k}^3 - 3\,\bar{k}^2\,(W_1\,a_1 + W_2\,a_2) \\ &- 3\,\bar{k}\,(w_1\,a^2_1 + W_2\,a^2_2) - (W_1\,a^3_1 + W_2\,a^3_2) \\ &- C_L\,[2\,W\,\bar{k} + 2\,(W_1\,a_1 + W_2\,a_2) - 3\,W\,\bar{k}^2 - 6\,\bar{k}\,(W_1\,a_1 + W_2\,a_2) \\ &- 3\,(W_1\,a^2_1 + W_2\,a^2_2) + W\,\bar{k}^3 + 3\,\bar{k}^2\,(W_1\,a_1 + W_2\,a_2) \\ &+ 3\,\bar{k}\,(W_1\,a^2_1 + W_2\,a^2_2) + W_1\,a^3_1 + W_2\,a^3_2]\} \end{aligned} \qquad (21)$$

In Eq. 21, let: The moment of inertia of the loading group about its own center of gravity equal—

$$\Sigma(W_n\,a^2_n\,L^2) = U \qquad (22a)$$

the modulus of dissymmetry of the loading group about its own center of gravity equal—

$$\Sigma(W_n\, a^3_n\, L^3) = S \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (22b)$$

and, by definition of the center of gravity—

$$\Sigma(W_n\, a_n\, L) = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (22c)$$

Eq. 21 may then be rewritten:

$$\begin{aligned} M_R = -\, W\, L\, A_R \Bigg[& -\bar{k}^3 + \bar{k} - \frac{3\,\bar{k}\,U}{W\,L^2} - \frac{S}{W\,L^3} \\ & - C_L \left(2\,\bar{k} - 3\,\bar{k}^2 - \frac{3\,U}{W\,L^2} + \bar{k}^3 + \frac{3\,\bar{k}\,U}{W\,L^2} + \frac{S}{W\,L^3} \right) \Bigg] \ldots\ldots (23) \end{aligned}$$

Now according to the ordinary definition, the radius of gyration of the loading group is equal to the square root of the quantity U/W. Similarly, the radius of dissymmetry of the loading group may be defined by the equation:

$$\epsilon^3 = \frac{S}{W} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (24)$$

Making these substitutions in Eq. 23:

$$\begin{aligned} M_R = -\, W\, L\, A_R \{ & -\bar{k}^3 + \bar{k}\,(1 - 3\,\lambda^2) - \epsilon^3 \\ & - C_L\,[\bar{k}^3 - 3\,\bar{k}^2 + \bar{k}\,(2 + 3\,\lambda^2) - 3\,\lambda^2 + \epsilon^3]\} \ldots\ldots\ldots (25a) \end{aligned}$$

Similarly:

$$\begin{aligned} M_L = -\, W\, L\, A_L \{ & \bar{k}^3 - 3\,\bar{k}^2 + \bar{k}\,(2 + 3\,\lambda^2) - 3\,\lambda^2 + \epsilon^3 \\ & - C_R\,[-\,\bar{k}^3 + \bar{k}\,(1 - 3\,\lambda^2) - \epsilon^3]\} \ldots\ldots\ldots\ldots (25b) \end{aligned}$$

Eqs. 25 are perfectly general and will give the bending moments at the supports of a restrained beam induced by any combination loading except a pure couple.

The equations are entirely usable in the form in which they appear. Their use is greatly simplified, however, by the aid of a curve which obviates the necessity of determining the detailed values of the k-functions and λ-functions for each case. In Fig. 5, the following quantities have been platted for all ordinary values of λ:

$$B = -\,\bar{k}^3 + \bar{k}\,(1 - 3\,\lambda^2) \ldots\ldots\ldots\ldots\ldots\ldots (26a)$$

and

$$D = \bar{k}^3 - 3\,\bar{k}^2 + \bar{k}\,(2 + 3\,\lambda^2) - 3\,\lambda^2 \ldots\ldots\ldots\ldots (26b)$$

With these substitutions, Eqs. 25 reduce to:

$$M_R = -\, W\, L\, A_R\,[B - \epsilon^3 - C_L\,(D + \epsilon^3)] \ldots\ldots\ldots (27a)$$

and

$$M_L = -\, W\, L\, A_L\,[D + \epsilon^3 - C_R\,(B - \epsilon^3)] \ldots\ldots\ldots (27b)$$

in which

$$A_R = \frac{C_R}{1 - C_R\,C_L} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (28a)$$

and

$$A_L = \frac{C_L}{1 - C_R C_L} \quad \text{(28b)}$$

The curves in Fig. 5 are readable accurately to three places, and as a result Eqs. 27 are entirely usable, with satisfactory accuracy, when only slide-rule work is involved.

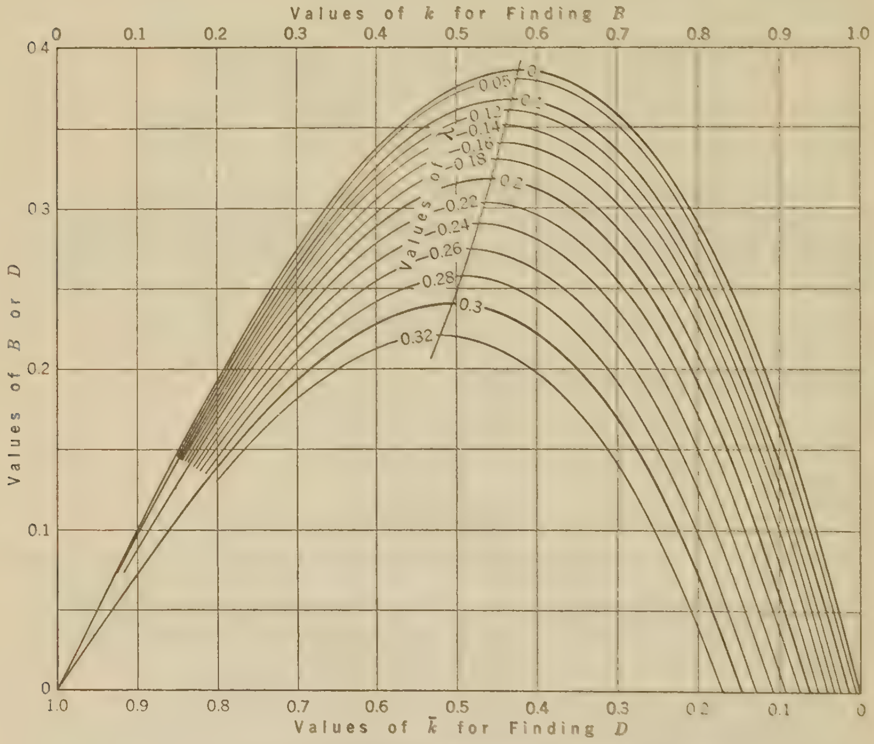


Fig. 5.—Curves for the Solution of Eq. 26

As an example of the use of Eqs. 27, let it be required to find the bending moments at the right and left supports of a beam with fixed ends, which is 30 ft long, and which carries a pair of truck concentrations spaced at 12 ft, weighing, respectively, 4 tons and 8 tons. Let the 8-ton concentration be placed at the span center with the 4-ton wheel to the left.

Let the center of gravity be a distance z from the 8-ton wheel; then: $z = \frac{4 \times 12}{12} = 4$ ft; $U = 4 \times 8^2 + 8 \times 4^2 = 256 + 128 = 384$ ton-ft²; $S = 256 \times (-8) + 128 \times 4 = -2{,}048 + 512 = -1{,}536$ ton-ft³; $\lambda^2 = \frac{384}{12 \times 900} = 0.0355$; $\lambda = 0.189$; $\epsilon^3 = -\frac{1{,}536}{12 \times 30^3} = -0.0047$; and $\bar{k} = \frac{15 - 4}{30} = 0.367$.

From Fig. 5: $B = 0.276$; and $D = 0.310$. Substituting in Eqs. 27: $M_R = -12 \times 30 \frac{0.5}{1 - 0.5^2} [0.276 + 0.0047 - 0.5 (0.310 - 0.0047)] = -360 \times 0.667 \times 0.1281 = -30.8$ ft-tons; and $M_L = -360 \times 0.667 [0.310 - 0.0047 - 0.5 (0.276 + 0.0047)] = -360 \times 0.667 \times 0.1650 = -39.6$ ft-tons.

6. Maximum Bending Moment Due to Moving Loads

In Eqs. 25 only two variables are involved, when the load is moved on the span, if it retains fixed spacing in movement—the bending moment, and the ratio $\bar{k}$. The designer may take a derivative of M with respect to $\bar{k}$, equate this to zero, and solve for $\bar{k}$, in the resulting quadratic equation. When this is done for each of the two equations, the following two criteria are obtained for placing a load in a restrained span in order to produce maximum bending moment at either of the two restrained supports: For maximum bending moment at the right support—

$$\bar{k}_R = \frac{C_L}{1 + C_L} \pm \sqrt{-\lambda^2 + \frac{1 - 2C_L}{3(1 + C_L)} + \left(\frac{C_L}{1 + C_L}\right)^2} \quad \ldots\ldots (29a)$$

and, for maximum bending moment at the left support—

$$\bar{k}_L = \frac{1}{1 + C_R} \pm \sqrt{-\lambda^2 + \frac{C_R - 2}{3(1 + C_R)} + \left(\frac{1}{1 + C_R}\right)^2} \quad \ldots\ldots (29b)$$

Eqs. 29 give the proper locations of the center of gravity of any fixed spacing loading in any restrained span, in order to produce maximum bending moment at the respective supports to which they apply. It is significant that these locations involve only two variables. For maximum moment at the right support the location depends upon the span distribution ratio, and the character of the support at the left end of the beam, as expressed through the coefficient C_L. Similarly, for maximum bending moment at the left support, the location of the load involves only the span distribution ratio and the character of the support at the opposite or right end as expressed through C_R. The dissymmetry of the load is not a factor. The support at the right end may vary in its restraint from fixity to a free support; yet the location of the moving load for maximum bending moment at this support will remain unchanged if the character of the left-hand support is unaltered. Of course, this fact does not prevent the actual maximum from varying in value, as the degree of restraint varies.

Eqs. 29 apply to all kinds of loads, with the exception of a pure couple, which will be treated separately as a special case. Most frequently it is necessary to use the equations in connection with loads, all of which act in the same direction against the beam upon which they are placed. For convenience, the equations have been platted in the form of curves for such cases, in Fig. 6, applying to either of the equations. These curves might well have included only the transmission coefficients from 0 to 0.5, if they were to be used only with beams of uniform section. The curves are also useful in connection with beams of variable section, and for that reason the coefficients represented have been extended to unity.

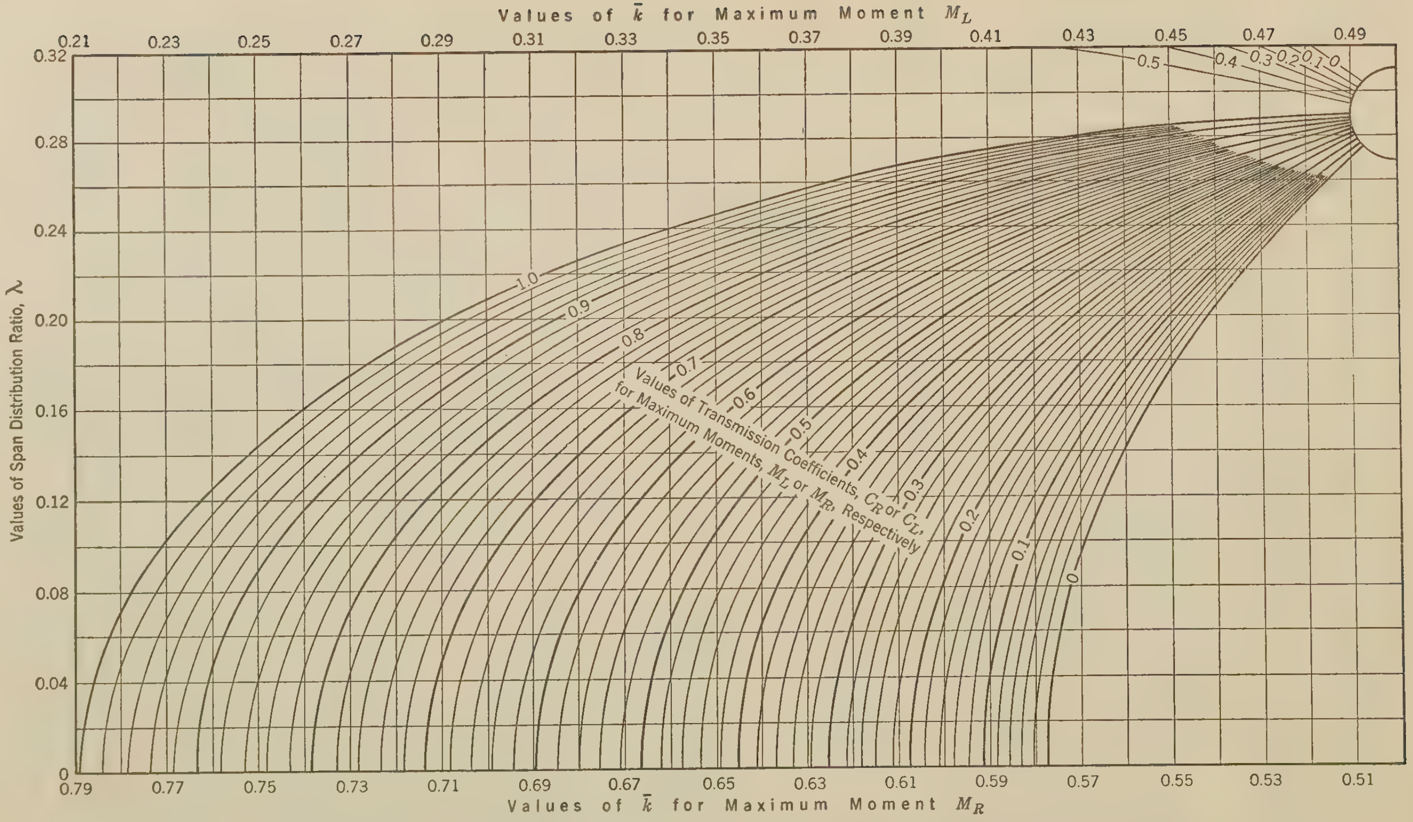


Fig. 6.—Curves for $\bar{k}$ to Give Maximum Bending Moment, for Various Loads, at the Left and Right Supports of Any Span of Any Restrained Beam (for Values of λ Greater than 0.289 the $\bar{k}$-Scales Should be Interchanged)

Example.—Let it be required to find the position of the load, in the problem given in Section 5, in order to produce maximum bending moment at the right support, and to compute this bending moment: $\lambda = 0.189$; and $C_L = 0.5$.

Enter Fig. 6 at $\lambda = 0.189$ and follow across horizontally to the value of $C_L = 0.5$, then follow vertically down to the lower horizontal scale, to read: $\bar{k}_R = 0.6075$. For this value of $\bar{k}$ and the value of λ given, the designer next finds, from Fig. 5, that $B = 0.318$ and $D = 0.289$. Then $M_R = -12 \times 30 \frac{0.5}{1 - 0.5^2} [0.318 + 0.0047 - 0.5 (0.289 - 0.0047)] = -360 \times 0.667 \times 0.1805 = -43.3$ ft-tons.

This represents an increase of about 40% over the bending moment obtained with the heavy wheel at the span center.

Fig. 6 represents only a part of the range of results possible with Eqs. 29. When the loads do not all act in the same direction (as would be the case when a column rests on a girder and carries a bending moment at its base in addition to the direct compression in the column), one of the two forces that make up the bending couple acts in a direction opposite to that of the direct compression and the other force composing the couple. In such a case, particularly when the couple is large and the direct compression small, the center of gravity lies outside the loading group, even outside the span in many cases. When such loads occur, λ^2 may be either positive or negative and much greater than represented in Fig. 6. The equations must be solved directly for $\bar{k}$ in such a case. Any value of $\bar{k}$ either positive or negative is useful for computing the moment that will cause the loading group to lie entirely on the span.

7. SIGNIFICANCE OF SPAN DISTRIBUTION RATIO AND DISSYMMETRY SPAN RATIO λ

When loads act in one direction only, the span distribution ratio, λ, becomes a measure of the distribution of the load throughout the span. If λ is small, the load lies bunched near a point. If it is large the load is distributed more generally like a uniform load. In fact, the point at the upper right of Fig. 6 through which all the curves pass represents the value of λ of 0.289 for a uniform load throughout the span. In practice, for loads in one direction only, the value of λ will not exceed this value.

As previously mentioned, the span distribution ratio is a controlling factor in determining the position of the load for maximum bending moment at either of the supports. This is evident from Eqs. 29. A large value of λ will reduce the quantity under the radical in either equation.

As will be shown subsequently, λ is also an important factor in determining the position of a moving load in order to produce maximum positive bending moment under one of the concentrations in the span.

The dissymmetry span ratio, as its name indicates, measures the degree of dissymmetry of the load with respect to its own center of gravity. When perfect symmetry exists the modulus of dissymmetry is always equal to zero. The converse is not always necessarily true—that, when the modulus is zero, perfect symmetry, in the ordinary sense of the word, must necessarily exist.

In determining the position of the load for maximum bending moment at the supports, the modulus of dissymmetry gives the proper orientation, in order to produce the greatest possible maximum. An examination of Eq. 27*a* will readily show that, for a particular load, M_R is greatest when B and ϵ^3 are additive. This occurs when ϵ^3 is negative. Likewise the greatest possible maximum occurs at the left support when ϵ^3 is positive. The sign of ϵ^3 reverses when the load reverses on the span, although the sign of λ does not change.

It is true that judgment might place the heavy concentration nearest the support at which a maximum is being sought, as the foregoing rules do. Nevertheless, in many cases of mixed loading, the proper orientation is not surely evident. The sign of ϵ^3 is an infallible criterion for the proper orientation.

The value of ϵ^3 is ordinarily small for loads which all act in the same direction against the beam. In cases of bastard couples (parallel unequal forces in opposite directions), however, where the center of gravity falls considerably outside the loading group, the value of ϵ^3 may become quite large and of determining importance in the magnitude of the bending moment.

8. Maximum Positive Bending Moment in Span, at a Given Concentration

Eqs. 27 give the bending moments at the restrained supports, and Eqs. 29 give the proper positions of the load to produce maximums. It is also often desirable to locate the load properly, so that it will produce a maximum positive bending moment under a given concentration out in the span. The designer is interested only in the moment under a concentration, since maximum bending moment occurs at a point of zero shear and shear may only change, in a straight beam, at a load concentration.

In Fig. 7(*a*), a span of length L is loaded with three fixed spacing concentrations W_1, W_2, and W_3, whose center of gravity lies at a distance $\bar{k}$ from the left support. It is required to place the combination load in the proper position for maximum bending moment under the concentration W_2. The simple beam bending moment diagram is shown in Fig. 7(*b*), with the moment under W_2 indicated as $WL(1 - \bar{k})(\bar{k} + a_2)$ − a constant. This represents the simple beam left reaction, times the distance to W_2, minus the constant, which is W_1 times the distance from W_1 to W_2. The separate complete bending moment diagrams, produced on the simple beam, by the two restraining couples at the ends, are represented in Figs. 7(*c*) and 7(*d*) by the two triangles, whose vertical legs are labeled M_L and M_R. The bending moment produced by each of these restraining couples

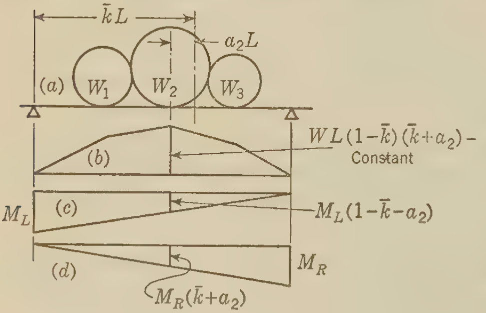


Fig. 7

under W_2 is marked on the respective triangles as $M_L\,(1 - \bar{k} - a_2)$ in the case of the diagram for M_L, and as $M_R\,(\bar{k} + a_2)$ for M_R.

The net bending moment under W_2 will now be the sum of the three separate ordinates, marked on the three bending moment diagrams. It is given by the following equation:

$$M_{W2} = W\,L\,(1 - \bar{k})\,(\bar{k} + a_2) - \text{a constant} + M_L\,(1 - \bar{k} - a_2) + M_R\,(\bar{k} + a_2) \quad \text{(30)}$$

There are potentially only two variables in Eq. 30, when the load is moved. These are (1) the bending moments under W_2 and (2) the value of $\bar{k}$. Taking the first derivative of M_{W2} with respect to $\bar{k}$, equate this expression to zero, and solve for $\bar{k}$, thus locating the position of the load for maximum bending moment under W_2:

$$\frac{dM_{W2}}{d\bar{k}} = W\,L\,(1 - 2\,\bar{k} - a_2) + (M_R - M_L) + \frac{dM_R}{d\bar{k}}\,(\bar{k} + a_2) + \frac{dM_L}{d\bar{k}}\,(1 - \bar{k} - a_2) \quad \text{(31)}$$

Eq. 31 may be set equal to zero after proper substitutions are made for the various quantities (which have been only indicated in this paper). Thus the values of M_R and M_L are the right-hand sides of Eqs. 25*a* and 25*b*, respectively. The values of $dM_R/d\bar{k}$ and of $dM_L/d\bar{k}$ are determined by taking the derivatives of Eqs. 25, and these values are substituted in Eq. 31. The process is not difficult but the algebra is tedious. It will not be given herein, but the result will be indicated.

The following substitutions can be made for combinations of factors:

$$q_1 = (C_R + 2\,C_R\,C_L + C_L) \quad \text{(32a)}$$

$$q_2 = (C_L + C_R\,C_L) \quad \text{(32b)}$$

$$f_3 = 4\,q_1 \quad \text{(33a)}$$

$$f_2 = 3\,(q_1\,a - 4\,q_2) \quad \text{(33b)}$$

$$f = 6\,q_1\,\lambda^2 + q_2\,(4 - 6\,a) + 2\,(3\,C_L - C_R - 1) \quad \text{(33c)}$$

and

$$\kappa = 2\,C_L - 1 + a\,(1 + C_R) + q_2\,(6\,\lambda^2 - 2\,a) - q_1\,(3\,a\,\lambda^2 + \epsilon^3) \quad \text{(34)}$$

It is to be noted that the symbol a in Eqs. 33 and 34 denotes that value which applies to the particular load under which a maximum bending moment is to be sought. This is a_2 in the case under discussion. When Eq. 31 is equated to zero, with the indicated substitutions, it reduces to:

$$f_3\,\bar{k}^3 + f_2\,\bar{k}^2 + f\,\bar{k} = \kappa \quad \text{(35)}$$

When the values of C_R and C_L each become zero, reducing the case to a simple beam, Eq. 35 reduces to the ordinary criterion for placing a load on a simple beam for maximum bending moment under a given concentration.

Because of limitations of space no example of the use of Eq. 35 is offered; but its solution is not difficult if a table of squares and cubes is used in connection with a cut-and-try method of solution. It is sufficient to state that Eq. 35 gives results very little different from those of the ordinary criterion for simple beams, in cases where the values of C_R and C_L are equal, or approximately equal. In such cases the criterion for simple beams may be used for the load placing, with satisfactory accuracy.

Even in cases where there is a decided difference in the restraints at the two ends of the beam, the simple beam criterion is useful for a preliminary estimate of load position. If it is remembered that the criterion of Eq. 35 will always place the load somewhat nearer the end of low restraint than does the simple beam criterion, and if such a shift is made arbitrarily according to acquired judgment, it is possible in most cases to avoid the use of the more complicated criterion of Eq. 35.

The criterion of Eq. 35 gives the position of the load for maximum bending moment under a particular load, where this moment is due to the moving load only, and is not a combination with any dead load that may exist on the span. It is easily possible to extend the foregoing procedure to the case of the absolute maximum produced by combining live-load and dead-load bending moments. It is usually sufficiently accurate and safe to add the maximums due to the two causes as separately obtained.

After the load is placed, it is desirable to find the actual bending moment under the wheel, by first using Eqs. 27 to find the restraining moments at the supports, and then determining the positive bending moments from these, using the laws of equilibrium.

9. Bending Moments at Supports of a Restrained Beam Due to a Pure Couple

Eqs. 27, for the restraining bending moments at the supports of a loaded span, do not apply in the case of a pure couple. The center of gravity of a pure couple lies at an infinite distance to one side of the center point between the forces that comprise the couple. As a result the moment of inertia and the modulus of dissymmetry of the load about its own center of gravity become meaningless. The span distribution ratio and the dissymmetry span ratio also lose their meaning. Since these values may not be computed they cannot be substituted in the equations and these are therefore rendered valueless for computing the restraining moments, resulting from a couple on the span.

Eqs. 14, however, may be modified in such a manner as to be valid for a couple. Fig. 8 shows a clockwise couple, composed of two equal forces W, acting in opposite directions against a span of length L, and spaced at a distance $2\,b\,L$ apart. The center point between forces is placed at a distance $k\ L$ from the left support. The load nearest to a support, $-W$, is at a distance $(k - b)\ L$ from the left support, and the farther load is at a distance $(k + b)\ L$. The value $(k - b)$ may now be substituted for k in Eq. 14a. The resulting equation will give the bending effect of the left-hand load in terms of the location of the center point between couples. The value $k + b$ may next be substituted in the same equa-

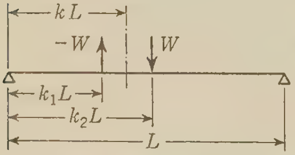


Fig. 8

tion with a second equation resulting which gives the independent effect of the right-hand load, in inducing bending moment at the left support.

If the first equation is now subtracted from the second (since the effect of the left-hand load is opposite to that of the right-hand load), the resulting equation will give the net effect of the couple in inducing bending moment at the left support.

When these steps are taken, remembering that the value T of the couple is $T = 2\,W\,b\,L$, there results:

$$M_L = -\,T\,A_L\,[2 - 6\,k + 3\,k^2 - C_R\,(1 - 3\,k^2) + b^2\,(1 + C_R)] \quad \text{..(36}a\text{)}$$

and

$$M_R = -\,T\,A_R\,[1 - 3\,k^2 - C_L\,(2 - 6\,k + 3\,k^2) - b^2\,(1 + C_L)] \quad \text{..(36}b\text{)}$$

The two equations, Eqs. 36*a* and 36*b*, give the restraining moments at the two supports of any restrained beam loaded with a couple placed at any point in the span. A negative sign of the bending moment here means the same thing it has meant in all the other equations for restraining moment—that the radius of curvature of the beam as bent at the support extends from the beam in the same direction as that force of the couple, which is considered positive, presses against the beam. The couple T is considered positive when it is clockwise from the viewpoint of the observer. Otherwise it is negative.

As a practical matter, a couple must always be applied to the beam with a finite distance between the forces which make it up. Actually the distance may be so small with respect to the length of the span to which the couple is applied that its effect is negligible, and so the couple is assumed applied at a point. In such a case the designer is tacitly assuming that the force W is infinite and that the value of b is zero, but that the product of infinity and zero is a positive constant, the couple. When it is desired to use Eqs. 36 in this manner, b becomes zero and the last term on the right side drops out. The equations are otherwise unchanged.

Care should be exercised, in using the equations, to remember that they give the restraining moments at the supports. When the couple moves up to the restrained support it is necessary to distinguish carefully between the restraining moment at the support and the bending moment out in the beam at the couple, a differential distance from the support. They are not the same.

Eqs. 36 may now be differentiated with respect to k, equated to zero, and criterions so developed for placing the couple in proper position for maximum bending moment at either support; thus, for maximum M_L—

$$k = \frac{1}{1 + C_R} \quad \text{........................(37)}$$

and, for maximum M_R—

$$k = \frac{C_L}{1 + C_L} \quad \text{........................(38)}$$

Eqs. 37 and 38 give the proper position of the couple in order to produce a truly mathematical maximum bending moment at the support. Eqs. 36, of course, are formulas of influence lines for the bending moments at the supports.

The mathematical maximums to which Eqs. 37 and 38 refer are at the points of zero slope of these curves. It is important to remember that these mathematical maximums are not always greatest bending moments. Greater bending moments than the mathematical maximums sometimes occur when the couple is placed at the supports. This happens in the case of a beam with fixed ends.

The positions of a couple, in order to produce zero bending moment at the support, may also be found readily by equating Eqs. 36 to zero, and solving for k. It is of interest to note that a couple occupies the same position in the span in order to produce zero bending moment at a support as would be occupied by a single concentrated load in order to produce maximum bending moment at the same support. Such positions accordingly may be read directly from Fig. 6.

It is also possible to develop equations for the bending moment out in the span, at differential distances from the forces that make up the couple. From these follow other interesting conclusions similar to the foregoing. Space limitations do not permit their inclusion herein.

10. Frame Characteristics and Their Computation

Equations have been developed for computing the bending moments due to loads placed on beams with restrained ends. Criteria have also been derived which place the loads properly to produce various kinds of maximum bending moments. The use of all of these requires a knowledge of the value of the transmission coefficients of the loaded span. If the effects of the loads are to be distributed to any frame, of which the loaded beam is a part, the designer must also know the transmission coefficients of the connecting beams and the moment splits at the joints. Often sidesway and lateral forces are involved which make it imperative to know the value of torque splits at the joints. All these terms—transmission coefficient, moment split, restraint index, and torque split—are included under the covering term, "frame characteristics."

Expressions have been developed for the value of the transmission coefficient and the moment split. Expeditious methods for computing the remaining characteristics, in order that the foregoing equations may become useful, are developed as follows.

The torque split is related very closely to the moment split, the only difference being the number of members that affect its value. The torque-split value for a member is that fraction by which any torque applied to a joint of a frame (artificially, as by a crank) must be multiplied in order to obtain the induced bending moment in that particular resisting member. The moment split does the same thing. The difference between the two lies in the fact that, in the case of the moment split, one of the radiating members of the joint is used as a vehicle for applying the torque, which is its own bending moment.

The member that applies torque, since it actuates the joint, cannot also participate as a resisting member. In the case of torque applied artifically to the joint, all the members act as resisting members. Thus, in the case of the moment split, all the members except one act as resisting or receiving members, and in the case of torque split all the members act as receiving members.

It is desirable, from the standpoint of ease of computing frame characteristics, that the expressions for torque split and for moment split be interrelated.

Writing R_B for the restraint index in bending for member 1 of a four-member joint, and R_T for the restraint index in torsion for the joint, from analogy with Eq. 9 for moment split, t_1 for torque split is:

$$t_1 = \frac{R_T}{i_1} = \frac{1}{i_1\left(\frac{1}{i_1} + \frac{1}{i_2} + \frac{1}{i_3} + \frac{1}{i_4}\right)} \qquad \text{(39)}$$

However, when member 1 is the transmitting member, its restraint index is:

$$R_B' = \frac{1}{\frac{1}{i_2} + \frac{1}{i_3} + \frac{1}{i_4}} \qquad \text{(40)}$$

Therefore:

$$t_1 = \frac{1}{1 + \frac{i_1}{R_{1B}}} \qquad \text{(41)}$$

Thus, when the restraint index has been computed for a member, the torque-split value for the same member may be found simply by dividing the stiffness index by the restraint index, adding 1 mentally, and taking the reciprocal of the result.

It is possible, of course, to compute frame characteristics for a small frame by using Eq. 5 directly for coefficients, Eq. 9 for moment splits, and Eq. 41 for torque splits. Such a procedure lacks order, and is conducive to errors. It has been found more satisfactory to use a tabular form of computation. In this form the equations are used, but they are used by rule, and more or less unconsciously.

Table 1 with its accompanying sketch will be used to illustrate this method of computation. It is designed with a view to keeping the computations in order, having them in such form that errors are reduced to a minimum, and rendering the procedure routine, as follows:

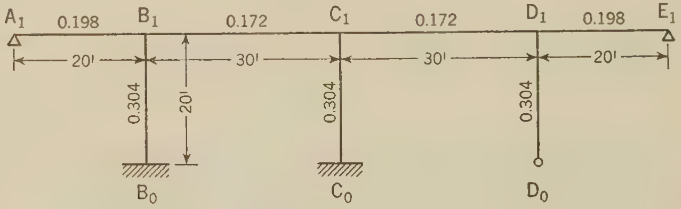
1. Enter all members of the successive joints in Col. 1, except joints at which the frame terminates at a fixed support, such as point B_0, or at a free support, such as point A_1. It is not necessary but is desirable to enter the members in the same order for all joints. Usually the member radiating downward is entered first, followed by successive members, clockwise around the joint in order. The letters that represent each member should read away from the joint, thus: C10 denotes the column $C_1 C_0$; 1CB denotes the member $C_1 B_1$; and 1CD denotes the member $C_1 D_1$—all originating at joint C_1. Since in general each member connects with two joints, each will appear twice in the table, once under each of its two connecting joints.

2. Enter the Q-values for each member in Col. 2. They appear at the centers of members in the diagram of Table 1. These Q-values, as previously defined, are the ratios of the span length divided by the moment of inertia. For some purposes (such as the determination of angular rotations at the joint, these ratios should be computed with length and moment of inertia of the section in the same units. For the purpose of computing frame characteristics,

however, they need not be in the same units. If all lengths are in feet and moments of inertia are in inches, the computed characteristics will be the same as if all were in the same units. The essential requirement is that all members and joints be treated on the same basis.

3. At the right side of each Q-value entered in Col. 2, Table 1, place the appropriate quantity $(2 - C)$ in so far as the C-values are known. The transmission coefficient to be used for a particular member is that value of C which transmits moment away from the joint in connection with which the member appears. In the case of the frame in Table 1 the coefficients of connecting members toward joints A_1, B_0, C_0, D_0, and E_1 are all known since these joints are either free or fixed. These known coefficients will be sufficient to solve the entire frame in this case. When sufficient coefficients are not so known, it is necessary to estimate some of them, as will be demonstrated subsequently.

TABLE 1.—COMPUTATION OF FRAME CHARACTERISTICS



Member	Stiffness Index		Re-straint index R	Transmission Coefficient C (Eq. 5)		Moment Split m (Eq. 9)			Torque split, t (Eq. 41)
	$Q(2-C)$ =	i		Value	Member				
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Joint B_1									
B10	0.304 (2 − 0.5)	0.456	0.1665	0.392	C_{B01}	(01)	0.387	0.465	0.268
1BA	0.198 (2 − 0)	0.396	0.1763	0.346	C_{1AB}	0.420	(AB)	0.535	0.308
1BC	0.172 (2 − 0.329)	0.287	0.212	0.310	C_{1CB}	0.580	0.613	(CB)	0.424
Joint C_1									
C10	0.304 (2 − 0.5)	0.456	0.146	0.403	C_{C01}	(01)	0.391	0.389	0.243
1CB	0.172 (2 − 0.310)	0.291	0.1785	0.329	C_{1BC}	0.502	(BC)	0.611	0.381
1CD	0.172 (2 − 0.294)	0.293	0.1775	0.330	C_{1DC}	0.498	0.609	(DC)	0.376
Joint D_1									
D10	0.304 (2 − 0)	0.608	0.1669	0.392	C_{D01}	(01)	0.395	0.321	0.215
1DC	0.172 (2 − 0.330)	0.288	0.240	0.294	C_{1CD}	0.579	(CD)	0.679	0.455
1DE	0.198 (2 − 0)	0.396	0.1955	0.334	C_{1ED}	0.421	0.605	(ED)	0.330

4. Compute the i-values for all members whose C-values away from the joint are known. In Table 1, this may be done at once for members B10 and 1BA for joint B_1; member C10 for joint C_1; and members D10 and 1DE for joint D_1 (Col. 3, Table 1).

5. At some of the joints all i-values except one should be known. For the frame in Table 1, this is true of joints B_1 and D_1. The R-value, for that member, whose i-value is missing at each joint, may now be computed, since it

depends upon all the i-values except its own. This is evident from an examination of Eq. 4 in Section 2. Eq. 40 may be written into an equivalent form:

$$R = \frac{i_2}{1 + \frac{i_2}{i_3}\left(1 + \frac{i_3}{i_4}\right)} \qquad (42)$$

The order in which the i-values appear in Eq. 42 is immaterial. It is convenient to use the lowest value in the numerator and the successively higher values in order. For three-member joints, such as the present example, the parentheses in the denominator of Eq. 42 drop out. Using joint B_1 as an example, the i-values for members B10 and 1BA are known. Under the number 0.396 on the slide rule the divisor 0.456 is set. To the quotient 0.869, unity is added mentally, and the divisor 1.869 is set under 0.396 to give the proper restraint index, 0.212 as entered in Col. 4, Table 1, for member 1BC.

6. For all members, except those at a two-member joint, the R-value is checked at once by computing the moment-split values for its member. In this case the R-value 0.212 is divided by the i-value of member B10 to give the moment-split value for this member as 0.465. It is next divided by 0.396 to yield the moment split for member 1BA as 0.535. These moment splits are for member 1CB, when it transmits moment toward the joint, as indicated in Col. 9. It is seen that the sum of 0.465 and 0.535 is unity. This would not have been true if there had been an error in computing the R-value from the two i-values.

7. The C-value for member 1CB may now be computed with confidence, since the R-value upon which it depends has been checked. The R-value computed for member 1BC was derived from i-values which involve coefficients away from joint B_1. The derivation of Eq. 5 will show that the transmission coefficient that results from this value of R is then toward the joint. The subscripts for C are accordingly reversed in entering them in Col. 6. The coefficient value 0.310 is computed by dividing the R-value 0.212 by the Q-value of the member, 0.172, adding 2 mentally, and taking the reciprocal of the result.

8. The value of C_{1CB} computed in step 7 is now substituted in its proper place under joint C_1. From this the i-value for the member 1CB may be computed. All the i-values at this joint are now known, except for member 1CD. Its R-value of 0.1775 may then be computed, and from it the moment splits, for the member, and the value of C_{1DC} just as was done in step 7. The new coefficient is substituted under joint D_1 and the operations repeated until all coefficients and moment splits ore obtained.

9. Torque splits may now be computed by Eq. 41, if the effects of sidesway and wind moments are to be analyzed. Otherwise, the torque-split values will not be needed. The value of 0.268 in Col. 10, Table 1, is found by taking the ratio (on the slide rule) of 0.456 to 0.1665, adding unity mentally, and taking the reciprocal of the result. The torque splits for the joint must add to unity just as the moment splits, which is a check on computations. Other torque splits are found in exactly the same manner, always dealing with the i-values and R-values in any single horizontal line.

11. THE ESTIMATION OF THE VALUES OF COEFFICIENTS

It frequently occurs that not enough coefficients are known in advance of computing the frame characteristics to permit the analyst to begin a tabular computation. Also it may be necessary occasionally to investigate the adequacy of a particular beam of a frame, without computing the characteristics of the entire frame. In such cases it is highly desirable to be able to make an accurate estimate of the value of the coefficients for a particular beam. This can be done readily with an accuracy which is ordinarily within 1%; and in any case the error is never great enough to affect the value of the coefficient beyond the third place to the right of the decimal point.

Eq. 5, for the value of the coefficient, may be expanded into the following form for a four-member joint:

$$C_1 = \cfrac{1}{2 + \cfrac{1}{Q_1}\,\cfrac{1}{\cfrac{1}{Q_2(2 - C_2)} + \cfrac{1}{Q_3(2 - C_3)} + \cfrac{1}{Q_4(2 - C_4)}}} \quad \ldots\ldots (43a)$$

The expressions for the *i*-values of the restraining members all contain the quantity $2 - C$. It is true that these C-values are all different. It is readily conceivable, however, that there is some kind of an equivalent C which, if inserted uniformly into the three parentheses in the denominator of Eq. 43*a*, would give the same result for the coefficient being computed by the equation as if the exact C-value for each member had been used. With this concept in mind, this equivalent coefficient may be called C_e, and Eq. 43*a* can be written:

$$C_1 = \cfrac{1}{2 + \cfrac{1}{Q_1}\,\cfrac{2 - C_e}{\cfrac{1}{Q_2} + \cfrac{1}{Q_3} + \cfrac{1}{Q_4}}} = \cfrac{1}{2 + \cfrac{1}{Q_1}\,\cfrac{Q_2(2 - C_e)}{1 + \cfrac{Q_2}{Q_3}\left(1 + \cfrac{Q_3}{Q_4}\right)}} \quad \ldots\ldots (43b)$$

Either part of Eq. 43*b* may be used for computation. The right-hand part has been arranged for convenient use with the slide rule, as was done for computing restraint indexes in Eq. 42, step 5, Section 10.

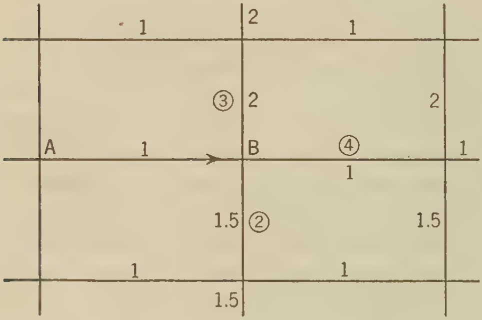


FIG. 9

Let it be required to estimate the value of the coefficient toward the right, of member AB, of the beam shown in Fig. 9. The Q-values of the beam itself, and of the restraining beams on the right, are shown by the numbers, 1, 1.5, 2, etc., in the centers of the spans. The restraining members have been given the numbers shown in circles.

For estimating coefficients in restraining members away from joint B, certain principles should be kept in mind:

1. A stiff member connecting into a limber member will have a low coefficient;
2. Many members, acting as a restraint, tend to raise the coefficient;
3. A member, restrained by a single member of its own stiffness, has a coefficient of approximately 0.29; and
4. The maximum value of the coefficient is 0.5 and the minimum value is 0.0.

By inspection note that restraining member 2 connects into members, at its end away from joint B, two of which are stiffer than, and one as stiff as, itself. One may judge its coefficient away from joint B to be approximately 0.4. Likewise, the coefficient for member 3 is perhaps about the same, and for member 1 about 0.3. Estimating that C_e is 0.35 and inserting it in Eq. 43*b*, in order to obtain a close approximation of C_{AB}:

$$C_{AB} = \frac{1}{2 + \frac{1}{1}\frac{1\,(2 - 0.35)}{1 + \frac{1}{1.5}\left(1 + \frac{1.5}{2}\right)}} = \frac{1}{2 + 1.65 \times 0.461} = 0.363.$$

This estimate will be accurate within the third decimal place. It can be seen readily that a considerable error may be made in C_e without great effect on C_{AB}. Thus, if the analyst had assumed C_e to be 0.4 instead of 0.35, the value of C_{AB} would be 0.366, and, if he had assumed 0.3 instead of 0.35, C_{AB} would be 0.359.

Such estimated coefficients may be used with entire confidence in computing frame characteristics. The exact value of the coefficient will be obtained, in the course of the computation, and finally may be used instead of the estimated value. No further correction need be made for errors in estimation, however, unless the exact value should differ from the estimated value beyond the third decimal place. In the writer's experience this does not occur.

These estimated coefficients are sufficiently accurate to be used in connection with the equations for restraining moment, in order to make spot checks of particular beams under special loads.

12. The Sense of the Bending Moment

It is obvious from the frame characteristics of Table 1 that bending moment may be distributed to the frame at a single operation. Before this is done, it is necessary to define the sense of the bending moment in such a manner as to avoid confusion. The concept of positive and negative bending moment has been abandoned for the purposes of making moment distribution, for the following reason: In courses in mechanics of materials, bending moment is defined as positive for horizontal beams when the beam is concave upward and negative when the beam is concave downward. These bending moments are also defined as positive when the algebraic sum of moments of forces on the left of a section is clockwise about the section and negative if counterclockwise, the definitions being reversed when forces on the right are considered.

Such definitions serve well for horizontal beams, but when the beams radiate in all directions, as they do in a frame, these definitions become confusing. Even though a proper distinction of sense of bending moment is developed for the frame in terms of plus signs and minus signs, such a distinction is likely to be confused with the old concept of plus and minus bending moment, which was so well learned in elementary studies.

For these reasons bending moment is designated as "normal" or "reverse." "Normal bending moment" is defined as follows (see Fig. 10(*b*)):

> I. Bending moment is said to be normal at the end of a beam if, when the beam is cut from its frame and considered as a free body, a clockwise torque must be applied at the end to hold the beam in the same condition of stress to which it was subjected in the frame.

"Reverse bending moment" is defined as follows (see Fig. 10(*a*)):

> II. Bending moment is said to be "reverse" at the end of a beam if, when the beam is cut from its frame and considered as a free body, a counterclockwise torque must be applied to the end in order to hold the beam in the same condition of stress to which it was subjected in the frame.

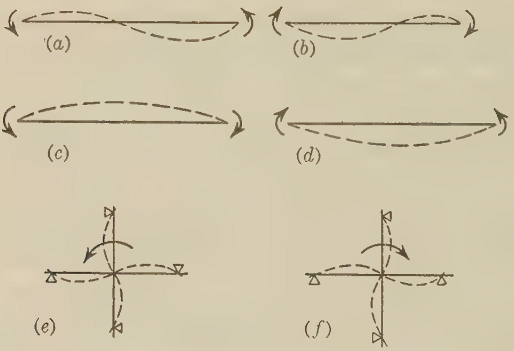


FIG. 10

Both definitions are consistent without regard to the end of the beam to which the torque is applied. Figs. 10(*c*) and Fig. 10(*d*) show bending moments of opposite senses at the two ends of the two members. These senses of the bending moment may be defined equally well in terms of the joint from which the member radiates. Thus, for normal bending moment (see Fig. 10(*e*)):

> III. Bending moment is said to be normal in the end of a member when the member is so flexed at that end as to resist the effect of a clockwise torque, artificially applied to its connecting joint.

For reverse bending moment (see Fig. 10(*f*)):

> IV. Bending moment is said to be reverse at the end of a member when the member is so flexed at that end as to resist the effect of a counterclockwise torque artificially applied to the joint to which the member end connects.

13. Distribution of Bending Moment to the Frame

Bending moment may be distributed to the frame at a single operation by the following rules:

1. Determine the sense of the bending moment at the ends of the beam of origin according to definitions I and II or III and IV.
2. Multiply the bending moment at the end of the beam of origin by the moment-split values for each of the receiving members, the member of origin being considered the transmitting member, and so obtain the part of the bending moment induced in each. Change the sense of the bending moment as it passes through the joint at the end of the member of origin and enter each portion of the moment for the proper receiving member.
3. Each of the bending moments, as obtained in rule 2, is now multiplied by that transmission coefficient, of the member in which it occurs, which transmits moment away from the joint, in order to obtain the induced bending moment at the opposite end of the member. It is entered here without change in sense.
4. At each succeeding joint proceed as in rule 2 and repeat rule 3 after the moment passes through the new joint.
5. The distribution of the moment is completed under the following conditions:

   (*a*) When it reduces to a negligible portion of its value at the origin;
   (*b*) When it arrives at a fixed or free support; and
   (*c*) When, as in most cases, it is carried only two joints beyond the joint of origin.

6. In order that all joints may remain balanced, distribution should cease just after the moment has passed through the last joint, not as it arrives.

An example will serve to illustrate the application of rules 1 to 6 for the distribution of bending moment, as well as to sum up the methods of computing bending moments in restrained beams. Let it be required to place two moving loads of the highway H-10 type on the two spans 1BC and 1CD in Table 1 each in position for maximum bending moment at joint C_1. Let it then be required to compute the two maximum bending moments and the simultaneous bending moments at the opposite ends of the respective beams, and to distribute these four bending moments to the frame. The loading type is shown in Fig. 11, with the center of gravity located. The computation follows: $U = 3 \times 11.2^2 + 12 \times 2.8^2 = 376.3 + 94.1 = 470.4$; $S = 376.3 \times (-11.2) + 94.1\,(2.8) = -4{,}214 + 263 = -3{,}951$; λ^2 (either span) $= \dfrac{470.4}{15 \times 900} = 0.0348$; $\lambda = 0.187$; and $\epsilon^3 = \pm \dfrac{3{,}951}{15 \times 27{,}000} = \pm 0.00975$ (sign depends on orientation).

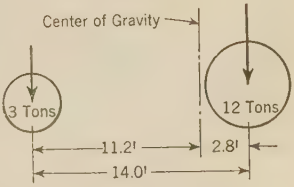


Fig. 11

From Fig. 6, using the coefficients from Table 1 (see also Fig. 12), the maximum bending at joint C_1 is: In span 1BC, $k_R = 0.580$; and in span 1DC,

$\bar{k}_L = 0.422$. Entering Fig. 5, with these values and the value of λ:

Symbol	Span 1BC	Span 1DC
B	0.323	0.302
D	0.302	0.323
A_L	$\dfrac{0.310}{1 - 0.310 \times 0.329} = 0.345$	$\dfrac{0.330}{1 - 0.330 \times 0.294} = 0.365$
A_R	$\dfrac{0.329}{0.898} = 0.366$	$\dfrac{0.294}{0.903} = 0.326$

The two loads have been shown, properly located with respect to the two joints B_1 and C_1, in the respective spans 1BC and 1CD in Fig. 12.

Next, with Eqs. 27, compute the bending moments induced at the right and left of the loaded spans: In span 1BC—

$$M_{1BC} = -15 \times 30 \times 0.345\,[0.302 - 0.0098 - 0.329\,(0.323 + 0.0098)]$$
$$= -28.4 \text{ ft-tons}$$
$$M_{1CB} = -15 \times 30 \times 0.366\,[0.323 + 0.0098 - 0.310\,(0.302 - 0.0098)]$$
$$= -39.9 \text{ ft-tons (due to placing for maximum moment)}$$

and in span 1CD—

$$M_{1CD} = -15 \times 30 \times 0.365\,[0.323 + 0.0098 - 0.294\,(0.302 - 0.0098)]$$
$$= -40.5 \text{ ft-tons (due to placing for maximum moment)}$$
$$M_{1DC} = -15 \times 30 \times 0.326\,[0.302 - 0.0098 - 0.330\,(0.323 + 0.0098)]$$
$$= -26.8 \text{ ft-tons.}$$

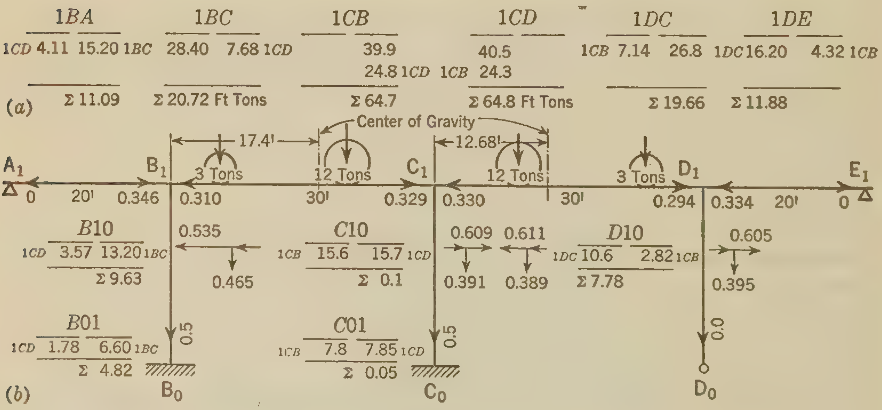

1BA
1BC
1CB
1CD
1DC
1DE
1CD 4.11 15.20 1BC
28.40 7.68 1CD
39.9
24.8 1CD
40.5
1CB 24.3
1CB 7.14 26.8 1DC
16.20 4.32 1CB
(a)
Σ 11.09
Σ 20.72 Ft Tons
Σ 64.7
Σ 64.8 Ft Tons
Σ 19.66
Σ 11.88
Center of Gravity
17.4'
12.68'
A_1
B_1
3 Tons
12 Tons
C_1
12 Tons
3 Tons
D_1
E_1
0
20'
0.346
0.310
30'
0.329
0.330
30'
0.294
0.334
20'
0
B10
1CD 3.57 13.20 1BC
Σ 9.63
0.535
0.465
C10
1CB 15.6 15.7 1CD
Σ 0.1
0.609 0.611
0.391 0.389
D10
1DC 10.6 2.82 1CB
Σ 7.78
0.605
0.395
0.5
0.5
0.0
B01
1CD 1.78 6.60 1BC
Σ 4.82
C01
1CB 7.8 7.85 1CD
Σ 0.05
(b)
B_0
C_0
D_0

Fig. 12

These bending moments may now be entered in Fig. 12, for distribution to the frame. For this purpose it is conventional to place normal bending moment on the right and reverse bending moment on the left. Two columns will be seen in Fig. 12 for each member end. For horizontal beams, under gravity loading, the bending moment is always reverse on the left of the loaded span and normal on the right. The bending moments computed will be seen in Fig. 12 according to this classification.

The moment-split values which are required for the purpose have been copied from Table 1, and are shown adjacent to the joints of the loaded beams, with arrows indicating the direction of moment flow.

As an example, consider the moment 28.4 ft-tons, at the left end of span 1BC. First multiply it by the moment-split value 0.465 to obtain 13.20 ft-tons as the moment at B10. This is entered as normal moment since it has passed through a joint. The same moment multiplied by 0.535 gives the value 15.20 in beam 1BA which is also entered as normal moment. The 13.20 is now multiplied by the coefficient 0.5 of its member away from B_1, to give 6.60 also normal at the support end B_0.

In a similar manner, 39.9 is multiplied by 0.609 and by 0.391 to give the reverse moments it induces in members 1CD and C10. These are entered, as are the values obtained from the 40.5 ft-ton moment times its appropriate splits, and those from the 26.8 ft-ton moment. It should be particularly noted that, in the beginning of its distribution, moment flows only outward from the loaded span of origin, and does not cross that span.

In placing the distributed moments, it will be seen that the point of origin has been noted beside each. This is only precautionary to provide for rechecking in case of possible necessity.

When the moments are all distributed, the algebraic sum is taken, at each member end, and noted by the summation sign below as the final bending moment at that point. For instance, the 20.72 ft-tons of reverse moment at member 1BC is equal to 28.4 − 7.68. The algebraic sum of all the total moments at any joint must add to zero. At joint B_1 the sum of the two normal moments 9.63 and 11.09 is equal to the reverse moment in member 1BC of 20.72. This is a check on the net bending moments as indicated by the summation signs.

The method outlined is entirely satisfactory for small frames. When large frames, involving many members and joints, are to be solved, a much more comprehensive method is used. This method involves a moment-distribution chart for each joint. Its bookkeeping arrangement is so designed that there is a minimum chance of error. One distribution also is sufficient for all purposes, including live-load and dead-load moment distribution, wind moments, and sidesway, as well as loading for maximum moment at every point in the frame. It is too lengthy to include in the present paper.

14. Coordination with the Method of Successive Approximations

The method of solving rigid frames by successive approximations, as proposed by Hardy Cross,[3] M. Am. Soc. C. E., starts, as does the slope deflection method, with fixed-end moments for loaded members. As a result there can be little coordination of the methods of computing moments in loaded members. The Cross method of distributing moments to a frame can be shown to be a special case of the method here outlined. That method consists essentially of the distribution, at a single joint, of an opposing torque equal to the unbalance at that particular joint; the outer ends of connecting members being considered as fixed, or in case they connect into a pinned support, as freely supported. This partial distribution is made successively, a single joint at a time, until error diminishes to a negligible amount.

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[3] "Analysis of Continuous Frames by Distributing Fixed-End Moments," by Hardy Cross, *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 1.

The Cross method then involves a torque split for a joint, whose members connect into either fixed or free supports at their outer ends, and transmission coefficients toward fixed ends or free ends, which were previously pointed out to be either 0.5 or 0. It will be remembered that, in the Cross method, the value of I/L is termed K, which is the reciprocal of the Q-value used in this paper.

The expression for the torque split, Eq. 39, Section 10, may be expanded as follows, for a four-member joint—

$$t_1 = \frac{1}{Q_1(2 - C_1)} \times \frac{1}{\frac{1}{Q_1(2 - C_1)} + \frac{1}{Q_2(2 - C_2)} + \frac{1}{Q_3(2 - C_3)} + \frac{1}{Q_4(2 - C_4)}} \quad \ldots (44a)$$

Assume that members 1, 2, and 3 connect to fixed or locked joints at their outer ends and that member 4 connects to a pinned or free support, the transmission coefficients or carry-over factors for the first three members will be 0.5 and for the fourth 0.0. Inserting these values in Eq. 44*a* and appropriate values of K for the reciprocals of $Q \left(= \frac{1}{L/I} = K \right)$: $t_1 = \frac{K_1}{1.5} \times \frac{1}{\frac{K_1}{1.5} + \frac{K_2}{1.5} + \frac{K_3}{1.5} + \frac{K_4}{2.0}}$; or:

$$t_1 = \frac{K_1}{K_1 + K_2 + K_3 + 0.75\,K_4} \quad \ldots\ldots\ldots\ldots\ldots (44b)$$

Eq. 44*b* will be recognized as the well-known Cross expression for what has been called torque-split value in this paper. The equation is written to apply to the particular joint under discussion, with three members fixed at the outer ends, and the fourth freely supported.

15. Other Applications of the Transmission Coefficient

The uses of the concept of the transmission coefficient, which have been outlined in this paper, represent only a very limited number of the purposes to which it has been adapted. It may be of interest to list some of its other applications.[3,4] It has been adapted to the placing of moving loads for greatest

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[3] "The Solution of Statically Indeterminate Structures," by R. C. Brumfield, 1933 (printed in mimeograph form by The Cooper Union, New York, N. Y.

[4] "Restrained Beams and Rigid Frames," by R. C. Brumfield, 1941 (revision and expansion of "The Solution of Statically Indeterminate Structures"[3]; printed in mimeograph form by The Cooper Union, New York, N. Y.).

shear both at the support and in the span, just as was done for maximum bending moment. An equation for elastic deformation has been developed from its use, which is perfectly general and applies to the entire beam, irrespective of the intervention of concentrated loads and supports. This equation has certain important uses. The transmission coefficient has been developed for straight beams of variable section, with restrained ends. For such beams its values theoretically may become infinite, but actually do not ordinarily exceed unity. It has been applied to curved beams, in which the coefficient is often negative. Curved beams, in fact, are reduced to the same status as straight beams, the formulas for the former reducing to those of the latter when curvature disappears. Moment splits, torque splits, and the effects of a movement of the supports may be computed for curved beams. The concept enables the beam which is a member of a noncoplanar frame to be solved by the formulas outlined herein. The number of factors that affect coefficients of such a beam is greater than that for coplanar beams; but, once the coefficients are computed, a beam in a noncoplanar frame may be loaded in quite the same manner as outlined for a coplanar straight beam, and moments computed by the same formulas.

16. ACKNOWLEDGMENT

The methods outlined, as well as most of those indicated, have been thoroughly tested in professional engineering practice, in and around the City of New York, N. Y. In this process a great many valuable suggestions have been received, for which acknowledgment is made.

The writer also wishes to express his thanks to Lawrence Perez, Assoc. M. Am. Soc. C. E., for carefully reading this paper.

DISCUSSION

R. W. Stewart,[5] M. Am. Soc. C. E.—The "transmission coefficient" and "restraint index" used in this paper are special beam and structure constants of a rather complex nature. The "moment of inertia of the loading group," "radius of gyration of a series of loads with fixed spacing," the "span distribution ratio" of the loads; and the "modulus of dissymmetry," "radius of dissymmetry," and "dissymmetry span ratio" constitute quite an array of rather complex functions all relating to the loads.

Using the theorem of three moments as the starting point and deriving the transmission coefficients by the use of Eqs. 1, 2, 3, and 4 involve algebraic work that can be avoided, and which, unless extensively supplemented, is not applicable to structures having members of variable section.

Why not make direct use of the basic constants of flexure[6,7] to obtain the transmission coefficients? This will simplify the analysis, increase the yield of valuable coefficients, and obtain an automatic check of the computations. Fig. 13 will then replace Table 1.

Fig. 13(*a*) shows the torque (998.4) necessary at joint B_1 to produce unit rotation at end E_1. All moments at the ends of members due to this torque are also shown. This enables not only all transmission coefficients for individual members in a direction away from point B_1 to be taken from the figure but it also yields the transmission coefficient from point B_1 to any other joint in the frame or from any joint to any other joint which is more remote from point B_1. The transmission coefficient across an individual member is obtained by dividing the moment at the end farther from point B_1 by the moment at the end nearer point B_1. In all cases these agree, to the last decimal place, with the values in Table 1. The transmission coefficient from any joint to any other joint which is more remote from point B_1 is obtained by dividing the moment at the more remote joint by the moment at the less remote joint. For example, the transmission coefficient from the left end of member B_1C_1 to the top of column D_1D_0 is $\frac{9.87}{424.0} = 0.0233$. To obtain the transmission coefficients in the opposite direction a computation is made (Fig. 13(*b*)) beginning with a rotation of unity at end A_1 and resulting in a torque of 998.4 at point D_1. This agreement with the torque obtained in computing Fig. 13 is an automatic check by Maxwell's theorem of reciprocal deflections (angular) and indicates that the computations for Figs. 13(*a*) and 13(*b*) are free from error. Fig. 13(*c*) shows the effect of a torque of 225.1 at point C_1. The right half of Fig. 13(*c*), including the center pier, is copied from Fig. 13(*a*). The left half is obtained by one setting of a slide rule. The torque of 225.1 is the sum of the resisting moments about joint C_1.

The "torque split" and the "moment split" at each joint can also be taken from Fig. 13. These agree with the author's values. A similar quick computa-

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[5] Bridge Engr., City of Los Angeles, Los Angeles, Calif.

[6] "Relative Flexure Factors for Analyzing Continuous Structures," by Ralph W. Stewart, *Transactions*, Am. Soc. C. E., Vol. 104 (1939), p. 521.

[7] "The Mechanics of Creep for Structural Analysis," by Joseph Marin, *ibid.*, Vol. 108 (1943), p. 477.

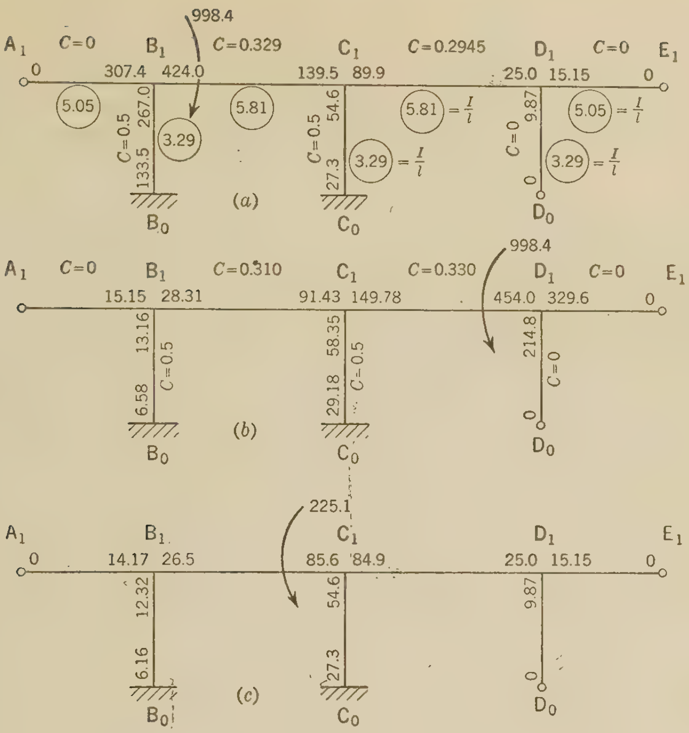
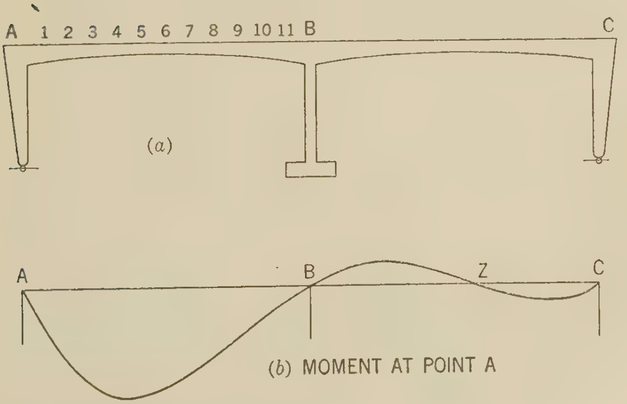
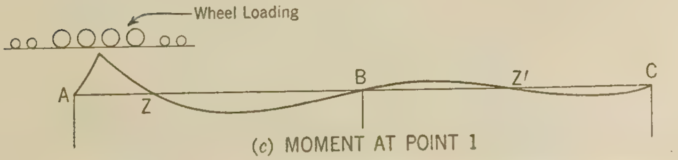

998.4
A₁
C=0
B₁
C=0.329
C₁
C=0.2945
D₁
C=0
E₁
0
307.4
424.0
139.5
89.9
25.0
15.15
0
5.05
C=0.5
267.0
133.5
3.29
5.81
C=0.5
54.6
27.3
3.29 = I/l
5.81 = I/l
C=0
9.87
0
3.29 = I/l
5.05 = I/l
(a)
B₀
C₀
D₀
998.4
A₁
C=0
B₁
C=0.310
C₁
C=0.330
D₁
C=0
E₁
15.15
28.31
91.43
149.78
454.0
329.6
0
13.16
6.58
C=0.5
58.35
29.18
C=0.5
214.8
0
C=0
(b)
B₀
C₀
D₀
225.1
A₁
B₁
C₁
D₁
E₁
0
14.17
26.5
85.6
84.9
25.0
15.15
0
12.32
6.16
54.6
27.3
9.87
0
(c)
B₀
C₀
D₀

FIG. 13


A
1
2
3
4
5
6
7
8
9
10
11
B
C
(a)
A
B
Z
C

(b) MOMENT AT POINT A


Wheel Loading
A
Z
B
Z′
C

(c) MOMENT AT POINT 1

FIG. 14.—INFLUENCE LINES FOR MOMENT, INCLUDING THE EFFECT OF SIDESWAY

tion of transmission coefficients and moment and torque splits for a structure involving variable moment of inertia has been published.[8]

The author's computation for placing two wheel-load assemblies on the structure illustrated by Fig. 12, so as to obtain the maximum moment over the center pier, is of mathematical interest. However, it neglects the effect of sidesway. For the problem illustrated in Fig. 12, the effect of sidesway is so small, due to the nearly symmetrical condition of both the structure and the loads about the joint of critical moment, that it can be neglected. If the maximum moment at the top of pier B_1B_0 were to be computed, it would be necessary to include the effect of sidesway in order to justify the refinement of finding the exact position of the load for the maximum moment.

The manner in which sidesway can affect the position of loads for maximum moment is illustrated in Fig. 14, which shows two of the complete set of influence lines that were computed for the actual design of a two-span rigid frame bridge of the type shown. In Fig. 14(*b*), the effect of the sidesway is to divide the right span at point Z into two sections in one of which the moment at point A produced by the load is positive and in the other negative. In order to obtain the maximum moment at point A not only span AB but also fractional span ZC would have to have loads properly placed therein. The writer does not find in the paper any means of locating the Z-points (intermediate points of zero influence) in a structure, or of determining whether such points exist. Fig. 14(*c*), which is the influence line for point 1 in Fig. 14(*a*), illustrates another complication which will occur if railway loading is used. The distance AZ is too short to accommodate the wheel base of a locomotive and tender, so that for maximum positive moment at point 1 some of the wheels of the assembly will be at the right of point Z no matter which way the locomotive is faced. A still further contingency which is not infrequent is the reversal of the direction of the bending moment in a column due to sidesway. How could the author's method of procedure be applied to such a condition?

For a good design of a concrete rigid frame bridge, in which both the maximum positive and maximum negative moments must be computed from point to point in order to properly locate ends of reinforcing bars, the writer sees no escape from the computation of complete influence lines. This can be done in a very moderate length of time by a computer who has a thorough knowledge of the work. There are not very many engineers who would approve constructing the $60,000 structure illustrated in Fig. 14 to a poorly detailed design rather than a well detailed design in order to save from two to three man-days of computing work.

Fortunately the operation of placing trucks having variable spacing on a structure is not required by the design specifications in general use. The truck trains which have been heretofore specified by the American Association of State Highway Officials all have fixed distances between trucks, and the trucks all face in the same direction. At present, either a single truck wheel assembly or a single concentration superimposed on a uniform load is specified. This eliminates the necessity for positioning independent wheel assemblies.

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[8] "Relative Flexure Factors for Analyzing Continuous Structures," by Ralph W. Stewart, *Transactions*, Am. Soc. C. E., Vol. 104 (1939), p. 527.

To give the author's method practical value it will be necessary to demonstrate its extension to variable moment of inertia and to the various contingencies due to sidesway introducing Z-points and reversing the direction of bending moment in a member.

NATHAN D. BRODKIN,[9] ASSOC. M. AM. SOC. C. E.—In this paper Professor Brumfield presents not only a new method for determining the position of moving loads to produce maximum moments in continuous beams, but—what seems of greater importance—he has developed a new and comparatively simple method for analyzing continuous frames so that moments may be distributed in one operation. The method described by Professor Brumfield finds its basis in certain structural concepts which, although well known, have seldom been as logically or lucidly integrated into a practical system of analysis. Utilizing these concepts, the author has developed a method of solution based on determining the exact values of the carry-over factors of the members of a structure.

In a rigid-frame structure, the elastic restraining effects of the various members are interrelated and the constants of the structure, dependent only on this interrelationship, govern the law of distribution of moments throughout the structure. The author terms these constants "frame characteristics" and gives simple equations and a procedure for finding and tabulating them. Distribution of moments from an originating member to the remaining members of the structure then becomes possible in a single arithmetical operation.

The author states in Section 4 that it is "possible to compute, accurately and without the concept of fixed-end moments, the induced restraining moments due to loads placed on the span." This is quite true. However, the writer would like to indicate certain simplifications that can be made in Eqs. 27 for use with static loading, such as is usually encountered in the design of building frames, through the introduction into these formulas of the fixed-end moment concept.

From an inspection of Eqs. 26 it is seen that the restraining conditions of the beam ends have no effect on the values of B and D, these values remaining constant for any given system of beam loading. Any known condition of restraint may therefore be applied to the ends of the beam, and the values of B and D then may be expressed in terms of such restraint. For convenience, a condition of total restraint will be applied. Under such a condition, $C_L = C_R = 1/2$, and the values of M_R and M_L in Eqs. 27 become the well-known fixed-end moments M_{fR} and M_{fL}, respectively. The values of A_R and A_L in Eqs. 28 become 2/3, and Eqs. 27 may then be written in the following form:

$$M_{fR} = -W\,L \times \tfrac{2}{3} \times \left[B - \epsilon^3 - \tfrac{1}{2}\,(D + \epsilon^3)\right] = -\frac{W\,L}{3}\,(2\,B - D - 3\,\epsilon^3) \quad \text{..(45a)}$$

and

$$M_{fL} = -W\,L \times \tfrac{2}{3} \times \left[D + \epsilon^3 - \tfrac{1}{2}\,(B - \epsilon^3)\right] = -\frac{W\,L}{3}\,(2\,D - B + 3\,\epsilon^3) \quad \text{..(45b)}$$

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[9] Civ. Engr., Board of Transportation, New York, N. Y.

Solving Eqs. 45 simultaneously for B and D yields the following:

$$B = -\frac{2\,M_{fR} + M_{fL}}{W\,L} + \epsilon^3 \qquad \text{(46a)}$$

and

$$D = -\frac{2\,M_{fL} + M_{fR}}{W\,L} - \epsilon^3 \qquad \text{(46b)}$$

Eqs. 46 express the values of B and D in terms of the fixed-end moments and the dissymmetry span ratio. When the loading is symmetrically placed on the beam the value of ϵ becomes zero. Substituting the values of B and D from Eqs. 46 into Eqs. 27 and simplifying, expressions for M_R and M_L in terms of the fixed-end moments M_f, and transmission coefficients are obtained as follows:

$$M_R = A_R\,[(2\,M_{fR} + M_{fL}) - C_L\,(2\,M_{fL} + M_{fR})] \qquad \text{(47a)}$$

and

$$M_L = A_L\,[(2\,M_{fL} + M_{fR}) - C_R\,(2\,M_{fR} + M_{fL})] \qquad \text{(47b)}$$

When one end of the beam is hinged and one end is restrained, Eqs. 47 reduce to either

$$M_R = 2\,M_{fRH}\,C_R \qquad \text{(48a)}$$

or

$$M_L = 2\,M_{fLH}\,C_L \qquad \text{(48b)}$$

in which M_{fRH} and M_{fLH} in Eqs. 48 are the fixed-end moments to be applied to the restrained end, the other end being hinged.

If the loading is symmetrically placed on the span and both ends are restrained, M_{fR} and M_{fL} in Eqs. 47 are equal. Substituting the single value M_f for M_{fR} and M_{fL}, Eqs. 47 become

$$M_R = 3\,A_R\,M_f\,(1 - C_L) \qquad \text{(49a)}$$

and

$$M_L = 3\,A_L\,M_f\,(1 - C_R) \qquad \text{(49b)}$$

It should be noted that the values of ϵ and λ do not enter into the solution of Eqs. 47 to 49 for M_R and M_L and need not be computed for static loading. They are, nonetheless, inherent in the values of the fixed-end moments M_{fR} and M_{fL}.

It is not intended that Eqs. 47 to 49 be used in place of Eqs. 27 when moving loads are under consideration. In such cases the values of ϵ and λ must be determined for finding maximum moments; but where the values of the fixed-end moments for static loads on building frames have been previously tabulated, Eqs. 47 to 49 afford a quick means for determining M_R and M_L. Such tabulations for the usual type of loading normally occurring in buildings may be found in various forms in texts and pamphlets.[10,11]

As an example of the application of Eqs. 47, the bending moments at the supports of beam B_1C_1 in the author's illustrative frame problem, with loading

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[10] "Continuity in Concrete Building Frames," 3d Ed., Portland Cement Assn., pp. 7–13.

[11] "Analysis of Rigid Frames," by A. Amerikian, U. S. Government Printing Office, Washington, D. C., 1942, pp. 389–392.

as shown in Fig. 12, will be computed. Beam B_1C_1 has a span of 30 ft and concentrations of 3 tons and 12 tons located 6.2 ft and 20.2 ft, respectively, from joint B_1, the transmission coefficients being $C_L = 0.310$ and $C_R = 0.329$. For this case, $A_L = 0.345$ and $A_R = 0.366$. For a single concentrated load,

$$M_{fL} = -\frac{P\,a\,b^2}{L^2} \qquad (50a)$$

and

$$M_{fR} = -\frac{P\,a^2\,b}{L^2} \qquad (50b)$$

in which a is the distance from the left support to the load. With these values inserted in Eqs. 50, the fixed-end moments are as shown in Table 2. Substituting in Eqs. 47:

TABLE 2.—EXAMPLE OF THE APPLICATION OF EQS. 47

Load P (tons)	Position of Load, in Feet, from Either End		Fixed-End Moments, in Foot-Tons	
	a	b	M_{fL}	M_{fR}
3	6.2	23.8	−11.70	−3.05
12	20.2	9.8	−25.84	−53.30
Total	....	....	−37.54	−56.35

$$M_{1CB} = 0.366\,[(-112.70 - 37.54) - 0.310\,(-75.08 - 56.35)] = -40.1 \text{ ft-tons}$$

and

$$M_{1BC} = 0.345\,[(-75.08 - 56.35) - 0.329\,(-112.70 - 37.54)] = -28.3 \text{ ft-tons}$$

These values check with the author's results within the limits of slide-rule accuracy.

It is often necessary and convenient to know the deflection angles at the supports of restrained members. Expressions for the end slopes of a beam may also be derived in terms of the concepts introduced by Professor Brumfield.

From the mechanics of materials the end slopes θ_L and θ_R due to a single concentration P are given by the following equations:

$$\theta_L = \frac{Q}{6\,E}\,(2\,M_L + M_R) + \frac{P\,Q\,L}{6\,E}\,(2\,k - 3\,k^2 + k^3) \qquad (51a)$$

and

$$\theta_R = \frac{Q}{6\,E}\,(2\,M_R + M_L) + \frac{P\,Q\,L}{6\,E}\,(k - k^3) \qquad (51b)$$

When the rotation of the tangent to the elastic curve with respect to the normal position of the beam is clockwise at the left support and counterclockwise at the right support, the signs of θ_L and θ_R are positive.

Where combination loading is under consideration the quantity $2\,k - 3\,k^2 + k^3$ in Eq. 51a may by analogy be replaced by $\bar{k}^3 - 3\,\bar{k}^2 + \bar{k}\,(2 + 3\,\lambda^2)$

$- 3\,\lambda^2 + \epsilon^3$; or $D + \epsilon^3$. Similarly $k - k^3$ in Eq. 51*b* may be replaced by $-k^3 + k\,(1 - 3\,\lambda^2) - \epsilon^3$; or $B - \epsilon^3$. Eqs. 51 may now be written for combination loading as follows:

$$\theta_L = \frac{Q}{6\,E}\,(2\,M_L + M_R) + \frac{\Sigma P\,Q\,L}{6\,E}\,(D + \epsilon^3) \ldots\ldots\ldots\ldots (52a)$$

and

$$\theta_R = \frac{Q}{6\,E}\,(2\,M_R + M_L) + \frac{\Sigma P\,Q\,L}{6\,E}\,(B - \epsilon^3) \ldots\ldots\ldots\ldots (52b)$$

Substituting the values of M_R and M_L from Eqs. 27 into Eqs. 52, and simplifying, yield the following expressions for the slopes:

$$\theta_L = \frac{\Sigma P\,Q\,L}{6\,E}\,\{D\,(1 - 2\,A_L + A_R\,C_L) + B\,(2\,A_L\,C_R - A_R) \\ + \epsilon^3\,[A_R\,(1 + C_L) - A_L\,(2 + 2\,C_R) + 1]) \ldots\ldots\ldots\ldots (53a)$$

and

$$\theta_R = \frac{\Sigma P\,Q\,L}{6\,E}\,\{B\,(1 - 2\,A_R + A_L\,C_R) + D\,(2\,A_R\,C_L - A_L) \\ + \epsilon^3\,[-\,A_L\,(1 + C_R) + A_R\,(2 + 2\,C_L) - 1]\} \ldots\ldots\ldots\ldots (53b)$$

Eqs. 53 may be written in a simpler form by substituting the values of B and D previously derived in Eqs. 46, yielding the following expressions:

$$\theta_L = \frac{Q}{6\,E}\,[(2\,A_L - A_R\,C_L - 1)\,(2\,M_{fL} + M_{fR}) \\ + (A_R - 2\,C_R\,A_L)\,(2\,M_{fR} + M_{fL})] \ldots\ldots\ldots\ldots (54a)$$

and

$$\theta_R = \frac{Q}{6\,E}\,[(2\,A_R - A_L\,C_R - 1)\,(2\,M_{fR} + M_{fL}) \\ + (A_L - 2\,C_L\,A_R)\,(2\,M_{fL} + M_{fR})] \ldots\ldots\ldots\ldots (54b)$$

The writer has applied Professor Brumfield's system of analysis in practice to the solution of problems involving restrained beams, frames, and arches of both constant section and varying section. In most cases he has found the author's method more simple and more expeditious than any of the so-called classical methods of solution. The method of analysis presented in the paper can be generalized, formulas can be derived for members of variable section, and it can be shown that the case of constant moment of inertia is simply a special case of the general solution.

T. F. HICKERSON,[12] M. AM. SOC. C. E.—The "frame characteristics" treated in this paper are derived from the theorem of three moments. Similar frame characteristics introduced by the writer[13] were derived from modified slope-deflection equations in which the "degree of fixity" is introduced. Both methods are theoretically exact, are entirely independent of the loading or deformations (if small), and lead to bending moments at a single operation.

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[12] Prof. of Applied Math., Univ. of North Carolina, Chapel Hill, N. C.

[13] "Statically Indeterminate Frameworks," by Thomas F. Hickerson, Univ. of North Carolina Press, 2d Ed., 1937, p. 204 (1st Ed., 1934).

The procedure for moving loads, and fixing their position on the framework to cause maximum bending moments, are treated thoroughly by the author. Necessarily these general bending-moment formulas for any loading combination (Eqs. 25) must be complicated even if they are supplemented with curves. Treating each individual case as a problem in itself (with fewer variables) offers an alternative procedure. One may arrive at a close approximation to the theoretical result after a few trial positions of the loads. This would be a relatively simple procedure. After solving several typical problems one would be in a better position to judge which of the two methods is preferable in practice.

Max W. Strauss,[14] Jun. Am. Soc. C. E.—Publication of this paper at long last makes available to the American engineering profession a method for the precise analysis of rigid frames that heretofore has been widely used in other countries. (A relatively small group in the United States, of which the writer is a member, had the privilege of studying under Professor Brumfield and thus was aware of this method prior to its present publication.)

The concept of an exact "carry-over factor" or "transmission coefficient" is not a new one. It can be traced back to the famous French engineer, Maurice Lévy, in whose great work[15] may be found the equivalent of many of the author's expressions. In the United States the concept was introduced in graphical form[16] by L. H. Nishkian and D. B. Steinman, Members, Am. Soc. C. E., as the method of conjugate points. More recently an analytic application of this concept was implied by Camillo Weiss,[17] M. Am. Soc. C. E. In his paper, Professor Brumfield presents a logical development of this analytic application.

Restraining Moments in a Loaded Span.—In Sections 4 and 5 of the paper, the author has derived expressions for the end moments in a loaded span due to the loads in that span. Eqs. 14, although quite simple, are limited in use to concentrated loads. For other loads or combinations of loads, the labor involved becomes somewhat tedious in that the analyst may either integrate Eqs. 14 (mathematically, if the particular loading so allows, as in Eq. 15, or otherwise mechanically), or use Eqs. 27 in conjunction with the curves of Fig. 5, Eq. 22*b*, and Eq. 24. However, much of this tedious labor can be avoided by a single expression that is applicable to any load or combinations of loads.

Eq. 14*b* may be rewritten—

$$M_R = -W L \frac{C_R}{1 - C_R C_L} [(2 k^2 - 2 k^3 + k - 2 k^2 + k^3) - C_L (2 k - 4 k^2 + 2 k^3 + k^2 - k^3)] \ldots\ldots (55)$$

Remembering that the fixed-end moments, M_{FR} and M_{FL}, for the right end and left end, respectively, for a concentrated load are given by

$$M_{FR} = (k^2 - k^3) W L \ldots\ldots (56a)$$

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[14] Asst. Structural Engr., Dept. of Water Power, Los Angeles, Calif.

[15] "La Statique Graphique et Ses Applications Aux Constructions," by Maurice Lévy, Gauthier-Villars & Co., Paris, France, 1886–1888.

[16] "Moments in Restrained and Continuous Beams by the Method of Conjugate Points," by L. H. Nishkian and D. B. Steinman, *Transactions*, Am. Soc. C. E., Vol. 90 (1927), p. 1.

[17] "Interrelation of Certain Structural Concepts," by Camillo Weiss, *ibid.*, Vol. 111 (1946), p. 391.

and

$$M_{FL} = (k - 2\,k^2 + k^3)\,W\,L \quad \text{(56b)}$$

and substituting these in Eq. 55, the latter becomes

$$M_R = -\frac{C_R}{1 - C_R\,C_L}\,[(2\,M_{FR} + M_{FL}) - C_L\,(2\,M_{FL} + M_{FR})] \quad \text{(57a)}$$

Although Eq. 57*a* is adequate, it can be simplified further. By collecting terms:

$$M_R = -\left[M_{FR}\,\frac{C_R\,(2 - C_L)}{1 - C_R\,C_L} + M_{FL}\,\frac{C_R\,(1 - 2\,C_L)}{1 - C_R\,C_L}\right] \quad \text{(57b)}$$

Substituting Eq. 4 for C_R and C_L, combining like terms, and then substituting Eq. 40, Eq. 57*b* reduces to

$$M_R = -\,[M_{FR}\,(1 - t_R) + M_{FL}\,t_L\,C_R] \quad \text{(58a)}$$

and, similarly,

$$M_L = -\,[M_{FL}\,(1 - t_L) + M_{FR}\,t_R\,C_L] \quad \text{(58b)}$$

in which t_R and t_L are obtained by Eq. 41 or from Col. 10 of Table 1.

Eqs. 58 have the advantage of being applicable to all types of loadings inasmuch as tables and curves for fixed-end moments for all types of loads are readily accessible in all standard reference books.

Stiffness and Restraint.—It seems to the writer that I/L, or K, affords a better mental image of the stiffness of a span than the author's L/I, or Q, since a stiffer beam is thus represented by a larger numerical value. Using larger numbers for stiffer beams avoids the mental juggling that must inevitably ensue if the reverse is adopted. Thus

$$i = \frac{2 - C}{K} \quad \text{(59)}$$

and if, furthermore, the stiffness index is defined as $1/i$ or r, instead of i, and the restraint index is defined as the sum of the stiffness indexes, then

$$r = \frac{1}{i} \quad \text{(60a)}$$

$$R' = \frac{1}{R} = S\,r \quad \text{(60b)}$$

and

$$C = \frac{1}{2 + \dfrac{K}{R'}} \quad \text{(60c)}$$

This is of more than academic interest since it would simplify the necessary calculations considerably. For example, if Col. 3 of Table 1 listed r, or $1/i$, instead of i, then Col. 4, which would list R' instead of R, would be obtained merely by the addition of the values in Col. 3 for the other members at that joint. The computer would not be required to obtain the reciprocal of the sum of the reciprocals of these values.

Transforming the author's expressions so as to utilize fixed-end moments serves the further purpose of illustrating the coordination of this method with the Cross method of successive approximations and the slope-deflection method. This is to be expected as all are based on, and derived from, the basic theorem of three moments.

D. A. MACKENZIE,[18] ASSOC. M. AM. SOC. C. E.—The analysis of continuous beams and frames supporting moving loads has been advanced considerably by the publication of this paper. Eqs. 29 and the curves of Fig. 6 are particularly useful. A precise mathematical formula now is available for locating the loads to produce maximum moments, instead of the more or less approximate methods commonly used in the past.

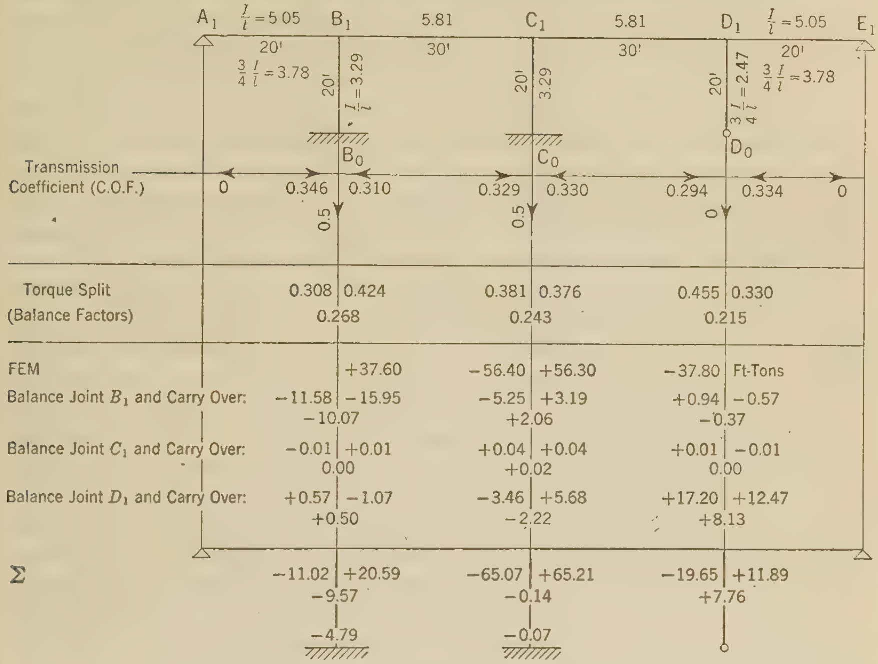
A_1
$\frac{I}{l} = 5.05$
B_1
5.81
C_1
5.81
D_1
$\frac{I}{l} = 5.05$
E_1
20'
$\frac{3}{4}\frac{I}{l} = 3.78$
20'
$\frac{I}{l} = 3.29$
30'
20'
3.29
30'
20'
$\frac{3}{4}\frac{I}{l} = 2.47$
20'
$\frac{3}{4}\frac{I}{l} = 3.78$
B_0
C_0
D_0
Transmission Coefficient (C.O.F.)
0
0.346
0.310
0.329
0.330
0.294
0.334
0
0.5
0.5
0
Torque Split (Balance Factors)
0.308
0.424
0.268
0.381
0.376
0.243
0.455
0.330
0.215
FEM
+37.60
−56.40
+56.30
−37.80
Ft-Tons
Balance Joint B_1 and Carry Over:
−11.58
−15.95
−10.07
−5.25
+3.19
+2.06
+0.94
−0.57
−0.37
Balance Joint C_1 and Carry Over:
−0.01
+0.01
0.00
+0.04
+0.04
+0.02
+0.01
−0.01
0.00
Balance Joint D_1 and Carry Over:
+0.57
−1.07
+0.50
−3.46
+5.68
−2.22
+17.20
+12.47
+8.13
Σ
−11.02
+20.59
−9.57
−65.07
+65.21
−0.14
−19.65
+11.89
+7.76
−4.79
−0.07

FIG. 15

The coefficients used for transmission (C), moment split (m), and torque split (t) may also be used as moment distribution factors and, once the loads are located on the span, the negative moments may then be found by fixed-end moments and direct moment distribution.

The author's example of Fig. 12 may be arranged as shown in Fig. 15 and the end moments readily found by that method.

Joint B_1 is first balanced to make $\Sigma M_{B1} = 0$. The moment (15.95) allocated to B_1C_1 is then multiplied by the carry-over factor ($C = 0.329$) to produce a moment of 5.25 ft-tons at joint C_1 which is then distributed to the other two members in proportion to their torque-split factor (t) or their moment-split factor (m), and the signs changed to make $\Sigma M_{C1} = 0$. These are carried over progressively to the other joints in the same manner; and the other joints are then balanced likewise.

The sign convention used is one which considers that a moment at the end of a member is positive if it tends to rotate the adjacent joint clockwise.

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[18] Design Engr., Buffalo-Niagara Elec. Corp., Buffalo, N. Y.

The computations in this discussion were done by slide rule, which probably accounts for the slight difference from the author's values.

Of course, the moments could also be found by the Hardy Cross method, but, having already obtained the direct-moment factors, the amount of work involved in making the balancing and distributing computations is very small. The principal advantage of the direct method is its flexibility for determining maximum moments where some spans are loaded and others not loaded.

CHARLES E. SCHAFFNER,[19] JUN. AM. SOC. C. E.—The most serious handicap of the Brumfield method is that it is comparatively difficult to learn without the help of an instructor. This fact, in addition to its limited distribution, heretofore has retarded the adoption of the method. One naturally chooses an instrument that is easy to adopt, rather than one which requires much study, and perhaps the assistance of other engineers.

One point concerning the paper, as published, is open to criticism. This is that it was begun with complete definitions of the special nomenclature to be used. Because of the newness of the approach, it is necessary to use unfamiliar nomenclature. However, psychologically, it might be better if the author had defined the terms and symbols as needed, throughout the text, and then summarized them at the end of the paper, or at the end of each section.

The only familiar symbol that has been changed is the Q-value which is the reciprocal of the familiar "K-value." It is well that this change has been made because, for this particular method, it is more advantageous to have the value in the numerator than in the denominator. The algebraic manipulations are simplified. In addition, it must be remembered that all writing, derivations, and solutions are done initially by hand rather than by the typewriter. The use of Q thus eliminates the possibility of confusing K with the k frequently used to denote a proportionate part of the span length.

The actual simplicity of the method, once one is familiar with the fundamentals, will be apparent upon examination of Eqs. 14, 16, and 17, which are derived in straightforward fashion from the theorem of three moments. The author has verified the accuracy of each by simple examples.

Many engineers have a phobia against using any equations for which curves are plotted. The curves, shown in Figs. 5 and 6, aid in the solutions of Eqs. 26 and 29. However, they are not indispensable, as the equations can be solved easily should the curves be unavailable. In fact, as the author notes, Eqs. 25 need not be used. The separate effects of a series of concentrations may be computed by the use of Eqs. 14, 16, or 17, and these effects can be added algebraically. However, the computing time is reduced considerably by the use of Eqs. 25; and it is reduced much more if Eqs. 27 and Fig. 5 are used.

In Section 7, the author shows good judgment by explaining the significance of a number of the new terms. One of the faults inherent in most involved systems of computation is that the computer gradually tends to do things by rote, and thus loses understanding of the different steps through which he passes. These terms are not just invented terms, but have a definite physical meaning. Thorough understanding of their use and significance is essential.

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[19] Instructor, Civ. Eng. Dept., The Cooper Union, New York, N. Y. (on military leave).

In Sections 6 and 8, the author succeeds where the proponents of all other methods have failed. He has shown how to locate the position of a moving load on a restrained beam for maximum bending moment, either at the support, or under one of the concentrations. This will provide a great reduction in the work and time necessary in analyzing this type of problem.

The derived equations, although not simple, can be handled competently, as the author shows in his illustrative example in Section 6. It is regrettable that no example of the use of Eq. 35 has been given, and that some of the steps in its development have been omitted. However, it is obvious that in a short paper, such as this, some things, ordinarily included, must be omitted. Thus some excellent algebraic manipulations have been eliminated from this treatment. One idea which tends to inspire confidence in the equations is the manner in which the author shows how each equation reduces to a well-known formula under simple beam conditions. Actually then, related simple beam formulas are only special cases of the more general formulas developed in this paper.

It is imperative to become familiar with the frame characteristics, as explained in Section 10. These new terms—transmission coefficient, stiffness index, restraint index, moment split, and torque split—and their interrelations are the basis of the method. A complete understanding of this material is essential to the proper use of the method. The arrangement, shown in Table 1, includes all the characteristics of the frame that are necessary for the analysis. In addition, the value of the table is enhanced by its self-checking features which eliminate practically all chances of error.

In the larger, more complicated frames, it is impossible to complete this table without resorting to the estimation of the value of a few of the coefficients. This operation (see Section 11), although quite simple, is invaluable. It enables the analyst to check key members rapidly without conducting a complete analysis.

One of the most frequent sources of error and confusion in the analysis of rigid frames has always been the sign or sense of the bending moment. The traditional "positive or negative" of courses in mechanics of materials is usually abandoned because of the confusion created by other than horizontal members. Directional signs or the terms, "clockwise or counterclockwise," usually are adopted; and actually that is what the author's "normal and reverse system" is. However, his concept, as given by his definitions, "pins down" the exact meaning of these terms, thus clarifying the physical picture in the mind of the user. These definitions, together with the other fundamentals, enable the analyst to picture exactly how each member of the frame reacts to different loading conditions. This gives the analyst a tremendous advantage over one who uses a system of positive and negative bending moment without attempting to picture the resultant bending of the frame.

In Section 13, Professor Brumfield mentions a moment-distribution chart for each joint. This is an excellent bookkeeping device. It must not be misconstrued, however, as an essential part of the method. Direct moment distribution, although a little cumbersome in the analysis of large frames, is no more so than that in the Hardy Cross system. The charts are something that the author has devised to facilitate computation. It has been the experience of the writer, working without these charts, that the method is easily adaptable to complicated reentrant frames. In one particular case it was used successfully on an airplane hangar upon which other men had worked, unsuccessfully, for much longer periods, with a number of other methods. When each of three other methods "bogged down" at a different sopt, this method was finally tried and the analysis was completed in a surprisingly short period. As the author states, one distribution is sufficient for all purposes, including sidesway.

In conclusion, the author remarks that the method has been extended to beams of variable section and restrained ends, and also to curved beams. The writer has done some work in this respect in arch analysis. Probably the most popular method of arch analysis is the "elastic center method." A comparison of the two methods would therefore be important.

The "elastic center method" is derived from the laws governing elastic displacements in ribs and frames—a fairly involved subject. This method is based upon some of the fundamental concepts of the moment-area method, familiar to all students of elementary mechanics of materials. It is developed from these concepts, in direct fashion, mainly through the use of the laws of statics. Therefore, where the Brumfield method would seem to be comparatively difficult to learn as applied to straight beams, it becomes the simpler method when applied to the more complex subject of curved beams of variable section. Every equation used is transformed into the corresponding orthodox straight beam formula as the radius of curvature of the beam approaches infinity.

These same equations, applied to arches, sewer rings, etc., can be used with equal facility for the analysis of ordinary frames, such as rectangular bents, saw-tooth roof frames, etc. This means, of course, great versatility and adaptability. Finally, the Brumfield method reduces the arithmetical work necessary in the elastic center method by at least one third. This is of inestimable value, as it is this part of the analysis that presents the greatest chance of error.

To summarize, the writer believes that this method will gain rapidly in popularity among engineers when they realize that its accuracy, ease of usage, and extreme adaptability far outweigh the initial time spent in learning it.

J. Kalb,[20] Esq.—As examples of unusual structures to which the Brumfield method has been applied successfully, two interesting frames are presented herein to advance the discussion of Section 15. For example, Fig. 16 represents, diagrammatically, one of the seven reinforced-concrete rigid-frame bents designed for a vocational high school to be constructed in New York, N. Y. The spans, sizes, loads, Q-values, and transmission coefficients for the frame, as well as the moment split at joint 2G, are shown in Fig. 16. These were

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[20] Asst. Civ. Engr. (Structural), Board of Education, New York, N. Y.

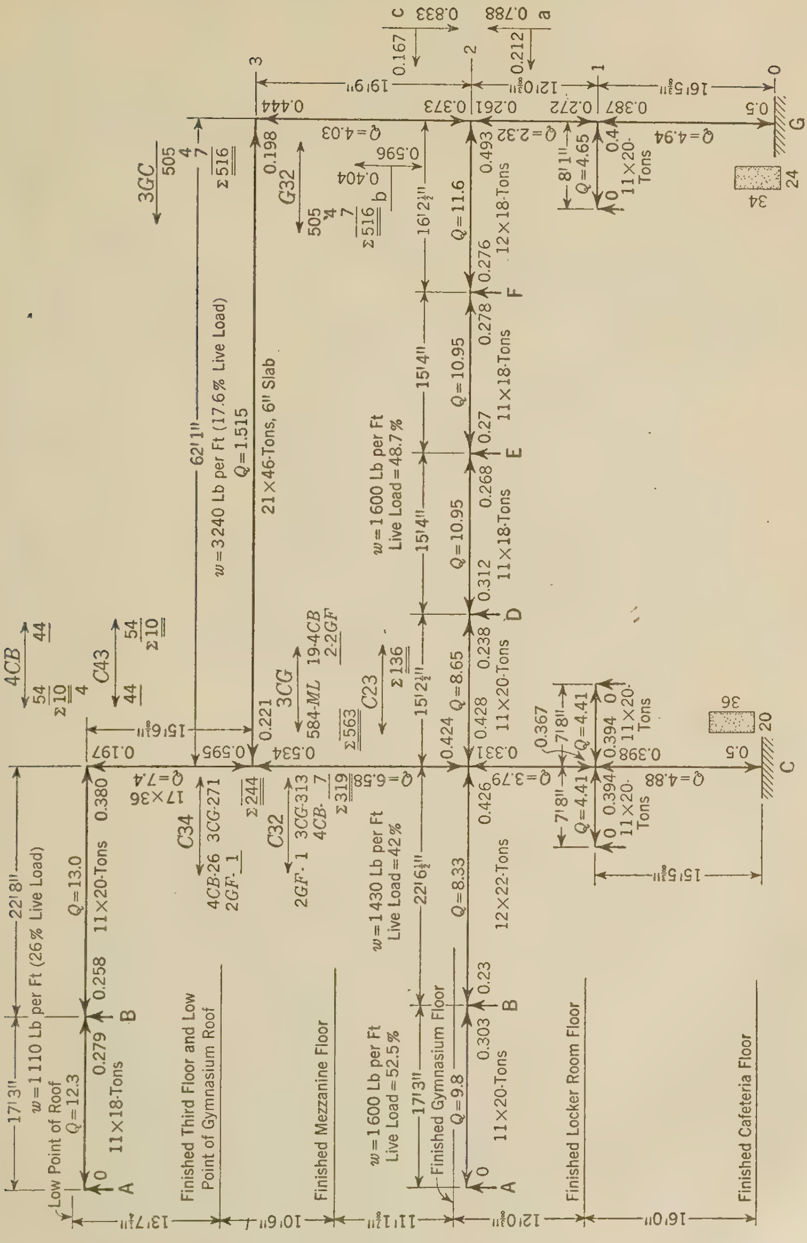
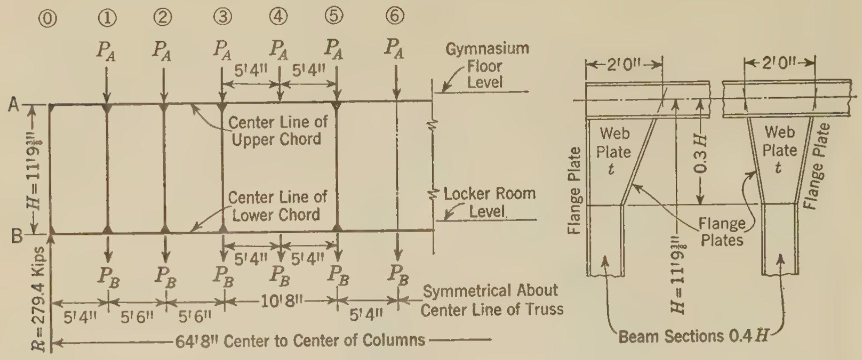

Low Point of Roof
Finished Third Floor and Low Point of Gymnasium Roof
Finished Mezzanine Floor
Finished Gymnasium Floor
Finished Locker Room Floor
Finished Cafeteria Floor
13'7¼''
10'9''
11'1⅞''
12'0⅝''
16'0''
17'3''
22'8''
62'1''
w=1 110 Lb per Ft (26% Live Load)
w=3 240 Lb per Ft (17.6% Live Load)
w=1 600 Lb per Ft Live Load=52.5%
w=1 430 Lb per Ft Live Load=42%
w=1 600 Lb per Ft Live Load=48.7%
Q=12.3
Q=13.0
Q=7.4
Q=1.515
Q=6.58
Q=9.8
Q=8.33
Q=8.65
Q=10.95
Q=10.95
Q=11.6
Q=4.03
Q=2.32
Q=4.65
Q=4.94
Q=3.79
Q=4.41
Q=4.41
Q=4.88
11×18-Tons
11×20-Tons
17×36
21×46-Tons, 6'' Slab
12×22-Tons
11×20-Tons
11×18-Tons
12×18-Tons
11×20-Tons
A
B
C
D
E
F
G
a
b
c
0
1
2
3
0.279
0.258
0.380
0.197
0.595
0.221
0.534
0.198
0.444
0.303
0.23
0.426
0.424
0.428
0.238
0.312
0.268
0.27
0.278
0.276
0.493
0.373
0.261
0.596
0.404
0.212
0.788
0.167
0.833
0.331
0.367
0.394
0.394
0.398
0.5
0.272
0.387
0.4
0.5
15'6⅜''
22'6½''
15'2½''
15'4''
15'4''
16'2½''
19'9''
12'0⅝''
16'5¾''
7'8''
7'8''
8'1''
15'5¾''
4CB 54/Σ10/4 44
C43 44 54/Σ10
C34 4CB·26 3CG·271 2GF·1 Σ244
C32 2GF·1 3CG·313 4CB·7 Σ319
3CG 584·ML 19·4CB 2·2GF Σ563
C23 Σ136
3GC 505 4 7 Σ516
G32 505 4 7 Σ516
34 24
36 20

Fig. 16

TABLE 4.—ANALYSIS OF VIERENDEEL TRUSS



⓪ ① ② ③ ④ ⑤ ⑥
P_A
5'4" 5'4"
Gymnasium Floor Level
A
B
H = 11'9⅜"
Center Line of Upper Chord
Center Line of Lower Chord
Locker Room Level
P_B
R = 279.4 Kips
5'4" 5'6" 5'6" 10'8" 5'4"
Symmetrical About Center Line of Truss
64'8" Center to Center of Columns
2'0"
Flange Plate
Web Plate t
0.3 H
Flange Plates
H = 11'9⅜"
Beam Sections 0.4 H

Member (see diagrams)	PHYSICAL CHARACTERISTICS			BENDING MOMENTS, IN FOOT-KIPS[a]			DIRECT FORCE, IN KIPS[b]	
	Beam section (1)	Thickness (t in.) of web plates (2)	Size of flange plates (in.) (3)	Direct force neglected (4)	Direct force considered (5)	Live load[c] and direct force (6)	Direct force neglected (7)	Direct force considered[d] (8)
Joint A0:								
0-AB	12 WF 120	⅞	14 × 1	411 N	384 N	....	−279.4	
A-01	14 WF 167	....	....	411 R	384 R	....	− 69.5	− 65.0
Joint A1:								
1-AB	12 WF 120	⅞	14 × 1	653 N	609 N	....	− 2.3	
A-10	14 WF 167	....	....	356 R	325 R	....	− 69.5	
A-12	14 WF 167	....	....	297 R	284 R	....	−180.5	−168.0
Joint A2:								
2-AB	12 WF 120	⅞	14 × ¾	498 N	480 N	....	− 2.3	
A-21	14 WF 167	....	....	339 R	360 R	....	−180.5	
A-23	14 WF 211	....	....	159 R	120 R	....	−265.0	−249.5
Joint A3:								
3-AB	12 WF 120	⅞	14 × ¾	492 N	495 N	....	− 3.5	
A-32	14 WF 211	....	....	289 R	320 R	....	−265.0	
A-35	14 WF 211	....	....	203 R	175 R	225 R	−349.0	−333.5
Joint A5:								
5-AB	12 WF 79	¾	14 × ½	169 N	164 N	70 R 206 N	− 4.6	
A-53	14 WF 211	....	....	329 R	368 R	....	−349.0	
A-55^1	14 WF 211	....	....	160 N	204 N	344 N	−377.5	−368.1
Joint B0:								
0-BA	14 WF 120	⅞	14 × 1	411 N	384 N	....	−279.4	
B-01	....	....	....	411 R	384 R	....	+ 69.5	
Joint B1:								
B-10	....	....	....	356 R	346 R	....	+ 69.5	(0.1483
1-BA	12 WF 120	....	....	653 N	607 N	....	− 2.3	
B-12	....	....	....	297 R	261 R	....	+180.5	
Joint B2:								
B-21	....	....	....	338 R	354 R	....	+180.5	(0.3293)
2-BA	12 WF 120	⅞	14 × ¾	498 N	480 N	....	− 2.3	
B-23	....	....	....	160 R	126 R	....	+265.0	
Joint B3:								
B-32	....	....	....	294 R	325 R	....	+265.0	(0.4953)
3-BA	12 WF 120	⅞	14 × ¾	492 N	495 N	....	− 3.5	
B-35	....	....	....	198 R	170 R	225 R	+349.0	
Joint B5:								
B-53	....	....	....	336 R	375 R	....	+349.0	(0.7815)
5-BA	12 WF 79	¾	14 × ½	169 N	164 N	70 R 206 N	− 4.6	
B-55^1	....	....	....	167 N	201 N	335 N	+377.5	

[a] N = normal moment and R = reverse moment. [b] Tension is positive. [c] Bending moments for maximum live load, and with direct force considered. [d] The last four items in this column (in parentheses) are the deflections at the joints indicated, in inches.

taken from computations such as those in Table 3 for joints 2G, 3G, and 3C. The values of Q are to be multiplied by 100 and the moments are in kip-feet. Members in the frame are identified by a coordinate system in which columns and supports are lettered and floors and roofs are numbered. Thus, joint C2 is in column C at floor level 2. The writer investigated the intrusion of the members in accordance with the principles developed by the author in his text,[4] because at several joints relatively limber members are framed into very stiff members.

TABLE 3.—Frame Characteristics for Three Joints of Fig. 16

Member	Q		R	Member	C	Moment split[a]		
				Joint 2G				
G-21	2.32	0.668	0.710	G-12	0.261	(a)	0.596	0.833
2-GF	11.60	3.35	0.399	2-FG	0.454	0.212	(b)	0.167
G-23	4.03	0.902	0.556	G-32	0.373	0.788	0.404	(c)
				Joint 3G				
G-32	4.03	0.764	0.448	G-23	0.444	(a)	1	....
3-GC	1.515	0.448	0.764	3-CG	0.198	1	(b)	....
				Joint 3C				
C-32	6.58	1.20	0.343	C-23	0.534	(a)	0.275	0.536
C-34	7.40	1.39	0.330	C-43	0.595	0.247	(c)	0.464
3-CG	1.515	0.455	0.642	3-GC	0.221	0.753	0.725	(d)

[a] The lower-case letters in the moment-split column represent the possible radiating members at the joint under consideration, thus: (a), (b), (c), and (d) always refer to the members radiating downward, to the left, upward, and to the right, respectively, from the joint.

In connection with this design three facts are particularly noteworthy:

(*a*) The original design of member 3CG called for a haunched T-beam, increasing from 46 in. at its center to 72 in. at the center line of the columns. The analysis indicated that the columns, limited in size by the architect, would be overstressed. Changing this girder to one of constant depth required recomputations of frame characteristics only for those members framing into the joints at the ends of the girder.

(*b*) The results of the transmission and collection of the moments from the various joints are indicated for joint 3C. For example, the uniform load on 4BC transmitted 19 ft-kips to member 3CG, the sense of the moment being the same. Therefore, changes in loadings resulted only in changing the value of M_R and M_L for that particular load and noting its effect on the various joints to which it is transmitted.

(*c*) The critical values of the column moments were obtained by collecting the moments to give the maximum or minimum values, using the live-load percentages of the total loads.

Table 4 demonstrates the analysis of a Vierendeel truss used in the structural steel framing of New York City school buildings, the loadings, in kips, being as follows:

Load	P_A	P_B
Dead	13.7	17.9
Live	14.0	5.2
Total	27.7	23.1

The analysis of such a truss may be summarized into the following basic steps: (1) Compute the frame characteristics; (2) prepare torque transfer strips; (3) prepare torque-distribution charts; (4) compute moments due to loads between panel points and distribute; (5) distribute and collect artificial and neutralizing torques; (6) set up and solve shear bending moment equations; (7) obtain primary moments by substituting in the collections of artificial and neutralizing torques; (8) compute the direct stress; (9) find the artificial moments due to changes in length and distribute the neutralizing torques; (10) compute the vertical forces necessary to hold the lower chords horizontal and find the moments caused by the shears by substituting as in step 7; and (11) the total moment in any member is the algebraic sum of the moments found in steps 7, 9, and 10. For ordinary purposes, the practice of using two-joint distribution and neglecting the secondary stresses will limit the analysis to the first seven steps and reduce the time required approximately one half. The analysis for secondary stresses and the effect of placing the live load to give the maximum moments are included in Table 4. The deflections of this truss (see last four items in Col. 8, Table 4) are practically the same as those for the conventional type. Fig. 17 shows the distorted frame and the horizontal and vertical displacements of the panel points.

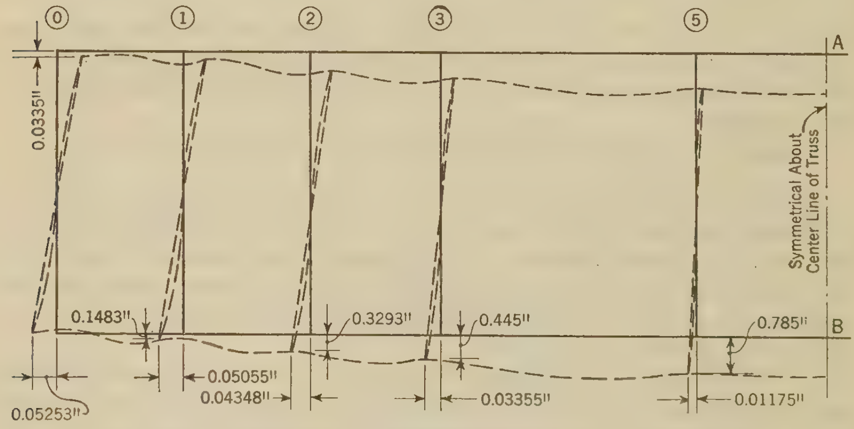


Fig. 17.—Distorted Frame

As a check on the accuracy of Professor Brumfield's method, the writer analyzed a Vierendeel truss whose solution[21] has appeared elsewhere. The results were in exact agreement.

Summary.—The adaptation of frame characteristics, which is the heart and soul of Professor Brumfield's system, cloaks each frame with scientific exactness. These frame characteristics make entirely clear every detail of the flow of moment through the frame, and they readily facilitate computations and the examination of computations in case of changes in loading on any member. They simplify the computations for critical values of moment and shear for any member, regardless of type of loading, by indicating the necessary pattern of loading required to give these values, thus enabling the designer to trace their effects to that member. By the formula and the principles stated in Section 11, the writer has estimated the transmission coefficients for non-reentrant frames with gratifying results both as to time saved and accuracy of results. The concept of normal and reverse bending moment introduced by Professor Brumfield yields a clear picture of the distorted frame and eliminates the possibility of error and confusion arising in complicated frames when the plus and minus signs are used.

The writer has applied Professor Brumfield's system, successfully, not only to frames with straight beams of variable sections but also to curved beams with variable sections and to noncoplanar frames. The entire system is thorough, well integrated, and readily applied.

G. ROLSMA,[22] ESQ., AND F. W. GIESEKING,[23] ESQ.—The concept of the transmission coefficient, as presented in this paper, provides a means for analyzing rigid frames regardless of shape or symmetry. Frames can be solved directly and completely for wind sidesway, moving loads, deflections, and settlement of supports. Expansion of the fundamental derivations furnishes the solution of each succeeding step. No additional theorems or assumptions are necessary. The fundamentals should be readily understood by the average engineer who is familiar with the three-moment theorem or the method of moment areas.

As a comment on Section 15 of the paper several "other applications" can be cited briefly. For example, the frame in Fig. 18(*a*) has been analyzed in the design of several stadia in New York, N. Y. A thorough study was desirable and the slight extra time required for computing coefficients was considered well spent as they were used in each succeeding step of the analysis. The frame characteristics in Fig. 18(*c*) were computed from the physical constants of the frame as outlined in Section 10 of the paper.

End moments were calculated, for all members of the frame, using Eqs. 16 and 17. For example, in the computations for joint C, $L_{CD} = 18.6$ ft and $L_{CB} = 16.0$ ft. The total design load, w, on member DB was 2.8 kips per ft, half of which was live load. Values of C_R and C_L are shown in Fig. 18(*b*). Substituting these values in Eqs. 16 and 17, $M_{CD} = \frac{28 \times 18.6^2}{4} \times \frac{0.41}{1 - 0.41 \times 0.284} \times 0.716 = 81$ ft-kips. Similarly, $M_{CB} = 56$ ft-kips. The end moments were then distributed to the frame in accordance with Section 13, except that moment-distribution charts were used, such as those for joint C in Fig. 19. Letters U, L, and R refer to the "under," "left," and "right" members, respectively, of the joint under consideration. The letters in the central column of each part of the chart indicate the joint of origin, and the lower-case letters at the sides show the particular member from which the distributed bending moment originates; thus: "a" denotes the under members, "b" denotes the left members, "c" the over members, and "d" the right members. Quantities next to the lower-case letters are live-load moments taken as a percentage of

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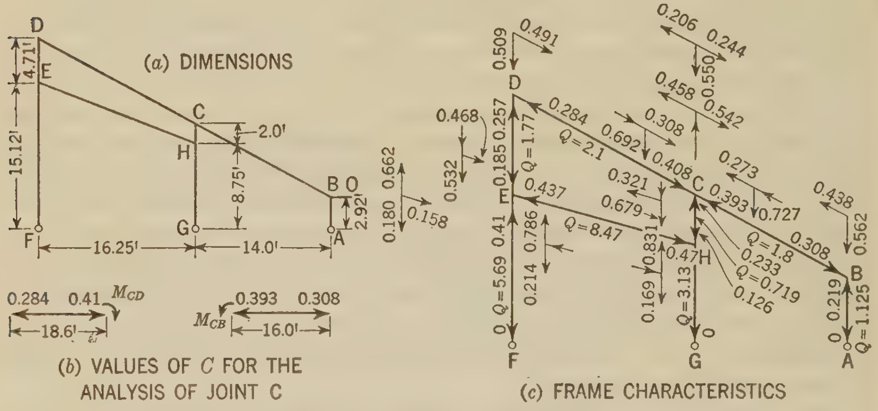
[21] *Transactions*, Am. Soc. C. E., Vol. 102 (1937), Table 3, p. 934.

[22] Asst. Civ. Engr., Board of Education, City of New York, Brooklyn, N. Y.

[23] Asst. Architect, Board of Education, City of New York, Brooklyn, N. Y.


(a) DIMENSIONS
D
E
C
H
B
O
G
F
A
4.71'
15.12'
2.0'
8.75'
2.92'
16.25'
14.0'
0.284
0.41
M_{CD}
18.6'
0.393
0.308
M_{CB}
16.0'
0.509
0.491
0.206
0.244
0.550
0.458
0.542
0.468
0.257
0.185
Q=1.77
0.532
0.662
0.180
0.158
0.284
Q=2.1
0.692
0.308
0.408
0.273
0.321
0.437
0.679
0.393
0.727
0.438
Q=8.47
0.831
0.169
0.47
Q=3.13
0.41
Q=5.69
0.786
0.214
Q=1.8
0.233
Q=0.719
0.126
0.308
0.562
0.219
Q=1.125
0
0
(b) VALUES OF C FOR THE ANALYSIS OF JOINT C

(c) FRAME CHARACTERISTICS

Fig. 18.—Analysis of Stadium Bent

L

1.1d	3	B		
		C	15	d 7.5
		C	81	b
		H	2	b 0.7
	D+L	Σ	95	
		Max	96	
		Min	87	

R

		B	10	d 3.8
d	56	C		
12.5 b	25	C		
		H	2	b 0.7
	69	Σ	D+L	
	74	Max		
	56	Min		

U

2.7d	7	B		
b	56	C	41	d 21
15 b	4	H		
	26	Σ	D+L	
	47	Max		

Fig. 19.—Example of Load Bending-Moment Collection Chart for Members at Joint C

the total moment, and are used for computing maximum and minimum moments. Moments of reverse sense are recorded in the left-hand column and normal moments in the right-hand column.

For example, at the U-member of joint C the 21 ft-kips (normal in sense) constitute 50% of the total moment from the d-member of joint C indicating removal of live load from member CB to produce maximum moment of 47 ft-kips. From the moment-distribution charts, live-load plus dead-load moments, and maximum moments and minimum moments, may be obtained rapidly for each member at all joints. The frame is now statically determinate for all possible loading conditions; and the reactions, column shears, and maximum positive bending moments are readily computed.

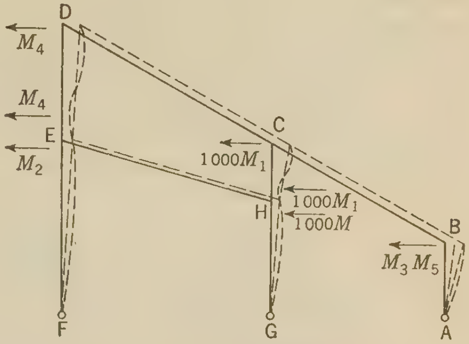


FIG. 20.—EFFECT OF SIDESWAY

Sidesway on an unsymmetrical frame, such as that in Fig. 18, may now be investigated easily by continuing the analysis. In general, the method is to assume that each joint has moved laterally to its final but unknown position as indicated by dashed lines in Fig. 20. At each end of all columns the designer places unknown bending moments sufficient to rotate the columns parallel to their original unstressed position. In this case 1,000 units of M were placed in top of column GH, 1,000 M_1 were placed at the top and the bottom of column HC. By comparing column FE to column GH: $M_2 = \frac{Q\,L}{Q_2\,L_2} \times 1{,}000\,M = \frac{3.13 \times 8.75}{5.69 \times 15.12} \times 1{,}000\,M = 318\,M$. Similarly, for the ends of the other columns, $M_3 = 8{,}350\,M$, $M_4 = 173\,M_1$, and $M_5 = 219\,M_1$.

Neutralizing torques, which are numerically equal but of opposite sense to the artificial bending moments placed at ends of columns, are applied at each joint and distributed to the frame. An example of a torque-distribution chart for joint C, in which 1,000 units of torque at joint C are distributed to the frame, is shown in Fig. 21. Such a chart is a valuable aid in distributing the neutralizing moments at a joint to the frame. After distribution, these neutralizing torques and the artificial bending moments are summarized for each member at all joints. Shears in the columns may now be expressed in terms of M and M_1, and, by the laws of equilibrium, simultaneous equations may be set up and solved for numerical values of M and M_1. These values of M and M_1 are then substituted back into the joint summary for each member, thus obtaining values of bending moments due to lateral deflection. A summary chart for joint C, similar in arrangement to Figs. 19 and 21, can then be prepared to show the artificial bending moments and neutralizing torques, and final moments due to sidesway.

Most engineers do not have time to study, thoroughly, all methods proposed; also, personal preferences often dictate the method to be used in solving rigid frames. Actually, one method may be more suitable for a particular problem than another, depending on the desired degree of accuracy and extent of analysis. The concept of transmission coefficients as presented, therefore, offers the following advantages:

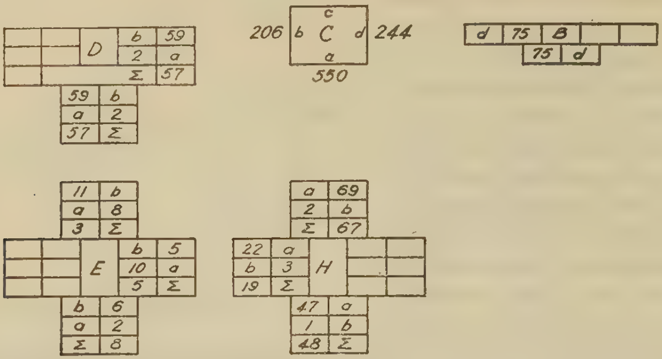


FIG. 21.—EXAMPLE OF CHART FOR THE DISTRIBUTION OF TORQUE FROM A GIVEN JOINT (JOINT C) TO ALL MEMBERS OF THE FRAME

(1) It is a perfectly general method of analysis and not a special means of solving a limited number of cases due to form or symmetry. In the frame presented by the writers the members are of uniform section. If some or all were of variable section, the procedure would be the same except that then the restraint triangles require modification.

(2) No approximations of the degree of end restraints of the members are necessary anywhere in the structure except at the base or foundation levels.

(3) The manner of bookkeeping suggested offers opportunity for detecting errors speedily and for observing the paths of moments through the frame. The resulting moments are obtained directly and accurately.

(4) The formulas, readily understood, lead to short cuts expressing, in a brief mathematical expression, ideas which would require many lines of written explanation. This is particularly true of the formula for moving loads which, together with the prepared graph, provides a direct and rapid method for placing moving loads of any composition upon restrained beams in position for the required maximum condition. Therefore, influence lines are not required. The case of frames loaded with uniform live load is illustrated in Fig. 19, the use of loading patterns and the attendant separate studies required for each pattern being obviated.

(5) The use of the concept of normal and reserve bending moment in place of the plus and minus signs removes much of the confusion in the old use of signs. The distortion of the frame is also more readily visualized.

The paper is a noteworthy contribution to engineering literature pertaining to the analysis of rigid frames. A study of the "concept of transmission coefficients" and the manner of approaching various problems cannot help but result in a greater understanding of the action of rigid frames under all types of loading and the method should not prove too difficult to master and remember.

Stanley S. Schure,[24] Esq.—The Brumfield method of analysis has several advantages over other comparable methods. For example, it is more direct and, in a number of cases, more simple than the Cross method. After he has computed the frame characteristics, the designer will be familiar with the structure. The effect of different loadings, or of changes in the original loading, can easily be traced.

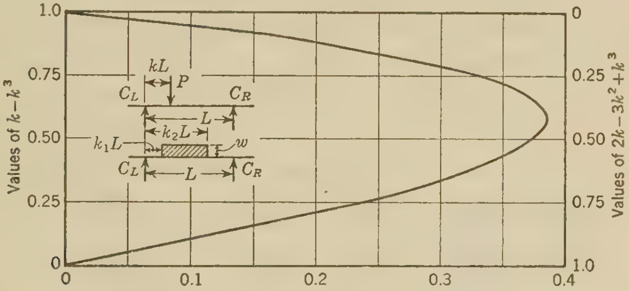


Fig. 22

To demonstrate this point, consider an ordinary continuous beam resting simply on five supports, with the four spans unequal in length. Assume the end supports to be partly fixed so that finite values of C will occur at the extremities by the beam. Formulas for the fixed moments in each span (see inset, Fig. 22) are computed as follows:

For concentrated loads—

$$M_R = -P\,L\,A_R\,[(k-k^3) - C_L\,(2\,K - 3\,K^2 + K^3)] \quad \ldots\ldots (61a)$$

and

$$M_L = -P\,L\,A_L\,[(2\,k - 3\,k^2 + K^3) - C_R\,(k-k^3)] \quad \ldots\ldots (61b)$$

For uniform loads—

$$M_R = -w\,L^2\,A_R\left[\frac{k^2_2 - k^2_1}{2}(1 - 2\,C_L) + (k^3_2 - k^3_1)\,C_L - \frac{k^4_2 - k^4_1}{4}(1 + C_L)\right] \quad \ldots\ldots (62a)$$

and

$$M_L = -w\,L^2\,A_L\left[\frac{k^2_2 - k^2_1}{2}(2 - C_R) - (k^3_2 - k^3_1) + \frac{k^4_2 - k^4_1}{4}(1 + C_R)\right] \quad \ldots\ldots (62b)$$

When $k_1 = 0$ and $k_2 = 1$, Eqs. 62 become

$$M_R = -\frac{w\,L^2}{4}\,A_R\,(1 - C_L) \quad \ldots\ldots (63a)$$

---

[24] Structural Designer, Standard Oil Development Co., Elizabeth, N. J.

and

$$M_L = -\frac{w\,L^2}{4}\,A_L\,(1 - C_R)\ldots\ldots\ldots\ldots\ldots\ldots(63b)$$

which may also be derived by substituting A_R-values and A_L-values in Eqs. 16 and 17, respectively. Values of $k - k^3$ and $2\,k - 3\,k^2 + k^3$ can be selected from curves such as Fig. 22. The succeeding step is to compute the frame characteristics similar to Table 1. For the example presented herein these characteristics are shown in Table 5. Finally, the restrained moments for each load, and the unbalanced moments from each span, are tabulated and the total moment is determined as shown in Table 6. As stated, the method is direct and simple. In a large percentage of cases the loads are concentrated or uniformly distributed, as in this example. For special loadings, such as locomotives, trucks, and variable distributions, the method described in Section 5 is applicable.

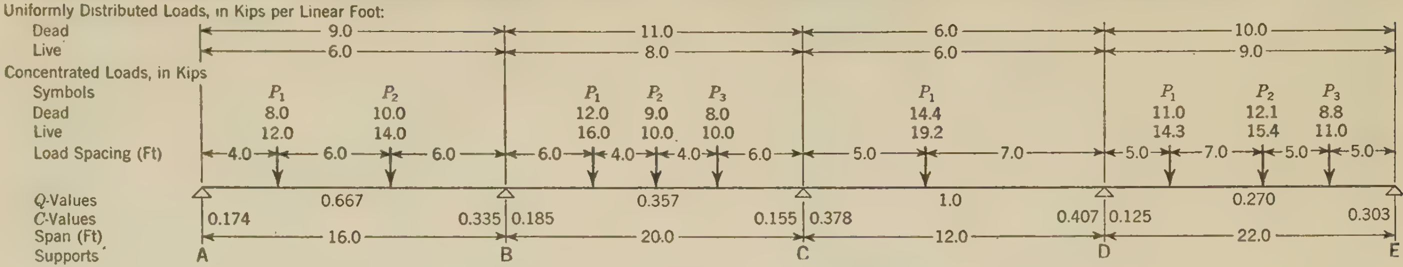
TABLE 5.—COMPUTATION OF FRAME CHARACTERISTICS, CONTINUOUS BEAM ON FIVE SUPPORTS

Member	STIFFNESS INDEX		Restraint index R	TRANSMISSION COEFFICIENT C (EQ. 5)	
	$Q\,(2 - C) = i$			Value	Member
(1)	(2)	(3)	(4)	(5)	(6)
Joint B					
BA	0.667 (2 − 0.174)	1.22	0.660	0.335	AB
BC	0.357 (2 − 0.155)	0.660	1.22	0.185	CB
Joint C					
CB	0.357 (2 − 0.184)	0.648	1.593	0.155	BC
CD	1.0 (2 − 0.407)	1.593	0.648	0.378	DC
Joint D					
DC	1.0 (2 − 0.378)	1.622	0.456	0.407	CD
DE	0.270 (2 − 0.303)	0.456	1.622	0.125	ED

In computing frame characteristics the writer suggests a modification which produces the same results as the author's method. This procedure avoids the use of a slide rule and thus eliminates errors. It involves the use of tables of reciprocals such as those found in most handbooks. For instance, the writer would add the following data in a column to be inserted between Cols. 3 and 4, Table 1:

Member	$\frac{1}{i}$
Joint B₁	
B10	2.1930
1BA	2.5253
1BC	3.4843
Joint C₁	
C10	2.1930
1CB	3.4384
1CD	3.4130
Joint D₁	
D10	1.6448
1DC	3.4722
1DE	2.5253

TABLE 6.—ANALYSIS OF A CONTINUOUS FOUR-SPAN BEAM RESTING ON FIVE SUPPORTS (END SUPPORTS PARTLY FIXED)


Uniformly Distributed Loads, in Kips per Linear Foot:
Dead
Live
Concentrated Loads, in Kips
Symbols
Dead
Live
Load Spacing (Ft)
Q-Values
C-Values
Span (Ft)
Supports
9.0
6.0
11.0
8.0
6.0
6.0
10.0
9.0
P_1
8.0
12.0
P_2
10.0
14.0
P_1
12.0
16.0
P_2
9.0
10.0
P_3
8.0
10.0
P_1
14.4
19.2
P_1
11.0
14.3
P_2
12.1
15.4
P_3
8.8
11.0
4.0
6.0
6.0
6.0
4.0
4.0
6.0
5.0
7.0
5.0
7.0
5.0
5.0
0.667
0.357
1.0
0.270
0.174
0.335
0.185
0.155
0.378
0.407
0.125
0.303
16.0
20.0
12.0
22.0
A
B
C
D
E

A (Eq. 19)	0.185			0.356		0.195			0.160		0.447			0.481		0.130			0.315	
λ-values			λ					λ					λ					λ		
Load	Dead	Live		Dead	Live	Dead	Live		Dead	Live	Dead	Live		Dead	Live	Dead	Live		Dead	Live
Uniform	7.1	4.7		16.8	11.2	18.1	13.2		14.3	10.4	5.7	5.7		6.5	6.5	11.0	9.9		33.4	30.1
P_1	6.0	9.0	0.25	7.9	11.9	14.8	19.7	0.3	8.0	10.7	18.6	24.8	0.417	17.2	23.0	8.1	10.5	0.227	13.2	17.1
P_2	5.9	8.3	0.667	18.0	25.2	11.1	12.4	0.5	9.1	10.0	....	....		....	....	7.4	8.8	0.545	28.6	36.4
P_3	....	....		....	....	6.8	8.5	0.7	7.8	9.8	....	....		....	....	2.9	4.0	0.775	17.2	21.5
Fixed-end moment	← 19.0	← 22.0		→ 42.7	→ 48.3	← 50.8	← 53.8		→ 39.2	→ 40.9	← 24.3	← 30.5		→ 23.7	→ 29.5	← 29.4	← 33.2		→ 92.4	→ 105.1
Distribution:																				
From AB	....	....		....	....	← 42.7	← 48.3		← 6.6	← 7.5	→ 6.6	→ 7.5		→ 2.7	→ 3.1	← 2.7	← 3.1		← 0.8	← 0.9
From BC	→ 8.9	→ 9.4		→ 50.8	→ 53.8	....	....		....	....	← 39.2	← 40.9		← 16.0	← 16.7	→ 16.0	→ 16.7		→ 4.9	→ 5.1
From CD	← 0.8	← 1.0		← 4.5	← 5.6	→ 4.5	→ 5.6		→ 24.3	→ 30.5	....	....		....	....	← 23.7	← 29.5		← 7.2	← 9.0
From DE	→ 0.3	→ 0.4		→ 2.1	→ 2.3	← 2.1	← 2.3		← 11.1	← 12.6	→ 11.1	→ 12.6		→ 29.4	→ 33.2	....	....		....	....
Final Moments:																				
Dead load	← 10.6			→ 91.1		← 91.1			→ 45.8		← 45.8			→ 39.8		← 39.8			→ 89.3	
Live load—																				
Maximum		← 23.0			→ 104.4		← 104.4			→ 71.4		← 71.4			→ 65.8		← 65.8			→ 110.2
Minimum		→ 9.8			← 5.6		→ 5.6			← 20.1		→ 20.1			← 16.7		→ 16.7			← 9.9

The proper values of $1/i$ are then added and the co-reciprocal of their sum is as the necessary R-value. For example, in member 1BC, joint B_1, 2.1930 + 2.5253 = 4.7183. The co-reciprocal of this value (0.212) is the value of $R_{1BC}\left(=\frac{1}{\frac{1}{i_1}+\frac{1}{i_2}}\right)$. Similarly for member 1DC, joint D_1, 1.6448 + 2.5253 = 4.1701, whose co-reciprocal (0.240) is the required value of R_{1DC}.

ALEXANDER DODGE,[25] ASSOC. M. AM. SOC. C. E.—The analysis of the frame shown in Table 1 was predicated on the assumption that no sidesway existed, which would hold true only if the ends A_1 and E_1 were hinged. However, this assumption is in conflict both with the sketch, in which the ends A_1 and E_1 are shown to be freely supported, and with the statement (see Section 10) "A free support, such as point A_1." It is inconceivable that the term "a free support" could ever be applied to the hinged end of a member. Furthermore, in modern rigid frames of the type shown, the bearing ends A_1 and E_1 are usually made free to move in a horizontal direction (to minimize shrinkage and temperature stresses) by the introduction of rockers or metal sliding plates; and the problem of determining the degree of lateral support, as a rule, is given due consideration in design. Therefore, with the information at hand, the degree of lateral support for this frame is an unknown quantity, depending on the position of a load and the amount of the downward reactions, as well as the coefficients of friction at points A_1 and E_1. This point should be clarified, so that no doubt is left in the reader's mind as to the type of a structure that is being analyzed. With loads placed as shown in Fig. 12, both ends A_1 and E_1 will be raised from their supports, provided this movement is not prevented by dead-load reactions, which, however, were not taken into consideration in the analysis. Therefore, the transmission coefficients will be changed and unbalanced shears in the columns will cause lateral deflections. Consequently, the characteristics of the frame will be altered to such an extent that an entirely new solution will result, although the changes will be slight because of the almost perfect symmetry of the frame and the loading.

The stated purpose of the paper (see "Synopsis") is to develop methods "for distributing the moments to the frame in a single operation," but the author has failed to show how this could be done because sidesway, present in the analysis of this frame, is disregarded. Moreover, the analysis of any rigid frame without sidesway can be reduced to an analysis of a continuous beam, and Eqs. 10 to 38 represent very special cases of structural concepts, belonging to the "beam class" rather than to the "frame class" of problems. The designer is interested not only in the bending moment under concentrated loads, but also (even for the very simple case treated in the paper) in negative moments at intermediate points; and these latter moments must be determined by some method not treated in the paper. A pure couple requires an entirely different set of computations using a very different concept, which makes the Brumfield analysis lengthy, nonuniform, and cumbersome.

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[25] Engr., U. S. Engrs., Portland, Ore.

The derivation of exact partly restrained end moments due to loads on a member has been demonstrated by A. Strassner.[26] He showed the practical applications of the concept of transmission coefficients and presented a method of distributing the moment to the frame of a general type in two operations—(1) by distributing the moment in a single operation on the assumption that no sidesway exists and (2) finding the sidesway corrections (also a single operation). Mr. Strassner's original work is mostly graphical—probably the reason why his method has never attained popularity in the United States. Subsequently, an entirely algebraic method, called the "coefficients of unit moment" method,[27] was developed which took variable moment of inertia, as well as sidesway adjustments, into consideration.

On the assumption that there is no sidesway, the general equation of the exact partly restrained end moment at end A of a member AB, due to applied load on the member, is given by:

$$M'_A = M_A\,(1 + t_{ab}) + M_B\, t_{ba}\, Z'_{ab} \ldots\ldots\ldots\ldots\ldots\ldots (64)$$

in which t_{ab} and t_{ba} (the writer's "coefficients of unit moment" and Professor Brumfield's "torque split") represent the exact moment induced in a member at a joint due to a unit moment applied at the same joint; and Z'_{ab} is the coefficient of unit moment, or the moment at the partly restrained end A of the beam AB due to a unit moment at the hinged end B. Furthermore,

$$M_A = \frac{\dfrac{\phi_\beta\, 6\, I}{L^2}\,(M_{sa}) - \dfrac{\phi_\alpha\, 12\, I}{L^2}\,(M_{sb})}{4\,\phi_\alpha\,\phi_{ab} - \phi^2{}_\beta} \ldots\ldots\ldots\ldots\ldots (65a)$$

and

$$M_B = \frac{\dfrac{\phi_{ab}\, 12\, I}{L^2}\,(M_{sa}) - \dfrac{\phi_\beta\, 6\, I}{L^2}\,(M_{sb})}{4\,\phi_\alpha\,\phi_{ab} - \phi^2{}_\beta}; \ldots\ldots\ldots\ldots\ldots (65b)$$

In Eqs. 65: ϕ_β, ϕ_α, and $\phi_{\alpha b}$ are form factors, or constants of integration depending upon the variation in moment of inertia of a beam; and M_{sa} and M_{sb} are numerical values of the moment of the simple-beam, M/I-area, due to external loading on beam AB taken about points A and B, respectively. For a constant moment of inertia, Z'_{ab} in Eq. 64 is equal to:

$$Z'_{ab} = \tfrac{1}{2} f = C_{BA} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (66a)$$

After a slight rearrangement, Eq. 4 may be written:

$$C_{BA} = \tfrac{1}{2}\left[\frac{1}{1 + \dfrac{1}{2\left(\dfrac{Q}{i_1} + \dfrac{Q}{i_2}\right)}}\right] \ldots\ldots\ldots\ldots\ldots\ldots (66b)$$

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[26] "Neuere Methoden zur Statik der Rahmentragwerke," by A. Strassner, Vol. 1, Ernst, Berlin, 1925–1927.

[27] "Solution of Rigid Frames by Coefficients of Unit Moment," by Alexander Dodge, thesis presented to the University of Washington at Seattle, Wash., in 1939, in partial fulfilment of the requirements for the professional degree of Civil Engineer.

combining Eqs. 66:

$$\frac{1}{1+\dfrac{1}{2\left(\dfrac{Q}{i_1}+\dfrac{Q}{i_2}\right)}} = f \text{.....................} (67)$$

The left side of Eq. 67 is the mathematical expression of the symbolic definitions, but the "end fixity," f, has been shown to represent the clear physical concept, simply derived from the knowledge of the stiffnesses of the joining beams only;[28] that is,

$$f = \frac{\Sigma S_p}{\Sigma S_p + S} \text{.......................} (68)$$

A very simple relation also exists between the "end fixity" and the "transmission coefficient." When f is equal to unity (100% end fixity for the fixed end), the value of C equals 0.5; and, when f is equal to zero (0% end fixity for the hinged end), the value of C becomes zero also. For any variation of the moment of inertia and for a partial fixity, the simple linear relation still exists between f and C. Eq. 64 was developed with the expectation that it might be used on a broad scale in engineering practice, but this expectation has never been realized because in rigid frame analyses sidesway adjustments may modify, considerably, the results found by Eq. 64. Eqs. 18 through 38 belong to the same category and from the practical standpoint can scarcely prove satisfactory to designers.

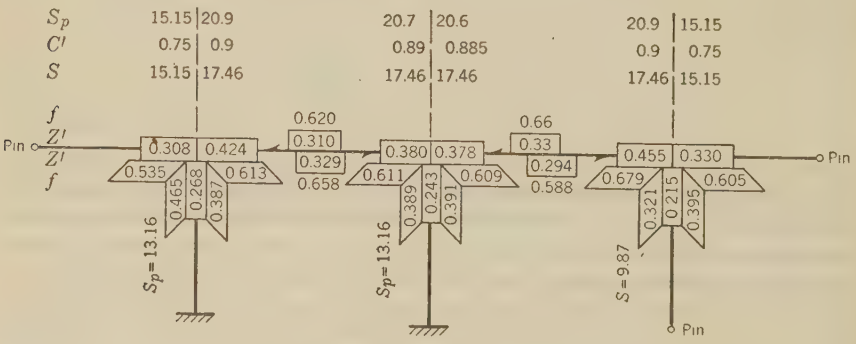
S_p
C'
S
f
Z'
Z'
f
Pin
15.15
20.9
0.75
0.9
15.15
17.46
0.308
0.424
0.535
0.613
0.465
0.268
0.387
$S_p = 13.16$
0.620
0.310
0.329
0.658
20.7
20.6
0.89
0.885
17.46
17.46
0.380
0.378
0.611
0.609
0.389
0.243
0.391
$S_p = 13.16$
0.66
0.33
0.294
0.588
20.9
15.15
0.9
0.75
17.46
15.15
0.455
0.330
0.679
0.605
0.321
0.215
0.395
$S = 9.87$
Pin
Pin

FIG. 23.—COEFFICIENTS OF UNIT MOMENT

The exact partly restrained end moment in a member due to end translation is expressed by Eq. 64, except that, in this particular case (compare Eq. 65a):

$$M_A = \frac{M_B\,(\phi_\beta + 2\,\phi_\alpha)}{\phi_\beta + 2\,\phi_{\alpha b}} \text{....................} (69)$$

In Eq. 65, Eq. 69, and following discussion the modulus of elasticity is assumed to be constant; or $E = 1$.

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[28] *Transactions*, Am. Soc. C. E., Vol. 111 (1946), p. 426.

Computation of "symbolic" quantities is a tedious process that has no physical meaning in a designer's mind. It will always be the source of numerous errors, especially if the computations are not performed as a matter of daily routine work. The introduction of unusual terms and definitions, such as S, e, ϵ, i, and R, which have no clear physical concepts, complicates the analysis unnecessarily, serves no particular purpose, and could be avoided. For example, if the frame in Fig. 12 were analyzed on the assumption that ends A_1 and E_1 were hinged, the necessary paper work would be limited to computations shown in Fig. 23, in which the author's basic Q-values have been replaced by the exact stiffnesses of the members as shown in Table 7.

TABLE 7.—EQUIVALENT S-VALUES, FRAME IN FIG. 12

Stiffness	A_1B_1	B_1B_0	B_1C_1	C_1C_0	C_1D_1	D_1D_0	D_1E_1
Q..........	0.198	0.304	0.172	0.304	0.172	0.304	0.198
S..........	15.15	....	17.46	....	17.46	9.87	15.15
S_p.........	....	13.16	....	13.16	....	....	....

Since the Cross method of analysis has been developed, it has been known that for members with fixed ends $S_p = 4\,I/L$ and for members with hinged ends $S = 3\,I/L$. Although Fig. 23 contains all characteristics that are recorded in Fig. 12, computation of the values shown in Table 1 is not necessary, and the procedure begins directly with step 6 of Section 10. The "moment split" m (the writer's "moment joint coefficient," J) for the members B_1A_1 and B_1B_0, respectively, will be

$$m_{ba} = \frac{S_p}{\Sigma S_p} = \frac{15.15}{15.15 + 13.16} = 0.535$$

$$m_{bb} = \frac{13.16}{28.31} = 0.465$$

$$\Sigma m_b = 1.000$$

The computations in step 7 would then be, by Eq. 68, $f = \frac{28.31}{28.31 + 17.46} = 0.620$; and $C_{1CB} = Z' = \frac{1}{2} f = 0.310$. In step 8 with $f = 0.620$ and the corresponding value[28] of $C' = 0.89$, the exact stiffness of the member B_1C_1 at joint C_1 is readily determined by the relation—

$$S_p = \frac{C'\,S}{C_d} \quad \text{.........................} (70)$$

—that is, $S_p = \frac{0.89 \times 17.46}{0.75} = 20.7$. In Eq. 70 the symbol C_d is a constant that can be read from design charts prepared by Thor Germundsson,[29] Assoc. M. Am. Soc. C. E. For a member of uniform section this constant is equal to 0.75. The value of S_p having been determined, the moment splits are computed thus: $m_{cb} = \frac{20.7}{20.7 + 13.16} = 0.611$; and $m_{cc} = \frac{13.16}{33.86} = 0.389$. The operations are repeated in this very simple and direct way until all characteristics are obtained.

---

[29] "Design of Continuous Frames Having Variable Moment of Inertia," by Thor Germundsson *Civil Engineering*, October, 1932, p. 647.

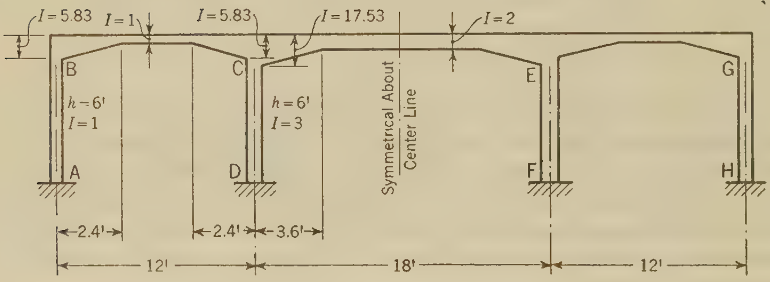
The torque splits, or "coefficients of unit moment," are also evaluated very simply and directly. For example, for joint B_1,

$$t_{ba} = \frac{S_p}{\Sigma S_p} = \frac{15.15}{15.15 + 13.16 + 20.9} = 0.308$$

$$t_{bb} = \frac{13.16}{49.21} = 0.268$$

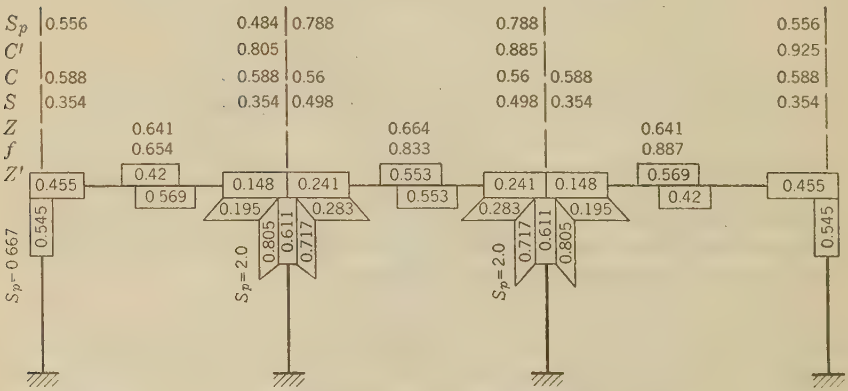
$$t_{be} = \frac{20.9}{49.21} = 0.424$$

$$\Sigma t_b = 1.000$$



I = 5.83
I = 1
I = 5.83
I = 17.53
I = 2
B
C
E
G
h = 6'
I = 1
h = 6'
I = 3
Symmetrical About Center Line
A
D
F
H
2.4'
2.4'
3.6'
12'
18'
12'

(a) GENERAL DATA



S_p
C'
C
S
Z
f
Z'
0.556
0.588
0.354
0.641
0.654
0.484
0.805
0.588
0.354
0.788
0.56
0.498
0.664
0.833
0.788
0.885
0.56
0.498
0.588
0.354
0.641
0.887
0.556
0.925
0.588
0.354
0.455
0.42
0.569
0.148
0.241
0.553
0.553
0.241
0.148
0.569
0.42
0.455
0.195
0.283
0.283
0.195
0.545
0.805
0.611
0.717
0.717
0.611
0.805
0.545
$S_p = 0.667$
$S_p = 2.0$
$S_p = 2.0$

(b) COEFFICIENTS OF UNIT MOMENT

FIG. 24.—SOLUTION OF A RIGID FRAME

Other "coefficients of unit moment" or "torque splits" are found in a similar manner.

The writer objects strongly to the term "torque split" for the reason that, in a structural analysis, the word "torque" is associated with twisting action, causing primarily shear stresses in a member. Moreover, the design of structures often incorporates the analysis of noncoplanar members, in which a moment acting at the joint may be resisted partly by the bending of some members and partly by the twisting of the other members. The term "torque split" would apply more consistently to the analysis of a spandrel girder receiving a twisting moment when this moment is split in unequal amounts to two segments of the girder; but, for the sake of clearness as well as uniformity in engineering literature, it would be advisable to stay within the customary definitions of bending and twists.

To distribute a moment to a frame of the general type (that is, a frame with a sidesway and variable moment of inertia) in a single operation, it is necessary to determine "true frame characteristics" or "true moment coefficients." At the designer's discretion, there are several optional methods of computing the "true moment coefficients." The following discussion does not infer that it presents the only procedure available but, rather, that it offers the simplest and the most systematic treatment of the problem. The true moment coefficients due to a unit moment m_{BC} may be derived as follows:

TABLE 8.—COMPUTATION OF INFLUENCE ORDINATES

Moments	COEFFICIENTS OF UNIT MOMENT, FOR POSITIVE UNIT VALUES OF:						INFLUENCE ORDINATES FOR MOMENT[a]		
	m_{BC}	m_{CB}	m_{BC}	m_{CB}	m_{BC}	m_{CB}	$M_{Bf} = -2.18$	$M_{Cf} = 0.44$	Total (kip-ft)
	Beam		Sidesway		True Moment				
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
M_{AB}	− 0.273	0.031	+ 0.043	0.051	− 0.230	0.082	0.50	0.04	0.54
M_{BC}	+ 0.545	− 0.062	− 0.033	− 0.039	+ 0.512	− 0.101	− 1.12	− 0.04	− 1.16
M_{CB}	− 0.259	0.852	− 0.034	− 0.040	− 0.293	0.812	0.64	0.36	1.00
M_{CD}	+ 0.186	− 0.611	+ 0.089	0.104	+ 0.275	− 0.507	− 0.60	− 0.22	− 0.82
M_{DC}	+ 0.093	− 0.305	+ 0.126	0.146	+ 0.219	− 0.159	− 0.48	− 0.07	− 0.55
M_{CE}	+ 0.073	− 0.241	− 0.054	− 0.064	+ 0.019	− 0.305	− 0.04	− 0.13	− 0.17
M_{EC}	+ 0.040	− 0.133	− 0.054	− 0.064	− 0.014	− 0.197	0.03	− 0.09	− 0.06
M_{EF}	− 0.032	0.107	+ 0.089	0.104	+ 0.057	0.211	− 0.12	0.09	− 0.03
M_{FE}	− 0.016	0.054	+ 0.126	0.146	+ 0.110	0.200	− 0.24	0.09	− 0.15
M_{EG}	− 0.008	0.026	− 0.034	− 0.040	− 0.042	− 0.014	0.09	− 0.01	0.08
M_{GE}	− 0.003	0.011	− 0.033	− 0.039	− 0.036	− 0.028	0.08	− 0.01	0.07
M_{HG}	+ 0.001	− 0.005	+ 0.043	0.051	+ 0.044	0.046	− 0.10	0.02	− 0.08

[a] Ordinates of the influence line for moment (in kip-ft) caused by a unit load at the quarter points of beam BC.

1. Compute the torque split ("coefficients of unit moment") shown in Fig. 24;

2. Compute "beam coefficients" due to $m_{BC} = +1$ on the assumption that no sidesway exists and record them in Col. 2, Table 8 (this step is the equivalent of the author's procedure outlined in Section 13);

3. Find the horizontal reaction R_{mB} due to m_{BC}, which prevents the translation of the joints ($R_{mB} = -\frac{1}{6}(0.273 + 0.545 - 0.186 - 0.093 + 0.032 + 0.016 - 0.003 - 0.001) = -0.097$ kips);

4. Compute sidesway moments in the frame due to $H = 1$, as shown in Table 9;

5. Find "sidesway coefficients" due to $-R_{mB}$, and record them in Col. 4, Table 8; and

6. Find the "true moment coefficients" due to m_{BC} by adding the values of Cols. 2 and 4, and record them in Col. 6, Table 8.

TABLE 9.—Sidesway Moments Caused by Various Fixed-End Moments

Sidesway moments	$M_{AB} = 1$ and $M_{BA} = 1$	$M_{DC} = 3$ and $M_{CD} = 3$	$M_{FE} = 3$ and $M_{EF} = 3$	$M_{HG} = 1$ and $M_{GH} = 1$	$H = -1.793\ k$	$H = +1\ k$
M_{AB}	0.727	0.093	− 0.016	0.001	0.805	− 0.448
M_{BA}	0.455	0.186	− 0.033	0.003	0.611	− 0.341
M_{CB}	− 0.259	− 0.444	0.078	− 0.008	− 0.633	+ 0.353
M_{CD}	0.186	1.167	0.323	− 0.032	1.644	− 0.916
M_{DC}	0.093	2.083	0.161	− 0.016	2.321	− 1.295
M_{CE}	0.073	− 0.723	− 0.401	0.040	− 1.011	+ 0.563
M_{EC}	0.040	− 0.401	− 0.723	0.073	− 1.011	+ 0.563
M_{EF}	− 0.032	0.323	1.167	0.186	1.644	− 0.916
M_{FE}	− 0.016	0.161	2.083	0.093	2.321	− 1.295
M_{EG}	− 0.008	0.078	− 0.444	− 0.259	− 0.633	+ 0.353
M_{GH}	0.003	− 0.033	0.186	0.455	0.611	− 0.341
M_{HG}	0.001	− 0.016	0.093	0.727	0.805	− 0.448

The Germundsson charts were used for the computation of beam constants, or the elastic properties of members which are independent of the framed structure. These computations are shown in Table 10. Steps 1 and 2 are computed by slide rule.[30] Computations for step 4 are similar to those for step 2. The sidesway moments from any horizontal force applied at the top of columns are found first. These values are reduced to moments generated by the force $H = 1$, as shown in Table 9.

TABLE 10.—Computation of Beam Constants

Members	a	b	$Z = r$	C	$S = C_s\, I/L$ [a]
BC and EG	2.4/12 = 0.2	1/5.83 = 0.171	0.641	0.588	4.25/12 = 0.354
CE	3.6/18 = 0.2	2/17.55 = 0.114	0.664	0.56	4.48 × 2/18 = 0.498
AB and HG	....	....	0.5	0.75	4/6 = 0.667
DC and EF	....	....	0.5	0.75	4 × 3/6 = 2.0

[a] In this formula the symbol C_s is a constant that can be read from the design charts.[30]

Thus far the discussion has been limited strictly to the computation of the "true moment coefficients" due to $m_{BC} = +1$. True moment coefficients will be changed, of course, with the point of application of a unit moment, but for any new position, such as will be defined by the subscript of the unit moment m_{CB}, the additional work recorded in Cols. 3, 5, and 7, Table 8, is negligible. After the sixth step, it is possible to distribute a moment to the frame in a single operation. Nevertheless, because the derivation of these coefficients may be divided into two distinct operations—an analysis of a frame without sidesway and evaluation of sidesway corrections—it would be more correct to describe the analysis as moment distribution in two operations. Thus, for a two-story frame, there will be two sidesway corrections, and the analysis will require three distinct operations. A rigid frame with N sidesway forces will have N corrections and $N + 1$ distinct operations.

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[30] *Transactions*, Am. Soc. C. E., Vol. 111 (1946), p. 430.

For practical purposes, contrary to the author's opinion, the writer still believes that the design of structures for moving loads is rendered much easier by influence lines than by any other method. This will be even more true of a structure that must be designed for various conditions of loading; but any important structure may be expected to have a set of design conditions. Even in a preliminary design of a continuous beam, the author's formulas could scarcely compete with the old fixed-end moment formulas, because, in the former, as a result of a series of abstract computations, the chances for mistakes are innumerable, and there is no way to "feel" the errors. However, in the latter the maximum limit of moments is given at once, and the values of the exact moments, depending on the end fixity, may be correctly estimated without the complete analysis. It will now be shown how simple moments due to a unit load placed, say, at the quarter point of beam BC may be determined. The fixed-end moments from this load will be: $M_{Bf} = - 0.182 \times 12 = - 2.18$ kip-ft; and $M_{Cf} = 0.037 \times 12 = 0.44$ kip-ft.

These moments multiplied by the values of Cols. 6 and 7, Table 8, will yield the products recorded in Cols. 8 and 9, respectively. Summation of Cols. 8 and 9 in Col. 10 gives the final distributed moments and, at the same time, the values of influence ordinates. If the computations of Cols. 8, 9, and 10 were repeated for a unit load placed at the center line and three-quarter point of the span, the complete values of the influence ordinates, for a load on beam BC, would be obtained. From a practical standpoint, these load positions were found to be entirely satisfactory for the accurate construction of influence lines. The writer, at least, has never had occasion to use more points than those suggested herein.

The definitions of "normal" and "reverse," as applied to bending moments, are definitely inferior to the brief and simple definition: "Clockwise moments are positive." The value of this simpler definition may be realized and appreciated to the greatest extent by one familiar with an analysis of frames in three dimensions, such as the twist analysis of a dam or the analysis of a structure on an elastic foundation. How difficult it would be to perform these analyses with the definitions proposed by the writer may be realized by consideration of the equation of joint equilibrium, which would be stated as follows: The summation of normal bending and clockwise twisting moments, minus reverse bending and counterclockwise twisting moments, plus or minus an externally applied couple at the joint, is equal to zero. In contrast to this phraseology is the simplicity of the commonly adopted statement of joint equilibrium, namely: "The summation of moments is equal to zero; clockwise moments are positive." Contrary to the author's opinion, the old definition of "clockwise moments are positive" fits the analysis of beams radiating in all directions exceptionally well—especially when many loading conditions must be investigated. The old definition fixes the sense of the bending moment in such a manner as to avoid confusion.

The author's statement (Section 13) that "One distribution also is sufficient for all purposes, including live-load and dead-load moment distribution, wind moments, and sidesway * * *" is not clear. Because the allowable unit stresses, as a rule, vary with the particular combination of loadings, a separate distribution should be made for each of the aforementioned loads.

Of incidental interest, but nonetheless important, is the question of nomenclature. Every new paper on this subject yields its own set of nomenclature and abbreviations. If writers would take full advantage of the commonly accepted symbols and restrict the introduction of new symbols to the absolute minimum, they would render the engineering profession a priceless service as well as facilitate the task of education. For example, there are only three really important structural concepts that need to be known for analysis of structures. They have been established previously and again are presented in this paper but, being differently derived, are given new definitions and symbols. Excluding foreign sources, the equivalent symbols for these concepts are known as follows: Transmission coefficient C equals carry-over factor f equals coefficient of unit moment Z'; torque split t equals distribution factor d equals coefficient of unit moment K; and moment split m equals joint moment coefficient J.

It is not clear why the span of a simple structure should be designated by three symbols such as 1BC, and why the same subscript should be needed to define the bending moment. In most structural analyses in two dimensions, two letters (or two subscripts in the case of a moment) are sufficient. If the first subscript were always selected to designate the location, the term M_{AB} would define the moment at point A in span (or member) AB. The same system would hold true for shear, angle, deflection, and twisting moment. In the case of an angle, two subscripts will be needed only when the efficiency of a joint is less than 100%. For isolated members, or for single spans, the second subscript, of course, would be omitted because the first subscript completely defines the location of the point. A similar system of lower-case letter subscripts could be adopted for the description of other properties of members. The selection of lower-case letters, although not absolutely necessary, would facilitate recognition of apparently equal terms, such as S_{AB} and S_{ab}, at a glance. The first term would refer to the shear and the second to the stiffness at point A of member AB. The transmission coefficient, or the exact carry-over factor with the subscript ab in the same system, would signify the exact moment at point A due to a unit moment at point B, which, evidently, would be contrary to the author's notation.

Sometimes a distinction may need to be made between hinged, fully fixed, or partly fixed, end conditions. In such cases the writer suggests the use of lower-case letters o, f, and p, respectively, for the third subscript. This appears to be a contradiction of the foregoing statement, but in reality it is not, because in the latter case three independent structures are being analyzed. Instead of verbal descriptions, a simple subscript is utilized for this distinction. For example, M_{ABf} would define the fixed-end moment at point A of member AB. Were one or both of the capital subscripts missing, M_f would still be recognized as a fixed-end moment. Not all the three subscripts o, f, or p would have to be retained in the same analysis; the predominating subscript might be omitted at the discretion of the designer.

For analyzing multistoried frames, with many members and joints, a grid pattern of numbering may be used to advantage. In this pattern, a joint is defined by the intersection of two lines and, therefore, one letter and one number will be needed for identification. The grid pattern of numbering will not be in conflict with the foregoing described system of abbreviations, if it is remembered that the first two subscripts define the point and the third subscript, the direction. For instance, M_{1BC} will define the moment at point 1B in the member BC.

Frederick S. Merritt,[31] Assoc. M. Am. Soc. C. E.—Inevitably, in practice, simplifications of theoretical solutions will occur to the designer. Such simplifications of Professor Brumfield's method, made by the writer as a result of experience with practical design problems, have been so numerous that only the basic assumptions have been untouched. For the present discussion Professor Brumfield's "transmission coefficients" may be compared with the writer's "moment ratios."[32]

The action of a beam in a continuous structure depends upon its shape and size relative to the remainder of the structure. Constants evaluating these relative shape and size characteristics may be selected and defined almost at will; but, of course, if they are to be of practical value, they should be few in number and sufficiently descriptive of the action of the member. For the purpose of this discussion, three fundamental constants—K, $m^F{}_R$, and $m^F{}_L$—will be used. Clockwise end moments and end rotations will be considered positive and counterclockwise moments and rotations negative.

Consider an unloaded member removed from a monolithic structure and simply supported instead. Then, K is the moment that must be applied at one end of this beam to produce unit rotation at the other end. Now, fix the right end of the member and apply a unit moment at the left end. For this condition, the moment that will be produced at the right, or fixed, end is $1/m^F{}_R$. Similarly, the carry-over ratio for a fixed end at the left is $1/m^F{}_L$. For beams of constant moment of inertia, $m^F{}_R = m^F{}_L = 2$ and $K = 6\,E\,I/L$.

The coefficients K and m^F are sufficient to describe the action of this member when it is once again returned to its position in the continuous structure. Stiffness s of the end of an unloaded beam in a monolithic structure is defined as the moment producing a rotation of unity at the end where it is applied, when the other end of the beam is restrained against rotation by other members of the structure. For an unloaded member of a continuous structure the moment ratio m is defined as the ratio of the moment introduced at one end of the span to the moment carried over, or transmitted, to the other end.

Then, the stiffness of the right end of a beam can be shown to be:

$$s_R = \frac{K}{m^F{}_R - \dfrac{1}{m_L}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (71a)$$

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[31] Editorial Asst., *Engineering News-Record*, New York, N. Y.

[32] "Moment Ratios Simplify Continuity Analysis," by Frederick S. Merritt, *Engineering News-Record*, November 1, 1945, p. 112 and November 15, 1945, p. 102.

Similarly, the stiffness of the left end is:

$$s_L = \frac{K}{m^F{}_L - \dfrac{1}{m_R}} \quad \text{(71}b\text{)}$$

For beams of constant moment of inertia:

$$s_R = \frac{K}{2 - \dfrac{1}{m_L}} \quad \text{(72}a\text{)}$$

and

$$s_L = \frac{K}{2 - \dfrac{1}{m_R}} \quad \text{(72}b\text{)}$$

Eqs. 71 and 72 should be compared with Professor Brumfield's stiffness index formulas (Eqs. 3).

The restraint r at the end of an unloaded beam at a joint is defined as the moment applied at that end to produce a unit rotation in all the members of the joint.

Since, by definition of continuity, the end rotations of all the members of a joint are equal, restraint of the member under consideration is equal to the sum of the stiffnesses of the adjacent ends of the other members of the joint. Furthermore, the moment induced in any of these other members bears the same ratio to the applied moment as the stiffness of the member does to the restraint. Consequently, end moments are distributed at a joint in proportion to the stiffnesses of the members. These relationships should be compared with Professor Brumfield's restraint index formulas, moment splits, torque splits, and technique of distribution (Eqs. 9, 39, 40, and 41).

Moment ratios can be computed in terms of K and, through r, the moment ratios of preceding spans. It can be shown that:

$$m_R = m^F{}_R + \frac{K}{r_R} \quad \text{(73}a\text{)}$$

and

$$m_L = m^F{}_L + \frac{K}{r_L} \quad \text{(73}b\text{)}$$

For beams of constant moment of inertia,

$$m_R = 2 + \frac{K}{r_R} \quad \text{(74}a\text{)}$$

and

$$m_L = 2 + \frac{K}{r_L} \quad \text{(74}b\text{)}$$

These formulas should be compared with Professor Brumfield's formulas for transmission coefficients (Eq. 5).

End moment formulas may be given in terms of moment ratios as follows:

$$M_R = -\frac{K\,(m_L\,\phi_R + \phi_L)}{m_R\,m_L - 1} \quad \text{(75}a\text{)}$$

and

$$M_L = -\frac{K\,(m_R\,\phi_L + \phi_R)}{m_R\,m_L - 1};\ldots\ldots\ldots\ldots\ldots\ldots(75b)$$

in which ϕ is the end rotation under load considering the member as simply supported. Fixed-end moments may be obtained by setting the moment ratios in Eqs. 75 equal to $m^F{}_R$ and $m^F{}_L$, respectively. These formulas, also, should be compared with the corresponding ones given in the author's paper (Eqs. 14 and 27).

When the suggested comparisons are made, it may be seen that in several of the formulas the use of moment ratios decreases the number of arithmetical steps involved, particularly for the cases of variable moment of inertia. There is no arithmetical advantage in using transmission coefficients, which are reciprocals of moment ratios, since multiplication is as easy as division. Similarly, computation of stiffness as the ratio of two factors involves no more steps than that of stiffness index, which is the product of two factors. However, calculation of restraint as the sum of stiffnesses is considerably easier than that of restraint index as the reciprocal of the sum of reciprocals of stiffness indexes. Furthermore, there is a considerable advantage in moment distribution in using K, s, and r, instead of Q, i, and R, respectively, because the former bear direct relationships to the properties of the structure, the latter reciprocal. Thus, much work may be saved and a better visualization of structural action obtained by changing the reciprocal terms used by Professor Brumfield to direct ones.[32]

Alfred B. Heiser,[33] M. Am. Soc. C. E.—For a number of years the writer has searched among the indeterminate stress theories such as the Cross method and the method of fixed points to find a quick and comparatively easy method for computing indeterminate stresses; but disappointment seemed to result from these excursions until he encountered Professor Brumfield's theory as described in the present paper. Few, if any, of the theories make it possible to place loads definitely in order to obtain the maximum moments or shears either at the supports or at intermediate points in the span. It is also very difficult to decide by these theories just which spans must be loaded to obtain the maximum results. Professor Brumfield's paper fills this need. His method permits the effects at any point of a load or group of loads to be summed up, readily, at any other point throughout the entire structure however complicated the structure or grouping of loads may be.

Does this seem impossible? Not if one peruses the full theory developed by Professor Brumfield since his system has been developed to cover wind stresses, sidesway, beams of any type and shape, arches, and, best of all, structures having spans that are noncoplanar.[3] He has conquered stress analysis through three-dimensional space. The writer has personally checked many of the formulas for this advance theory and has found that, although they are complicated, they are workable.

In all cases, these formulas cover the general as well as the special cases. As the span member or load conditions are simplified, the equations are reduced to forms familiar to all—that is, the formulas used in ordinary beam design.

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[33] Chf. Draftsman, Am. Can Co., New York, N. Y.

T. B. Rights,[34] Assoc. M. Am. Soc. C. E.—This method is not new or untried. The Cooper Union, in New York, N. Y., has offered this course for graduate credit, since 1938. Professor Brumfield filed his paper, "Moving Loads on Beams with Restrained Ends," in the Engineering Societies Library in 1937. The task of placing a moving load for maximum moment at any point on a restrained beam is quite difficult by older methods. In the Hardy Cross method,[2] it meant that unit loads must be placed successively across the beam and their influence calculated, which is quite time consuming. By the use of Anger's tables,[35] it is comparatively simple to select positions for maximum moment. These tables are for uniform cross sections, however, and any haunching would affect the results. Values for the usual range of span lengths in two, three, and four spans are given, but any unusual span ratios would prevent the use of these tables. Furthermore, these tables, having been printed in Germany, are not available in the United States. The tables by Walter Ruppel, Assoc. M. Am. Soc. C. E., should also be mentioned.[36]

It would be interesting if the author would show how his method, applied to tall building frames, gives a fast and accurate solution to this difficult problem.

Once this method is mastered, it will be found to be a quick solution for most indeterminate stress analyses. The writer can set up a three-span or a four-span problem faster by the Cross method; but, for any problems of involved conditions, the Brumfield method is faster because the moment at the joint is only "transmitted once." For moving loads on a continuous span, this method is a "must."

Phil M. Ferguson,[37] M. Am. Soc. C. E.—In so far as the writer knows, this paper is the first presentation in English of an analytical method for locating moving loads so as to cause maximum moment. That the criteria developed are not simple does not in any way detract from the definite advance in thought that this method represents. The key to this advance lies in certain advantages inherent in designating the location of wheels in a load system in terms of distances from the resultant of the system, or, as the author states, from the center of gravity. The writer regards this as a development of considerable importance, even if it is, as now presented, limited strictly to beams of uniform section.

The use of Professor Brumfield's criteria does not require adoption of the author's general procedure for frame analysis. The criteria (Eqs. 29, 32, 33, 34, and 35) involve loading constants and the so-called transmission factors C_L and C_R. Only C_L and C_R involve the author's method of frame analysis. However, these transmission factors are identical with the "modified carry-over factors" introduced by T. Y. Lin,[38] Jun. Am. Soc. C. E., and, of course, are related to the fixed points that are usually determined graphically. Hence, the methods are also available in connection with the loading criteria.

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[34] Cons. Engr., Roselle, N. J.

[35] "Zehnteilige Einflusslinien," by Anger, Ernst and Son, Berlin, 1939.

[36] *Transactions*, Vol. 90 (1927), p. 167.

[37] Chairman, Civ. Eng., Univ. of Texas, Austin, Tex.

[38] "A Direct Method of Moment Distribution," by T. Y. Lin, *Transactions*, Am. Soc. C. E., Vol. 102 (1937), p. 561.

The development of the criteria for the position of loads might not have resulted from any other approach to frame analysis than that of the author. Nevertheless, the writer considers this particular approach to the direct method of moment distribution definitely inferior to the Lin method in several respects and would urge that the author's excellent ideas be used for moving loads applying either the Brumfield or the Lin idea of the direct method of moment distribution (basically the same method) and that the author's method of calculating the frame constants be discarded entirely.

The writer has three basic criticisms of the author's method of frame analysis:

1. A new vocabulary of specialized terms is required, most of which are not essential to this type of analysis and the remainder of which could be clarified by correlation with the basic Cross nomenclature;
2. As given, the method is not directly applicable to members of variable section, and such adaptation would seem to require extensive alterations; and
3. The method abandons the principle that has been most productive in American methods of frame analysis—namely, the use of fixed-end moments for the initial step.

In addition, the writer feels that the system of subscripts used by the author is nonspecific enough to create unnecessary difficulty for the student of this method.

The author prefaces his paper (see heading, "1. Definitions and the Nomenclature of Unusual Terms") with the statement:

> "Inherent in the usefulness of the paper * * * are certain symbolisms peculiar to the method itself, and the nomenclature needs to be understood clearly to grasp the full import of the paper."

This nomenclature is an unnecessary handicap because it is truly an entirely new vocabulary. Several symbols represent concepts applicable only to this method. A new symbol Q is even introduced to designate the reciprocal of the stiffness K—apparently only because it looks simpler to write Q than to write $\frac{1}{K}$ or $\frac{L}{I}$. In contrast, Mr. Lin has presented an equivalent method of distributing moments to a frame in a single operation utilizing only one new symbol and two "modified" terms—modified stiffness and modified carry-over factor—both closely related to the original moment distribution terms developed by Hardy Cross,[2] M. Am. Soc. C. E. Since it may be presumed that American engineers will first know something of Professor Cross' basic method, notation related to it will always be preferable in introducing new ideas in moment distribution. Not only does Mr. Lin's[38] method involve simpler notation, but it is applicable, without modification, to members of variable sections.

In addition to a new terminology, the author introduces a notation difficult to follow in that subscripts are not self-explanatory and are not uniform. For example, the rather definite form of subscript used in C_{AB} (Eqs. 3*a* and 3*b*) is dropped in favor of the indefinite form C_1 and C_2 for other members, and becomes C_L and C_R in connection with moments in loaded beams. There are no subscripts to identify the particular R to be used in Eqs. 5 and 8. With such uncertain notation it is difficult to picture the author's process properly as it is gradually developed. As a result, close study of Sections 10 and 13, near the end of the paper, is required to clarify Section 2.

Mr. Lin develops essentially the same method of moment distribution from short proofs based upon the original moment distribution process. The frame-constant equations in the Lin method are stated for comparison as follows:

K_{ab} is the relative stiffness of member AB (Fig. 25) based upon rotation of end A when end B is fixed. When the member has uniform section K_{ab} equals I/L.

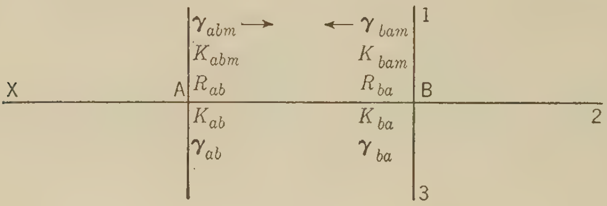

FIG. 25

K_{abm} is the modified relative stiffness of member AB based upon rotation of end A when end B is considered restrained only by the remainder of the frame.

γ_{ab} is the carry-over factor from end A to end B when end B is fixed (0.5 when beam has uniform section).

γ_{abm} is the modified carry-over factor when end B is considered restrained only by the remainder of the frame. Factor γ_{abm} is identical with the author's C_{AB}.

R is a ratio defined by the expression:[39]

$$R_{ba} = \frac{K_{ba} + K_{b1m} + K_{b2m} + K_{b3m}}{K_{ba}} \qquad (76)$$

R_{ba} is infinite if end B is fixed, and is unity if end B is pinned.

Mr. Lin derives the following relations:

$$K_{abm} = K_{ab}\left(1 - \frac{\gamma_{ab}\,\gamma_{ba}}{R_{ba}}\right) = K_{ab}\left(\frac{R_{ba} - \gamma_{ab}\,\gamma_{ba}}{R_{ba}}\right) \qquad (77a)$$

and

$$\gamma_{abm} = \gamma_{ab}\left(\frac{R_{ba} - 1}{R_{ba} - \gamma_{ab}\,\gamma_{ba}}\right) \qquad (77b)$$

For members of constant section, $\gamma_{ab} = \gamma_{ba} = 0.5$, and these equations become simply:

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[39] "A Direct Method of Moment Distribution," by T. Y. Lin, *Transactions*, Am. Soc. C. E., Vol. 102 (1937), Eq. 1, p. 562.

$$K_{abm} = K_{ab}\left(\frac{R_{ba} - 0.25}{R_{ba}}\right); \dots\dots\dots\dots\dots\dots (78a)$$

and

$$\gamma_{abm} = 0.5\left(\frac{R_{ba} - 1}{R_{ba} - 0.25}\right) \dots\dots\dots\dots\dots\dots (78b)$$

The quantities in parentheses in Eq. 78*a* and Eq. 78*b* are susceptible to simple tabulation for various values of R, but the equations are also simple to use with the slide rule. It should be noted that R in Mr. Lin's paper is not the same as that in the Brumfield paper, but is to be evaluated from Eq. 76.

Since R_{ba} at a fixed end B is infinity and at a pinned end B is unity (and at other terminals can be estimated), it is easy to use Eqs. 78 or Eqs. 77 to find K_{abm} and γ_{abm} at the opposite or inside end of any terminal member. The resulting value of K_{abm} and similar modified stiffness values of other terminal members meeting at joint A can then be used in Eq. 76 to find R_{ax} in the next interior member AX; and R_{ax} is used in turn to find the modified stiffness carry-over factor at end X of member AX. It must be kept in mind that the sequence of subscripts used with carry-over factors indicates the direction in which the carry-over moment is being transmitted—that is, γ_{ab} or γ_{abm} indicates that the direction is from end A to end B.

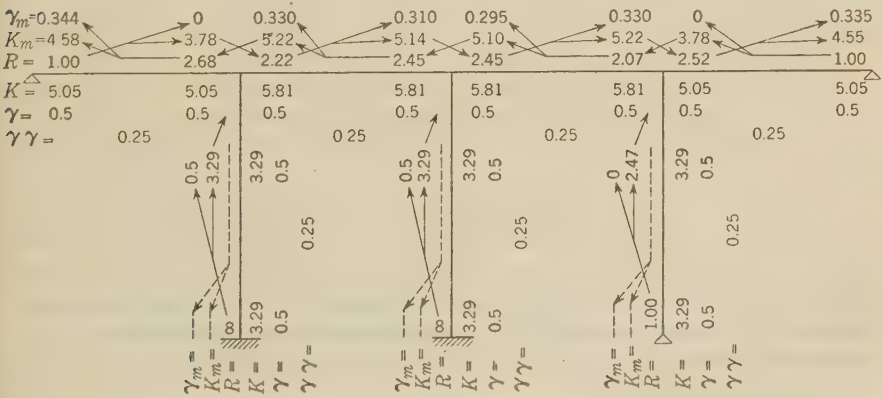
γ_m=0.344
K_m=4 58
R= 1.00
0
3.78
2.68
0.330
5.22
2.22
0.310
5.14
2.45
0.295
5.10
2.45
0.330
5.22
2.07
0
3.78
2.52
0.335
4.55
1.00
K= 5.05
γ= 0.5
$\gamma\gamma$=
0.25
5.05
0.5
5.81
0.5
0 25
5.81
0.5
5.81
0.5
0.25
5.81
0.5
5.05
0.5
0.25
5.05
0.5
0.5
3.29
3.29
0.5
0.25
∞
3.29
0.5
γ_m=
K_m=
R=
K=
γ=
$\gamma\gamma$=
0.5
3.29
3.29
0.5
0.25
∞
3.29
0.5
γ_m=
K_m=
R=
K=
γ=
$\gamma\gamma$=
0
2.47
3.29
0.5
0.25
1.00
3.29
0.5
γ_m=
K_m=
R=
K=
γ=
$\gamma\gamma$=

FIG. 26

To illustrate the sequence of these frame-constant calculations, the Lin constants for the frame in Table 1 of the Brumfield paper are developed in Fig. 26. The arrows indicate the sequence of calculations, not the direction of carry-over factors. Some constants not needed by the author's problem are omitted, but are indicated by dotted lines. Since the constants are tabulated with the same arrangement at each joint, no table is needed. Each constant is tabulated at the joint corresponding to its first subscript letter—that is, K_{ab} is tabulated at joint A. The K-values for beams of uniform stiffness are I/L-

ratios and are the reciprocals of the author's Q-values. Except to maintain a form suitable for variable sections, there is no need to tabulate, separately, K_{ab} and K_{ba} or γ_{ab} and γ_{ba}. For members of variable section, the unmodified K-values and γ-values would be calculated for fixed ends, exactly as in the basic moment-distribution method, and then used in Eqs. 77. No modification of the Lin method is needed for these members of variable section.

The Lin modified carry-over factor γ_{abm} can be substituted directly for its equivalent C_R in the author's criteria for the location of loads; and likewise γ_{bam} can be substituted for C_L. Fixed-end moments for the critical position of loading can then be calculated and used with the regular Lin method. This method begins with the equivalent of the author's torque split applied to fixed-end moments. The Lin method proceeds to calculate the numerical terms that would make up the restraining moments used by the author, but Mr. Lin does not segregate and label these as such.

It is also possible to calculate the restraining moments from formulas and thus eliminate the first steps of the Lin method. This would agree with the author's procedure. To develop such formulas, the moment-distribution method (Fig. 27) can be used. Let M_F represent the fixed-end moment. The distribution factor or torque split for this moment at joint A is

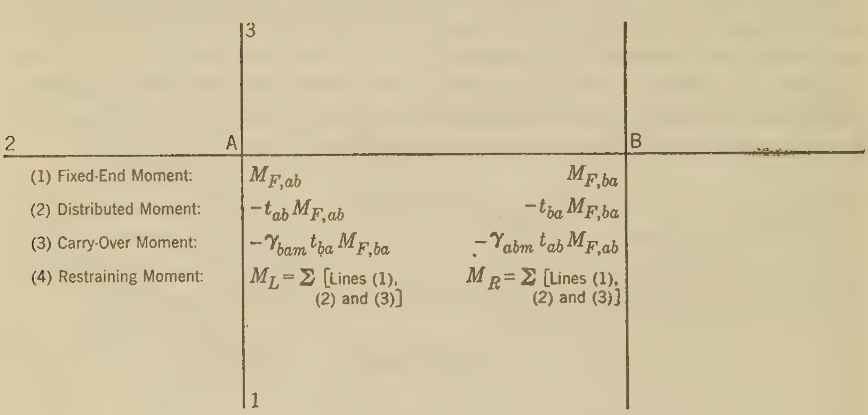

FIG. 27

$$t_{ab} = \frac{K_{abm}}{\Sigma K_{anm}} \quad \text{.........................(79)}$$

in which ΣK_{anm} represents the summation of modified stiffness for all members meeting at joint A. The distribution and carry-over terms in Fig. 27 can be written into formulas for M_L and M_R:

$$\begin{aligned} M_L &= M_{F,ab} - t_{ab}\, M_{F,ab} - \gamma_{bam}\, t_{ba}\, M_{F,ba} \\ &= M_{F,ab}\,(1 - t_{ab}) - M_{F,ba}\,\gamma_{bam}\, t_{ba} \quad \text{..............(80a)} \end{aligned}$$

and

$$\begin{aligned} M_R &= M_{F,ba} - t_{ba}\, M_{F,ba} - \gamma_{abm}\, t_{ab}\, M_{F,ab} \\ &= M_{F,ba}\,(1 - t_{ba}) - M_{F,ab}\,\gamma_{abm}\, t_{ab} \quad \text{..............(80b)} \end{aligned}$$

In the Lin method, in the first distribution and carry-over step, this operation is performed by simple arithmetic rather than by algebra. Thereafter, there is no difference whatever between the Lin and the Brumfield procedures. Mr. Lin calculates the moment split to member A1 (without naming it) as $\frac{K_{a1m}}{\Sigma K_{am}}$ in which ΣK_{am} includes the modified stiffness of all members meeting at joint A except that of member AB, the member from which the moment originates.

Even if one does not accept the writer's thesis that a method more closely related to the basic moment distribution method, and already adapted to members of variable section, is preferable to the author's method, Eqs. 80 are still worthy of consideration. They are preferable to Eqs. 14, 16, and 17, because they permit the use of existing tables for M_F. For those who follow the author's method, C would be substituted for γ_m, as noted previously. The author's modified end-moment formulas written directly in terms of k seem necessary to develop criteria for placing loads, but seem unnecessarily complex for other purposes.

In conclusion, the writer again wishes to state that the treatment of moving loads in this paper is a most worthy one. Those who may have difficulty with the background method upon which it is based should be attracted by the relatively simple concepts of Mr. Lin's approach to the necessary frame constants. The Lin concepts are closely correlated with those used in the Cross method. The actual work involved in applying either the Brumfield method or the Lin method is not greatly different, but such difference as exists seems slightly in favor of the Lin procedure in that Mr. Lin does not use reciprocals of constants.

LEON BESKIN,[40] ASSOC. M. AM. SOC. C. E.—It is customary to make a distinction between "classical" and "modern" methods of solving statically indeterminate structures. The writer has attempted a comparison of these methods in a previous discussion,[41] and he feels that undue emphasis has been placed on "modern" methods. For this reason, it was especially gratifying to read Professor Brumfield's description of a "classical" method that had been developed for the special case of continuous beams and later was extended to frames. Regrettably, however, Professor Brumfield has omitted references to many of his predecessors—thus tending to obscure his personal contribution to the subject.

The concept of a transmission factor (also called nodal ratio, intercept ratio, or carry-over factor) was developed fully by O. Mohr and by Maurice Lévy [42] for continuous beams and by more recent authors, some of them previously indicated by the writer,[41] for frames. Formulas similar to Eqs. 5 and 43*a* can be found in these references—all defining relationships between transmission factors in adjacent spans. Eqs. 14, 16, and 17, defining moments at the supports of a continuous beam under the action of loads placed on a span with the abscissa corresponding to the maximum support moment, seem to have been first established by Weyrauch and by Mr. Lévy.[43]

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[40] Design Specialist, Consolidated Vultee Aircraft Corp., Vultee Field, Calif.

[41] *Transactions*, Am. Soc. C. E., Vol. 111 (1945), p. 416.

[42] "La Statique Graphique, et Ses Applications aux Constructions," by Maurice Lévy, Gauthier-Villars & Co., Paris, France, 2d Ed., Vol. 2, 1886

Professor Brumfield extends the formula defining the abscissa corresponding to the maximum support moment to combination loads, which leads to the introduction of concepts of radius of dissymmetry, span-distribution ratio, and dissymmetry-span ratio. These results represent a definite contribution to the theory of continuous beams and frameworks; they can be generalized and could be applied as well to the case of distributed loads or to combinations of concentrated and distributed loads.

However, their practical value appears somewhat limited, because great care should be exercised when applying the formulas established by the author because they seldom define true maxima. Consider, for instance, the simple case of two concentrated loads. Depending on the ratio of distance to length of the span, the maximum moment on a support may be obtained either when the two loads are on the same span or when they are on the two sides of a support. Professor Brumfield's formulas are valid only in the first case, and the restriction is extremely inconvenient because no criterion is given to determine when the formulas are not valid. This important limitation is not mentioned by the author. In the special case of two equal loads, placed on a continuous beam with spans of equal stiffness ("*K*-value"), the author's formulas are valid only if $a_1 = a_2 < 0.37$ (see Fig. 4).

Another limitation results from the fact that, in general, diagrams for several loads are longer than the length of one span. Professor Brumfield's method cannot be applied in this case, since the effects of loads acting on one span only are considered. In addition, the method may be misleading even if the loading diagram is shorter than the span, because it may lead to such a location of the diagram that some of the loads are in the adjacent span. In this case, the formula gives an analytical maximum that has no physical significance.

Thus, contrary to the statement, "Eqs. 29 apply to all kinds of loads, with the exception of a pure couple," Professor Brumfield's method seems to have a very limited field of application. The writer feels that the method applies principally to the design of girders for overhead cranes, because the loads are generally distributed on a short length (small a-constants). For this case, however, very complete tables are available, which make any general method superfluous.

BRUCE JAMEYSON,[44] ASSOC. M. AM. SOC. C.E.—The paper dealing with a method of solution for continuous rigid frames composed of prismatic members and with criteria for moving loads is of academic value only because (1) the usual case of moving loads occurs in bridge design and involves frames composed of nonprismatic and often unsymmetrical members; (2) the paper does not fully include the effect of broken loads on adjacent and other spans; (3) the author's method already seems complicated and would become unduly so if modified to apply to frames composed of nonprismatic members; and (4) better and more general methods of approach to the problem are available.

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[43] "La Statique Graphique, et Ses Applications aux Constructions," by Maurice Lévy, Gauthier-Villars & Co., Paris, France, 2d Ed., Vol. 2, 1886, pp. 187 and 282.

[44] Prof., Civ. Eng., Univ. of California, Berkeley, Calif.

The writer considers one-cycle moment distribution, the method advanced by T. Y. Lin,[38] Jun. Am. Soc. C. E., more general and superior to any other method yet proposed, for this particular case of one-story rigid continuous frames carrying moving loads. In fact, a considerable part of the author's procedure is very similar to the Lin method—that is, his "transmission coefficients" and his "methods for distributing the moment to the frame in a single operation." The author has taken the difficult way to develop and explain this procedure. Since the loads may be broken loads, influence lines are necessary if high accuracy is expected.

The author's "transmission coefficient," "moment split," and "torque split" ("frame characteristics") are not new but are simply carry-over factors and joint distribution factors of moment distribution for the general case of any end condition of members. It would have been better to have used this latter nomenclature since it is in general use. For the same reason, the well-known stiffness-factor symbol K should have been used rather than the reciprocal, the author's Q-value. Moment distribution becomes a more powerful tool if these terms, carry-over factors (C'), stiffness factors (K'), and distribution factors, are considered as modified factors applying to members with end conditions as they exist in the structure—that is, with fixed, restrained, or free ends. Formulas and diagrams have appeared in recent literature[8,38,46] for these modified factors and have been used for some years in bridge design in conjunction with one-cycle balance of moment distribution.[38]

In addition to the final one-cycle balancing by moment distribution, the writer prefers to obtain the foregoing modified factors by moment distribution rather than by formulas or diagrams regardless of whether or not the frame is composed of prismatic members. The method is probably faster. Fig. 28 indicates this procedure and is self-explanatory to any one familiar with moment distribution. The modified carry-over factor ($C' = 0.310$) checks the author's value of the "transmission coefficient" in Table 1. Continuing

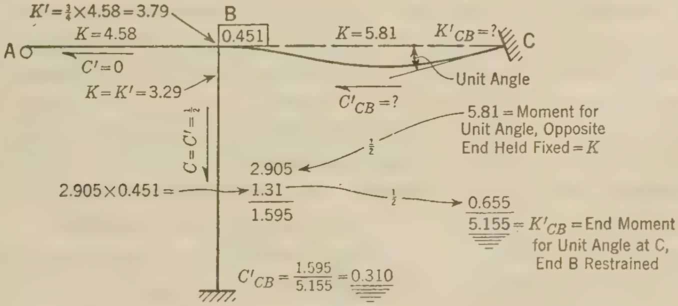
$K' = \frac{3}{4} \times 4.58 = 3.79$
B
$K = 4.58$
0.451
$K = 5.81$
$K'_{CB} = ?$
C
A
$C' = 0$
Unit Angle
$K = K' = 3.29$
$C'_{CB} = ?$
5.81 = Moment for Unit Angle, Opposite End Held Fixed = K
$C = C' = \frac{1}{2}$
$\frac{1}{2}$
2.905
$2.905 \times 0.451 =$
1.31
$\frac{1}{2}$
0.655
1.595
5.155 = K'_{CB} = End Moment for Unit Angle at C, End B Restrained
$C'_{CB} = \frac{1.595}{5.155} = 0.310$

Fig. 28

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[46] "Theory of Modern Steel Structures," by L. E. Grinter, The Macmillan Co., New York, N. Y., Vol. 2, 1936–1937, p. 192.

to the right and then proceeding back to the left in a similar manner, all values of C' and K' are found for the actual end conditions of the members in the structure. Distribution values follow from the K'-values. The distribution constants are better shown in a diagram than in a table, as the author has done, since more than one set may be required at a joint. For example, at joint B four sets are obtained, one set for each possible combination of members. These are shown for joint B in Fig. 29 and can be identified for use at a glance

Any one fixed-end moment is now balanced in one cycle throughout the structure using the modified carry-over factors (C') and distribution factors. One precaution must be taken—a fixed-end moment is balanced into all members at a joint whereas a moment carried over is the final correct end moment. This final moment must remain unchanged and therefore is balanced into only the remaining members at the joint, which explains why different sets of distribution factors are obtained for each joint. Some of these sets may never be used, however, depending upon the loading and where the fixed-end moments originate.

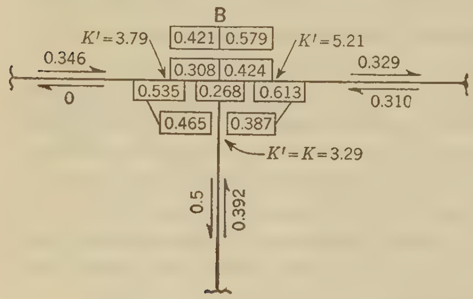

Fig. 29

Should the structure be composed of nonprismatic members, the K-values and C-values, for fixed ends and for most types and shapes of members used in practice, can readily be obtained from excellent tables or diagrams well known to structural engineers. Using these values the modified values are obtained in the simple, quick, and understandable manner outlined herein. The fixed-end moments are also taken from these tables or diagrams. One-cycle moment distribution then yields final end moments. Sidesway corrections, if necessary, follow. Influence lines may then be computed and plotted. This method is exceedingly rapid since the modified carry-over factors, stiffness values, and the distribution factors are independent of the loading and remain unchanged. This method is as "exact" as any other.

A clear knowledge of the fundamental definition of "stiffness factor" and "carry-over factor" as applied to any end condition is really all that is needed for an understanding of this method. In moment distribution the stiffness factor should never be defined as I/L even for a prismatic member.

The author's statement that moment distribution "can be shown to be a special case" of his method is not supported by his explanation of only a part of the moment-distribution concept. The Cross method of moment distribution is based upon fundamental concepts, independent of other methods; is more general than the Brumfield method which is involved with criteria, and, in its broader application, is a powerful tool of analysis. When the subject matter pertaining to criteria is separated from the Brumfield procedure, there remains the valuable part of this article—that part which is very similiar to the method of direct moment distribution.[38]

CHARLES W. DOHN,[46] JUN. AM. SOC. C. E.—The art of designing statically indeterminate structures, including the determination of maximum moments caused by concentrated loading groups on continuous frame structures, has been advanced materially by the publication of this paper. Although a new method may simplify the design of a great number of statically indeterminate structures, there is always some resistance to its adoption, particularly if it is not widely recorded, discussed, and understood. Design agencies that could use such a method are limited by their respective authorities, usually governmental, to widely published and accessible methods which have been accepted and applied, more or less, as office standards. In one instance, known to the writer—which involved bridge cranes operating on rigid-frame bents with haunched beams, and with a trussed jib that had a variable-section upper chord—the Brumfield method of transmission coefficients[4] was rejected in favor of the Cross method of successive approximations (moment distribution)[2] because: (1) Time would not permit teaching, organizing, and instituting a new method; and (2) the clients wanted the design made by a widely known method that could be accepted quickly, and could be checked and approved in successive steps by a number of other engineers, at different times and places.

When many designs and detailed investigations of the statically indeterminate type must be made for a great many different loadings and loading conditions, the method of transmission coefficients holds some advantages over those of conventional moment distribution. The busy design office handling a multitude of such investigations cannot afford to ignore its advantages. On the other hand, a design agent or office, which handles only a limited number of statically indeterminate investigations and designs, may well prefer the conventional moment-distribution method.

Computations can be expedited using transmission coefficients in conjunction with computing devices to determine frame characteristics, and to distribute moments, according to the procedure developed by Professor Brumfield. His method is well-adapted to machine computation, and the mechanics of calculation could thus easily be done by a group of computers working under the supervision of a design engineer. The speed of applying the method to any particular design could be increased greatly merely by employing more computers and machines. Furthermore, the method tends to relieve the design engineer of the arduous tasks involved in changes in design and loadings; and it allows him more time and inducement to change the design to a more efficient one, whatever the purpose. The value of the method in redesigns and investigations will also be apparent.

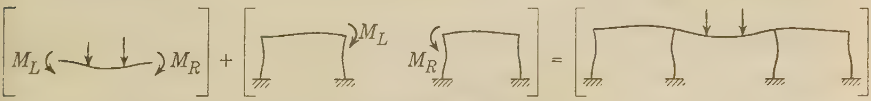

FIG. 30.—OUTLINE OF THE BRUMFIELD METHOD OF ANALYSIS

In the method of transmission coefficients, the only artificially restrained joints are those at the ends of the loaded span and these are only partly locked or restrained. At the beginning (see Fig. 30) the condition can best be pictured as that of a loaded span removed from the structure, with artificial restraint applied at each end of an amount equal to that which would have existed had the loaded span not been removed from the structure. Equal and opposite artificial restraints are then applied to the structure at joints corresponding to the application of the artificial restraints at the ends of the loaded span. The loaded span is then inserted in the structure and the analysis is complete. The method is "exact" and will give the same result as other accepted methods.

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[46] With Basic Design Dept., George G. Sharp, Naval Architect and Cons. Engr., New York, N. Y.

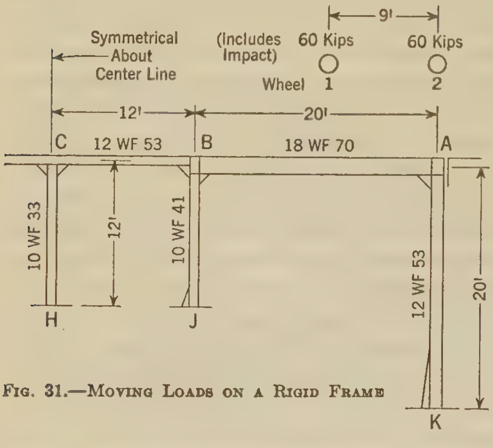


FIG. 31.—MOVING LOADS ON A RIGID FRAME

Moving Loads on a Rigid Frame.—To support a fair appraisal of the method of transmission coefficients, the writer presents, herewith, the solution of a frame with moving loads (Fig. 31). The problem is treated by the Brumfield method and, for comparison, by the Cross method of moment distribution.

TABLE 11.—ANALYSIS OF MEMBER AB, FIG. 31, BY MOMENT DISTRIBUTION; LOADED AT THE ENDS

Description	(a) END A ROTATED					(b) END B ROTATED				
	Joint C		Joint B		Joint A	Joint C		Joint B		Joint A
Factor $K = I/L$	0.415	0.415	0.317	0.508	0.730	0.415	0.415	0.317	0.508	0.730
Fixed-end moment	....	....	....	....	−1,000	....	....	....	+1,000	....
Distribution:										
Balance......	....	....	....	....	+730	....	....	−317	−508	....
Carry-over....	....	....	....	+365	0	....	−158	0	0	−254
Balance......	....	....	−116	−185	0	+66	+ 66	0	0	+185
Carry-over....	....	−58	0	0	−92	....	0	+33	+92	0
Balance......	+24	+24	0	0	+67	....	0	−40	−63	0
Carry-over....	0	0	+12	+33	0	....	....	0	0	−31
Balance......	0	0	−14	−23	0	....	....	0	0	+22
Carry-over....	....	....	0	0	−11	....	....	....	....	....
Balance......	....	....	0	0	+8	....	....	....	....	....
Carry-over....	....	....	....	+190	−298	....	....	....	+521	−78
Final unit moment	....	....	....	−190	−298	....	....	....	−521	−78

By Moment Distribution.—The formal distribution of unit, fixed-end moments at joints A and B, Fig. 31, is presented in Table 11. By a similar procedure, unit ordinates are computed (for the construction of influence lines) of 1-kip loads placed successively at each fifth-point beginning at end A.

Table 12 contains a summary of the results, for negative moment, as applied to beam AB, Fig. 31. The maximum negative moment M_A (see Fig. 32(*a*)) equals $-0.81 \times 60 - 0.70 \times 60 = -90.6$ ft-kips. Similarly (see Fig. 32(*b*)), the maximum negative moment M_B equals $-1.46 \times 60 - 1.40 \times 60 = -171.6$ ft-kips. The final step is the preparation of influence lines for positive moment, from data in Tables 11 and 12. For successive positions of wheel 1 the positive moments (see Fig. 32(*c*)) are:

Fifth-point	Computation	Ft-kip
0.2 L	$\frac{1}{20} \times 60 \times 16 \times 4 - \frac{4}{20} \times 0.82 \times 60 - \frac{16}{20} \times 0.81 \times 60$	143
0.4 L	$\frac{1}{20} \times 60 \times 12 \times 8 - \frac{8}{20} \times 1.55 \times 60 - \frac{12}{20} \times 1.01 \times 60$	215
0.6 L	$\frac{1}{20} \times 60 \times 12 \times 8 + \frac{8}{17} \times \frac{1}{20} \times 60 \times 17 \times 3 - \frac{12}{20} \times 60\,(0.62 + 1.86) - \frac{8}{20} \times 60\,(0.66 + 0.79)$	236
0.8 L	$\frac{1}{20} \times 60 \times 4 \times 16 + \frac{4}{13} \times \frac{1}{20} \times 60 \times 13 \times 7 - \frac{16}{20} \times 60\,(1.46 + 1.40) - \frac{4}{20} \times 60\,(0.39 + 1.02)$	122

The maximum ordinate (Fig. 32(*c*)) is 239 ft-kips. Similarly, for successive positions of wheel 2, the positive moments are:

Fifth-point	Computation	Ft-kips
0.2 L	$\frac{1}{20} \times 60 \times 16 \times 4 + \frac{4}{13} \times \frac{1}{20} \times 60 \times 7 \times 13 - \frac{4}{20} \times 60\,(0.82 + 1.83) - \frac{16}{20} \times 60\,(0.81 + 0.70)$	172
0.4 L	$\frac{1}{20} \times 60 \times 12 \times 8 + \frac{8}{17} \times \frac{1}{20} \times 60 \times 3 \times 17 - \frac{8}{20} \times 60\,(1.55 + 1.20) - \frac{12}{20} \times 60\,(1.02 + 0.28)$	247
0.6 L	$\frac{1}{20} \times 60 \times 8 \times 12 - \frac{12}{20} \times 1.86 \times 60 - \frac{8}{20} \times 0.79 \times 60$	202
0.8 L	$\frac{1}{20} \times 60 \times 4 \times 16 - \frac{16}{20} \times 1.46 \times 60 - \frac{4}{20} \times 0.39 \times 60$	117

The maximum ordinate in this case (Fig. 32(*d*)) is 248 ft-kips, when load 2 is at a distance of 10.74 ft from joint B.

By Transmission Coefficients.—The computation of frame characteristics is presented in Table 13. The next step is to compute the maximum negative moments at the supports in span AB (Fig. 33) caused by the moving loads. By Eq. 28, $A_R = \frac{0.165}{1 - 0.165 \times 0.268} = 0.173$; and, similarly, $A_L = \frac{0.268}{0.956}$

TABLE 12.—UNIT MOVING LOADS ON MEMBER AB, FIG. 31; SUMMARY OF COMBINED UNIT MOMENTS

Description	1-KIP LOAD AT THE FOLLOWING POINTS (FROM END A):									
	0.8 L		0.6 L		0.5 L		0.4 L		0.2 L	
	M_B	M_A	M_B	M_A	M_B	M_A	M_B	M_A	M_B	M_A
Fixed-end moment, unit moving load...	2.56	0.64	2.87	1.92	2.50	2.50	1.92	2.87	0.64	2.56
Distributed unit load combined with:										
Rotation effect, end A[a].........	0.12	0.19	0.36	0.57	0.47	0.75	0.55	0.86	0.49	0.76
Rotation effect, end B[a].........	1.34	0.20	1.50	0.22	1.31	0.19	1.00	0.15	0.33	0.05
Combined unit moment, moving load and end rotation..................	1.46	0.39	1.86	0.79	1.78	0.94	1.55	1.01	0.82	0.81

[a] From Table 11, 1 kip at end A yields $M_A = -0.298$ and $M_B = -0.190$; and 1 kip at end B yields $M_A = -0.078$ and $M_B = -0.521$.

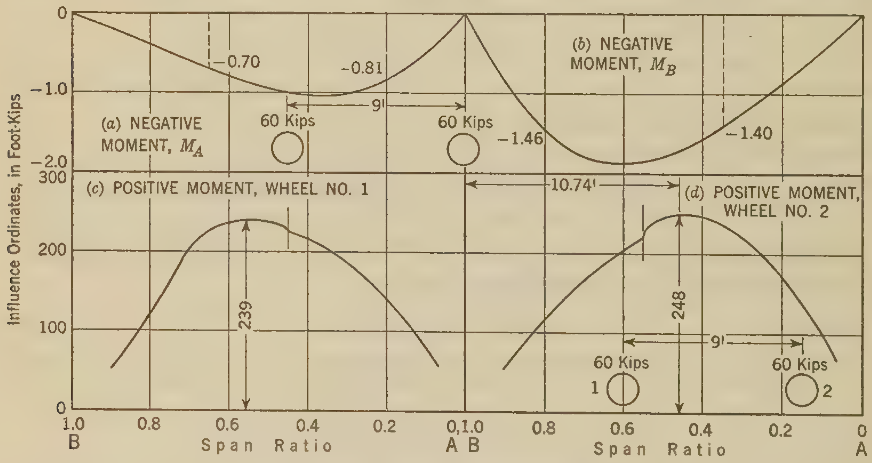


FIG. 32.—INFLUENCE LINES FOR MAXIMUM MOMENTS

TABLE 13.—BRUMFIELD METHOD OF COMPUTING FRAME CHARACTERISTICS

$$Q = \frac{L}{I}; \quad R = \frac{i^2}{1 - i^2/i^3}; \quad C = \frac{1}{2 + R/Q}; \quad m = \frac{R}{i}; \quad t = \frac{1}{1 + i/R}$$

Member	STIFFNESS INDEX		Restraint index R	TRANSMISSION COEFFICIENT C		Moment split m			Torque split t
	$Q\ (2 - C)$ =	i		Value	Member				
Joint A:									
AK	0.562 (2 − 0.5)	0.843	0.361	0.378	KA	KA	1.0	..	0.30
AB	0.208 (2 − 0.268)	0.361	0.843	0.165	BA	1.0	BA	..	0.70
Joint B:									
BA	0.208 (2 − 0.165)	0.384	0.358	0.268	AB	AB	0.593	0.72	0.482
BJ	0.650 (2 − 0.5)	0.975	0.228	0.435	JB	0.367	JB	0.28	0.190
BC	0.337 (2 − 0.319)	0.568	0.275	0.355	CB	0.633	0.407	CB	0.328
Joint C:									
CB	0.337 (2 − 0.355)	0.556	0.382	0.319	BC	BC	0.50	0.69	..
CH	0.842 (2 − 0.5)	1.263	0.273	0.430	HC	0.31	HC	0.31	..
CD = CB	0.337 (2 − 0.355)	0.556	0.382	0.319	DC = BC	0.69	0.50	DC	..

= 0.282. The span distribution ratio $\lambda = \frac{r}{L} = \frac{4.5}{20} = 0.225$. The remainder of the solution yields:

Quantity	Left	Right
C	0.165	0.268
K	0.455	0.555
B	0.29	0.29
D	0.30	0.28
M	−170	−90

—in which: K, for maximum moments M_L and M_R, is selected from Fig. 6; B and D are selected from Fig. 5; and M_L and M_R, respectively (in foot-kips), are computed from Eqs. 27. The dissymmetry span ratio ϵ is zero in this case because the modulus of dissymmetry of the particular loading group used is zero.

The foregoing values compare favorably with those obtained by the Cross method (see Figs. 32(*a*) and 32(*b*)).

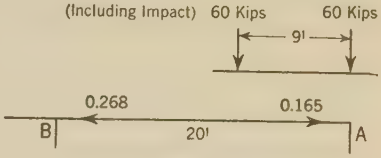


FIG. 33

The maximum negative moments having been computed, the maximum positive moment under the moving loads in the span AB is computed as follows: By Eq. 32, $q_1 = 0.165 + 2 \times 0.165 \times 0.268 + 0.268 = 0.522$; and $q_2 = 0.268 + 0.165 \times 0.268 = 0.312$. Eqs. 33 then yield: $f_3 = 4 \times 0.522 = 2.088$; $f_2 = 3\left(\frac{4.5 \times 0.522}{20} - 4 \times 0.312\right) = -3.339$; and f (Eq. 33*c*) $= 6 \times 0.522 \times 0.225^2 + 0.312\left(6 - \frac{6 \times 4.5}{20}\right) + 2\,(3 \times 0.268 - 0.165 \times 1) = -0.112$.

The application of the foregoing equations, as shown, gives the value of κ for use in placing the loads to produce maximum positive moment. Substituting in Eq. 34: $\bar{\kappa} = 2 \times 0.268 - 1 + 0.225\,(1 + 0.165) + 0.312\,(6 \times 0.225^2 - 2 \times 0.225) - 0.522 \times 3 \times 0.225 \times 0.225^2 = -0.339$. Finally, by Eq. 35, $\kappa = 2.088 \times -0.339^3 - 3.339 \times -0.339^2 - 0.112 \times 0.339 = -0.312$. This represents the ratio of the distance of the mass center of the loading group from the left support. Hence, wheel 2 is $0.312 \times 20 + 4.5 = 10.74$ ft from joint B for maximum positive moment. As in the determination of maximum negative moment the remainder of the solution, with $\lambda = 0.225$, yields:

Quantity	Left	Right
C	0.165	0.268
κ	0.312	0.312
B	0.24	0.24
D	0.25	0.25
M	−142	−70

in which, as before, B and D are selected from Fig. 5, and M_L and M_R, respectively (in foot-kips), are computed from Eqs. 27. The composite maximum moment is computed by applying the laws of statics, as: $\frac{60 \times 10.74 \times 9.26}{20} + \frac{60 \times 1.74 \times 18.26}{20} \times \frac{9.26}{18.26} - \frac{142 \times 9.26}{20} - \frac{70 \times 10.74}{20} = 246$ ft-kips. This value compares favorably with $M = 248$ ft-kips as computed by the Cross method (see Figs. 32(*c*) and 32(*d*)).

Summary of Comparisons.—The actual design of the bent illustrated in Fig. 31 was better proportioned to resist the moments with minimum excess of material, which was an important requirement of the design. Hence, haunched beams and columns were proportioned to resist the moments as indicated by the preliminary investigation, and another investigation was made. Sidesway was neglected in the preliminary design since the column sections were large and there were enough of them to keep sidesway effect to a minimum in any case. Another condition of loading—a horizontal thrust—required bracing in the interior bay for the final design. The exterior columns were also increased 50% in section for other reasons, chiefly impact on the column itself.

Although the Brumfield method has a short distribution, it requires comparable work in order to obtain the frame characteristics. The Cross method requires little more than a pencil and something to write on. It can be applied any time and anywhere. The Brumfield method with its formulas, constants, and charts is better adapted to advanced design work in a well-established office.

One great advantage of using the Cross method rather than the Brumfield method in the analysis of moving loads should not be overlooked—the principle of separately distributing an assumed fixed-end moment of, say, 1,000 ft-kips at each end of all loaded spans. It is then only necessary to multiply the distributed moments by the ratio of the actual fixed-end moment to the assumed fixed-end moment. The influence lines can then be plotted; maximum and minimum moments for any loading are obtained therefrom. The influence lines are a useful part of any analysis using live loads, and an American design engineer would not want to analyze any involved structural design problem without their aid.

The method of using influence lines will be preferred as the "picture" can be applied to different loading conditions with ease. The Brumfield method can be used to determine influence lines also, although not with the same ease and convenience as the Cross method; the latter requires only two distributions for any number of loadings and corresponding fixed-end moments in any one span (see the foregoing example of moving loads).

In any actual continuous structure subjected to moving loads, the loads outside the span as well as the loads in the span must be considered for maximum and minimum moments. For most practical problems, influence lines are the best guide to the placing of loads for maximum and minimum moments due to loads both within the span and outside the span. Hence, except as in the special case of the span in question loaded and all other spans unloaded (the dead load remaining, nevertheless), there is no point in determining the maximum moments within a span of a restrained beam, especially when the loading group occupies more than two spans. Most actual maximum moments in design work are due to loadings both inside and outside the span in question. Hence, influence lines must be used, and the Cross method is probably the best adapted to their use.

The Brumfield method is not so complicated as might appear. It is probably a bit difficult to assimilate and apply at first. With constant practice, the operations become automatic, and the time and effort necessary for each step gradually diminish. It might be well to note that the frame characteristics are constants. In standard multiple designs they are similar and hence, once calculated, they can be used over and over again for all different conditions of loadings, for secondary stress investigations, and for sidesway, settlement of supports, temperature stress, degree of fixity, erection, damage, replacement, modification, alteration, and conversion.

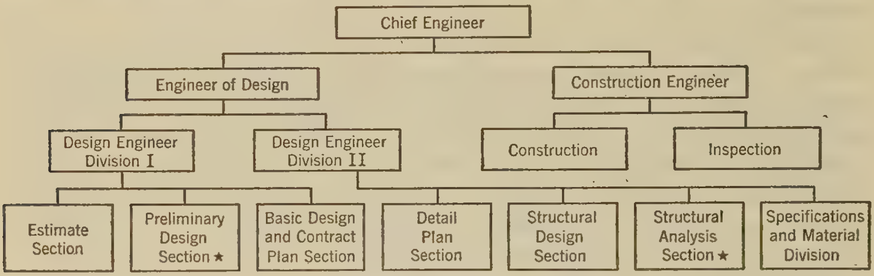
The Hardy Cross method is more easily recalled to use after long periods of disuse. It requires quite an extensive review to apply the Brumfield method after a long lapse of time. Furthermore, practically minded designers will question the niceties of this method. It is probably only in the unusual problems, or in the case where detailed investigation is required, and where otherwise it is unnecessary to obtain influence data, that the Brumfield method is most applicable. The method is also helpful as a check or alternative method to the Cross method.

In the study and application of moment-distribution methods, the individual designer will usually evolve tables, curves, and nomographic charts as an aid to computation. Several of these have been published, and their usefulness should not be overlooked. It might be possible to construct a nomograph to read M_L and M_R and a maximum positive moment directly based on Professor Brumfield's equations and constants. One of the factors in this nomograph could be fixed-end moments.

The similarity of some of the factors used in determining the "frame characteristics" to equations of electrical circuits suggests that perhaps it would be possible to set up an electronic computing device for "frame characteristics." Then all one would need to do would be to press some buttons and read the frame characteristics on a dial, enter a nomograph, and read M_L, M_R, positive M_{max}, and make a simple distribution on prepared forms. By another electronic computing device it might also be possible to draw influence lines if necessary.

In the hands of the unenlightened, this method, and other useful tools of engineering, can become "sledges" to drive engineers deeper into the mud of tedium. However, in the hands of enlightened direction, with sufficient care in distributing the labor of computation over a large group of people, working short daily hours under healthful working conditions with adequate compensation, and with sufficient rest periods, something worthwhile can be accomplished without straining humanity.

Moment distribution is but one step in the design procedure for any continuous structure. To illustrate the proportionately small but nevertheless important part that moment analysis occupies in an engineering organization doing continuous-frame structural work, a simplified organizational chart is presented in Fig. 34. Sections where moment analysis and the Brumfield method may be applied are indicated as the "Preliminary Design" section in division I and the "Structural Analysis" section in division II. The design organization illustrated is one engaged in preparing both contract and working plans, as well as supervising and inspecting construction. In an organization engaged only in preparing contract plans, moment analysis assumes a proportionately larger part of the organization.


Chief Engineer
Engineer of Design
Construction Engineer
Design Engineer Division I
Design Engineer Division II
Construction
Inspection
Estimate Section
Preliminary Design Section ★
Basic Design and Contract Plan Section
Detail Plan Section
Structural Design Section
Structural Analysis Section ★
Specifications and Material Division

★ Areas Primarily Concerned with Moment Analysis

FIG. 34.—ORGANIZATION CHART FOR ENGINEERS ENGAGED IN THE DESIGN AND CONSTRUCTION OF CONTINUOUS FRAMES

CLIFFORD PETERSEN,[47] JUN. AM. SOC. C. E.—It would be well to state that Professor Brumfield's method is flexible to the utmost and is readily applicable to the solution of problems in rigid frames outside the scope of conventional rectangular building frames. Also, once a basic understanding of the method has been absorbed, it is possible to pick up the method and employ it on a short notice even if a considerable interval has passed since last it was used.

The writer first became familiar with the method during his undergraduate days. In the succeeding years he has been engaged entirely in the aeronautical engineering field and has had no occasion to apply the method to building frames. There have been a few isolated yet important instances, however, where it has been necessary for the writer to investigate moving air loads and other loads in the design of multiangular frames which occur so often in aircraft structures. After the briefest perusal of Professor Brumfield's mimeographed notes,[3] the method was applied successfully to the exact solution of the problems with a considerable over-all saving in time and effort. The saving in time resulted primarily from the fact that, once the characteristics of the frame were determined, it was a comparatively simple and rapid process to evaluate the effect of the many different loading conditions.

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A. A. EREMIN,[48] ASSOC. M. AM. SOC. C. E.—The distribution moments in continuous frames by the Brumfield method is similar in many respects to the procedure for distributing moments graphically. Therefore, the author's method can be simplified easily by introducing graphical constructions.

For illustration consider a member AB with a constant section and with the ends restrained by the continuity of members. The member AB is subjected to a concentrated load P as shown in Fig. 35. The distances, a and b, to the inflection points in member AB, are

$$a = \frac{C_{AB}}{1 + C_{AB}} \times L \ldots (81a)$$

and

$$b = \frac{C_{BA}}{1 + C_{BA}} \times L \ldots (81b)$$

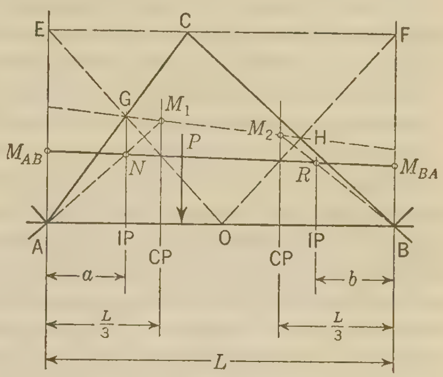

FIG. 35

The constants C_{AB} and C_{BA} are determined from Eq. 4. The moments in member AB may be determined by the graphical construction as shown in Fig. 35. Draw the verticals from the inflection points and from the central points at the third-points of span AB. Line EF is drawn through the vertex C of the M-diagram of span AB, as shown. Connect the points E and F with the mid-span point O. Draw line GH through the points cut on the M-diagram. Line GH gives the central moments M_1 and M_2 at the third-points. The closing line of the final moments M_{AB} and M_{BA} is drawn through the points cut by lines AM_1 and BM_2.

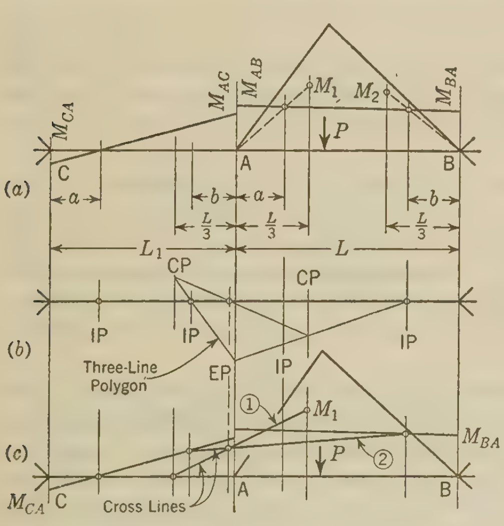

FIG. 36.—GRAPHICAL DISTRIBUTION OF MOMENTS

Fig. 36 illustrates the solution of a continuous beam CAB with a constant section and with the ends restrained by continuity of members. The span AB and the loading are the same as those shown in Fig. 35. At point A the beam is supported on a rigid column. The moments in span CA may be determined either by the author's moment-split factors, Eq. 39, or by the cross lines as shown in Fig. 36(*c*). The location of the inflection points in span CA are determined by Eq. 40. The location of the elastic-point vertical is determined[49] by the three-line polygon as shown in Fig. 36(*b*). The central moments in span CA are each zero; therefore, line 1 of the cross lines is drawn through the third-point in span CA and M_1 in span AB. Line 2 of the cross lines is drawn through the closing point on the inflection-point vertical in span AB and the point cut by line 1 on the elastic-point vertical. The intersection point of line 2 and the inflection-point vertical in span CA give the closing point at that span, thus deriving the points by which the closing line of moments M_{CA} and M_{AC} are constructed.

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[48] Associate Bridge Engr., Bridge Dept., Div. of Highways, State Dept. of Public Works, Sacramento, Calif.

In calculating the maximum moment Professor Brumfield has placed the moving load on one span only. It is known that the maximum moment near a support is produced by a moving load placed on two or more spans; and it would be valuable to have Professor Brumfield demonstrate his method for this type of loading. Evidently the method of influence lines is still the simplest for computing such moments. The graphical procedure illustrated by Figs. 35 and 36, in conjunction with Eq. 4 of the paper, is a time-saving method of drawing influence lines. Therefore, it is gratifying that Professor Brumfield has developed the algebraic formulas for the physical properties of structures which may also be applied in the graphical distribution of moments.

R. C. Brumfield,[50] M. Am. Soc. C. E.—It will not be possible to comment in detail on all the questions raised in discussion. Some were asked by more than one discusser. The writer will accordingly try to answer first those which are frequently duplicated and then proceed to comment on the individual discussions.

At the start the writer wishes to express his sincere appreciation for the many complimentary statements. Compensation for the time and effort of developing the type of material represented by the paper must of necessity be more or less spiritual in character. No small part of such compensation is the knowledge that others have benefited from the effort involved. It is gratifying, therefore, to read that the principles developed have been found useful in practice.

The writer was somewhat puzzled by the number of discussers who stressed terminology and nomenclature and by others who devoted their comments to the subject of moment distribution. Difference in viewpoint may account for this insistence on discussing subjects which were considered only auxiliary to the main purpose of the paper—the handling of moving loads on restrained beams. Accordingly, it seems desirable to give a brief history of the development of this material in the hope of somewhat clarifying the diversity of viewpoint. Nomenclature and moment distribution will then each be commented on separately.

The essential part of the material on moving loads, which is contained in the paper, was developed about 1925–1926 and has been available since that time together with the nomenclature which accompanied it. About 1930 the material on moment distribution, in its elementary form as it appears in the paper, was added and the whole, with additional material on sidesway and secondary stresses in trusses, was published in mimeographed form, in 1933.[3] The procedure has been taught continuously to both undergraduate and graduate students at The Cooper Union in New York, N. Y., from then to the present time. The methods were adapted to beams of variable section and to curved beams about 1936. The material was republished in photo-offset form in 1941.[4] Since that time it has been extended to include beams of noncoplanar frames.

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[49] "Analysis of Continuous Frames by Graphical Distribution of Moments," by A. A. Eremin, Sacramento, Calif., 1943 Ed.

[50] Associate Prof., Civ. Eng. Dept., The Cooper Union, New York, N. Y.

Nomenclature and terminology, in technical writing, bear some analogy to the language of ordinary speech or written material. It is natural that each reader should desire to read in his own language, since less effort is involved. It would also be a courtesy on the part of the writer to express his material so as to accommodate the greatest number, even though the truth is the same expressed in any nomenclature. Unfortunately, basic nomenclatures dealing with beams are as diverse as are the textbooks from which the engineer learns fundamentals in mechanics of materials.

The writer therefore has no apologies for the nomenclature used in the paper; it was fully embedded, in the method presented, long before such papers as that by Hardy Cross,[2] M. Am. Soc. C. E., and most of the others cited were published. From the standpoint of serving the majority, the nomenclature performed that service, as far as the discussers were concerned, since more than half of them were men who had previously studied the writer's courses in indeterminate structures. Also, a writer should not be closely restrained in his choice of nomenclature. Since he is responsible for accuracy of statement, he is most likely to meet this requirement when thinking and writing in a familiar language—his own nomenclature.

Some of the discussers chose to comment on the method of distribution of bending moment, and several introduced, briefly, methods of their own, for comparison. These are interesting, of course. It is significant that the methods are not alike, which indicates that preferences in the methods of distribution of moment to a frame are likely to be determined to a large extent by the habits and prejudices of the individual. Few men are sufficiently familiar with all available methods through practice to be qualified to make a fair comparison of their merits. Although the writer has a bowing acquaintance with some of those cited, he confesses to a lack of the necessary intimate familiarity to make profitable comparisons on the bases of convenience and serviceability. Again, the writer gave only the essentials of his method of distribution, and so stated in the paper. In fact, in the method actually used for extensive frames, the frame characteristics developed are only a means to an end and do not appear as such in the final method. Messrs. Rolsma and Gieseking touch briefly on the more general method in their discussion. Since space is not available for introducing the method here, the writer must refuse various invitations to make comparisons in methods of moment distribution and confine his closing comment closely to the matter of moving loads.

Mr. Stewart makes the observation that the method is not applicable to beams of variable section. This is true of the formulas shown in the paper since they have been adapted to the special case of beams of uniform section throughout the span. The method has been adapted to members of variable section, however, giving formulas which reduce to those for members of uniform section when the nonuniformity of the beam disappears. Thus, the formulas in the paper are special cases of the more general ones which apply to nonuniform beams, and the nonuniform beam formulas in turn are special cases of those which apply to curved beams. In fact, a paper was filed in the Engineering Societies Library in 1936 which contained many of the formulas that apply to beams of nonuniform section.

Mr. Brodkin has transformed the formulas of the paper to apply to the computations of fixed-end bending moments at beam ends. Mr. Strauss also does essentially the same thing. These are interesting applications and no doubt will be useful to engineers interested in fixed-end moments. Mr. Brodkin also brings out the fact that angular rotations at the supports may readily be computed from the B-values and the D-values. This is an interesting comment on a matter not included in the paper.

Professor Hickerson speculates as to whether or not it might be easier to determine the position of load for the maximum bending moment and the value of the maximum bending moment by influence lines, rather than by the formulas of the paper. At least one other discusser raised the question as to the difference between the method of influence lines and the formulas of the paper. Considering them as separate methods of handling moving loads, there should be no argument possible. This is true since Eqs. 14*a* and 14*b* are exact formulas for influence lines at the two supports, caused by a single load, when the value of W is unity. Similarly, Eqs. 25*a* and 25*b*, as well as Eqs. 27*a* and 27*b*, are formulas for combined influence lines when the value of W is unity, the unit load being considered as distributed on the span in any manner chosen, as dictated by λ^2 and ϵ^3. Since the equations of the paper are in essence those of influence lines, it would seem strained to attempt to make the distinctions cited.

Mr. Strauss has suggested that it would be more convenient to use the reciprocals of Q, i, and R, rather than the symbols in the paper. Mr. Merritt makes essentially the same suggestion. In support of his suggestion, Mr. Strauss states that, in computing the R-values of Table 1 from the i-values of Col. 3, this procedure would avoid taking the reciprocals of the i-values before adding them to obtain the R-value needed in Eq. 40. The writer readily admits that this is true and commends the suggestion to those who compute R-values in this way. In his own work, however, he has always followed the suggestion made in connection with the transformed Eq. 42. In Eq. 42, there is no conscious addition of the i-values at all, but rather ratios of i-values are taken on the slide rule and unity is added mentally. Thus, the entire computation of a R-value, from any number of component i-values, consists of a single continuous slide-rule operation leading to the final result.

Mr. Mackenzie illustrates the use of the frame characteristics in the paper for distribution of fixed-end moments, indicating an interesting combination of the methods of the paper and the concepts of the Cross approach to the problem of rigid frames.

The comments of Mr. Schaffner on the use of the methods of the paper in practical engineering should be of interest to those not familiar with them, since the procedure must be judged on such testimony. The practical illustrations given by Mr. Kalb and Messrs. Rolsma and Gieseking are further examples of such testimony, and are particularly valuable in showing adaptations to studies on unusual frames and the flexibilities obtained with the methods. Messrs. Rolsma and Gieseking touch briefly on the general method which the writer uses for distribution of bending moment but it is not likely that the explanations, because of their brevity, will be clear to the uninitiated.

Mr. Schure gives the analysis of a five-span continuous beam. It appears that Eqs. 61*a* and 61*b* are identical to Eqs. 14*a* and 14*b* of the paper. The curve in Fig. 22 is identical with the outer curve ($\lambda = 0$), in Fig. 5 of the paper. Mr. Schure also suggests the use of a table of reciporocals in computing frame characteristics. This is in line with the suggestions made by Messrs. Strauss and Merritt and the same answer applies. If reciprocals are to be taken in the computation, the suggestion of Mr. Strauss had better be followed and the inversions of Q, i, and R used.

Mr. Dodge makes considerable effort to show that the supports A_1 and E_1, in Fig. 12, will permit uplift and that hence the frame characteristics computed are invalid. He also states that sidesway would result in any case and that this would render inaccurate the solution given for the frame and loading. He buttresses these observations with the statement that, if the supports were pins, no lateral movement would be permitted and thus expansion due to temperature would be impossible.

The writer has long been familiar with the fact that most textbooks on the mechanics of materials, from which engineers obtain their first concepts of beams, deal only with the fixed support and free support as being the two limiting cases of a large group of restrained supports. For mathematical purposes, the free support is defined as one which permits no translation but does allow perfect freedom of rotation. Although this is an idealized definition, it serves mathematical purposes very well. In this sense the writer used the definition of free support in the paper. With such a definition, of course, there is no question of sidesway or uplift. In fact, the very symbol, used in the paper to designate the support, is the one ordinarily used in textbooks to show a free support conforming to this definition.

Again, if it be granted, in deference to the practical man, that such supports must permit lateral motion to provide for expansion, it must at the same time be granted that the practical man would design his support to prevent uplift, to avoid hammer, just as is done at the anchor pier of a cantilever bridge. Such a provision would render the frame characteristics accurate. Since only one such support is required to care for expansion, sidesway is eliminated by fixing the other support against lateral motion. The writer has no desire to avoid the question of sidesway; but, since it was not covered in the paper, he feels it would be presumptuous to introduce it in the discussion.

Mr. Dodge's distinction between the terms "beam class" and "frame class" is obscure. Since beams are the units from which all frames are built, no distinction would appear to be justified. Whether the frame is to be analyzed with or without sidesway, the fundamental approach to the problem of the beam is the same as to that of the frame. Mr. Dodge also states that, as presented by the writer, the computation for the effects of a pure couple involves a concept that is different from those for other loads. This is not true as may be verified from the paper. The formulas for the couple expressed by Eqs. 36 were obtained directly from Eqs. 14*a* and 14*b*, and hence have the same underlying concept.

Mr. Dodge "objects strongly to the term 'torque split'" to designate the division of resisting bending moment among the receiving members of a joint which is actuated by an artificial couple. The writer used this term to distinguish such moment from similar bending moment induced in the members, when the joint is actuated by a bending moment transmitted to it by one of the radiating members. Mr. Dodge would reserve the term torque split for those cases where "primarily shear stresses" are caused in the members.

The term "primarily" is not defined. Primarily may not refer to the order of induction since, when both shear and bending moment are induced in a member, their induction is simultaneous. It may perhaps then be inferred that Mr. Dodge means that shear must be the greatest or predominating influence and the bending influence must be lesser in numerical value. Even this interpretation is difficult to make consistent. In such a case as that of a shaft, which has by long usage been associated with the term torque, when torsion is applied by some procedure such as a belt and pulley, the question of whether bending moment or torsion predominates in the shaft is wholly a matter of distribution of bearings, pulley-size belt pull, and so forth. Similarly, in a noncoplanar frame if some members become quite short, it is entirely conceivable that shear may predominate in amount. Thus, accepting Mr. Dodge's criterion, it would seem that the use of terms is somewhat relative and a matter of individual taste.

The writer regrets that space does not permit a full explanation of the meaning of his statement (see Section 13) that: "One distribution is sufficient for all purposes, including live-load and dead-load moment distribution, wind moments, and sidesway * * *." Mr. Dodge suggests that a separate distribution is required for each operation. Under the writer's procedure this is not the case. Although separate collections or partial collections are made for the various purposes they all follow from one distribution. The discussion by Messrs. Rolsma and Gieseking touches very briefly on this question.

The testimonies of Messrs. Rights and Heiser as to the usefulness of the method and its flexibility and versatility in practice are most gratifying coming from men actively engaged in a broad field of general engineering practice.

The writer appreciates the complimentary comments of Professor Ferguson. The confusion which Professor Ferguson feels in the terminology and nomenclature would seem to arise from unfamiliarity and perhaps from the too brief explanations which must accompany a short paper. For many years, the writer has taught these concepts, expressed in this terminology, to undergraduate engineering students and has noted no confusions. It is harder to supplant a familiar nomenclature with a new one than to become familiar with the new one from the start, which may account for some of Professor Ferguson's difficulty.

Mr. Beskin suggests that the writer has omitted reference to his predecessors. The writer is most conscious of the duty to give credit where credit is due. However, he confesses total lack of knowledge of the writings of O. Mohr and Maurice Lévy[42] which are stated to have been published about 1886. As for the other references cited, they all appeared after the methods of the paper were complete. The writer has a general familiarity with most of them but they had no influence on any procedures in the paper.

Messrs. Beskin, Eremin, and Jameyson all mention, in one way or another, the opinions that the equations for moving loads are not valid when the load occupies more than one span and that therefore this is a distinct limitation on their use. Mr. Beskin particularly refers to the fact that the greatest bending moment at the common support of two contiguous spans occurs when load lies in each of them. This is true and the equations cover this case.

A continuous beam of six spans is shown in Fig. 37. A load W is shown, which is too extensive to be contained wholly in span CD. It then becomes a "broken load," in Professor Jameyson's terminology, with a part W_1 lying in span CD and a part W_2 lying in span DE. There are two statements to be made in connection with this case. The first is that such a load is not in general the most suitable type for use as a criterion to determine the maximum bending moment at a given point—say, the right end of span DC. If the loads W_1 and W_2 are fixed in spacing with respect to each other, it is obvious that one of them, say, W_1, may be placed, by Fig. 6 or Eq. 29*a*, in position to produce maximum bending M_{DC}. It is also evident that the load W_2 in span DE will occupy any companion position dictated by the fixed spacing. Although, fortuitously, this could also be the position that would produce maximum bending moment M_{DE}, it is likely to be some position that will produce less than maximum at the left end of span DE. Thus, it can obviously be concluded that a fixed-space loading that occupies more than one of two contiguous spans simultaneously may not ordinarily produce as great a bending moment at the common support as would the two sections of the load if they were free to move independently of each other and that, in any case, the bending moment produced may not be greater than the one that would be produced by the independent movement of the two sections. This leads to the conclusion that loads which may not be wholly contained within a span should not be used as criterions for maximum bending moment.

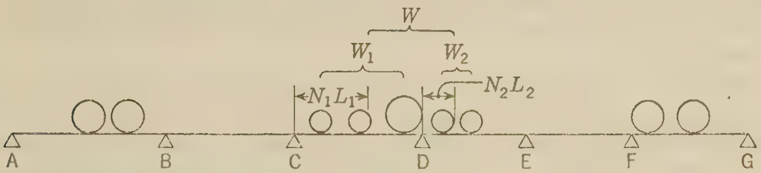

FIG. 37

The second statement to be stressed is that if, in spite of the undesirability of fixed-space loads too widely spaced to be contained in the span as criterions, it is necessary to use such loads, the procedure may be as follows: Referring to Fig. 37, let the major part of the load be placed in the span which has the greatest product of $L\ A$ (see Eq. 27), in position for maximum in its own span at the common support—W_1 in Fig. 37. Then, W_2 will lie in span DE in whatever position the fixed spacing dictates. The maximum bending moment at point D due to W_1 and the simultaneous bending moment due to W_2 will ordinarily be all that is required. If greater accuracy is desired, a few other bending moments may be computed from Eqs. 27*a* and 27*b* in a trial-and-error location of an absolute maximum. The equations are those of combined influence lines and this is exactly the procedure that would be required with influence lines.

It may be stated in passing that the writer has derived exact equations for this case; but, since these formulas involve the characteristics of both spans and hence are quite complicated, they have not been considered practical.

Fig. 37 also illustrates the possibility of placing the load in distant spans for maximum at a given point. In addition to bending moment M_{DC} induced from the loads in the adjacent spans CD and DE, loads placed in spans AB and FG in positions for maximum at point B and point F will also contribute to the maximum M_{DC} in greatest possible amount. Mr. Dohn has referred to this condition.

Commenting on the opinion of Professor Jameyson that the method would be impossibly complicated when applied to beams of variable section, the writer can state that, on the contrary, it is quite as simple for these cases as for those outlined for beams of constant section and the formulas for beams of variable section reduce directly to those of straight section when the nonuniformity disappears.

The observation by Mr. Petersen that he has found the methods useful in airplane structures is quite interesting, indicating practicability in fields outside those ordinarily considered structural.

In closing the writer again wishes to express his appreciation of the privilege of reading the extended comment on his paper and of presenting a topic of interest to structural engineers.

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Paper No. 2285

SEDIMENTATION AND THE DESIGN OF SETTLING TANKS

BY THOMAS R. CAMP,[1] M. AM. SOC. C. E.

WITH DISCUSSION BY MESSRS. NORVAL E. ANDERSON, R. A. MULHOLLAND, P. CHARLES STEIN, LYNN PERRY, E. SHERMAN CHASE, ROLF ELIASSEN, AND THOMAS R. CAMP.

SYNOPSIS

Sedimentation has been used for generations to clarify liquors and to concentrate solids in many widely diversified fields. It is the most commonly used process in the field of water and sewage treatment. The investments for settling tanks in this field are probably about one third of the total capital investment for treatment. Despite the importance of the process, current understanding of the principles involved has progressed so slightly that there is as yet no such thing in practice as the economic design of tanks from a functional viewpoint. The dimensions of most settling tanks are fixed on the basis of standard detention periods recommended by state health departments. This practice prevails in disregard of the fact that evidence was presented as early as 1904[2] that removal of suspended matter depends upon surface area and not upon tank volume. The settling characteristics of the suspensions to be clarified are rarely considered in the design of settling tanks.

The purpose of this paper is to collect in one compendium the known principles of sedimentation essential to the development of design theory, and to present the theory of design developed to a stage which will permit its use in practice. The paper is divided into ten interrelated topics. Much of the subject matter is the work of others, and due credit is given by means of references. The use of the theory is illustrated by a number of practical examples. It has been necessary to omit a discussion of the design of inlets and outlets because of lack of space.

Notation.—In so far as they apply, the letter symbols used in this paper conform essentially to American Standard Letter Symbols for Hydraulics (ASA—Z10.2—1942) prepared by a Committee of the American Standards

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NOTE.—Published in April, 1945, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

[1] Cons. Engr., Boston, Mass.

[2] "On Sedimentation," by Allen Hazen, *Transactions*, Am. Soc. C. E., Vol. LIII (December, 1904), p. 63.

Association, with Society representation, and approved by the Association in 1942.

1. Settling Velocities of Individual Particles

When a particle is released in a still fluid it will move vertically due to gravity if its density differs from that of the fluid. The particle will accelerate until the frictional "drag" of the fluid approaches the value of the impelling force, after which the vertical velocity of the particle with respect to the fluid at rest will be substantially constant. The terminal velocity is known as the "settling velocity" of the particle and it is of great importance in sedimentation.

The law for drag was first proposed by Sir Isaac Newton[3] on the assumption that the drag is due to inertia only and hence is proportional to the square of the velocity. Newton's law is now usually written:

$$F_D = C_D\, A\, \frac{\rho\, v^2}{2} \quad \ldots\ldots\ldots\ldots (1)$$

in which: F_D = the drag force; C_D = a dimensionless number called the "drag coefficient"; A = the projected area of the body in the direction of motion $\left(\frac{\pi}{4} D^2 \text{ for spheres}\right)$; v = the relative velocity between the body and the fluid; $\frac{\rho\, v^2}{2}$ = the dynamic pressure; and ρ is the mass density of the fluid. Instead of being constant, as Newton assumed, the drag coefficient C_D has been found by experiment to vary widely. Experiments demonstrate, however, that when geometric similarity exists, as in the case of objects of similar shape, similarly oriented, the drag coefficient is a function of the Reynolds number, as shown in Fig. 1.[4,5]

The general equation for the settling velocity of spheres of diameter D in terms of the drag coefficient may be obtained by equating the impelling force to the drag, from which the settling velocity is

$$v = \sqrt{\frac{4}{3}\,\frac{g}{C_D}\,\frac{(\rho_1 - \rho)\, D}{\rho}} \quad \ldots\ldots\ldots\ldots (2)$$

in which g is the gravity constant and ρ_1 is the mass density of the sphere. The solution of settling velocity problems by means of Eq. 2 and experimental values of C_D from Fig. 1 requires trial computations involving the Reynolds number $\mathbf{R} = \frac{v\, D}{\nu}$ in which ν is the kinematic viscosity of the fluid. To avoid trial calculations, the curve of Fig. 1 has been replotted in Fig. 2 in terms of

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[3] "Mathematical Principles of Natural Philosophy," by Sir Isaac Newton, Vol. 2, 1910, Sections 2 and 7.

[4] "Hydro- und Aero-Dynamik," by Ludwig Schiller, *Handbuch der Experimental Physik*, Bd. 4, Teil 4, Leipzig, 1932, p. 369.

[5] "Nomograph for the Settling Velocity of Spheres," by Hunter Rouse, *Report of the Committee on Sedimentation for 1936–1937*, National Research Council, Div. of Geology and Geography, Washington, D. C., 1937, p. 57.

$\frac{C_D}{\mathrm{R}}$ and $C_D\,\mathrm{R}^2$ as ordinates against R as abscissas.[6] It will be noted from Eq. 2 that

$$\frac{C_D}{\mathrm{R}} = \frac{4}{3}\,\frac{\rho_1 - \rho}{\rho^2}\,\frac{g\,\mu}{v^3} \quad \text{(3a)}$$

and

$$C_D\,\mathrm{R}^2 = \frac{4}{3}\,\rho\,(\rho_1 - \rho)\,\frac{g\,D^3}{\mu^2} \quad \text{(3b)}$$

in which μ is the absolute viscosity of the fluid. For obtaining the diameter in terms of a measured settling velocity, $\frac{C_D}{\mathrm{R}}$ may be computed from Eq. 3*a* and R can be obtained from the curve; D is then computed from the value of R taken from Fig. 2. Similarly, for computing the settling velocity of a sphere of size D, $C_D\,\mathrm{R}^2$ is calculated from Eq. 3*b* and the corresponding value of R is taken from Fig. 2; v is then computed from R.

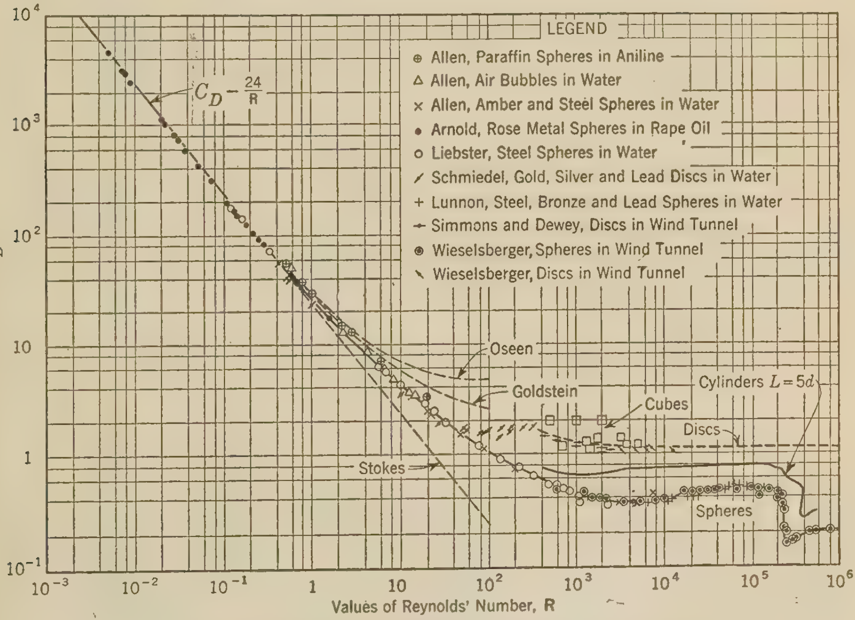

FIG. 1

The drag for a small sphere in an incompressible viscous fluid of infinite extent was first developed by Sir G. G. Stokes,[7] completely neglecting the inertia forces, as follows:

$$F_D = 3\,\pi\,\mu\,D\,v \quad \text{(4)}$$

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[6] "Principles of Chemical Engineering," by W. H. Walker, W. K. Lewis, W. H. McAdams, and E. R. Gilliland, McGraw-Hill Book Co., Inc., New York, N. Y., 1937, p. 299.

[7] "On the Theories of the Internal Friction of Fluids in Motion, and of the Equilibrium and Motion of Elastic Solids," by Sir G. G. Stokes, *Transactions*, Cambridge Philosophical Soc., Vol. VIII, 1845, p. 287; also in "Mathematical and Physical Papers," by Sir George Gabriel Stokes, Univ. Press, Cambridge, England, Vol. I, 1880, p. 75.

Comparison of Eq. 4 with Eq. 1 reveals that, in the region of viscous settling, the drag coefficient is $\frac{24}{R}$. The corresponding settling velocity is given by Stokes' law:

$$v = \frac{1}{18}\frac{g}{\mu}(\rho_1 - \rho)\, D^2 \quad (5)$$

which has been verified experimentally by numerous investigators for fluids and

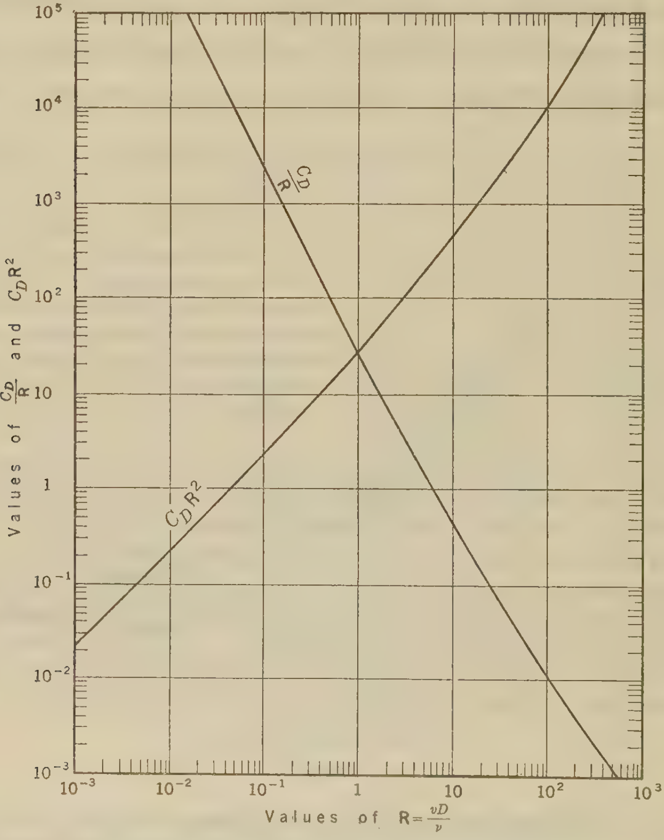

FIG. 2

spheres of various types. The data (see Fig. 1) are in good agreement with Stokes' law for values of R from 10^{-4} to about 0.5. Although Stokes' expression for the drag force, Eq. 4, was developed for incompressible fluids, the effects of compressibility are so small that the relation may be used safely with gases. In fact, the coefficient of drag throughout the entire range of Fig. 1 is not appreciably influenced by the compressibility of the fluid.

Particles to be removed from water and sewage by sedimentation are seldom truly spherical and are usually quite irregular in shape. The influence of irregularities in shape upon the drag is much greater as the velocity increases, as may be noted from Fig. 1, where size is taken in terms of the equivalent diameter of a sphere of equal volume.

A single particle settling in a container is subject to the influence of the walls at which the velocity of the fluid is zero. Empirical correction factors have been developed by A. W. Francis[8] for viscous settling and by H. S. Munroe[9] for turbulent settling. Computed values of the Francis and Munroe correction factors indicate that the influence of the wall is negligible if the diameter of the particle is less than 1% of the cylinder diameter.

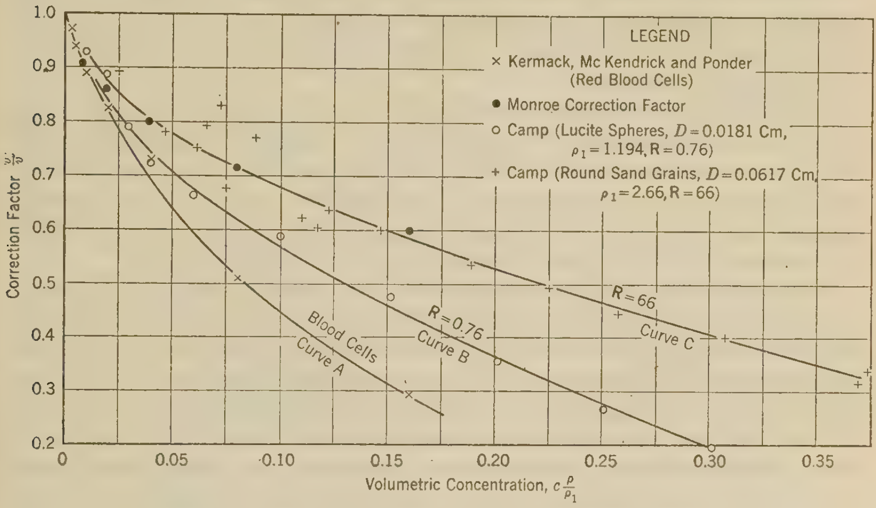

FIG. 3.—REDUCTION IN VELOCITY DUE TO HINDERED SETTLING

When a number of particles are settling in a fluid in close proximity, their velocity fields interfere and the settling is hindered. As in the case of a particle settling in a cylinder of limited cross section, there is an appreciable upward displacement of the fluid; and a correction factor analogous to the wall correction factor may be applied to estimate the "hindered" settling velocity. A theoretical analysis of the problem is lacking, however, and experimental data are not numerous.

The experiments of W. O. Kermack, A. G. McKendrick, and E. Ponder[10] on the hindered settling of red blood corpuscles are shown as curve A in Fig. 3, in which c is the concentration by weight. The writer's experiments on the viscous settling of lucite spheres in still water (curve B, Fig. 3) indicate that the correction factor $\frac{v'}{v}$, in which v' is the "hindered settling velocity," increases as the Reynolds number increases. No data are available for hindered settling in a still fluid in the transition region between viscous and turbulent settling, but A. M. Gaudin[11] has applied the Munroe correction factor as a rough approximation as shown in Fig. 3. The writer's experiments on round sand grains (curve C, Fig. 3) suspended in a tube of rising water may also be accepted as an approximation.

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[8] "Wall Effect in Falling Ball Method for Viscosity," by Alfred W. Francis, *Physics*, Vol. 4, 1933, pp. 403–406.

[9] "The English Versus the Continental System of Jigging—Is Close Sizing Advantageous?" by H. S. Munroe, *Transactions*, A.I.M.M.E., Vol. 17, 1888–1889, pp. 637–659.

[10] "The Stability of Suspensions. III. The Velocities of Sedimentation and Cataphoresis of Suspensions in a Viscous Fluid," by William Ogilvy Kermack, Anderson Gray McKendrick, and Eric Ponder, *Proceedings of the Royal Society of Edinburgh*, Vol. XLIX, 1928–1929, pp. 170–177.

2. Nature of Settling Processes in Water and Sewage Treatment

The particles to be removed in water and sewage treatment plants consist of mineral and organic solids and grease with varying quantities of entrained water and occasionally gas. The settling velocity of a particle is controlled by its size and its net density ρ_1.

The suspended matter in turbid river water consists primarily of finely divided silica, clay, and silt. The density of the particles will range from about 2.6 for fine sand grains down to about 1.03 for flocculated mud particles containing 95% water. Water from wooded regions may contain suspended vegetable matter with particle densities ranging from about 1.5 down to about 1.0 depending upon the quantity of entrained and adsorbed water.

In water treated by coagulants in rapid filter plants, the suspended particles consist of precipitated Al_2O_3 or Fe_2O_3 floc with its adsorbed water and the enmeshed solids of the raw water. Alum or iron floc may contain as much adsorbed water as 20 mols per mol of oxide when freshly precipitated, the corresponding densities being 1.18 for $Al_2O_3 \cdot 20\,H_2O$ and 1.34 for $Fe_2O_3 \cdot 20\,H_2O$. The floc particles may also contain entrained water up to about 99% of the volume, with a corresponding reduction in density to about 1.002. The density of the floc particles is further modified by the amount and character of solids entrapped from the raw water. Clay and silt should increase the density, whereas entrapped organic coloring matter might reduce the density. The size of floc particles ranges from submicroscopic to 1 mm or more.

In softening plants where the water is treated with lime and soda ash, the suspended particles are composed principally of precipitated $CaCO_3$ crystals. Peter Homack,[12] Jun. Am. Soc. C. E., has shown that the pure precipitate is calcite having a density of 2.7 and a maximum crystal size of 15 to 20 microns. The curve of correction factors for hindered settling of these particles should be slightly above curve A, Fig. 3. A study of Lieutenant Homack's experiments on hindered settling of calcite indicates that the crystals settle in small clusters, about 0.1 mm in size, which contain about 75% water and have a net density of about 1.2. The indicated "free settling" velocity of these aggregates is about 0.06 cm per sec. Magnesium hydroxide, $Mg(OH)_2$, forms a flocculent precipitate and adsorbs water like alum floc. Hence, when the excess-lime process is used the density of the floc particles will be less as the relative amount of magnesium precipitate is increased. Calcium carbonate, $CaCO_3$, flocs are small, but floc particles composed partly of $Mg(OH)_2$ may be large because of the adsorbed water. Settling is less rapid when $Mg(OH)_2$ is present than when only $CaCO_3$ is present. Alum is sometimes used in lime softening plants treating turbid waters to speed flocculation and clarification. Alum floc Al_2O_3 is not formed in this case due to the high pH-value. Lieutenant Homack's experiments indicate that the effect produced is probably due to the flocculation of the negatively[13] charged colloidal calcite crystals by the Al^{+++} ions forming large aggregates.

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[11] "Principles of Mineral Dressing," by A. M. Gaudin, McGraw-Hill Book Co., Inc., New York and London, 1939, p. 190.

[12] "A Study of the Precipitates Formed in Lime Softening," by Peter Homack, thesis submitted to Mass. Inst. Tech., Cambridge, Mass., in 1941 in partial fulfilment of the requirements for the degree of Master of Science.

In American type grit chambers for sewage treatment it is desired to remove heavy granular material such as sand in order to protect pumps, inverted siphons, or treatment units from undue abrasion and heavy deposits. Sewage grit may properly include coal dust, coffee grounds, fruit seeds, egg shells, and many other substances which are not subject to anaerobic decomposition. The density of these materials will vary from about 1.2 to 2.7. Practice indicates that chambers effectively removing sand 0.2 mm in size and larger (material retained on 65-mesh sieves) will usually perform the protective function satisfactorily. The settling velocity of 0.2 mm sand grains is from 2 to 2.4 cm per sec.

Some large sewage solids will also settle in a grit chamber, but because of their putrescibility they are not proper constituents of grit and should pass on to be disposed of with the other organic solids. Grit with no more than 3.5% washable organic matter[14] as measured by the "Dazey Churn" test can usually be disposed of as fill without nuisance, but washed grit with less than 1% washable organic matter can be obtained with careful design and operation. Washable putrescible solids may be separated from grit by "bed-load" movement in the grit chamber, by means of a mechanical "classifier" which washes the organics out as the grit is removed, or by grit washers.

Sewage organic suspended solids to be settled in primary tanks consist mainly of proteins and fats with some cellulose and varying amounts of entrained water and some gas. Thus, the net density of these particles will vary from less than 1.0 to perhaps 1.2. De-emulsified soaps, oils, grease, and some fatty particles tend to rise and form scum, carrying with them some proteins and cellulose. In chemical precipitation plants these sewage solids are combined in flocs with the chemicals used for coagulation. Suspended sewage particles range in size from colloidal to several centimeters.

In final settling tanks for trickling filters or activated sludge, the suspended particles consist of gelatinous flocs composed of a zoogleal matrix, numerous living bacteria and microorganisms, and some undecomposed organic matter. The particles settle rapidly despite their low density.

All the aforementioned suspended particles except grit flocculate during settling. Two particles that coalesce during settling lose their original settling velocities and thenceforth settle at another velocity characteristic of the combined particle which is usually faster than the original velocities. In a suspension of discrete particles, such as grit, each particle maintains its individuality and characteristic settling velocity during sedimentation; and an analysis of the suspension in terms of settling velocities may be used to predict removal and design basins. This is not the case in a suspension of flocculating particles, however, since settling velocities are changing continually.

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[13] "Theoretical Limits of the Lime-Soda Method of Water Softening," by T. E. Larson and A. M. Buswell, *Industrial and Engineering Chemistry*, Vol. 32, 1940, p. 132.

[14] "Grit Removal from Sewage," by W. A. Kivell and N. B. Lund, *Water Works & Sewerage*, Vol. 87, 1940, pp. 176–185.

From Fig. 3 it appears that the volumetric concentration of suspended solids must exceed about 0.5% before there is an appreciable reduction in settling velocity due to hindered settling. This is comparable to from 1,000 to 5,000 ppm of silt in a turbid river water, about 2,000 ppm of $CaCO_3$ in a lime-softening plant, and about 1,000 ppm of sewage suspended solids. Hence, sedimentation in all these processes may be considered "free settling" without appreciable error.

In the final tank of the activated sludge process, however, the suspended solids content will range from less than 800 to about 5,000 ppm, and the volumetric concentration of the floc particles is so great that the initial subsidence is a process of hindered settling. The particles are so close together and so flocculent that they appear to settle as a blanket, all going down at about the same velocity at the start and leaving a clear supernatant above the top of the sludge blanket. There is doubtless some free settling above the top of the sludge blanket of small floc particles which escaped entrapment in the blanket; but in tanks that are functioning well the concentration of these particles should be small enough to permit withdrawal of the supernatant from almost any place at the liquid surface beyond the inlet zone. The nature of the settling of activated sludge is illustrated in Fig. 4.

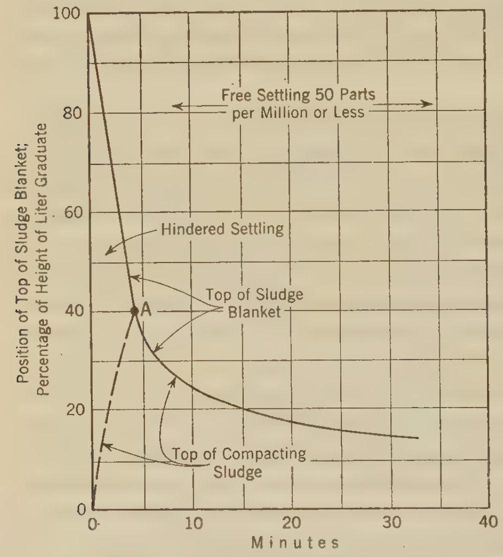

FIG. 4.—SETTLING AND COMPACTING OF TYPICAL ACTIVATED SLUDGE MIXED LIQUOR WITH AN INITIAL SUSPENDED SOLIDS CONTENT OF ABOUT 2,000 PPM

The initial subsidence of the activated sludge blanket is a process analogous to upward filtration through the sludge such as occurs in the Spaulding "Precipitator" and other similar units in which the sludge is virtually at rest with respect to the tank, being held in position by the drag of the water which flows upward between the particles. In the settling of activated sludge, however, the liquid is virtually at rest with respect to the test container as regards vertical motion and the sludge blanket moves downward. As the particles in the lower part of the blanket reach the bottom, others above settle upon these particles, and so on progressively until all floc particles in the sludge are supported upon the particles below. The supported particles do not come to rest when they make contact with the particles below, however. The weight of the sludge accumulating above distorts and compresses the supporting particles, slowly squeezing the liquid out of the sludge in an upward direction. The process is one of "sludge compacting" in which each particle above the bottom subsides at a decreasing velocity. In most existing final settling tanks the influent mixed liquor flows as a "density current"[15] along the bottom of the tank, and the process described takes place within this current.

A study of experiments[16] made by W. Rudolfs, M. Am. Soc. C. E., and I. O. Lacy indicates that the hindered settling velocity of activated sludge conforms fairly well with curve B of Fig. 3. The volumetric concentration of good activated sludge in a mixed liquor containing 2,000 ppm of suspended solids appears to be about 10%, indicating 98% water in the floc particles and a net density of about 1.005. The experiments of Messrs. Rudolfs and Lacy indicate that the hindered settling velocity, at room temperature, of a 20,000 ppm sludge is from 0.08 to 0.15 cm per sec. Fig. 3 indicates a free settling velocity of from 0.14 to 0.27 cm per sec for these particles. The equivalent diameter of the floc particles is about 1 mm.

3. Settling Analyses of Suspensions

A settling analysis of a suspension is a laboratory-scale test of its settling qualities made for the purpose of predicting removals or subsidence rates in a plant-scale settling tank. The rate of subsidence in a settling tank is a function of both the settling quality of the suspension and the hydraulic characteristics of the tank. Settling tests made for design purposes or for checking settling tank performance should yield analyses which are characteristic of the suspension when it is subjected during settling to only such tank influences as are readily reproducible in the test container.

The tank characteristics that may be reproduced approximately in a properly designed settling test container are:

(1) Flocculation due to differences in settling velocity of flocculent particles;
(2) Retardation in settling due to turbulence;
(3) Flocculation due to velocity gradients in the liquid; and
(4) Scour from the bottom.

Geometric models of plant-scale tanks cannot be adapted to reproduce these effects, but are quite useful in the measurement of hydraulic short-circuiting. Settling analyses may be corrected for short-circuiting to predict performance in plant-scale settling tanks.

A quiescent analysis of a suspension of discrete particles will yield results that are characteristic of the suspension only. A quiescent settling analysis of a suspension of flocculent particles will yield results which include the effects of flocculation due to differences in particle settling velocity. Stirring is required to reproduce tank characteristics (2), (3), and (4).

To obtain reliable quiescent settling analyses, the following precautions should be taken:

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[15] "Stratified Flow in Reservoirs and Its Use in Prevention of Silting," by Hugh S. Bell, *Miscellaneous Publication No. 491*, U. S. D. A., September, 1942.

[16] "Settling and Compacting of Activated Sludge," by Willem Rudolfs and I. O. Lacy, *Sewage Works Journal*, Vol. VI, No. 4, 1934, p. 647.

1. The sample should be representative of the suspension to be settled and should be collected in the test container in a manner least likely to change its character. Since settling and flocculation are continuous processes, the time interval between the collection of the sample and the start of the settling test must be small enough to be a negligible factor in the result. With flocculent suspensions, the collection process must not subject the floc particles to greater velocity gradients than prevail in the mother suspension lest they be broken up. Settling analyses of delicate floc are best made in the container in which the floc is formed. Settling analyses made on sewage samples collected from a sewer are of little value for predicting removal in primary tanks if the sewage must first pass through comminutors or meters or be pumped to the tanks.
2. The settling sample should be kept at a constant temperature throughout the depth of the sample by means of a water bath in order to minimize convection currents.
3. If practicable, the settling analysis should be made in a sample whose depth is equal to or greater than the depth of the tank. If the process is largely sludge concentration, a depth equal to the depth of the settling tank is preferable. Settling analyses made in shallow containers are useful in predicting performance in deep tanks, but the predictions are based on extrapolations.

For convenience, a hypothetical settling tank in which settling takes place in exactly the same manner as in a quiescent settling container of the same depth will be referred to as an "ideal basin." In an actual tank settling will differ from the quiescent analysis because of short-circuiting, eddies due to turbulence, bed-load movement or scour, faster flocculation, convection currents, and other causes. Most of these factors retard settling but faster flocculation will promote faster settling. Hence the performance of an actual settling basin will differ from that of the ideal basin, and the removal may be greater or less depending upon the relative magnitude of the disturbing factors.

Quiescent settling tests of the type here advocated have not been used widely in general practice. The Imhoff cone test has long been used in sewage plant operation, but it is inadequate for design. Another test now commonly used for the control of the activated sludge process is the quiescent settling of the "mixed liquor" in liter graduates. A liter of mixed liquor is permitted to settle and the position of the top of the sludge blanket is noted at intervals during a period of 30 min or more. The results of such a test are shown in Fig. 4.

If readings are made at more frequent intervals at the start of settling, this test may be used to make approximate predictions of settling of activated sludge in deep tanks, although a deep container is preferred in order to avoid extrapolations. In the example given in Fig. 4, the slope of the line at the start of settling is a measure of the velocity of hindered settling of the particles in the top of the sludge blanket. Hindered settling is complete at point A, Fig. 4. The rate of compacting thereafter, in terms of the ratio of the volume occupied by the sludge blanket, seems to be little influenced by the depth of the sludge or the temperature, according to the experiments by Messrs. Rudolfs and Lacy.[16] Experiments indicate that sludge compacting begins at a concentration of from 4,000 to 7,000 ppm for good activated sludge. The foregoing principles can be illustrated by two examples, as follows:

Example 1.—For the mixed liquor of Fig. 4, how long will it take for hindered settling to be complete in an ideal tank 10 ft deep, and what will be the depth of the sludge blanket and the solids concentration at the end of this period?

Since the height of the liter graduate in which the test is made is about 40 cm, the velocity indicated is about 0.11 cm per sec. In a 10-ft tank, the settling period corresponding to point A, Fig. 4, is $\frac{10 \times 30.48}{40} \times 3.6 = 27.5$ min. The depth of sludge is 4 ft and the average concentration of suspended solids in the sludge is $\frac{2,000}{0.4} = 5,000$ ppm.

Example 2.—For the same mixed liquor, and the same tank as in Example 1, what settling period is required to produce a sludge with a suspended solids content of 12,500 ppm?

The corresponding volume is $\frac{2,000}{12,500}$ or 16% of the initial volume, and the time required to reach this volume in the test (Fig. 4) was 25 min. Because of the greater depth, however, there is a time lag of 27.5 − 3.6 = 23.9 min for the completion of hindered settling. Hence a settling period of 25 + 23.9 = 48.9 min is required.

Quiescent settling analyses of suspensions of either discrete or flocculent particles may be made in a container of the type illustrated in Fig. 5. The temperature of the water bath should be the same as that of the sample; and at the start of the analysis the suspension should be distributed with a minimum of residual disturbance as uniformly as possible throughout the depth of the container. The interpretation of the analysis and its application to settling tank studies are much simplified if the initial concentration is constant throughout the depth. The analysis consists of determining the initial concentration of suspended matter and the concentration at one or more depths at measured time intervals throughout a period of settling in excess of that required in the settling tank.

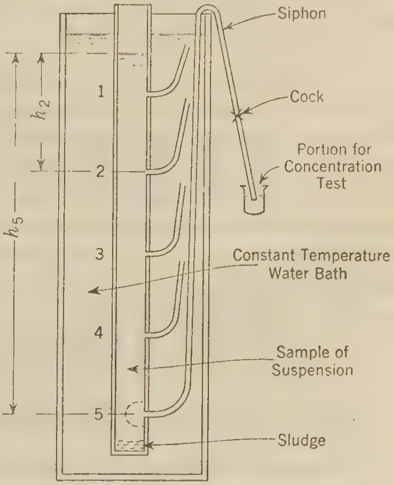
Siphon
Cock
Portion for Concentration Test
Constant Temperature Water Bath
Sample of Suspension
Sludge
h_2
h_5
1
2
3
4
5

Fig. 5.—Apparatus for Quiescent Settling Analyses

The concentration of suspended matter may be measured in terms of suspended solids, turbidity, iron, alumina, calcium, hardness, color, biochemical oxygen demand (B.O.D.), or any other index that is reduced by settling. Fig. 5 indicates means for removal of portions for concentration tests. Portions may be withdrawn quite safely for tests for iron, alumina, calcium, hardness, suspended solids, total solids, and B.O.D.; but this method cannot be used when the turbidity test is used and the results are doubtful with color determinations. Turbidity and color may be measured directly in the settling container by photoelectric or comparator methods. When portions are to be withdrawn for analysis the size of the portion required depends upon the test used and accuracy desired. The cross-section area of the settling container should be large enough to permit withdrawal of all necessary portions without lowering the water surface excessively.

The method to be used for collecting samples in the settling container must be determined for each problem, and considerable ingenuity may be required to obtain satisfactory results. In some cases, the mother suspension may be used as the water bath. If the mother suspension or a representative sample of it can be directed through the water bath so that the concentration is substantially uniform with depth, the test sample may be collected by lowering a bottomless cylinder vertically through the water bath to the bottom, thus trapping the sample within the cylinder.

TABLE 1.—QUIESCENT SETTLING ANALYSES OF SUSPENSIONS

Point[a]	Settling Time, in Minutes[b]												
	0	15		30		45		60		90		180	
	h	c_r	h	c_r	h	c_r	h	c_r	h	c_r	h	c_r	h
(a) Discrete Particles													
1	42	0.65	40	0.23	38	0.10	36	0.05	34	0.03	32	0.01	30
2	122	0.96	120	0.81	118	0.62	116	0.46	114	0.23	112	0.06	110
3	202	0.98	200	0.94	198	0.85	196	0.73	194	0.52	192	0.16	190
4	282	0.99	280	0.97	278	0.93	276	0.86	274	0.70	272	0.32	270
5	362	1.0	360	0.98	358	0.96	356	0.92	354	0.83	352	0.46	350
(b) Flocculent Suspension													
1	42	0.65	40	0.23	38	0.10	36	0.05	34	0.03	32	0.01	30
2	122	0.77	120	0.50	118	0.28	116	0.17	114	0.09	112	0.02	110
3	202	0.81	200	0.54	198	0.33	196	0.21	194	0.11	192	0.03	190
4	282	0.85	280	0.57	278	0.38	276	0.25	274	0.13	272	0.04	270
5	362	0.90	360	0.60	358	0.40	356	0.27	354	0.14	352	0.04	350

[a] Sampling points in Fig. 5. [b] The depth below the surface in centimeters, as shown in Fig. 5, is represented by h; and c_r is the ratio of measured concentration to initial concentration. At 0 minutes c_r is equal to 1.

Table 1(*a*) shows a hypothetical settling analysis of a suspension of discrete particles such as would result with unhindered settling and ideal test conditions. The concentration has been recorded for each sampling point and each settling period as the ratio c_r of the measured to the initial concentration. The depth h recorded for each portion is the depth below the surface just before the first portion of the set is withdrawn. To minimize error, portions should be withdrawn in sequence from top to bottom and as rapidly as possible at the end of the measured time interval. Each portion withdrawn comes from a bulb-shaped region as shown at point 5, Fig. 5. The suspension above, except in the immediate vicinity of the sampling point, is drawn down vertically a distance corresponding to the volume removed, with practically no change in the relative position of the suspended particles. Hence the values of h for any period are less than the previous values by an amount corresponding to the volume of all portions withdrawn at the end of the previous period.

Each portion contains no particles whose settling velocities are high enough to carry them from the surface past the sampling point during the settling period. Hence the maximum settling velocity v for each portion is approximately $\frac{h}{t}$, in which t is the settling period. The values of v thus computed for the top samples are correct, but values for the other portions are slightly in error because the values of h do not represent the actual distance settled by the largest particles. The error is greatest for the last sample taken at the lowest sampling point. In the example, the actual distance settled for the last portion taken (point 5 at 180 min, Table 1(a)) is 352.4 cm instead of 350 as recorded.

Since the initial concentration was uniform from top to bottom, the concentration c_r of all particles in a sample whose settling velocities are less than v is the same as the initial concentration. Therefore a "settling velocity curve" may be plotted as a characteristic of the suspension. Fig. 6 represents the data in Table 1(a), the points marked with crosses and circles corresponding to samples from points 2 and 4, respectively. It will be noted that the curve may be drawn from data obtained at only one sampling point, but that data from all sampling points fall on the same curve.

Under ideal test conditions with unhindered settling, if flocculation is taking place, particles that coalesce will thenceforth settle at other velocities, usually faster. This results in more rapid clearing near the bottom than with discrete particles. A hypothetical settling analysis of a flocculent suspension is shown in Table 1(b). A settling velocity curve cannot be drawn for flocculent suspensions because the particles do not retain constant settling velocities. If an attempt is made to draw a settling velocity curve, a separate curve will result for each sampling point. An examination of the tabulated data from a settling analysis to determine if the values of c_r are about the same for all portions having the same value of v will reveal whether the suspension is discrete or flocculent.

4. Clarification Theory for an Ideal Basin

Since an "ideal basin" has been defined as a hypothetical settling tank in which settling takes place in exactly the same manner as in a quiescent settling container of the same depth, an "ideal rectangular continuous flow basin" for unhindered settling has the following characteristics:

1. The direction of flow is horizontal and the velocity is the same in all parts of the settling zone (hence, each particle of water is assumed to remain in

the settling zone for a time equal to the detention period—namely, the volume of the settling zone divided by the discharge rate);

2. The concentration of suspended particles of each size is the same at all points in the vertical cross section at the inlet end of the settling zone; and

3. A particle is removed from suspension when it reaches the bottom of the settling zone.

Any continuous flow basin may be considered as being comprised of four zones (see Fig. 7) according to function as follows: (1) An inlet zone in which the suspension is dispersed uniformly over the cross section of the basin; (2) the settling zone in which all settling takes place; (3) an outlet zone in which the clarified liquid is collected uniformly over the cross section of the basin and directed to an outlet conduit; and (4) a sludge zone at the bottom. The present discussion is confined to the settling zone.

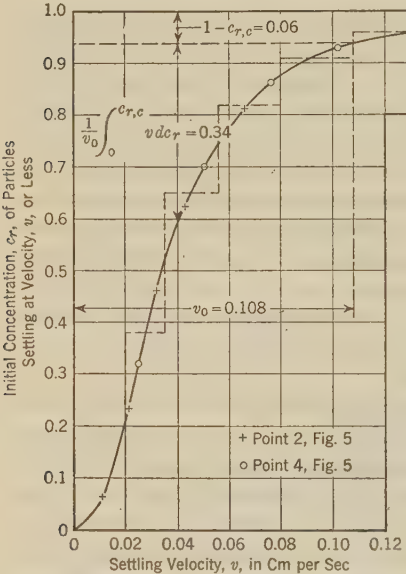

Fig. 6.—Settling Velocity Analysis of Suspension of Discrete Particles (Illustrating Example 5)

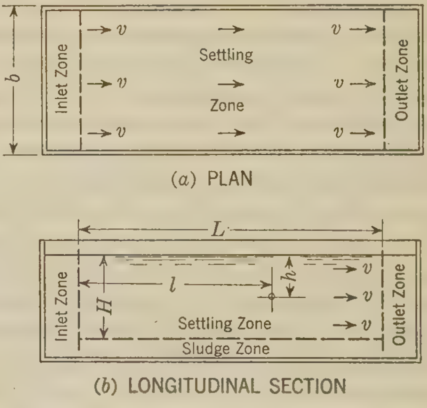

Fig. 7.—Ideal Rectangular Settling Basin

The concentration of suspended matter in any vertical column in an ideal basin will be distributed in precisely the same manner as in the quiescent settling analysis after an equal period of settling. Hence the settling analysis may be used to predict the concentration at any point in the settling zone, in accordance with the following example:

> *Example 3.*—What will be the concentration of suspended matter at a depth of 194 cm in an ideal basin after 60 min of settling of the suspension represented by Table 1(*a*); also of the suspension represented by Table 1(*b*)?
>
> Referring to Table 1, the concentrations are 73% and 21%, respectively, of the original concentration.

In the vertical cross section through any point in an ideal basin the concentration of suspended matter will be distributed vertically as indicated by the values of c_r in the analysis after an equal settling period; and the average concentration is approximately equal to the arithmetical average of the values of c_r throughout the depth H. Therefore, the residual concentration and the removal for any period of settling may be estimated from the analysis, as follows:

> *Example 4.*—What will be the removal in the settling zone of an ideal basin after 60 min of settling of the suspension of Table 1(*a*) if the depth is 390 cm; also if the depth is only 150 cm?
>
> For a depth of 390 cm, the average concentration from Table 1(*a*) is about 0.60; and the removal is therefore 40%. For a depth of 150 cm, the average concentration is about 0.26 and the removal is 74%.

It is evident therefore that a depth and settling period may be selected to secure any removal within the limits of the settling analysis of the suspension. In design the length L and depth H of the settling zone are determined from the removal required. The distribution of solids along the bottom may be estimated by computing the removal in partial lengths l.

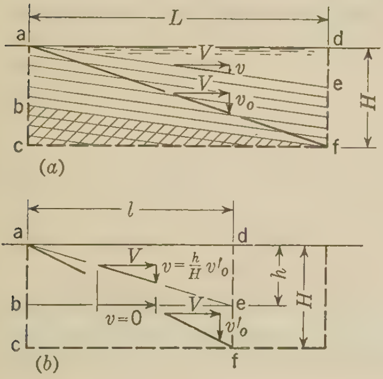

FIG. 8

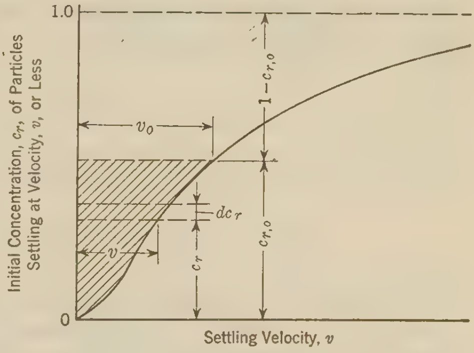

FIG. 9.—TYPICAL SETTLING VELOCITY ANALYSIS CURVE OF SUSPENSION FOR DISCRETE PARTICLES

Any particle settling in a moving liquid will move in a direction and at a velocity, with reference to the basin, which is the vector sum of its own settling velocity and the velocity of the surrounding liquid. In an ideal basin, the settling paths of discrete particles may be used to develop a clarification theory that is helpful in grit-chamber design and in understanding settling phenomena. In an ideal rectangular basin the paths of all discrete particles will be straight lines, and all particles with the same settling velocity will move in parallel paths as illustrated by lines parallel to ae in Fig. 8(*a*). Moreover, the settling pattern will be the same in all longitudinal sections of the settling zone and any longitudinal section, as shown in Fig. 8(*a*), may be considered in a study of the process.[17,18] All particles having settling velocities equal to or greater than v_o will settle out; but the removal of particles having any settling velocity v which is less than v_o will be equal to the ratio $\frac{\overline{\mathrm{bc}}}{\overline{\mathrm{ac}}}$. From similar triangles, this removal ratio is

$$r_r = \frac{v}{v_o} = \frac{b\,L\,v}{Q} \quad \text{(6)}$$

in which $b\,L =$ the surface area of the settling zone of width b and length L; $Q =$ the rate of discharge; and $v_o = \frac{Q}{b\,L} =$ the "overflow rate" or the discharge per unit of surface area of the settling zone.

Eq. 6 appears to have been discovered by the late Allen Hazen,[2] M. Am. Soc. C. E. The area $b\,L$ is the surface area only for basins with vertical side-walls. For basins with sloping walls, $b\,L$ is the average area of horizontal sections throughout the depth of settling.

If the settling velocity analysis of the original suspension is represented by a curve such as Fig. 9, the value of c_r (here designated as $c_{r,o}$) corresponding to the overflow rate v_o of the basin may be read from the curve. Since all particles which settle faster than v_o will be 100% removed, the removal of such particles in terms of the total suspension will be $1 - c_{r,o}$. The removal of particles having any settling velocity v which is less than v_o is given by Eq. 6; and, since the ratio of such particles to the total suspension is dc_r, the removal in terms of the total suspension is

$$r_r\,dc_r = \frac{1}{v_o}\,v\,dc_r \quad \text{(7a)}$$

The total removal of all particles is, therefore,

$$r_r = 1 - c_{r,o} + \frac{1}{v_o}\int_0^{c_{r,o}} v\,dc_r \quad \text{(7b)}$$

It is not necessary to assign an equation to the velocity analysis curve in order to solve for the removal. The last term of Eq. 7b is the average vertical distance from the curve (Fig. 9) to the horizontal line for $c_r = c_{r,o}$. It is the shaded area in Fig. 9 divided by v_o.

The concentration of suspended matter at any point in the settling zone, such as point e, Fig. 8(b), is the initial concentration of all sizes whose settling paths are such as to permit them to reach the point. The only particles that can reach point e are those whose settling velocities are between zero and $\frac{h}{H}\,v'_o$, in which v'_o is the overflow rate for the length l. Therefore, the concentration at point e is c'_r, the value of c_r corresponding to the velocity $\frac{h}{H}\,v'_o$, which may be read directly from the settling velocity analysis curve.

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[17] "A Study of the Rational Design of Settling Tanks," by Thomas R. Camp, *Sewage Works Journal*, Vol. VIII, No. 5, September, 1936, p. 742.

[18] Discussion by Thomas R. Camp of "Sedimentation in Quiescent and Turbulent Basins," by J. J. Slade, Jr., *Transactions*, Am. Soc. C. E., Vol. 102 (1937), p. 306.

The foregoing theory also applies to the settling of discrete particles in ideal radial flow basins, in which case the basin velocity V varies inversely as the radial distance, and consequently the paths of the particles are curved. The application of Eq. 7*b* is demonstrated by Example 5, as follows:

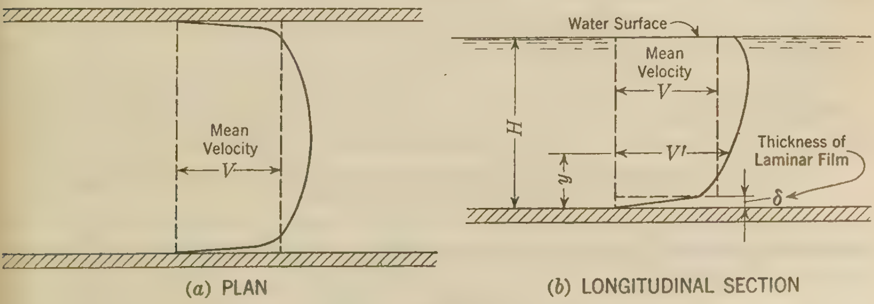
> *Example 5.*—By means of Eq. 7*b*, estimate the removal in the settling zone of an ideal basin after 60 min of settling of the suspension of Table 1(*a*) and Fig. 6 if the depth is 390 cm.
>
> The overflow rate, v_o, of the basin is $\frac{390}{60 \times 60} = 0.108$ cm per sec, and the corresponding value of $c_{r,o}$ from Fig. 6 is 0.94. Since the value of the last term of Eq. 7*b*, from Fig. 6, is about 0.34, the removal is $1 - 0.94 + 0.34 = 0.40$, or 40% as in Example 4.

As a consequence of the foregoing theory (Eqs. 6 and 7), the following conclusions may be drawn with respect to settling of discrete particles in an ideal basin:

1. For any given discharge, the removal is a function of the surface area and is independent of the depth of the basin; or, the removal is a function of the overflow rate and, for a given discharge, is independent of the detention period; and
2. The concentration of suspended matter at any cross section in the settling zone increases with the depth below the surface, and decreases with the proximity of the cross section to the outlet end of the basin. The settling path of a particle of settling velocity v from the surface at the inlet end of the settling zone is a line of equal concentration, the concentration being the corresponding value of c_r from the settling velocity analysis curve of the suspension. Hence, lines of equal concentration intersect at the surface at the inlet end, are straight lines for rectangular basins, and are curved lines for radial flow basins.

5. Tractive Force and Bed-Load Movement

In an actual settling basin the velocity is not uniform over a cross section as assumed for an ideal basin. In a rectangular tank with the best of conditions the distribution will approximate that shown in Fig. 10. The value of the Reynolds number $\frac{VR}{\nu}$, in which R is the hydraulic radius, will in most cases



(*a*) PLAN (*b*) LONGITUDINAL SECTION

Fig. 10

greatly exceed 500. Hence, in most settling tanks the flow will be turbulent. If the walls are smooth, it may be assumed that the flow in a thin "laminar film" of thickness δ, adjacent to the walls and floor, will be streamlined.

If the bottom is covered with a leveled sand bed, A. Shields[19, 20] reasoned that the turbulent flow theory as developed by J. Nikuradse, L. Prandtl, and Theodor von Kármán, M. Am. Soc. C. E., should be applicable to the beginning of bed-load movement. Accordingly, he developed the following relation:

$$\frac{T_c}{(\gamma_s - \gamma)\,D} = \phi\left(\frac{D}{\delta}\right) = \beta \ldots\ldots\ldots\ldots\ldots\ldots (8)$$

in which T_c is the critical tractive force required to produce motion along the bottom of particles of diameter D and specific weight γ_s, γ is the specific weight of the liquid, and $\phi\left(\frac{D}{\delta}\right)$ is some function of $\frac{D}{\delta}$ which may be assumed equal to a constant β without great error.

The results of experiments by Mr. Shields and others on the critical tractive force of unigranular materials of varying specific gravity are plotted in Fig. 11 in accordance with Eq. 8. For the beginning of bed-load movement, the experiments show that

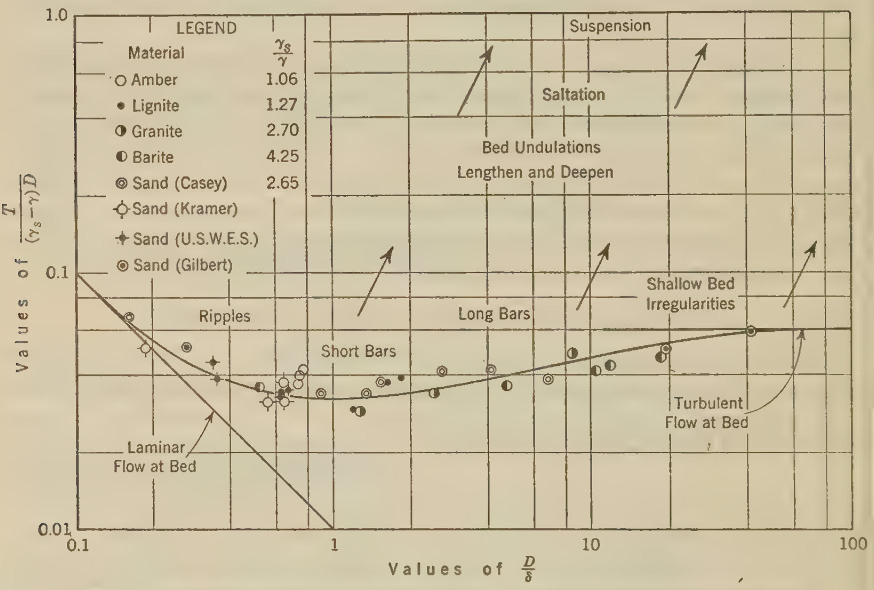


FIG. 11.—BEGINNING OF BED-LOAD MOVEMENT AS A FUNCTION OF GRAIN DIAMETER AND BOUNDARY-LAYER THICKNESS

$$T_c = \beta\,(\gamma_s - \gamma)\,D \ldots\ldots\ldots\ldots\ldots\ldots (9)$$

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[19] "Anwendung der Aehnlichkeitsmechanik und der Turbulenz forschung auf die Geschiebebewegung," by A. Shields, *Mitteilungen der Preussischen Versuchsanstalt für Wasserbau und Schiffbau*, Heft 26, 1936, Berlin.

[20] "An Analysis of Sediment Transportation in the Light of Fluid Turbulence," by H. Rouse, Sedimentation Div., U. S. D. A., Washington, D. C., July, 1939.

in which the value of β, or $\phi\left(\frac{D}{\delta}\right)$, is about 0.04. For nonuniform material or for sticky and flocculent material, the value of β may be larger. Fine nonuniform sands tend to exhibit two values[21] for the critical tractive force, a lower value (β equal to about 0.04) for impending motion from smooth beds and a higher value (β equaling from 0.10 to 0.25) for impending motion from sand ripples formed from the smooth bed; the values of β, in both cases, corresponding to the mean grain size.

It is evident that the average value of the tractive force T in a unit length of open channel is $\gamma R h_f$, in which R is the hydraulic radius and h_f the head loss. Since, from the Weisbach-Darcy formula,

$$h_f = \frac{f}{4R}\frac{V^2}{2g} \qquad (10)$$

in which f is the well-known friction factor and V is the mean velocity in the channel, it follows that:

$$T = \frac{f}{8}\frac{\gamma V^2}{g} \qquad (11)$$

From Eqs. 9 and 11, the channel velocity required to start motion of particles of size D is

$$V_c = \sqrt{\frac{8\beta}{f} g (s-1) D} \qquad (12)$$

in which s = the specific gravity of the particles.

Eq. 12 indicates that the velocity required to start scour is independent of the size and depth of tank; and that it depends only on the friction factor and the mean particle size and specific gravity. Actually, the value of the tractive force T in Eq. 11 is the mean value around the wetted perimeter. It is constant only for circular pipes flowing full. In open channels, the tractive force is greater at the invert than on the sides, and hence its value at the bottom is influenced by the shape of the cross section. The maximum velocity that may be used in a settling tank to avoid scour is given by Eq. 12.

Eq. 12 is of value in the investigation of sewage grit chambers. Both grit and some organic matter will reach the bottom, but since organic solids are lighter than grit the chamber acts as a "classifier" to separate the organics by bed-load movement. The greater the difference in specific gravity the better is the separation. An application of Eq. 12 is as follows:

Example 6.—Estimate the limiting velocity in a grit chamber to avoid scour of sand particles 0.2 mm in size. What size sand will be started in motion if the velocity is 1.0 ft per sec? What is the relative size of sand and organic matter (assume $s = 1.2$) that will be scoured?

If β is taken at 0.06 and f is assumed to be 0.03, the value of V_c for sand ($s = 2.65$) 0.2 mm in size, from Eq. 12, is 0.75 ft per sec. If V_c is

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[21] *Paper No. 17*, U. S. Waterways Experiment Station, Vicksburg, Miss., January, 1935.

1.0 ft per sec, the sand size $D = \left(\frac{1.0}{0.75}\right)^2 \times 0.2 = 0.36$ mm. The critical size of organic matter is $\frac{2.65 - 1}{1.2 - 1}$ or eight times as large as the sand.

Eqs. 8 and 12 yield no clue as to the quantity of sediment moved by traction, nor upon the vertical distribution of material picked up from the bottom. As the tractive force is increased more material is thrown into suspension, as shown by Fig. 11; and if the tractive force is great enough all the material appears to be in suspension, as in the case of a clean sewer. Transportation in suspension is not of itself an equilibrium condition. For the concentration in suspension to remain constant at any depth, the rate of pickup from the bottom must equal the rate of settling on to the bottom. If all particles are fixed in position or removed when they reach the bottom, the stream will eventually clarify.

The effect of turbulence on settling of material already in suspension is independent of scour and may or may not be accompanied by bed-load movement.

6. Effects of Turbulence on Sedimentation

The turbulent mixing process works in a manner similar to diffusion. The rate of transport of any substance or fluid property is proportional to the space rate of change in concentration or magnitude of the substance or fluid property, the flow being in the direction of decreasing concentration or magnitude. In the transfer of suspended matter due to mixing, the rate of upward transport at a point y above the bottom (Fig. 10(*b*)) is $-\epsilon\frac{dc}{dy}$, in which $-\frac{dc}{dy}$ is the concentration gradient in the direction of y and ϵ is the mixing coefficient.

The intensity of shear is zero at the water surface and is distributed vertically in two-dimensional open channel flow according to the expression,

$$\tau = T\left(1 - \frac{y}{H}\right) \quad \text{(13)}$$

Therefore, the assumptions made by Prandtl[22] in deriving the theory of turbulent mixing will yield the following relation:

$$\epsilon = \frac{\frac{\tau}{\rho}}{\frac{dV'}{dy}} = \frac{T}{\rho}\,\frac{1 - \frac{y}{H}}{\frac{dV'}{dy}} \quad \text{(14)}$$

in which V' is the velocity at height y. This expression was not expected by Prandtl to hold at the boundaries or at the center of a full pipe where it yields a value of zero for ϵ. Mixing is known to take place across the axis of a full pipe and a finite value for ϵ at the bed seems necessary to start scour.

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[22] "Bericht über Untersuchungen zur ausgebildeten Turbulenz," by L. Prandt., *Zeitschrift für Angewandte Mathematik und Mechanik*, Vol. 5, No. 2, 1925, p. 136.

The value of ϵ for any depth y in the intermediate zone between bottom and water surface may be estimated by means of Eq. 14 from the slopes of measured velocity distribution curves. The trend of ϵ from Nikuradse's measurements of velocity distribution in both smooth and rough pipes is shown[20] by curve A of Fig. 12. If a logarithmic velocity distribution is assumed, the resulting trend of ϵ is shown by curve B. A more complex expression as developed by Professor von Kármán is shown by curve C,[20] Fig. 12. A parabolic velocity distribution results in a constant value of ϵ throughout the depth:

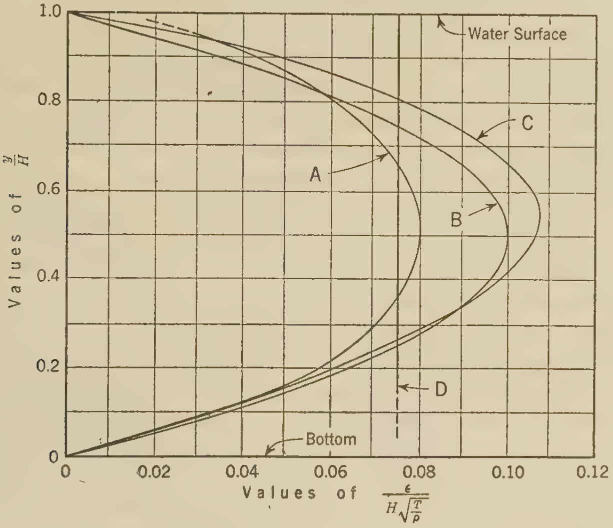

FIG. 12

$$\epsilon = 0.075\, H \sqrt{\frac{T}{\rho}} \quad \text{(15)}$$

as shown by curve D, Fig. 12. The constant 0.075 results in a parabola which approximates Nikuradse's measured velocity distribution from the pipe axis to within 0.9 of the radial distance to the wall. The assumption of a parabolic velocity distribution simplifies computations; but this approximation cannot be used for studying turbulence near the wall since it implies a finite wall velocity.

If the rate of scour from the bed is at equilibrium with the rate of settling in steady uniform flow, the suspended load transport at any height y above the bed is also at equilibrium with settling as follows:

$$\epsilon \frac{dc}{dy} = -v\,c \quad \text{(16)}$$

v being the settling velocity of the particles, and c the concentration of the particles at height y. Integration of Eq. 16 results in an expression for the relative concentration at height y with respect to any other height a:

$$\log_e \frac{c}{c_a} = -v \int^{y} \frac{dy}{\epsilon} \quad \text{(17)}$$

The validity of Eq. 17 has been verified experimentally by Hunter Rouse,[23] M. Am. Soc. C. E., and by W. E. Dobbins,[24] Jun. Am. Soc. C. E., for constant values of ϵ by means of vertical cylindrical tanks equipped with mechanical agitators designed to produce a constant intensity of mixing throughout the depth. It has also been verified by Vito A. Vanoni,[25] Assoc. M. Am. Soc. C. E., for open channel flow with a value for ϵ as shown by curve B, Fig. 12. If a parabolic velocity distribution is assumed as an approximation of the logarithmic distribution in which ϵ may be determined by Eq. 15, and if a is assumed very small compared with H, computed values of $\frac{c}{c_a}$ are substantially correct at 0.4 H. They reach a maximum deviation at 0.1 H and 0.7 H, being too large at 0.1 H and too small at 0.7 H.

In a settling tank the equilibrium condition postulated by Eq. 16 is approached by only those particles which are scoured after they reach the bottom. Eqs. 16 and 17 provide the means, therefore, of estimating the vertical distribution of discrete particles thrown into suspension by bed-load transportation. The equations permit computations of only relative concentrations.

Most of the particles settling in a tank are undisturbed after they reach the bottom, and the effect of turbulence on the settling of these particles is limited to a delaying action. This effect may be investigated for discrete particles with two-dimensional flow in a rectangular tank by studying the concentration changes at a point in the tank such as m, Fig. 13.

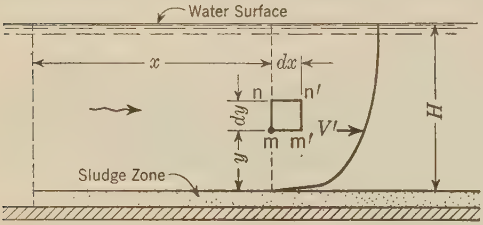

FIG. 13

For the steady state in which the concentration at a fixed point in the tank remains unchanged with time, Ensign Dobbins[24] found that

$$V' \frac{\partial c}{\partial x} = \epsilon_y \frac{\partial^2 c}{\partial y^2} + \left(v + \frac{\partial \epsilon_y}{\partial y} \right) \frac{\partial c}{\partial y} + \epsilon_x \frac{\partial^2 c}{\partial x^2} \quad \text{(18)}$$

in which ϵ_x is the mixing coefficient in the direction of x. No solution of Eq. 18 has been obtained in its general form. Before a solution can be found, V', ϵ_y, and ϵ_x must be expressed as functions of y. The equation can be greatly simplified by making certain approximations. If the assumptions are made that $\epsilon_x \frac{\partial^2 c}{\partial x^2}$ is zero and that both V' and ϵ_y are constant throughout the depth H, Eq. 18 is reduced to

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[23] "Experiments on the Mechanics of Sediment Suspension," by H. Rouse, *Proceedings*, Fifth International Cong. for Applied Mechanics, Cambridge, Mass., 1938.

[24] "Effect of Turbulence on Sedimentation," by William E. Dobbins, *Transactions*, Am. Soc. C. E., Vol. 109 (1944), p. 629 (condensed from "A Study of the Effect of Turbulence on Sedimentation," presented to Mass. Inst. Tech., Cambridge, Mass., in partial fulfilment of the requirements for the degree of Doctor of Science, October, 1941).

[25] "Experiments on the Transportation of Suspended Sediment by Water," by Vito A. Vanoni, Sedimentation Div., U. S. D. A., Washington, D. C., September, 1940.

$$V \frac{\partial c}{\partial x} = \epsilon \frac{\partial^2 c}{\partial y^2} + v \frac{\partial c}{\partial y} \qquad (19)$$

Now, if the point m is assumed to move in the direction of x at velocity $V = \frac{\partial x}{\partial t}$, the time rate of change in concentration at m is

$$\frac{\partial c}{\partial t} = \epsilon \frac{\partial^2 c}{\partial y^2} + v \frac{\partial c}{\partial y} \qquad (20)$$

Eq. 20 also expresses the rate of change in concentration at any point m in a settling container in which turbulence of uniform intensity is imposed throughout.

A solution for Eq. 20 has been obtained by Ensign Dobbins[24] and has been verified experimentally. Since no solution has been found for the general form of the differential equation as given by Eq. 18, it is necessary to use Ensign Dobbins' solution of Eq. 20 to obtain an approximation of the effect of turbulence in retarding settling. This integration is also a solution for Eq. 19 and corresponds with the assumption of uniform concentration at the inlet end and constant values for ϵ and tank velocity throughout the tank. For the case of no scour, the writer[26,27] has transformed Ensign Dobbins' solution so that the results are given in terms of the removal as follows:

$$r_r = 1 - 8 \left(\frac{v\,H}{2\,\epsilon}\right)^2 e^i \sum_{n=1}^{\infty} \frac{\alpha^2{}_n\, H_n\, e^j}{\left[\left(\frac{v\,H}{2\,\epsilon}\right)^2 + \alpha^2{}_n + 2\,\frac{v\,H}{2\,\epsilon}\right]\left[\left(\frac{v\,H}{2\,\epsilon}\right)^2 + \alpha^2{}_n\right]^2} \qquad (21)$$

in which $\alpha_1, \alpha_2, \alpha_3, \cdots, \alpha_n$ are the successive real positive roots of the transcendental equation

$$2 \cot \alpha = \frac{\alpha}{\frac{v\,H}{2\,\epsilon}} - \frac{\frac{v\,H}{2\,\epsilon}}{\alpha} \qquad (22)$$

H_n is $+1$ when α is in the first and second quadrants and -1 when α is in the third and fourth quadrants; v_o is the overflow rate; and to simplify typography

$$i = \frac{v\,H}{2\,\epsilon} \qquad (23a)$$

and

$$j = -\,(i^2 + \alpha^2{}_n)\,\frac{v}{2\,i\,v_o} \qquad (23b)$$

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[26] "The Effect of Turbulence in Retarding Settling," by T. R. Camp, *Bulletin 27*, *Proceedings*, 2d Hydraulics Conference, Iowa Inst. of Hydr. Research, Univ. of Iowa, Iowa City, 1942, p. 307.

[27] *Transactions*, Am. Soc. C. E., Vol. 109 (1944), p. 660.

The removal in Eq. 21 is expressed as a function of only three variables, $\frac{v\,H}{2\,\epsilon}$, α_n, and $\frac{v}{v_o}$, all of which are dimensionless. Fig. 14 is a dimensionless plot for the graphical solution of Eq. 21. The successive values of α required for each computed point on this graph were determined to six decimal places by trial and error solutions of Eq. 22. Convergence of the series is rapid for low values of $\frac{v\,H}{2\,\epsilon}$, only one term being required for a value of 0.1; but fourteen terms were required for $\frac{v\,H}{2\,\epsilon} = 30$, and no solution could be obtained for $\frac{v\,H}{2\,\epsilon} = 100$.

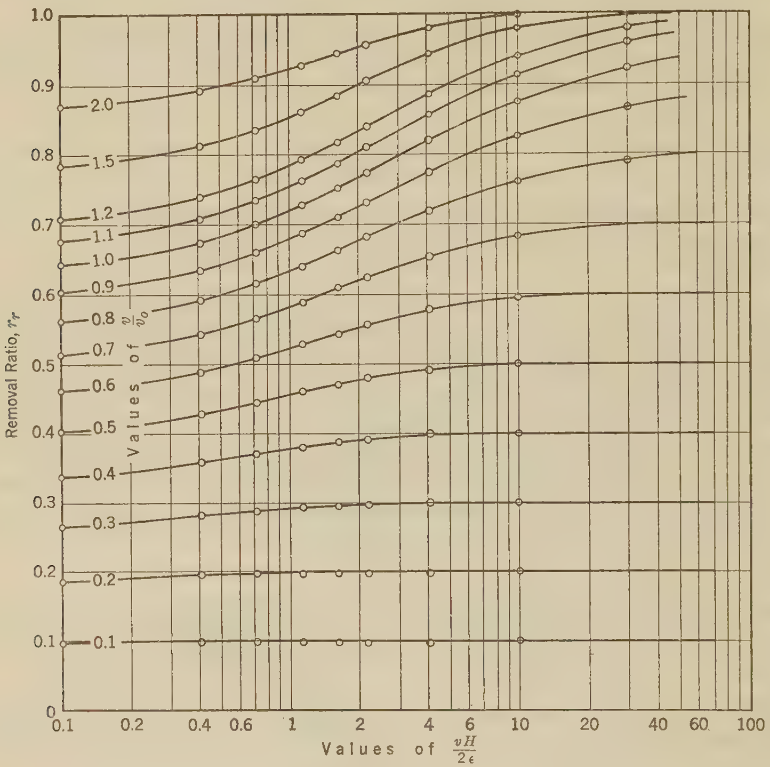

Fig. 14

In order to use Fig. 14 to estimate the removal in an actual settling tank with a free liquid surface, it is convenient to use values of ϵ arising from the assumption of parabolic velocity distribution as given by Eq. 15. With this value for ϵ and the friction factor f taken arbitrarily at 0.024:

$$\frac{v\,H}{2\,\epsilon} = 122\,\frac{v}{V} = 122\,\frac{v_o}{V}\,\frac{v}{v_o} = 122\,\frac{H}{L}\,\frac{v}{v_o} \ldots\ldots\ldots\ldots\ldots (24)$$

For values of $\frac{L}{H}$ ranging from about $\frac{2}{3}$ to 200 as they do in practice, and for values of $\frac{v}{v_o}$ from 0.1 to 2.0; Eq. 24 indicates a range from 0.06 to 81 for values of $\frac{v\,H}{2\,\epsilon}$.

The direct application of Fig. 14 in practice is limited to the determination of the length of grit chambers or other tanks settling discrete particles. The method of designing grit chambers on the basis of the simple theory of sedimentation for ideal basins (Eqs. 6 and 7*b*) has been described elsewhere.[28] An example of the application of Fig. 14 is as follows:

Example 7.—Determine the length of the settling zone of a grit chamber 4 ft deep which is required to effect a 95% removal of grit settling at 2 cm per sec when the chamber velocity is 1.0 ft per sec.

From Eq. 24, $\frac{v\,H}{2\,\epsilon} = 122\,\frac{v}{V} = 122\,\frac{2}{30.48 \times 1} = 8.0$. From Fig. 14 for 95% removal, $\frac{v}{v_o} = 1.3$ and $v_o = \frac{2}{30.48 \times 1.3} = 0.05$ ft per sec. Since $\frac{L}{H} = \frac{V}{v_o}$, the required length of the settling zone is $L = 40\,\frac{1.0}{0.05} = 80$ ft. If there were no retardation of settling by turbulence, $\frac{v}{v_o}$ would be 0.95 and the required length would be only 58.5 ft.

7. FLOCCULENT SUSPENSIONS

Flocculation in a settling tank is due to two causes:

1. Differences in the settling velocities of the particles whereby faster settling particles overtake those which settle more slowly and coalesce with them; and
2. Velocity gradients in the liquid which cause particles in a region of higher velocity to overtake those in adjacent stream paths moving at slower velocity.

In the ideal basin and in quiescent settling tests only the first cause is present. The degree of flocculation is difficult to measure except in terms of the effect on settling; but this effect may be measured by concentration tests at intervals of depth and time. Some experimental data,[17,18,29] mostly qualitative, are available.

Where flocculation occurs, settling velocities are usually increased. It is difficult to obtain a correlation between the rate of change of settling velocities and the rate of flocculation, because, when particles coalesce, size, density, and shape are all changed. The rate of flocculation is a function of the number of contacts between particles in unit time. Hence the number of contacts per unit time in unit volume may be used as a measure of the rate of flocculation, and where the rate is highest it may be assumed that the effect on settling is greatest.

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[28] "Grit Chamber Design," by Thomas R. Camp, *Sewage Works Journal*, Vol. 14, No. 2, 1942, pp. 368–381.

[29] *Transactions*, Am. Soc. C. E., Vol. LIII (December, 1904), pp. 53 and 72.

In quiescent settling, a particle of diameter D_1 and settling velocity v_1 will contact in unit time all particles of diameter D_2 and settling velocity v_2 in a cylinder whose volume is $\frac{\pi}{4}(D_1 + D_2)^2(v_1 - v_2)$. If n_2 is the number of particles of diameter D_2 per unit volume, the number of contacts per unit of time per unit of volume with each 1-particle is $n_2 \frac{\pi}{4}(D_1 + D_2)^2(v_1 - v_2)$. If n_1 is the number of particles of diameter D_1 per unit of volume, the total number of contacts per unit of time per unit of volume due to settling velocity[30] is

$$N_s = n_1 n_2 \frac{\pi}{4}(D_1 + D_2)^2(v_1 - v_2) \qquad (25a)$$

If the consideration is limited to viscous settling, the Stokes law value of v from Eq. 5 may be substituted in Eq. 25*a* so that

$$N_s = n_1 n_2 \frac{\pi g (s - 1)}{72 \nu}(D_1 + D_2)^3(D_1 - D_2) \qquad (25b)$$

in which all particles have the same density.

It will be noted that there will be no union of particles having the same settling velocity, and that the rate of flocculation is proportional to the difference in settling velocities. The effect of flocculation will thus be more apparent near the bottom of deep tanks after a period of settling. Also it will be noted that since the rate of flocculation is proportional to the relative number of contacts, $\frac{N_s}{n}$ (in which n is the total number of particles per unit volume, and both n_1 and n_2 are proportional to n), the rate of flocculation is directly proportional to the concentration.

From a similar development,[30] the total number of contacts per unit of time per unit of volume due to a velocity gradient in the fluid is

$$N_v = n_1 n_2 \tfrac{1}{6} G (D_1 + D_2)^3 \qquad (26)$$

in which G represents the mean temporal velocity gradient in the fluid. Thus the rate of flocculation due to velocity gradient is directly proportional to the velocity gradient.

The velocity gradients in a settling tank are due both to the residual turbulence at the inlet and to the drag on the walls and floor. This discussion will be limited to velocity gradients caused by drag, the mean value of which can be evaluated as a function of tank shape and velocity as follows.

The shearing force on the opposite faces of an elementary cube of fluid in which the velocity gradient is $\frac{dv'}{dz}$ is $\mu \frac{dv'}{dz}\,dx\,dy$, and the work done per unit of time is $\mu \left(\frac{dv'}{dz}\right)^2 dx\,dy\,dz$. Hence, the work done by shear per unit of volume per unit of time[30] is

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[30] "Velocity Gradients and Internal Work in Fluid Motion," by T. R. Camp and P. C. Stein, *Journal*, Boston Soc. of Civ. Engrs., October, 1943, p. 219.

$$W = \mu \left(\frac{dv'}{dz} \right)^2 \qquad (27a)$$

or

$$W = \mu G^2 \qquad (27b)$$

in which G is the root-mean-square velocity gradient throughout the fluid. The work done in a tank of length L in unit time, by drag, is

$$Q g \rho h_f = Q g \rho \frac{f L}{4 R} \frac{V^2}{2 g} = \frac{f L b H}{8 R} \rho V^3 \qquad (28)$$

from which the work done per unit of volume per unit of time is

$$W = \frac{f}{8 R} \rho V^3 \qquad (29)$$

Equating the values of W from Eqs. 27 and 29, the mean velocity gradient in the tank due to drag on the walls and floor is found to be

$$G = \sqrt{\frac{f}{8 \nu R} V^3} \qquad (30)$$

If this value of G is substituted in Eq. 26, the total number of contacts per unit of time per unit of tank volume due to velocity gradients is

$$N_V = \frac{n_1 n_2}{6} \sqrt{\frac{f}{8 \nu R} V^3} (D_1 + D_2)^3 \qquad (31)$$

From Eq. 31, it may be seen that the rate of flocculation due to velocity gradients is proportional to $V^{1.5}$ and inversely proportional to $\sqrt{R}$. Hence this type of coagulation is greatest for shallow tanks with relatively high velocities.

Computations by means of Eqs. 25*b* and 31 of the relative rates of flocculation due to settling and velocity gradients in water and sewage tanks indicate that:

(1) Flocculation in deep tanks at low velocity ($R = 20$ ft and $V = 1$ ft per min) is due almost entirely to differential settling velocities; and

(2) The rates of flocculation by the two processes are about the same in a tank 2 ft deep with a velocity of 10 ft per min.

The particles of flocculent suspensions are fragile and subject to rupture by shear, the shearing strength being less as the particle size increases. Although velocity gradients in the fluid bring small particles together to form larger ones, it is evident that there is a limiting size of particle which is determined by the mean shearing force and the strength of the floc. Since the mean shearing force is proportional to the mean velocity gradient, there is a limiting size corresponding to each value of the mean velocity gradient G; and the size is greater the smaller the velocity gradient. If the particles of a flocculent suspension entering the settling tank have not reached this limiting size, they will continue to coalesce until they have either reached this size or passed from the settling zone.

The flocculation of suspensions dealt with in water and sewage treatment is rarely complete when the particles enter the settling tanks, because they have been subjected to much higher velocity gradients during conveyance to the tanks. The additional flocculation caused by turbulence in the tanks may speed up the settling to a greater extent than turbulent mixing retards it. There is much qualitative experimental evidence that most of the suspensions are very much under-flocculated and that additional flocculation will greatly improve settling.

In a number of cases in the writer's studies in which two similar samples of iron floc were permitted to settle in liter graduates after a 30-min mixing period, one settling undisturbed and the other gently stirred, the stirred sample invariably clarified much more rapidly. In studying the flocculation and settling of iron floc in battery jars, Darrell A. Root[31] found that, with a total time of mixing and quiescent settling in excess of 75 min, any increase in mixing time at the expense of settling time up to 60 min of mixing would improve the settling.

In laboratory experiments on nine raw sewages from widely separated plants, A. J. Fischer, Assoc. M. Am. Soc. C. E., and A. Hillman have shown[32] that 20 min of self-flocculation prior to settling increases the removal of suspended solids after 60 min of settling by 4% to 36% over results obtained without flocculation, an average improvement of 20%. Plant-scale tests at Cedar Rapids, Iowa, and Sioux Falls, S. Dak., have demonstrated that the improvement expected from preflocculation can be obtained. Many primary sewage tanks have been built with provisions for preflocculation, and it is probable that this will become standard practice.

It is evident from the foregoing discussion that increased removal should be expected from tanks with higher velocities than are commonly used. In order to determine the optimum velocity and depth for a particular suspension, settling tests with turbulence superimposed are desirable. Already mechanical mixing grids have been used successfully to reproduce particular values of the mixing coefficient ϵ. It is quite likely that the dimensions, amplitudes, and mixing speeds of these devices can be adjusted so as to produce, at the same time, the desired values of ϵ and the mean velocity gradient G. The magnitude of G can be computed from the measured work input by Eq. 27*b*. Thus the effects of both turbulent retardation and coagulation can be taken account of in the settling tests.

8. OVERFLOW RATE, DETENTION PERIOD, VELOCITY, AND TANK DIMENSIONS

In the simple theory of sedimentation of discrete particles in an ideal basin it was demonstrated that the removal is a function of the overflow rate and is independent of the tank depth and the detention period. It follows from the simple theory that the most economical tank will have the least possible depth for the required overflow rate.

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[31] "The Effect of Temperature Upon the Rate of Floc Formation," by Darrell A. Root, thesis submitted to Mass. Inst. Tech., Cambridge, Mass., in 1939, in partial fulfilment of the requirements for the degree of Master of Science.

[32] "Improved Sewage Clarification by Preflocculation Without Chemicals," by A. J. Fischer and A. Hillman, *Sewage Works Journal*, Vol. 12, No. 2, p. 280.

For a given overflow rate and fixed horizontal dimensions, the tank velocity varies inversely as the depth. Hence, the least possible depth is fixed by the maximum velocity that can be used without excessive scour. The magnitude of this velocity can be determined experimentally or estimated by Eq. 12, a relation which holds for either discrete or flocculent particles with the proper value of β.

The effect of turbulent mixing on these conclusions may be studied by means of Eqs. 21 and 24 and Fig. 14. It is evident from Eq. 24 that $\frac{v H}{2 \epsilon}$ is directly proportional to the depth, and since the removal is a function of $\frac{v H}{2 \epsilon}$ it is not independent of the depth as indicated in the simple theory. The effect of depth, however, is small. Fig. 14 indicates that a 50% decrease in depth will decrease the removal of discrete particles by not more than about 5% on the average; that is, a decrease in volume of 50% through a decrease in depth may be compensated for by a 5% increase in volume through increased surface area. For economy, therefore, the conclusion still holds that the depth should be made as small as is consistent with no scour.

Since flocculation in a settling tank produces a smaller concentration gradient, the retardation of settling due to turbulence will be even less than for discrete particles. Hence, from the standpoint of turbulent retardation of settling, shallow high-velocity tanks are more economical than deep tanks for both discrete and flocculent suspensions. Flocculation, however, also changes the settling velocity of the particles. With the data available, it is not possible to draw conclusions regarding the effect of flocculation upon removal, except that removal is generally increased. It is possible, however, to draw conclusions relating to the effect of depth upon flocculation and indirectly upon removal. Two limiting cases may be considered for argument—(1) deep tanks with low velocities in which coagulation is due entirely to differences in settling velocities; and (2) shallow tanks with high velocities in which the rates of coagulation by settling and velocity gradient are about equal.

Eq. 25*b* indicates that the rate of flocculation in deep tanks with low velocities is independent of the velocity or depth. If it is assumed that flocculation continues throughout the settling period (in other words, that the limiting size of particle is not reached), the amount of flocculation will be directly proportional to the detention period. Hence, an increase in depth for the same overflow rate will produce a proportional increase in the amount of flocculation. An increase in depth will thus increase removal; but it is not possible to state, without settling tests, whether the same increase in removal might not be obtained more economically by a decrease in overflow rate.

Eq. 31 indicates that the rate of flocculation due to velocity gradients is proportional to $\sqrt{\frac{V^3}{H}}$ in tanks whose depth is small compared to width. For the same horizontal dimensions V varies inversely with H. Hence, the rate of flocculation is proportional to $\frac{1}{H^2}$; and, assuming continuous flocculation, the amount of flocculation is inversely proportional to the depth. If the rates of flocculation by settling and velocity gradients are about equal, an increase in the amount of flocculation by velocity gradients due to a decrease in depth should be compensated for by a decreased flocculation due to settling. Therefore, for this case, the amount of flocculation should be independent of the depth for the same overflow rate. In shallow tanks with high velocities, removal of flocculent suspensions should be relatively independent of tank depth for the same horizontal dimensions and overflow rate, provided there is no scour, and provided the particles do not reach the limiting size. Hence, for economy, the same conclusion may be drawn, as with discrete suspensions, that the depth should be made as small as is consistent with no scour.

The minimum depth will result from the least practical value of the hydraulic radius. This economic value of R may be computed from Eq. 30 by using the optimum mean velocity gradient, G_o, which will yield the desired removal with the greatest overflow rate and the maximum velocity, V_c, which will not scour. The economic hydraulic radius is

$$R_e = \frac{\frac{f}{8\nu} V^3_c}{G^2_o} \qquad (32)$$

In a tank with a rectangular cross section, the hydraulic radius is, for a free water surface,

$$R = \frac{b\,H}{b + 2\,H} = \frac{\frac{Q}{V}}{\frac{Q}{V\,H} + 2\,H} \qquad (33a)$$

and, for a submerged cover,

$$R = \frac{b\,H}{2\,b + 2\,H} = \frac{\frac{Q}{V}}{\frac{2\,Q}{V\,H} + 2\,H} \qquad (33b)$$

It follows from Eqs. 33 that the economic tank depth is, for a free water surface,

$$H_e = \frac{Q}{4\,V_c\,R_e} - \sqrt{\left(\frac{Q}{4\,V_c\,R_e}\right)^2 - \frac{Q}{2\,V_c}} \qquad (34a)$$

and, for a submerged cover,

$$H_e = \frac{Q}{4\,V_c\,R_e} - \sqrt{\left(\frac{Q}{4\,V_c\,R_e}\right)^2 - \frac{Q}{V_c}} \qquad (34b)$$

It will be noted that Eqs. 34 yield unreal results if $\frac{Q}{2\,V_c}$ and $\frac{Q}{V_c}$ are greater than $\left(\frac{Q}{4\,V_c\,R_e}\right)^2$. This indicates the existence of a minimum or limiting discharge; and, if the actual value of Q is less than the limiting value, Eqs. 34

are not applicable. The limiting discharge is, for a free water surface,

$$Q_{\text{lim}} = 8\,V_e\,R^2_e \qquad (35a)$$

and, for a submerged cover,

$$Q_{\text{lim}} = 16\,V_e\,R^2_e \qquad (35b)$$

For this discharge the width of the tank is twice the depth for a free water surface and is equal to the depth for a submerged cover.

If the actual discharge is less than Q_{lim}, for best economy the width must be twice the depth for a free water surface and equal to the depth for a submerged cover. For this case, therefore, for a free water surface,

$$Q = 8\,V\,R^2 \qquad (36a)$$

and, for a submerged cover,

$$Q = 16\,V\,R^2 \qquad (36b)$$

in which V must be less than V_e, R must be less than R_e, or both V and R must be reduced. If V_e is maintained and R reduced, the optimum velocity gradient will be exceeded and the overflow rate must be decreased to obtain the required removal. If V is reduced and R_e maintained, the velocity gradient will be less than the optimum, and a decrease in overflow rate will be required. No decrease in overflow rate will be required if the optimum velocity gradient is maintained, but both V and R must be reduced. If the values of R from Eqs. 36 are introduced into Eq. 30, the value of V required to maintain the optimum velocity gradient is, for a free water surface,

$$V = \sqrt[7]{8\,Q\left(\frac{\nu}{f}\right)^2 G^4_o} \qquad (37a)$$

and, for a submerged cover,

$$V = \sqrt[7]{4\,Q\left(\frac{\nu}{f}\right)^2 G^4_o} \qquad (37b)$$

The corresponding values of R may be computed from Eqs. 36 with values of V given by Eqs. 37.

A direct procedure for the economic design of settling tanks for flocculent suspensions is to make settling analyses of the suspensions in containers equipped with stirring devices designed to produce any desired values of ϵ and G and to control the rate of scour from the bottom. If it is anticipated that a shallow high velocity tank can be used, the depth of container is not important. The method of designing stirring devices for this purpose has not yet been developed, but studies[33] with this objective have been made in the Sanitary Engineering Laboratory of the Massachusetts Institute of Technology, at Cambridge, Mass.

In the first series of tests, there should be no scour from the bottom and the magnitude of G should be increased progressively until a gradient is found that will produce the desired removal with the greatest overflow rate. The overflow rate is the depth of container divided by the time of settling. The values of ϵ to be used in this series may be computed by means of Eq. 15, and hence it is necessary to anticipate the approximate values of V and H. The stirring device may be calibrated for ϵ with a suspension of discrete particles of known settling velocity by means of Eq. 17.

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[33] "Measurement of Work Input in Liquids by Reciprocating Mixing Devices," by K. A. Ignacio, thesis submitted to Mass. Inst. Tech., Cambridge, Mass., in 1943, in partial fulfilment of the requirements for the degree of Master of Science.

The next series of tests should be made at the optimum velocity gradient, G_o, previously found; or, if this value is too high, at the maximum practical value of G. In this series, scour from the bottom should be increased progressively until it just begins to reduce the removal. The amount of scour may be controlled by regulating the mean position of the stirring device relative to the bottom of the container. The device may be calibrated to determine the relation between incipient scour and tank velocity V by means of Eq. 12 and a bed of granular material of known size and density.

These settling tests should yield the best values of G and V, and the overflow rate v_o which is required for the desired removal. The tank dimensions may then be determined with the foregoing equations. The procedure is best illustrated by a number of examples, as follows:

Example 8.—Assume that settling analyses of a flocculent suspension at 10° C have resulted in the following test data for the required removal:

Velocity gradient G (sec^{-1})	Overflow rate v_o (gal per day per sq ft)	Non-scouring velocity V_e (cm per sec)
2.0	1,000	5.08
6.0	1,500	....
9.0	1,800	....
12.0	2,000	5.08
15.0	1,800	....
18.0	1,200	....

The problem is to determine the economic dimensions of the settling zone in rectangular tanks for (*a*) $Q = 30$ cu ft per sec $= 19.4$ mgd; and (*b*) $Q = 0.1$ cu ft per sec $= 64{,}600$ gal per day. The test data indicate that G_o is 12 sec^{-1} and that V_e is 5.08 cm per sec or $\frac{1}{6}$ ft per sec. Assuming that $f = 0.032$, the economic hydraulic radius, from Eq. 32, is: $R_e = \dfrac{\dfrac{0.032}{8 \times 0.01} \times 5.08^3}{12^2} = 0.366$ cm $= 0.144$ in. This value corresponds with tank depths of 0.14 in. and 0.28 in. for open and covered tanks, respectively, and it is obviously impracticable to construct such tanks. A tank of the tray clarifier type is indicated, however. Let it be assumed that the minimum depth of pass practical for construction and sludge removal is 6 in. (15.2 cm). Then R is about 7.6 cm and, from Eq. 30, the corresponding velocity gradient is

$$G = \sqrt{\frac{f}{8\,\nu\,R}\,V^3{}_e} = \sqrt{\frac{0.032}{8 \times 0.01 \times 7.6} \times 5.08^3} = 2.64 \text{ sec}^{-1}.$$

The test data indicate that, with this velocity gradient, the overflow rate must be reduced to about 1,000 gal per day per sq ft or 1.547×10^{-3} ft per sec to obtain the required removal.

(*a*) For $Q = 30$ cu ft per sec, $b = \frac{Q}{VH} = \frac{30}{\frac{1}{6} \times 0.5} = 360$ ft; and $L = \frac{V}{v_o} \times H = \frac{\frac{1}{6}}{1.547 \times 10^{-3}} \times 0.5 = 54$ ft. The width of pass for this discharge is obviously too great, and it is necessary to divide the discharge through a number of passes whose width is satisfactory for construction and sludge removal purposes. Let it be assumed that a width of 18 ft is satisfactory. There will then be twenty parallel passes, each 18 ft wide by 6 in. deep by 54 ft long and passing 1.5 cu ft per sec. For convenience, they may be stacked vertically in two tanks, each with ten trays. For each pass, from Eq. 33*b*: $R = \frac{18 \times 0.5}{2\,(18 + 0.5)} = 0.243$ ft, which is sufficiently close to the assumed value (7.6 cm = 0.25 ft) so that there will be no appreciable change in the computed value of the velocity gradient, 2.64 sec^{-1}. The detention period, $T = \frac{H}{v_o} = \frac{0.5}{1.547 \times 10^{-3}} = 323$ sec = 5.4 min.

(*b*) For $Q = 0.1$ cu ft per sec, the width of the tank will be much less than for $Q = 30$ cu ft per sec and will have an appreciable effect on R. If it is again assumed that 6 in. is the minimum practical depth, $b = \frac{0.1}{\frac{1}{6} \times 0.5} = 1.2$ ft.

From Eq. 33*b*, $R = \frac{1.2 \times 0.5}{2\,(1.2 + 0.5)} = 0.177$ ft = 5.4 cm, and from Eq. 30, $G = \sqrt{\frac{0.032}{8 \times 0.01 \times 5.4} \times 5.08^3} = 3.12\ \text{sec}^{-1}$. From the test data, the required removal can be obtained with an overflow rate of about 1,100 gal per day per sq ft ($v_o = 1.7 \times 10^{-3}$ ft per sec). Therefore, $L = \frac{\frac{1}{6}}{1.7 \times 10^{-3}} \times 0.5 = 49$ ft, and the detention period $T = \frac{0.5}{1.7 \times 10^{-3}} = 294$ sec = 4.9 min.

This tank may be made shorter by dividing the total length into a number of passes in series, stacked vertically or side by side.

It is not practical to use an open surface tank for this case because, to obtain the same velocity gradient, the depth would be only 3 in.

Example 9.—Assume that settling analyses of a flocculent suspension at 10° C have resulted in the following test data for the required removal:

Velocity gradient G (sec^{-1})	Overflow rate v_o (gal per day per sq ft)	Non-scouring velocity V_s (cm per sec)
0.2	200	....
0.5	400	....
1.0	600	3.81
1.5	400	....

Determine the economic dimensions of the settling zone in rectangular tanks for: (*a*) $Q = 30$ cu ft per sec = 19.4 mgd; and (*b*) $Q = 0.1$ cu ft per sec = 64,600 gal per day.

The test data indicate that $G_o = 1.0\ \text{sec}^{-1}$, $V_c = 3.81$ cm per sec, or $\frac{1}{8}$ ft per sec, and the overflow rate required is 600 gal per day per sq ft or 0.928×10^{-3} ft per sec. Assuming that $f = 0.032$, the economic hydraulic radius from Eq. 32 is $R_e = \dfrac{\dfrac{0.032}{8 \times 0.01} \times 3.81^3}{1^2} = 22.2$ cm $= 0.728$ ft.

(*a*) For $Q = 30$ cu ft per sec, $\dfrac{Q}{V_c R^2_e} = \dfrac{30}{\frac{1}{8} \times 0.728^2} = 453$. From Eqs. 34, H_e is about 0.73 ft for an open tank and 1.46 ft for a submerged cover. The corresponding widths are $\dfrac{30}{\frac{1}{8} \times 0.73} = 328$ ft and 164 ft. The tank with the free water surface is the cheaper because its length will be half that of the tank with the submerged cover. Therefore, $L = \dfrac{V_c}{v_o} \times H_e = \dfrac{\frac{1}{8}}{0.928 \times 10^{-3}} \times 0.73 = 98.3$ ft and the detention period is $T = \dfrac{H_e}{v_o} = \dfrac{0.73}{0.928 \times 10^{-3}} = 786$ sec $= 13.1$ min.

If it is desired to divide the width into parallel passes about 18 ft wide for sludge removal mechanism, there will be eighteen passes each with a discharge of 1.67 cu ft per sec. For each pass, $\dfrac{Q}{V_c R^2_e} = \dfrac{1.67}{\frac{1}{8} \times 0.728^2} = 25.2$. From Eq. 34*a*, $H_e = 0.8$ ft; and $b = 20.8 \times \dfrac{1}{0.8} = 16.65$ ft. Also, $L = \dfrac{\frac{1}{8}}{0.928 \times 10^{-3}} \times 0.8 = 108$ ft; and the detention period is $T = \dfrac{108}{7.5} = 14.4$ min.

(*b*) For $Q = 0.1$ cu ft per sec, $\dfrac{Q}{V_c R^2_e} = \dfrac{0.1}{\frac{1}{8} \times 0.728^2} = 1.51$.

It is evident that Q is less than Q_{lim}. Therefore, for economy, the velocity as given by Eq. 37*a* is $V = \sqrt[7]{8 \times 0.1 \times \overline{30.48}^3 \times \left(\dfrac{0.01}{0.032}\right)^2 \times \overline{1.0}^4} = 3.0$ cm per sec $= 0.0983$ ft per sec; and, from Eq. 36*a*, $R = \sqrt{\dfrac{Q}{8\,V}} = \sqrt{\dfrac{0.1}{8 \times 0.0983}} = 0.357$ ft. Finally: $H = 2 \times 0.357 = 0.715$ ft; $b = 2 \times 0.715 = 1.43$ ft; $L = \dfrac{V}{v_o} \times H = \dfrac{0.0983}{0.928 \times 10^{-3}} \times 0.715 = 75.8$ ft; and $T = \dfrac{75.8}{0.0983} = 772$ sec $= 12.8$ min.

Example 10.—Assume that settling analyses of a flocculent suspension at 10° C have resulted in the following test data for the required removal:

Velocity gradient G (sec^{-1})	Overflow rate v_o (gal per day per sq ft)	Non-scouring velocity V_c (cm per sec)
0.1	200	....
0.2	300	....
0.3	400	3.81
0.4	300	....
0.5	200	....

Determine the economic dimensions of the settling zone in rectangular tanks for: (a) $Q = 30$ cu ft per sec $= 19.4$ mgd; and (b) $Q = 0.1$ cu ft per sec $= 64{,}600$ gal per day. The test data indicate that G_o is 0.3 sec^{-1} and V_c is 3.81 cm per sec, or $\frac{1}{8}$ ft per sec, and that the required overflow rate is 400 gal per day per sq ft or 0.618×10^{-3} ft per sec. Assuming that $f = 0.032$, the economic hydraulic radius, from Eq. 32, is $R_e = \dfrac{\frac{0.032}{8 \times 0.01} \times 3.81^3}{0.3^2} = 247$ cm $= 8.1$ ft.

(a) For $Q = 30$ cu ft per sec, $\dfrac{Q}{V_c R^2_e} = \dfrac{30}{\frac{1}{8} \times \overline{8.1}^2} = 3.66$. It is evident that Q is less than Q_{lim}. Therefore, for economy, the velocity must be reduced as given by Eq. 37*a* to $V = \sqrt[7]{8 \times 30 \times \overline{30.48}^3 \times \left(\dfrac{0.01}{0.032}\right)^2 \times \overline{0.3}^4} = 3.41$ cm per sec $= 0.112$ ft per sec and, from Eq. 36*a*, $R = \sqrt{\dfrac{Q}{8\,V}} = \sqrt{\dfrac{30}{8 \times 0.112}} = 5.8$ ft. Finally: $H = 2 \times 5.8 = 11.6$ ft; $b = 2 \times 11.6 = 23.2$ ft; $L = \dfrac{V}{v_o} \times H = \dfrac{0.112}{0.618 \times 10^{-3}} \times 11.6 = 2{,}100$ ft; and $T = \dfrac{2{,}100}{0.112} = 18{,}800$ sec $= 5.22$ hr. The 2,100-ft length may be divided into a number of passes in series, placed side by side.

For this case it may be more economical to divide the 30 cu ft per sec between a number of parallel passes and thus reduce the depth and detention period. For example, if six passes are used, each with a discharge of 5 cu ft per sec, the dimensions of each will be: $H = 5.36$ ft; $b = 10.72$ ft; and $L = 754$ ft. The detention period will be reduced to 2.42 hr. The most economical number of passes can be determined only by considering the construction costs.

(b) For $Q = 0.1$ cu ft per sec, the discharge is again less than Q_{lim}, and, from Eq. 37*a*, $V = \sqrt[7]{8 \times 0.1 \times \overline{30.48}^3 \times \left(\dfrac{0.01}{0.032}\right)^2 \times \overline{0.3}^4} = 1.51$ cm per sec $= 0.0495$ ft per sec.

From Eq. 36*a*, $R = \sqrt{\dfrac{Q}{8\,V}} = \sqrt{\dfrac{0.10}{8 \times 0.0495}} = 0.502$ ft. Finally: $H = 2 \times 0.502 = 1.004$ ft; $b = 2 \times 1.004 = 2.008$ ft; $L = \dfrac{V}{v_o} \times H = \dfrac{0.0495}{0.618 \times 10^{-3}} \times 1.004 = 80.5$ ft; and $T = \dfrac{80.5}{0.0495} = 1{,}625$ sec $= 27.1$ min. The tank may be shortened by dividing the total length, 80.5 ft, into a number of passes in series placed side by side.

Comment.—A study of Examples 8, 9, and 10 reveals that the magnitude of the optimum velocity gradient G_o is of great influence on the type of tank required for economy. The possible range in values of G_o may be divided into three regions, each with a different method of tank design as follows:

(1) High values of G_o which result in values of R_e that are too small to be practicable. (The economic velocity V_c may be used, but it is impossible to

realize the optimum velocity gradient, and lower values must be used in design. Tanks of the tray clarifier type are indicated, and the depth H for each pass is dictated by construction and sludge removal requirements.)

(2) Intermediate values of G_o which can be realized in tank design with reasonable values of R_e and H_e. (Shallow tanks with free water surfaces are indicated.)

(3) Low values of G_o which are accompanied by values of Q less than Q_{lim}. (The optimum velocity gradient can be realized in tank design, but only by using values of V and R less than V_c and R_c. Tanks with a free water surface are indicated, with the width always twice the depth.)

There is a dearth of experimental data concerning the optimum values of G for the suspensions encountered in practice. A study of tank velocities used in practice will reveal the magnitudes of G in use. It has been common practice to limit the design velocity in settling tanks to about 3 ft per min. There is increasing evidence that higher velocities are accompanied by better removal, but it is doubtful whether any tanks have been built to operate at velocities and velocity gradients approaching V_c and G_o.

Reporting on a tunnel section with a flow velocity of 12 ft per min, W. F. Langelier[34] stated that settling had been unusually effective in removing turbidity and color from an alum-coagulated water. For this suspension it is evident that V_c was greater than 12 ft per min. Since a velocity of about 18 ft per min (0.3 ft per sec) is considered a minimum for baffled mixing chambers to avoid too much settling of floc in the chamber, it appears that 18 ft per min is greater than V_c for alum and iron floc.

The value of G for Langelier's tunnel computed by Eq. 30 is 0.96 sec^{-1}. Similar computations for baffled mixing chambers reveal that good floc is produced at values of G as high as 25 sec^{-1}. Hence, the value of G_o for alum and iron oxide floc is probably much higher than 1 sec^{-1}. The estimated value of G in the pore constrictions of a rapid sand filter during normal operating conditions is from 200 sec^{-1} to 400 sec^{-1}. This is too high to promote further coagulation of the fine floc particles within the bed, according to an unpublished paper on microscopic studies by P. C. Stein, Assoc. M. Am. Soc. C. E., but it is obviously not high enough to prevent the floc from sticking to the sand grains. The estimated value of G in the pore constrictions of slow sand filters is from 5 sec^{-1} to 40 sec^{-1}.

The primary tanks at the Detroit (Mich.) sewage treatment works[35] are designed for higher velocities than are in general use, the velocity at peak flow being 7.2 ft per min. The corresponding value of G is about 0.3 sec^{-1}. In view of the fact that preflocculation of sewage at velocity gradients of the same order of magnitude as those used for coagulation of water has shown marked improvement in settling, it seems likely that the values of G_o for sewage suspensions are much greater than the value of 0.3 sec^{-1} used at Detroit.

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[34] "Shallow Sedimentation Basins," by Wilfred F. Langelier, *Journal*, A.W.W.A., Vol. 22, No. 11, 1930, p. 1484.

[35] "Report on Hydraulic Model Inlet Tests—Primary Sedimentation Tanks," by George E. Hubbell, Detroit Sewage Disposal Project, December 15, 1937.

Pending the development of methods of design of stirring devices to produce any desired values of ϵ and G in settling test containers, the parameters V_c, v_o, and the maximum practical value of G may be measured in a small continuous flow rectangular tank so designed that both depth and velocity may be changed between runs. This method of attack requires the construction and operation of a pilot treatment plant and, therefore, it is more costly than the aforementioned laboratory method. The pilot plant method, however, has the advantage that the pilot settling tank is an actual demonstration of the shape of tank required for best economy.

9. Short-Circuiting and Stability

Some of the fluid passes through the settling zone of a tank in less time than the detention period T. This "short-circuiting" is exhibited by all tanks and is due to differences in the velocities and lengths of stream paths. Short-circuiting is accentuated by mixing of the tank contents, by high inlet velocities, and by density currents.

A "density current"[15] is a flow of a fluid into a relatively quiet fluid having a different density. A density current that is heavier than the main body of fluid will sink and flow along the bottom of the tank at a relatively high velocity. If the density current is lighter it will flow along the top. Differences in density may be caused by differences in temperature, salt content, or suspended matter content. If the tank velocity is sufficiently great, density currents will mix with the tank contents with only slight effect on the flow pattern through the tank. Such a tank has a stable flow pattern. If density currents maintain their identity through a tank, the flow pattern is unstable.

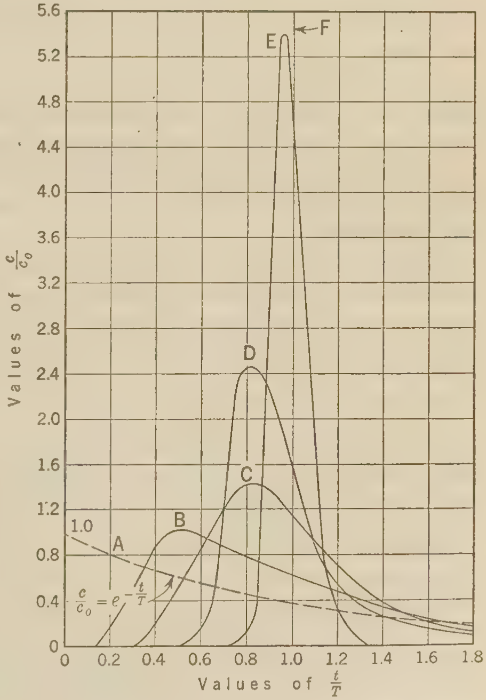

Fig. 15.—Typical Dispersion Curves for Tanks

Short-circuiting studies are usually made on model tanks operating in accordance with Froude's law. A dose of dye or salt solution is injected into the influent and the concentration of the added substance in the effluent is observed at the end of various time intervals until substantially all the substance has passed from the tank. From the data thus obtained a curve of concentration against time may be plotted. The shape of the dispersion curve is a measure of the flow pattern through a tank; and if these curves are plotted in dimensionless terms, as in Fig. 15, they are a means for comparing the hydraulic characteristics of different shapes of tanks or of the same tank at different rates of discharge.

If the flow pattern is relatively unstable, the density of the dye or salt solution must be sufficiently close to that of the water in the model to avoid the formation of a density current.

The abscissa for a point on the dimensionless curve is the relative time $\frac{t}{T}$—that is, the actual time t divided by the detention period T. The ordinate is the relative concentration $\frac{c}{c_o}$ computed by dividing the actual concentration c of salt or dye in the effluent by the concentration c_o which would be obtained if the dose were dispersed instantaneously throughout the tank volume. This type of plot is independent of the amount of dye or salt used and is independent of the detention period. If all the charge is accounted for in the effluent the area under the curve will be unity. If the flow pattern is stable and there are no dead spaces the value of $\frac{t}{T}$ to the center of gravity will also be unity.

If duplicate runs made under the same conditions yield dispersion curves which vary in shape, the flow pattern in the tank is unstable. A stable flow pattern is indicated when the same curve can be reproduced with fair precision. If a tank is not stable, short-circuiting studies will yield results which are not dependable and the performance of the tank in removing suspended matter will be erratic.

If there are dead spaces in a tank in which the liquid plays little or no part in the displacement through the tank, the effective tank volume is less than the true volume. The effective detention period will be less than the true detention period and the relative time to the center of gravity of the dispersion curve $\frac{t_G}{T}$ will be less than unity. If there is any interchange between the dead spaces and the main body of the liquid there will be flow through the dead spaces, but the resulting "tail" of the dispersion curve may be so long and the ordinates may be so small that the end of the tail will be lost and an apparent value of $\frac{t_G}{T}$ less than unity will result. If there are dead spaces, the recorded values of $\frac{c}{c_o}$ will be greater than the effective values and all ordinates to the dispersion curve will be increased in proportion to the decrease in $\frac{t}{T}$, such that the area will remain unity.

Since the relative time to the center of gravity is unity except for tanks with dead spaces, $\frac{t_G}{T}$ is an indicator of the amount of the dead space; but it is not a suitable index to short-circuiting in tanks with no dead spaces. The relative time to the center of area $\frac{t_A}{T}$ is usually less than unity and is smaller the worse the short-circuiting. Since half the particles of fluid pass in less time, t_A is the "probable flowing through time" of the fluid. The dispersion of the fluid is measured approximately by the time of initial appearance t_i of the charge in the effluent. The smaller the relative time $\frac{t_i}{T}$ the more rapid the dispersion. Small values of $\frac{t_i}{T}$ and $\frac{t_A}{T}$ indicate serious short-circuiting.

The shapes of dispersion curves found in practice for tanks without dead spaces will be intermediate to those characteristic of two limiting types of tanks: (1) The ideal dispersion tank, in which the influent is dispersed instantaneously and uniformly throughout the tank volume, characterized by curve A of Fig. 15; and (2) the ideal basin in which the velocity is the same throughout, characterized by a vertical line at $\frac{t}{T}$ equal to unity. The dispersion tank is the least efficient and the ideal basin the most efficient for settling. The dispersion curve for a cubical mixing chamber approximates curve A of Fig. 15.

Curves B, C, D, and E, Fig. 15, are characteristic of progressively better types of settling tanks. These curves are the results of measurements with models under stable flow conditions. Curve B is the dispersion curve for a radial-flow primary tank 200 ft in diameter proposed for the Detroit Sewage Treatment Works.[35] This curve represents the flow pattern with the conventional type of inlet and at the design detention period of 90 min. Curve C is the dispersion curve for the wide rectangular sedimentation basins at the Springwells Filtration Plant at Detroit.[36] Curve D is the dispersion curve[35] for the narrow rectangular primary tanks constructed for the Detroit Sewage Treatment Works in preference to the circular tanks characterized by curve B. Curve E is the dispersion curve for a model of a round-the-end type baffled mixing chamber with fifteen passes.[37] The velocity is too high in this chamber for ordinary settling, but the curve was inserted to indicate what might be expected of well-designed tray clarifiers. The significant data for the tanks represented by the curves of Fig. 15 are listed in Table 2.

TABLE 2.—HYDRAULIC CHARACTERISTICS OF TYPICAL TANKS

Curve (see Fig. 15)	Type of tank	b (ft)	H (ft)	L (ft)	V (ft per min)	$F = \frac{V^2}{Rg}$	$\frac{t_i}{T}$	$\frac{t_A}{T}$
A	Ideal dispersion tank....	....	....	....	....	....	0	0.693
B	Radial-flow circular....	....	14.2	....	1.11	7.5×10^{-7}	0.14	0.831
C	Wide rectangular......	135	18.5	330	3.12	5.8×10^{-6}	0.30	0.925
D	Narrow rectangular....	16	14	273	5.17	4.53×10^{-5}	0.52	0.903
E	Baffled mixing chamber.	2	22.3	1,056	66.6	4×10^{-2}	0.74	0.988
F	Ideal basin............	....	....	....	....	....	1.0	1.0

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[36] "Sedimentation Basin Research and Design," by Arthur B. Morrill, *Journal*, A.W.W.A., Vol. 24, No. 9, 1932, p. 1442.

[37] "Comparative Studies of Shortcircuiting in Mixing Chambers by Means of Models," by Jorge Echarte, thesis submitted to Mass. Inst. Tech., Cambridge, Mass., in 1940, in partial fulfilment of the requirements for the degree of Bachelor of Science.

It has been shown previously by the writer[17] that the hydraulic characteristics of long narrow tanks are superior to those of wide low velocity tanks. This is indicated clearly by Fig. 15 and Table 2. The flow pattern is better for tanks with high values of Froude's number. Fig. 16 shows how the minimum and probable "flowing-through times" increase with higher values of Froude's number. It has also been shown by the writer[17] that tanks with high values of Froude's number are more stable. George E. Hubbell,[35] Assoc. M. Am. Soc. C. E., has shown that radial flow basins are characteristically unstable in their flow patterns. In the Detroit studies, stability was improved in the circular tank by directing the influent by inlet vanes to flow spirally around the tank for two or three circuits before the fluid reached the outlet weir. Since the velocity was thus increased the value of **F** for the circular tank was also raised. The value of $\frac{t_i}{T}$ was increased to 0.24 by this device and by other improvements in the design of the inlet.

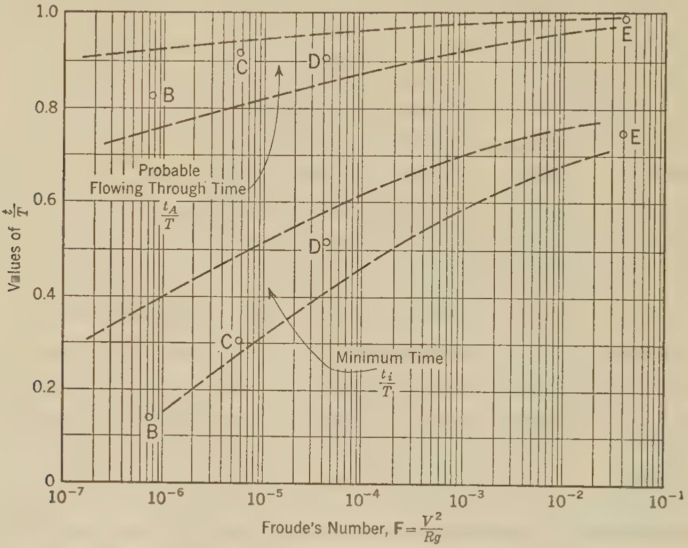

FIG. 16

A rough estimate of the effect of short-circuiting on removal may be had if it is assumed that the suspension is subjected to varying settling times, distributed as indicated by the dispersion curve. This method has been described previously by the writer.[38]

10. CONCLUSIONS

Fig. 17 illustrates the details for a tentative design of a primary settling tank based upon the principles developed in this paper. Each unit of this design is for a maximum discharge of 10 mgd, and provides a flocculation period of 60 min. The velocity of flow in the tray settling tank is 9.3 ft per min, the detention period is 10.8 min, the overflow rate is 667 gal per day per sq ft, and the mean velocity gradient is about 2 $\sec^{-1}$. Sludge and scum are scraped laterally from the trays. Gates must be provided at the sides of each tray which will prevent short-circuiting through the sludge compartments, and will open to permit the ejection of sludge and scum. In current practice, return bends in the flow path of primary settling tanks are generally frowned upon because of the turbulence set up by the bends. This objection is of minor importance if the length occupied by the bend is small as compared with the length of the passes.

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[38] Discussion by Thomas R. Camp of "Grit Chamber Model Tests for Detroit, Michigan, Sewage Treatment Project," by George E. Hubbell, *Transactions*, Am. Soc. C. E., Vol. 104 (1939), p. 232.

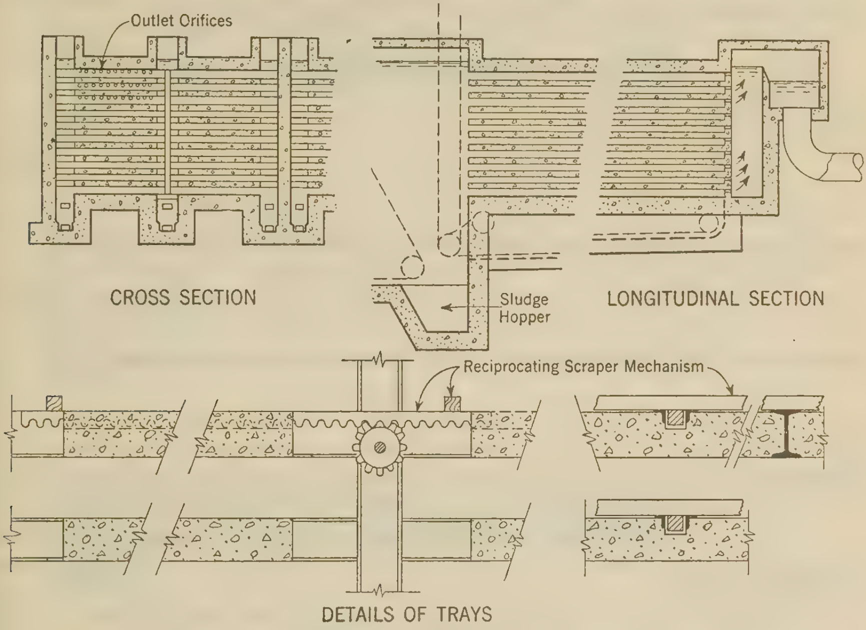

Fig. 17.—Details of Proposed Type of Tray Settling Tank

Fig. 18 illustrates current trends in the design of final settling tanks[39,40] for the activated sludge process. In many final settling tanks, the depth is so great and the mean velocity is so low that the mixed liquor falls at the inlet end as a density current and flows at high velocity toward the outlet end where it impinges on the end wall and rises as shown in Fig. 18(*a*). The flow is so rapid that hindered settling in the density current may not be complete until it reaches the end wall of the tank. The usual practice of withdrawing sludge from the bottom at the inlet end results in a mixture of fresh sludge short-circuited from the density current and stale sludge scraped from the effluent end. Short-circuiting of the sludge may be avoided if it is withdrawn from the outlet end as shown in Fig. 18. If the tank is made shallow and designed for a high velocity, the density current may be avoided and the sludge will settle as illustrated in Fig. 18(*c*). The hindered settling process will be completed at some point A which corresponds with point A of Fig. 4, and the re-

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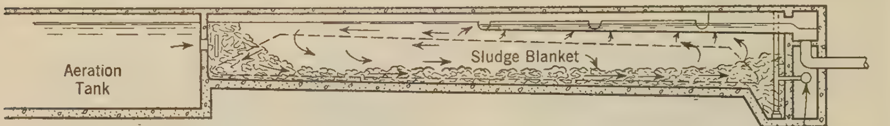
[39] "Operating Experiences in New York City," by Richard H. Gould, *Sewage Works Journal*, Vol. 14, No. 1, 1942, p. 77.

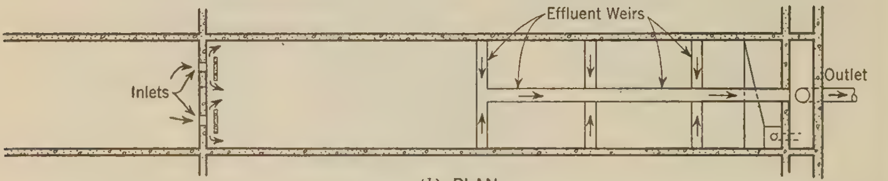
[40] "Developments and Trends in Sewerage Practices of 1941," by Samuel A. Greeley and Paul Hansen, *Water Works & Sewerage*, Vol. 89, No. 2, 1942, p. 53.

mainder of the length of the tank may be devoted to the task of concentrating the sludge.

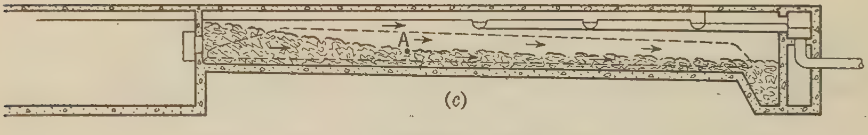
In Fig. 18, the final settling tank is shown adjoining the aeration tank without connecting conduits. This is contrary to current practice, but is desirable in order to avoid high inlet velocities and the destruction of floc by excessive velocity gradients. The currents that usually prevail in final settling tanks, as illustrated in Fig. 18(*a*), produce the clearest liquor at the surface near the inlet. The effluent launders, which in early practice were placed at the effluent end of the tanks as with primary tanks, are now being extended toward the inlet end in order to pick up the clearer liquor. Weirs of great length are considered desirable to avoid high approach velocities and the danger of scour from the sludge blanket. Multiple V-notches are usually provided in these weirs to increase the head and assure uniform collection of the discharge.



(*a*) SECTION



(*b*) PLAN



(*c*)

FIG. 18.—CURRENT TREND IN THE DESIGN OF FINAL SETTLING TANKS FOR THE ACTIVATED SLUDGE PROCESS

ACKNOWLEDGMENT

The writer wishes to acknowledge the assistance of the many independent investigators whose work is collected to make up the bulk of this paper. Much of the original work published for the first time in this paper was done by the writer's students in the laboratories of the Massachusetts Institute of Technology. The writer is particularly indebted to his former associate, Ensign Dobbins, for the studies on the effect of turbulence, and to his associate, Mr. Stein, for the fundamental theory of flocculation, the concept of mean velocity gradient, the studies on short-circuiting, and for many helpful suggestions on other portions of the theory.

DISCUSSION

Norval E. Anderson,[41] M. Am. Soc. C. E.—The mathematical analyses presented by Mr. Camp should prove useful in further studies of settling tanks. Such theoretical analyses are informative. However, the "Conclusions" appear abrupt, since they imply recommendation of types of tanks for primary settling and for final settling of activated sludge, without reference to the available operating and experimental data on plant-size units and with little relation to the theory in the case of final tanks.

Multiple-tray settling tanks, similar in principle to the tank proposed by the author for primary settling, have been patented and promoted for many years. The author might have shown why these tanks have had such limited acceptance and in what way his proposed tank may be more acceptable.

Most of Mr. Camp's conclusions on final settling tanks for the activated sludge process may be questioned on the basis of an extensive study of plant-size experiments and operating data, some results of which were presented[42] by the writer in October, 1944.

It is doubtful if the statement in Section 10 that "Fig. 18 illustrates current trends in the design of final settling tanks[39,40] * * *" is based on a general view of the field.

The author's statements to the effect that density currents would be less with shallow tanks are not confirmed by actual velocity measurements. Experiments[42] on 126-ft diameter tanks at the Chicago (Ill.) Southwest Plant (allowing the sludge blanket to build up to various depths, thereby giving the effect of different tank depths) showed that the velocity of the density current was in an approximately inverse ratio to the flow depth. Also, relatively higher velocities of density current were found[42] in the rectangular final settling tanks at Columbus, Ohio, and at Wards Island, New York, where the ratio of length to depth is relatively large.

The author's argument for withdrawal of sludge at the effluent end of the tank is open to some question. He states (Section 10) that:

> "The usual practice of withdrawing sludge from the bottom at the inlet end results in a mixture of fresh sludge short-circuited from the density current and stale sludge scraped from the effluent end. Short-circuiting of the sludge may be avoided if it is withdrawn from the outlet end * * *."

Why should one avoid this so-called short-circuiting of fresh sludge when one of the most desirable features of a final settling tank is the withdrawal of fresh return sludge? The writer agrees[42] that activated sludge settles near the inlet of the tank. Thus, with a long rectangular tank and flight-conveyer-type sludge removal mechanisms as indicated in Fig. 18, the length of time that the dense sludge remains in the tank would be governed largely by the speed of the mechanism, resulting in sludge of greater average age than would be the case if the drawoff were near the inlet. With the drawoff near the inlet, the dense sludge is required to move a much shorter distance and only the lighter particles that settle at the effluent end need to move the full length of the tank. Also, it is possible that greater flocculation is obtained by withdrawing sludge near the inlet, since this causes the sludge to flow against the density current and affords more opportunities for contacts between particles, as stated by the author under "7. Flocculent Suspensions."

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[41] Engr. of Treatment Plant Design, The San. Dist. of Chicago, Chicago, Ill.

[42] "Design of Final Settling Tanks for Activated Sludge," by Norval E. Anderson, *Sewage Works Journal*, January, 1945, p. 50.

Elimination of connecting conduits between aeration tanks and final settling tanks "in order to avoid high inlet velocities and the destruction of floc" may be practical in a small plant. However, in a large plant such an arrangement results in an uneconomical layout with sacrifice of flexibility in operation, as shown by layouts and studies of similar arrangements made by The Sanitary District of Chicago in 1933 for plants with 136-mgd capacity and more. Furthermore, it appears that "destruction of floc by excessive velocity gradients" (last paragraph of Section 10) is not an important factor with aeration tank effluent. Comparative tests made at three activated sludge treatment works of The Sanitary District of Chicago showed practically no change in the "settleability" of aeration tank effluent—even after a free fall over a weir and passage through both a venturi meter and a considerable length of conduit and aerated channel.

R. A. MULHOLLAND,[43] ASSOC. M. AM. SOC. C. E.—Although most consulting engineers, from time to time, are faced with the problems of designing sedimentation tanks, it seems that in most cases these consultants do not delve completely into the theories involving sedimentation.

As a result of this method of design there is the continual problem of inadequate settling. This is particularly true of sewage treatment plants. In most industrial processes using sedimentation, such as those involving chemical reaction, the designers seem to have a much better understanding of sedimentation. This can be seen by the results obtained in such plants. Concerning the design of grit chambers, emphasis should be placed on the sixth and seventh paragraphs of Section 2. With the exception of the very largest class of sewage treatment plants, one of the weakest links in the entire process is that of the grit chamber. This is particularly true of sewage treatment plants serving small towns and villages. Here the designers have been faced with controlling financial outlay to eliminate every cost possible. In many instances an excuse for a grit chamber has been installed which could not possibly function to collect any grit whatsoever, and has only been an added expense to the construction of the plant itself.

Mr. Camp states that geometric models for plant-scale tanks cannot be adapted to produce those effects, but are quite useful in the measurement of hydraulic short-circuiting; also, that settling analysis may be corrected for short-circuiting to predict performance in plant-scale settling tanks. It has been the writer's experience with model tanks that their flow characteristics can be correlated to full-scale flow characteristics and zones of sedimentation can be reproduced. It remains a matter of judgment and experience upon the part of the designer to adapt model results to full-scale operation. In regard to Mr. Camp's statement concerning the building of an adjoining tank without connecting conduits, the writer has knowledge of at least a dozen plants built along this pattern in recent years. Virtually all of the recently adapted Hay's process plants have been so arranged in current years. Such construction results in a decided economy, since dividing walls are merely web members of a rigid structure of sufficient strength to withstand the hydraulic load.

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[43] Consultant, Austin Eng. Co., Austin, Tex.

P. Charles Stein,[44] Assoc. M. Am. Soc. C. E.—It is obvious that a heavy particle suspended in a liquid will tend to settle. The difficulty encountered in applying this casual concept of settling to the design and operation of sedimentation units furnishes enough proof of the need for understanding the principles of settling and related phenomena. A technical basis for the design of settling processes must depend upon specific knowledge of the properties of the suspensions to be treated and of the physical phenomena associated with flow through tanks.

This paper should make more engineers aware of the mechanics of settling tanks. Its scope extends beyond that mission, however, in that the concepts involved in the more important parts of the problem are developed and definite design principles are formulated.

It is to be hoped that the paper will help dispel the erroneous notion that detention time is the important factor in the removal of suspended matter. For a given settling velocity both depth and detention time govern degree of removal since, with a deeper tank, a particle will require a longer time to reach the bottom. Thus, with a given settling velocity, the ratio of depth to detention time determines the number of particles reaching the bottom of an ideal tank. Simple algebra will show that this ratio is equal to the rate of flow divided by the surface area of the tank, which is the definition of overflow rate given by the author following Eq. 6. When one considers a particle settling at a given rate in a tank, it is simple to grasp the fact that depth divided by settling time is the proper criterion for removal and that neither depth nor time alone has real significance.

The tests mentioned by the author are not simple routine ones because standardized methods have not been formulated. Thus far, data on physical properties of suspensions encountered in water and sewage treatment are extremely meager, as the author states. This scarcity of information means not only that there are no acceptable normal values at present to serve as comparators but also that little has been done in the way of developing standard techniques. This lack of development does not nullify the value of the method of design described by the author. Instead, realization of the shortcoming should spur sanitary engineers toward a more scientific study of the treatment methods that they use.

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[44] Cons. Engr. (Kuchler & Stein), Sharon Hill, Pa.

For new units it is necessary, before design, to determine the nature of the suspension that will be obtained when the unit is in operation. If the new unit is an addition to an existing plant, design information may be secured by suitable interpretation of tests conducted on suspensions in existing units. If the unit is to be a part of an entirely new plant, then recourse should be had to sanitary engineering laboratory design methods or to a pilot plant. In the laboratory method a proposed process is examined critically to determine the variables entering into each stage of the process. Then, the process is completed step by step on batch samples, the important variables in each step such as chemical characteristics, mixing time, degree of turbulence, velocity gradient, etc., being controlled with a view toward determining the optimum values of the several variables. In the course of this procedure, the settling velocity analysis can be made at the proper time in the treatment routine. It is assumed that the steps prior to sedimentation will have been finished in a container of adequate design and dimensions so that there will be no necessity for transferring the sample to another apparatus for the settling analysis.

The paper exposes the neglect with which investigators have treated the mechanism of flocculation and its relationship to the settling problem. Eq. 26, which was introduced by M. von Smoluchowski,[45] and Eqs. 25 are based on rather crude assumptions, since their purpose in this paper is to show relative rather than absolute effects. In their development, the actual hydrodynamics of fluid flow about the particles has been ignored. Microscopic studies by the writer indicate that the process of flocculation by settling is much more complex than the simple mechanism pictured by the assumptions leading to Eqs. 25. Similar comments apply to Eq. 26 but analysis of the results of Darrell A. Root[31] along with torsion dynamometer measurements by the writer to determine work input on the Root apparatus indicate that the proportionalities between rate of flocculation and concentration and between rate of flocculation and velocity gradient are valid.

Despite the very approximate nature of Eq. 26, it discloses important relationships concerning the effect of flocculation during settling. Although actual problems involve polydisperse suspensions, which have been extensively studied by the writer, for simplicity, the considerations will be limited to particles of uniform size. The conclusions with such a restriction are qualitatively valid.

For spherical particles, it is evident that the number of particles per unit volume of the suspension will equal the volumetric concentration divided by the cube of the diameter times a constant. Introduction of this relationship into Eq. 26 discloses that the number of contacts per unit volume per unit of time is proportional to the square of the concentration, and that the percentage of particles involved in contacts is proportional to the volumetric concentration and is independent of the size of the particles. For a velocity gradient of 10 sec^{-1} and a volumetric concentration of 2 parts in 10,000 (roughly equivalent to 200 ppm suspended solids in sewage), the initial flocculation rate given by

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[45] "Versuch einer mathematischen Theorie der Koagulations-kinetic kolloider Losungen," by M. von Smoluchowski, *Zeitschrift für physikalische Chemie*, Vol. 92 (1917), p. 155.

Eq. 26 is equivalent to contacts involving 12% of the particles per minute. For lower or higher concentrations, the rate of flocculation will be proportional.

From the foregoing, it should be clear that the effect of flocculation on removal should be particularly pronounced where the volumetric concentration of suspended matter is high. With the volumetric concentrations present in water and sewage treatment processes, the effect of flocculation on settling can be substantial. It is only with low concentrations of the order of 10 ppm or 20 ppm that the flocculation effect would be negligible unless both velocity gradient and detention time are large.

The application of the meager available information concerning flocculation in settling tanks certainly indicates that the shallow tanks dictated by the general settling theory are also more favorable from the standpoint of flocculation during settling. It will remain for pilot plant testing to demonstrate the advantages of the method of design presented by the author. Testing should be done also in units of existing plants. The results might well indicate that a necessary increase in settling capacity can be attained by inserting horizontal diaphragms in existing tanks at considerably less cost than would be required for building additional tanks.

The writer would like to point out what he believes to be an error in Example 2, since it might lead to faulty design of final settling tanks for activated sludge. In this example, it is assumed that the concentration of the sludge in the sludge blanket is a function of the time of compacting only and is independent of the depth of the blanket. This means that, for a given degree of compaction, the rate of subsidence of the sludge blanket is proportional to the thickness of the blanket. That this cannot be a valid conclusion is indicated by a consideration of settling and compaction in a very deep tank. The assumption that rate of compaction, in terms of the ratio of the volume occupied by the sludge blanket, is independent of depth will lead to the conclusion that in a very deep tank the sludge blanket must sink at a velocity greater than the hindered settling velocity. This, obviously, is not possible. If, in Example 2, for a tank 10 ft deep, the position of the top of the blanket for a sludge concentration of 6,000 ppm were calculated, it would be found that the rate of subsidence from point A of Fig. 4 to the depth corresponding to 6,000 ppm would be about 0.14 cm per sec, whereas the hindered settling velocity is only 0.11 cm per sec.

There is a similarity between the problem of compaction of sludge and of the consolidation of clays as found in soil mechanics problems. In both cases, the rate of consolidation in a lamina is proportional to the normal gradient of the hydrostatic excess pressure. In a sludge blanket at a given degree of consolidation, it may be argued that the hydrostatic excess pressure at any point is roughly proportional to the depth of sludge over that point. Thus, the pressure gradient causing consolidation will be roughly independent of the depth in the early stages of the compaction process. This means that the rate of drainage of interstitial fluid will be independent of depth and that two blankets, one shallow and the other deep, would compact with approximately the same absolute velocity of subsidence. In the final stages of consolidation the compaction in deep tanks will be greater than in shallow tanks because of the increased weight of sludge over the bottom layers.

In general, the results reported by Willem Rudolfs, M. Am. Soc. C. E., and I. O. Lacy,[16] cited by the author, are at variance with the author's conclusions respecting the effect of depth on sludge compaction. To secure data on this specific problem, Messrs. Rudolfs and Lacy made tests of settling and consolidation in glass tubes with initial depths of 117 in., 93 in., and 45 in. The results taken from curves[16] presented by Messrs. Rudolfs and Lacy have been calculated to give a nondimensional plot, devised by the writer. In this plot, presented in Fig. 19(a), a value of v = 2.5 in. per min was used in computing the abscissa. If the settling velocity were the same for the three samples and the absolute rates of subsidence were equal, the curves for the three depths would coincide. (In Fig. 19 the settling time modulus $\frac{vt}{H}$ is plotted against the ratios and D/H as ordinates, definition being as follows: v = settling velocity; t = time; H = tank depth; and D = depth of sludge.

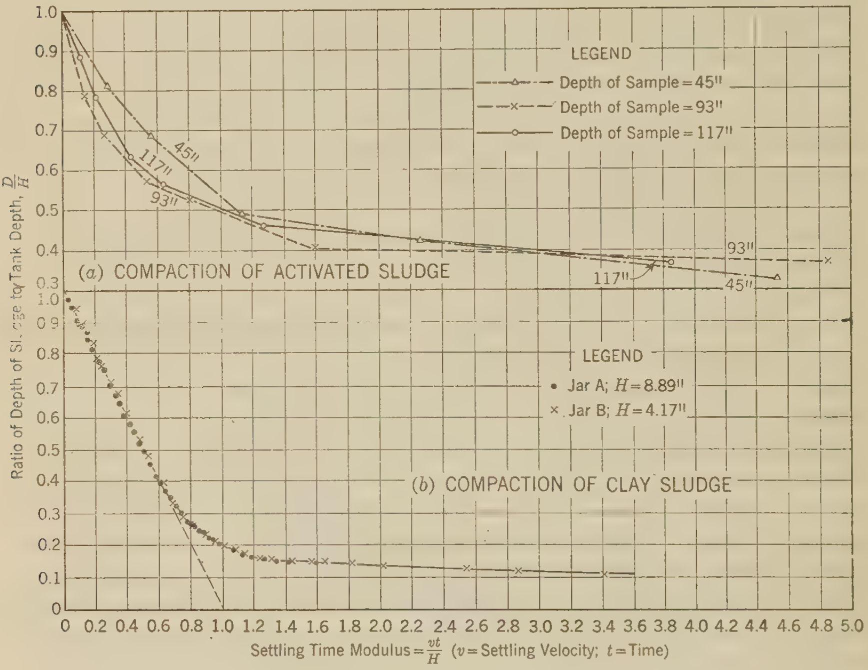

FIG. 19.—EFFECT OF DEPTH ON RATE OF COMPACTION

Actually, the plotted points show a substantial scattering. However, it will be observed that the placement of the curves does not represent a consistent order, the results from the deepest tube being between those of the intermediate and lowest depth. After 19 hr of settling the results were consistent, the sludge depths being, respectively, 12%, 14%, and 16% of the liquid depths for the 117-in., 93-in., and 45-in. samples. The writer believes that the tests shown in Fig. 19(*a*) constitute a rough check of his contention that the absolute rate of consolidation in the early stages is independent of sludge depths. The failure to achieve perfect coincidence of the curves is due to experimental error. From the breaks in the curves for the individual samples, the writer would conclude that there were arching effects present due to the use of settling tubes which did not have a sufficient diameter. The writer has found, in consolidation tests made with calcium carbonate and with clay suspensions, that the settling tube must be large with respect to the depth of the sludge if arching effects are to be negligible.

To secure data of greater precision, the writer performed consolidation tests with two jars containing a floc made by coagulating a suspension of red clay with ferric chloride, ammonia, and sodium chloride. Both jars were 5.2 in. in internal diameter. Jar A had an initial depth of 8.89 in. and the depth in jar B was 4.17 in. After coagulation, the floc was poured back and forth in the two jars to insure equality in condition. It was found that the hindered settling velocities in the two jars, as well as the time rate of subsidence at equal degrees of compaction, were identical within the limits of experimental error as shown on the nondimensional plot presented in Fig. 19(*b*). The hindered settling velocity of the floc used was found to be 0.125 in. per min and the total duration of the test was 9 hr. Although only the first quarter of the test is shown in Fig. 19(*b*), the results of the two samples gave identical curves even for the end of the test. It is to be expected that there would be some disparity between the two samples eventually because of the larger final intergranular pressure in the sludge blanket for the deeper sample.

This material is presented because it serves to throw a small amount of light on the problem of sludge compaction, which is a part of final settling tank design. The data presented were obtained with a clay floc and not with floc from activated sludge plants. Unless the floc from activated sludge plants is radically different from other types of materials, the general conclusions of the test should be valid. Comparison of Fig. 19(*a*) with Fig. 19(*b*) shows that the general types of curves obtained with activated sludge and with a clay floc are the same.

It would appear, then, that better compaction in a given detention time would be possible with shallow final settling tanks than with deep tanks; or, for equal degrees of compaction, less time is required with thin sludge blankets than with thick ones. Thus, shallow final settling tanks would require less detention time to obtain a specified concentration of sludge. With activated sludge, this is an important biological consideration because long absence from a medium rich in oxygen will reduce the viability of the active organisms.

LYNN PERRY,[46] M. AM. SOC. C. E.—Concepts of the fundamental principles of flow and fluids have changed radically during the past generation. Capable investigators with laboratory equipment, frequently inadequate, have correlated widely scattered bits of information, checked over the fundamentals, and secured an immense amount of supporting laboratory data. This work is by no means complete.

The more mature men of the engineering profession who were brought up on "Merriman" and "Hughes and Safford," with their detail about Kutter and other early investigators, find it extremely difficult to imagine that the film of a moving fluid in contact with the wall of a conduit is stationary. They have been engaged in design and construction during the most active and progressive generation in the history of the human race; and they will hold to the empirical methods of design which have been tried and found fairly satisfactory for their purpose. It is the younger generation who, with proper training, will base their fluid analyses on rational fundamentals.

Dimensional analysis gives the proper form for an equation but, unfortunately, it does not yield a numerical result. Numerical results, sufficiently accurate for design, are required by the designer, and are required promptly. Usually, theoretical analysis must be supplemented by considerable laboratory work before it can be put into form for the designer to use. Unfortunately, most of the data that have been made and published in support of empirical formulas are not exactly what is required.

Designing engineers are constantly facing conditions where a deposition of solids is necessary, where no deposition is permitted, or where a partial deposition is desired to separate heavier suspended solids from the lighter. Some years ago, the writer came into the possession of information which led to the curve presented in Fig. 20, the equation of which is

$$V = 424\,(G_r - 1)\,D^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (38)$$

In Eq. 38, V is the velocity of water in which sand of various particle sizes will settle; G_r is the specific gravity; and D is the diameter of a particle. This curve has been used and found to be trustworthy. The line is for sand—not spheres, cubes, or any other geometrical figures—plain sand ($G_r = 2.35$). It has been checked from time to time, and the small circles indicate the results of laboratory observations at Lafayette College, Easton, Pa. For heavier or lighter suspended solids other such curves are available. For velocities and sizes below the curve, deposition will occur; for velocities above the line, the solids will be carried along with the current of water. In many industrial processes the velocity of the water carrying the solids in suspension can be controlled. The necessary controls for maintaining a constant velocity of water or sewage passing through a municipal treatment plant would be a subject for another paper. Engineers are competent to design a satisfactory water or sewage treatment plant for a predetermined rate, and for a specific quantity and type of suspended and dissolved solids which require a given type, quantity, and quality of chemical. Few designers ever become the operators of the plants they have designed and built. Although they have an academic knowledge of operation, they seldom face the trying problems that confront the operator. This trend in engineering development has produced a rift between the designers and the operators. An unequal settlement of a few hundredths of a foot is the source of much more operating anxiety than a failure to secure a slightly higher efficiency in sedimentation. The designer is always seeking perfection even though he knows that goal is unlikely. He deals almost entirely with the forces of nature. The operator, on the other hand, is constantly confronted with the frailties of human nature.

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[46] Designing Engr., Dept. of Water and Sewers, City of Miami, Miami, Fla.

Theoretically, engineers should be able to develop the desired effect in the laboratory or in the pilot plant, and attain the same effect in the prototype. When this cannot be done, it indicates that there is something about the model law that is not quite familiar, and another set of empirical constants must be developed to give fairly accurate numerical results. Either engineers should be able to compute these constants or their dimensional analysis needs a review.

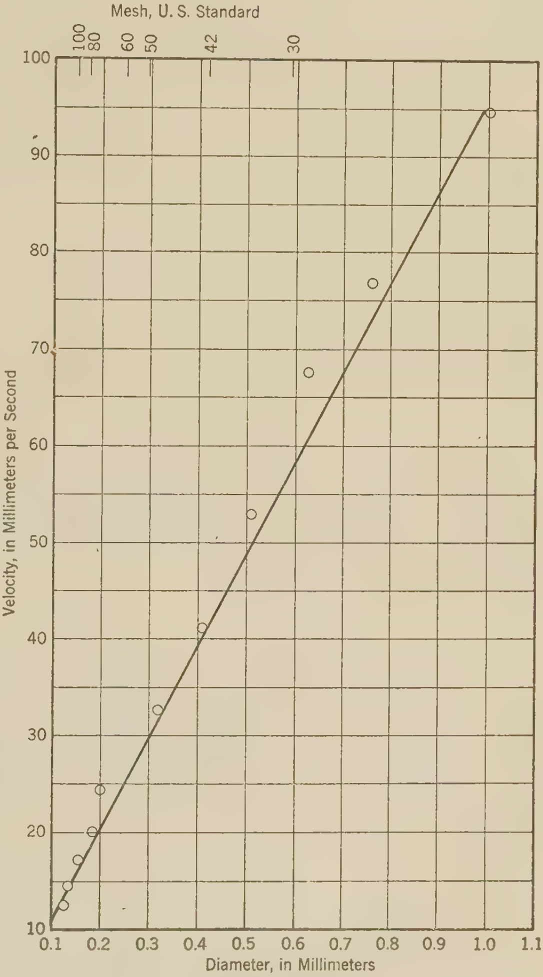

Fig. 20.—Velocity of Water in Which Sand of Various Sizes Will Settle

The conventional circular radial-flow sedimentation tank has several serious faults. Not the least of these is the tendency toward unequal flow in the different quadrants. In some cases it is serious; and in relatively few cases has the problem been solved to the satisfaction of the operator. Wind velocities and wave action may be involved, but it is difficult to imagine any considerable uniform and constant effect from prevailing low wind velocities.

The results of an exhaustive series of observations made on full size units at the Southwest Sewage Treatment Plant, Chicago, Ill., have been reported by Norval E. Anderson,[47] M. Am. Soc. C. E. A careful examination of the results reported would seem to indicate that deep tanks are unnecessary, and that there is plenty of room for improvement in the geometrical figure. It may be that, in order to conserve ground space and economize in construction, many tanks have been built deeper than necessary. Perhaps undue weight has been given to the theoretical period of detention while some of the more essential features have been overlooked.

The writer cannot refrain from entering a protest against the practice of drawing the sludge from the effluent end of a tank. Provision should be made to draw sludge, once settled out of the water, just as far away from the overflow weir as possible. This will provide for resedimentation, if necessary—a feature that is desirable.

The subject of trays is not quite new. Such provisions have not met with marked enthusiasm in the past. Further development in the details of design may bring more favor. In this connection, it might be timely to mention, once more, that handling a hundred million gallons of water or sewage per day presents more problems than merely building a number of smaller units. A multitude of factors, which may be overlooked as of negligible importance or easily met in a small plant, grow to major difficulties in large installations.

The author deserves a great deal of commendation for the logical and attractive manner in which the paper has been arranged; and for gathering, in one place, the pertinent elements on this subject. It serves to focus attention to the present status and recent developments in this extremely active and progressive field. It deserves amplification to several times its present length, and time for discussion.

E. Sherman Chase,[48] M. Am. Soc. C. E.—Thirty odd years' experience has demonstrated to the writer the difficulties involved in reducing to formulas the various phenomena encountered in sanitary engineering practice; particularly those relating to that nonconstant, unpredictable, exasperating material known as "sewage." However, these difficulties have not discouraged inquiring minds like that of the author, from trying, and commendation is due Mr. Camp for the effort.

The function of settling tanks is to remove suspended matter from the liquid passing through them. The character of this matter varies with the type and condition of the liquid. In the case of raw sewage the solids in suspension are of such a heterogeneous and variable nature that no formula could possibly be devised of general applicability. Furthermore, the effluents from various types of secondary sewage treatment differ from each other as regards the nature of the solids carried in suspension. Even the effluents from the same type of secondary treatment will contain suspended solids which vary from place to place and time to time as regards shape, size, specific gravity, and settling characteristics.

The relatively greater importance of overflow area as compared with detention period, as demonstrated by the author, has long been appreciated, and overflow rates have been used in the design of settling tanks for many years. In nature, the clarification of sewage-polluted waters in streams of little depth is a common example of the effectiveness of shallow flowage.

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[47] *Sewage Works Journal*, Vol. XVII, No. 1, January, 1945, p. 50.

[48] Partner, Metcalf and Eddy, Boston Mass.

The tray clarifier, somewhat similar to that advocated by the author, was developed some time ago, but its practicability has yet to be demonstrated. In a recent communication from the company that has promoted this type of tank, the writer was advised that the company had decided to abandon this piece of equipment. Apparently in practice it failed to fulfil its theoretical advantages.

An interesting type of settling tank for flocculated water has been described by Ian M. E. Aitken.[49] This tank contains a series of annular baffles so spaced as to give alternate downward and upward flows with the upward flows at velocities considerably less than those of the downward, and with the upward velocities decreasing toward the effluent weir at the perimeter.

Intriguing as the pursuit of elusive formulas may be to those so fortunate as to be possessed of a precise mathematical turn of mind, the design of practicable settling tanks will continue to be based upon judgment, experience, and common sense seasoned with a moderate amount of theoretical computations used as aids to judgment.

Rolf Eliassen,[50] Assoc. M. Am. Soc. C. E.—The expressed purpose of the paper is to develop the theory of sedimentation to such an extent that all the basic factors affecting the sedimentation of water and sewage solids may be so evaluated that it is possible to use mathematical functions to predict the economic design of settling tanks. Mr. Camp has developed mathematical functions for some of the variables affecting sedimentation. The questions that the designing engineer must ask are whether all the important factors have been considered, whether these factors have been evaluated properly, and whether the conclusions drawn are valid for conditions encountered in practice.

In the "Synopsis," the author makes a statement that "The settling characteristics of the suspensions to be clarified are rarely considered in the design of settling tanks." On the basis of his experimental work, Mr. Camp defines these settling characteristics in terms of overflow rates of from 200 gal per min to 2,000 gal per min per sq ft—which are equivalent to settling velocities of from 1.1 ft per hr to 11.1 ft per hr.

An analysis of the design of settling tanks selected at random from sewage treatment plants throughout the United States is shown in Table 3. These plants have been designed on the basis of overflow rates well within the limits prescribed by Mr. Camp. Some of them are close to the value of 667 gal per min per sq ft mentioned in Section 10 as a desirable overflow rate for the primary sedimentation of sewage. Thus, one would tend to believe that sanitary engineers, state boards of health, and equipment manufacturers have taken into account the settling characteristics of the suspensions involved.

The major difference between the practices of designing engineers and the conclusions of Mr. Camp is not in the matter of settling velocities and overflow rates, but in detention times. Modern settling tanks in water treatment and sewage treatment plants are designed on the basis of settling periods of from 1 hour to 4 hours and involve the use of tanks with depths from 5 ft to 15 ft. The author comes to no definite conclusions as to settling times, since these are involved in mathematical functions which may be solved for each condition of flow and character of suspension. However, he does cite examples wherein detention periods of from 0.1 hour to 2.4 hours are calculated for tanks with depths ranging from 0.5 ft to 11.6 ft. Mr. Camp places particular emphasis on the need for tanks with short detention periods, shallow depths, and relatively high velocities of the liquids flowing through the tanks.

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[49] "A Fresh Approach to the Problem of Precipitation Tank Design," by Ian M. E. Aitken, *Water and Water Engineering*, Midsummer, 1945, p. 320.

[50] Prof. of San. Eng., New York Univ., New York, N. Y.

TABLE 3.—OVERFLOW RATES FOR SETTLING TANKS AT SEWAGE TREATMENT PLANTS

City	Type of treatment	Design flow (mgd)	Area of tanks (Sq ft)	Gallons per day per Square foot
Huntington Beach, Calif.	Trickling Filter	0.60	804	746
Sayreville, N. J.	Chemical precipitation	0.75	1,232	609
Birmingham, Ala. (Shades Valley)	Chemical precipitation	2.0	3,180	1,590
Dayton, Wash.	Trickling filter	3.6	1,964	545
Olean, N. Y.	Primary	5.0	3,927	786
Neenah-Menasha, Wis.	Primary	10.	8,836	1,132
New York, N. Y. (Coney Island)	Chemical precipitation	35	25,448	1,372
Atlanta, Ga. (Clayton)	Primary	42	39,408	1,065
Denver, Colo.	Chemical precipitation	54	61,576	880
Cleveland, Ohio (Easterly)	Activated sludge	123	157,632	780
Washington, D. C.	Primary	130	105,900	1,230
Buffalo, N. Y.	Primary	150	80,424	1,868
Chicago, Ill. (Southwest)	Activated sludge	400	395,000	1,010

Much attention is given to the derivation of equations involving the effect of velocity gradients on the flocculation of particles suspended in water and sewage. From these equations, the author proceeds to a consideration of the most economical settling tank design. Examples are presented leading to numerical values for tank dimensions. Some of the intermediate steps in the process should be recognized by the designing engineer so that he is cognizant of the limitations of this method of analysis.

In the first place, only the velocity gradients caused by drag on the floor and walls of a tank are considered by the author—the factors of inlet turbulence and density currents being neglected. Both of the latter may have an appreciable influence in tanks where the ratio of $\frac{L}{H}$ is comparatively low, in the range of from 2 to 10. These low ratios are found in most circular tanks and many of the shorter rectangular tanks in common use throughout the United States. Thus, any numerical or quantitative results derived from these mathematical equations must be examined critically to determine whether the basic assumptions made by the author are valid for the tank under consideration.

Within the limitations of long shallow tanks in which the drag on the tank floor and walls will have a dominating influence on velocity gradients, the author develops a sound theory leading to quantitative results in several examples. However, the suspensions given in the examples are hypothetical ones as far as velocity gradients and mixing coefficients are concerned. The author has specified the type of laboratory equipment that should be developed to obtain the necessary analyses of suspensions; but, as of early 1946, the equipment is not available. The logical approach to a correct analysis of the problem is by the construction and testing of pilot plants. Engineers frequently have difficulty in securing funds from their clients for pilot-plant work. For this reason, it is not easy to evaluate many of the factors influencing sedimentation. Therefore, designing engineers proceed on an empirical basis.

In Section 9 Mr. Camp departs from considerations of the theoretical aspects of sedimentation and discusses short-circuiting and stability in tanks. The writer believes that Mr. Camp places too much emphasis on short-circuiting as the criterion of settling tank performance. In the third paragraph of Section 9, the author states, "Short-circuiting studies are usually made on model tanks operating in accordance with Froude's law." This law is applied in an attempt to secure dynamic similarity between model and prototype when gravity forces predominate and leads to the equation known as Froude's number, $\left(\frac{V^2}{l\,g}\right)_p = \left(\frac{V^2}{l\,g}\right)_m$, in which the subscripts p and m refer to the prototype and the model, respectively. Gravity forces may predominate in the study of some settling tank inlets in the case where high velocities exist and head becomes a factor. However, Mr. Camp has chosen to neglect the effect of inlet turbulence and to consider only the case in which the forces of drag on the floor and walls of the tank predominate. These are viscous forces which also predominate in the settling of particles in water and sewage, as mentioned by Mr. Camp in Section 1. Consequently, if dynamic similarity is to exist with the predominance of viscous forces, the relationship between prototype and model should be governed by Reynolds' number, $\left(\frac{V\,l\,\rho}{\mu}\right)_p = \left(\frac{V\,l\,\rho}{\mu}\right)_m$. Mr. Camp presents no data based on model studies operated in accordance with Reynolds' law. Thus, the results as depicted in Fig. 15 are open to question.

Furthermore, the selection of an arbitrary hydraulic efficiency as the criterion of settling tank efficiency is not consistent with the theory previously developed in the earlier sections of the paper. In Section 4 Mr. Camp concludes that, in an ideal basin:

> "For any given discharge, the removal is a function of the surface area and is independent of the depth of the basin; or, the removal is a function of the overflow rate and, for a given discharge, is independent of the detention period * * *."

In Section 9 he states that:

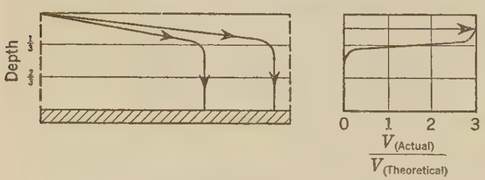
> "A rough estimate of the effect of short-circuiting on removal may be had if it is assumed that the suspension is subjected to various settling times, distributed as indicated by the dispersion curve."

All the liquid passing through the tank is subjected to the same overflow rate, but various fractions of the liquid are given different detention times. If overflow rate is the criterion, as claimed in Section 4, then no appreciable effect should be noted on the efficient removal of settleable solids in the tank. If detention time is the criterion, as suggested in Section 9, short-circuiting should have a marked effect. The former possible criterion is based on sound theory and the latter is based on an opinion, shared by many other engineers, but not borne out by experimental evidence.

Tests by the writer have shown that many circular settling tanks experience greater short-circuiting than do rectangular tanks; but the average effect on settling efficiency seems to be inappreciable. This statement is borne out by results in existing water treatment and sewage treatment plants. With similar overflow rates and theoretical detention periods, no great difference in settling efficiency is noted between the performance of rectangular and circular tanks at many plants throughout the United States. On the whole, average values of removals are quite consistently alike, as may be indicated by perusal of the wealth of operating data in the literature on the subject.[51] Selection of circular or rectangular settling tanks seems to be based on preferences of designing engineers and economy of plant design.

Consider a settling tank, either circular or rectangular, with a dispersion curve similar to curve B in Fig. 15. This has a "probable flowing through time," $\frac{t_A}{T}$, of 0.831. The author claims that the other curves shown in Fig.15, each having higher values of $\frac{t_A}{T}$, and thus greater hydraulic efficiencies, "are characteristic of progressively better types of settling tanks."

For the sake of argument assume that the flow is concentrated in the upper third of the tank with the entire lower two thirds dead space. The average velocity will be three times the theoretical and the "flowing through time," $\frac{t_A}{T}$, will approximate 0.33. Since the lower part of the tank is assumed to be quiescent water, the paths of particles will be as shown in Fig. 21. On a hydraulic basis the tank would only be considered 33% efficient. However, a glance at Fig. 21, together with consideration of the significance of settling velocities and overflow rates, shows that particles will be removed as well as if the entire tank were utilized. This tank is merely the shallow high velocity tank recommended by Mr. Camp in Section 4 as the economical tank for settling.



(a) PATHS OF PARTICLES

(b) VELOCITY DISTRIBUTION

Fig. 21

The main argument against this type of flow pattern might be that the time theoretically available for flocculation is not utilized to the fullest extent. However, this might not even be correct if the statement by Mr. Camp following Eq. 31 holds true. He considers the effect of velocity gradients on flocculation and states,

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[51] "Sewage Treatment Works," by C. E. Keefer, McGraw-Hill Book Co., Inc., New York, N. Y., 1940, p. 125.

> "From Eq. 31, it may be seen that the rate of flocculation due to velocity gradients is proportional to $V^{1.5}$ and inversely proportional to $\sqrt{R}$. Hence this type of coagulation is greatest for shallow tanks with relatively high velocities."

It must be admitted that short-circuiting can have an adverse effect on settling efficiency if strong currents travel along the bottom of the tank at such a rate that the tractive force will dislodge particles already settled, or will carry the particles along as part of the bed load. The effect of velocities on bed loads is discussed in Section 5. Inlet baffles should be so designed that this type of short-circuiting is avoided. As long as bottom velocities are low enough, short-circuiting in the upper regions of the tank should have no appreciable effect since overflow rate is the governing factor.

This discussion of the effect of short-circuiting is not meant to imply that no attempt should be made by designing engineers to secure good flow distributions in settling tanks. It is merely a plea for a rational approach to the determination of settling tank efficiencies. These should be determined by actual tank performance, as measured by solids removals, and not by an arbitrary assumption that settling efficiency is almost a direct function of hydraulic efficiency. By theoretical analysis in another section of his paper, Mr. Camp proves, and emphasizes time and again, that overflow rate is the controlling function in removal of solids. This is sound practice and should be the main criterion of settling tank design.

The conclusions in Section 10 do not constitute a summary of the salient factors developed from theoretical considerations in the main body of the paper. In his closing discussion Mr. Camp should summarize the most important conclusions regarding the clarification theory, the effects of tractive force and bed-load movement, and the effects of turbulence and flocculation on sedimentation. Such a summary would make the paper much more valuable to the designing engineer.

As presented, the conclusions are opinions of the author on the design of mechanisms and general arrangements for settling tanks. Several factors not mentioned under theoretical considerations and principles are introduced when the author abandons theory and writes about practice.

To utilize shallow tanks with high velocities, Mr. Camp suggests the design of a tray settling tank as shown in Fig. 17. Such tanks have been used for many years in the chemical and metallurgical industries for the sedimentation of particles somewhat different from those encountered in water treatment and sewage treatment plants. This type of tank has been made available to the sanitary engineering profession by a number of equipment manufacturers but has been accepted for relatively few installations.

In 1940 the writer saw one such unit installed in a sewage treatment plant at Springfield, Mo. Considerable difficulty had been experienced when the sludge that had settled on the trays began to digest. Entrapped gas caused some of the sludge to rise as scum and accumulate on the underside of the tray above where it must have collected in considerable quantity and solidified to a degree. Further digestion of this scum released gas which could not escape readily. Pressure must have built up within the mass of scum and sludge to the extent that the concrete slab forming the tray was broken and lifted, with the virtual destruction of the tray. There is no assurance that the same condition might not develop in the design proposed by Mr. Camp. The mechanism would be required to scrape the sludge that rises as well as that which settles. Much experimental work must be conducted before a practical design can be developed. The design of final settling tanks for the activated sludge process does not necessarily result from the material developed in the paper.

In summarizing, it should be emphasized that Mr. Camp has accomplished one part of his announced objective—namely, "to collect in one compendium the known principles of sedimentation essential to the development of design theory." However, he has only partly accomplished his other goal, which he announced was "to present the theory of design developed to a stage which will permit its use in practice." Some of the numerical examples are based on hypothetical analyses, the laboratory equipment for whose determination is yet to be developed. Other factors influencing sedimentation are too complex for mathematical analyses or cannot be evaluated on the basis of the present state of knowledge of the art.

Mr. Camp's paper should be valuable to the designing engineer because it provides for a better understanding of the principles affecting sedimentation. It should also encourage pilot-plant studies of settling tank design, applying those principles developed in the paper. The accumulation of much data on the operation of shallow tanks using high velocities at appropriate overflow rates, together with the interpretation of these data by engineers and mathematicians, is necessary before many of the factors influencing sedimentation can be evaluated and formulated to such an extent that they are directly applicable to the problems of the designing engineer.

THOMAS R. CAMP,[52] M. AM. SOC. C. E.—To those who have found time during these busy days to study and discuss the paper the writer wishes to express his appreciation. The paper was intended to be a comprehensive treatise on a subject which has long been neglected, but because of space limitations some important aspects of the subject had to be omitted and explanatory notes had to be kept to a minimum.

The writer and the profession are indebted to Mr. Stein for his careful scientific analysis of the contents of the paper—particularly for indicating the error in Example 2 which was based on the writer's erroneous statement regarding the compacting of activated sludge. Based on a much too hasty analysis of the experiments by Messrs. Rudolfs and Lacy, the writer concluded that the rate of compacting in terms of the ratio of the volumes occupied by the sludge blanket is a function of time only and is independent of the depth of the blanket. Mr. Stein has shown that this is not the case. He has demonstrated conclusively that the rate of compaction is also a function of the depth of the blanket. His experiments and analysis indicate that the absolute rate of subsidence of the top of the sludge blanket is substantially independent of the depth of the blanket. Thus, the shallower the depth of the blanket, the greater will be the rate of compacting in terms of the ratio of the volumes occupied by the blanket. This finding is very important because it demonstrates the desirability of shallow tanks for sludge compaction. The writer has shown that shallow tanks are both desirable and economical for free settling. Mr. Stein indicates that this is also the case for sludge concentration.

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[52] Cons. Engr., Boston, Mass.

Mr. Perry suggests that the paper might have been amplified to several times its present length and Professor Eliassen states that the paper lacks a concise summary. Mr. Anderson objects because the conclusions appear abrupt. The original manuscript on which this paper was based was nearly three times the printed length. It contained an extensive development of the theory of design of inlets and outlets as well as other important details—all of which were omitted to hold the length of the paper within permissible limits. In this manner, a full treatment of the subject of turbulence created by inlets (which Professor Eliassen believes to have been neglected) was omitted for lack of space.

Mr. Chase states that:

> "* * * the design of practicable settling tanks will continue to be based upon judgment, experience, and common sense seasoned with a moderate amount of theoretical computations used as aids to judgment."

This attitude has undoubtedly been that of most of the designers of settling tanks for water and sewage treatment purposes and should be sufficient to explain to Mr. Anderson why multiple-tray settling tanks have had such limited acceptance. The moderate amount of theoretical computations which Mr. Chase suggests for seasoning the judgment has unfortunately been so moderate that it has been almost completely absent. Design has consisted almost wholly of conformity to previous practice in size and shape with little regard for the principles involved or for the nature and settling characteristics of the particles to be settled.

A tray-type settling tank was constructed at Springfield, Mo., as mentioned by Professor Eliassen. A careful analysis of the removals effected in this plant before and after the trays were introduced will reveal that the improvement which should be expected by such an increase in the settling area was actually obtained.[53] Professor Eliassen's criticism of the Springfield tray settling tanks deals with details of mechanical design rather than with the principles involved. Obviously, if the sewage contains entrained gas and scum, they will rise to the roof of a tray; and a competent designer will consider skimming in this type of tank just as it is customarily provided for in conventionally designed tanks.

Mr. Anderson suggests that the writer might have shown in what way his proposed design for multiple-tray settling tanks may be more acceptable than those which have been patented and promoted for many years. The reason why the proposed design should be more acceptable than previously constructed tray-type tanks was carefully stated in the paper. Any tank that will produce the same removal as other tanks at less cost should be more acceptable. All previously constructed multiple-tray tanks that have come to the writer's attention have tray depths of about 2 ft and are designed with such low velocities that the flow is unstable. The Springfield tanks, moreover, are of the radial-flow type with the high inlet velocities and unstable flow conditions which characterize all radial-flow tanks. The design proposed suggests only rectangular tanks and high velocity shallow trays. The writer does not propose, however, that any such tank should be designed without experimental knowledge of the settling and flocculating characteristics of the suspension to be settled.

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[53] "Report of Research Committee," *Sewage Works Journal*, March, 1942, p. 323.

Mr. Anderson states that the conclusions on the trends in design of final settling tanks for the activated sludge process may be questioned on the basis of an extensive study of plant-size experiments and operating data. In support of this statement Mr. Anderson submits the results of his own experiments.[42] On the basis of these experiments, Mr. Anderson argues that activated sludge should be withdrawn from the inlet end of the final settling tank. In a discussion of Mr. Anderson's paper,[54] R. H. Gould, M. Am. Soc. C. E., draws precisely the opposite conclusions and from the same experimental data—that is, Mr. Gould believes that activated sludge should be withdrawn from the outlet end of final settling tanks to counteract, partly, the effect of density currents. The writer believes that Mr. Gould's arguments are sound.

Mr. Anderson presents the results of experiments on a circular tank at Chicago, Ill., in which the sludge blanket was allowed to build up to great depths, as evidence that density currents are not avoided by the use of shallow high velocity final settling tanks. In this experiment the mixed liquor was introduced in a vertical flow pipe at the center of the tank and was discharged into the tank above the sludge blanket. The mixed liquor, being heavier than the supernatant liquor in the tank, must fall to the bottom of the supernatant liquor and thus produce the density current which was observed. A shallow high velocity tank is not in itself sufficient to avoid density currents. The inlet to the tank must be placed low enough, as indicated by Fig. 18(*c*), so that the mixed liquor enters below the top of the sludge blanket. It must also be introduced at a low inlet velocity.

An examination of all the final settling tanks referred to by Mr. Anderson reveals that the mixed liquor is introduced near the surface of the tank, and in many cases it may fall through a depth of as much as 10 ft. A rough estimate of the velocity of this fall may be made by assuming that the velocity head which induces the velocity of the density current is equal to the height of the fall multiplied by the net specific gravity of the mixed liquor. The specific gravity of the mixed liquor is approximately 1.0005; and, since the specific gravity of the supernatant is substantially equal to about 1.0000, the net specific gravity is the difference, or 0.0005. For a fall of 10 ft the corresponding velocity of a density current is 34 ft per min. The actual velocity, of course, is somewhat less because of the resistance to fall imposed by the supernatant liquor.

Mr. Anderson asks why this so-called short-circuiting of fresh sludge should be avoided when one of the most desirable features of a final settling tank is the withdrawal of fresh return sludge. The withdrawal of fresh return sludge is a desirable feature, of course, but it is only one of many desirable features that the designer should endeavor to secure. The tank should be so designed that all the sludge remains fresh and contains some dissolved oxygen. It should also be so designed that sludge is withdrawn only after it has received a maximum amount of compaction so as to reduce the volume of sludge that must be pumped. The sludge blanket after compacting is not a "flocculent suspension" as inferred by Mr. Anderson since the particles are resting on one another.

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[54] *Sewage Works Journal*, January, 1945, p. 63.

Mr. Anderson states that:

> "* * * it appears that 'destruction of floc by excessive velocity gradients' * * * is not an important factor with aeration tank effluent."

It is well known that certain types of floc such as those in water treatment plants, particularly color floc, are easily destroyed by moderately high velocity gradients. The writer suspects that activated sludge flocs are much tougher and less easily destroyed, and it is reassuring to learn that the Sanitary District of Chicago has made tests which show practically no change in the settleability of aeration tank effluent even after a free fall over a weir and passage through both a venturi meter and a considerable length of conduit and aerated channel. The results of these tests at Chicago, however, should not be taken as an indication that all activated sludge flocs are tough. One cause of bulking of sludge in final settling tanks may be the destruction of floc at the inlet of such tanks. Since it is a comparatively simple matter to avoid any danger of destruction by designing the inlets as suggested in Fig. 18 (*c*), there appears to be no valid argument against such design.

Mr. Mulholland calls attention to the fact that the design of grit chambers is one of the weakest links in the entire process of sewage treatment plant design. This is true not only of sewage treatment plants serving small towns and villages but also of the largest plants because the principles involved have not been well understood by the designer.

Geometric scale models for studying sedimentation are not practicable because the laws of hydraulic similitude require that all dimensions should be scaled down proportionately, and obviously this is not practicable for the particles to be settled. Geometric scale models are useful in studying hydraulic short-circuiting because the flow patterns that are obtained in plant-scale tanks can be reproduced in the model. When a prototype suspension is introduced into a model settling tank, the results obtained are not an indication of the performance of the prototype settling tank as inferred by Mr. Mulholland. This statement should not be interpreted, however, as an argument against the use of pilot plants for sedimentation experiments. A well-designed and well-operated pilot plant should yield all the information the investigator needs regarding the settling and the flocculation of the suspension to be dealt with in a prototype plant. A pilot settling tank, however, is not a model tank in the true sense.

Mr. Chase states that:

> "In the case of raw sewage the solids in suspension are of such a heterogeneous and variable nature that no formula could possibly be devised of general applicability."

The theory developed in the paper applies not only to sewage solids but to all liquid suspensions. In fact, much of the basic theory applies to many other types of disperse systems such as solids in gas, liquid in gas, liquid in liquid, and gas in liquid. It is obviously impossible to develop any man-made theory which is complete and correct in all respects. It is the duty of engineers, nevertheless, to endeavor to place their analyses and designs on as rational a basis as can be obtained. If sanitary engineers do not follow this philosophy generally, in due time the business of designing treatment plants will be taken over by more competent experts. Many of the real advances in treatment plant design have been brought about through the efforts of equipment manufacturers and even now there is a tendency to look to manufacturers, rather than to consulting engineers, for expert service.

Professor Eliassen's principal criticism seems to be that the writer did not go far enough in his development. This is obviously true of any presentation of new theory or new experimental facts. One should not be condemned for starting the rationalization of settling tank design on the grounds that he has not presented a completed and fully tested theory. The responsibility for progress belongs to the profession as a whole.

Professor Eliassen attempts to show that practicing engineers have taken into account the settling characteristics of the suspension involved because they have employed overflow rates which are within the ranges used by the writer in some of his examples. On the contrary, the ranges in overflow rates mentioned by the writer were taken from current practice and not from any experimental measurements of the settling characteristics of suspension. These overflow rates were not recommended by the writer as desirable, but were simply used in examples for the purpose of illustrating the theory and methods of application.

Professor Eliassen suggests that some of the intermediate steps in the process of analysis of tank dimensions in terms of velocity gradients should be recognized by the designing engineer so that he is cognizant of the limitations of this method of analysis. Surely the obligations of the designing engineer extend further than this. As one limitation Professor Eliassen suggests that the velocity gradients produced by inlet turbulence and density currents have been neglected. Both density currents and inlet turbulence may be reduced to negligible proportions. In fact, the implications of the paper are that economical design makes this mandatory. Professor Eliassen's arguments are based on a knowledge of settling tanks currently in use rather than on a consideration of how tanks should be designed.

He states that:

> "Within the limitations of long shallow tanks in which the drag on the tank floor and walls will have a dominating influence on velocity gradients, the author develops a sound theory leading to quantitative results in several examples."

The natural corollary to this statement is that this type of tank should be used provided that experimental tests on the suspensions to be settled indicate that the optimum velocity gradients are within the required range. There is plenty of evidence in the literature that the flow conditions are unstable in short low velocity, deep tanks and that removal of suspended solids per unit volume of tank is not what it should be. This is particularly true of radial-flow circular tanks.

Professor Eliassen, questioning the use of Froude's law in model studies of short-circuiting, states that: "Mr. Camp presents no data based on model studies operated in accordance with Reynolds' law." To be strictly correct, geometric model studies should conform both to the Reynolds and Froude model laws. This is physically impossible, however, and the experimenter must use the law that yields flow patterns in the model which are most nearly similar to the flow patterns in the prototype. Both experience and tests indicate that Froude's law gives acceptable results.

Model tests were made by the Department of Water Supply of the City of Detroit (Mich.) on short-circuiting in the settling tanks in the water works park filtration plant as a preliminary to the design of guide vanes to reduce short-circuiting. These basins are of the "round-the-end" baffled type, and the change in direction of flow resulted in considerable short-circuiting and reduced settling efficiency. The model was operating in accordance with Froude's law, and the results obtained in the model were found to agree with those obtained in the prototype.

An experimental study was made at Massachusetts Institute of Technology at Cambridge, Mass., in 1939 under the writer's direction by William Christensen and Sylvester Morabito,[55] Jun. Am. Soc. C. E., to determine the applicability of Froude's model law in connection with model studies of the flow patterns in cubically-shaped mixing chambers operating with and without mechanical agitation by paddles. Two tanks, one four times the size of the other, were used in these experiments. The experiments revealed that, within the limits of experimental accuracy, Froude's law may be employed in model studies of the flow pattern in cubically-shaped mixing chambers and that, when paddles are employed, Froude's law also applies to the peripheral velocity of the paddles. No similarity in flow patterns was observed when Reynolds' law was used.

Professor Eliassen states that the writer has placed too much emphasis on short-circuiting as a criterion of settling tank performance. He feels that, since removal is determined by overflow rates, short-circuiting should have no marked effect. As commonly defined, overflow rate is the discharge per unit of surface area of the tank. For the purpose of this discussion, the definition given by Mr. Stein is preferable—that is, overflow rate is equal to the ratio of depth to detention time. Since all the particles of water do not have the same detention time, they do not have the same overflow rate. In other words, short-circuiting affects the overflow rate in precisely the same manner as it does the detention time.

The literature is full of experimental evidence that short-circuiting impairs the removal in settling tanks. Nearly all the papers describing improvements made to inlets and the baffling of tanks reveal that the effect of such improvements was to increase the removal of suspended solids. These improvements have no effect on the ratio of discharge to surface area. The improvements in removal must therefore have been caused by reduced short-circuiting.

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[55] "Model Studies of Short-Circuiting in Mixing Chambers," by William Christensen and Sylvester Morabito, thesis presented to Massachusetts Institute of Technology at Cambridge, Mass., in 1939, in partial fulfilment of the requirements for the degree of Bachelor of Science.

Professor Eliassen states that tests made by himself indicate that settling efficiency is as great in circular tanks as in rectangular tanks despite the fact that short-circuiting is known to be worse in circular tanks. This is contrary to the experience of others. Operating experience on the long rectangular tanks at Detroit, Minneapolis, Minn., and Elizabeth, N. J., show much better suspended solids removal than does operating experience on the radial-flow circular tanks at Buffalo, N. Y.

In conclusion, the writer wishes to repeat for emphasis the statement made in the "Synopsis," relative to sedimentation, as follows:

> "Despite the importance of the process, current understanding of the principles involved has progressed so slightly that there is as yet no such thing in practice as the economic design of tanks from a functional viewpoint."

The responsibility for the development of the art of water treatment and sewage treatment rests squarely with the sanitary engineering profession since the public considers the sanitary engineer as the legitimate expert in this field. If he is to maintain this position he must participate in research and development. More participation in the field of sedimentation research is badly needed.

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MATHEMATICAL EXAMINATION OF THE HIGHWAY TRANSITION SPIRAL

BY JOHN ORAN EICHLER,[1] ASSOC. M. AM. SOC. C. E., AND
HOWARD W. EVES,[2] ESQ.

WITH DISCUSSION BY MESSRS. JOSEPH BARNETT, RALPH W. STEWART, DONALD THOMPSON, T. F. HICKERSON, ALLEN H. BROWNFIELD, RENÉ S. PULIDO Y MORALES, C. C. WILEY, MAX C. KOEHLER, AND WILLIAM T. PRYOR.

SYNOPSIS

The growing importance of superhighways demands that increased attention be given to the inherent problems of construction and design. One of the most important of these problems is that of horizontal curvature and the attendant superelevation. A rational solution of this problem is the adaptation of a transition spiral, whose curvature varies inversely as the arc length. From considerations of speed, safety, and comfort it is found that two cases arise—that in which a central piece of circular arc is required, and that in which the curve is transitional throughout. The second case is desirable on grounds of relative simplicity of calculations, formulas, and field work. The question then naturally arises: "When, under certain given limiting conditions, can a curve be transitional throughout?" An answer to this question (and to other office and field problems related to the spiral) necessitates an examination of the mathematics of the transition spiral. Most existing mathematical treatments seem to suffer shortcomings due to lack of proof, or to failure to consider comfort and safety, or to loss of generality, or to confusion between exact and approximate relations, or to failure to be concise. Therefore, it is felt that an exposition of the curve and its properties obviating these faults and conducted primarily as a basis for examining questions, such as the foregoing, is eminently desirable. Such an attempt is offered in this paper.

1. INTRODUCTION

It seems safe to predict that the road engineers of the postwar period will have to give ever-increasing attention to the problems connected with the

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development of superhighways. The aim will be to construct a roadway capable of carrying high-speed traffic, designed for maximum safety and comfort to drivers. Of the many problems involved, that of horizontal curvature, accompanied with proper superelevation, is one of the most important, and it is essentially with this feature of the future road that this paper is concerned. In particular, the writers would like to offer a simple and definite answer to the question: "When can a curve be transitional throughout?" Considerable theoretical research, table construction, and discussion have already taken place, as a complete bibliography of the subject would testify; but, because of the scattered nature of these references, the complete theory of easement curves is not readily available to the average road engineer. It would seem desirable, therefore, to have a clear, concise, and easily accessible account of the mathematical theory of the ideal easement curve, since it is only through a sound grasp of the mathematical properties of this curve that the dependent practical field work of laying out easement curves can be conducted intelligently. A representative bibliography is included as an Appendix to the paper.

2. The Transition Spiral

The forces acting on a vehicle and its passengers are altered when the vehicle passes from a straight to a curved path. The value of the centrifugal force P acting on a body of weight W, traveling at a speed V, and rounding a curve of radius r, is

$$P = \frac{W\,V^2}{g\,r} \qquad \text{(1)}$$

in which g is the acceleration due to gravity. If the path of the vehicle changes immediately from a straight line to a circle of radius r, the force P will be applied instantaneously, and the vehicle and passengers will be subjected to a proportionate shock. However, if the radius of curvature of the path is gradually reduced from infinity to its minimum value, the centrifugal force P will be increased gradually and little shock will be experienced by the vehicle and its passengers. Any curve that provides this transition is called an "easement curve." J. Glover (9)[3] and W. H. Shortt (27) have argued that the rational way to achieve this transition is to make the rate of gain of the "centrifugal ratio," P/W, constant throughout the curve. This is equivalent to the demand that the driver sharpen his curve by turning his steering wheel at a constant rate. Since, for a fixed speed V, P/W is proportional to $1/r$, it follows that the mathematical requirement for the perfect easement curve is that the radius of curvature, r, at any point should be inversely proportional to the distance, s, of that point from the beginning of the curve. Such an easement curve may be termed a "transition spiral." For a fixed speed, superelevation slope is proportional to curvature and hence, for the transition spiral, to s. It follows, therefore, that this curve is also ideal for the application of superelevation.

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[3] Numerals in parentheses, thus: (9), refer to corresponding items in the Bibliography (see Appendix).

Many other forms of easement curves have been suggested and used, such as empirical or bastard transitions (possessing little to recommend them), the quadratic parabola (advocated by E. L. Leeming (16)), the cubic parabola (used extensively for short transitions in railroad work), the lemniscate (developed and used by F. G. Royal-Dawson (22) in England), the multi-compound spirals (such as the one evolved by W. H. Searles (25)), and the ten-chord spiral (13) adopted by the American Railway Engineering Association (which is essentially a transition spiral laid out in ten equal chord lengths). All these curves are modifications or approximations of the theoretically ideal transition spiral. It is not proposed here to enter into the arguments favoring any one easement curve over the others, but to develop the mathematics of the transition spiral and then, in the interests of economy in calculation and field work, to examine briefly some approximate formulas derived from the exact ones.

3. INTRINSIC EQUATION OF SPIRAL

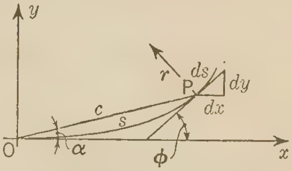


FIG. 1

In Fig. 1, let O be the beginning of the spiral on the initial tangent or x-axis. Then, if s is the arc length of the spiral from O to any point P on the curve, r is the radius of curvature at point P and ϕ is the angle that the tangent at point P makes with the x-axis, by the definition of a transition spiral:

$$\frac{1}{r} = k\,s \qquad (2)$$

in which k is a constant of proportionality; but, since curvature is the arc rate of change of ϕ: $\frac{1}{r} = \frac{d\phi}{ds} = k\,s$; whence, at the end of the spiral,

$$s = K\sqrt{\phi} \qquad (3)$$

in which

$$K = \sqrt{\frac{2}{k}} = \sqrt{2\,R\,S} \qquad (4)$$

R and S being the final radius of curvature and length of the spiral, respectively.

4. PARAMETRIC CARTESIAN REPRESENTATION

Letting ds, dx, and dy be differential changes in s, x, and y, respectively:

$$dx = ds\cos\phi = 0.5\,K\,\phi^{-1/2}\cos\phi\,d\phi \qquad (5a)$$

and

$$dy = ds\sin\phi = 0.5\,K\,\phi^{-1/2}\sin\phi\,d\phi \qquad (5b)$$

whence

$$x = 0.5\,K\int_0^{\phi}\phi^{-1/2}\cos\phi\,d\phi$$

$$= K\,\phi^{1/2}\left[1 - \frac{\phi^2}{5(2!)} + \frac{\phi^4}{9(4!)} - \frac{\phi^6}{13(6!)} + \cdots\right] \qquad (6a)$$

and

$$y = 0.5\,K \int_0^{\phi} \phi^{-1/2} \sin\phi \, d\phi$$

$$= K\,\phi^{1/2}\left[\frac{\phi}{3} - \frac{\phi^3}{7(3!)} + \frac{\phi^5}{11(5!)} - \frac{\phi^7}{15(7!)} + \cdots\right] \qquad (6b)$$

5. Polar Deflection Angle α

Let α be the polar deflection angle of point P with respect to O as origin and the tangent to the curve at O as x-axis. Then, by Eqs. 6,

$$\tan\alpha = \frac{y}{x} = \frac{\dfrac{\phi}{3} - \dfrac{\phi^3}{7(3!)} + \dfrac{\phi^5}{11(5!)} - \dfrac{\phi^7}{15(7!)} + \cdots}{1 - \dfrac{\phi^2}{5(2!)} + \dfrac{\phi^4}{9(4!)} - \dfrac{\phi^6}{13(6!)} + \cdots}$$

$$= \frac{\phi}{3} + \frac{\phi^3}{105} + \frac{\phi^5}{5{,}997} - \frac{\phi^7}{198{,}728} - \cdots \qquad (7)$$

Solving for α and setting

$$C = \frac{\phi}{3} - \alpha \qquad (8)$$

one may now compute the following values, expressing C, in minutes, corresponding to given values of ϕ, in degrees:

ϕ, in degrees	C, in minute
15	0.2
20	0.4
25	0.8
30	1.4
35	2.2
40	3.4
45	4.8
50	6.6

Such a table is given by J. Barnett, M. Am. Soc. C. E. (2), and is valuable for quickly obtaining the corrections C which must be subtracted from one third of any given ϕ in order to obtain the corresponding α. Use of this table will be made in section 9, where is developed the program for finding the control data of a required spiral.

6. Chord Length c

Denoting the chord length corresponding to arc s by c:

$$c = (x^2 + y^2)^{1/2} \qquad (9)$$

Substituting from Eqs. 6:

$$c = K\,\phi^{1/2}\left[1 + \frac{\phi^4}{25(2!)^2} + \frac{\phi^8}{81(4!)^2} - \frac{2\,\phi^2}{5(2!)} + \frac{2\,\phi^4}{9(4!)} - \frac{2\,\phi^6}{13(6!)} + \frac{2\,\phi^8}{17(8!)} - \frac{2\,\phi^6}{45(2!)(4!)} + \frac{2\,\phi^8}{65(2!)(6!)} + \cdots + \frac{\phi^2}{9} + \frac{\phi^6}{49(3!)^2} - \frac{\phi^4}{21(3!)} + \frac{2\,\phi^6}{33(5!)} - \frac{2\,\phi^8}{45(7!)} - \frac{2\,\phi^8}{77(3!)(5!)} + \cdots\right]^{1/2} \ldots\ldots\ldots(10a)$$

or

$$c = K\,\phi^{1/2}\left[1 + \phi^2\left(\frac{1}{9} - \frac{2}{5(2!)}\right) + \phi^4\left(\frac{1}{25(2!)^2} + \frac{2}{9(4!)} - \frac{2}{21(3!)}\right) + \phi^6\left(-\frac{2}{13(6!)} - \frac{2}{45(2!)(4!)} + \frac{1}{49(3!)^2} + \frac{2}{33(5!)}\right) + \cdots\right]^{1/2} \ldots(10b)$$

Eq. 10*b* is in the form

$$c = s\,[1 + (A\,\phi^2 + B\,\phi^4 + C\,\phi^6 + \cdots)]^{1/2} \ldots\ldots\ldots\ldots(11a)$$

or, expanding Eq. 11*a* by the binomial theorem,

$$c = s\left[1 + \frac{(\quad)}{2} - \frac{(\quad)^2}{8} + \frac{(\quad)^3}{16} - \cdots\right] \ldots\ldots\ldots\ldots(11b)$$

This then gives

$$c = s\left[1 + \phi^2\left(\frac{A}{2}\right) + \phi^4\left(\frac{B}{2} - \frac{A^2}{8}\right) + \phi^6\left(\frac{C}{2} - \frac{A\,B}{4} + \frac{A^3}{16}\right) + \cdots\right] \ldots(12a)$$

or

$$c = s\,[1 - 0.0444444\,\phi^2 + 0.0007054\,\phi^4 - 0.0000025\,\phi^6 + \cdots] \ldots(12b)$$

in which ϕ, of course, is given in radians. If ϕ is to be given in degrees:

$$c = s\,[1 - 1.354\,(10^{-5})\,\phi^2 + 6.6\,(10^{-11})\,\phi^4 - \cdots] \ldots\ldots\ldots(13)$$

Setting

$$p = \frac{c}{s} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(14)$$

one may use Eq. 12*b* or Eq. 13 for constructing a useful table (or a graph, as has been done by H. J. Collins and C. A. Hart, Assoc. Members, Am. Soc. C. E. (5)) giving the values of p corresponding to values of ϕ.

7. SAFETY AND COMFORT REQUIREMENTS FOR DRIVING THE SPIRAL

(*A*) The centrifugal ratio $\frac{P}{W} = \frac{V^2}{g\,r}$ must have an upper limit ρ, for the following reason: Let μ be the angle of superelevation and ν the angle of friction for the tires on the road. Then, from mechanics,

$$\frac{V^2}{g\,r} = \tan(\mu + \nu) = \frac{\tan\mu + \tan\nu}{1 - \tan\mu\,\tan\nu} = \frac{e + f}{1 - e\,f} \ldots\ldots\ldots(15)$$

in which $e = \tan \mu$ and $f = \tan \nu$. For practical reasons the superelevation slope e must be limited, and f (which is the coefficient of friction for transverse sliding of tires on the roadway) seems to depend on the speed. According to present American practice e seldom exceeds about 0.10, and, from a number of driving tests (19), it seems that f may be assumed at about 0.14 for a speed of 70 miles per hr to perhaps 0.08 for 100 miles per hr. According to Messrs. Collins and Hart (5):

> "The maximum centrifugal ratio for railways is normally taken as 1/8, but owing to the lower centre of gravity of motor vehicles, it may be safely increased to 1/4 for road work. This value is quite arbitrary and the 'factor of safety' will depend on a great number of considerations such as degree of superelevation, adhesion between road and tyres, bumpiness of road, etc."

(*B*) The rate of change of radial acceleration must have an upper limit γ. Let t be the time required to travel the entire length S of the spiral. Then

$$\gamma = \frac{V^2/R}{t} = \frac{V^2/R}{S/V} = \frac{V^3}{R\,S}\left(\frac{\text{ft}}{\text{sec}^3}\right).$$

This average rate varies for different drivers and many tests are needed to determine the maximum average rate. From considerations of comfort, Mr. Shortt (27) has fixed γ at $1\ \frac{\text{ft}}{\text{sec}^3}$, but more recent experiments by R. A. Moyer, Assoc. M. Am. Soc. C. E. (19), indicate that $\gamma = 2\ \frac{\text{ft}}{\text{sec}^3}$ is both safe and comfortable.

Paragraphs (*A*) and (*B*) yield the formulas,

$$R = \frac{V^2}{g\,\rho} \qquad \text{(16}a\text{)}$$

$$S = \frac{V^3}{R\,\gamma} = \frac{V\,g\,\rho}{\gamma} \qquad \text{(16}b\text{)}$$

and

$$\Phi = \frac{S^2}{K^2} = \frac{S}{2\,R} = \frac{V^3}{2\,R^2\,\gamma} = \frac{g^2\,\rho^2}{2\,\gamma\,V} \qquad \text{(16}c\text{)}$$

in which Φ is the deflection angle, R the radius of curvature, and S the arc length of the spiral for the point where the maximum centrifugal ratio is reached.

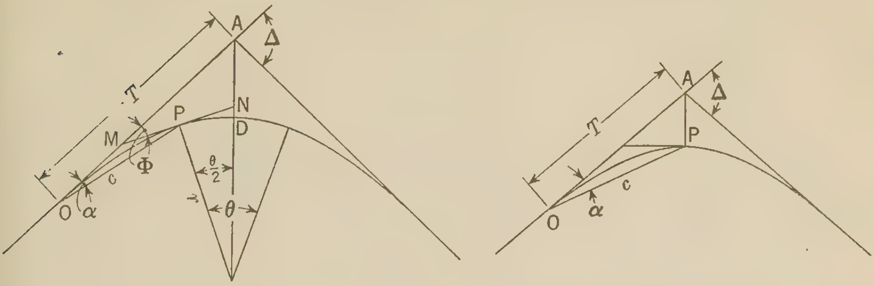
8. COMPUTATION OF TANGENT DISTANCE *T* AND EXTERNAL DISTANCE *E*

Let Δ be the intersection angle of the highway curve. For a given velocity V and selected values of ρ and γ, Eq. 16*c* indicates that Φ cannot exceed $\frac{g^2\,\rho^2}{2\,\gamma\,V}$. Hence, in designing a curve for a given speed V and selected values of ρ and γ, two cases need to be considered—that for which $\Phi < 0.5\,\Delta$ and that for which $\Phi \geqq 0.5\,\Delta$. In the first case the curve must contain a central circular arc; and, in the second case, the curve will be transitional throughout.

Thus, as an example, for a velocity of 100 miles per hr, $g = 32 \frac{\text{ft}}{\text{sec}^2}$, $\rho = 0.25$, and $\gamma = 2 \frac{\text{ft}}{\text{sec}^3}$, Eq. 16c yields $\Phi = 0.1091$ radian, or about 6.25°. Hence, if $\Delta \leqq 12.5°$ the curve, under these conditions, will be transitional throughout; otherwise a central circular arc will be needed.

Case 1. *$\Phi < 0.5\,\Delta$ (Central Circular Arc Required).*—In Fig. 2(*a*), angle MAN $= 90° - 0.5\,\Delta$ and angle MNA $= 90° + 0.5\,\Delta - \Phi$. Also,

$$T = \overline{\text{OM}} + \overline{\text{MA}} = \frac{c \sin (\Phi - \alpha)}{\sin \Phi} + \frac{\overline{\text{MN}} \sin (90° + 0.5\,\Delta - \Phi)}{\sin (90° - 0.5\,\Delta)} \quad \text{. . (17}a\text{)}$$



(*a*) CASE 1, $\Phi < 0.5\,\Delta$ (*b*) CASE 2, $\Phi \geqq 0.5\,\Delta$

FIG. 2

Again,

$$\overline{\text{MN}} = \overline{\text{MP}} + \overline{\text{PN}} = \frac{c \sin \alpha}{\sin \Phi} + R \tan 0.5\,\theta \quad \text{. . . . . (17}b\text{)}$$

Therefore,

$$T = \frac{c \sin (\Phi - \alpha)}{\sin \Phi} + \frac{\left(\dfrac{c \sin \alpha}{\sin \Phi} + R \tan 0.5\,\theta\right) \sin (90° + 0.5\,\Delta - \Phi)}{\sin (90° - 0.5\,\Delta)} \quad \text{. . (18)}$$

Furthermore,

$$\overline{\text{AN}} = \overline{\text{MN}} \frac{\sin \Phi}{\sin (90° - 0.5\,\Delta)} = \frac{c \sin \alpha + R \tan 0.5\,\theta \sin \Phi}{\sin (90° - 0.5\,\Delta)} \quad \text{. . . . (19}a\text{)}$$

and

$$\overline{\text{ND}} = R \sec 0.5\,\theta - R \quad \text{. . . . . (19}b\text{)}$$

Combining Eqs. 19,

$$E = \overline{\text{AN}} + \overline{\text{ND}} = \frac{c \sin \alpha + R \tan 0.5\,\theta \sin \Phi}{\sin (90° - 0.5\,\Delta)} + R \sec 0.5\,\theta - R \quad \text{. . (20)}$$

Case 2. *$\Phi \geqq 0.5\,\Delta$ (Transitional Throughout).*—In Fig. 2(*b*), angle OPA $= 90° + 0.5\,\Delta - \alpha$. Therefore,

$$T = \frac{c \sin (90° + 0.5\,\Delta - \alpha)}{\sin (90° - 0.5\,\Delta)} \quad \text{. . . . . (21}a\text{)}$$

and

$$E = \frac{c \sin \alpha}{\sin (90° - 0.5\,\Delta)} \quad \text{. (21}b\text{)}$$

9. Curve Control Data for Spiral

It is desirable to know the curve data for a transition spiral for which V, ρ, γ, and Δ are given. After computing Φ by Eq. 16*c*, such data may be found, for the two cases, in six steps, as follows:

Step	Case 1	Case 2
1	Compute R by Eq. 16*a*	Compute K by Eqs. 4, 16*a*, and 16*b*
2	Compute S by Eq. 3, 4, or 16*b*	Compute S by Eq. 3
3	Find α by referring to Section 5	Compute R by Eq. 4 or 16*b*
4	Find c by referring to Section 6	In Section 5, find α corresponding to $\phi = 0.5\Delta$
5	Compute $\theta = \Delta - 2\,\Phi$	Find c by referring to Section 6
6	Compute T by Eq. 18	Compute T by Eq. 21*a*

It must not be thought that there is only the one transition spiral suitable for a given velocity V and intersection angle Δ. On the contrary, because of the freedom of choice in selecting the limits ρ and γ, there are any number of permissible spirals. For example, in case 1, it is not necessary for the vehicle actually to achieve the maximum centrifugal ratio. By reducing ρ, the radius and the central angle of the central circular section are increased, and the lengths of the bounding spirals are reduced. In both cases γ can be reduced, thus increasing the total length S. As a matter of fact, in the three formulas, Eqs. 16, there are six variables, Φ, ρ, γ, V, R, and S. This means that any three of them may be chosen arbitrarily (within practical limits) and the other three thereby determined. In forming tables of curve data it may be considered convenient to fix, say, V, S, and R. For a given V and an accompanying variety of suitable values of S and R the designer would thus be able to give data for a number of allowable spirals for each degree of Δ. This has been done, for example, by Mr. Barnett (2*a*).

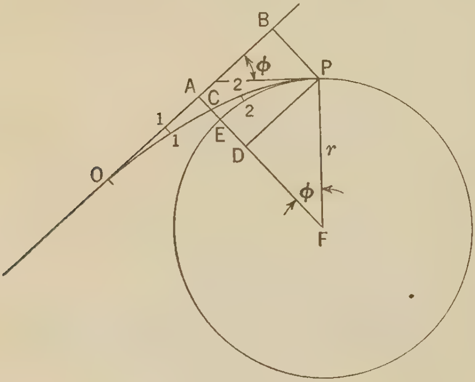


Fig. 3

10. Two Useful Relations

Let O (Fig. 3) be the origin and P any point of the spiral, and let F be the center of the osculating circle for P. Let the perpendicular FA from F to the tangent line OA cut the circle at point E and the spiral at point C. Denote length AE by σ and length OA by τ. Let D be the base of the perpendicular from P on FA and B the base of the perpendicular from P on OA (see Fig. 3). Then, $\sigma = \overline{\mathrm{BP}} - \overline{\mathrm{ED}} = y - r\,(1 - \cos\phi) = K\,\phi^{1/2}\left(\dfrac{\phi}{3} - \dfrac{\phi^3}{42} + \cdots\right)$

$$-r\left(\frac{\phi^2}{2}-\frac{\phi^4}{24}+\cdots\right) = s\left(\frac{s^2}{3\,K^2}-\frac{s^6}{42\,K^6}+\cdots\right)-\left(\frac{K^2}{2\,s}\right)\left(\frac{s^4}{2\,K^4}-\frac{s^8}{24\,K^8}+\cdots\right),$$

since $\phi = \frac{s^2}{K^2}$ and $K^2 = 2\,r\,s$. Finally

$$\sigma = s\left(\frac{s^2}{12\,K^2}-\frac{s^6}{336\,K^6}+\cdots\right)\ldots\ldots\ldots\ldots\ldots(22a)$$

Similarly, $\tau = \overline{\text{OB}} - \overline{\text{DP}} = x - r\sin\phi = K\,\phi^{1/2}\left(1-\frac{\phi^2}{10}+\ldots\right)$

$$-r\left(\phi-\frac{\phi^2}{6}+\cdots\right) = s\left(1-\frac{s^4}{10\,K^4}+\cdots\right)-\frac{K^2}{2\,s}\left(\frac{s^2}{K^2}-\frac{s^4}{6\,K^4}+\cdots\right).$$

Finally,

$$\tau = s\left(\frac{1}{2}-\frac{s^4}{60\,K^4}+\cdots\right)\ldots\ldots\ldots\ldots\ldots\ldots(22b)$$

11. APPROXIMATE RELATIONS GOOD FOR SPIRALS OF SMALL DEFLECTION

There are a number of approximate relations that apply for small values of ϕ, and, within their ranges of negligible error, are extremely useful for both computation and field work. In this section, a few of these relations are developed. Nine such approximations are presented, as follows:

1. For $\phi < 6°$, offsets from the initial tangent vary approximately as the cubes of the distances along the tangent;
2. Angle α is approximately equal to one third of ϕ; that is, $\alpha \approx \frac{\phi}{3}$;
3. Angle α varies approximately as the square of s;
4. For $\phi < 10°$, the tangent distance τ is approximately equal to $0.5\,x$; that is, $\tau \approx 0.5\,x \approx 0.5\,s$;
5. For $\phi < 10°$, the tangential offset AC, Fig. 3, is equal approximately to $0.5\,\sigma$;
6. When an upper limit ρ is developed, $\sigma \approx \frac{g^3\,\rho^3}{24\,\gamma^2}$, which is a constant;
7. The tangent distance T is equal approximately to $0.5\,S + (R + \sigma)\tan 0.5\,\Delta$;
8. The external radial distance E, from the point of intersection of two tangents to the curve, is approximately equal to $(R + \sigma)\sec 0.5\,\Delta - R$; and
9. Offset distances (1-1, Fig. 3) from the tangent to the spiral are approximately equal to the distance (2-2, Fig. 3) from the spiral to the tangent osculating circles, at equal distances from the opposite ends of a spiral.

Approximation 1.—For $\phi < 6°$, offsets from the initial tangent vary approximately as the cubes of the distances along the tangent. By Eqs. 6, $y \approx \frac{1}{3}\,K\,\phi^{3/2}$ and $x \approx K\,\phi^{1/2}$. Eliminating ϕ:

$$y \approx \frac{x^3}{3\,K^2}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(23)$$

Thus, for small deflection angles, the transition spiral is approximated by a cubic parabola. The cubic parabola can be used successfully as an easement curve (6) for deflections up to about 8.5°, because all cubic parabolas reach their maximum curvature at $\phi = \text{arc tan } \frac{\sqrt{5}}{15} \approx 8.5°$.

Approximation 2.—Eq. 8, Section 5, indicates that when $\phi = 15°$ the error in assuming $\alpha \approx \frac{\phi}{3}$ is about 10″ (see list of C-values in Section 5).

Approximation 3.—Angle α varies approximately as the square of s because $\phi = \frac{s^2}{K^2}$; and therefore $\alpha \approx \frac{s^2}{3 K^2}$.

The formula for the lemniscate (see Fig. 4) is

$$c^2 = m^2 \sin 2\, \alpha \quad \text{.......................(24)}$$

in which m is the value of c at the midpoint of the lobe. For this curve the angle α equals exactly $\frac{\phi}{3}$. A special study of the lemniscate was made by Mr. Royal-Dawson (22). It can be shown that for the lemniscate $c\, r$, and not $s\, r$, is a constant. The objections to the lemniscate seem to be that s bears no simple relationship to any of the angles involved and hence the curve does not lend itself readily to calculations concerning s and r. Moreover, it is not a simple matter to lay out a lemniscate from a point on itself.

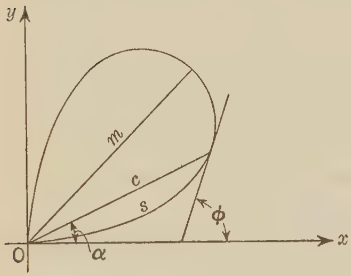


FIG. 4

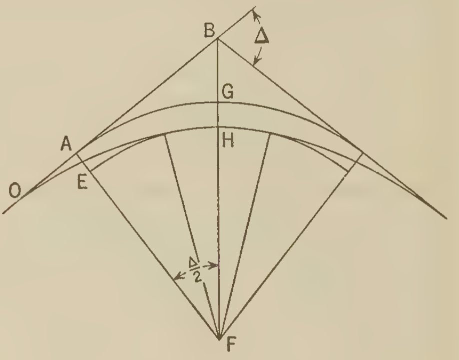


FIG. 5

Approximation 4.—For $\phi < 10°$, the tangent distance $\tau \approx 0.5\, x \approx 0.5\, s$ as can be concluded from Eq. 22*b*.

Approximation 5.—For $\phi < 10°$, the distance AC $\approx 0.5\, \sigma$ because (see Section 10):

$$y = s \left(\frac{s^2}{3 K^2} - \frac{s^6}{42 K^6} + \cdots \right) \quad \text{................(25)}$$

Therefore,

$$\overline{\text{AC}} = \frac{s}{2} \left(\frac{s^2}{12 K^2} - \frac{s^6}{2{,}688 K^6} + \cdots \right) \approx \frac{\sigma}{2} \quad \text{............(26)}$$

Thus, by approximations 4 and 5, it is clear that s and σ bisect each other approximately when ϕ is small.

Approximation 6.—When ρ is developed, $\sigma \approx \frac{g^3\,\rho^3}{24\,\gamma^2}$ (which is a constant) because (see Eq. 26) $\frac{\sigma}{2} \approx \overline{\mathrm{AC}} \approx \frac{\overline{\mathrm{BP}}}{8} = \frac{y}{8}$. Therefore,

$$\sigma \approx \frac{y}{4} \approx \frac{s^3}{12\,K^2} = \frac{s^2}{24\,r} = \frac{g^3\,\rho^3}{24\,\gamma^2} \qquad (27)$$

Approximation 7.—From Fig. 5,

$$T = \overline{\mathrm{OA}} + \overline{\mathrm{AB}} = \tau + (R + \sigma)\tan 0.5\,\Delta \approx 0.5\,S + (R + \sigma)\tan 0.5\,\Delta \qquad (28)$$

Approximation 8.—In Fig. 5, $\overline{\mathrm{BH}} = E$, $\overline{\mathrm{HF}} = R$, and $\overline{\mathrm{AE}} = \sigma$. Then, by geometry:

$$E = (R + \sigma)\sec 0.5\,\Delta - R \qquad (29)$$

Approximation 9.—In Fig. 3, let 1-1 be an offset from the tangent (perpendicular to the tangent) and let 2-2 be an offset from the osculating circle (normal to the osculating circle), and further assume that spiral arc O1 equals spiral arc P2. Then 1-1 ≈ 2-2, because, from Section 3, $\frac{d\phi}{ds} = k\,s$, whence $\frac{d^2\phi}{ds^2} = k$. Thus, the arc rate of change of the curvature for the spiral is constant; and the spiral departs from any osculating circle at the same arc rate as from the initial tangent. Hence, if ϕ is small, 1-1 ≈ 2-2. This property is convenient when using the method of deflection angles and the entire spiral cannot be set out from the origin, necessitating moving up along the spiral (21).

12. STAKING THE TRANSITION SPIRAL

There are three field methods establishing intermediate points on the spiral—the method of deflection angles, the method of tangential offsets, and the method of middle ordinates. Of these the first is the one most generally used in highway work and this section will be devoted to a brief examination of the method. For the remaining two procedures the reader is referred to the work of the late G. W. Pickels and C. C. Wiley, Members, Am. Soc. C. E. (21).

The problem is to lay out the spiral by polar deflection angles and constant chord lengths. The constant chord length will be referred to as the "unit chord." This unit chord must approximate closely the corresponding arc length, and should therefore not exceed 0.1 R. Denoting the unit chord by $\bar{s}$ and its deflection angle by $\bar{\phi}$, since ϕ varies as s^2,

$$\bar{s} = S\sqrt{\frac{\bar{\phi}}{\Phi}} \qquad (30)$$

To determine $\bar{s}$, set ϕ in Eq. 30 equal to 3′, 12′, 27′, or 48′, selecting that value which makes $\bar{s}$ of most suitable length. Having thus chosen $\bar{s}$ and $\bar{\alpha} = \bar{\phi}/3$, proceed to set out the spiral by polar deflection angles. In Table 1, the polar deflection angles have been computed by the fact that ϕ varies as

s^2 and that (see Eq. 8) $\alpha = \phi/3 - C$, and then recorded to the nearest 0° 0′ 20″, this being the limit of graduation of instruments in normal use (5).

It may be considered desirable to divide the spiral into a number of equal chords and then to set out the points from the formula,

$$\bar{\alpha} \approx \alpha \left(\frac{s}{S} \right)^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (31)$$

which is sufficiently accurate for values of ϕ to about 18°. Extensive tables, corrected for the approximation, have been prepared for this method (2), yielding the so-called ten-chord and twenty-chord spirals, ten or twenty usually being the adopted number of equal chords.

Both of these methods are open to the objection that the points fixed do not coincide with the even chain lengths along the curve; but in the absence of complete tables for the spiral the saving of time and computation is considered worth while.

TABLE 1.—POLAR DEFLECTION ANGLES FOR STAKING A SPIRAL BY CONSTANT CHORD LENGTHS

Values of α:[a]				Polar deflection
0° 1′	0° 4′	0° 9′	0° 16′	
1	....	....	....	00° 01′ 00″
2	1	....	....	00° 04′ 00″
3	....	1	....	00° 09′ 00″
4	2	....	1	00° 16′ 00″
5	....	....	....	00° 25′ 00″
6	3	2	....	00° 36′ 00″
7	....	....	....	00° 49′ 00″
8	4	....	2	01° 04′ 00″
9	....	3	....	01° 21′ 00″
10	5	....	....	01° 40′ 00″
11	....	....	....	02° 01′ 00″
12	6	4	3	02° 24′ 00″
13	....	....	....	02° 49′ 00″
14	7	....	....	03° 16′ 00″
15	....	5	....	03° 45′ 00″
16	8	....	4	04° 16′ 00″
17	....	....	....	04° 48′ 40″
18	9	6	....	05° 23′ 40″
19	....	....	....	06° 00′ 40″
20	10	....	5	06° 39′ 40″
21	....	7	....	07° 20′ 20″
22	11	....	....	08° 03′ 20″
23	....	....	....	08° 48′ 00″
24	12	8	6	09° 34′ 40″
25	....	....	....	10° 23′ 40″
26	13	....	....	11° 14′ 00″
27	....	9	....	12° 06′ 40″
28	14	....	7	13° 01′ 00″
29	....	....	....	13° 57′ 20″
30	15	10	....	14° 55′ 20″
31	....	....	....	15° 55′ 20″
32	16	....	8	16° 57′ 20″
33	....	11	....	18° 01′ 00″
34	17	....	....	19° 06′ 20″

[a] Number of unit chord lengths from the beginning of the curve, for the values of α shown.

13. DEGREE OF CURVATURE

Without entering into the arguments for or against (23) the use of the idea of "degree of curvature," it remains a fact that it is the present American practice to treat curves in terms of this concept; hence a word or two will be offered in this section relating the equations of the previous sections to those involving the degree of curvature. Many definitions of degree of curvature exist, but that which seems most acceptable at present is the following (21):

Definition 1.—The degree of curvature, d, of a circular curve is the central angle of an arc of 100.007 ft.

Definition 2.—Extending Definition 1, the degree of curvature, d, of a transition spiral at any point is the degree of curvature of its osculating circle at that point. The final degree of curvature of the spiral is D.

Definition 3.—Let κ represent the rate of change of the degree of curvature of the transition spiral per 100 ft of arc length. The spiral can then be classified as a $\kappa°$-spiral (21). Relations connecting r and d, K and κ may be found as follows: The circumference of a circle equals $2\,\pi\,r = \dfrac{100.007 \times 360}{d}$; whence,

$$r = \frac{100.007 \times 360}{2\,\pi\,d} = \frac{5{,}730}{d} \quad\ldots\ldots\ldots\ldots\ldots\ldots(32)$$

From Eq. 4, $2\,r\,s = K^2$; and $\dfrac{2\,(5{,}730)\,s}{d} = K^2$. Making $s = 100$ ft and $d = \kappa$:

$$\kappa = \frac{200 \times 5{,}730}{K^2} = 100 \times 5{,}730\,k \quad\ldots\ldots\ldots\ldots\ldots(33)$$

By use of Eqs. 32 and 33, r (or R) and K can be substituted in any of the formulas of this paper by d (or D) and κ. Messrs. Pickels and Wiley (21) have presented a treatment along these lines.

14. Conclusion

One of the many problems encountered in highway engineering is that of curve construction and the consequent superelevation. The staking of the superelevation on unspiraled curves was a hit-or-miss affair, placing most of the burden on the motorist. In addition to speed, considerations of safety and comfort must also be treated attentively in curve design. The answer to the problem is a continuous spiral whenever that is possible. In determining when a curve may be transitional throughout it became necessary for the writers to examine the mathematical theory of the transition spiral. The first step was to examine the existing readily obtained literature, where it was found that the mathematical theory of the spiral was often obscured for one or more of the following reasons:

1. Although numerous formulas were offered, often no proof, or very little proof, was given;
2. Comfort and safety requirements were not sufficiently considered;
3. Generality was lost because field men were soon weaving into formulas specially adopted or favored values for certain parameters (for example, numerical values of ρ and γ might be adopted early in the treatment with the result that the significance of these parameters is lost forever in the sequel);
4. In an effort to be practical the essential mathematics was obscured by the mathematically extraneous idea of "degree of curvature";
5. Confusion and inconsistency were created by neglecting to make clear which mathematical properties of the curve are exact and which are approximate, or by neglecting to inform the reader to what extent a formula is approximate;
6. Often the mathematics was developed with the principal object of explaining or developing a particular set of tables, or the treatment was otherwise (purposefully, perhaps) one-sided; and

7. In a few cases (British), the treatment was too massive or exhaustive.

Perhaps this paper obviates some of these faults and will better serve the growing number of persons who need a clear and concise introduction to the fundamental principles of the curve.

Historically, the transition spiral was first investigated by the Swiss mathematician, Leonard Euler, and seems to be a mathematically rich curve. R. C. Archibald has presented an excellent historical sketch concerning the curve and its earlier applications to the study of elastic springs and to the theory of optics (35). An amazingly rich bibliography of the mathematical development of the properties of this curve is given. There is no duplication between this bibliography and the remaining items in the Appendix. In all likelihood the Euler curve possesses some still unknown properties (exact or approximate) which perhaps could be valuable to the engineer. Who knows, for example, how well the engineer might improve on the present osculating circle method of laying out the spiral when he faces the need for moving up along the curve (15)? The mathematical pole of the spiral can be found easily; can it be used? What new problems will the spiral present? Will there be any need for a rational method of applying vertical transition at the ends of the spiral? How can the engineer best devise such a method? Can he improve upon existing methods of describing right of ways for these curves (8)?

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DISCUSSION

Joseph Barnett,[4] M. Am. Soc. C. E.—In several respects this paper is valuable, and not the least of these is its importance in calling the attention of highway engineers to a neglected phase of geometric highway design. Despite the surprisingly large bibliography, for which engineers are indebted to the authors, the use of transition approaches to curves on highways has not been generally accepted even though greater safety, smoother riding, and a more graceful alinement are attained without extra cost. Only about half the state highway departments now use transitions in highway alinements.

Among the reasons for this are reluctance to change existing practices and to revise standards; a belief that a vehicle using the highway is not confined to a track and is able to effect the change from a straight to a circular motion smoothly; a belief that widening at a sharp curve together with the gradual attainment of this widening accomplishes a satisfactory transition; and a belief that transitions involve tedious calculations both in the office and field, resulting in increased costs of engineering. There has also been considerable confusion as to what factors should determine the minimum length of a transition.

Some of these reasons were valid when road speeds were slower than at present but they do not apply now. At moderate speeds the average driver can traverse a satisfactory short transition path within the limits of a vehicle lane, but with increased speeds longer transitions are required. These can be traversed only by hazardous crowding or occupation of adjoining lanes. The increase in the speeds at which vehicles are operated, the desire for increased speed with safety, and the mounting accident toll make it imperative that highways be constructed in such manner that a driver traveling at the safe speed for which the highway is designed will not only find it possible to confine his vehicle to the occupied lane but will be encouraged to do so.

A discussion of the highway transition spiral might be divided into two general sections—theoretical and practical. Too much already has been written regarding theory. Several forms of curve have been advocated. The form used by the authors is that in which the radius varies inversely as the distance from the beginning of the transition curve. It is the same form of transition used by the late Arthur N. Talbot, Past-President and Hon. M. Am. Soc. C. E. (29), except that Professor Talbot developed a set of tables around the symbol a which represents the rate of change of degree of curve per 100 ft of length, and which equals the degree of curve at a point 100 ft from the "TS," the common point of tangent and spiral. T. F. Hickerson, M. Am. Soc. C. E. (13), used the same form of transition as the authors and gave the mathematical derivation for the coordinates x and y, but extended the mathematical derivation no further, since almost all values required for tables can be computed from x and y. W. H. Searles (25) developed a transition effected by a series of compound circular curves. In England, F. G. Royal-Dawson (23) advocated the use of the lemniscate, a form of curve which permits a certain type of field simplification. Several other forms are available. As regards riding qualities, safe speeds, or rates of change of superelevation, no one form of transition has any particular advantage over other forms.

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There have been differences of opinion regarding the length of transition which should be used in highway design. Suggested lengths have been developed by theory but unfortunately there have been differences regarding the basis on which theoretical lengths should be developed. In developing lengths of transition for railways, W. H. Shortt used an average rate at which the vehicle on the transition approaches the final constant centripetal acceleration on the circular curve (27), but the rate to be used on highways is largely a matter of opinion. In a similar approach the time to turn the steering wheel to attain a certain degree of curvilinear motion at a certain speed has been advocated as a basis for length determination. Rate of runoff (that is, the change of superelevation in a unit of length) has many advocates, but there are differences of opinion regarding both the rate of runoff and the rate of superelevation. In all these determinations the speed of the vehicle is assumed and that is the basic difference of approach, particularly between designers in England and in the United States.

In England most designers assume that the geometric features of each curve, the radius, superelevation, and length of transition, determine the speed of the vehicle, an idea which has many advocates in the United States. Under this assumption a driver will change his speed at each curve. Although scientifically controlled observations are needed for a proper determination, most engineers have observed that once the average driver escapes the influences of urban conditions that tend to force a wide range of speeds, he is inclined to settle down to a uniform speed which, in his mind, is as great as the traffic and the physical characteristics of the highway permit. The speed is seldom the maximum speed of the vehicle since almost all passenger vehicles and some buses and trucks can attain speeds greater than that considered safe on existing highways and greater than that for which the public and highway engineers deem it economically feasible to construct most highways at the present time. For purposes of design it is assumed that traffic is not dense enough to slow down a vehicle and that the steady speed adopted by the driver depends largely upon the physical characteristics of the highway.

The personal factor cannot be ignored. Some drivers choose higher speeds than others. Some passengers feel comfortable at higher speeds than others. Those physical characteristics of the highway that tend to make a fast driver slow down also tend to make a slow driver reduce speed; but an approximately uniform speed, rather than a variable speed changed at each curve, is generally the aim of all drivers.

The difference in thinking regarding speed to be used in design has led to still another difference—one which relates to the use of a curve transitional throughout. The idea that each curve is traversed at its maximum permissible speed leads to use of curves transitional throughout for the smaller values of the intersection angle, Δ, and has been advocated chiefly in England. Highway designers in the United States incline to the use of two transitions and a central circular curve each taking up about a third of the central angle.

These differences can be reconciled only by the application of the results of tests of speeds on highways generally, tests of speed in relation to curvature and superelevation, and tests of driving comfort around curves. Results of tests on speed and comfort have been reported by D. W. Loutzenheiser (36), O. K. Normann (37)(38), and A. Taragin (39). The tests by John J. Leeming and A. N. Black (40) constitute one of the few attempts to measure comfort and driving practice around curves and, although they leave much to be desired to make them completely scientific and conclusive, they point the way toward conclusions which the writer feels are significant and which he has assumed for many years as the result of sheer logic. In their conclusions, Messrs. Leeming and Black state:

> "* * * it is tentatively suggested that curves should be designed to have about two-thirds of their length transitional, which would mean that the transition angle would be about one-quarter of the deviation angle. Curves designed according to this rule would then conform to what appears to be the average practice of drivers. As far as the information goes at present, it does not appear that any better compromise is available. As, however, the length of transition is not very important, it will be amply good enough for all practical purposes to make the transition angle as near as convenient to one-quarter of the deviation angle. In general, transitions should be short rather than long, and the main object should be simplicity and ease of layout and construction."

It is evident that precision in the form or length of transition curve is not necessary. The differences of opinion regarding the factors that determine length and type (particularly the factor of speed and its relation to curvature and superelevation) prove this. Then the problem of encouraging the adoption of transition curves for highway alinements is not one of persuading highway engineers to use a particular type of curve or particular relationship that results in a particular length, but in persuading engineers to use some kind of a transition curve. The mathematics presented by the authors should be simple to any one who has completed the average college course in calculus, or even advanced algebra; but it will "scare the shirt off" many surveyors on highway work. In addition the calculations must be made frequently in the field and supervisors are discouraged from adopting any procedure that requires a survey party to await the results of tedious calculations.

The control on most highway alinement problems, once the deviation angle Δ is measured, is either the tangent distance, T, or the external distance, E, from the point of intersection to the center of the curve, or both. If the common practice is followed of first designing and locating a circular curve and later adjusting the curve for inclusion of transition approaches, it results in odd values of the degree of curve and length of transition, and tedious calculations are necessary. If the uselessness of precision in length of transition is recognized, however, tables can be prepared which give the tangent distance T_s for curves with transitions, and the external distance E_s for curves with transitions, directly, and it becomes as easy with such tables to design and locate curves with transitions as to design and locate simple curves, provided the curves are designed initially as curves with transitions. Such tables, referred to by the authors (2), were prepared by the writer in 1938.

Several years of experience with these tables have revealed wherein they have proved their worth and where there are shortcomings. Where they have been tried on actual work and the designer or locator has jumped the hurdle of the explanatory mathematics which was kept to the minimum, no difficulty was encountered. In fact, the writer now advises that initiates omit the text and proceed to actual trial using the numerical examples to learn the simple operations. The choice of lengths of transition is limited to multiples of 50 ft and the degrees of curvature to even degrees, in some cases half degrees, so that except for straight-line interpolation no calculations are necessary that cannot be made, for all practical purposes, mentally; and, for every combination of the length of spiral L_s and the degree of curve D_c additional tables give all salient features of the transition curve including deflection angles to points on the transition divided into ten equal lengths.

Incidentally, although mathematicians do not take kindly to the concept of the degree of curvature used by the writer, it is a mighty handy tool to the man who must make calculations quickly. The length of arc subtending a central angle to be known as the degree of curve should be an even 100 ft and not 100.007 ft which the authors apparently chose in order to make the value representing the relationship between radius and degree of curve come out even. The necessity for having this value even is obscure. However, the use of radius instead of degree of curve does not minimize the usefulness of the type of tables proposed by the writer. In their Spanish translation in 1941, M. San Miguel, E. F. Tagle, and A. J. L. Bolognesi used the radius of curve in recomputing the writer's tables (2) to the metric system.

The principal shortcomings revealed by use of the tables in "Transition Curves for Highways" (2) are two in number: (*a*) For the larger values of Δ small values of D_c will seldom be used and corresponding values of T_s and E_s are unnecessary. It would be better to devote the limited space to giving a greater choice of the larger values of D_c; that is, values corresponding to degrees and half degrees instead of degrees only; and (*b*) for the very large values of Δ used in mountainous areas, and sometimes in turnouts, values for L_s = 100 ft and possibly values for sharper curvatures should be added.

RALPH W. STEWART,[5] M. AM. SOC. C. E.—As a mathematical examination of the transition spiral this paper is very thorough and complete. The bibliography, although unusually well prepared, omits what is probably the first presentation for practical use in America of the mathematics of the true transition spiral whose curvature uniformly increases from point to point. This was a paper (41) by the late Elliot Holbrook, M. Am. Soc. C. E., which appeared in 1880. The same mathematical procedure was used by A. N. Talbot, Past-President and Hon. M. Am. Soc. C. E. (42), in a paper published in 1891, later to be included in Professor Talbot's book, "The Railway Transition Spiral" (29). In 1899, F. K. Vial used the law of constant increase in curvature of the transition spiral to devise a universal table of coefficients for deflections for laying out the curve. By the use of this table of coefficients (43) (29*a*), an instrument could be set at any chord point of any spiral which was divided into uniform chord lengths and the deflections to all other chord points could be computed by multiplying the spiral constant by the series of coefficients.

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There were, in former years, many engineers who boycotted such mathematical procedures as the expansion of a function into an infinite series of terms by Maclaurin's theorem or otherwise, and who were annoyed by formulas such as Eqs. 10 and 11. This led to the use of the curve composed of a series of finite circular arcs whose properties could be computed by trigonometry.

The author states that he does not propose to enter into arguments favoring any one type of easement. The writer, however, wishes to call attention to the disadvantages (discovered through actual field experience) of the curve composed of a series of finite circular arcs; and also to show how to lay out a true spiral curve which, for all practical purposes, will coincide with a multi-arc curve.

The chief disadvantage of the multi-arc curve is that, with the transit set at the beginning of the first arc, which is called the beginning of the curve, accurate deflections are known only to the points where the arcs are compounded. If it is necessary to set points for the bents of a trestle bridge, a common procedure is first to set stakes at the compound points of the arcs and then move the transit from compound point to compound point to set the intermediate points at the bent locations. This and similar situations are not rare. It is sometimes necessary to set points that do not coincide with the compound points in order to investigate side clearances and to locate track crossings or other intersecting lines. The accumulation of all such incidents is quite appreciable. At one time the writer had to lay out the points of a multi-compound-arc spiral on the floor beams of a high steel bridge which was under construction. Not knowing at that time how to run the equivalent true spiral it was necessary for the bridge construction crew to lay temporary plank platforms at the compound points of the curve.

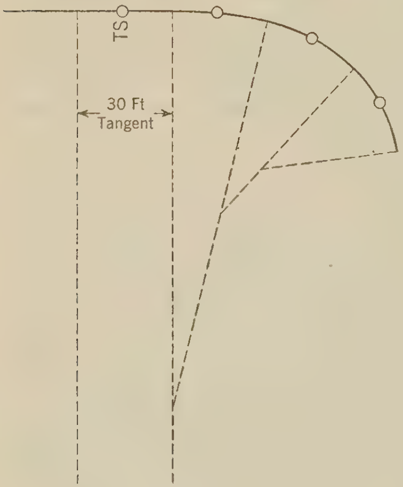

Fig. 6.

The true spiral, which (for all practical purposes) will coincide with a multi-arc substitute curve, can be determined easily. It is only necessary to lay out one arc length of tangent behind the beginning of the first arc. The center point of this tangent and the center points of the consecutive arcs will then define a true spiral of ample accuracy for practical use which will have the same rate of increase in curvature per unit arc length as the multi-compound substitute curve.

This is best illustrated by a numerical example. Let Fig. 6 represent a compound curve composed of arcs each 30 ft long. Let the first arc be a 0° 30 curve, the second a 1° curve, etc. Note that this provides for an increase in the rate of curvature of 0° 30′ in 30 ft or 1′ per ft. The radial lines in the figure indicate the ends of the arcs. The small circles indicate the centers of the arcs, and also the center point, marked TS (change from tangent to spiral), of the arc length of tangent which precedes the curve. If the accumulated curvature from TS to the center of each arc is computed, it will be found to vary as the square of the distance along the center line which is the intrinsic equation of the true spiral. It will also be found that a true spiral having a rate of curvature increase of 1′ per ft and beginning at the TS point will pass through the center points of the arcs, subject to negligible errors depending on whether chord lengths or arc lengths are measured and other causes of minute differences.

Since ample tables are available for laying out spirals, it is not necessary to cope with equations that involve an infinite series of terms.

The following analogy is useful in general engineering practice: A transition spiral represents uniform increase in rate of curvature. Uniformly accelerated motion represents uniform increase in velocity. This makes the total curvature in a spiral for any distance x along the curve equal to $0.5\ a\ x^2$, in which a is the rate of increase in curvature. This is analogous to the formula $0.5\ g\ t^2$ which represents the distance that a body falls (starting from rest) in any elapsed time t under the rate of acceleration g. Also, the elastic curve of a cantilever beam of uniform section caused by a concentrated load at its free end is a transition spiral as long as the deflection is small, because the bending moment, and consequently the elastic curvature, increase uniformly with the distance from the load.

Donald Thompson,[6] Esq.—The review of the mathematical theory of the highway transition spiral, as presented in this paper, should prove interesting and instructive to engineers designing high-speed highways. Recent publications on transition curves, notably the work of Joseph Barnett, M. Am. Soc. C. E., cited by the authors (2), have avoided an exposition of the mathematical theory on which the true transition curve is based. Thomas F. Hickerson, M. Am. Soc. C. E. (13), gives the derivation of most of the spiral formulas but omits the mathematical theory for the formula for the deflection angle.

Earlier publications on the true transition curve, however, gave the derivation of all the fundamental equations developed by Professors Eichler and Eves. In 1893 the late C. L. Crandall, M. Am. Soc. C. E. (44), developed all the formulas for the true transition curve. Professor Crandall credits the late Elliot Holbrook, M. Am. Soc. C. E., with first describing the true transition curve in which curvature increases directly with the distance (41), and developing the formulas and practice for small angles.

It is true, of course, that knowledge of the mathematical theory is not essential either for designing or for staking transition curves. With either Mr. Barnett's or Professor Hickerson's book, the use of transition curves will involve scarcely any more work than the use of circular curves. The authors' method of calculating the elements of a particular spiral by substituting in the fundamental formulas, although interesting from a mathematical standpoint, is not practicable. The spiral elements can be taken directly from tables in either of the afore-mentioned books (2) (13) in a fraction of the time consumed in substituting appropriate values in the formulas of this paper.

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The derivation of the formula for the deflection angle α in Section 5 is not complete. Eq. 7 was given as $\tan \alpha = \frac{\phi}{3} + \frac{\phi^3}{105} + \frac{\phi^5}{5{,}597}$, in which ϕ is in radians. To derive a formula for α (not $\tan \alpha$) the deflection angle must be expanded in terms of its tangent. Thus,

$$\alpha = \tan \alpha - \frac{1}{3} \tan^3 \alpha + \frac{1}{5} \tan^5 \alpha \ldots\ldots\ldots\ldots\ldots (34a)$$

Substituting Eq. 7 in Eq. 34*a*, dropping higher powers than the third, and converting to degrees, the correct formula for the deflection angle is

$$\alpha^\circ = \frac{\phi^\circ}{3} - 0.000000859\ (\phi^\circ)^3 \ldots\ldots\ldots\ldots\ldots\ldots (34b)$$

The formula for the correction C that must be subtracted from one third of any given value of ϕ in order to obtain the corresponding value of α is, therefore,

$$C = 0.000000859\ (\phi^\circ)^3 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (35a)$$

In calculating the deflection angles for a particular spiral it is necessary to express the deflection angle α in terms of the length s along the curve to the point for which the deflection is desired, the total length S of the spiral, and the total spiral angle Φ. In terms of these variables

$$\alpha = \frac{\Phi^\circ}{3} \left(\frac{s}{S} \right)^2 - 0.000000859\ (\Phi^\circ)^3 \left(\frac{s}{S} \right)^6 \ldots\ldots\ldots\ldots (35b)$$

The formula for the centrifugal ratio, Eq. 15, is given as $\frac{V^2}{g\,R} = \frac{e + f}{1 - e\,f}$ in which e is the superelevation in feet per foot, f is the coefficient of friction, R is the radius in feet, g is acceleration of gravity, and V is the velocity in feet, per second. This is the exact formula for superelevation with friction considered. It would be useful in designing a racing track where large values of e might be necessary, but it is never used in highway design. Since the maximum values of e and f permitted in the design of highways are quite small, $1 - e\,f$ will be very nearly equal to 1 and the formula for the centrifugal ratio can be simplified to

$$\frac{V^2}{g\,R} = e + f \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (36a)$$

Converting V to miles per hour, Eq. 36*a* becomes

$$\frac{0.067\ V^2}{R} = e + f \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (36b)$$

which is the approximate formula for the centrifugal ratio and the one always used in highway design. Since the approximate formula will give slightly lower velocities than the exact, it is on the side of safety.

A value of $f = 0.14$ for the unbalanced centrifugal ratio (which is mentioned by the authors in Section 7 as being the accepted standard) seems too great for comfort at high speeds. Ralph A. Moyer, Assoc. M. Am. Soc. C. E. (45), has recommended that the unbalanced centrifugal ratio f should not exceed 0.10 at 60 miles per hr. Charles M. Noble, M. Am. Soc. C. E. (46), after exhaustive high-speed tests on the Pennsylvania Turnpike, recommended that, for velocities of 70 miles per hr, f should not exceed 0.10. In designing parkways for the Bureau of Engineering of the City of Los Angeles, Calif., the writer uses a value for f which varies with the velocity. The variation is considered to be linear and, expressed as a formula, is

$$f = 0.16 - 0.001\,V \qquad (37)$$

in which V is in miles per hour. At 60 miles per hr, f would be 0.10. Using a maximum value of 0.10 for e, the maximum value of the centrifugal ratio would be 0.20. For the maximum superelevation of 0.10 the minimum radius will be $R = \frac{0.067 \times 60^2}{0.20} = 1{,}200$ ft. For curves larger than the minimum the superelevation should vary inversely as the radius so that the general formula for superelevation for a design speed of 60 miles per hr is

$$e = \frac{120}{R} \qquad (38)$$

in which e is in feet per foot, and R is in feet. For the length of the transition spiral and the superelevation transition the writer is using a value of 1.5775 ft per sec^3 for γ instead of the commonly accepted 2 ft per sec^3 which results in a value for the length of the transition of

$$S = \frac{2\,V^3}{R} \qquad (39a)$$

in which V is in miles per hour and S and R are in feet. For a design speed of 60 miles per hour, Eq. 39a can be expressed as

$$S = \frac{432{,}000}{R} \qquad (39b)$$

This length of transition will limit the difference in grade throughout the superelevation transition between the center line of a 36-ft roadway and the outside edge of a 36-ft roadway to 0.5% or 1 in 200 provided the superelevation is attained by rotating about the center line of the roadway.

It is the present practice in the City of Los Angeles to first project a circular alinement making proper allowances for external distances and lengths of tangents between reversing curves and then to insert the transition curves. With this method it becomes a very simple matter to determine whether a curve will be transitional throughout or have a remaining circular arc. If any circular arc on the projected alinement has a length equal to 432,000/R, the transition curves when inserted will take up all the circular arc and the resulting curve will be transitional throughout. If a circular curve on the original alinement has a length greater than 432,000/R, then there will be some circular arc left after the transitions have been inserted. For the sake of appearance, the minimum length of any circular curve has been set at 500 ft. Since the maximum length of transition would be 432,000/1,200 = 360 ft, occurring on the minimum radius of 1,200 ft, all curves will have a portion of a circular arc remaining and will not be transitional throughout.

In their conclusion the authors state that they hope their paper will serve as a clear and concise introduction to the fundamental principles of the true transition curve. The paper has accomplished its purpose and should stimulate interest in, and promote, the wide acceptance of the true transition curve by highway engineers.

T. F. HICKERSON,[7] M. AM. SOC. C. E.—In their mathematical examination of the highway transition spiral the authors have cited some, but not by any means all, of the contributions that have been made in adapting this curve to the layout of highways. For example, the writer has prepared a table containing the deflections from a tangent to any point on a spiral (13*a*). This table meets the objection mentioned by the authors, in their description of two methods of staking the spiral (Section 12), that "the points fixed do not coincide with the even chain lengths along the curve * * * ." In connection with the curve that is transitional throughout, the writer prepared a table (13*b*) giving "Tangents and Externals to the Unit Double-Spiral Curve of Length 100 Ft."

Knowing the intersection angle Δ and the actual length of this curve which is transitional throughout, the tangents and externals can be written at once. Thus, if $\Delta = 40°$ and the length of each spiral is 600 ft (total length of curve 1,200 ft), then $T_s = 102.987 \times 6 = 617.92$ ft and $E_s = 12.275 \times 6 = 73.65$ ft. These tabular values of T_s and E_s for the unit curve were computed by the writer from the formulas:

$$T_s = T + k + p \tan 0.5\,\Delta \qquad (40a)$$

and

$$E_s = E + p \sec 0.5\,\Delta \qquad (40b)$$

in which T and E are the tangents and external distances for the osculating circular curve;

$$k = x_c - R_c \sin \phi_c \qquad (41a)$$

and

$$p = y_c - R_c \operatorname{vers} \phi_c \qquad (41b)$$

In Eqs. 41 the subscripts c refer to point P (Fig. 1) at the end of the spiral where it coincides with the circle.

Eqs. 40 are more simple than Eqs. 21 presented by the authors inasmuch as the data required for their solution are already known to a high degree of accuracy (13*c*). The tabular values of T_s and E_s have been checked by the writer by means of the following independent pairs of formulas:

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$$T_s = x_c + y_c \tan \phi \dots\dots (42a)$$

$$T_s = c\,(\cos \alpha + \sin \alpha \tan \phi) \dots\dots (42b)$$

$$E_s = y_c \sec \phi \dots\dots (43a)$$

$$E_s = c \sin \alpha \sec \phi \dots\dots (43b)$$

The values of x and y for any point of the spiral are:

$$x = s\left(1 - \frac{\phi^2}{10} + \frac{\phi^4}{216} - \frac{\phi^6}{9{,}360} + \cdots\right) \dots\dots (44a)$$

and

$$y = s\left(\frac{\phi}{3} - \frac{\phi^3}{42} + \frac{\phi^5}{1{,}320} - \frac{\phi^7}{75{,}600}\right) \dots\dots (44b)$$

in which ϕ is measured in radians.

Expressing ϕ in degrees, Eqs. 44 become:

$$x = s\,[1 - 0.304617\,(10)^{-4}\,\phi^2 + 0.429592\,(10)^{-9}\,\phi^4 - 0.301987\,(10)^{-14}\,\phi^6 + \cdots] \dots\dots (45a)$$

and

$$y = s\,[0.581776\,(10)^{-2}\,\phi - 0.126585\,(10)^{-6}\,\phi^3 + 0.122691\,(10)^{-11}\,\phi^5 - 0.65256\,(10)^{-17}\,\phi^7 + \cdots] \dots\dots (45b)$$

Calculating machines and eight-place tables of logarithms and trigonometric functions are needed to compute the tabular values correct to the sixth significant figure. Such accuracy would establish the parts of a spiral to the nearest hundredth of a foot even when the length of the spiral is 1,000 ft. Now that these tables have been compiled and checked independently by different parties it is to be hoped that engineers will use them in good faith just as they do tables of logarithms.

The authors brought up a good point in stating that there are any number of permissible spirals suitable for a given velocity V and intersection angle Δ. In Eq. 16b, if $\gamma = 2 \frac{\text{ft}}{\text{sec}^3}$ (a safe limiting value) and if $V \frac{\text{ft}}{\text{sec}}$ is expressed as $v \frac{\text{miles}}{\text{hr}}$:

$$S = \frac{1.58\,v^3}{R} \dots\dots (46)$$

Eq. 46 is often misinterpreted. It gives the minimum permissible length of transition curve that may be used without exceeding the maximum allowable rate of change of acceleration while driving a car from a straight path to the full curvilinear path.

Assume that $v = 60$ miles per hr; then if $R = 2{,}000$ ft, $S = 170.6$ ft; and if $R = 4{,}000$ ft, $S = 85.3$ ft. These values of S are strictly according to the arbitrary formula, but if a much longer transition is feasible the change from a straight to a curved path would be all the easier and the general appearance of the curve all the more improved.

ALLEN H. BROWNFIELD,[8] ASSOC. M. AM. SOC. C. E.—This paper and its extensive bibliography should prove quite useful in highway design for establishing departmental standards involving transition curves. In addition to the authorities cited previously the following should be included: R. C. Archibald (47), the late D. E. Hughes, M. Am. Soc. C. E. (48) (49), the late William Cain, M. Am. Soc. C. E. (50), the late C. L. Crandall, M. Am. Soc. C. E. (44), the late S. W. Robinson, M. Am. Soc. C. E. (51), who cites the work of the late Elliot Holbrook, M. Am. Soc. C. E. (41), Southern Pacific Railroad Company (52), J. C. Nagle (53), who cites the work of Mr. Crandall (44), the late W. B. Lee, M. Am. Soc. C. E. (54), John H. Fletcher (55), the late N. B. Kellogg, M. Am. Soc. C. E. (56), A. L. Higgins (57), H. Criswell (58), and William T. Pryor, Assoc. M. Am. Soc. C. E. (59). Each of these articles deals directly with Euler's spiral (sometimes referred to as the American spiral) and will help to trace its development in the United States.

A complete treatment of the Euler spiral will involve three steps: (*a*) A discussion of the background, as has been done so ably by the authors; (*b*) analyses by tables or graphs showing the authors' recommendations regarding length of curve and superelevation (also a more extensive list of useful formulas correlating all parts of the spiral to be used by highway planners); and (*c*) solutions of numerical examples for field and office use in staking this curve.

There seems to be considerable hesitancy on the part of some highway departments to adopt the Euler spiral. Maybe this is due to the apparent complexity of the formulas and difficulty in finding the proper tables. Several of the references have quite extensive tables and one of these is a seven-place table of natural functions at 3-min intervals.

From the following it should be clear that the spiral, with tables, is nearly as simple as a circular curve, and more simple than a compounded curve. Evidently the circle would be difficult without extensive tables of natural and logarithmic functions.

Since most of the spiral tables are developed in terms of s and ϕ, it is convenient to compare them with a circle using the x-axis along a tangent, as follows:

Dimension	Circle (Fig. 7(*a*))	Euler's spiral (Fig. 7(*b*))
s	$r\,\phi$	$2\,R\,\phi$
y	$s\,\frac{\text{vers}\,\phi}{\phi}$	$s\,\text{vers}_s\,\phi$
x	$s\,\frac{\sin\phi}{\phi}$	$s\,\sin_s\phi$
c	$\frac{2\,s}{\phi}\sin\frac{\phi}{2}$	$s\,f(\phi)$
t (and t_1)	$\frac{s}{\phi}\tan\frac{\phi}{2}$	$s\,\tan_s\phi$
α	$\frac{\phi}{2}$	$\frac{\phi}{3}$ − correction

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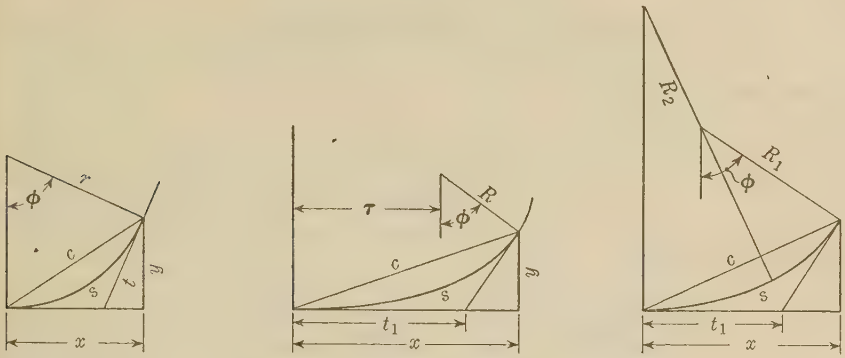
The terms $\sin_s$, vers_s, and $\tan_s$ are introduced to show the parallelism and are defined as "sine spiral," "versine spiral," and "tangent spiral," respectively. They are similar to the circular functions of $\frac{\sin \phi}{\phi}$, etc.



(a) CIRCLE (b) EULER'S SPIRAL (c) COMPOUND CIRCLE

FIG. 7

From an examination of Figs. 7(*b*) and 7(*c*) it is evident that the tables for Euler's spiral are faster to use than those for a compound circle.

In the following discussion the writer has not included the derivations as they may detract from, rather than enhance, the use of spirals.

Apparently the authors followed the same procedure as F. G. Royal-Dawson (23) in deriving Eq. 7. If the division is made as indicated in the foregoing a formula is evolved which is nearly identical to Eq. 7:

$$\tan \alpha = \frac{y}{x} = \frac{\phi}{3} + \frac{\phi^3}{105} + \frac{26\,\phi^5}{155{,}925} - \frac{17\,\phi^7}{3{,}378{,}375} - \cdots\cdots\cdots\cdots \quad (47)$$

or

$$\alpha = \frac{\phi}{3} - c = \frac{\phi}{3} - \frac{8\,\phi^3}{2{,}835} - \frac{32\,\phi^5}{467{,}775} - \frac{128\,\phi^7}{83{,}284{,}288} - \cdots\cdots\cdots \quad (48)$$

The last term of Eq. 48 is nearly correct; thus:

$$\phi = 3\,\alpha + \frac{8}{35}\,\alpha^3 + \frac{1{,}376}{13{,}475}\,\alpha^5 + \frac{207{,}488}{3{,}678{,}675}\,\alpha^7 + \cdots\cdots\cdots\cdots \quad (49)$$

and

$$c = s\left(1 - \frac{2\,\phi^2}{45} + \frac{2\,\phi^4}{2{,}835} - \frac{316\,\phi^6}{127{,}702{,}575} + \cdots\right)\cdots\cdots\cdots \quad (50)$$

Solution of Eqs. 7 to 13 will seldom be needed in practice since all values necessary for their solution have been tabulated by several authors.

The writer has found the following equations useful on several occasions (one or two terms will usually suffice). Let σ equal the offset AE in Fig. 3; then:

$$n = \frac{6\,\sigma}{r}\text{; and } \phi = \sqrt{n + \frac{n^2}{28} + \frac{29\,n^3}{16{,}170} + \frac{1{,}391\,n^4}{13{,}582{,}800} + \cdots}\cdots\cdots \quad (51)$$

The area under the curve between s, x, and y equals

$$A = s^2 \left(\frac{\phi}{12} - \frac{\phi^3}{42} + \frac{23\,\phi^5}{10{,}395} - \frac{2\,\phi^7}{17{,}325} + \cdots \right) \ldots\ldots\ldots\ldots (52)$$

In Fig. 2(b),

$$T = s\left(1 + \frac{7\,\phi^2}{30} + \frac{139\,\phi^4}{1{,}512} + \frac{8{,}927\,\phi^6}{240{,}240} + \cdots\right) \ldots\ldots\ldots\ldots (53)$$

As compared with Eq. 22b,

$$\tau = s\left(\tfrac{1}{2} - \frac{\phi^2}{60} + \frac{\phi^4}{2{,}160} - \cdots\right) \ldots\ldots\ldots\ldots (54a)$$

or

$$\tau \approx \frac{2\,s + x}{6} \ldots\ldots\ldots\ldots (54b)$$

Similarly (see Eq. 22a),

$$\sigma = \frac{s}{2}\left(\frac{\phi}{3(2!)} - \frac{\phi^3}{7(4!)} + \frac{\phi^5}{11(6!)} - \frac{\phi^7}{15(8!)} + \cdots\right) \ldots\ldots\ldots\ldots (55)$$

Many simple and useful relations can be derived from the following, some of which have been published by Mr. Hughes (48) (49):

$$s^2_2\,\phi_1 = s^2_1\,\phi_2 \ldots\ldots\ldots\ldots (56a)$$

$$R^2_1\,\phi_1 = R^2_2\,\phi_2 \ldots\ldots\ldots\ldots (56b)$$

$$R_1\,s_1 = R_2\,s_2 = \text{constant} \ldots\ldots\ldots\ldots (56c)$$

Typical formulas for s are:

$$s = x\left(1 + \frac{\phi^2}{10} + \frac{29\,\phi^4}{5{,}400} + \frac{127\,\phi^6}{702{,}000} + \cdots\right) \ldots\ldots\ldots\ldots (57a)$$

$$s \approx x_1 + \frac{x^5_1}{40\,R^2_1\,s^2_1} \pm \ldots\ldots\ldots\ldots (57b)$$

and

$$s = c\left(1 + \frac{2}{45}\,\phi^2 + \frac{2\,\phi^4}{1{,}575}\cdots\right) \ldots (57c)$$

Eq. 57b is derived from the cubic parabola. Referring to Fig. 8,

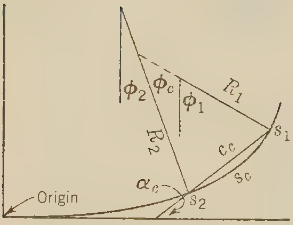


FIG. 8

$$s_c = s_1 - s_2 = 2\,R_c\,\phi_c \ldots\ldots\ldots\ldots (58)$$

in which s_c is any part of the spiral arc, R_c is its mean radius, and ϕ_c is the subtended angle. Also:

$$R_c = \frac{R_1\,R_2}{R_1 + R_2} \ldots\ldots\ldots\ldots (59a)$$

$$\phi_c = \phi_1 - \phi_2 \ldots\ldots\ldots\ldots (59b)$$

and

$$s_m = \frac{s_1 + s_2}{2} = 4\,R_c\,\phi_m \ldots\ldots\ldots\ldots (60)$$

Some useful approximations can be derived from Eqs. 58 to 60:

$$c_c \approx 4\,R_c \sin\frac{\phi_c}{2}; \qquad (61a)$$

$$\alpha_c \approx \frac{\phi_1 + \sqrt{\phi_1\,\phi_2} + \phi_2}{3} = \frac{\phi_1 + 4\,\phi_m + \phi_2}{6}; \qquad (61b)$$

and

$$c_c = s_c - \text{correction} \qquad (61c)$$

The correction in Eq. 61*c* is nearly proportional to $(\phi_c)^2$, which fact makes it possible to traverse a spiral along its chords.

By Eq. 50 the first chord correction (see Table 2) is:

$$\text{Correction} = -\,s \times \frac{2\,(\phi^\circ)^2}{45} \times \frac{\pi^2}{180^2} \qquad (62)$$

That is,

$$-\,s \times \frac{2\,(\phi^\circ)^2}{45} \times \frac{\pi^2}{180^2} = -\,50 \times 0.5^2 \times 0.0000135386 = -\,0.00016923 \text{ ft.}$$

Assume the following conditions as an example: $s_1 = 300$ ft, $\phi_1 = 18°$, $R_1 = 477.465$, $x_1 = 297.053$ ft, $y_1 = 31.195$ ft, $\tau = 149.508$ ft (by Eq. 54*b*, $\tau = 149.509$ ft), $\sin_s \phi = 0.9901754$, and $\text{vers}_s\,\phi = 0.1039838$. Referring to Table 2, x_1 is computed as 297.050 ft, which checks the foregoing value of 297.053. Similarly, 31.195 checks the foregoing value of y. The values of $\phi°$ in Col. 2, Table 2, are proportional to $(\text{point})^2$ in Col. 7; thus, for point 5, $\phi° = \frac{25}{36} \times 18° = 12.5°$, etc. In Cols. 8 and 9, a is the difference between adjacent values of $(\text{point})^2$, the values of a^2 being factors used in determining the chord correction in Col. 10. Eq. 61*b* is used for computing the values of $\alpha°_c$ in Col. 4. The length of any given chord is equal to s_s minus the correction for that chord computed by Eq. 62 and Col. 9.

TABLE 2.—COMPUTATIONS FOR TRAVERSING ALONG SPIRAL CHORDS

Point	Station (1)	$\phi°$ (2)	Chord (ft) (3)	$\alpha_c°$ (4)	$\cos\alpha_c$ (5)	$\sin\alpha_c$ (6)	$(\text{Point})^2$ (7)	a (8)	a^2 (9)	Correction (ft) (10)
0	0 + 00	0.0					0			
			50.000	0.167	0.999996	0.002909		1	1	−0.00016923
1	0 + 50	0.5					1			
			49.998	1.167	0.999793	0.020361		3	9	−0.002
2	1 + 00	2.0					4			
			49.996	3.167	0.998473	0.055241		5	25	−0.004
3	1 + 50	4.5					9			
			49.992	6.167	0.994214	0.107421		7	49	−0.008
4	2 + 00	8.0					16			
			49.986	10.167	0.984298	0.176512		9	81	−0.014
5	2 + 50	12.5					25			
			49.980	15.167	0.965169	0.261628		11	121	−0.020
6	3 + 00	18.0					36			

Many trick problems can be solved directly. For example, given s and σ, let $z = \frac{12\,\sigma}{s}$. Then

$$\phi = z + \frac{z^3}{28} + \frac{397\,z^5}{129{,}360} + \frac{55{,}437\,z^7}{162{,}993{,}600} + \cdots\ldots\ldots\ldots\ldots \text{(63}a\text{)}$$

Another case arises when a given curve center and a given spiral s are fixed. Let R equal the radius of the existing circle; it is required to compute the new radius R_n. If $q = \frac{s}{2\,R}$—

$$\phi = q + \frac{q^3}{6} + \frac{25\,q^5}{504} + \frac{20{,}167\,q^7}{1{,}164{,}240} + \cdots\ldots\ldots\ldots\ldots \text{(63}b\text{)}$$

from which, knowing ϕ and s, $R_n = \frac{s}{2\,\phi}$ can be computed readily.

As indicated by the authors, the spiral and its involute wind about a point (35) (47) (49) that lies on a 45° line from the origin and is distant from it a length $\sqrt{\frac{\pi\,R\,s}{2}}$. This is the value of c as ϕ approaches ∞. The maximum chord and maximum ϕ-values have been calculated. The involute of Euler's spiral is not the hyperbolic spiral; nor is the evolute of a hyperbolic spiral a simple transition curve.

The idea of a curve that is transitional throughout has occurred to many engineers and, as indicated in the paper, it presents a grade transition problem where the curves meet. Compound spirals similar to compound circles are feasible and the writer has seen this arrangement on a set of plans only once.

The authors are to be commended for their extensive research as indicated by the bibliography. Their list of "approximate relations" will be found useful and may be verified by plotting corrections to s on either log-log paper or alinement graphs.

It should be obvious from the foregoing that Euler's spiral is an exact curve and that tables can be used for most of the work. Some writers use the degree-of-curve system. This should be a personnel choice because either system works if one can establish a definition for it. Since s, R, and ϕ (or Δ) are quantities used by all surveyors, there seems to be little need for any particular one of a dozen degree systems.

Right of ways can be described as parallel to a center-line spiral and this center-line curve can be retraced from any two parts such as s and R or R and ϕ. Areas can be computed by Eq. 52, and they can be checked approximately by the traverse.

The use of tables has been emphasized because the writer has seen many men shrink from the extensive formulas and derivations. The body of data presented herein is only a small portion of the material available and it should be needed only on rare occasions. Eqs. 51 to 63 should prove useful particularly to any designer trying to aline a transition around a bridge pier. The subject of computing a spiral between two circles has not been given much attention in this discussion. Most cases are quite simple if Eqs. 56 are combined with Eqs. 58, 59, and 60. These are exact equations and will not require extensive tables.

RENÉ S. PULIDO Y MORALES,[9] ESQ.—The use of any of the easement curves to provide a safe and comfortable transition around a curve is only a rough method of obtaining the maximum possible high speed with safety and comfort. It is necessary to study more intently the path and characteristics of the center of gravity of the vehicle, endeavoring to keep it approximately at the same height, and on a predetermined trajectory. Another cause of discomfort is the acceleration of the center of gravity of the vehicle in a transverse direction. This effect cannot be eliminated, but it can be reduced.

The spiral that is transitional throughout is composed of two symmetrical spirals of a length:

$$S = R_m \Delta \ldots\ldots\ldots\ldots (64)$$

in which Δ, the total deflection of the curve, is in radians, and R_m is the radius at the point of maximum curvature. Therefore, the total length is equal to:

$$S_t = 2\, R_m \Delta \ldots\ldots\ldots\ldots (65)$$

Also,

$$S = 1.6 \frac{V^3}{R_m} \ldots\ldots\ldots\ldots (66)$$

Combining Eqs. 64 and 66, and solving for R_m:

$$R_m = \sqrt{\frac{1.6\, V^3}{\Delta}} \ldots\ldots\ldots\ldots (67)$$

Eq. 67 is a valuable guide for determining the minimum radius of a spiral that is transitional throughout.

A mathematical expression whose derivation is readily available in standard texts is

$$\gamma = -\frac{V^3}{R^2}\frac{dR}{dS} \ldots\ldots\ldots\ldots (68)$$

Eq. 68 demonstrates that the spiral is the most advantageous of all types of transition curves. It yields the spiral of minimum length and is particularly useful when space is restricted.

Integrating Eq. 68:

$$S = \frac{V^3}{R\,\gamma} + C \ldots\ldots\ldots\ldots (69)$$

Noting that $\frac{V^3}{\gamma} = K$ is a constant:

$$S = \frac{K}{R} + C \ldots\ldots\ldots\ldots (70)$$

which is the typical spiral equation. The formula for the determination of the transition length—$S = \frac{V^3}{R\,\gamma}$ (Eq. 16*b*)—is not precisely applicable to other transitions than the spiral. It is derived by the following reasoning: In the path of a vehicle traveling on a transition curve at constant speed, the centripetal acceleration is zero at the beginning where the radius is equal to ∞, and $\frac{v^2}{R}$ at the end where the radius is the minimum for that case. If t is the time of travel, the rate of acceleration along the curve is expressed by:

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$$\frac{v^2}{R} \times \frac{1}{t} = \gamma . : \ldots\ldots\ldots\ldots\ldots\ldots (71)$$

Substituting $t = \frac{S}{v}$ in Eq. 71, $S = \frac{v^3}{R\,\gamma}$; but this is only an expression for the mean value and cannot be accepted as an elegant conclusion, although it is admitted that mean acceleration is equal to maximum acceleration.

The mathematical requirement for the perfect easement curve—that the radius of curvature r at any point should be inversely proportional to the distance s to that point—is not precise, since these two quantities vary approximately (but not precisely), in inverse ratio, as stated in the paper.

In the writer's opinion, all the easement curves—the cubic parabola, the lemniscate, the Talbot, Searles, Crandall, and Ericson curves—are satisfactory and give substantially the same results in the field. The lemniscate advocated by F. G. Royal-Dawson (24) in England is the most rational at the single point where the radius of spiral curvature is greatest; but it is not rational for any other point along the spiral. This easement curve requires more length than the spiral.

The paper is a good contribution to the literature on easement curves.

C. C. Wiley,[10] M. Am. Soc. C. E.—Every one who uses the spiral must, at some time, learn how. Consequently, in addition to being mathematically sound and practical in application, the treatment should be such that it can be comprehended easily. The first requirement is that the basic geometry shall be presented simply and clearly, and that it be kept constantly in the foreground. The second is that formulas, computations, and operations shall be simple, direct, universally applicable without auxiliaries, and conform to accepted engineering practices.

Most writers on spiral transitions have failed in these things—partly, because they seem to have been too anxious to plunge into mathematical derivations, but more particularly, because they have attacked the problem in such a manner, and from such premises, as to result in cumbersome formulas, tedious computations, and awkward field work, which have obscured the geometry and given the spiral an unmerited reputation of complexity.

The purpose of a curve is to provide a safe, convenient, and comfortable change of direction. The critical factor in traveling on curves is the centrifugal force, which must be kept within satisfactory limits. Basically, centrifugal force is a function of the angular velocity which, in turn, is a function of curvature. The conventional formulas for centrifugal force, however, have been written in terms of linear velocity and radius of curvature. This may explain why so many writers have started with the radius of curvature instead of curvature itself.

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[10] Prof., Civ. Eng., Univ. of Illinois, Urbana, Ill.

The circle is universally used as the basic curve because its geometry is simple; it is symmetrical with respect to traffic; and conditions of travel on it are constant. Its defect lies in that, mathematically, it requires instant change from rectilinear to curvilinear motion, which is mechanically impossible. Consequently, spirals are introduced at the ends of the circular arc to provide a gradual transition in curvature to control the centrifugal force. Any change of direction made by the spiral is only a by-product.

A circle may be defined properly as "a curve whose angular change of direction per unit of arc is a constant." When expressed in ordinary engineering units this is nothing more nor less than the familiar "degree of curve" whose merits have been recognized by railroad engineers for about a century, but which many highway engineers still ignore. The degree of curve expresses the curvature of the circle directly, whereas the radius is merely an incidental linear dimension useful in certain computations. Since the spiral is designed to provide transition in curvature, it would seem that the "rational" attack would be on the basis of curvature rather than radius.

All writers on this subject seem to agree that the ideal spiral should be a curve whose curvature increases uniformly with the distance from the origin. Therefore, the intrinsic equation of the spiral may be written directly from the definition in the form

$$d = k\,s \qquad \text{(72)}$$

which, in engineering terms, makes d the degree of curve at any point; k the rate of change of the degree of curve in degrees per 100 ft (station); and s the distance from the origin in stations (100 ft). Every term now has a simple geometric conception which is easily comprehended and easily learned. An examination of the writings of the late A. N. Talbot, Past-President and Hon. M. Am. Soc. C. E., and those of others using his method shows that it has many intrinsic advantages.

In practice, the curvature of the basic circle is selected with respect to expected speed subject to topographic restrictions. A length of arc is then cut from it to provide the required total change of direction. Rarely, if ever, is the length chosen and the curvature made to conform. In contrast, most writers have chosen the length of spiral and made the curvature to fit; probably because the first spirals were to be superimposed on existing arbitrary lengths of superelevation runoff. This mode of attack, especially when combined with the radius of curvature, inevitably leads to cumbersome formulas, complicated computations, and other troubles, which have obscured, and sometimes almost obliterated, the basic geometry. As stated, the basic geometry must be understood if the spiral is to be used intelligently.

Under the Talbot method, when k is chosen, a spiral of indefinite length is "fixed" from which arcs of proper length may be cut to fit between a tangent and any circle, or between two circles as in the case of the compound curve. Thus, this method becomes fully "rational" and of simple and universal application.

Various writers have shown that the curvature characteristics of a satisfactory spiral are a function of the cube of the speed. The authors express this in Eq. 16*b* which is not readily usable, whereas R. A. Moyer, Assoc. M. Am. Soc. C. E. (19), gives it in the simple arithmetical form

$$L = \frac{1.58\,V^3}{R} \qquad (73)$$

stating that this is the minimum length (L) for safety. In order to provide for both safety and comfort, the writer and the late G. W. Pickels, M. Am. Soc. C. E., have recommended a desirable highway spiral in the form

$$k = \frac{230{,}000}{V^3} \qquad (74)$$

which is practically equivalent to a coefficient of 2.5 in Eq. 73 and gives a spiral one half as long as that considered desirable on the railroads (21).

Since moderate variations of the speed are permissible on any curve, and since the same speeds will apply over moderate variations in curvature, it follows that a comparatively small number of different spirals having simple and convenient values of k will fully cover all ranges of speed and curvature to be found in highway alinement. On the other hand, the use of arbitrary lengths, each covering a range of curvature, results in inconvenient values and requires the computation of an entirely new spiral for each change of curvature. Thus the Talbot method of fixing the spiral curvature and selecting a length to meet the given circle provides simple formulas, a minimum of computations, and a maximum flexibility in application.

The term ϕ, total change of direction at any point on the spiral, appears in many of the authors' formulas, but none of the formulas seem to give an expression for evaluating it. Perhaps this is a result of the idea of making $\phi = \frac{1}{2}\Delta$; but if Eq. 4 is extended a step or two further, the result is:

$$\phi = \tfrac{1}{2}\,k\,s^2 \qquad (75)$$

which is identical in form with the Talbot formula but is in different units. In the Talbot formula ϕ is given directly in degrees, whereas the authors give it in radians, which is undesirable.

Various writers show that α, the spiral deflection angle, is very nearly $\frac{1}{3}\,\phi$ and, in Table 1, the authors give corrections from which they show that in most cases the difference is very small, and may be omitted. Professor Talbot came to the same conclusion. As a matter of fact, in practice α can always be taken as $\frac{1}{3}\,\phi$ because the sole effect of omitting the correction is to move the circular curve bodily a very small distance outward. In a very extreme case, with $\phi = 30°$, this movement was only about 1 in., which is utterly insignificant.

In Eq. 75, α becomes $\frac{1}{6}\,k\,s^2$ in degrees which, when converted to minutes, becomes the ridiculously simple equation

$$\phi = 10\,k\,s^2 \qquad (76)$$

Obviously, with uniform chord lengths, it is necessary to compute only the first value of α by the formula and then find the others by multiplying the first by the square of the digits. If simple values of k are used with stakes at the usual distances of 25 ft, 50 ft, or 100 ft, the spiral deflections may be written almost as fast as those of a circular curve (with possibly a last one equal to $\frac{1}{3}\,\phi$ if the spiral has an odd length). This is so simple, and so in accord with basic geometry, that it is easily learned.

The general solution for the tangent distance of a spiraled curve, in Section 8, is interesting. It is cumbersome because it involves the calculation of the otherwise useless spiral chord c and the solution of one right triangle and two oblique triangles. The geometry of the solution is well shown in the figure, but the authors "spoiled the show" by writing out all the mechanical operations, culminating in the appalling appearance of Eq. 18, which completely obliterates the geometry. The day is past when either engineers or students want, or need, a formula like this which only adds to the forbidding appearance of the spiral treatment.

A few additions to Fig. 3 would have shown directly that

$$T = (R + \sigma)\ tan\ \tfrac{1}{2}\Delta + \tau \ldots\ldots\ldots\ldots\ldots\ldots\ldots (77)$$

which is far simpler as it involves the solution of only one right triangle plus two spiral elements often wanted for other purposes. This solution is shown by the authors in approximation 7, Section 11, in which the only approximation is the very slight one of using $\tau = \frac{1}{2}\,S$.

Obviously, safety and comfort are highly important but it must not be inferred that they have not received, or are not receiving, careful attention. The railroads originally derived lengths of runoff from studies of safety and comfort, and later derived their spirals in a similar manner. Highway engineers, however, had little factual data to aid them until the work of Professor Moyer appeared (19). Unfortunately, present data are scarcely adequate and those that are available must be used with care and judgment.

Eq. 15 may be reduced to the slightly approximate "safe speed" formula often used by highway engineers

$$V^2 = 15\,R\,(e + f) \ldots\ldots\ldots\ldots\ldots\ldots\ldots (78)$$

in which: e is the actual superelevation cross slope built into the road; and f is usually defined as the safe coefficient of friction against lateral skidding. Actually, it is a coefficient of unbalanced centrifugal force expressed in terms of cross slope. Thus, the quantity $e + f$ becomes the equivalent theoretical superelevation for the maximum speed V. If the maximum value of f against skidding only is used, no allowance is made for comfort; hence the latter must be introduced merely by judgment in choosing f since, at present, factual data are not available.

Shortly after the appearance of Professor Moyer's work (19), the U. S. Public Roads Administration suggested a value of $f = 0.16$. Several highway departments objected and several made test runs. The Illinois Division of Highways, on the basis of both safety and comfort, reached the conclusion that a maximum value of $f = 0.10$, for speeds of approximately 50 miles per hr, was satisfactory. An examination of available data indicates that, for both safety and comfort, f may be taken as about 0.10 at 40 miles per hr and about

0.04 at 100 miles per hr. This agrees well with the railroad rule for excess speed which is equivalent to about $f = 0.05$.

The idea of curves "transitional throughout" is an old one. It seems to result from a misinterpretation of "transitional" and the failure to give due consideration to the elements of comfort and convenience on curves. The primary function of the spiral is to make a transition in curvature to control centrifugal force—not to provide change of direction. To require also that it provide change of direction is to impose a duty for which it is not fitted.

When traveling on a tangent both driver and passengers are subject to constant conditions. In traveling on a circle they are also subjected to constant conditions although these conditions are not the same as on a tangent. The function of the spiral is to provide a transition between these different conditions. With proper spirals the change is made with comfort and convenience as well as safety, and the driver and passengers must adjust themselves to changing conditions only on the spirals. If the curve is "transitional throughout" there is no part of its entire length where travel conditions are constant.

Furthermore, since a spiral is twice as long as the circle with the same final curvature and total central angle, making the curve transitional throughout adds greatly to the length of curve over all of which travel conditions are constantly changing. For example, assume a speed of 60 miles per hr on a 4° curve ($R = 1{,}432.5$) with $\Delta = 64°$. A spiral with $k = 1$ is suitable, giving spirals 400 ft long. The unspiraled circle would have a length of 1,600 ft, with bad conditions at the end; but most of its length would offer constant travel conditions. When spiraled, the total length would be 2,000 ft, of which 1,200 would have uniform travel conditions with 400 ft at each end to make the transitions. If "transitional throughout," the length would be 3,200 ft with no part having constant conditions. If the latter curve is reduced to 2,000 ft its central curvature would be increased to 6° 24′ ($R = 895.3$), which is entirely too sharp for the given speed.

If, for a given speed, curvature, and change of direction, it should happen that $\phi = \frac{1}{2}\Delta$, the curve is short and automatically and properly is transitional throughout. If the computed value of ϕ is greater than $\frac{1}{2}\Delta$ then, because of speed, the same spiral should be used but it should merely be shortened until $\phi = \frac{1}{2}\Delta$. If the computed value of ϕ is somewhat less than $\frac{1}{2}\Delta$, speed would again call for the same spiral and the residual circular arc, no matter how short, should be retained for simplicity. Making $\phi = \frac{1}{2}\Delta$ would not improve the curve and would require completely new calculations and field notes. For greater values of Δ the benefits of uniform travel conditions on the curve would become increasingly evident.

In 1923 the writer had a graduate student who spent much time in trying to develop a curve "transitional throughout." Several such curves were laid out and test runs made over them. Except for short curves they were disliked by both drivers and passengers. If they were made of suitable middle curvature, the lengths became unreasonably long, and if they were shortened to reasonable length, the middle curvature became too sharp for the speeds. In addition, the geometry was complex, the formulas ponderous, the calculations long and tedious, and the field work awkward. In short, the results were so unsatisfactory that they were never published.

The spiral is intrinsically so simple that it is a misfortune that it should have been so mutilated as to be almost unrecognizable, and to acquire a forbidding if not sinister appearance. When properly presented, its applications are so simple that an average field engineer can make complete computations and field notes in very little more time than is required for an unspiraled curve and the field operations are practically identical. In some schools students are required to be able to spiral, completely, either a simple or a compound curve with no auxiliaries whatever (not even the textbook) except the usual tables of the trigonometric functions. Until something simpler than this is devised the writer is perfectly contented with his present methods.

To summarize the writer's honest view of this paper: It contains nothing really new in mathematical treatment. The general mode of attack is the same as that used by a number of other writers, as are many of the forms of mathematical derivations. It adds nothing to expedite or improve the use and applications of the spiral; if anything, it confuses them more than before. The use of ϕ in the formulas is an innovation, but it is objectionable for several reasons. The principal objection is that it is based on a false premise since ϕ, the total change of direction on the spiral, is an incidental result and not a basic function. Some of the incidental items such as "safety and comfort" have merit but might have been given more attention. The "approximations" have all been long recognized whereas the manner in which they are given still leaves a hazy impression that they should not be tolerated.

What the highway spiral needs is some way of eradicating all the old awkward treatments of the spirals as given in most field manuals, and of avoiding the need for tabular data. State highway departments are wasting plenty of persons, pencils, paper, patience, and profanity on a problem which, actually, is little, if any, more complex than circular curves.

MAX C. KOEHLER,[11] ASSOC. M. AM. SOC. C. E.—With the nation returning to a normal economy, reconstruction of existing highways and the construction of new superhighways will be one of the major programs of the postwar period, and a discussion of the spiral easement curve is timely. Prior to World War II this refinement of design was going through its "growing pains"; it is still in that process and will possibly continue so for ten years or more. It has followed, and will follow, the same processes of development as the railroad easement curves in the nineteenth century; wherein every one from roadmaster to college professor advanced his pet idea on how to make the transition from a tangent to a circular curve without throwing the locomotive off the track or the passengers in the aisles. The smooth gliding ride of the modern "streamliners" at from 70 to 100 miles per hr has forced the passengers again to rely on the weather as a topic of conversation. A short history of this period may reveal the pattern that will be followed in highway design.

Writing at the University of Glasgow in Glasgow, Scotland, in 1862, W. J. M. Rankine (60) stated that a Mr. Gravatt, in 1828, was using the sine curve for railroads in England. This curve was transitional throughout, but was objected to by many because there was no simple curve. Rankine recommended the use of a transition curve developed by William Froude in 1841. Froude made "short shift" of curve staking as follows:

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(1) Locate the center of the railroad, using simple curves and tangents.

(2) Determine the length for the "curve of adjustment" by the formula $L = 300\,x$ the greatest change of cant (superelevation) in feet.

(3) Compute the shift b, in terms of the length L and the radius R, as follows:

$$b = \frac{L^2 R}{24} \qquad (79)$$

(4) Set out the curve by use of formula (a cubic parabola):

$$y = 4\,b\,\frac{l^3}{L^3} \qquad (80)$$

in which l is the distance measured on the circular arc and y is the ordinate. For further information, Rankine refers to a technical article (61) published in 1860 and 1861. Rankine also advised raising the outer rail above the center line for one half the cant, and depressing the inner rail an equal amount. Happily, the outer rail is now adjusted for the full cant and "grade" is carried on the inner rail. The highways passed through this same experience, complicated by parabolic crowns and extra widening on the inside. It is now fairly uniform practice to omit the crown as soon as the superelevation begins, carry the grade on the inside edge of the pavement, and "throw" all the superelevation to the outside of the curve. Where the curve is located on a "ruling grade" the grade is "compensated"; that is, it is reduced so that the grade of the high edge does not exceed the "ruling grade."

In the United States the late Elliot Holbrook, M. Am. Soc. C. E. (41), seems to have been the first to publish a comprehensive article on the transition spiral. It contains a rigorous proof that the spiral angle θ, when the ratio of degree of curve to length L is unity, is:

$$\theta = \frac{L^2}{200} \qquad (81)$$

This is the first published account the writer has been able to find of the "American Spiral," in which the degree of curve varies directly as the distance along the curve. The editor remarks at the end of Holbrook's article, that "In Germany, and on some roads in France, the tangent is connected with the curve proper, or circular curve by what is called a 'connecting-curve,' which in Germany is made parabolic." However, the late William Cain, M. Am. Soc. C. E. (50), claims that the late A. M. Wellington, M. Am. Soc. C. E. (62), was the first man to define the true transition spiral. Wellington himself refers to it as a cubic parabola in which the length is proportional to the square root of the offset.

Holbrook prepared a table for a spiral increasing 10 min in degree of curve for every 10 ft of length along the spiral up to 210 ft where $\theta = 3^\circ\,30'$. This same table contains a column for deflection angles from the beginning of the spiral. For lengths as great as 50 ft, d (deflection) seems to be $\frac{\theta}{2}$; but after that $d = \frac{\theta}{3}$ which has since been proved true for any deflection, using a slight correction when θ exceeds 15°. A short but rigorous proof of the intrinsic equation of the spiral has been presented by the late D. E. Hughes, M. Am. Soc. C. E. (63), as follows: Referring to Fig. 9, R varies as $\frac{1}{L}$; that is, $R = \frac{K}{L}$. Also, $R = \frac{dL}{d\theta}$, from which $\frac{dL}{d\theta} = \frac{K}{L}$, or

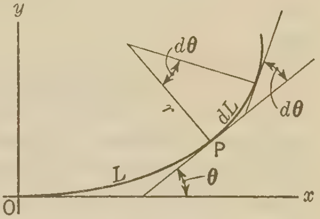

FIG. 9

$$L\,dL = K\,d\theta \ldots\ldots\ldots\ldots (82)$$

Integrating Eq. 82, $\frac{1}{2} L^2 = K\,\theta$; or

$$L = \sqrt{2\,K\,\theta} \ldots\ldots\ldots\ldots (83)$$

In other words, θ varies as L^2 (see Eq. 83) and $L = 2\,R\,\theta$, or twice a circular arc of the same radius and angle. Hughes then expands the sine and cosine series of $d\theta$ to derive:

$$x = \sqrt{2\,K\,\theta}\left(1 - \frac{\theta^2}{5^2} + \frac{\theta^4}{9^4} - \frac{\theta^6}{13^6} + \cdots\right) \ldots\ldots\ldots\ldots (84a)$$

and

$$y = \sqrt{2\,K\,\theta}\left(\frac{\theta}{3^1} - \frac{\theta^3}{7^3} + \frac{\theta^5}{11^5} - \frac{\theta^7}{15^7} + \cdots\right) \ldots\ldots\ldots\ldots (84b)$$

suggesting that a table be prepared for values of x and y. It is to be noted that Hughes does not use the degree of curve. Messrs. Eichler and Eves prefer this same reasoning. They call K a constant of proportionality which is equal to $\sqrt{2\,R\,L}$.

In 1901 the late Arthur N. Talbot, Past-President and Hon. M. Am. Soc. C. E. (29), gave the definition of this spiral as: "The Transition Spiral is a curve whose degree-of-curve increases directly as the distance along the curve from the point of spiral," which is expressed mathematically as

$$D = a\,L \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (85)$$

in which D = degree of curve $= \frac{5{,}730}{R}$; a is the constant ratio $\frac{D}{L}$; and L is the length of the spiral in 100-ft stations. Eq. 85 is the same expression as $\frac{1}{R} = K\,S$ "dressed in different clothing."

In computing and staking curves it is always necessary to know how much central angle has been consumed by any given length of curve. In the simple curve it is the degree of curve times the length of arc in stations, since, by definition, 100 ft of arc subtends 1° of central angle. All other arcs are directly proportional, and therefore their degree is an expression of the number of times their radius will divide into the unit radius of 5,730 ft. For convenience, these values are usually in simple whole numbers. In the spiral curve, since one starts from a tangent and progresses at a uniform rate of change of curvature a until the desired degree of curve is reached, the amount of central angle consumed is exactly half that of the simple curve of the same length. This is the application of the law of averages.

Since $R = \frac{dL}{d\theta} = \frac{5{,}730}{D}$,

$$d\theta = \frac{D\,dL}{5{,}730} \qquad \text{(86a)}$$

From Eq. 85, $D = a\,L$; but L is in stations, and

$$d\theta = \frac{a\,L\,dL}{57.30} \qquad \text{(86b)}$$

Also,

$$\theta = \int \frac{a\,L\,dL}{57.30} = \frac{a\,L^2}{114.6}\ \text{(radians)} = \frac{a\,L^2}{2}\ \text{(degrees)} \qquad \text{(87)}$$

The 100-ft arc was chosen to subtend 1° of central angle because the radius of such a circle is 5,730 ft (nearly) and is not, by coincidence, the number of degrees in one radian. Eq. 87 is exact, even though two incommensurable numbers appear in the solution. They are the same and divide equally into each other.

Talbot seems to have been the first to recognize that "A spiral diverges from its osculating circle, at any point, at the same rate that the spiral deflects from the initial tangent" and as a result devised the best method of computing the backward and forward deflections to any point on a spiral from any other point in the spiral. This is such a valuable tool, to the locating engineer, when moving up along the line in heavy timber or brush, that it is worth repeating:

> "The deflection angle between the local tangent, at any point on the spiral, to any other point on the spiral is the sum or difference of the deflection angle of the osculating circle for a length equal to the distance between the two points and the deflection angle for the spiral from the P.S. for a length equal to the distance between the two points."

Referring to Fig. 10, PP′ = PC = AB; $\phi = \frac{\theta_1}{3}$; and $\delta = \frac{i}{2}$. According to the rule, the forward deflection is $(\delta + \phi)$ and the backward deflection is $(\delta - \phi)$. A more comprehensive treatment of this rule has been published by T. F. Hickerson, M. Am. Soc. C. E. (13).

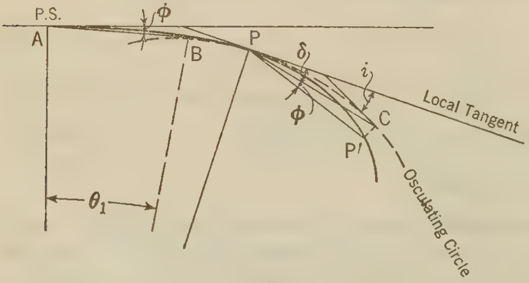

FIG. 10

Orville Kofoid, Assoc. M. Am. Soc. C. E., computed the deflection angle from any point back on the spiral to any point forward without reference to the local tangent by the following formula (15):

$$(L_1 + L_2 + L_3)\,(L_3 - L_1)\,\frac{\theta}{3\,(L)^2} \qquad \text{(88)}$$

in which L_1 is the distance from point of spiral PS to backsight; L_2 is the distance from PS to transit point; L_3 is the distance from PS to foresight; L is the total length of spiral; and θ is the total spiral angle. Although it is useful, Eq. 88 is not well suited for field work because of the multiplication and division involved; and the use of logarithms by field men is becoming a lost art since the introduction of the computing machine.

Most field men prefer the "ten-chord spiral" because of available tables in which all ten deflections from the PS may be read directly. "Moving up along the spiral" is accomplished simply by multiplying the first deflection by a suitable coefficient which is also selected from a table. Since the coefficients are mostly two-digit numbers, and the first deflection $\left(\frac{\theta}{300} \text{ in all cases}\right)$ is also a small number, the multiplication is not involved (25). The writer once edited a table (not published) for staking a "ten-chord spiral" backward—that is, from the sharp end toward the tangent. The table was prepared by several of the field engineers in the Portland (Ore.) office of the U. S. Public Roads Administration so that staking could proceed along the spiral in the direction of the stationing at all times. The objection to the "ten-chord spiral" is the tendency of the field men to omit staking the "breaks" unless they happen to fall at a chord point.

It seems fairly certain that the type of spiral to be used on highway work is finally accepted as a curve whose degree of curvature or radius varies as the distance along the curve, and that sine curves, series of compound curves, lemniscates, logarithmic and Archimedes spirals, etc., have been tested and found wanting in one respect or another. The tempest in the teapot now raging is whether to abandon the degree of curve in favor of the even-radius curves, such as 1,000 ft, 2,000 ft, etc. One argument in favor of the latter is that the radius of the curve could be posted on the highway and would be understood readily by the layman, whereas the degree of curve means nothing to him. Time alone will decide that issue.

In applying the spiral to highway design, the present tendency is to combine it with superelevation, since both begin at the PS and both progress uniformly until the central circular curve is reached where both curvature and superelevation become some predetermined constant. The degree of curve is determined by the designing engineer to fit the topography of the ground; the superelevation is determined by the "design speed" of the highway and varies with the degree of curve. These conditions are usually set by the county, state, or bureau doing the design. L. I. Hewes, M. Am. Soc. C. E., has prepared a useful table (11*a*) which also recommends the length of spiral to be used for each combination of "design speed" and degree of curve.

It is evident from the foregoing data that the question of whether a curve is transitional throughout will be self-determined by the various restrictions of design.

At the conclusion of the paper by Messrs. Eichler and Eves, the authors raise several questions for further study and discussion. While studying the locus of the center of radius of the spiral, the writer seemingly stumbled on a conclusion that the spiral is restricted to 90° of central angle. It would be interesting to discover if this theory can be either proved or disproved.

The question was raised as to how to improve upon existing methods of describing right of ways, a subject that the writer has treated in some detail elsewhere (64).

Attention is invited to the fact that, in Eq. 7, the correction C is subtracted from $\frac{\phi}{3}$ when ϕ exceeds 15°.

William T. Pryor,[12] Assoc. M. Am. Soc. C. E.—Messrs. Eichler and Eves have presented their paper for the purpose of clarifying certain features regarding the spiral curve. They recognized that it is the aim of highway engineers to construct a highway capable of carrying high-speed traffic with maximum safety and driving comfort. They wrote in Section 1:

> "Of the many problems involved, that of horizontal curvature, accompanied with proper superelevation, is one of the most important, and it is essentially with this feature of the future road that this paper is concerned."

The large Bibliography given by the authors, although not complete, will show that they selected the "clothoid"—the transition spiral—the curve which has a uniform change in length of radius in exact inverse ratio to its length. There are other forms of curvature transition or easement; but, upon careful analysis and by actual use, none have proved to be as easy to handle and as adaptable to highway alinement design and to field staking.

A moving vehicle cannot, safely and comfortably, change immediately from straight-line motion to constant angular motion. Circular curves possess a constant angular change per unit length of arc (the degree of circular curve, D). The transition spiral does not have a constant angular change per unit length, but it does have a uniform rate of angular increase (29) per unit length (the spiral degree of curvature increase, k). A logical way of using these angular changes to design the spiral curve in conformity with the speed of the vehicle and the superelevation and dependable friction factor on the highway are of vital importance.

At the end of the spiral curve the rate of angular change attained produces a computable centrifugal force for any given vehicle speed; and for the same speed the rate of angular change and centrifugal force are halved at a point midway between the beginning and end of the spiral curve. This persists proportionally throughout its entire length. On the circular curve the centrifugal force and angular change are constant for any given vehicle speed.

An easy procedure for applying the basic formulas in the geometric design of highway alinement is demonstrated by the diagrams in this discussion. Fundamentally, this discussion has been prepared to show, in a graphical manner, the general application that can be made of the authors' developments to serve as a guide in choosing spiral curves for transition in highway alinement.

Tables are available (2)(59) which will be found very useful for the solution of most of the functions of spiral curves and circular curves for use in design layout and field staking, thus avoiding the labor of solving long-series equations. The use of tables that have been calculated and found sound in actual practice should be emphasized. Field and office men alike hesitate to begin the use of long formulas, and recoil quickly from such formulas if some other method will yield the same results.

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The superelevation, the friction factor (expressed as a cross slope to be in the same units as superelevation), degree of circular curve, and the spiral degree of curvature increase are all intrinsically related. These, as well as the geometry of the spiral curve, must be clearly understood if intelligent use is to be made of all principles. Then full application can be made of the flexible features of the basic formulas.

If the velocity, Eq. 1, is measured in v miles per hour instead of in V feet per second:

$$\frac{P}{W} = \frac{0.0669\, v^2}{r} \qquad (89)$$

The ratio P/W is illustrated in Fig. 11(*a*). The centrifugal force P must be opposed equally or the moving vehicle will slide outward off the curve. Opposition may be accomplished by inclining the curving path so that its cross slope is everywhere normal (at right angles) to the component force of weight W and the centrifugal force P, Fig. 11(*b*). Angle CAB may be thought of as being the sum of two angles, s and t, Fig. 11(*c*). The tangent of the sum of these two angles is e (superelevation) plus f (friction factor). Considering the length of line AB as unity, by similar triangles $\frac{(e+f)}{1} = \frac{P}{W}$, and from Eq. 89 if D, in degrees per one hundred foot arc, is used instead of $1/r$:

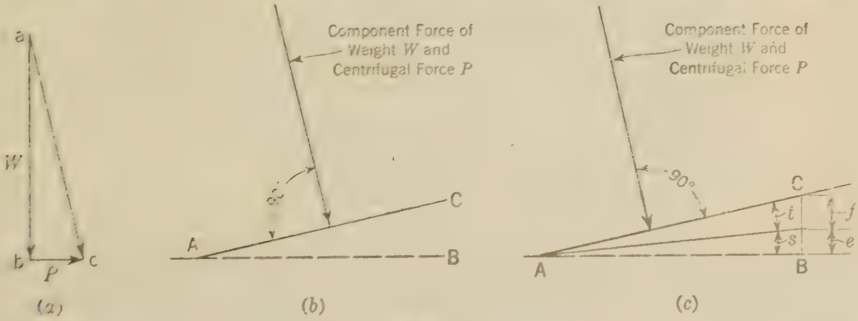
a
W
b
c
P
(a)
Component Force of Weight W and Centrifugal Force P
90°
A
C
B
(b)
Component Force of Weight W and Centrifugal Force P
90°
C
t
f
s
e
A
B
(c)

FIG. 11

$$(e + f) = 0.0000117\, D\, v^2 \qquad (90)$$

Use of Eqs. 89 and 90 made it easy to prepare Fig. 12 to show the relationship of degree and radius of circular curve for vehicle speeds, superelevation, and friction factors. The sum of e plus f, Fig. 11(*c*), is shown as the abscissa in Fig. 12.

In the use of Fig. 12, line SSSS represents the superelevation rate suggested for circular curves in climatic regions where snow and icy conditions may be expected on the road. It was computed by Eq. 90, setting f equal to zero and v equal to 30 miles per hr. Line SSS′S′ represents the superelevation rate suggested for circular curves in climatic regions where no snow or ice is expected on the roadway. Line SSS′S′ has an upper limit of 0.12 ft per ft for a 12° circular curve, and all sharper circular curves. Use of this upper limit to the superelevation rate is considered the maximum for safety and comfort where road surface conditions are likely to be good throughout the entire year, with no snow or ice on the road at any time. Line ffff represents 0.08 ft per ft of friction added to the line SSSS, and line fff′f′ the same amount to line SSS′S′. Likewise, line FFFF represents 0.16 ft per ft of friction added to line SSSS, and line FFF′F′ the same amount to line SSS′S′.

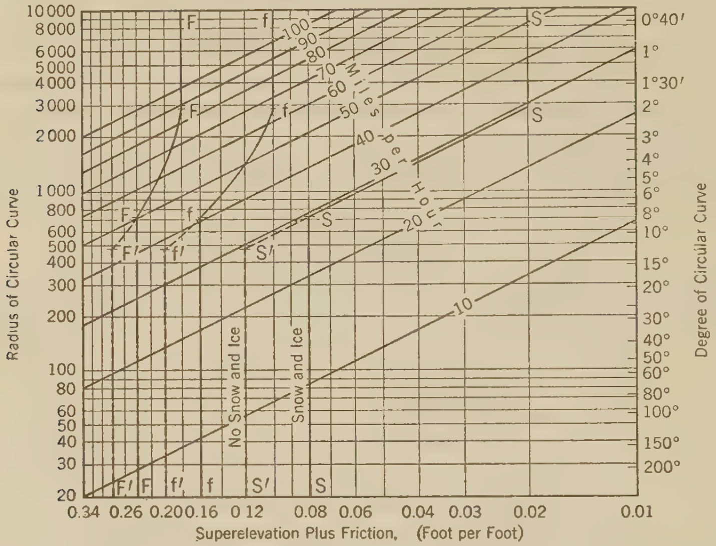

FIG. 12.—DEGREE AND RADIUS OF CIRCULAR CURVE FOR VEHICLE SPEEDS, SUPERELEVATION, AND FRICTION FACTORS

A circular curve may be selected easily and quickly for any combination of conditions within the practical limits of e, f, $(e + f)$, and maximum vehicle speeds of from 10 to 100 miles per hr. This represents a graphical method of showing the results obtainable by the full use of Eq. 1 of the paper. For spiral curve design the superelevation-plus-friction factor, the degree of circular curve, vehicle speed, and rate of change of radial acceleration (or the spiral degree of curvature increase) are combined.

Eq. 16b can be reduced to more easily used terms. Change from $1/R$ to D in degrees per 100 ft-station of arc and from V in feet per second to v in miles per hour; and:

$$S = \frac{0.0005506\,D\,v^3}{\gamma} = \frac{D\,v^3}{1{,}816\,\gamma}\ldots\ldots\ldots\ldots\ldots\ldots(91a)$$

and

$$\phi = \frac{0.000002753\,D^2\,v^3}{\gamma} = \frac{D^2\,v^3}{363{,}200\,\gamma} = 20{,}202\,\frac{(e+f)^2}{\gamma\,v}\ldots\ldots(91b)$$

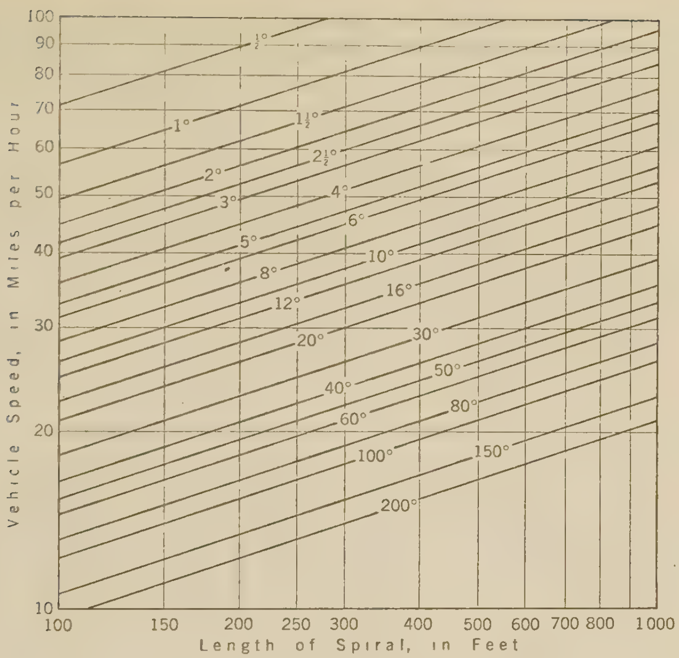

Fig. 13.—Length of Spiral Curve, According to Vehicle Speed and Degree of Circular Curve

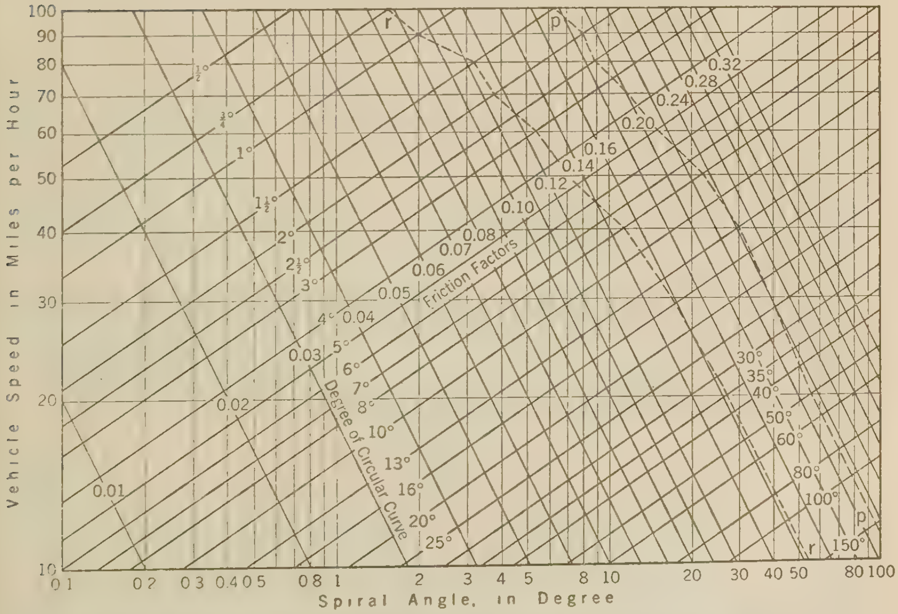

Fig. 14.—Spiral Angles Corresponding to Various Vehicle Speeds, Degree of Curve, Superelevation, and Friction Factors

Fig. 13 was constructed simply by substituting appropriate values for the respective variables in Eq. 91a and plotting the S-answers. The rate of change of radial acceleration (γ) in feet per second^3 was set at one, as explained subsequently herein. Fig. 14 was constructed by use of Eq. 91b with $\gamma = 1$. In design, if a value of $\gamma = 2$ is used, spiral angles will be exactly one half of those shown. Line pp in Fig. 14 indicates the limit of the sharpest permissible circular curves; and the line rr indicates the limit of recommended curves. The diagonal lines marked "Friction Factors" are $(e + f)$-factors. They apply to either superelevation or friction alone, or to the sum of the two, whichever the engineer may wish to consider, as e, f, and $(e + f)$.

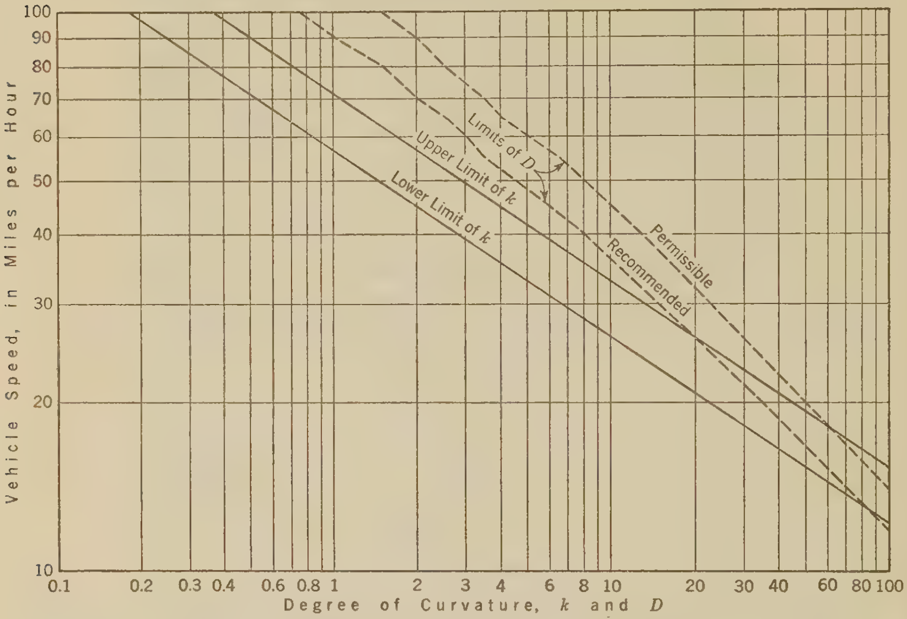

FIG. 15.—DEGREE OF SPIRAL AND CIRCULAR CURVES FOR DIFFERENT VEHICLE SPEEDS

Fig. 15 shows the relationship between the upper limits of degree of circular curve D and the spiral degree of curvature increase k for vehicle speeds of from 10 to 100 miles per hr. The limits of D were chosen from critical points in Fig. 12, and the limits of k were calculated by simple manipulation of Eq. 91a. Since $k = D/S$, if S is measured in 100-ft stations:

$$k = \frac{181{,}600}{\gamma\, v^3} \qquad (92)$$

For the upper limit of k a value of $\gamma = 2$ was chosen, and for the lower limit, $\gamma = 1$.

In order to ascertain, rationally, the value for γ to be used in Eqs. 91a to 92 so that they will satisfy the requirements for safety, and provide comfort and ease in gaining the radial acceleration for the driver while the vehicle is moving around the spiral curve, four items of fact should be kept in mind, as follows:

(1) Vehicles will be driven over the highway at speeds of from 10 to 20 miles per hr faster than the speed rating of the highway, especially for passing;

(2) As a factor of safety, designers should plan for this faster moving vehicle when and wherever it is possible, and then the slower moving vehicles will be provided for;

(3) Steering wheels must be turned much more when the vehicle is being driven considerably faster than the speed for which the curve is superelevated, in order to generate sufficient friction to counteract the unbalanced centrifugal force; and

(4) Although R. A. Moyer, Assoc. M. Am. Soc. C. E. (19), indicates that γ should be equal to 2, W. H. Shortt (27) uses a value of $\gamma = 1$.

By casual examination of the V^2/r-values in Table 3 it is evident that, for $\gamma = 2$ ft per sec^3, the time to attain these rates of radial acceleration would be from 1 sec to 5 sec. Actually the circular curves and vehicle speeds giving V^2/r-values greater than 9 ft per sec^2 require $(e+f)$-values that are too high, relatively, for safety and comfort and ease in traveling around the curves. Therefore, if γ is fixed at 2 ft per sec^3, all acceleration rates would be attained in less than 5 sec. This is not much time, but perhaps the efficient and alert vehicle driver will not experience any difficulty and will experience not too much discomfort in the process. In the light of the foregoing analysis, most engineers will undoubtedly agree that $\gamma = 2$ should be the largest value used in design.

TABLE 3.—SUPERELEVATION PLUS FRICTION $(e+f)$ FOR GIVEN RATES OF RADIAL ACCELERATION

V^2/r (ft per sec^2)	$(e+f)$ (ft per ft)
2.0	0.06
3.0	0.09
4.0	0.12
5.0	0.16
6.0	0.19
7.0	0.22
8.0	0.25
9.0	0.28
10.0	0.31

It is believed, however, that a smaller value of γ (say, $\gamma = 1$) would be preferable. In Table 4 the vehicle speed in miles per hour (Col. 1), the degree of circular curve (Col. 2), and recommended length of spiral curve (Col. 3) were taken from Figs. 12 and 13, respectively, and other columns were computed for: The vehicle speed rating of the highway (Cols. 4 and 5); the rated speed plus 10 miles per hr (Cols. 6, 7, and 8); and the rated speed plus 20 miles per hr (Cols. 9, 10, and 11).

It is quite obvious that the vehicle driver who attempts to exceed the safe speed of the highway, either at constant speed or by increasing his speed,to pass another vehicle quickly creates $(e+f)$-conditions that are not safe and comfortable, but hazardous. This is especially so for vehicles moving at speeds exceeding the vehicle speed rating of a slow-speed highway. Fixing a value of $\gamma = 1$, however, keeps the value of γ in Col. 8 (Table 4) between limits of 1.3 at 110 miles per hr and 3.4 at 30 miles per hr, and, in Col. 11, between the limits of 1.7 at 120 miles per hr and 4.7 at 50 miles per hr. These resulting values for γ are then near or within the maximum value of $\gamma = 2$ established previously, except in those cases where the highway speed ratings of 10 miles per hr and 20 miles per hr (Col. 1, Table 4) are exceeded by 10 miles per hr (Cols. 6, 7, and 8) and those of from 10 to 40 miles per hr (Col. 1), are exceeded by 20 miles per hr (Cols. 9, 10, and 11). In view of this analysis, the graphs, Figs. 13, 14, and 15, were prepared by making $\gamma = 1$ ft per sec^3, which should be the desirable upper limit.

TABLE 4.—COMPARATIVE EFFECT OF EXCEEDING SAFE DESIGN SPEEDS ON A HIGHWAY

Design speed (miles per hr)	Maximum curve[a] (degrees)	Spiral length[b] (ft)	At Design Speed[c]		10-Mile Excess[d]			20-Mile Excess[e]		
			V^2/r	$(e+f)$	V^2/r	$(e+f)$	γ	V^2/r	$(e+f)$	γ
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
100	1.5	820	5.6	0.175	6.8	0.212	1.3	8.1	0.252	1.7
90	2	800	6.1	0.189	7.5	0.234	1.4	9.1	0.283	1.8
80	2.5	700	6.0	0.187	7.6	0.236	1.4	9.4	0.292	2.0
70	3.5	660	6.4	0.200	8.4	0.261	1.5	10.6	0.331	2.1
65	4	600	6.3	0.197	8.4	0.262	1.5	10.9	0.337	2.3
60	5	600	6.8	0.210	9.2	0.286	1.6	12.0	0.373	2.3
55	6.5	600	7.4	0.229	10.3	0.320	1.6	13.7	0.426	2.5
50	8	550	7.5	0.234	10.8	0.336	1.7	14.7	0.458	2.7
45	10	500	7.6	0.237	11.4	0.353	1.8	15.9	0.493	3.0
40	13	460	7.8	0.243	12.2	0.379	1.9	17.6	0.546	3.4
30	23	340	7.8	0.242	13.8	0.430	2.4	21.6	0.671	4.7
20	50	220	7.5	0.234	16.9	0.525	3.4	30.1	0.934	8.0
10	200	110	7.5	0.234	30.0	0.935	8.0	67.5	2.10	27.0

[a] The sharpest permissible curve. [b] Recommended length of spiral curve. [c] For the speeds given in Col. 1, $\gamma = 1$ in each case. [d] Results for 10 miles per hr in excess of speeds shown in Col. 1. [e] Results for 20 miles per hr in excess of speeds shown in Col. 1.

Fig. 14 is a graph designed to give the total spiral angle, in degrees, for vehicle speeds of from 10 to 100 miles per hr, the degree of circular curve, and superelevation-plus-friction factor, as well as the sharpest circular curve permissible and recommended. Because of these features, it is a composite graph which may be used in lieu of Figs. 12 and 13. For example, if the vehicle speed rating of the highway is to be 60 miles per hr and a spiraled 3° 00′ circular curve will fit the topography, the spiral angle should be 5.4°, and only 0.126 ft per ft of superelevation plus friction would be required to balance the centrifugal force. Moreover, for example, let us assume that the central angle ($\triangle$) between two tangents is 20° 00′, and that it is desired that the curve joining these straights be transitional throughout, and the vehicle speed rating of the highway is to be 60 miles per hr. (It is not a recommended practice to design curves transitional throughout, but it is useful as a means of demonstrating the easy application of a flexible procedure.) Proceed from the line marking the spiral angle of 10° $\left(\frac{20^\circ}{2}\right)$ on the graph and follow the line upward until it intersects the vehicle speed line of 60 miles per hr. These two lines intersect at almost the same point as the 4° circular curve and 0.17 ft per ft superelevation-plus-friction factor line. Use of a superelevation rate of 0.04 ft per ft would mean 0.13 ft per ft of friction would be required. Almost at a glance, practical results are obtainable in spiral curve design for a transition to any circular curve usable for the intended vehicle speed rating of the highway. It is a simple graphical means of combining Eqs. 89, 90, and 16*b* for ready use.

Messrs. Eichler and Eves expressed an interest in the possibility of improving on the present osculating circle method of laying out the spiral whenever the transit must be set up at any point, *t*, on the curve. This is a simple problem to solve.

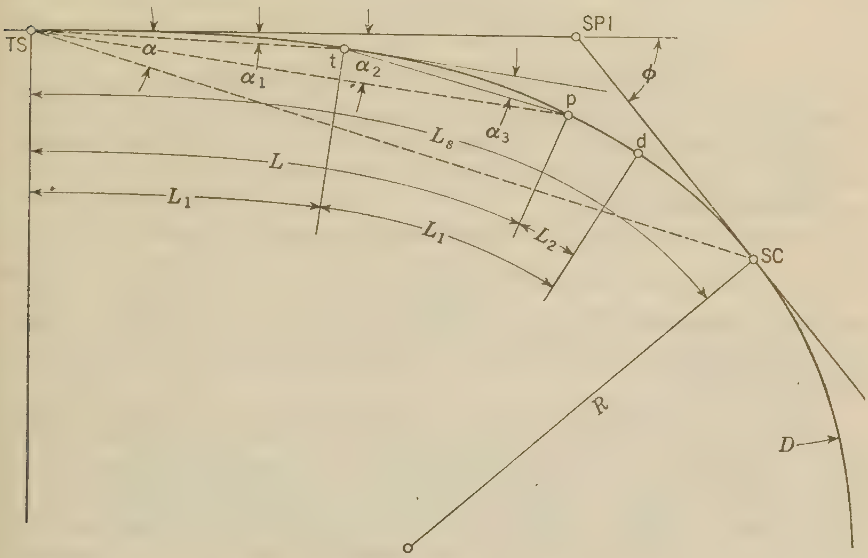

FIG. 16.—DEFLECTION ANGLES ON A SPIRAL CURVE

Derivations will not be given here; but the deflection angle α_2 (in minutes) from the beginning of the spiral (TS, Fig. 16) to any point p is:

$$\alpha_2 = \frac{D\,L^2}{10\,L_s} - c \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (93)$$

in which D is the degree of circular curve connected to the spiral curve at a distance L_s (feet) from the TS; and L is the distance in feet along the spiral curve from the TS to point p. A table of corrections c, in minutes, corresponding to the given spiral angles ϕ, in degrees, at 0.1 increments of the total spiral length, beginning at 0.3 and ending at 1.0 of the spiral length, is available (59).

The deflection angle from the tangent to the spiral curve at any transit point, t, to any point, p, is equal to the deflection calculated from the TS to that point, plus a straight-line correction. This correction is:

$$C' = \frac{D\,L_1\,L_2}{10\,L_s} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (94)$$

The constant $\frac{D}{10\,L_s}$, as used in the formula for the deflection from the TS to point p, is also used in this expression. The length L_1 is the distance in feet from the TS to the transit, and L_2 is the distance, in feet, from a point d to the deflection point p. Point d is at a distance L_1, in feet, from the transit in direction of the sharpest curvature. When distance L_2 to point p is measured from point d toward the TS it is negative, and when measured from point d in a direction away from TS the distance L_2 is positive. This fact must be remembered when calculating deflections for any transit position not on the TS.

The simple formula—

$$\alpha_3 = \frac{D\,L^2}{10\,L_s} + \frac{D\,L_1\,L_2}{10\,L_s} - c \qquad (95)$$

—can be used to solve the minutes of deflection from any transit setup to any deflection point. In field work, it is customary first to calculate the α_2-deflections from the TS to all spiral curve points, and to record them in the field notebook. When it is necessary to move the transit forward the simple corrections are easily made.

If the length of spiral L_s is divisible into the degree of circular curve D an even number of times the constant $\frac{D}{10\,L_s}$ and the constant $\frac{D\,L_1}{10\,L_s}$ are simple decimal fractions with definite digit endings, which is of considerable advantage to the transitman. This point can be easily kept in mind when the spiral curve is designed.

The paper has served to assemble, in one place, many of the salient points regarding the clothoid (Euler's spiral), which is a curve that follows an exact law. Approximations should be avoided as much as practicable because the inexperienced engineer will eventually be embarrassed when he uses approximate formulas where more exact results are required. Once tables have been prepared on an exact basis, the labor of calculating geometric parts (functions) of the spiral curve by the use of long-series equations, or by using approximations of such equations, has been eliminated. Then the office and field work of designing and staking spiral curves as transitions in highway alinement becomes simple and easy.

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STRUCTURAL SKEW PLATES

BY F. L. EHASZ,[1] JUN. AM. SOC. C. E.

WITH DISCUSSION BY MESSRS. JOHN W. ALLEN, WILLIAM A. CONWELL, JULIUS SIGMAN, AND F. L. EHASZ

SYNOPSIS

In the theory of elasticity it is sometimes feasible by means of approximate methods to obtain analyses of problems which do not lend themselves readily to exact treatment. This is true in the study of the structural behavior of plates and slabs. It is particularly valid for skew plates and slabs which, so far as the writer knows, have not been analyzed by other than approximate methods in technical literature. The object of this paper is to present a comparison of results acquired for certain skew plates by means of approximate methods involving finite differences, trigonometric series, and power series.

INTRODUCTION

Some consideration has been given by various investigators in the United States and abroad to the analysis and design of skew plates and slabs. In the field of reinforced concrete these studies have been of considerable interest and practical importance, especially in the case of highway bridge slabs that cross streams, railways, or other highways below at an oblique angle. It is common knowledge that modern traffic and speed requirements in the United States stipulate the construction of many skew structures. Whereas bridge slabs usually are provided with integral curbs and rails, which tend to make the structural behavior more complex, it is nevertheless of interest to ascertain the manner in which the more simple slabs perform. Structural engineers have been studying the skew plate, simply supported on all four sides, the clamped skew plate, and skew slabs simply supported on two sides with two free edges, in addition to the more practical bridge slabs with the integral curbs mentioned previously.

A skew plate or slab may be defined as one having the shape of a parallelogram which is not a rectangle nor a square. Some engineers designate the

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rectangular or square plate as a plate with a skew angle of zero. In any case, the term "degree of skew," in this paper, is defined as the angle of distortion from the rectangular or square shape. This designation corresponds in general with the practice and the technical literature pertaining to other types of skew structures. As in the case of skew arches,[2] the skew effect in plates depends upon the ratio of the lengths of sides as well as on the angle of skew.

It was the purpose of the investigation upon which this paper was based to study theoretically the structural action of skew plates simply supported on all four sides, subjected to a uniformly distributed lateral load, and a clamped skew plate similarly loaded. In the course of these studies various systems of coordinates were used to determine whether the skew boundaries would lend themselves to treatment more readily by one system than by the others. It is to be observed, moreover, that, although the results reported herein pertain directly to metal plates (Poisson's ratio approximately 0.3), they can be modified easily to include other materials, such as concrete, in which case Poisson's ratio may be varied from 0.15 to 0.20.

In this work simply supported skew plates were investigated by the method of finite differences, by trigonometric series satisfying the plate equation throughout and the boundary conditions only at certain points, and by power series. A clamped skew plate was also treated by these methods. A preliminary review was made of solutions for triangular plates with a view to ascertaining the effect of the density of difference-equation networks and to determining some measure of the accuracy to be expected for similar meshes in the case of skew plates.

PREVIOUS STUDIES

The structural behavior of skew plates and slabs has been investigated mainly by the method of finite differences. The first publication on skew plates that has come to the writer's attention is a paper by Cecilia Vittoria Brigatti.[3] Results were obtained by difference equations for uniformly loaded skew plates simply supported and clamped, the sides being of the same length. The fundamental considerations in this work appear to be open to question.

Adolf Anzelius[4] has given some attention to a uniformly loaded skew plate, simply supported on two opposite sides and free on the other two. His solution is in the form of series involving hyperbolic and trigonometric functions, which yield an infinite system of linear equations. The method is approximate and appears to be quite cumbersome. As in other similar approaches, the accuracy of the method depends upon the number of coefficients taken in the series. Mr. Anzelius gave, qualitatively, only twisting moments for a 45° skew slab.

A summary of results obtained by difference equations for uniformly loaded skew slabs, in particular, with two opposite sides simply supported and the other two edges free, has been abstracted from the doctoral work of Helmut Vogt[5] and promulgated in a paper[6] by the same author. Professor Vogt has given particular attention to the arrangement of the reinforcement in bridge slabs. Brief mention was made of skew slabs simply supported on four sides and uniformly loaded.

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[2] "The Design and Construction of a Skew Arch," by S. C. Hollister, *Proceedings*, A.C.I., Vol. XXIV, 1928, p. 371.

[3] "Applicazione del metodo di H. Marcus al calcolo della piastra parallelogrammica," by Cecilia Vittoria Brigatti, *Ricerche di Ingegneria*, Vol. XVI, No. 2, March–April, 1938, p. 42.

[4] "Über die elastische Deformation parallelogrammförmiger Platten," by Adolf Anzelius, *Der Bauingenieur*, Vol. 20, No. 35–36, September, 1939, p. 478.

Studies of skew slabs by means of difference equations have also been made by V. P. Jensen,[7] M. Am. Soc. C. E. Uniformly loaded skew slabs simply supported on four sides, and similarly loaded skew slabs simply supported on two opposite edges and free on the other two sides, were analyzed by use of difference equations developed in a form readily applicable to networks made up of lines parallel to the sides, regardless of the degree of skew. In these slabs the ratio of the length of the long side to that of the short span was kept nearly constant and equal to a value of 2.0. Poisson's ratio was taken to be 0.2. Particular attention was given by Professor Jensen to a simple-span slab bridge with curbs and a 45° skew. The method of obtaining influence surfaces by means of difference equations as recommended by Nathan Newmark,[7] Assoc. M. Am. Soc. C. E., proved to be useful in these studies. Also, Professor Jensen considered the effectiveness of reinforcement for various arrangements of the main and transverse steel.

ORDINARY THEORY OF PLATES

Notation.—The letter symbols in this paper are identified in the text, or by diagram, where they first appear and, for convenience of reference, are assembled alphabetically in the Appendix.

Assumptions and Limitations.—In the ordinary theory of plates several simplifying assumptions are made, resulting in concomitant limitations. As a matter of convenience the plate is considered to be horizontal, homogeneous, elastic, isotropic, uniform in thickness, and subjected to lateral loads only. In addition to being uniform, the thickness must be small compared with the lateral dimensions. The deflections and the energy of deformation in the medium-thick plate are not affected by the vertical stresses (tensions, compressions, and shears) as they are in the case of thick plates. Likewise, the energy due to shortening and stretching of the middle surface may be neglected, which is tantamount to stating that the deflections are small relative to the thickness of the plate. This work of deformation, naturally, must be considered in the theory of thin plates. It is taken for granted, furthermore, that Hooke's law applies to the horizontal strains. In the theory of beams, it will be recalled, plane cross sections before bending are assumed to remain plane after bending. Similarly, in the ordinary theory of plates, straight lines normal to the middle surface of the plate before bending remain straight and normal after bending. Analogous to the concept of straight-line variation of tensile and compressive stresses in the cross section of the beam, there follows in the case of the plate the postulation that horizontal unit stresses (tensions, compressions, and shears) in vertical sections are distributed linearly.

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[5] "Beitrag zur Berechnung schiefwinkliger Platten, nebst Anwendung bei der Berechnung und Anordnung der Bewehrung schiefwinkliger Brückenbauwerke," by Helmut Vogt, Dissertation, Technische Hochschule, Hannover, 1940.

[6] "Die Berechnung schiefwinkliger Platten und plattenartiger Brückensysteme," by Helmut Vogt, *Beton und Eisen*, Vol. 39, No. 17, September, 1940, pp. 243–245.

[7] "Analyses of Skew Slabs," by V. P. Jensen, *Bulletin No. 332*, Univ. of Illinois Eng. Experiment Station, Urbana, Ill., September, 1941.

Fundamental Differential Equations.—The derivation of the differential equations for the ordinary theory of plates in Cartesian coordinates has been given by various writers in the technical literature[8,9,10,11] and therefore will not be given in this paper. It is pertinent, however, to outline the basic equations for later reference.

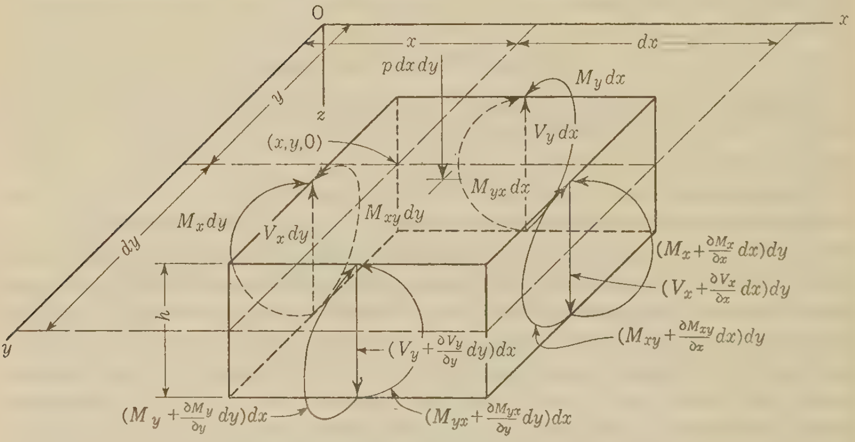
The bending and twisting moments in the distorted plate (see Fig. 1) are given by the expressions

$$M_x = -N\left(\frac{\partial^2 w}{\partial x^2} + \mu\frac{\partial^2 w}{\partial y^2}\right) \qquad \text{(1a)}$$

$$M_y = -N\left(\frac{\partial^2 w}{\partial y^2} + \mu\frac{\partial^2 w}{\partial x^2}\right) \qquad \text{(1b)}$$

and

$$M_{xy} = -(1-\mu)\,N\,\frac{\partial^2 w}{\partial x\,\partial y} \qquad \text{(1c)}$$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$M_{yx}\,dx$
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$V_x\,dy$
$M_{xy}\,dy$
dy
h
y
$(M_x+\frac{\partial M_x}{\partial x}dx)dy$
$(V_x+\frac{\partial V_x}{\partial x}dx)dy$
$(M_{xy}+\frac{\partial M_{xy}}{\partial x}dx)dy$
$(V_y+\frac{\partial V_y}{\partial y}dy)dx$
$(M_y+\frac{\partial M_y}{\partial y}dy)dx$
$(M_{yx}+\frac{\partial M_{yx}}{\partial y}dy)dx$

FIG. 1.—RECTANGULAR ELEMENT OF A PLATE (THE DIRECTIONS SHOWN ARE POSITIVE)

in which w designates the deflection, μ is Poisson's ratio for the material, and N, the flexural rigidity, is a function of the modulus of elasticity, E, of the material, the plate thickness, h, and Poisson's ratio, as is manifested from the equation

$$N = \frac{E\,h^3}{12\,(1-\mu^2)} \qquad \text{(2)}$$

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[8] "Theory of Plates and Shells," by S. Timoshenko, 1st Ed., McGraw-Hill Book Co., Inc., New York and London, 1940, p. 85.

[9] "An Introduction to the Theory of Elasticity for Engineers and Physicists," by R. V. Southwell, Oxford Univ. Press, 1936, p. 228.

[10] "Die elastischen Platten," by A. Nadai, Julius Springer, Berlin, 1925, p. 18.

[11] "Die Theorie elastischer Gewebe und ihre Anwendung auf die Berechnung biegsamer Platten," by H. Marcus, 2d Ed., Julius Springer, Berlin, 1932.

The Lagrange equation for the flexure of plates produced by a uniformly distributed lateral load p may be shown to be

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2\,\partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{p}{N} \qquad (3)$$

or

$$\nabla^4 w = \nabla^2\nabla^2 w = \frac{p}{N} \qquad (4)$$

in which

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \qquad (5)$$

is the Laplace operator. From Eqs. 1 it is clear that the moment sum is expressible in the form

$$M_x + M_y = -(1+\mu)\,N\,\nabla^2 w \qquad (6)$$

The vertical shears are given by the equations

$$V_x = -N\frac{\partial}{\partial x}\nabla^2 w \qquad (7a)$$

and

$$V_y = -N\frac{\partial}{\partial y}\nabla^2 w \qquad (7b)$$

Maximum stresses, occurring at the surface, are

$$\sigma_x = \pm\frac{6\,M_x}{h^2} \qquad (8a)$$

$$\sigma_y = \pm\frac{6\,M_y}{h^2} \qquad (8b)$$

and

$$\tau_{xy} = \pm\frac{6\,M_{xy}}{h^2} \qquad (8c)$$

From the shears and the twisting moments it is possible to deduce the vertical reactions

$$R_x = V_x + \frac{\partial M_{xy}}{\partial y} = -N\left[\frac{\partial^3 w}{\partial x^3} + (2-\mu)\frac{\partial^3 w}{\partial x\,\partial y^2}\right] \qquad (9a)$$

and

$$R_y = V_y + \frac{\partial M_{xy}}{\partial x} = -N\left[\frac{\partial^3 w}{\partial y^3} + (2-\mu)\frac{\partial^3 w}{\partial x^2\,\partial y}\right] \qquad (9b)$$

These reactions are defined in the same manner as the vertical shears.

Polar Coordinates.—The differential equations for the bending of plates may be readily placed into polar form by some simple transformations. It will be necessary later to use the concept of average curvature at a point of the distorted plate and, consequently, it is introduced here. Curvatures in the xz-plane and the yz-plane are

$$\frac{1}{r_x} = \frac{\partial}{\partial x}\left(\frac{\partial w}{\partial x}\right) = \frac{\partial^2 w}{\partial x^2} \qquad (10a)$$

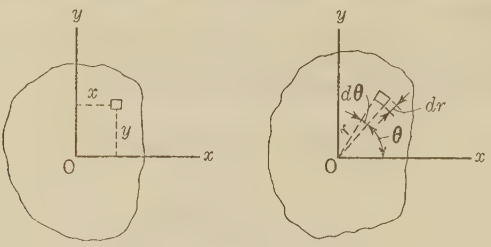
and

$$\frac{1}{r_y} = \frac{\partial}{\partial y}\left(\frac{\partial w}{\partial y}\right) = \frac{\partial^2 w}{\partial y^2} \qquad (10b)$$

respectively. It can be shown that at the same point the sum of the curvatures in any two perpendicular directions n and t is equal to the sum of Eqs. 10 or, symbolically,

$$\frac{1}{r_n} + \frac{1}{r_t} = \frac{\partial^2 w}{\partial n^2} + \frac{\partial^2 w}{\partial t^2} = \frac{1}{r_x} + \frac{1}{r_y} = \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \qquad (11)$$

r_n and r_t are defined in a manner similar to r_x and r_y. The invariant sum defined by Eq. 11 has been designated as the average curvature of the plate's surface at a point. Transformation from Cartesian to polar coordinates (see Fig. 2) yields the result



(a) CARTESIAN (b) POLAR

FIG. 2.—TRANSFORMATION TO POLAR COORDINATES

$$\nabla^2 w = \frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2} \qquad (12a)$$

By repetition of this operation it is possible to obtain the Lagrange plate equation in the form

$$\nabla^2\nabla^2 w = \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}\right)\left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2}\right) = \frac{p}{N} \qquad (12b)$$

Eqs. 12 will be used subsequently in the approximate trigonometric-series solution for a skew plate.

From simple transformation equations, expressions for moments in polar form may be deduced by stipulating that the x-axis coincides with the radius r. If t is the direction (tangential) perpendicular to the radial direction r, M_r and M_t are the moments per unit length acting in sections perpendicular to r and t, respectively. These moments and the corresponding twisting moments, M_{rt}, may be shown to be

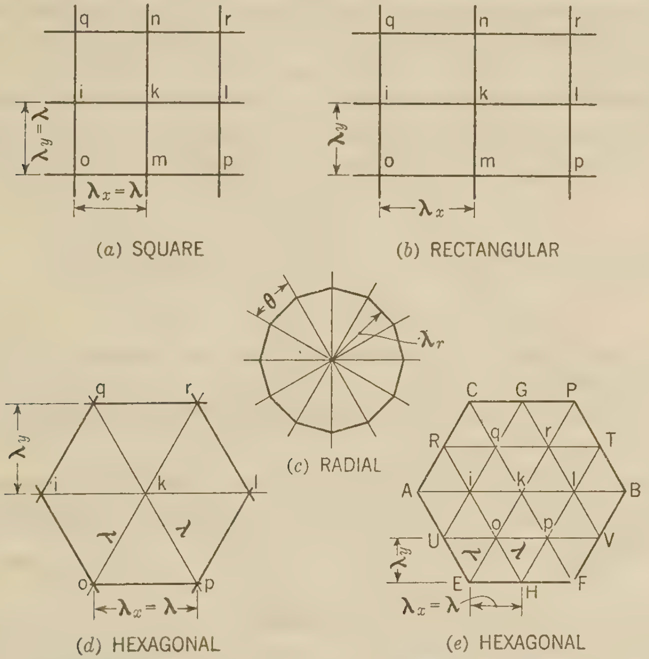
$$M_r = -N\left[\frac{\partial^2 w}{\partial r^2} + \mu\left(\frac{1}{r}\frac{\partial w}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2}\right)\right] \qquad (13a)$$

$$M_t = -N\left(\mu\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2}\right) \qquad (13b)$$

and

$$M_{rt} = -(1-\mu)\,N\left(\frac{1}{r}\frac{\partial^2 w}{\partial r\,\partial\theta} - \frac{1}{r^2}\frac{\partial w}{\partial\theta}\right) = -(1-\mu)\,N\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial w}{\partial\theta}\right) \quad ..(13c)$$

It is possible to obtain a power series in quadrilinear form that would yield an approximate solution of skew plate problems. Since this procedure would have entailed considerably more work than a similar series approach in Cartesian form, solutions were not sought by quadrilinear coordinates. In this work the power-series approach, useful for attacking a wide variety of plate problems, was dealt with only in Cartesian coordinates and was applied to a few plate problems which were treated also by other methods.



(*a*) SQUARE (*b*) RECTANGULAR (*c*) RADIAL (*d*) HEXAGONAL (*e*) HEXAGONAL

FIG. 3.—TYPES OF NETWORKS FOR DIFFERENCE EQUATIONS

Difference Equations.—By means of difference equations it is possible to obtain approximate solutions for many plate problems which cannot be treated easily by exact methods. Skew plates and slabs fall into this category. Many writers have applied this method to plate problems.[8–17] Since finite squares, rectangles, and triangles (hexagons) (see Fig. 3) are used in the formation of

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[12] "Bestemmelse af Spaendinger i Plader ved anvendelse af Differensligninger," by N. S. Nielsen, Kopenhagen, 1920.

[13] "Die elastischen Platten," by A. Nadai, Julius Springer, Berlin, 1925, p. 205.

[14] "The Calculation of Flat Plates by the Elastic Web Method," by Joseph A. Wise, *Proceedings*, A.C.I., 1928, p. 408.

[15] "Design of Reinforced Concrete Slabs," by Joseph A. Wise, *ibid.*, 1929, p. 712.

[16] "Analysis of Plate Examples by Difference Methods and the Superposition Principle," by D. L. Holl, *Journal of Applied Mechanics, Transactions*, A.S.M.E., Vol. 58, September, 1936, p. A-81.

[17] "Cantilever Plate with Concentrated Edge Load," by D. L. Holl, *ibid.*, Vol. 59, March, 1937, p. A-8.

difference-equation networks, rather than the corresponding infinitesimal elements associated with differential equations, only approximate results are obtained. The approximations become better as the elements become smaller. An attempt was made in this investigation to ascertain the effect of the density of the networks for triangular plates as well as skew plates.

The procedure for the method of finite differences as presented by H. Marcus,[11] M. Am. Soc. C. E., was followed throughout this work. According to Mr. Marcus, the elastic web is said to consist of a network of elastic strings attached to the edges of the plate in such a way as to conform with the boundary conditions of the actual plate or slab. In the case of a plate that supports a concentrated load, it is desirous to arrange a network that has a point of intersection at the position of the load. If the load is uniformly distributed, it is replaced by a statically equipollent system of concentrated forces applied at the intersection points contiguous to the loaded area. The difference equations relating the deflections of the web usually yield a system of simultaneous linear equations in normal form. A function of the moment sum can be written

$$\bar{M} = -N\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) \dots\dots\dots\dots\dots\dots (14)$$

From Eqs. 4 and 14 the equation of the elastic surface becomes

$$\nabla^2 \bar{M} = -p \dots\dots\dots\dots\dots\dots\dots\dots (15)$$

The sum of the moments in the x-direction and the y-direction is

$$M_x + M_y = -N(1+\mu)\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) = (1+\mu)\,\bar{M} \dots\dots (16)$$

The deflections, $\bar{w}$, of a membrane subjected only to fiber stresses parallel to the surface, are related to the horizontal component, S, of the surface stresses by the differential equation

$$\nabla^2 \bar{w} = \frac{\partial^2 \bar{w}}{\partial x^2} + \frac{\partial^2 \bar{w}}{\partial y^2} = -\frac{p}{S} \dots\dots\dots\dots\dots\dots (17)$$

Manifestly, S must be constant over the entire membrane because of conditions of static equilibrium. If $S = 1$, comparison of Eqs. 15 and 17 suggests the theorem:[16]

> "The deflection of a membrane subjected to loads proportional to those on a given plate may be considered as the sum of the principal moments of the actual plate."

Similar to the procedure in the case of beams, it is possible to derive another theorem:

> "A second membrane may be loaded with elastic weights proportional to these moment sums and, subject to appropriate boundary conditions, the deflections of the latter membrane will be proportional to the deflections of the actual plate under the given loading system."

The elastic weights in the second theorem are actually $\frac{\bar{M}}{N}$.

It is obvious that the solution of the plate problem is manifested by the equations

$$\nabla^2 \bar{M} = -p \qquad (18a)$$

and

$$\nabla^2 w = -\frac{\bar{M}}{N} \qquad (18b)$$

By analogy and equilibrium considerations one may obtain the relations

$$\bar{M}_k = S_1 \bar{w}_k \qquad (19)$$

in which S_1 is the quantity S referred to the first membrane, and

$$w_k = \bar{z}_k \frac{S_1 S_2}{N} \qquad (20)$$

in which $\bar{z}_k$ designates the deflection of the nodal point k in the second membrane. Equations relating the web deflections for the square mesh may be written as follows:

$$4\,\bar{w}_k - (\bar{w}_i + \bar{w}_l + \bar{w}_m + \bar{w}_n) = \frac{p_k \lambda^2}{S_1} \qquad (21a)$$

and

$$4\,\bar{z}_k - (\bar{z}_i + \bar{z}_l + \bar{z}_m + \bar{z}_n) = \frac{\bar{M}_k \lambda^2}{S_1 S_2} \qquad (21b)$$

in which the subscripts refer to the points in Fig. 3(*a*). For the hexagonal network, the following expressions apply (see Figs. 3(*d*) and 3(*e*)):

$$4\,\bar{w}_k - \frac{2}{3}(\bar{w}_i + \bar{w}_l + \bar{w}_o + \bar{w}_p + \bar{w}_q + \bar{w}_r) = \frac{p_k \lambda^2}{S_1} \qquad (22a)$$

and

$$4\,\bar{z}_k - \frac{2}{3}(\bar{z}_i + \bar{z}_l + \bar{z}_o + \bar{z}_p + \bar{z}_q + \bar{z}_r) = \bar{M}_k \frac{\lambda^2}{S_1 S_2} \qquad (22b)$$

From the simultaneous linear equations the values of $\bar{w}_k$ and $\bar{z}_k$ are obtained. Once these are found the $\bar{M}_k$-functions and deflections of the actual plate result from Eqs. 19 and 20.

Bending moments and twisting moments in the case of rectangular meshes (see Fig. 3(*b*)) are given by the expressions[18]

$$M_x = S_1 S_2 \left[\frac{2\,\bar{z}_k - \bar{z}_i - \bar{z}_l}{\lambda^2_x} + \mu \frac{2\,\bar{z}_k - \bar{z}_m - \bar{z}_n}{\lambda^2_y} \right] \qquad (23a)$$

$$M_y = S_1 S_2 \left[\frac{2\,\bar{z}_k - \bar{z}_m - \bar{z}_n}{\lambda^2_y} + \mu \frac{2\,\bar{z}_k - \bar{z}_i - \bar{z}_l}{\lambda^2_x} \right] \qquad (23b)$$

and

$$M_{xy} = (1 - \mu)\, S_1 S_2 \left[\frac{(\bar{z}_p + \bar{z}_q) - (\bar{z}_o + \bar{z}_r)}{4\,\lambda_x \lambda_y} \right] \qquad (23c)$$

---

[18] "Die Theorie elastischer Gewebe und ihre Anwendung auf die Berechnung biegsamer Platten," by H. Marcus, 2d Ed., Julius Springer, Berlin, 1932, p. 18.

values of λ being defined in Fig. 3. The vertical shearing forces are to be found from the equations

$$V_x = S_1 \frac{\partial \bar{w}}{\partial x} = \frac{S_1}{2\,\lambda_x} (\bar{w}_l - \bar{w}_i) \qquad (24a)$$

and

$$V_y = S_1 \frac{\partial \bar{w}}{\partial y} = \frac{S_1}{2\,\lambda_y} (\bar{w}_n - \bar{w}_m) \qquad (24b)$$

Bending moments in the directions 1, 2, and 3 (in which $M_2 = M_x$ and directions 1 and 3 are −60° and +60°, respectively) may be obtained from the following:

$$M_{1k} = (1 - \mu)\, S_1\, S_2 \frac{2\,\bar{z}_k - \bar{z}_p - \bar{z}_q}{(\lambda_y \sec \alpha)^2} + \mu\, S_1\, \bar{w}_k \qquad (25a)$$

$$M_{2k} = (1 - \mu)\, S_1\, S_2 \frac{2\,\bar{z}_k - \bar{z}_i - \bar{z}_l}{\lambda^2_x} + \mu\, S_1\, \bar{w}_k \qquad (25b)$$

and

$$M_{3k} = (1 - \mu)\, S_1\, S_2 \frac{2\,\bar{z}_k - \bar{z}_o - \bar{z}_r}{(\lambda_y \sec \alpha)^2} + \mu\, S_1\, \bar{w}_k \qquad (25c)$$

in which $2\,\alpha$ equals 60°. Vertical shearing forces are given by the terms

$$V_{1k} = S_1 \frac{\bar{w}_p - \bar{w}_q}{2\,\lambda_y \sec \alpha} \qquad (26a)$$

$$V_{2k} = S_1 \frac{\bar{w}_l - \bar{w}_i}{2\,\lambda_x} \qquad (26b)$$

and

$$V_{3k} = S_1 \frac{\bar{w}_r - \bar{w}_o}{2\,\lambda_y \sec \alpha} \qquad (26c)$$

The twisting moments result from the expression (see Fig. 4(*a*)):

$$M_{xy} = -(1 - \mu)\, N \frac{\partial^2 w}{\partial x\, \partial y} = (1 - \mu) \frac{S_1\, S_2}{2\,\lambda_x\, \lambda_y} [(\bar{z}_p + \bar{z}_q) - (\bar{z}_o + \bar{z}_r)] \qquad (27)$$

As stated previously, $M_x = M_2$. In order to find M_y, it is possible to use the simple equation (from Eq. 16):

$$M_y = (1 + \mu)\, \bar{M} - M_x \qquad (28)$$

The maximum and minimum bending moments may be shown to be

$$M_{\max} = \frac{M_x + M_y}{2} + \frac{1}{2} \sqrt{(M_x - M_y)^2 + 4\, M^2_{xy}} \qquad (29a)$$

and

$$M_{\min} = \frac{M_x + M_y}{2} - \frac{1}{2} \sqrt{(M_x - M_y)^2 + 4\, M^2_{xy}} \qquad (29b)$$

The maximum value of the twisting moment is

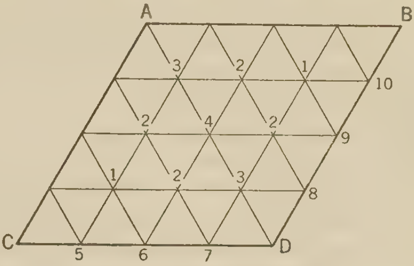
$$M_t = \frac{1}{2} \sqrt{(M_x - M_y)^2 + 4\, M^2_{xy}} \qquad (30)$$

To obtain the trajectories of the principal moments (see Fig. 2(*b*)), use is made of the equation

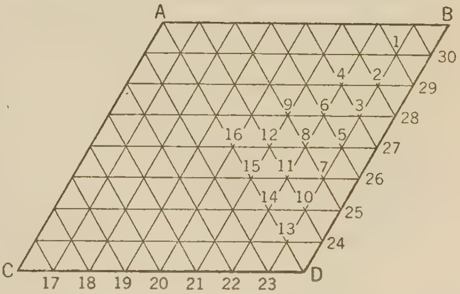
$$\tan 2\,\theta = \frac{2\,M_{xy}}{M_x - M_y} \qquad (31)$$

PRELIMINARY SURVEY OF TRIANGULAR PLATES

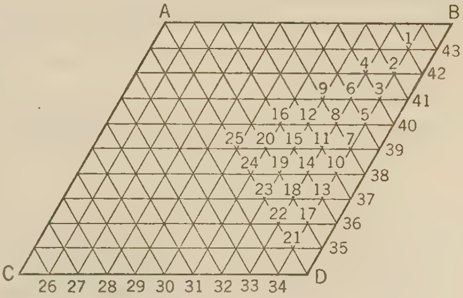
A brief preliminary study of the effect of density of difference-equation networks on certain deflections and moments was made for the equilateral triangular plate, the 30°–60°–90° triangular plate, and an isosceles triangular plate having a right angle, all plates being simply supported and subjected to a uniformly distributed load. Comparison with published results obtained by more exact methods indicated that a relatively small number of nodal points gives values that are in fair agreement with those of the more accurate solutions. This is especially true in the case of bending moments. In general, moments and deflections for points nearer the center of gravity of the plates showed better agreement than those for points nearer the boundaries. Net works considered involved from two to twenty-one coefficients. Because of lack of space, however, detailed observations for triangular plates are not given herein.



(*a*) FOUR COEFFICIENTS



(*b*) SIXTEEN COEFFICIENTS



(*c*) TWENTY-FIVE COEFFICIENTS

FIG. 4.—NETWORKS FOR 30° SKEW PLATES (AB = AC = 2 *c*)

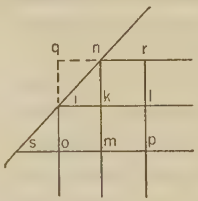
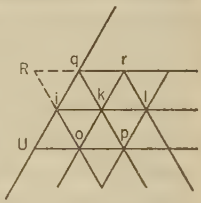
SIMPLY SUPPORTED 30° SKEW PLATE WITH EQUAL SIDES SUBJECTED TO A UNIFORMLY DISTRIBUTED LOAD

Analysis by Method of Finite Differences.—The most convenient network for the analysis of a 30° skew plate by the difference-equation method is a triangular (hexagonal) arrangement formed by drawing lines parallel to the sides and the shorter diagonal of the plate (see Figs. 3(*d*) and 4). To determine the effect of the mesh intervals on the results for deflections and moments in the case of the uniformly loaded, simply supported, 30° skew plate having equal sides (length = 2 c), three networks of varying density ($\lambda = c/2, c/4, c/5$) were analyzed by this method.

The difference equations relating the $\bar{w}$-values and $\bar{z}$-values at the intersections of the lines forming the networks result in a system of simultaneous linear equations. The number of those equations depends upon the number of intersection points or nodal points. In the particular plate under consideration, there is symmetry about both diagonals and, consequently, only one quarter of the plate need be analyzed.

If the length of the sides is divided into four parts ($\lambda = c/2$), it is apparent that four nodal points result (some of them are repeated), and, therefore, four simultaneous linear equations must be written and four coefficients are to be found. For $\lambda = c/4$ and $\lambda = c/5$, sixteen and twenty-five equations, respectively, must be written. As stated previously, the equations relating the deflections of the membranes involved in this method are usually in normal form, and they may be solved conveniently by the "Doolittle Method" of solving simultaneous equations.[19] The boundary conditions for the membranes associated with simply supported plates are indicated in Table 1. It is to be noted that the relations given for $\bar{z}$ in this table are also valid for $\bar{w}$.

TABLE 1.—BOUNDARY CONDITIONS FOR SKEW PLATES

Condition	 (a) SQUARE MESH		 (b) HEXAGONAL MESH	
	$\bar{z}_s$	$\bar{z}_q$	$\bar{z}_u$	$\bar{z}_R$
Simply supported edge	$\bar{z}_s = \bar{z}_n = 0$	$-\bar{z}_k$	$\bar{z}_i = \bar{z}_q = 0$	$-\bar{z}_k$
Clamped edge	$\bar{z}_i = \bar{z}_n = 0$	$+\bar{z}_k$	$\bar{z}_i = \bar{z}_q = 0$	$+\bar{z}_k$

Substitution in Eqs. 22 gives the following equations for the sixteen-coefficient solution—

$$\left.\begin{aligned} 4\,\bar{w}_1 - \frac{2}{3}\,(2\,\bar{w}_2) &= \frac{p\,\lambda^2}{S_1} \\ 4\,\bar{w}_2 - \frac{2}{3}\,(\bar{w}_1 + \bar{w}_2 + \bar{w}^3 + \bar{w}_4) &= \frac{p\,\lambda^2}{S_1} \end{aligned}\right\}\ldots\ldots\ldots\ldots\ldots(32)$$

—etc.; and

$$\left.\begin{aligned} 4\,\bar{z}_1 - \frac{2}{3}\,(2\,\bar{z}_2) &= \bar{w}_1\,\frac{\lambda^2}{S_2} \\ 4\,\bar{z}_2 - \frac{2}{3}\,(\bar{z}_1 + \bar{z}_2 + \bar{z}_3 + \bar{z}_4) &= \bar{w}_2\,\frac{\lambda^2}{S_2} \end{aligned}\right\}\ldots\ldots\ldots\ldots\ldots(33)$$

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[19] "Application of the Theory of Least Squares to the Adjustment of Triangulation," by Oscar S. Adams, *Special Publication No. 28*, U. S. Coast and Geodetic Survey, Government Printing Office, Washington, D. C., 1915.

etc. Eqs. 32 and 33 may be put into the form given in Table 2. The last two columns in this table are the terms that appear on the right side of Eqs. 32 and 33. Hence, it is to be remembered that after each numerical value in these two columns (Table 2) there are actually an equal sign and zero, since a minus sign precedes each one. The "Doolittle Method" was adopted for simultaneous equations that happened to occur in normal form.[19]

TABLE 2.—NORMAL FORM OF DIFFERENCE EQUATIONS, EQUAL-SIDED, 30° SKEW PLATE; UNIFORMLY DISTRIBUTED LOAD, SIMPLY SUPPORTED

COEFFICIENT NO.:																$\frac{p\,\lambda^2}{S_1} = A$	$\frac{p\,\lambda^4}{S_1\,S_2} = B$
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16		
3	−1															−0.750000	−0.463973
−1	5	−1	−1													−1.500000	−1.658843
	−1	6	−1	−1	−1											−1.500000	−2.192174
	−1	−1	3		−1											−0.750000	−1.462041
		−1		6	−1	−1	−1									−1.500000	−2.523863
		−1	−1	−1	5		−1	−1								−1.500000	−3.796233
				−1		6	−1		−1	−1						−1.500000	−2.636190
				−1	−1	−1	6	−1		−1	−1					−1.500000	−4.268592
					−1		−1	3			−1					−0.750000	−2.411222
						−1			6	−1		−1	−1			−1.500000	−2.469393
						−1	−1		−1	6	−1		−1	−1		−1.500000	−4.305309
							−1	−1		−1	5			−1	−1	−1.500000	−5.277498
									−1			3	−0.5			−0.750000	−0.915260
									−1	−1		−0.5	3	−0.5		−0.750000	−1.897150
										−1	−1		−0.5	3	−0.5	−0.750000	−2.567936
											−1			−0.5	1.50	−0.375000	−1.401323

TABLE 3.—SOLUTION OF EQS. 32 AND 33 WITH REFERENCE TO TABLE 2

Point number:	$\bar{w}$	$\bar{z}$	Point number:	$\bar{w}$	$\bar{z}$
1	0.618631 A	0.811050 B	9	3.214962 A	8.697569 B
2	1.105895 A	1.969178 B	10	1.646262 A	3.956232 B
3	1.461449 A	3.107640 B	11	2.870206 A	7.690210 B
4	1.949388 A	4.268346 B	12	3.518332 A	9.904091 B
5	1.682575 A	3.949917 B	13	1.220346 A	2.718592 B
6	2.530822 A	6.266184 B	14	2.529533 A	6.568515 B
7	1.757560 A	4.290623 B	15	3.423915 A	9.605294 B
8	2.845728 A	7.511201 B	16	3.736860 A	10.738709 B

The solution yields the values for $\bar{w}_1$, $\bar{w}_2$, $\bar{z}_1$, $\bar{z}_2$, etc., given in Table 3, in which, to simplify typography,

$$A = \frac{p\,\lambda^2}{S_1} \qquad (34a)$$

and

$$B = \frac{p\,\lambda^4}{S_1\,S_2} \qquad (34b)$$

When all the $\bar{w}$-coefficients and $\bar{z}$-coefficients are found, it is an easy matter to obtain the deflections and moments. These are indicated for the sixteen-coefficient solution in Table 4. It is necessary to refer to Figs. 3(*d*) and 4(*b*) in the preparation and interpretation of these results. Similar analyses were made for the uniformly loaded, simply supported, 30° skew plate or slab, having equal sides, by utilizing four-coefficient and twenty-five-coefficient difference-equation networks (see Figs. 4(*a*) and 4(*c*)).

Table 5 affords a comparison of the moments and deflections at the center of the equal-sided, uniformly loaded, simply supported, 30° skew plate for three different networks. It is seen that the moments M_1, M_2, M_3, M_x, and M_y are in fairly good agreement even in the four-coefficient case. The twisting moments, M_{xy}, are in fair agreement only for the sixteen-coefficient and twenty-

TABLE 4.—UNIFORMLY LOADED, SIMPLY SUPPORTED, 30° SKEW PLATE HAVING EQUAL SIDES

(Sixteen Coefficients, $\lambda = c/4$ and $\mu = 0.3$)

Point	Deflection $\frac{w N}{p c^4}$	Moments								Degrees, θ
		$\frac{M_1}{p \lambda^2}$	$\frac{M_2}{p \lambda^2}$	$\frac{M_3}{p \lambda^2}$	$\frac{M_x}{p \lambda^2}$	$\frac{M_y}{p \lambda^2}$	$\frac{M_{xy}}{p \lambda^2}$	$\frac{M_{\max}}{p \lambda^2}$	$\frac{M_{\min}}{p \lambda^2}$	
1	0.003168	1.321059	−0.057366	−0.057366	−0.057366	0.861586	−0.795830	1.321057	0.516837	30
2	0.007692	1.710194	0.100776	0.345535	0.100776	1.336888	−0.787882	1.720206	−0.282542	26
3	0.012139	1.801289	0.402802	0.645765	0.402802	1.497082	−0.667139	1.812749	0.087135	25.5
4	0.016673	2.209804	0.795747	0.795747	0.795747	1.738457	−0.816402	2 209804	0.324400	30
5	0.015429	1.648328	0.776816	0.855873	0.776816	1 410532	−0.457522	1.650204	0.537145	27.5
6	0.024477	2.380634	1.268258	1.286222	1.268258	2.021811	−0.631856	2.380698	0.909369	29.5
7	0.016760	1.276270	1.150963	0.999806	1.150963	1.133735	−0.159615	1.302199	0.982499	−43.5
8	0.029341	2.277665	1.671594	1.599924	1.671594	2.027852	−0.391292	2.279652	1.419794	33
9	0.033975	2.625404	1.821893	1.821893	1.821893	2.357588	−0.463905	2.625396	1.554056	30
10	0.015454	0.649457	1.436043	1.126153	1.436043	0.704098	0.275219	1.527981	0.612163	18
11	0.030040	1.925130	1.900214	1.771555	1.900214	1.831054	−0.088666	1.960806	1.770462	−34.5
12	0.038688	2.605217	2.146290	2.109223	2.146290	2.427542	−0.286360	2.605942	1.967889	32
13	0.010620	−0.425828	1.402770	1.402770	1.402770	0.183680	1.055736	2.012292	−0.425842	30
14	0.025658	1.328061	1.802272	1.802272	1.802272	1.487121	0.273784	1.960338	1.328054	30
15	0.037521	2.359530	2.158576	2.158576	2.158576	2.292514	−0.116020	2.359507	2.091583	30
16	0.041948	2.707839	2.289523	2.289523	2.289523	2.568395	−0.241513	2.707833	2.150086	30
17	0	....	....	....	0	0	−0.655560	0.655560	−0.655560	45
18	0	....	....	....	0	0	−0.936099	0.936099	−0.936099	45
19	0	....	....	....	0	0	−0.920203	0.920203	−0.920203	45
20	0	....	....	....	0	0	−0.680801	0.680801	−0.680801	45
21	0	....	....	....	0	0	−0.275388	0.275388	−0.275388	45
22	0	....	....	....	0	0	0.270284	0.270284	−0.270284	45
23	0	....	....	....	0	0	1.000367	1.000367	−1.000367	45
24	0	....	....	....	0.866348	−0.866348	0.500184	1.000367	−1.000367	15
25	0	....	....	....	0.234074	−0.234074	0.135142	0.270284	−0.270284	15
26	0	....	....	....	−0.238494	0.238494	−0.137694	0.275388	−0.275388	15
27	0	....	....	....	−0.589594	0.589594	−0.340401	0.680801	−0.680801	15
28	0	....	....	....	−0.796923	0.796923	−0.460101	0.920203	−0.920203	15
29	0	....	....	....	−0.810690	0.810690	−0.468049	0.936099	−0.936099	15
30	0	....	....	....	−0.567735	0.567735	−0.327780	0.655560	−0.655560	15

TABLE 5.—COMPARISON OF MOMENTS AND DEFLECTIONS AT THE CENTER OF THE UNIFORMLY LOADED, SIMPLY SUPPORTED, 30° SKEW PLATE FOR DIFFERENT NETWORKS

Description	Twenty-five coefficients	Sixteen coefficients	Four coefficients
λ	$c/5$	$c/4$	$c/2$
Moments:			
M_1	$0.169566\ p\ c^2$	$0.169240\ p\ c^2$	$0.166256\ p\ c^2$
M_2	$0.142812\ p\ c^2$	$0.143095\ p\ c^2$	$0.146451\ p\ c^2$
M_3	$0.142812\ p\ c^2$	$0.143095\ p\ c^2$	$0.146451\ p\ c^2$
M_x	$0.142812\ p\ c^2$	$0.143095\ p\ c^2$	$0.146451\ p\ c^2$
M_y	$0.160650\ p\ c^2$	$0.160524\ p\ c^2$	$0.159654\ p\ c^2$
M_{xy}	$-0.015446\ p\ c^2$	$-0.015095\ p\ c^2$	$-0.011434\ p\ c^2$
Deflections:			
w	$0.041583\ p\ c^4/N$	$0.041948\ p\ c^4/N$	$0.045291\ p\ c^4/N$

five-coefficient solutions. The deflections at the center of the plate agree fairly well again only for the sixteen-coefficient and twenty-five-coefficient cases.

Solutions in Trigonometric Series.—Another approximate method for determining deflections and moments in a plate is one in which the deflection function is set up in the form of a trigonometric series, satisfying the plate equation. This series contains a set of arbitrary constants that can be computed in such a manner as to satisfy the boundary conditions at an arbitrary number of points. The degree of approximation, naturally, depends upon the number of points selected on the boundary, but it has been found that in many cases a relatively small number of points is sufficient to give fairly reliable results. There is reason to believe that the use of approximate methods is justified, since the assumptions made in theories are very seldom entirely true in practice.

The approximate determination of the stress function for the prismatic bar in torsion and the deflection function in a clamped square plate has been indicated by J. Barta.[20] These problems had been solved previously by more exact methods, and the agreement found by Mr. Barta was fairly good.

The trigonometric-series approach may be used quite readily in the solution of both simply supported and clamped skew plates. Both of these cases are treated for a 30° skew in this paper. It is to be noted (see Fig. 5) that a new set of Cartesian-coordinate axes is chosen in order to take advantage of symmetry.

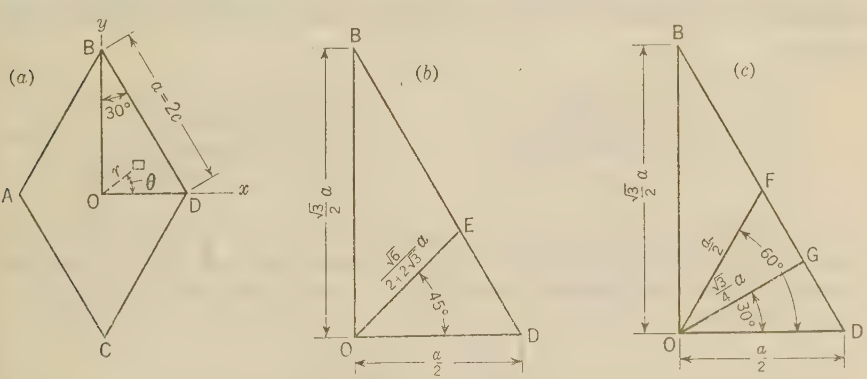

FIG. 5.—DIAGRAMS FOR TRIGONOMETRIC-SERIES ANALYSES OF SKEW PLATES (AB = AC = BD = CD)

In Fig. 5(*c*) it is seen that the number of points chosen along the entire boundary is 12. It is sufficient, therefore, to satisfy the boundary conditions at points B, D, F, and G. Because of symmetry, the assumed deflection function is taken to be

$$w = \frac{p\,r^4}{64\,N} + a_0 + r^2\,a_2\cos 2\,\theta + r^2\,c_2 + r^4\,a_4\cos 4\,\theta + r^4\,c_4\cos 2\,\theta$$
$$+ r^6\,a_6\cos 6\,\theta + r^6\,c_6\cos 4\,\theta + r^8\,c_8\cos 6\,\theta \ldots\ldots\ldots\ldots (35)$$

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[20] "Über die näherungsweise Lösung einiger zweidimensionaler Elastizitätsaufgaben," by J. Barta, *Zeitschrift für Angewandte Mathematik und Mechanik*, Vol. 17, No. 3, 1937, pp. 184–185.

The boundary conditions for a simply supported edge (length of side = a) are: $w = 0$; and

$$\nabla^2 w = \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = 0 \qquad (36)$$

which, in polar form, becomes Eq. 12a equated to zero.

From Eqs. 35, 36, and 12a it is clear that, at the boundary points, the following relations must be valid:

$$w = \frac{p\,r^4}{64\,N} + a_0 + r^2 a_2 \cos 2\,\theta + r^2 c_2 + r^4 a_4 \cos 4\,\theta + r^4 c_4 \cos 2\,\theta$$
$$+ r^6 a_6 \cos 6\,\theta + r^6 c_6 \cos 4\,\theta + r^8 c_8 \cos 6\,\theta = 0 \qquad (37a)$$

and

$$\nabla^2 w = \frac{p\,r^2}{4\,N} + 4\,c_2 + 12\,r^2 c_4 \cos 2\,\theta$$
$$+ 20\,r^4 c_6 \cos 4\,\theta + 28\,r^6 c_8 \cos 6\,\theta = 0 \qquad (37b)$$

It may be shown easily that the assumed deflection function (Eq. 35) satisfies the plate formula, Eq. 4.

By substituting the values of r and θ for points B, D, F, and G in Eqs. 37, and allowing a and its powers to equal unity, it is possible to obtain eight simultaneous equations, the solution of which yields the values of the arbitrary constants:

$$\left.\begin{array}{ll} a_0 = 0.0027059329\,p/N & c_4 = 0.0057873004\,p/N \\ a_2 = -0.0038678560\,p/N & a_6 = 0.0041150544\,p/N \\ c_2 = -0.0148808677\,p/N & c_6 = -0.0138895211\,p/N \\ a_4 = 0.0131565067\,p/N & c_8 = -0.0068034953\,p/N \end{array}\right\} \qquad (38)$$

These values, multiplied by the corresponding powers of a (length of side), may be inserted in Eq. 35 to give the approximate solution for the deflection function. From the resulting expression the maximum value of deflection may be obtained by letting $r = 0$:

$$w_{\max} = 0.0432949264\,\frac{p\,c^4}{N} \qquad (39a)$$

in which $a = 2\,c$. The corresponding value obtained by the method of finite differences is

$$w_{\max} = 0.041583\,\frac{p\,c^4}{N} \qquad (39b)$$

which indicates a difference of about 4%, based upon the difference-equation value.

From Eqs. 13 and 35, expressions may be deduced which, for the center of the plate ($r = 0$ and $\theta = 0$), reduce to:

$$M_r = -2\,N\,[a_2 + c_2 + \mu\,(c_2 - a_2)] \qquad (40a)$$

and

$$M_t = -2\,N\,[\mu\,(a_2 + c_2) - a_2 + c_2] \qquad (40b)$$

At the center of the plate these moments are found to be

$$M_r = 0.1764210176\, p\, c^2 \tag{41a}$$

and

$$M_t = 0.1331010304\, p\, c^2 \tag{41b}$$

which compare fairly well with the corresponding difference-equation moments

$$M_r = 0.169566\, p\, c^2 \tag{42a}$$

and

$$M_t = 0.133894\, p\, c^2 \tag{42b}$$

The differences based upon the difference-equation values for these radial and tangential moments are 4% and 0.6%, respectively. The agreement for the deflections and moments in this case is fairly good.

Power-Series Method.—Several power-series approaches may be found for the approximate solution of plate problems. One of these is presented in this paper, whereas others studied in the investigation are given in a more detailed report filed in the Engineering Societies Library. In the detailed report plates with unequal sides are also treated.

In the case of the uniformly loaded, simply supported, 30° skew plate having equal sides, it is desirable to establish a set of Cartesian-coordinate axes as shown in Fig. 6(*a*). This enables one to take advantage of the symmetry prevailing about both axes. The deflection function may be written in the form

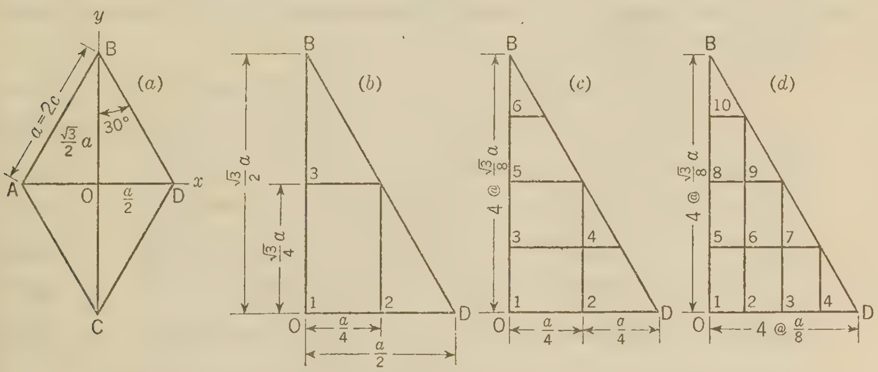
y
B
(a)
a=2c
$\frac{\sqrt{3}}{2}a$
30°
A
O
$\frac{a}{2}$
D
x
C
B
(b)
$\frac{\sqrt{3}}{2}a$
$\frac{\sqrt{3}}{4}a$
3
1
2
D
O
$\frac{a}{4}$
$\frac{a}{2}$
B
(c)
6
5
3
4
1
2
4 @ $\frac{\sqrt{3}}{8}a$
D
O
$\frac{a}{4}$
$\frac{a}{4}$
B
(d)
10
8
9
5
6
7
1
2
3
4
4 @ $\frac{\sqrt{3}}{8}a$
D
O
4 @ $\frac{a}{8}$

FIG. 6.—POWER-SERIES NETWORKS FOR SKEW PLATES

$$w = \frac{p}{N}\left(y + \sqrt{3}\,x + \frac{\sqrt{3}}{2}a\right)\left(y - \sqrt{3}\,x - \frac{\sqrt{3}}{2}a\right)\left(y + \sqrt{3}\,x - \frac{\sqrt{3}}{2}a\right) \times \left(y - \sqrt{3}\,x + \frac{\sqrt{3}}{2}a\right)\sum_{m=0,2,\ldots}^{\infty}\ \sum_{n=0,2,\ldots}^{\infty} C_{m,n}\left(\frac{x}{a}\right)^m\left(\frac{y}{a}\right)^n \tag{43}$$

Since Eq. 43 involves the product of expressions for the sides, it is apparent that one of the boundary conditions, $w = 0$, is already satisfied.

Other conditions to be imposed on Eq. 43 are that $\nabla^4 w = \frac{p}{N}$ (Eq. 4) throughout the entire plate, and that $\nabla^2 w = 0$ (see Eq. 36) along the boundary.

It is to be expected that an approximate solution of the deflection function should result, if Eqs. 4 and 36 are satisfied only at certain points within the plate and certain other points along the boundary, respectively. One particular case involving points B, D, and E, along the boundary, and points 1, 2, 4, and 5 (see Fig. 6(*c*)) yielded the following values for the center of the plate ($a = 2\,c$):

$$w = 0.045007 \frac{p\,c^4}{N} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (44)$$

$$M_x = 0.196316\,p\,c^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (45a)$$

and

$$M_y = 0.110744\,p\,c^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (45b)$$

which are about 8%, 16%, and 17% different from the corresponding difference-equation values.

Clamped 30° Skew Plate with Equal Sides Subjected to a Uniformly Distributed Load

Analysis by Method of Finite Differences.—In solving the problem of the clamped plate, it is convenient to relate the membrane deflections in such a manner that only one set of simultaneous equations need be solved. From Eqs. 19, 22*b*, and Fig. 3(*e*),

$$4\,\bar{z}_k - 2/3\,(\bar{z}_i + \bar{z}_l + \bar{z}_o + \bar{z}_p + \bar{z}_q + \bar{z}_r) = \bar{w}_k\,\lambda^2/S_2 \ldots\ldots (46a)$$

$$4\,\bar{z}_i - 2/3\,(\bar{z}_A + \bar{z}_k + \bar{z}_R + \bar{z}_q + \bar{z}_U + \bar{z}_o) = \bar{w}_i\,\lambda^2/S_2 \ldots\ldots (46b)$$

$$4\,\bar{z}_l - 2/3\,(\bar{z}_k + \bar{z}_B + \bar{z}_r + \bar{z}_T + \bar{z}_p + \bar{z}_V) = \bar{w}_l\,\lambda^2/S_2 \ldots\ldots (46c)$$

$$4\,\bar{z}_q - 2/3\,(\bar{z}_R + \bar{z}_r + \bar{z}_C + \bar{z}_G + \bar{z}_i + \bar{z}_k) = \bar{w}_q\,\lambda^2/S_2 \ldots\ldots (46d)$$

$$4\,\bar{z}_r - 2/3\,(\bar{z}_q + \bar{z}_T + \bar{z}_G + \bar{z}_D + \bar{z}_k + \bar{z}_l) = \bar{w}_r\,\lambda^2/S_2 \ldots\ldots (46e)$$

$$4\,\bar{z}_o - 2/3\,(\bar{z}_U + \bar{z}_p + \bar{z}_i + \bar{z}_k + \bar{z}_E + \bar{z}_H) = \bar{w}_o\,\lambda^2/S_2 \ldots\ldots (46f)$$

$$4\,\bar{z}_p - 2/3\,(\bar{z}_o + \bar{z}_V + \bar{z}_k + \bar{z}_l + \bar{z}_H + \bar{z}_F) = \bar{w}_p\,\lambda^2/S_2 \ldots\ldots (46g)$$

From Eqs. 22*a* and 46 the more convenient form is found as follows:

$$42\,\bar{z}_k - 10\,(\bar{z}_i + \bar{z}_l + \bar{z}_o + \bar{z}_p + \bar{z}_q + \bar{z}_r) + (\bar{z}_A + \bar{z}_B + \bar{z}_C + \bar{z}_D + \bar{z}_E + \bar{z}_F)$$
$$+ 2\,(\bar{z}_R + \bar{z}_T + \bar{z}_U + \bar{z}_V + \bar{z}_G + \bar{z}_H) = \frac{9}{4}\frac{p\,\lambda^4}{S_1\,S_2} \ldots\ldots\ldots (47)$$

For this particular clamped plate, sixteen coefficients were used (see Fig. 4(*b*)). Table 6 contains the deflections and moments at the intersection points of the network and at certain boundary points.

Solutions in Trigonometric Series.—The trigonometric-series approach, as mentioned previously, applies to the clamped skew plate as well as to the simply supported case. Again, a plate having a 30° skew is selected, the sides being clamped and the lengths equal to *a*. The assumed function for the deflections must satisfy the plate formula, Eq. 4, throughout the entire plate and the boundary conditions, as before, are to be met at an arbitrary number of points (twelve in this case).

TABLE 6.—UNIFORMLY LOADED, CLAMPED, 30° SKEW PLATE HAVING EQUAL SIDES

(Sixteen Coefficients, $\lambda = c/4$ and $\mu = 0.3$)

Point	Deflection, $\frac{w\,N}{p\,c^4}$	Moments		
		$\frac{M_x}{p\,\lambda^2}$	$\frac{M_y}{p\,\lambda^2}$	$\frac{M_{xy}}{p\,\lambda^2}$
1	0.000440	−0.084288	0.075654	−0.138513
2	0.001339	−0.226305	0.209767	−0.156285
3	0.002410	−0.369557	0.201683	−0.073520
4	0.003919	0.107912	0.488398	−0.329509
5	0.003291	−0.431873	0.089358	0.061223
6	0.006665	0.277831	0.720474	−0.260080
7	0.003664	−0.362506	−0.060125	0.208976
8	0.008551	0.458341	0.744322	−0.119157
9	0.010656	0.939737	1.699625	−0.244719
10	0.003240	−0.177311	−0.216376	0.347719
11	0.008760	0.585401	0.611740	0.054672
12	0.012802	0.958844	1.123920	−0.131579
13	0.001782	−0.082983	−0.527702	0.385134
14	0.006963	0.524883	0.390833	0.202693
15	0.012224	0.958416	0.989663	−0.027060
16	0.014384	1.112409	1.250479	−0.119571
17	0	−0.045034	−0.150112	0
18	0	−0,182127	−0.607089	0
19	0	−0.383836	−1.279452	0
20	0	−0.583738	−1.945795	0
21	0	−0.712181	−2.373603	0
22	0	−0.706964	−2.356548	0
23	0	−0.514224	−1.714081	0
24	0	−1.414117	−0.814189	0.519550
25	0	−1.944152	−1.119360	0.714287
26	0	−1.958497	−1.127620	0.719557
27	0	−1.605281	−0.924252	0.589784
28	0	−1.055548	−0.607740	0.387811
29	0	−0.500849	−0.288367	0.184013
30	0	−0.123842	−0.071303	0.045500

Boundary conditions at points B, D, F, and G (see Fig. 5(*c*)), which must be satisfied are: $w = 0$; $\partial w/\partial r = 0$; and $\partial w/\partial \theta = 0$.

Because of symmetry the deflection function may be put in the form

$$w = \frac{p\,r^4}{64\,N} + a_0 + r^2\,a_2\cos 2\,\theta + r^2\,c_2 + r^4\,a_4\cos 4\,\theta + r^4\,c_4\cos 2\,\theta + r^6\,a_6\cos 6\,\theta + r^6\,c_6\cos 4\,\theta + r^8\,a_8\cos 8\,\theta + r^8\,c_8\cos 6\,\theta + r^{10}\,c_{10}\cos 8\,\theta \ldots\ldots\ldots\ldots\ldots\ldots \quad (48)$$

Differentiation of Eq. 48 results in

$$\frac{\partial w}{\partial r} = \frac{p\,r^3}{16\,N} + 2\,r\,a_2\cos 2\,\theta + 2\,r\,c_2 + 4\,r^3\,a_4\cos 4\,\theta + 4\,r^3\,c_4\cos 2\,\theta + 6\,r^5\,a_6\cos 6\,\theta + 6\,r^5\,c_6\cos 4\,\theta + 8\,r^7\,a_8\cos 8\,\theta + 8\,r^7\,c_8\cos 6\,\theta + 10\,r^9\,c_{10}\cos 8\,\theta \ldots\ldots\ldots \quad (49a)$$

and

$$\frac{\partial w}{\partial \theta} = -2\,r^2\,a_2 \sin 2\,\theta - 4\,r^4\,a_4 \sin 4\,\theta - 2\,r^4\,c_4 \sin 2\,\theta$$
$$- 6\,r^6\,a_6 \sin 6\,\theta - 4\,r^6\,c_6 \sin 4\,\theta - 8\,r^8\,a_8 \sin 8\,\theta$$
$$- 6\,r^8\,c_8 \sin 6\,\theta - 8\,r^{10}\,c_{10} \sin 8\,\theta \quad \text{(49b)}$$

By substituting values of r and θ for points B, D, F, and G (see Fig. 5(*c*)) and setting a equal to unity, it is feasible to derive ten simultaneous equations, the solution of which gives the values

$$\left.\begin{array}{ll} a_0 = 0.0007979061\,p/N & a_6 = -\,0.0168885960\,p/N \\ a_2 = -\,0.0022057884\,p/N & c_6 = -\,0.0493657788\,p/N \\ c_2 = -\,0.0065078196\,p/N & a_8 = 0.0177867736\,p/N \\ a_4 = 0.0128847660\,p/N & c_8 = 0.0297909016\,p/N \\ c_4 = 0.0057365111\,p/N & c_{10} = 0.0276254724\,p/N \end{array}\right\} \quad (50)$$

These values, it must be remembered, are to be multiplied by appropriate powers of a before they are inserted in Eq. 48 to give the approximate solution for the deflection function. The maximum deflection for this plate occurs at the center ($r = 0$), and its value is found to be

$$w_{\max} = 0.0127664976\,\frac{p\,c^4}{N} \quad \text{(51a)}$$

The corresponding value obtained by means of difference equations is

$$w_{\max} = 0.014384\,\frac{p\,c^4}{N} \quad \text{(51b)}$$

which indicates a difference of about 11% based on the latter value. Referring to Eqs. 13, it is possible to deduce expressions for the radial and tangential moments, which for the center of the plate ($r = 0$ and $\theta = 0$) reduce to the same form as that found for the simply supported plate, Eqs. 40, except that the values of the constants are different. Substitution of the values obtained for a_2 and c_2 in Eqs. 40 gives:

$$M_r = 0.0800337388\,p\,c^2 \quad \text{(52a)}$$

and

$$M_t = 0.0553289088\,p\,c^2 \quad \text{(52b)}$$

The corresponding difference-equation values are

$$M_r = 0.082470\,p\,c^2 \quad \text{(53a)}$$

and

$$M_t = 0.065211\,p\,c^2 \quad \text{(53b)}$$

indicating, for the radial and tangential moments at the center, a difference of about 3% and 15%, respectively, based on the difference-equation moments. Obviously, in the case of the clamped plate, the boundary conditions should be satisfied for a greater number of points (for the same degree of approximation) than in the case of the simply supported skew plate.

For the equal-sided clamped 30° skew plate with the boundary conditions being satisfied at only eight points along the entire boundary (see Fig. 5(*b*)), the results are naturally less reliable than those obtained when twelve points are taken into consideration.

The maximum deflection of the plate obtained by the trigonometric-series approach for this case (eight boundary points considered) is approximately 17% too low based on the difference-equation result. The difference between the radial and tangential moments as obtained by the trigonometric and difference-equation methods average about 20%. Clearly, the degree of approximation in this case is not satisfactory.

Power-Series Method.—The clamped 30° skew plate having equal sides and subjected to a uniformly distributed load may also be analyzed by power-series methods. The deflection function may be written in the form (see Fig. 6),

$$w = \frac{p}{a^4 N}\left(y + \sqrt{3}\,x + \frac{\sqrt{3}}{2}a\right)^2\left(y - \sqrt{3}\,x - \frac{\sqrt{3}}{2}a\right)^2\left(y + \sqrt{3}\,x - \frac{\sqrt{3}}{2}a\right)^2$$

$$\times\left(y - \sqrt{3}\,x + \frac{\sqrt{3}}{2}a\right)^2 \sum_{m=0,2,4,\ldots}^{\infty}\ \sum_{n=0,2,4,\ldots}^{\infty} C_{m,n}\left(\frac{x}{a}\right)^m\left(\frac{y}{a}\right)^n \ldots (54)$$

from which it is apparent that the boundary conditions $w = 0$, $\partial w/\partial x = 0$, and $\partial w/\partial y = 0$ are valid. The plate formula, Eq. 4, must also be satisfied. By doing this for only a certain number of interior points, it is possible to solve the problem approximately. It is convenient to select the number of points to correspond to the number of constants, $C_{m,n}$, chosen for the series; in this manner one obtains as many equations as there are unknowns in the series. Eq. 4 may be satisfied for a larger number of points by normalizing the resulting equations. This process yields a system of linear equations in normal form (symmetry about the diagonal) which may easily be solved by the Doolittle Method.[19]

Six points are now selected within the plate (see Fig. 6(*c*)) and, correspondingly, six coefficients are taken in the power series. Obviously, from symmetry, these are $C_{0,0}$, $C_{2,0}$, $C_{0,2}$, $C_{4,0}$, $C_{2,2}$, $C_{0,4}$. From Eq. 54, by a few simple operations, one may derive the differential equation presented (for compactness) in Table 7.

Substitution of values of x and y in Table 7 for the six points gives rise to a system of equations, the solution of which yields the values

$C_{0,0} = 2.6184138565 \times 10^{-3}$ $C_{4,0} = 20.1633240889 \times 10^{-3}$
$C_{2,0} = 11.8165756479 \times 10^{-3}$ $C_{2,2} = -10.3051787620 \times 10^{-3}$
$C_{0,2} = -2.6453532509 \times 10^{-3}$ $C_{0,4} = -1.1029769807 \times 10^{-3}$

The maximum deflection of the plate is found to be

$$w_{\max} = 0.0008284825\,\frac{p\,a^4}{N} \ldots\ldots\ldots\ldots (55a)$$

which is about 7.8% less than the difference-equation value—

$$w_{\max} = 0.000899\,\frac{p\,a^4}{N} \ldots\ldots\ldots\ldots (55b)$$

From Eqs. 1 and 54 it is possible to determine the bending moments at the center of the plate

$$M_x = 0.084732\, p\, c^2 \qquad (56a)$$

and

$$M_y = 0.051492\, p\, c^2 \qquad (56b)$$

TABLE 7.—DIFFERENTIAL EQUATION DERIVED FROM EQ. 54, WITH $a = 1$

$$\frac{N}{p}\nabla^4 w =$$

$$\begin{aligned}
&+ 864\, C_{0,0} - 135\, C_{2,0} - 81\, C_{0,2} + \frac{243}{32} C_{4,0} + \frac{81}{32} C_{2,2} + \frac{243}{32} C_{0,4} \\
&+ x^2 \left(-27{,}648\, C_{0,0} + 11{,}340\, C_{2,0} + 1{,}620\, C_{0,2} - \frac{3{,}807}{2} C_{4,0} - \frac{567}{2} C_{2,2} - \frac{243}{2} C_{0,4}\right) \\
&+ x^4 (124{,}416\, C_{0,0} - 132{,}624\, C_{2,0} - 9{,}072\, C_{0,2} + 51{,}921\, C_{4,0} + 3{,}807\, C_{2,2} + 729\, C_{0,4}) \\
&+ x^6 (385{,}344\, C_{2,0} + 15{,}552\, C_{0,2} - 401{,}976\, C_{4,0} - 17{,}496\, C_{2,2} - 1{,}944\, C_{0,4}) \\
&+ x^8 (924{,}696\, C_{4,0} + 26{,}568\, C_{2,2} + 1{,}944\, C_{0,4}) \\
&+ y^2 \left(324\, C_{2,0} + 2{,}268\, C_{0,2} - \frac{81}{2} C_{4,0} - \frac{405}{2} C_{2,2} - \frac{1{,}701}{2} C_{0,4}\right) \\
&+ y^4 (1{,}536\, C_{0,0} - 144\, C_{2,0} - 3{,}312\, C_{0,2} + 81\, C_{4,0} + 567\, C_{2,2} + 7{,}209\, C_{0,4}) \\
&+ y^6 (-64\, C_{2,0} + 3{,}648\, C_{0,2} - 72\, C_{4,0} - 456\, C_{2,2} - 12{,}456\, C_{0,4}) \\
&+ y^8 (24\, C_{4,0} + 72\, C_{2,2} + 8{,}856\, C_{0,4}) \\
&+ x^2 y^2 (-27{,}648\, C_{0,0} + 11{,}232\, C_{2,0} - 18{,}144\, C_{0,2} + 3{,}402\, C_{4,0} + 14{,}094\, C_{2,2} + 11{,}178\, C_{0,4}) \\
&+ x^2 y^4 (12{,}480\, C_{2,0} - 20{,}160\, C_{0,2} + 1{,}080\, C_{4,0} + 11{,}160\, C_{2,2} + 5{,}400\, C_{0,4}) \\
&+ x^2 y^6 (-2{,}976\, C_{4,0} + 8{,}352\, C_{2,2} - 26{,}784\, C_{0,4}) \\
&+ x^4 y^2 (-146{,}880\, C_{2,0} + 77{,}760\, C_{0,2} + 50{,}760\, C_{4,0} - 113{,}400\, C_{2,2} - 48{,}600\, C_{0,4}) \\
&+ x^4 y^4 (70{,}800\, C_{4,0} - 104{,}400\, C_{2,2} + 32{,}400\, C_{0,4}) \\
&+ x^6 y^2 (-476{,}064\, C_{4,0} + 282{,}528\, C_{2,2} + 69{,}984\, C_{0,4})
\end{aligned}$$

$$= 1$$

These are about 3% and 21% different from the corresponding difference-equation values.

SUMMARY AND CONCLUSIONS

In this paper a theoretical study of uniformly loaded, simply supported, and clamped skew plates has been made by means of approximate methods. No exact solution of skew plate problems has come to the attention of the writer, and consequently comparisons were made between approximate methods.

A survey was made of a few systems of coordinates in order to ascertain their convenience in the treatment of triangular and skew plates. The trilinear and quadrilinear systems appear to have no particular advantages, except possibly as regards the question of symmetry. Both seem to require more effort than the Cartesian-coordinate system in the solution of the same problems. The trilinear system has been suggested by an investigator who solved plate problems that have also been analyzed by Cartesian coordinates. The quadrilinear system, in the case of power-series methods, would definitely involve more work. Polar-coordinate equations, on the other hand, have been very useful in the trigonometric-series solutions.

From a preliminary survey of triangular plates and a more extensive investigation of the uniformly loaded, simply supported, 30° skew plate having equal sides, a measure of the reliability of difference-equation results for various densities of networks was obtained. In general, it may be stated that a relatively small number of nodal points gives values which are in fair agreement with those of more dense and, hence, more accurate networks. This is especially true in the case of bending moments. More specifically, in this connection, the following observations are noteworthy:

(1) In the case of simply supported equilateral, triangular plates, subjected to uniformly distributed loads, reasonably good results are obtainable by the method of finite differences even for a relatively small number of coefficients—say, five. The maximum deflection given by the five-coefficient solution and the eight-coefficient solution varied from the exact solution by 2.7% and 1.2%, respectively.

(2) A comparison of deflections at certain points of the simply supported, 30°-60°-90° plate subjected to a uniformly distributed load obtained by networks having three and twenty-one nodal points showed a difference of about 6% or 7%.

(3) For the simply supported isosceles, right triangular plate subjected to a uniformly distributed load, the deflections as obtained by A. Nadai's approximation and a difference-equation network having twelve coefficients are in fair agreement at points near the center of gravity of the plate, but nearer the boundaries there is considerable disagreement.

(4) A comparison of bending moments and twisting moments at similar points of the uniformly loaded, simply supported, 30° skew plate (equal sides) obtained by the four-coefficient solution and the sixteen-coefficient solution indicated reasonably good agreement. At one pair of corresponding points the results agreed well within 1%. Two other pairs, for which a comparison was made, showed a maximum difference of about 5% for bending moments, whereas the twisting moments varied by about 10% in one case and by about 22% in the other. The deflections differed by about 8% or 9% for the three points investigated.

(5) At the center of the uniformly loaded, simply supported, 30° skew plate having equal sides, the bending moments and deflections resulting from the networks having twenty-five and sixteen coefficients agreed well within 1%. The corresponding twisting moments differed by only 2%.

(6) A similar comparison (see conclusion (5)) between these results as obtained by the twenty-five-coefficient and the four-coefficient solutions showed good agreement (within 3%) for the bending moments. The twisting moments and deflections at the center differed by 19% and 9%, respectively.

The uniformly loaded, simply supported, 30° skew plate with equal sides was analyzed by the trigonometric-series and power-series methods, as well as by the difference-equation method, and certain deflections and moments were compared. The following comparisons may be enumerated:

(*a*) The maximum deflection given by the trigonometric-series method for this case (eight coefficients) differs by about 4% from the difference-equation value. At the center of the plate, the radial and tangential moments differ by 4% and 0.6%, respectively, for these same two methods. The agreement here is seen to be good.

(*b*) At the center of the plate, the deflection, w, and the moments M_x and M_y, as obtained by the method using power series (only three terms and seven points considered), differed by 8%, 16%, and 17%, respectively, from the corresponding difference-equation values. By increasing the number of terms in the series and the number of points for which the conditions are satisfied, it would appear to be possible to increase the accuracy.

The clamped 30° skew plate with equal sides, subjected to a uniformly distributed load, was studied by similar methods, and the following observations were made:

(*A*) A difference of 11% was found between the maximum deflections as obtained by trigonometric series (ten coefficients) and by difference equations. The radial and tangential moments at the center differed (for these two methods) by about 3% and 15%, respectively.

(*B*) The values for the maximum deflection as obtained by power series (six coefficients) and difference equations differed by 7.8%. The moments M_x and M_y differed by 3% and 21%, respectively.

Structural action of skew plates and slabs in the region of corners was not treated. Singularities prevailing in these sections probably necessitate the use of more rigorous, mathematical methods.

ACKNOWLEDGMENT

The source of the material for this paper was a thesis by the writer, entitled "A Theoretical Study of Skew Plates," presented to Lehigh University, Bethlehem, Pa., in June, 1941, in partial fulfilment of the requirements for the degree of Doctor of Philosophy.

This investigation was conducted under the general supervision of Hale Sutherland, M. Am. Soc. C. E., Director of the Fritz Engineering Laboratory of Lehigh University, while the writer served as Instructor in Theoretical and Applied Mechanics at the University of Illinois, Urbana, Ill. The suggestions and criticisms of Professors Newmark and Jensen, previously mentioned, are gratefully acknowledged.

APPENDIX

NOTATION

The following letter symbols conform essentially to American Standard Letter Symbols for Mechanics, Structural Engineering and Testing Materials (ASA—Z10a—1932) prepared by a Committee of the American Standards

Association, with Society representation, and approved by the Association in 1932:

A = a substitution coefficient in Table 2;
a = a horizontal dimension of plate (for example, see Fig. 5);
a_n = a coefficient in a trigonometric series;
B = a substitution coefficient in Table 2;
C = coefficient; $C_{m,n}$ = a coefficient in a power series;
c = a horizontal dimension of a plate (for example, see Fig. 5);
c_n = a coefficient in a trigonometric series;
E = the modulus of elasticity of a material in tension and compression;
G = the modulus of elasticity of a material in shear;
h = thickness of plate;
M = moment (positive directions shown in Fig. 1):

- M_x, M_y = bending moments per unit of length in sections perpendicular to the x-axis and the y-axis, respectively;
- M_{xy} = twisting moment per unit of length in sections perpendicular to the x-axis;
- M_r, M_t = bending moments per unit of length in sections perpendicular to radial and tangential directions (positive when producing compression in the top fibers of a plate);
- $M_{r,t}$ = twisting moments per unit of length in sections perpendicular to radial directions;
- M_1, M_2, M_3 = bending moments per unit of length in sections perpendicular to directions 1, 2, and 3, respectively;
- M_t = maximum value of a twisting moment (Eq. 30);

$\bar{M}$ = a function of the moment sum (Eq. 14); $\bar{M}_k$ = value of $\bar{M}$ at the nodal point k (Eq. 19);
N = flexural rigidity of the plate (Eq. 2);
n = a direction in any given plane, perpendicular to direction t (n = "normal" and t = "tangential");
p = uniformly distributed lateral unit load;
R = reactions; R_x and R_y = vertical reaction per unit of length in sections normal to the x-axis and the y-axis, respectively;
r = radius = polar coordinate with angular displacement θ; r_x, r_y = radii of curvature of the middle surface of a plate in planes parallel to the xz-plane and the yz-plane, respectively;
S = horizontal component of the surface stresses; S_1 = S referred to the first membrane, etc. (Eq. 19);
t = a direction perpendicular to n;
u = displacement in the x-direction;
V = vertical shear:

- V_x, V_y = vertical shear per unit of length in sections normal to the x-axis and the y-axis, respectively;

V_{1k} = vertical shear referring to the nodal point k and the plane 1;

v = displacement in the y-direction;

w = displacement in the z-direction; $\bar{w}$ = deflection of first membrane ($\bar{w}_k$ = values of $\bar{w}$ referred to the nodal point k);

x = distances in the x-direction;

y = distances in the y-direction;

z = distances in the z-direction ($\bar{z}_k$ = the deflection of the nodal point k in the second membrane);

θ = angular displacement; polar coordinate with the radius r;

λ = intervals in difference-equation networks (see, for example, Fig. 3), subscripts x and y referring to respective axes;

μ = Poisson's ratio (lateral contraction to longitudinal elongation);

σ = bending stress; σ_x, σ_y = normal components of bending stress in the x-direction and the y-direction, respectively;

τ = shear stress; τ_{xy}, τ_{xz}, τ_{yz} = shearing components of stress, the first subscript in each case indicating the axis to which the plane (in which the stress is acting) is normal, and the second subscript indicating the direction of the shearing stress;

∇ = the Laplace operator.

DISCUSSION

JOHN W. ALLEN,[21] JUN. AM. SOC. C. E.—An interesting comparison of three approximate methods of solving the problem of skew plates is presented in this paper. For the average engineer the difference-equation solution is the most direct of the three. The physical meaning of the difference equations can be understood readily and applied to various structures. This method often provides a powerful tool when exact solutions are not available.

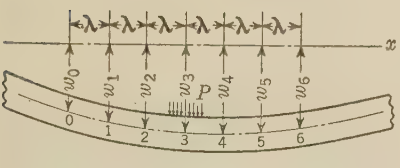

FIG. 7.—DEFLECTED BEAM

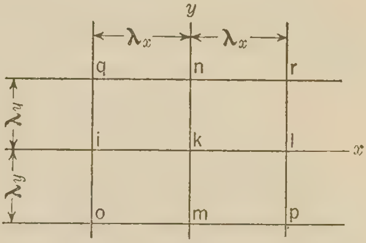

FIG. 8.—NETWORK OF POINTS IN A SLAB

If the structure under consideration were to deflect in only one direction such as the beam in Fig. 7, these equations can be set up in simplified form. The basic equations can then be transposed easily to a structure that is deflecting in two directions.

The difference equations are merely approximate expressions for the slope, and the rate of change of slope, of a curve such as a deflected surface or a moment curve. The theorem is basic, in ordinary structural analysis, that:

(I) The moment in a structure is proportional to the rate of change of slope of the deflected structure; and

(II) The load applied to a structure at a point is proportional to the rate of change of the slope of the moment curve.

The corresponding difference equations for the foregoing theorems can be written as follows: The rate of change of slope of a structure, at point 3, Fig. 7, is proportional to—

$$\frac{\text{(Slope midway between points 2 and 3)} - \text{(slope midway between points 3 and 4)}}{\text{Distance between the midpoints}}.$$

In other words,

$$\left(\frac{d^2w}{dx^2}\right)_3 \cong \frac{1}{\lambda}\left(\frac{w_2 - w_3}{\lambda} - \frac{w_3 - w_4}{\lambda}\right) \cong \frac{w_2 - 2\,w_3 + w_4}{\lambda^2} \ldots\ldots (57)$$

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[21] Engr., Chicago Bridge & Iron Co., Chicago, Ill.

but $\frac{d^2w}{dx^2} = -\frac{M}{E\,I}$; and therefore

$$\left(\frac{M}{E\,I}\right)_3 \overset{\frown}{=} -\frac{w_2 - 2\,w_3 + w_4}{\lambda^2} \qquad (58)$$

These same equations can be written similarly for the second theorem. For example, let P = the load on the structure at the point under consideration. Then $P = \frac{d^2M}{dx^2}$; and:

$$(P)_3 \overset{\frown}{=} \frac{M_2 - 2\,M_3 + M_4}{\lambda^2} \qquad (59)$$

By combining Eqs. 58 and 59 load P can be expressed in terms of the deflection w:

$$\left(\frac{P}{E\,I}\right)_3 \overset{\frown}{=} -\frac{w_1 - 4\,w_2 + 6\,w_3 - 4\,w_4 + w_5}{\lambda^4} \qquad (60)$$

From the foregoing it can be seen that difference equations involve finite distances to express approximately that which differential equation expresses in minute distances. With difference equations the expressions are solved simultaneously for a few points instead of a complete solution for an infinite number of points.

It is also interesting to note the geometrical meaning of the twisting moment in the slab. Referring to Fig. 8, the following expression for the twisting moment can be obtained. Since the twisting moment is proportional to the rate of change of slope in the x-direction with respect to the y-direction, it can be expressed approximately for point k as follows:

$$\frac{(\text{Slope at point } n \text{ in the } x\text{-direction}) - (\text{slope at point } m \text{ in the } x\text{-direction})}{\text{Distance between the points } n \text{ and } m \text{ in the } y\text{-direction}}.$$

In other words,

$$-\left[\frac{M_{xy}}{(1-\mu)N}\right]_k \overset{\frown}{=} \frac{1}{2\,\lambda_y}\left(\frac{w_q - w_r}{2\,\lambda_x} - \frac{w_o - w_p}{2\,\lambda_x}\right) \overset{\frown}{=} \frac{w_q + w_p - w_r - w_o}{4\,\lambda_x\,\lambda_y} \qquad (61)$$

The foregoing demonstrates that difference equations supply a direct, even though approximate, method of computing quantities that are important in the design of a structure. An equation must be written for each point on the network selected, and solved simultaneously.

Application of these equations and tests of models of structures[22] proves that the conclusions concerning the number of points on a network required to obtain reasonable accuracy are substantially correct.

William A. Conwell,[23] M. Am. Soc. C. E.—In cataloging and demonstrating the various methods available for the solution of plate problems, the author has performed a remarkable service to the profession. In particular, the presentation of the method for applying difference equations to a determination of the bending moments in a plate, whether skew or rectilinear, will be appreciated by most engineers, who prefer physical concepts to involved mathematics. The paper will serve, without doubt, as a text for future study by those engineers who have long been disturbed by rule-of-thumb, semitheoretical, and semi-empirical methods and have sought a practical analytical approach.

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[22] "Analysis of Skew Slabs with Curbs," by John W. Allen, thesis presented to the Univ. of Illinois, Urbana, Ill., in 1941, in partial fulfilment of the requirements for the degree of Master of Science in Theoretical and Applied Mechanics.

[23] Pittsburgh, Pa.

The writer wishes to call attention to some of the properties of the square network as shown in Fig. 3(*a*) and Eqs. 21*a* and 21*b*. The simplicity of the equations becomes more apparent when the equations are solved for $\bar{w}_k$ and $\bar{z}_k$, respectively. The values of $\bar{w}_k$ and $\bar{z}_k$ are then evidently merely the averages of the four adjacent values in the network, adjusted by one fourth of the quantity appearing on the right-hand side of Eqs. 21*a* and 21*b*.

This property of simplicity permits the use of numerical methods[24] in the solution of the problem. Reasonable values for $\bar{w}$ and $\bar{z}$ are first assumed for each nodal point on the network and then adjusted by the application of the equations to each point in turn. When a value thus computed differs from the assumed or previously computed value, it is substituted for the assumed or previously computed value and used in all the computations which follow until another application of the equation results in a further adjustment. Each assumed value is thus adjusted and readjusted until the equations are satisfied at each point. For large networks, this process would be a laborious one if it were not for the fairly quick convergence which may be secured by a method outlined by George H. Shortley, Royal Weller, and Bernard Fried.[25]

There is an old engineering practice of using an even number for the last digit in a dividend when the division results in a fraction of a half. Used strictly in numerical procedures, this practice assures accuracy in the results even though only a few significant figures are used in the computations. Because of this fact, elaborate machine computations are avoided and the calculations are reduced to simple arithmetical procedures and slide-rule work. Furthermore, it is possible to obtain values rapidly, which are correct to two significant figures, for rough designs or estimates. The use of such values is fully justified by the fact that the values represent a considerable improvement over values obtained from rule-of-thumb or pseudo-analytical processes. In its final form, furthermore, a slab or plate does not ordinarily fulfil all the assumed conditions of homogeneity, elasticity, "isotropicity," uniformity of thickness, etc.; and, hence, more elaborate computations may be warranted only for detailed analysis or research.

In addition to its simplicity, the square network can be applied universally to either rectilinear plates or those having any degree of skew. The hexagonal network, as demonstrated by the author, is convenient in the solution of plates having a 30° skew. Not only is the square network the natural one to use for a 45° skew but, again because of its simplicity, it is a good choice for any skew. Its application results in a solution in which two irregular edges are substituted for two straight-line edges; but, as the writer will demonstrate, this has little effect on the final results. Values adjacent to the irregular edge are in error, of course; but, for an equal-sided symmetrical plate, this error presents no problem as accurate values are available along the straight edge, symmetrical to the irregular edge. For other than symmetrical, equal-sided plates two solutions may be necessary—one with the network based on one of the edges and another with the network based on the edge that was irregular in the first solution. The solution of such plates by other than the square network would also result in a considerably more complicated procedure than that required for symmetrical equal-sided plates.

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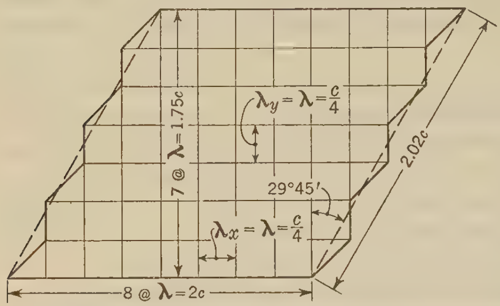
[24] "A Method of Obtaining Moments in a Slab by the Numerical Solution of Differential Equations," by William A. Conwell, thesis submitted to the Carnegie Inst. of Technology, Pittsburgh, Pa., in 1937 in partial fulfilment of the requirements for the degree of Master of Science in Civil Engineering.

[25] "Numerical Solution of La Place's and Poisson's Equations," by George H. Shortley, Royal Weller, and Bernard Fried, *Bulletin No. 107*, Eng. Experiment Station, Ohio State Univ., Columbus, September, 1940.



$7 @ \lambda = 1.75c$
$\lambda_y = \lambda = \frac{c}{4}$
$2.02c$
29°45′
$\lambda_x = \lambda = \frac{c}{4}$
$8 @ \lambda = 2c$

FIG. 9.—SQUARE NETWORK FOR THE SOLUTION OF A 30° SKEW PLATE WITH EQUAL SIDES

To demonstrate the foregoing statements, the writer has prepared a solution of the simply supported, equal-sided 30° skew plate supporting a uniform load. Fig. 9 shows the square network used in the solution, with its irregular edges and its span of 1.75 c (as against the actual 1.73 c) in the direction of Y—with the resulting discrepancies of 15 min in skew and 0.02 c in the length of the irregular edges. That these irregularities do not influence results beyond the accuracy required for engineering design is evident from comparison of the results with those of the author's sixteen-coefficient solution, as shown in Table 8. Inasmuch as the network used did not have a nodal point at the center of the plate, the values there were obtained by means of a supplementary network which included the six points adjacent to the center and which permitted direct computation of the center values.

TABLE 8.—COMPARISON OF MOMENTS AND DEFLECTIONS AT THE CENTER OF A UNIFORMLY LOADED, SIMPLY SUPPORTED, 30° SKEW PLATE FOR VARIOUS NETWORKS

Network	λ	Moment M_x	Moment M_y	Moment M_{xy}	Deflection w
Hexagonal........	$c/4$	$0.143095\ p\,c^2$	$0.160524\ p\,c^2$	$-0.015095\ p\,c^2$	$0.041948\ p\,c^4/N$
Square............	$c/4$	$0.144\ p\,c^2$	$0.159\ p\,c^2$	$-0.0144\ p\,c^2$	$0.0420\ p\,c^4/N$
Square............	$c/3$	$0.12\ p\,c^2$	$0.14\ p\,c^2$	$-0.016\ p\,c^2$	$0.038\ p\,c^4/N$

Prior to the more accurate computation (in general, to three significant figures), the writer made a rough calculation using only two significant figures and taking a network in which $\lambda = c/3$. In that instance, the span of the network in the direction of Y was 1.667 c; the skew, 31°; and the length of the irregular edge, 1.94 c. That such rough computations, which produce results very quickly, have engineering value is again evident from a comparison of the results given in Table 8.

Julius Sigman,[26] Esq.—Various methods of analysis are used in this interesting study of skew plates. These methods involve solution by finite differences, by trigonometric series, and by power series. It is very evident that the author has done some painstaking research, and the results obtained seem to agree fairly closely.

When practical considerations are taken into account, it becomes apparent that, because of the limitations of the basic assumptions, skew slabs and structures present a vast field still to be explored. Such an exploration would help in connection with many problems that commonly occur in practice.

Five general methods of studying the structural action of concrete slabs and frames may be listed:

1. Experimental tests of prototypes;
2. Laboratory tests of large-scale models;
3. Model analyses (plaster models and photoelastic tests);
4. Semi-empirical analyses such as those developed by C. Bach,[27] and John R. Nichols; [28] and
5. Theoretical analyses utilizing (a) differential equations, (b) difference equations, (c) "single-infinite" series of hyperbolic functions, (d) "doubles-infinite" Fourier series, (e) power series, (f) trigonometric series, (g) W. Ritz's [29,30] approximate energy method involving the principle of energy minimum, (h) Bessel [31] functions, and (i) short series of polynomials.

In ordinary theory, the plates are assumed to be uniform in thickness and the thickness is assumed to be small compared with the superficial dimensions. In the case of skewed reinforced concrete rigid frame bridges, the thickness of the deck cannot be considered small compared with the lateral dimensions; and the depth, furthermore, is variable so that ordinary theory does not apply.

In addition, the deflections and energy of deformation are assumed not to be affected by the vertical stresses. In the rigid frame bridge deck, deflections and strain energy are markedly affected by the vertical tensions, compressions, and shears.

The following problems with practical value, suggested for future investigation, are worthy of theoretical treatment as thorough as that presented by the author: (1) Skew plates with large deflections; (2) skew slabs with variable thickness; (3) continuous skew slabs; (4) effect of partial fixity of edges; (5) effect of curbs, sidewalks, and parapets on deck action in rigid frame bridges;

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[26] Brooklyn, N. Y.

[27] "Elastizität und Festigkeit," 7 Auflage, Abschn, 9.

[28] "Two way Slabs in the Proposed Building Code for Boston and New England," by John R. Nichols, *Proceedings*, A.C.I., May-June, 1934, p. 504.

[29] *J. reine angew. Math.*, Vol. 135, 1908.

[30] *Ann. Physik* (4), Vol. 28, 1909, p. 737.

[31] "The Theory of Sound," by J. W. Strutt, Baron Rayleigh, Dover Publications, New York, N. Y., 2d Ed., 1945.

(6) structural action at corners of skew slabs—a possible approach to the solution would be one using the theory of functions of complex variables; and (7) variable depth skewed slabs with curbs.

Also, since authorities do not agree as to proper procedure for skewed reinforced concrete rigid frame bridge analysis, a solution involving the theory of elasticity might accomplish much toward clarifying certain questions concerning which opinion differs.

F. L. Ehasz,[32] Jun. Am. Soc. C. E.—Discussion of the paper has served to emphasize the usefulness of approximate methods in the analysis of difficult slab and plate problems. In general, the comments made by the various discussers were favorable, and the writer is grateful for their contributions.

The directness and simplicity of difference-equation solutions are discussed by Mr. Allen. From his presentation it is clear that the method is a powerful one in cases not easily dealt with by so-called exact methods. The geometrical significance of structural functions as expressed by difference equations seems to be particularly lucid.

Mr. Conwell states that most engineers prefer physical concepts to purely mathematical treatment in his illuminating remarks. The writer agrees with Mr. Conwell's statements concerning the number of significant figures required for rough design and estimating purposes and the extent to which practice satisfies the ideal assumptions made in analysis. Mr. Conwell's study of the equal-sided 30° skew plate, uniformly loaded and simply supported, by difference equations applied to a square network is interesting. The close agreement between results for the networks given in Table 8 is to be expected, to the extent indicated, in regions near the center of the plate. In the case of the plate fixed at its edges, furthermore, the agreement at the various points throughout the plate would be less. As Mr. Conwell states, however, the engineering value of relatively rough computations is satisfactory for practical purposes. On the other hand, research investigations must be conducted with greater accuracy, since they serve as a basis for deciding the extent to which practicing engineers may abbreviate their studies.

Several interesting problems for future research have been listed by Mr. Sigman. The suggestion that the skewed reinforced concrete rigid frame bridge be analyzed by the theory of elasticity is one that has been in the minds of several investigators. It is common knowledge that this task would be a prodigious one. Admittedly, if such a problem could be thus analyzed successfully, it would go a long way toward clarifying the true structural behavior. Present methods of analysis appear to be satisfactory from a practical viewpoint, considering the uncertain loading and boundary conditions in this type of structure. The writer agrees, however, that such a solution would be a welcome one, even though the assumptions of restraint and loading (action of earth pressure, for example) would still be open to question. Extensive field tests of bridges under actual operation would be of much value in clarifying some of the uncertainties to which Mr. Sigman referred in his discussion. This type of research has found considerable favor in Europe.

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[32] Cons. Engr., New York, N. Y.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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TRANSACTIONS

Paper No. 2288

THE MECHANISM OF ENERGY LOSS IN FLUID FRICTION

By Boris A. Bakhmeteff,[1] M. Am. Soc. C. E., and
William Allan,[2] Assoc. M. Am. Soc. C. E.

With Discussion by Messrs. George R. Rich, J. C. Stevens, Leon Beskin, Hunter Rouse and A. A. Kalinske, and Boris A. Bakhmeteff and William Allan.

Synopsis

An account of the inner processes, by which the energy of fluid flow is lost through friction, is presented in this paper. Previous investigations have dealt largely with bulk appraisals, relating to over-all losses of head. The internal mechanism was investigated in pioneer fashion by Sir G. G. Stokes,[3] who first expressed the internal energy balance for motion in viscous media and formulated the so-called "dissipation function." Horace Lamb[4] broadened the Stokes reasoning, applying it to laminar motion in general. More recently, an attempt to interpret energy exchange in turbulent motion was made by G. I. Taylor.[5,6]

Although it is unimpeachable from a formal mathematical point of view, the traditional treatment does not elucidate the physical aspect of the problem, failing in particular to reveal the fundamental fact that the withdrawal of energy from the flow and its final dissipation into molecular heat do not coincide spatially and that they constitute the initial and the ultimate phase of a manifold process.

The paper reveals these consecutive phases, explaining their dynamic significance and the nature of the losses involved in the different stages. The facts disclosed are especially pertinent for gaining a more comprehensive understanding of the mechanism of turbulent flow. The analysis begins with friction in

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Note.—Published in February, 1945, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

[1] Prof., Civ. Eng., Columbia Univ., New York, N. Y.

[2] Associate Prof., Civ. Eng., College of the City of New York, New York, N. Y.

[3] "On the Effect of Internal Friction of Fluids on the Motion of Pendulums," by Sir G. G. Stokes, *Transactions*, Cambridge Philosophical Soc., Vol. IX, 1850, p. 8; also the collected papers in Vol. III, p. 7.

[4] "Hydrodynamics," by Horace Lamb, 6th Ed., Univ. Press, Cambridge, England, 1932, p. 579.

[5] "Statistical Theory of Turbulence," by G. I. Taylor, *Proceedings*, Royal Soc. of London, Series A, Vol. CLI, 1935, pp. 455–464.

[6] "Modern Developments in Fluid Dynamics," edited by S. Goldstein, Clarendon Press, Oxford, England, 1938, Vol. II, p. 395.

uniform established motion and subsequently is extended to boundary layer flow. The fundamental relations are established first for the simpler case of laminar motion, and subsequently are applied to the more complex turbulent flow.

NOTATION

The letter symbols in this paper, defined in the text where they first appear, conform essentially to American Standard Letter Symbols for Hydraulics (ASA—Z10.2—1942), prepared by a Committee of the American Standards Association, with Society representation, and approved by the Association in 1942.

1. INTRODUCTION TO UNIFORM RECTILINEAR MOTION

Fluid friction is revealed outwardly by the resistance slope $S = h_f/l$, or the pressure gradient

$$-\frac{dp}{dx} = \frac{p_1 - p_2}{l_{1,2}} = \gamma \frac{h_1 - h_2}{l_{1,2}} = \gamma S \left(\frac{\text{lb} \times \text{ft}}{\text{ft}^3 \text{ per ft}} \right) \qquad (1)$$

in which: p = pressure intensity; l = distance measured in the direction of flow; γ = specific weight (weight per unit volume); and h = head, measured from a datum line. In this instance Eq. 1 measures the rate of energy per unit volume lost equally by each and every element of a cross section, in its displacement over a unit length of the conduit. The same quantity (Eq. 1) serves to determine the mean wall friction stress

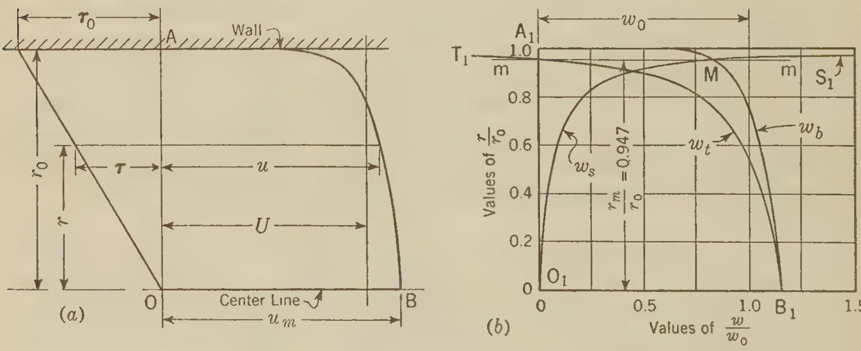

FIG. 1.—TURBULENT FLOW IN SMOOTH PIPE

$$\bar{\tau}_o = -\gamma S R = \frac{dp}{dx} R \left(\frac{\text{lb}}{\text{ft}^2} \right) \qquad (2)$$

and, with that, the internal shearing stress structure. The latter, in uniform motion, is linear.[7] Thus, for a pipe (Fig. 1)—

$$\tau = \tau_o \frac{r}{r_o} \qquad (3a)$$

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[7] "The Mechanics of Turbulent Flow," by B. A. Bakhmeteff, Princeton Univ. Press, Princeton, N. J., 1936, p. 3.

and, for two-dimensional motion (Fig. 2)—

$$\tau = \tau_o \frac{y'}{y'_o} \qquad (3b)$$

In Eqs. 2 and 3, r = the distance from the axis to an inner filament; A = the cross-sectional area; P_w = the wetted perimeter; $R = A/P_w$ the hydraulic radius; and y', in two-dimensional motion, will designate specifically the distance from the free surface (Fig. 2) or from the central axis as against y, which exemplifies a transverse coordinate in general.

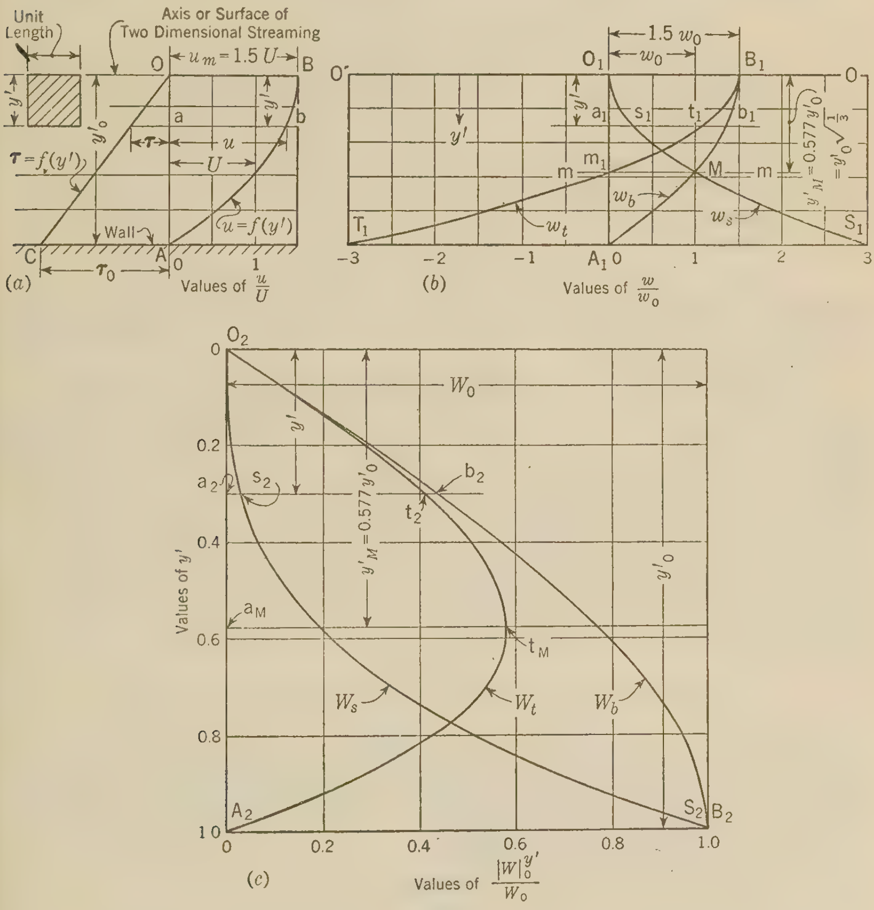


FIG. 2

Another familiar concept is the bulk loss of power:

$$W_o = \gamma S Q = -\frac{dp}{dx} Q \left(\frac{\text{lb} \times \text{ft}}{\text{sec per ft}} \right) \qquad (4)$$

or the total work done by the flow in overcoming frictional resistances during unit time and over a unit length of the conduit. In Fig. 3, the discharge Q enters the unit length block and subsequently leaves it, in the interim unit time suffering the loss W_o; or, one may state that within the unit length block, and during each second, mechanical energy at the rate of W_o is withdrawn from the flow and is spent to overcome resistances, by way of ultimate dissipation into molecular heat. Since the velocities do not change, all of W_o is withdrawn from the potential energy store carried by the flow, thereby causing a reduction of the piezometric head by $h_f = S \times 1$. The foregoing may be symbolized by writing

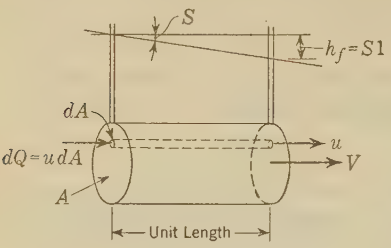


FIG. 3

$$W_o = (W_b)_o = (W_s)_o \qquad (5)$$

with the subscript b designating the process of withdrawal or the "borrowing" of the energy from the flow, and the subscript s indicating the spending of the energy on resistances.

The equality shown in Eq. 5 holds for bulk relations only. In fact, contrary to Eq. 5, the local values of w_b and w_s are generally not equal and do not balance each other. Indeed, the cardinal fact which dominates the entire energy loss mechanism is that the cross-sectional distribution of the local borrowing quantity w_b and the local spending rate w_s manifest altogether different, and in a sense opposite, outlines, and that for such reason the borrowing and the spending phases are necessarily linked by an intermediate process, the function of which is to transmit energy withdrawn at one part of the cross section to other parts, where the mechanism of energy exchange calls for its spending.

2. The Energy Withdrawing or Borrowing Function

The distribution of the local borrowing rate w_b is determined by noting that the energy withdrawn per unit time within an elementary tube dA (Fig. 3) is $\gamma\ S\ dQ = \gamma\ S\ u\ dA$, with u the local velocity. Dividing by the volume $dA \times 1$, one obtains the unit rate (that is, the work over a unit time interval, apportioned to a unit volume of fluid):

$$w_b = \gamma\, S\, u = -\frac{dp}{dx}\, u \left(\frac{\text{lb} \times \text{ft}}{\text{ft}^3 \times \text{sec}}\right) \qquad (6)$$

which shows that the spatial distribution of w_b follows the local velocity profile, copying the latter to the scale $\gamma\, S$ (curves BA and B_1A_1 in Figs. 2(a) and 2(b)).

In two-dimensional flow, for a block of unit width normal to the plane (Fig. 2), the total borrowed energy will be

$$(W_b)_o = \gamma\, S \int_0^{y'_o} u\, dy' = \gamma\, S\, U\, y'_o = \gamma\, S\, q_o = W_o \qquad (7)$$

and the average cross-sectional unit rate will be

$$w_o = \frac{W_o}{y'_o} = \gamma\, S\, U = -\frac{dp}{dx}\, U \ldots\ldots\ldots\ldots\ldots\ldots (8)$$

Furthermore, the aggregate energy amount "withdrawn" during a unit time from a partial cross-sectional block, such as that shown in Fig. 2(*a*) by the hatched area, is

$$\Big|W_b\Big|_0^{y'} = \int_0^{y'} w_b\, dy' = \gamma\, S \int_0^{y'} u\, dy' = \gamma\, S \Big|q\Big|_0^{y'} \ldots\ldots\ldots (9)$$

constituting the "cumulative" borrowing rate between the transverse limits, as indicated. The quantity $|q|_0^{y'}$, measured by the area OBba in Fig. 2(*a*), will be termed the "cumulative discharge." The corresponding quantities $|W_b|_0^{y'}$ are defined in Fig. 2(*c*) by curve $O_2b_2B_2$, with the coordinate $a_2\, b_2$ representing the area $O_1B_1b_1a_1$ under the part B_1b_1 of the local w_b-curve in Fig. 2(*b*). Obviously, for $y' = y'_o$, $|W_b|_0^{y'}$ equals W_o as defined in Eq. 7.

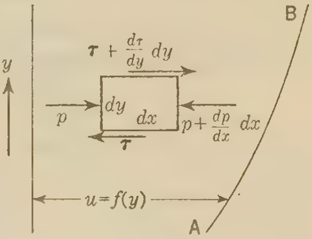


FIG. 4

Differential Form.—To express the energy borrowing process mathematically, note that for a two-dimensional element (Fig. 4), the resulting friction force is $\left(\tau + \frac{d\tau}{dy}\, dy\right) dx - \tau\, dx = \frac{d\tau}{dy}\, dx\, dy$. In uniform established motion frictional traction is in balance with the pressure resultant $\left(-\frac{dp}{dx}\, dx\right) dy$ which, apportioned to a volume unit, defines the equilibrium premises:

$$-\frac{dp}{dx} + \frac{d\tau}{dy} = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10a)$$

and

$$\frac{d\tau}{dy} + \gamma\, S = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10b)$$

The sign of the stress gradient $d\tau/dy$ is necessarily negative, for the friction resultant opposes the motion and compensates the positive pressure, or the gravity force, actuating the flow. The stress τ is essentially an internal agency. Action on the interfaces n and m of adjoining filaments (Fig. 5(*b*)) is equal in size, but opposite in direction. In determining which sign (plus or minus) is to be used for τ in Eqs. 10 and subsequently, one is guided by the direction of the traction on the face of a fluid block (Fig. 5(*a*)), terminated by the positive ordinate y. As is evident, the sign of τ will coincide appropriately with that of du/dy.

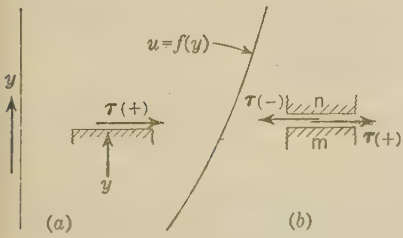


FIG. 5

The unit time work performed by friction on the element (Fig. 4) is $\frac{d\tau}{dy}\,dx\,dy\,u$, which is also a negative quantity to be compensated by the positive work of the pressure resultant $[(-\,dp/dx)\,dx\,dy\,u]$. The latter, in turn, is equal to the positive unit rate at which energy is withdrawn locally from the flow. Hence

$$w_b = -\frac{dp}{dx}u = -u\frac{d\tau}{dy} \qquad \text{(11)}$$

Note that, thus far, no limitations have been imposed on the particular nature of the stress-generating agencies. Therefore, Eq. 11 is quite general and will apply to all cases of two-dimensional flow, whether laminar or turbulent.

3. The Energy Spending Phase

It is expedient to consider first purely viscous (laminar) patterns, in which the mechanism is relatively simple. Indeed, in accord with Stokes,[3] the spending of energy has been identified with the work involved in viscous deformation. For two-dimensional motion (Fig. 6), when the element abcd is deformed into ab′c′d, during the time dt, the work done is the product of the shearing force $\tau\,dx$ and the displacement $du\,dt = \frac{du}{dy}\,dy\,dt$, and the local unit time rate of deformation work per unit volume is equal to

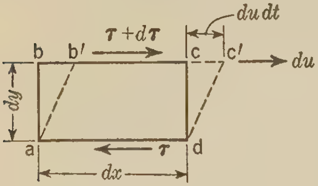


Fig. 6

$$w_s = \tau\frac{du}{dy} \qquad \text{(12}a\text{)}$$

or for laminar motion, with $\tau = \mu\frac{du}{dy}$:

$$w_s = \mu\left(\frac{du}{dy}\right)^2\left(\frac{\text{lb} \times \text{ft}}{\text{ft}^3 \times \text{sec}}\right) \qquad \text{(12}b\text{)}$$

Energy spending is thus concentrated in the region of high velocity gradients, which is principally near the walls of a conduit. In fact, at the axis, where $du/dy = 0$, the local loss is nil. The outline of the local energy spending curve w_s (curve $o_1\,s_1$, Fig. 2(*b*)) is thus "opposite" to that of w_b.

By analogy to Eq. 9, the cumulative spending rate—

$$\left|W_s\right|_0^{y'} = \int_0^{y'} w_s\,dy' \qquad \text{(13}a\text{)}$$

—is the aggregate work spent on resistances per unit time in the shaded block of Fig. 2(*a*), being represented in Fig. 2(*c*) by curve $O_2s_2S_2$, in which the coordinate a_2s_2 measures the area $O_1s_1a_1$ in Fig. 2(*b*). For the total reach $0 - y'_o$ obviously—

$$\left|W_s\right|_0^{y'_o} = \int_0^{y'_o} w_s\,dy' = \left|W_b\right|_0^{y'_o} = W_o \qquad \text{(13}b\text{)}$$

the cumulative curves W_b and W_s (Fig. 2(*c*)) terminating at the same point B_2. Naturally the areas $O_1B_1A_1$ and $O_1S_1A_1$ under curves w_b and w_s in Fig. 2(*b*) are equal. For any intermediary position, however, such as y', the cumulative values $|W_b|_0^{y'}$ and $|W_s|_0^{y'}$ shown in Fig. 2(*c*) by a_2b_2 and a_2s_2, do not coincide. Neither do the local values w_b and w_s represented in Fig. 2(*b*) by a_1b_1 and a_1s_1. Indeed in the central region of Fig. 2(*b*), between the axis and the intersection point M, the borrowed energy w_b exceeds the rate w_s at which energy is spent locally, whereas, nearer to the wall beyond M, the local energy rates, required for compensating the deformation work, exceed by far the comparatively small quantities w_b, available locally by borrowing. It follows, accordingly, that, between the "borrowing" and the "spending" phase, there must necessarily be an intermediate "transmittance" function, by means of which the excess of borrowed energy, available in the central region $0 - y'_M$, is conveyed to the wall zones to cover the requirements for spending.

4. The Energy Balance Equation

The local energy balance can be expressed by

$$w_b = w_s + w_t \quad \text{(14)}$$

with $w_t = w_b - w_s$ designating the local "transmittance energy rate." In the central reach, with $w_b > w_s$, w_t is positive, exemplifying the excess of energy subject to transmittance. In the wall region, where w_s exceeds the local supply of w_b, w_t becomes negative, signifying that the deficiency in locally borrowed energy is supplied from the store accumulated by transmittance from the central zone.

In Fig. 2(*b*), the (B_1T_1)-curve is obtained by subtracting O_1S_1 from B_1A_1, so that the local value for y' is $s_1b_1 = a_1t_1$. The resulting curve crosses the axis ($w_t = 0$) at the intersection ordinate y'_M, reaching its maximum negative value A_1T_1 at the solid boundary.

In Fig. 2(*c*), the curve $O_2t_MA_2$ features the corresponding cumulative quantity

$$|W_t|_0^{y'} = |W_b|_0^{y'} - |W_s|_0^{y'} \quad \text{(15)}$$

obtained by deducting $O_2s_2S_2$ from $O_2b_2B_2$. Obviously the coordinate $a_2t_2 = a_2b_2 - a_2s_2$ measures the area $O_1B_1b_1s_1 = O_1B_1t_1a_1$ in Fig. 2(*b*), indicating the aggregate excess energy, transmitted from the block toward the wall region. The maximum cumulative value a_Mt_M in Fig. 2(*c*), corresponding in position to the intersection point M of the local w_b-curve and w_s-curve, is the total excess energy, accumulated over the reach $0 - y'_M$ and made available through transmittance for spending in the wall zone. Obviously the coordinate a_Mt_M in Fig. 2(*c*) represents the positive excess area $O_1B_1MO_1$ in Fig. 2(*b*), equal to the negative deficiency area MS_1A_1M. The decline of W_t beyond point M signifies the gradual consumption in the wall region of the energy accumulated by transmittance. At the solid boundary the transmittance energy exhausts its functions. Accordingly, the cumulative W_t-curve goes through zero.

5. THE TRANSMITTANCE FUNCTION

To grasp the "transmittance" concept consider, in two-dimensional flow, an element A (Fig. 7) in its relation to the adjacent layers B and C (for clarity the interfaces are shown separated). In its motion relative to layer B, element A exercises a positive traction, performing work on layer B at the unit time rate of $\tau\,dx\,u$. In turn, element A receives work from layer C at the rate $(\tau + d\tau)\,dx\,(u + du)$. The resulting positive work performed by the element on the adjoining fluid, over what it receives, is $\tau\,dx\,u - (\tau + d\tau)(u + du)\,dx$ which, after omitting quantities of higher order of smallness, becomes

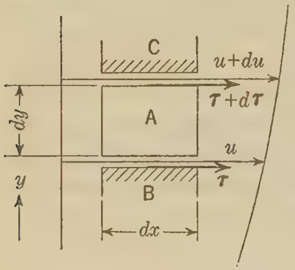


FIG. 7

$$-\,dx\,(\tau\,du + u\,d\tau) = -\frac{d}{dy}(\tau\,u)\,dy\,dx.$$

Dividing by the volume $dx\,dy$, the unit local rate of work transmitted to the adjacent fluid or the transmittance unit rate is

$$w_t = -\frac{d}{dy}(\tau\,u) = \frac{d}{dy}(-\,\tau\,u)\left(\frac{\text{lb}\times\text{ft}}{\text{ft}^3\times\text{sec}}\right)\dots\dots\dots\dots(16)$$

With Eqs. 11 and 12*a*, the local energy balance, Eq. 14, for two-dimensional laminar flow, takes the basic differential form

$$\underset{(w_b)}{-\,u\frac{d\tau}{dy}} = \underset{(w_s)}{\tau\frac{du}{dy}} - \underset{(w_t)}{\frac{d}{dy}(\tau\,u)}\dots\dots\dots\dots\dots\dots\dots(17)$$

Formally, by transposing the members, Eq. 17 may be changed into a complete differential of $\tau\,u$, which furnishes the starting point in the Stokes[3] analysis. However, to give Eq. 17 the proper physical significance, the members must be presented in the algebraic sequence as written. Note that, for established uniform motion, with $\frac{d\tau}{dy}$ necessarily negative (see Eq. 10), the quantity $-\,u\frac{d\tau}{dy} = w_b$ is positive. The quantity w_s is also positive, for in $\tau\frac{du}{dy}$ both members are of the same sign (see paragraph following Eqs. 10). The only quantity that changes in sign is the local transmittance rate w_t (Eq. 16).

Since w_t is the derivative of $-\,(\tau\,u)$, the latter must feature the cumulative transmittance rate W_t. In fact, for the block $0 - y'$ in Fig. 2(*a*), by Eq. 16,

$$\left|W_t\right|_0^{y'} = \int_0^{y'} w_t\,dy' = \int_0^{y'}\frac{d}{dy'}(-\,\tau\,u)\,dy' = \left|-\,u\,\tau\right|_0^{y'}\dots\dots(18)$$

represents the work done by the shaded block on the adjacent flow. The quantity $u\,\tau$ is negative, being the product of a positive velocity and a stress, which in terms of Fig. 8 is negative. Hence $(-\,\tau\,u)$ is positive, as indicated by curve $O_2t_MA_2$ in Fig. 2(*c*). The sign of the local value w_t, on the other hand, conditioned by the tangent to the W_t-curve, changes from positive to negative at point t_M.

As a matter of procedure, when dealing with the transmittance phase, it is expedient to start with the cumulative curve which, according to Eq. 18, is the product of the velocity curve (BA in Fig. 2(*a*)) and the stress line (OC in Fig. 2(*a*)). The maximum value of $-u\,\tau$ is reached at the intersection point M in Fig. 2(*b*) corresponding to $w_t = w_b - w_s = 0$. Point M is to be referred to as the "reversal point."

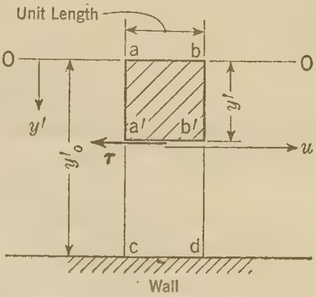


Fig. 8

Cumulative Energy Balance Equation.—The local equation (Eq. 17) is complemented for two-dimensional uniform flow by the cumulative expression

$$-\int_{y_1}^{y_2} u\,\frac{d\tau}{dy}\,dy = \int_{y_1}^{y_2} \tau\,\frac{du}{dy}\,dy - \int_{y_1}^{y_2} \frac{d}{dy}\,(\tau\,u)\,dy \,\ldots\ldots\ldots (19)$$

or, in relation to Fig. 2:

$$\int_0^{y'} \left(-\,u\,\frac{d\tau}{dy}\right) dy' = \int_0^{y'} \tau\,\frac{du}{dy}\,dy' + \Big|-\tau\,u\Big|_0^{y'} \,\ldots\ldots\ldots\ldots (20)$$

and

$$\gamma\,S\Big|q\Big|_0^{y'} = \int_0^{y'} w_s\,dy' \quad + \Big|-\tau\,u\Big|_0^{y'} \,\ldots\ldots\ldots\ldots (21)$$

all the members (see Fig. 2(*c*)) being positive.

Again, as in the case of Eq. 11 and Eq. 9, no restrictions were imposed on the nature of the stress-forming agency. The "transmittance" phase formulas, Eqs. 16 and 18, apply to all forms of flow, laminar or turbulent.

6. Mechanical Analogy

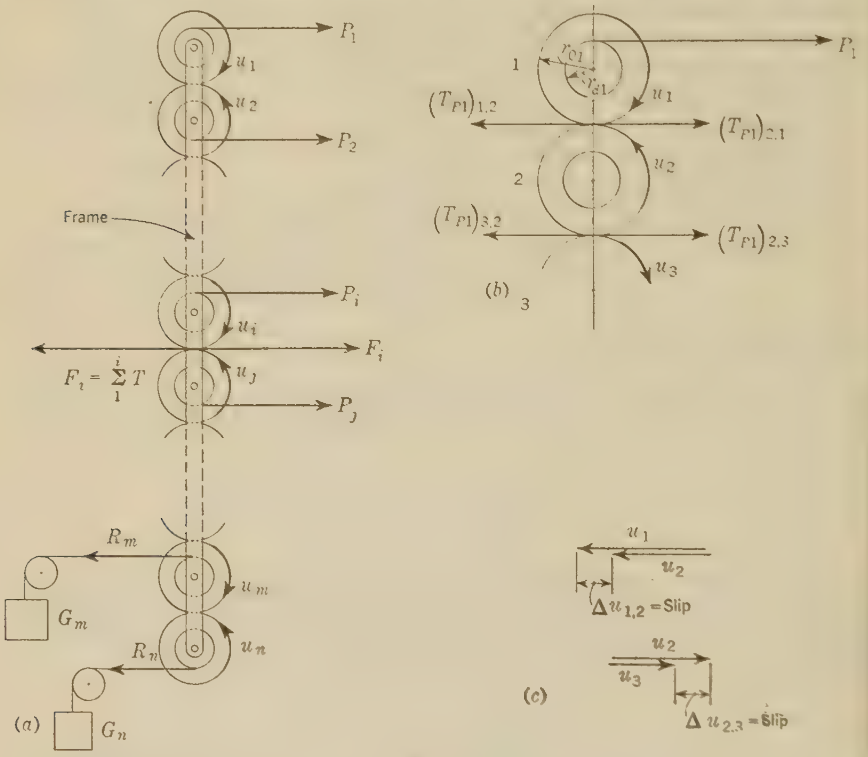
The essence of the "transmittance function" may be demonstrated by a mechanical model. In Fig. 9 a sequence of wheels is mounted on a common frame, with adequate friction between the contacting surfaces. Because of this friction, energy derived from the "driving" agencies $P_1, P_2, \ldots,$ is transmitted to where it is spent on the resistances R. Slipping between successive wheels will mean that, although the friction components such as $|T_{p1}|_{1,2} = P_1\,\frac{r_d}{r_e}$ and $|T_{p1}|_{2,1}$ (Fig. 9(*b*)) are equal, the velocities u_1 and u_2 differ, and thus only a part of the power $|T_{p1}|u_1$ derived from P_1 (namely, $|T_{p1}|u_2$) will be transmitted to the next wheel. The difference

$$|T_{p1}|u_1 - |T_{p1}|u_2 = |T_{p1}|\Delta u_{1,2} \,\ldots\ldots\ldots\ldots\ldots\ldots\ldots (22)$$

represents a local mechanical "energy loss," an amount dissipated directly into molecular heat. The work done cumulatively by all the preceding driving agencies (P_1 to P_i) through wheel i is

$$F_i u_i = u_i \sum_1^i |T_p| \ldots\ldots\ldots\ldots\ldots\ldots\ldots (23)$$

Of this work, however, only $F_i u_j$ is passed to the next wheel (j), the amount $F_i (u_i - u_j)$ being dissipated locally in the slip between (i) and (j). As a result, the "cumulative" energy balance for the wheel sequence is expressed by



P_1
u_1
u_2
P_2
Frame
P_i
u_i
F_i
$F_i = \sum_1^i T$
u_j
P_j
R_m
u_m
G_m
R_n
u_n
(a)
G_n
P_1
1
r_{01}
r_{d1}
u_1
$(T_{P1})_{1.2}$
$(T_{P1})_{2.1}$
u_2
2
$(T_{P1})_{3.2}$
$(T_{P1})_{2.3}$
u_3
(b)
3
u_1
u_2
$\Delta u_{1,2}$ = Slip
u_2
u_3
(c)
$\Delta u_{2.3}$ = Slip

Fig. 9

$$\sum_1^i |T_p| u = \sum_1^i (F \Delta u) + u_j F_i \ldots\ldots\ldots\ldots\ldots\ldots (24)$$

Eq. 24 is analogous to Eq. 21. The friction resultant F_i is the simile of the stress τ in Fig. 8; $\sum_1^i |T_p| u$ is equivalent to the borrowed energy $|W_b|_0^{y'}$; the local "slip" loss (Eq. 22) compares with w_s in Eq. 12a; and $\sum (F \Delta u)$ in Eq. 24 is the analogue of $|W_s|_0^{y'}$. Finally, the "transmittance" member $u_j F_i$ is the equivalent, both in meaning and form, of the product $|\tau u| = |W_t|_0^{y'}$.

7. ENERGY "SPENDING" AND "DISSIPATION"

Loss of energy signifies ultimate dissipation into molecular heat. Energy borrowed from the flow changes its mechanical aspects and is transformed into thermal form. Such transformation is caused finally by viscous attrition, which enhances intermolecular activity. The ultimate dissipation develops thus on an ultramicroscopic molecular scale.

In laminar patterns the process of dissipation is assumed to coincide with viscous deformation (see Section 3). Mechanical energy spent in deformation is dissipated through transformation into heat in loco.

In turbulent flow, "spending" and "dissipation" coincide only partly. The term "spending" will be used to signify only the idea that energy borrowed from the flow has lost, irretrievably, its original flow form; and "dissipation," under all circumstances, will designate the ultimate "degradation" of mechanical energy into heat.

8. EXAMPLES OF ENERGY BALANCES IN LAMINAR MOTION

In some of the simpler laminar patterns, the stress and velocity quantities can be expressed in definite mathematical form, permitting an appraisal of the energy exchange process by purely analytical procedure.

(*a*) *Two-Dimensional Motion, Between Parallel Plates* (*Fig. 2*).—With $q_o = U\,y'_o$ as the unit width discharge between the axis and the wall, the local velocity is

$$u = u_M \left[1 - \left(\frac{y'}{y'_o} \right)^2 \right] \qquad (25)$$

in which $u_M = \frac{3}{2}\,U$. Furthermore,

$$-\frac{dp}{dx} = \frac{3\,\mu\,U}{(y'_o)^2} \qquad (26a)$$

and

$$\tau_o = -\frac{3\,\mu\,U}{y'_o} \qquad (26b)$$

Accordingly in Eqs. 7 and 8,

$$W_o = \frac{\tau^2_o\,y'_o}{3\,\mu} \qquad (27a)$$

and

$$w_o = \frac{\tau^2_o}{3\,\mu} \qquad (27b)$$

By elementary algebraic procedure (which is omitted), one arrives at the following dimensionless expressions for the local and cumulative unit rates that appear in Eqs. 17 and 20:

$$w_b = \tfrac{3}{2}\,w_o \left[1 - \left(\frac{y'}{y'_o} \right)^2 \right]; \qquad w_s = 3\,w_o \left(\frac{y'}{y'_o} \right)^2;$$

$$w_t = \tfrac{3}{2}\,w_o \left[1 - 3 \left(\frac{y'}{y'_o} \right)^2 \right] \qquad (28a)$$

and

$$\left|W_b\right|_0^{y'} = \tfrac{3}{2}\, W_o \left[\frac{y'}{y'_o} - \frac{1}{3}\left(\frac{y'}{y'_o}\right)^3\right]; \qquad \left|W_s\right|_0^{y'} = W_o \left(\frac{y'}{y'_o}\right)^3;$$

$$\left|W_t\right|_0^{y'} = \tfrac{3}{2}\, W_o \left[\frac{y'}{y'_o} - \left(\frac{y'}{y'_o}\right)^3\right] \qquad (28b)$$

The curves of these equations are plotted in Fig. 2.

(b) *Cylindrical Pipe.*—The Poiseuille relation,

$$-\frac{dp}{dx} = \frac{8\,U\,\mu}{r^2_o} \qquad (29)$$

with $R = \dfrac{r_o}{2}$ and $u_m = 2\,U$, leads to the following expressions:

$$\tau_o = -\frac{4\,\mu\,U}{r_o} \qquad (30a)$$

$$u = -\frac{\tau_o\,r_o}{2\,\mu}\left[1 - \left(\frac{r}{r_o}\right)^2\right] \qquad (30b)$$

$$W_o = \frac{\pi\,r^2_o\,\tau^2_o}{2\,\mu} \qquad (30c)$$

and

$$w_o = \frac{\tau^2_o}{2\,\mu} \qquad (30d)$$

In establishing the local rates, it is expedient to operate with an elementary cylindrical shell. For example, the work involved in the deformation shown in Fig. 10(*a*) results in the formula:

$$w_s = \tau\,\frac{du}{dr} \qquad (31)$$

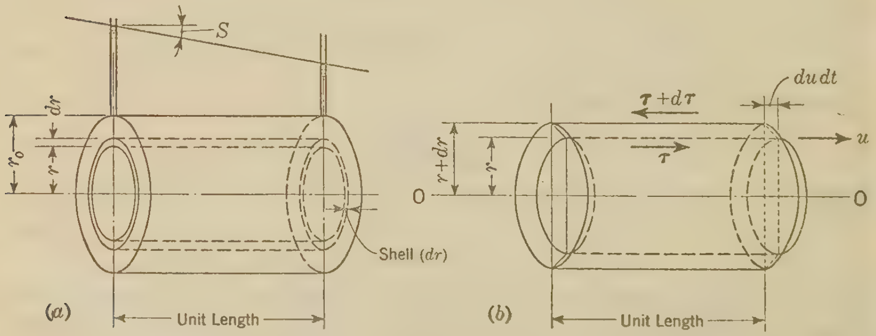


FIG. 10

The cumulative quantities should refer to an internal cylinder of radius r and unit length. The final dimensionless interrelationships are:

$$w_b = 2\,w_o\left[1 - \left(\frac{r}{r_o}\right)^2\right]; \quad w_s = 2\,w_o\left(\frac{r}{r_o}\right)^2; \quad w_t = 2\,w_o\left[1 - 2\left(\frac{r}{r_o}\right)^2\right] \qquad (32)$$

and

$$\left|W_b\right|_0^r = 2\,W_o\left(\frac{r}{r_o}\right)^2\left[1-\frac{1}{2}\left(\frac{r}{r_o}\right)^2\right]; \qquad \left|W_s\right|_0^r = W_o\left(\frac{r}{r_o}\right)^4;$$

$$\left|W_t\right|_0^r = 2\,W_o\left(\frac{r}{r_o}\right)^2\left[1-\left(\frac{r}{r_o}\right)^2\right] \quad \ldots\ldots\ldots\ldots\ldots (33)$$

The cumulative curves are reproduced for comparison, with the eventual turbulent outlines, in Fig. 11.

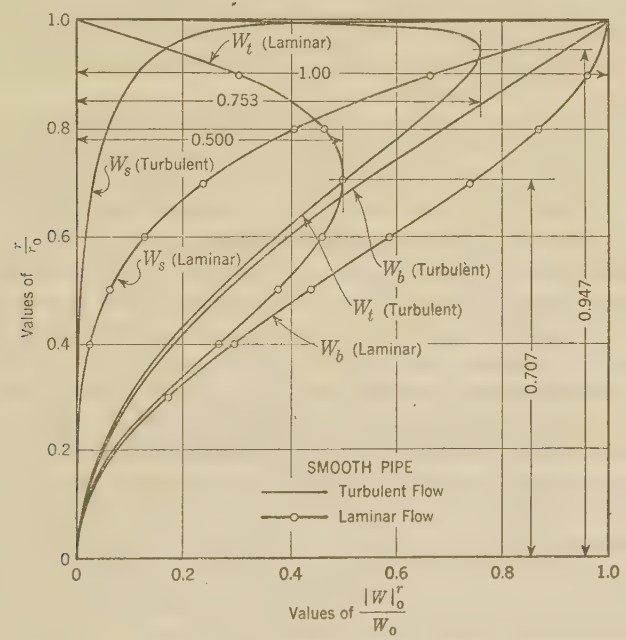


FIG. 11

9. TURBULENT FLOW PATTERNS

Since no restrictions were introduced to limit the expressions for the borrowing rate w_b (Eq. 11) and the transmittance rate w_t (Eq. 16), the equations are directly applicable to turbulent flow. Naturally, $\bar{\tau}$ will be the local Reynolds stress, and $\bar{u}$ the mean steady local velocity, whereas the quantities $\bar{w}_b = -\overline{u\frac{d\tau}{dy}}$ and $\bar{w}_t = \frac{d}{dy}(-\overline{\tau u})$ in Eq. 18 and Eq. 21 will feature the temporal average unit borrowing and transmittance rates. Because of the complex nature of turbulent motion, the spending rate w_s cannot be appraised a priori as in the case of Eq. 12*b* for laminar flow. On the other hand, from the standpoint of a complete differential, the mean local spending rate for two-dimensional turbulent flow—

$$\bar{w}_s = \bar{w}_b - \bar{w}_t = -\overline{u\frac{d\tau}{dy}} - \frac{d}{dy}(-\overline{\tau u}) = \overline{\tau\frac{du}{dy}} \ldots\ldots\ldots\ldots (34)$$

—must conserve the mathematical form of Eq. 12*a*. The w_s-quantity indicates the unit rate at which flow energy disappears under the action of agencies pertinent to the turbulent process.

Furthermore, the incumbent stress and velocity quantities cannot be derived from analytical expression as in Section 3, and must be taken directly from observation. Empirical data vary widely, depending on wall roughness, the Reynolds number, and other circumstances of the flow. Nevertheless, energy balances for turbulent flow display certain consistent characteristics, which happen to differ substantially from laminar outlines, and reflect the essence of turbulence dynamics. A typical example is given in Fig. 1, based on Göttingen data[8] relating to turbulent flow in a 10 cm smooth pipe, with $U = 876$ cm per sec, and $-dp/dx = 480$ g per cm^3.

By using in Eq. 34 the values for $\bar{\tau}$, $\bar{u}$, and $d\bar{u}/dy$ contained in the Nikuradse report, one computes the consecutive local unit rates $\bar{w}_b$, $\bar{w}_t$, and $\bar{w}_s$ as plotted in Fig. 1. The cumulative curves, obtained from the local outlines through step-by-step integration are presented, in dimensionless form, in Fig. 11, together with the comparative plottings for laminar flow.

The striking feature, characteristic of all turbulent patterns, is the intensive concentration of energy spending in a narrow zone next to the wall. For laminar flow the reversal point (M) is located at a distance 0.293 r_o from the wall, whereas in turbulent motion it is shifted to within 0.053 r_o from the solid boundary. Furthermore, over the reach $0-r_M$, constituting about 95% of the radial distance, relatively small amounts of energy are spent locally, the major part of the locally borrowed energy w_b being transmitted to the wall zone (r_M-r_o) where the accumulated energy is used to offset the unusually high requirements for spending. In fact, in Fig. 1, the calculated local values of w_s and w_t close to the solid boundary outreach by far the bounds of the figure. Accordingly, to appraise the phenomena, one must refer to the dimensionless cumulative diagram, Fig. 11, in which the specific characteristics of the energy exchange in turbulent motion become particularly lucid. Through the central region, the cumulative transmittance curve $|W_t|_0^r$ follows closely the borrowing outline $|W_b|_0^r$. In fact, over the reach $0-\text{M}$, in which more than 90% of the total energy W_o is withdrawn (borrowed) from the flow, the cumulative loss is less than 17%. By contrast over 83% of W_o is spent in the narrow (0.05 r_o) wall zone, where the quite insignificant local borrowing accounts for less than 8% of W_o. In other words, the concentrated "spending" in the wall zone takes effect principally at the expense of the transmitted energy.

One should infer, from the shape of the turbulent curves, that the energy exchange in the central zone and in the border zone must be actuated by entirely different physical factors. Conventionally, in fact, one may accept the premise that the "reversal radius" r_M separates the fluid body into two parts, of which the central cylindrical block $(0-r_M)$ will be called the "transmittance region," whereas the annular space near the wall (r_M-r_o) will be referred to as the "conversion zone" (see Section 12).

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[8] "Gesetzmässigkeiten der turbulenten Strömung in Glatten Rohren," by J. Nikuradse, *Forschungsheft*, Verein Deutscher Ingenieure, No. 356, 1932, pp. 20 and 31.

10. The Mechanics of Turbulent Friction Loss

Turbulent motion is characterized by the presence, in the flow, of an array of eddies that "swarm" in an apparently unpredictable manner and at each instant add their rotary components to the respective local velocities of the main motion. The ever-fluctuating turbulent pattern, superimposed over the steady flow structure, devolves thus from the composite action of a multitude of individual vorticing units.

The energy of a fluid in turbulent motion is compounded from the energy of the main flow, characterized locally by the energy head

$$e = z + \frac{p}{\gamma} + \frac{u^2}{2g} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (35)$$

and by the kinetic energy of the superimposed turbulent structure. The latter, being essentially the composite energy of the swarming eddies, is defined locally by value of $\rho\overline{(u'_t)^2}/2$, where

$$\overline{(u'_t)^2} = \overline{(u')^2} + \overline{(v')^2} + \overline{(w')^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (36)$$

is the temporal mean square of the local turbulent fluctuations in the three coordinate directions.

A statistically permanent state of turbulence implies the maintenance of a steady average eddying structure, which means that new vorticing units must be generated incessantly to replace the eddies "extinguished" by attrition in the course of their swarming motion through the flow. The generation of eddies infers that they are endowed with energy, which must necessarily be supplied from the energy store of the main flow. Indeed, the primary characteristic of the turbulence mechanism is that energy, at an apportioned rate, is withdrawn continuously from the main flow, and is "invested" into turbulent eddies, such "investment" involving the conversion of the borrowed flow energy into vortical eddying form. The generation of turbulence is thus synonymous with the producing of the turbulent eddies. Turbulent energy, manifest in turbulent fluctuations, is originally flow energy, transformed into vortical form. By its very nature the conversion is an irreversible process, because energy, once converted into eddies, can by no means be restored to the flow. For such reason, so far as the main flow is concerned, when energy withdrawn from the latter is "spent" in the creation of turbulent eddies, it may be considered "lost." Actually the "lost" flow energy reappears, at least partially, in the converted vortical form, which animates the turbulent structure. The final dissipation of such energy takes place after conversion, and at the time when, in the course of swarming, the mechanical energy with which the eddies were initially endowed is transformed (degraded) through viscous attrition into molecular heat.

According to prevailing views, the generation of turbulent eddies in established "friction" flow is largely concentrated in a narrow zone near the solid boundaries of a conduit. L. Prandtl[9] most appropriately qualifies his definition of such boundary zones as "the actual eddy producing mill," whereas the "spreading" of the eddies toward the central region is said to take place "rather passively." Indeed, the faculty of generating eddies seems to be a specific property, inherent to discontinuity "vortex sheets" that are subjected to exexcessive strain. Whenever they are overstrained, such vortex sheets, or layers of concentrated vorticity, become unstable; and after passing through a cycle of ever-increasing self-induced pendulations, they terminate by eventually "curling," or being "rolled" up, into "chains" of eddies or a sequence of intermittent lumps of condensed vorticity. The process is comparatively well defined in the case of "free" discontinuity sheets caused in the interior of a fluid by "separation." As illustrated by Fig. 12, the formation of the eddies, in such instances, may be observed visually. (In these observations by L. Schiller and his associates,[10] the original vortex sheet was made visible by the injection of dye.) In friction flow, past solid surfaces, the reach of excessive strain is supposedly associated with the boundary zone near the wall where, over an exceedingly thin "sublayer," the velocity drops abruptly to zero. Thus far, existing techniques have not permitted the direct observation of turbulence generation near solid boundaries. There is ample indirect evidence, nevertheless, pointing to the essential similarity of the case with that of "free" separation sheets. (For further information in this generally still obscure and mysterious realm, the reader is referred to the general literature in this field.)[11,12]

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[9] *Proceedings*, 5th International Cong. for Applied Mechanics, Cambridge, Mass., John Wiley & Sons, Inc., New York, N. Y., 1939, p. 367.

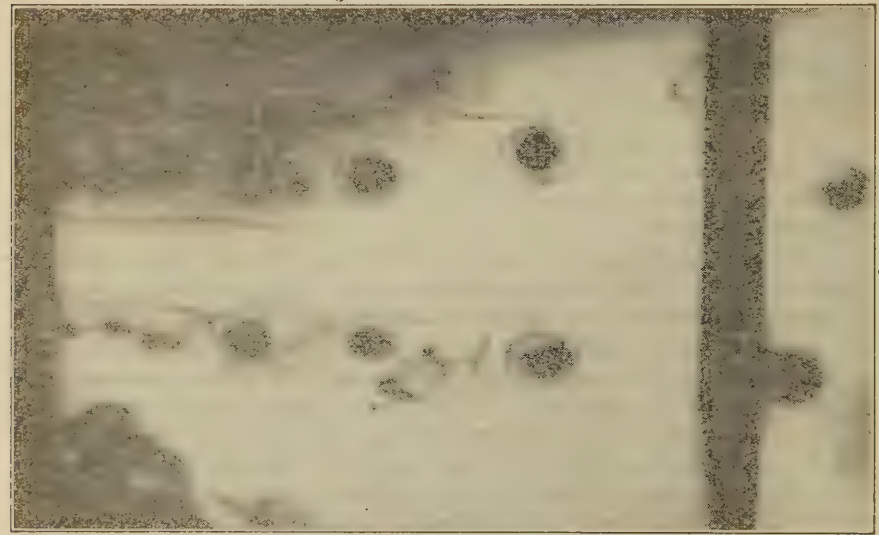

FIG 12.—THE "ROLLING UP" OF ANNULAR EDDIES, ON THE SURFACE OF A CONTRACTED VEIN ENTERING A PIPE

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[10] *Proceedings*, 5th International Cong. for Applied Mechanics, Cambridge, Mass., John Wiley & Sons, Inc., New York, N. Y., 1939, p. 315.

[11] "Fundamentals of Hydro- and Aeromechanics," based on lectures of L. Prandtl, by O. G. Tietjens, translated by L. Rosenhead, 1st Ed., McGraw-Hill Book Co., Inc., New York and London, 1934, p. 222.

[12] "The Formation of Vortices from a Surface of Discontinuity," by L. Rosenhead, *Proceedings*, Roya Soc. of London, Series A, Vol. CXXXIV, 1931-1932, pp. 170-192.

The pivotal phase in the sequence of events is conversion, in the course of which turbulent eddies are generated incessantly in the conversion zone. The preconversion phases (Section 9) deal with energy of the main flow. As illustrated by Fig. 11, flow energy is borrowed largely in the middle regions of a cross section and through transmittance is concentrated near the wall, being made available for conversion. In the post-conversion phase, the remainder of the originally borrowed mechanical energy is in turbulent eddying form. Individual vorticing units, assumedly of the Fig. 12 type, are "cast off" or "shed" from the generating zone into the adjacent flow, and as the eddies spread across the conduit on generally oblique paths, their initial energy is gradually dissipated into molecular heat. The possible interreaction between a vortical unit and the surrounding flow is illustrated by Fig. 13. On the top of the eddy, the rotary component u_t adds to the local velocity u_1, whereas at the bottom it subtracts from u_2. The resulting difference in velocity heads results in a transverse pressure differential, prompting the eddy to move in a crosswise direction away from the wall. The "shedding" of the eddies from the seat where they are originally generated may be thus considered as caused by a "Magnus effect," similar in nature to the "lift" of an airfoil or to the propelling force in a rotor ship. The transverse motion, prompted by such induced pressure forces, combines with the drag of the current which tends to wash the eddy downstream.

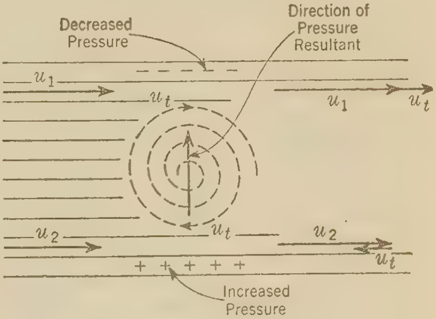

FIG. 13

The transformation of the mechanical vortical energy into thermal state is by viscous attrition. Indeed, the local dissipation rate may be expressed in terms of the Taylor formula[13,14]

$$w_d = \kappa\,\mu\,\frac{\overline{(u'_t)^2}}{\lambda^2} \qquad (37)$$

in which κ is a numerical coefficient, and λ is a characteristic dimension of the small-size eddying group in the vorticing composite. In an eddying composite, energy is dissipated in large part through the instrumentality of the smallest size eddies, or the so-called micro-turbulence fringe. Large eddies, in turn, are responsible for producing the turbulent stress and more generally for actuating convective transfer. The large eddies, accordingly, enhance turbulent diffusion, and their direct contribution to dissipation is comparatively small. It is surmised that, in the course of swarming, the larger eddies are subject to a process of "grinding," by which they are split continuously into smaller

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[13] "Statistical Theory of Turbulence," by G. I. Taylor, *Proceedings*, Royal Soc. of London, Series A, Vol. CLI, 1935, pp. 430–454.

[14] "Modern Developments in Fluid Dynamics," edited by S. Goldstein, Clarendon Press, Oxford, England, 1938, Vol. II, p. 221.

units. G. I. Taylor has treated this general subject and has presented a bibliography.[15,16]

The sequence involved in the mechanism of turbulent friction loss is presented schematically in Fig. 14. Of the total borrowed flow energy W_o, the amount W_c is made available in the conversion zone. Energy W_c is less than W_o by about the cumulative loss $\left|W_s\right|_0^M$ in the transmittance zone. In the absence of adequate experimental knowledge, one cannot judge as to what part of W_c is dissipated locally in the course of conversion. No doubt, however (see Section 12), the conversion process is accompanied by a considerable inherent loss. The latter, in Fig. 14, is symbolized by the ordinate CB. The remaining W (turbulent) = BA represents the part of the original W_o which survives conversion and which, liberated in eddying form, furnishes the "animating" source by which the state of turbulence, as revealed in its outward manifestations, is actuated and maintained. Finally, curve B′ − O portrays the gradual extinction of the eddying structure or the transverse "decay of turbulence," by the dissipation of mechanical energy into molecular heat (Eq. 37).

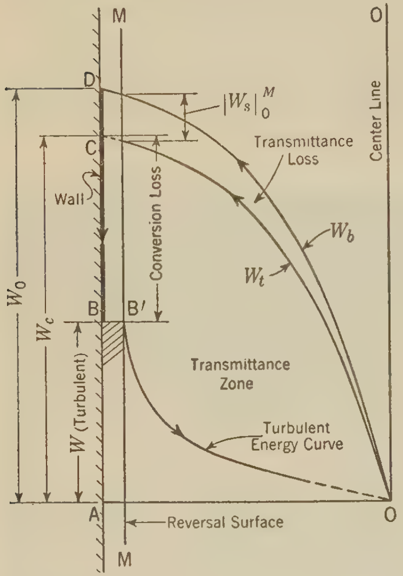

FIG. 14

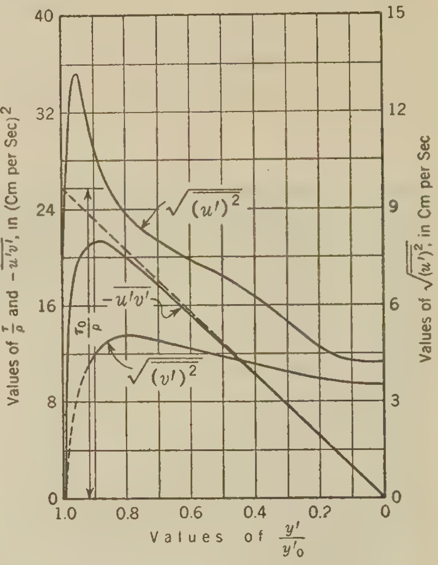

FIG. 15

An exact quantitative appraisal of the sequence would require experimental data of the type presented in Fig. 11 complemented by detailed records showing reliably, in three-dimensional fashion, the distribution of turbulence intensity across the cross section. No complete data of this nature are available. Fig. 15 shows, in part, the distribution of the axial and transversal turbulent com-

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[15] "Some Recent Developments in the Study of Turbulence," by G. I. Taylor, *Proceedings*, 5th International Cong. for Applied Mechanics, Cambridge, Mass., John Wiley & Sons, Inc., New York, N. Y., 1939, p. 301.

[16] "Turbulence Investigations at the National Bureau of Standards," by Hugh L. Dryden, *ibid.*, p. 365.

ponents in a rectangular conduit.[17] As seen, the axial intensity of turbulence, measured by the root mean square $\sqrt{\overline{(u')^2}}$, reaches a sharp maximum in the immediate proximity of the wall, readily corresponding to the outward boundary of the conversion region. From there on, the intensity recedes rapidly toward the center of the conduit.

11. EFFECT OF WALL ROUGHNESS

A basic premise underlying the Prandtl-von Kármán turbulence theory is the organic link between the stress structure and the shape of the velocity curves.[18] Experiments reveal that, if the roughness of the walls of a given conduit of radius r_o is modified consecutively, the flow Q being adjusted to yield in each instance the same resistance slope S, and hence the same stress structure (Eqs. 3 and 4), then the respective velocity profiles in the central "turbulent"

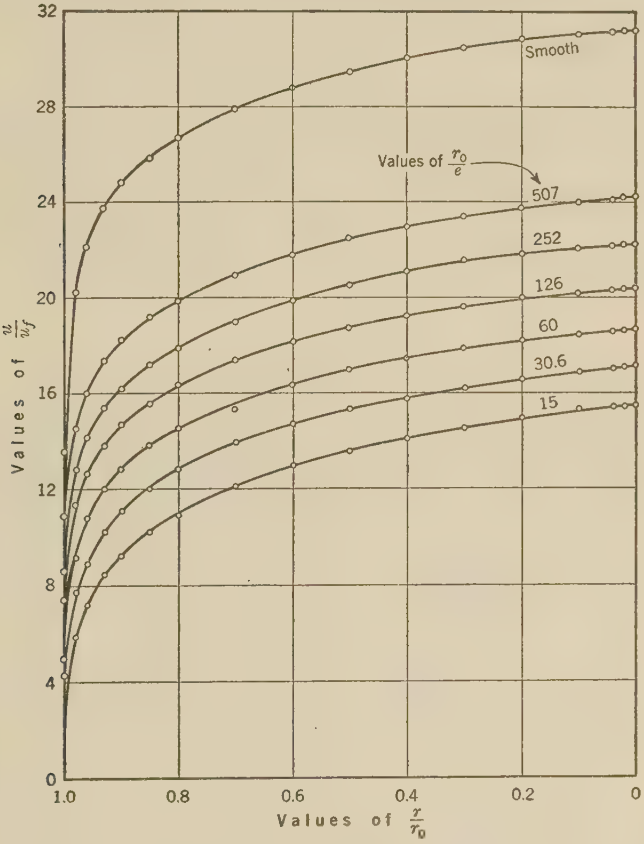

FIG. 16

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[17] "Beitrag zum Turbulenz Symposium," by L. Prandtl, *Proceedings*, 5th International Cong. for Applied Mechanics, Cambridge, Mass., John Wiley & Sons, Inc., New York, N. Y., 1939, p. 345.

[18] "The Mechanics of Turbulent Flow," by B. A. Bakhmeteff, Princeton Univ. Press, Princeton, N. J., 1936, p. 55.

zone will follow identically shaped parallel outlines (see Fig. 16). In fact, if the observed velocities were plotted from a common maximum point, the curves for the different roughnesses would coincide over the greater part of the cross section, the eventual differences becoming apparent only in the comparatively narrow boundary zones.[19]

In regard to the energy balances, an unvarying resistance slope makes the total energy loss $W_o = \gamma\, Q\, S$ proportional to the respective average velocity. Furthermore, congruent velocity profiles signify that the velocity gradients $d\bar{u}/dy$ in the central turbulent zone are the same for all degrees of roughness, and that therefore in such a zone the local losses w_s by Eq. 34 will be identical in all instances. The same obviously will pertain to the cumulative values $|W_s|_0^r$, as long as one does not transgress to a point where the velocity profiles cease to be congruent.

The inescapable conclusion is that the spending of energy in the transmittance region, the latitude of which may be broadly said to coincide with the central "turbulent" zone, is solely dependent on the stress structure, and that the spending process in the transmittance zone is implemented by the same agencies that shape the stress patterns (see Section 12). Deducting the common loss in the transmittance zone $|W_s|_0^M$ from the respective W_o-quantities (which are proportional to the varying bulk velocities), one obtains the energy quantities

$$W_c = W_o - |W_s|_0^{r_M} = \gamma\, S\, A\, U_. - |W_s|_0^{r_M} \ldots\ldots\ldots\ldots (38)$$

which for one or another degree of wall roughness are to be spent in the conversion (boundary) zones. Paradoxically, the largest value of W_c occurs in the case of a smooth surface. The rougher the walls, the smaller the value of W_o and therefore of W_c, always assuming an unvarying resistance slope.

The relation between the velocities and the wall roughness is summarized in terms of dimensionless ratios in Fig. 16.[20,21] Velocity profiles for pipes of different size and different relative roughness are plotted versus r/r_o in terms of the so-called "friction velocity"

$$u_f = \sqrt{\frac{\tau_o}{\rho}} = U\sqrt{\frac{c_f}{2}} = U\sqrt{\frac{f}{8}} \ldots\ldots\ldots\ldots\ldots\ldots (39)$$

c_f is the conventional turbulent friction factor in

$$\tau_o = \rho\, c_f\, \frac{U^2}{2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (40a)$$

and f is the coefficient of friction in the pipe formula

$$S = \frac{f}{D}\frac{U^2}{2\,g} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (40b)$$

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[19] "The Mechanics of Turbulent Flow," by B. A. Bakhmeteff, Princeton Univ. Press, Princeton, N. J., 1936, p. 56.

[20] "Strömungsgesetze in rauhen Rohren," by J. Nikuradse, *Forschungsheft*, Verein Deutscher Ingenieure, No. 361, 1933, p. 8.

[21] "The Mechanics of Turbulent Flow," by B. A. Bakhmeteff, Princeton Univ. Press., Princeton, N. J., 1936, p. 97.

(Fig. 16 refers to flow in pipes ranging from 1 in. to 4 in. in diameter, D, the walls of which were roughened artifically by uniform sand grains of different sizes. The inverse ratios of relative roughness r_o/e are the quotients of the radius of the pipe by the grain size. The shape of the roughness grains is assumed to be "similar.")

In the central reaches the velocity profiles in Fig. 16 show congruence in outline, which is to be anticipated, since reducing the velocities to the term u_f is equivalent to referring the velocity pattern to a common stress structure. Since the curves are dimensionless and imply no restrictions with regard to the absolute dimensions of the conduit or the size of the roughness grains, the diagram typifies universal velocity patterns for the "standard" Göttingen sand roughness type. Accordingly the experimental data, underlying Fig. 16 and explicitly tabulated in the Göttingen Report,[22] may be used for disclosing, in equally generalized manner, the comparative features of the internal energy exchanges for pipes of the respective relative wall roughnesses. In fact, in terms of the "friction" velocity (Eq. 39) the pressure gradient is

$$-\frac{dp}{dx} = \gamma S = \left|\frac{2\,\tau_o}{r_o}\right| = \frac{2\,\rho\,u^2_f}{r_o} \qquad (41)$$

and the total energy loss and the average spending rate are

$$W_o = 2\,\pi\,r_o|\tau_o|U = 2\,\pi\,r_o\,\rho\,u^3_f\,\frac{U}{u_f} \qquad (42a)$$

and

$$w_o = \frac{W_o}{\pi\,r^2_o} = \frac{2\,\rho\,u^3_f}{r_o}\,\frac{U}{u_f} \qquad (42b)$$

The analysis may be confined to the cumulative quantities only. In expressing borrowed energy, replace the pressure gradient in w_b (Eq. 11) by Eq. 41; substitute $u_f\,\dfrac{u}{u_f}$ for u; and $r_o\,\dfrac{r}{r_o}$ for r. Then:

$$\Big|W_b\Big|_0^r = -\frac{dp}{dx}\int_0^r 2\,\pi\,r\,u\,dr = 2\,\pi\,r_o\,\rho\,u^3_f\int_0^{r/r_o} 2\,\frac{r}{r_o}\,\frac{u}{u_f}\,d\left(\frac{r}{r_o}\right) \qquad (43a)$$

The subintegral quantity is twice the static moment, with regard to the axis, of the area bounded by the velocity curve u/u_f in Fig. 16, between the limits 0 and r/r_o.

The cumulative transmittance rate, for a cylinder of radius r, takes the form

$$|W_t|_0^r = 2\,\pi\,r(-\,\tau\,u)_0^r = 2\,\pi\,r_o\,\rho\,u^3_f\left[\left(\frac{r}{r_o}\right)^2\frac{u}{u_f}\right] \qquad (43b)$$

in which the quantity in brackets equals the product of the squared relative distance (r/r_o) in Fig. 16 by the corresponding u/u_f.

After the curves for the borrowing and for the transmittance phases are computed and plotted, the cumulative spending rate

$$|W_s|_0^r = |W_b|_0^r - |W_t|_0^r \qquad (44)$$

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[22] "Strömungsgesetze in rauhen Rohren," by J. Nikuradse, *Forschungsheft*, Verein Deutscher Ingenieure, No. 361, 1933, p. 12.

is determined by subtracting the outline for W_t from that of W_b. Eqs. 43*a* and 43*b* contain the quantity $2\,\pi\,r_o\,\rho\,u^3_f$ which, with Eq. 42*a*, becomes equal to $W_o \frac{u_f}{U}$; and this value is used as a reference basis in plotting the curves. The resulting diagrams (Fig. 17), of which Fig. 17(*b*) is reduced to dimensionless terms, are most instructive. The reversal M point is closest to the boundary in the instance of a smooth wall. As the roughness increases, the transverse latitude of the conversion zone also grows; in the extreme case for which

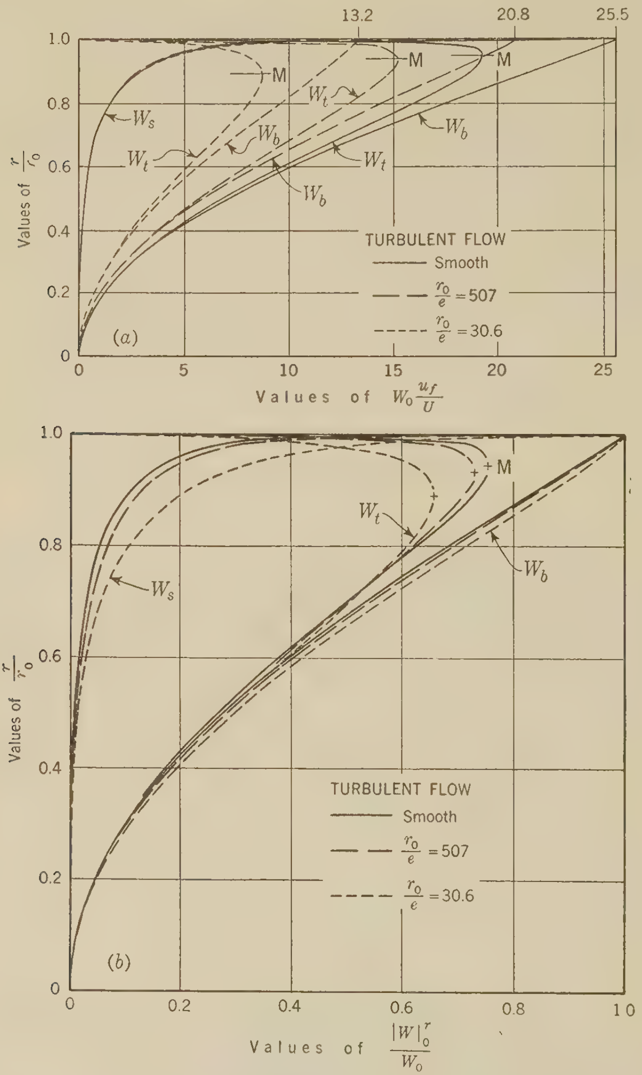

Fig. 17.—Cumulative Energy Balance in Turbulent Flow in Pipes

$r_o/e = 30.6$, it is about 10% of r_o. As expected, within the transmittance zones, the cumulative loss is shown by a single outline. In fact, W_s shows no appreciable discrepancies, up to the closest observable range ($r = 0.98\, r_o$) from the wall, which means that conversion is actually concentrated in a very narrow layer, the thickness of which is only a small portion of the reach $r_M - r_o$. In the case of the smooth pipe, the energy destined for conversion constitutes nineteen $\left(W_e \frac{u_f}{U}\right)$-units of a total of 25.5, or about 75%. For the roughest surface, the corresponding energy is 8.75 units of a total of 13.2, or 66%. On the whole, however, the dimensionless curves of Fig. 17 are a rather compact set, suggesting that, over a great variety of circumstances, energy exchange patterns in turbulent flow exhibit generically similar characteristics.

12. THE NATURE OF THE ENERGY SPENDING PROCESS

In treating the factors that shape the stress structure in turbulent patterns, it is customary to distinguish between the "viscous" and the "turbulent" agencies. Thus in

$$\tau = \mu \frac{d\bar{u}}{dy} - \left| \rho\, l^2 \left(\frac{d\bar{u}}{dy} \right)^2 \right| \qquad (45)$$

the quantity between the vertical bars represents (according to Prandtl) the effect of the Reynolds "momentum transfer," produced by transverse turbulent mixing.[23] One could expect that the same agencies would be instrumental in determining the manner in which energy, withdrawn from the flow, is "spent" in the different regions of a cross section.

Conversion Zone.—Paradoxically the function of conversion, which was said to be synonymous with the generation of turbulence (Section 10) reverts primarily to viscous action. The "casting off" of eddies appears to be the ultimate phase in a sequence of cyclic transformations suffered by viscous vortex sheets subjected to excessive strain. The intensity of the latter is affected by the fact that, in the immediate vicinity of the wall, the boundary stress by Eq. 40*a*, corresponding in magnitude to the general turbulent structure, must be "carried across" the laminar sublayer by viscous shear. This results in an extremely sharp velocity gradient

$$\left(\frac{du}{dy} \right)_{y \to 0} = \left| \frac{\tau_o}{\mu} \right| = \frac{\rho\, u^2{}_f}{\mu} = \frac{c_f}{\nu} \frac{U^2}{2} \qquad (46)$$

which exceeds by hundreds, if not by thousands, the ordinary "laminar" strain underlying the appraisal of deformation work in Fig. 6. The extreme character of the ensuing deformation is suggested by Fig. 18(*a*); Fig. 18(*b*) shows in turn the ultimate phase of the cyclic transformation, with the original viscous sheet rolled up into interspaced eddies, ready to be detached and "cast off" into the main flow.[11,12]

The deformation work involved in the initial straining of the vortex sheet can be appraised by means of Eqs. 12. Indeed, it is viscous deformation work, in the first instance, which absorbs the energy made available through transmittance and is destined for conversion.

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[23] "The Mechanics of Turbulent Flow," by B. A. Bakhmeteff, Princeton Univ. Press, Princeton, N. J., 1936, p. 44.

In the case of laminar motion, as stated in Section 7, all the deformation energy is dissipated in loco. In the generation of turbulence, on the other hand, part of the energy absorbed in deformation reappears in converted eddying form. Such an apparent difference in behavior can be explained at present only by hypothetical conjecture. The answer may lie in the dimensional scales of the respective phenomena. In laminar motion the action of viscosity, when one layer slides past another, is on a molecular scale. Any eddies formed in the interface between layers, which constitute the vortex sheets, would be of ultramicroscopic size and therefore (Eq. 37) would become subject to a most intensive dissipative action. As a result the tiny "molecular" eddies become extinguished in loco without having a chance of breaking away as individual units.

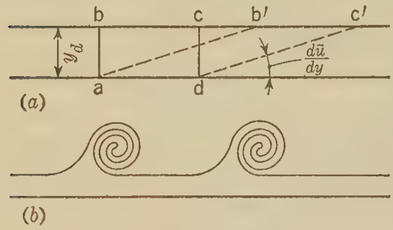

Fig. 18

By contrast in the boundary zone of Fig. 18, the pendulations, induced by the instability of the initial sheet, result in concentrated lumps of vorticity of a molar, macroscopic size. In accordance with Eq. 37, the very scale of the eddies militates against instantaneous dissipation in loco, giving the energy quantums invested into the vorticing units a certain period of life, before they are "ground up," in the process of swarming, and finally extinguished.

The nature of the phenomena involved in the deformation work during the conversion may be appraised quantitatively. Since, with Eq. 2, the energy loss in Eq. 4 equals $\frac{\tau_o}{R} A\, U = \tau_o\, P_w\, U$ the energy available in the conversion zone, per unit of wall surface, may be expressed as

$$\frac{W_c}{P_w} = |\alpha\, \tau_o\, U| \qquad (47)$$

in which $\alpha = W_c/W_o$, a fractional coefficient on the order of 0.8. The rate of deformation work per unit volume in the strained boundary vortex sheet (Fig. 18(*a*)), appraised on the basis of Eq. 12*b*, is

$$\mu \left(\frac{du}{dy}\right)^2_{\to 0} = \mu \left(\frac{\tau_o}{\mu}\right)^2 = \frac{\tau^2_o}{\mu} \qquad (48)$$

Since deformation work at such rate is called to absorb the energy amount indicated by Eq. 47, the volume of the deformable layer (Fig. 18) must measure transversely

$$y_d = \frac{\alpha\, \tau_o\, U}{\frac{\tau^2_o}{\mu}} = \frac{\alpha\, \mu\, U}{\tau_o} = \frac{\alpha\, U\, \nu}{u^2_f} = \frac{\nu}{u_f}\, \alpha \sqrt{\frac{2}{c_f}} \qquad (49a)$$

Eq. 49a acquires a special significance when reduced to the terms of the so-called "friction distance" similitude parameter[24]

$$y_f = \frac{y\, u_f}{\nu} \quad\quad (49b)$$

For smooth surfaces, flow is wholly shaped by the aforesaid similitude parameter. Indeed the thickness of the laminar sublayer δ corresponds to a parametrical value of $y_{f\delta} \approx 8$, and the transverse latitude of the entire boundary zone, between the wall and the fully turbulent inner region, to $y_{fb_z} \approx 30$. The in-between reach $(8 < y_f < 30)$ constitutes a "transitory" zone. The physical significance of this zone, which lies outward from the strictly laminar sublayer and inward from the turbulent reach where flow is fully "infested" with eddies, would be that of a turbulence generating region, within which eddies are formed and from which they are intermittently released.

In Eq. 49a, the thickness of the deformable viscous sheet, required for absorbing the convertible energy W_c in Eq. 47, corresponds to

$$y_{fd} = \frac{y_d\, u_f}{\nu} = \alpha \sqrt{\frac{2}{c_f}} \quad\quad (50a)$$

For smooth pipes, with c_f between 0.0015 and 0.0025, and $\alpha \approx 0.8$,

$$y_{fd} = 0.8 \sqrt{\frac{2 \cdot 10^4}{15 - 25}} \approx 29 - 23 \quad\quad (50b)$$

commensurate with the thickness of the boundary zone. About a third of y_{fd} in Eq. 50b is assignable to the laminar sublayer, in which dissipation occurs in loco in accord with Eq. 12b. This leaves about two thirds, or slightly more, of W_c to be spent in the transitory zone for actual conversion into turbulence. Again a portion of this remaining energy must be dissipated locally in the course of rolling up the eddies, which means that only the smaller part of the flow energy W_c, first available in the conversion zone, finally reappears in turbulent eddying form.

Transmittance Losses.—In the central "turbulent" region the contributions of the viscous shear $\mu \dfrac{d\bar{u}}{dy}$ in Eq. 45 are practically nil. Equally negligible are the energy losses $\mu \left(\dfrac{d\bar{u}}{dy} \right)^2$, computed from Eq. 12$b$ by using for $\dfrac{d\bar{u}}{dy}$ the observed local values of the main flow. It would be natural, on the other hand, to gage the w_s-rates, as they actually appear in Figs. 17 and 11, in terms of the same "mixing" agencies, which shape the "turbulence" stress. For example, one may appraise the "mixing action" in the "Prandtl" manner,[25] assuming that a molar element m_1 (Fig. 19(a)), initially belonging to filament (1) and traveling at a velocity u_1, is moved by a transverse impulse over a "mixing distance"

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[24] "The Mechanics of Turbulent Flow," by B. A. Bakhmeteff, Princeton Univ. Press, Princeton, N. J., 1936, pp. 43 and 47.

[25] *Ibid.*, p. 40.

l' into filament (2), where it becomes "embedded" acquiring the velocity u_2 and suffering a velocity change:

$$|\Delta u| = |u_1 - u_2| = \left| l' \frac{d\bar{u}}{dy} \right| \tag{51}$$

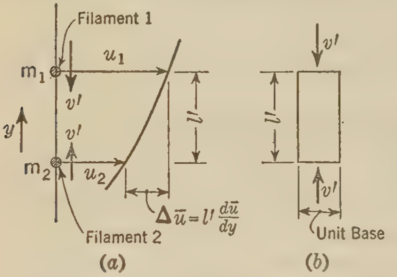
Using the latter for u' in the expression for Reynolds "momentum transfer"—

$$\tau = \rho \left| \overline{u' v'} \right| \tag{52}$$

—assuming further that the transverse exchange intensity v' is of the same order and indeed is proportional to u'; and finally absorbing the proportionality factor by a proper adjustment of the mixing scale, one ends with

$$\tau = \rho\, l' \frac{d\bar{u}}{dy} \times C\, l' \frac{d\bar{u}}{dy} = \rho\, l^2 \left(\frac{d\bar{u}}{dy} \right)^2 \tag{53}$$

in which l is the Prandtl "mixing length" and C is the proportionality factor.



(a) (b)

FIG. 19

Physically the change in the velocities due to the embedment is caused by viscous attrition that occurs during abrupt transverse displacements. Circumstances, in fact, are analogous to the case of impact with lasting constraints, as in the familiar instance of "inelastic bodies." Energy losses for the latter are expressed by the Carnot principle,[26] according to which the loss suffered by an element of mass m, the velocity of which is altered from u_1 to u_2, equals $m(u_1 - u_2)^2/2$.

In applying the Carnot principle to the block in Fig. 19(b), the aggregate mass exchanged per unit time across the upper and lower base is $2\,\rho|v'|$, and the lost velocities equal $\left| l' \frac{d\bar{u}}{dy} \right|$, thus making the energy loss (by substituting Eq. 53):

$$2\,\rho|v'|\,\frac{1}{2}\left(l' \frac{d\bar{u}}{dy} \right)^2 = \rho\, l^2 \left| \frac{d\bar{u}}{dy} \right|^3 l' = \tau \frac{d\bar{u}}{dy}\, l' \tag{54}$$

Dividing by the volume $l' \times 1$, one obtains the unit rate:

$$w\,(\text{Carnot}) = \tau\,(\text{turbulent}) \frac{d\bar{u}}{dy} = w_s \tag{55}$$

revealing that the "Carnot" loss, involved in the "mixing activity," is expressed by the same relation, which, in Eq. 34, determines the spending rate. In this light the quantity w_s (Eq. 34) in the central turbulent transmittance region, may be qualified as constituting a "mixing" or a "stress-forming" loss. Note, in particular, that the velocities altered through attrition in transverse embedment are those of the main flow. Physically, therefore, mixing action appears to imply a direct dissipation of flow energy into molecular heat, arising from the transporting action of the large-size eddies. Such convective transfer of molar masses by the large vorticing units constitutes specifically the function of so-called "turbulent diffusion." Accordingly, the local loss of flow energy in the inner turbulent zone of a cross section, determined by Eq. 55, is a dissipation rate attendant upon "turbulent diffusion" and dependent in magnitude on the intensity of the latter.

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[26] "Traité de Mécanique Rationnelle," by Paul Émile Appell, Paris, France, 1911, Vol. II, p. 508.

13. Applications to Engineering Practice

Broad Natural Watercourse.—Consider a river sufficiently broad and regular so that the bottom traction force on a unit-area column (Fig. 20) can be expressed by the customary du Boys formula

$$\tau_o = -\gamma S y'_o \quad \text{(56)}$$

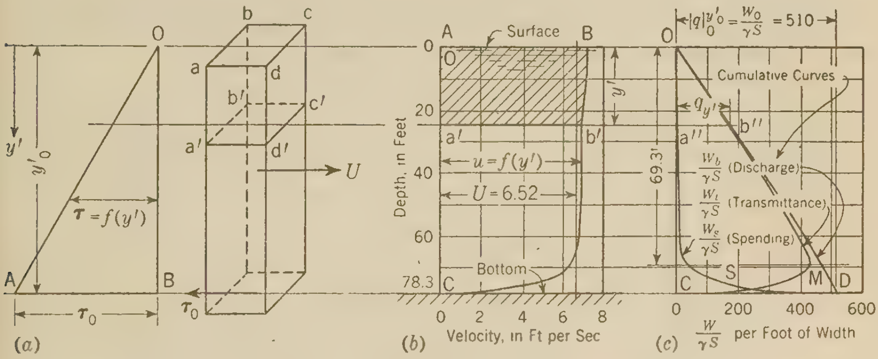
b
c
O
a
d
b′
c′
a′
d′
U
y′
y′o
τ = f(y′)
A
B
τ0
τ0
(a)
A
Surface
B
O
0
20
40
60
78.3
Depth, in Feet
a′
b′
u = f(y′)
U = 6.52
y′
Bottom
C
0 2 4 6 8
Velocity, in Ft per Sec
(b)
O
$|q|_0^{y'_o} = \frac{W_o}{\gamma S} = 510$
Cumulative Curves
$q_{y'}$
b″
a″
69.3′
$\frac{W_b}{\gamma S}$ (Discharge)
$\frac{W_t}{\gamma S}$ (Transmittance)
$\frac{W_s}{\gamma S}$ (Spending)
S
C
M
D
0 200 400 600
(c)
$\frac{W}{\gamma S}$ per Foot of Width

Fig. 20

which, with linear stress distribution, makes the unit shearing force in a plane a′b′c′d′ equal to

$$\tau = \tau_o \frac{y'}{y'_o} = \gamma S y' \quad \text{(57)}$$

In other words, the stress diagram τ is the product of the multiplier γS by the depth. The observed velocity profile $u = f(y')$ in Fig. 20(*b*) serves for plotting the cumulative discharge curve OD in Fig. 20(*c*), the ordinates of which (a″b″) for y' represent the volume of flow:

$$|q|_0^{y'} = \int_0^{y'} u\, dy' = y' U_{y'} \quad \text{(58)}$$

as indicated by the shaded area ABb′a′ in Fig. 20(*b*).

In the light of Eqs. 7 and 9, whereas the total flow energy borrowed and spent in the unit prism of Fig. 20(*a*) is

$$W_o = \gamma S |q|_0^{y'_o} = \gamma S y'_o U \quad \text{(59a)}$$

the cumulative borrowed quantity—

$$|W_b|_0^{y'} = \gamma S |q|_0^{y'} \quad \text{(59b)}$$

—is directly represented by the cumulative discharge curve OD in Fig. 20(c) to the scale $\frac{W_o}{\gamma\, S}$. Likewise, with Eqs. 18 and 57, the energy rate cumulatively transmitted from the block y' deep across a′b′c′d′ is

$$|W_t|_0^{y'} = -|\tau\, u|_0^{y'} = \gamma\, S\, y'\, u \qquad (60)$$

represented (except for a common multiplier $\gamma\, S$) in Fig. 20(c) by the curve OMC. The ordinates of this curve constitute the product of the local velocity by the respective depth, or the respective moment of the local velocity vector with regard to the surface of the streaming. The energy-spending curve W_s is obtained by subtracting, point by point, W_t from W_b, making

$$|W_s|_0^{y'} = \gamma\, S\, (|q|_0^{y'} - y'\, u) = \gamma\, S\, y'\, (U_{y'} - u) \qquad (61)$$

In Eq. 61 the quantity in parentheses represents the difference between the partial mean velocity $U_{y'}$ for the block (average ordinate of the shaded figure), and the local velocity for y' (ordinate a′b′ in Fig. 20(b)).

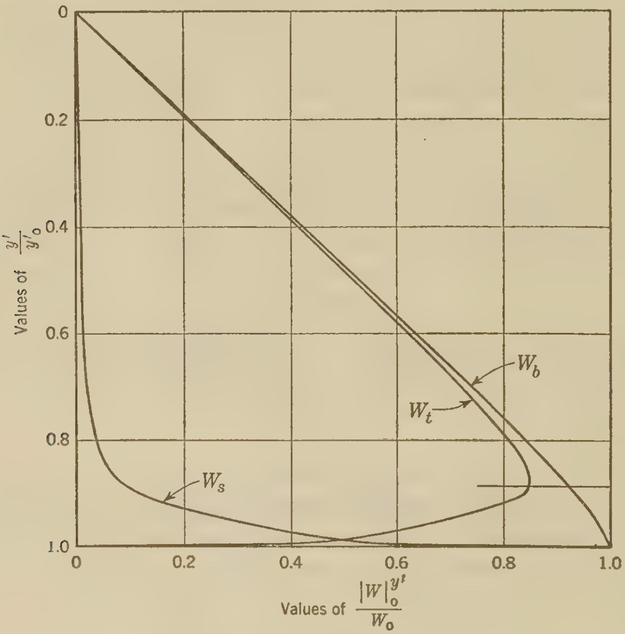

FIG. 21

The particular velocity profile, presented in Fig. 20(b), is for a large southern river during a moderately high stage. With a slope of 0.35 ft per mile, the multipler in Eqs. 57 to 61 is $\gamma\, S = 62.4 \times 0.66 \times 10^{-4} = 0.0041$; and its reciprocal is $\frac{1}{\gamma\, S} \approx 244$.

In Fig. 21, the curves of Fig. 20 are replotted in terms of dimensionless ratios. The features characteristic are typical and shed considerable light on the eventual circumstances which condition the "working" of a watercourse. In fact, the reversal point is located at a depth of 69 ft to 70 ft, or at about 11% to 12% of the total depth (78.3 ft) from the bottom. Of the total energy W_o (Eq. 59a), nearly 92% is borrowed within the transmittance zone and of this about 90% is conveyed by transmittance for spending in the lower 8 ft to 9 ft of the flow. The aggregate transmittance loss is about 8½% of the total. In fact, the relatively small "slip" loss is suggested a priori by the shape of the velocity curve. In the upper regions this curve exhibits only slight deviations from a vertical, and actually begins to show a pronounced decline of the local velocity only in the lower reaches, below $y' = 65$ ft. Actually over a depth of 70% of the total depth the cumulative loss is only 2% of W_o. It is less than 4% for $0.8\ y'_o$, and 12% for $0.9\ y'_o$. From there on, the spending increases rapidly. The most intensive exchange is in the closest proximity to the bottom, the spending in the last foot of the total $y'_o = 78.3$ ft constituting 51% of W_o. An analogous procedure applies to circular conduits.

14. General Treatment of Boundary Layers

It is assumed that the reader is familiar with the general concept and theory of flow in a boundary layer. A general account is given in several texts.[27,28,29]

To elucidate the process of energy loss in boundary layers, consider the two-dimensional motion next to a plate (see Fig. 22), under the customary premises that motion in the transverse sense is negligible, and that the pressures in a cross section do not differ from the pressure $p_b = f(x)$ at the border point b. In Fig. 22, and elsewhere in this paper, the reasoning is applied to a layer of unit width in the direction perpendicular to the plane of the figure.

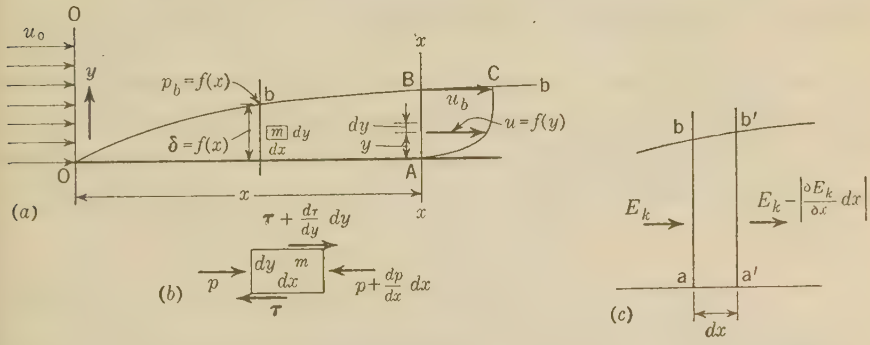

Fig. 22

The dynamic equilibrium for an element (Fig. 22(b)) is expressed by equating the mass-acceleration quantity $\rho\, dx\, dy \frac{du}{dt}$ to the resultant of the forces, of

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[27] "Fundamentals of Hydro- and Aeromechanics," based on lectures of L. Prandtl, by O. G. Tietjens, translated by L. Rosenhead, 1st Ed., McGraw-Hill Book Co., Inc., New York and London, 1934, p. 58.

[28] "The Mechanics of Viscous Fluids," Div. G in "Aerodynamic Theory," by L. Prandtl, W. F. Durand, editor in chief, Springer, Berlin, Germany, 1935, Vol. III, p. 80.

[29] "Fluid Mechanics for Hydraulic Engineers," by Hunter Rouse, 1st Ed., McGraw-Hill Book Co., Inc., New York and London, 1938, p. 192 et seq.

which $-\frac{dp_b}{dx}\,dx\,dy$ is the pressure component, and $\frac{d\tau}{dy}\,dy\,dx$ is the friction resultant. Reduced to a unit volume of fluid, the equation is

$$\rho\frac{du}{dt} = -\frac{dp_b}{dx} + \frac{d\tau}{dy} \qquad \text{(62)}$$

For the usual case of u_b = constant, and $\frac{dp_b}{dx} = 0$, Eq. 62 becomes

$$\rho\frac{du}{dt} = \frac{d\tau}{dy}; \qquad \frac{d\tau}{dy} + \left(-\rho\frac{du}{dt}\right) = 0 \qquad \text{(63)}$$

indicating that, in the absence of a pressure gradient, motion in the boundary layer is sustained by the inertia of the oncoming flow. The negative friction resultant $\frac{d\tau}{dy}$ (see Eqs. 10) is offset by an appropriate retarding of the motion, with the positive "inertial force" $\left(-\rho\frac{du}{dt}\right)$ balancing the negative friction traction. Obviously the work implied in overcoming frictional resistances is performed at the expense of the kinetic energy of the flow, the initial store of which is gradually reduced. With regard to an elementary cross-sectional block (Fig. 22(*c*)), the kinetic energy E_k, entering per unit time across ab, exceeds the energy leaving the block across line a′b′ by $\left|\frac{dE_k}{dx}\right|dx$, in which the quotient $\left|\frac{dE_k}{dx}\right|$ measures the energy withdrawn (borrowed) and spent per unit time in a boundary layer block of unit length. It has the same significance for boundary layer flow as W_o (Eq. 4) has for established uniform motion, except that $\left|\frac{dE_k}{dx}\right|$ will vary from section to section. Since the quotient is essentially negative,

$$-\frac{dE_k}{dx} = f(x) = (W_\delta)_x \qquad \text{(64)}$$

the inner subscript δ suggesting that W_δ covers the entire transverse extent of the layer, whereas the outer subscript indicates specifically the location of the particular unit block under consideration. On the other hand, $|W_\delta|_0^x$ will designate the aggregate energy borrowed and spent in a unit of time in the entire body of the boundary layer OBA between O and x. Obviously for $\frac{dp}{dx} = 0$, $|W_\delta|_0^x$ is the difference between the kinetic energy $\rho\int_0^\infty u\frac{u^2}{2}\,dy$ flowing across AB and the energy $\rho\int_0^\infty u\frac{u_o^2}{2}\,dy$, originally carried by the same volume of flow in the on-coming undisturbed state, across O–O. In other words,

$$|W_\delta|_0^x = \rho\int_0^\infty u\left(\frac{u_o^2 - u^2}{2}\right)dy = \frac{\rho u_o^3}{2}\int_0^\infty \frac{u}{u_o}\left[1 - \left(\frac{u}{u_o}\right)^2\right]dy \qquad \text{(65)}$$

The quantity E_k includes energy carried across the cross section from the plate to infinity. Inasmuch as there is no appreciable change of velocity outside of the boundary layer, one may substitute for the right side of Eq. 65: $\frac{\rho u^3_o}{2}\int_0^\delta \frac{u}{u_o}\left[1-\left(\frac{u}{u_o}\right)^2\right]dy$. Eq. 65 complements the well-known von Kármán expression

$$F_x = \rho\, u^2_o \int_0^\infty \frac{u}{u_o}\left(1-\frac{u}{u_o}\right)dy \dots\dots\dots\dots (66)$$

which serves to determine the skin friction resultant over the length x in terms of the momentum lost by the flow between 0 and x. Note that Eqs. 65 and 66 are quite general and therefore are applicable to boundary layers of all types whether laminar or turbulent, subject only to the limitation $\frac{dp_b}{dx} = 0$. Unfortunately, present knowledge of turbulent boundary layers is anything but complete, and an adequate analysis of the energy exchanges is not possible. On the other hand, for the case of a laminar layer next to a flat plate with $\frac{dp_b}{dx} = 0$, detailed theoretical calculations are available, and are in excellent accord with repeated observations. For example, theoretical and experimental aspects have been presented in detail by Hugh L. Dryden,[30] whose report, as well as subsequent papers by the same author,[16] contain the more recent interpretations of boundary layer motion, particularly with regard to "transition."

15. ENERGY BALANCE IN LAMINAR BOUNDARY LAYERS WITH $\frac{dp}{dx} = 0$

Basically, the flow pattern is conditioned by the parameter

$$\sqrt{\frac{x\,\nu}{u_o}} = x\sqrt{\frac{\nu}{x\,u_o}} = \frac{x}{\sqrt{\mathbf{R}_x}} \dots\dots\dots\dots (67)$$

in which $\mathbf{R}_x = \frac{x\,u_o}{\nu}$ is the so-called "distance Reynolds number" of the boundary layer. In particular, the transverse velocity profile $u = f(y)$ at a station x (Fig. 22(*a*)) is determined by the dimensionless quantity

$$\eta = \frac{y}{\sqrt{\frac{x\,\nu}{u_o}}} = \frac{y}{x}\sqrt{\mathbf{R}_x} \dots\dots\dots\dots (68)$$

so that the ratio ϕ of a local velocity u distant y from the plate, to the constant (border) velocity u_o $\left(\phi = \frac{u}{u_o}\right)$ is solely a function of η. In any two cross sections, if the ordinates were chosen to make $\eta_1 = \left(\frac{y_1}{x_1}\right)\sqrt{\mathbf{R}_{x1}}$ and

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[30] "Air Flow in the Boundary Layer Near a Plate," by Hugh L. Dryden, *Technical Report No. 562*, National Advisory Committee for Aeronautics, U. S. Government Printing Office, Washington, D. C., 1937, p. 339.

$\eta_2 = \left(\frac{y_2}{x_2}\right)\sqrt{\mathbf{R}_{x2}}$ identical, the respective velocity ratios $\phi_1 = \frac{u_1}{u_o}$ and $\phi_2 = \frac{u_2}{u_o}$ would be the same. Velocity profiles with such similarity of shape are said to be "affine," permitting the representation of the $u = f(y)$-curve for any cross section by the single generalized outline plotted as $\phi\left(\frac{u}{u_o}\right) = f(\eta)$ in Fig. 23. Affinity obviously applies also to the derivative quantities

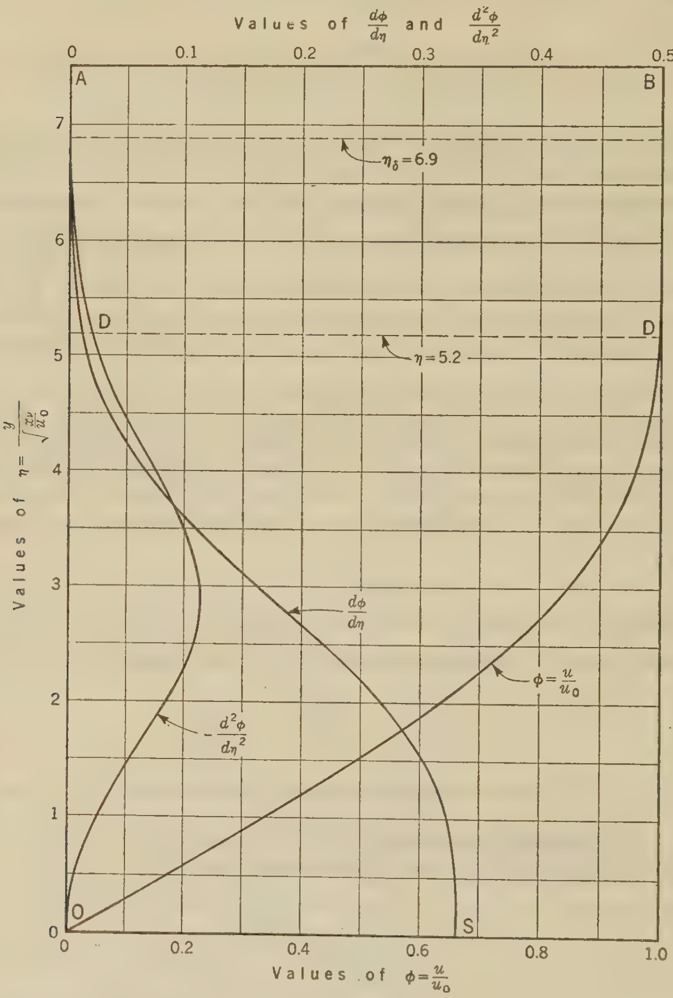

FIG. 23

$$\frac{d\phi}{d\eta} = \frac{du}{dy}\sqrt{\frac{\nu x}{u^3_o}}; \qquad \frac{d^2\phi}{d\eta^2} = \frac{d^2u}{dy^2}\frac{\nu x}{u^2_o} \quad \text{(69)}$$

the generalized outlines for which are also presented in Fig. 23 (based on a tabulation of the original Blasius data[31] included by the Dryden report.[30] The values of $\frac{d^2u}{dy^2}$ throughout the layer are negative. In Fig. 23 the positive of the curve corresponds to $-\frac{d^2\phi}{d\eta^2}$.) In particular the dotted line D–D in Fig. 23 is drawn at a transversal distance $\eta = 5.2$, suggested by Prandtl as a conventional measure for the boundary layer thickness $\delta = 5.2\sqrt{\frac{x\,\nu}{u_o}}$. At such distance, the local value of u differs from the border quantity u_o by the negligible amount of 0.5%. Fig. 23 discloses, however, that the derivative values at $\eta = 5.2$ are still quite substantial, and may weigh accordingly in the eventual energy balances. A more appropriate conventional value for the laminar boundary layer thickness would be $\eta_\delta = 6.9$ making $\delta = 6.9\sqrt{\frac{x\,\nu}{u_o}}$. The particular value of $\eta_\delta = 6.9$ is chosen, so as to give δ four times the so-called "displacement thickness" $\delta_d = 1.73\sqrt{\frac{\nu\,x}{u_o}}$. The Prandtl $\eta_\delta = 5.2$ corresponds to $\delta = 3\,\delta_d$.

For "affine" profiles, the energy equations take definite numerical form. In fact, substituting $\phi\,u_o$ for u, and $\eta\sqrt{\frac{x\,\nu}{u_o}}$ for y, the aggregate energy loss (Eq. 65) becomes

$$\left|W_\delta\right|_0^x = \frac{\rho\,u^3_o}{2}\sqrt{\frac{x\,\nu}{u_o}}\int_0^\infty \phi\,(1-\phi^2)\,d\eta = \frac{\rho\,x\,u^3_o}{2\sqrt{\mathbf{R}_x}}\,\omega = 0.522\,\frac{\rho\,x\,u^3_o}{\sqrt{\mathbf{R}_x}} \ldots (70)$$

in which ω is a dimensionless factor representing the value of the integral

$$\omega = \int_0^\infty \phi\,(1-\phi^2)\,d\eta \ldots\ldots\ldots\ldots (71)$$

Eq. 71 is dependent solely on the shape of the "affine" profile. For the laminar outline of Fig. 23, the quantity ω equals 1.044.

To obtain the local cross-sectional loss (Eq. 64), one proceeds by differentiating Eq. 70, which makes

$$(W_\delta)_x = \frac{d\left|W_\delta\right|_0^x}{dx} = \frac{\omega\,\rho\,u^3_o}{2}\,\frac{d}{dx}\sqrt{\frac{x\,\nu}{u_o}} = 0.261\,\frac{\rho\,u^3_o}{\sqrt{\mathbf{R}_x}} \ldots\ldots\ldots (72)$$

In Eqs. 70 and 72—

$$\left|W_\delta\right|_0^x = 2\,x\,(W_\delta)_x \ldots\ldots\ldots\ldots (73)$$

For establishing the interrelation between the energy loss and the skin friction resultant, note that, numerically, the von Kármán expression (Eq. 66) equals

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[31] "Air Flow in the Boundary Layer Near a Plate," by Hugh L. Dryden, *Technical Report No. 562*, National Advisory Committee for Aeronautics, U. S. Government Printing Office, Washington, D. C., 1937, p. 361.

$$F_x = 0.664 \frac{x}{\sqrt{\mathbf{R}_x}} \rho\, u^2_o \quad \text{.......................(74)}$$

and the local wall stress (friction force per unit length of plate at the distance x) is

$$\tau_o = \frac{dF_x}{dx} = 0.332 \frac{\rho\, u^2_o}{\sqrt{\mathbf{R}_x}} = \frac{F_x}{2\,x} \quad \text{..................(75)}$$

Substituting Eq. 75 into Eqs. 70 and 72, one finds that

$$|W_\delta|_0^x = \frac{\omega\, u_o}{2} \frac{\rho\, u^2_o\, x}{\sqrt{\mathbf{R}_x}} \approx \tfrac{3}{4}\, \omega\, F_x\, u_o = 0.786\, F_x\, u_o \quad \text{.........(76a)}$$

and

$$(W_\delta)_x = \frac{\omega\, u_o}{4} \frac{\rho u^2_o}{\sqrt{\mathbf{R}_x}} \approx \tfrac{3}{4}\, \omega\, \tau_o\, u_o = 0.786\, \tau_o\, u_o \quad \text{..........(76b)}$$

16. The Local Unit Rates

Local borrowing in a boundary layer is expressed by the same formula (Eq. 11) as in uniform established motion, although, with $\frac{dp_b}{dx} = 0$, the work absorbed by the friction tractions is supplied wholly from the kinetic energy store.

In particular, for a laminar layer, using Eq. 69 in Eq. 11, one obtains

$$w_b = -\mu \frac{d^2u}{dy^2} u = \frac{\rho\, u^3_o}{x} \left(-\phi \frac{d^2\phi}{d\eta^2} \right) \quad \text{...............(77)}$$

the multiplier of the parentheses group constituting (in view of Eq. 72):

$$\frac{\rho\, u^3_o}{x} = \frac{4}{\omega} \sqrt{\frac{u_o}{\nu\, x}}\, (W_\delta)_x = 3.83 \frac{\sqrt{\mathbf{R}_x}}{x} (W_\delta)_x \quad \text{.............(78)}$$

Furthermore, dividing $(W_\delta)_x$ in Eq. 72 by the transverse dimension of the layer $\delta = 6.9 \sqrt{\frac{x\, \nu}{u_o}}$ (see paragraph containing Eq. 69), one obtains an average unit rate of

$$(w_\delta)_x = \frac{(W_\delta)_x}{\delta} = \frac{\omega}{27.6} \frac{\rho\, u^3_o}{x} = \frac{1}{26.4} \frac{\rho\, u^3_o}{x} \quad \text{.............(79)}$$

which is a quantity analogous to w_o (Eq. 8) for uniform flow. Substituting Eq. 79 into Eq. 77:

$$w_b = (w_\delta)_x \frac{27.6}{\omega} \phi \left(-\frac{d^2\phi}{d\eta^2} \right) = 26.4\, (w_\delta)_x\, \epsilon'_b \quad \text{...........(80)}$$

in which

$$\epsilon'_b = \phi \left(-\frac{d^2\phi}{d\eta^2} \right) \quad \text{........................(81)}$$

is a dimensionless function, represented in Fig. 24 by the curve AaO This curve is obtained by multiplying the ordinates of the curves Ao and Bo in Fig. 23. Obviously, ϵ'_b, in terms of 26.4 $(w_\delta)_x$, denotes the distribution of the borrowing process in all laminar boundary layers of the type $\frac{dp_b}{dx} = 0$. The

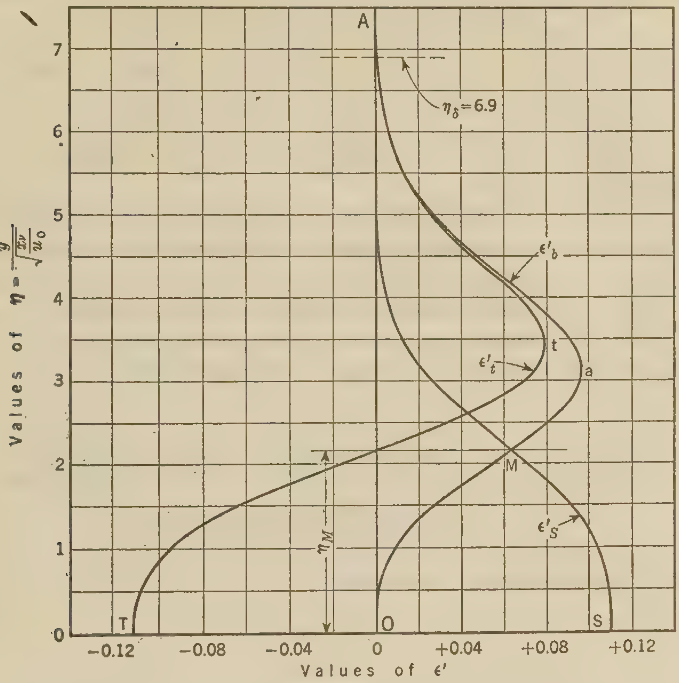

FIG. 24

local borrowing is zero near the wall, and it also declines into insignificance beyond $\eta_\delta = 6.9$. The maximum intensity of the borrowing function is reached in the intermediate region near $\eta = 3 - 3.25$.

The Spending Rate.—In a laminar layer, with the local spending measured by the work involved in viscous deformation (Eq. 12*b*), one obtains, with Eq. 69,

$$w_s = \mu \left(\frac{du}{dy}\right)^2 = \frac{\mu u^3_o}{\nu x}\left(\frac{d\phi}{d\eta}\right)^2 = \frac{\rho u^3_o}{x}\left(\frac{d\phi}{d\eta}\right)^2 = 26.4\,(w_\delta)_x\,\epsilon'_s \ldots (82)$$

Again, the dimensionless function

$$\epsilon'_s = \left(\frac{d\phi}{d\eta}\right)^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (83)$$

represented in Fig. 24 as the curve AS, describes the spending process in generalized universal terms. Similar to uniform motion, the spending of energy is concentrated near the solid boundary. With $\eta_M = y_M \sqrt{\frac{x\,\nu}{u_o}}$, the intersection point M, corresponding to $w_b = w_s$, indicates the region within which spending exceeds the energy amounts obtained by local borrowing.

The Transmittance Rate.—By subtracting curve AS (Fig. 24) from curve AaO, one obtains the outline curve AtT, or ϵ'_t which defines the local transmittance function. Expressed algebraically,

$$w_t = w_b - w_s = \frac{\rho\, u^3{}_o}{x}\left[-\phi\frac{d^2\phi}{d\eta^2} - \left(\frac{d\phi}{d\eta}\right)^2\right]$$
$$= \frac{\rho\, u^3{}_o}{x}\frac{d}{d\eta}\left(-\phi\frac{d\phi}{d\eta}\right) = 26.4\,(w_\delta)_x\,\epsilon'_t \quad\ldots\ldots(84)$$

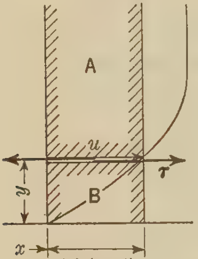

FIG. 25

Cumulative Energy Rates.—In presenting the cumulative energy quantities, it is expedient to add in the direction from the unaffected outward zone toward the plate, thus making the resulting outlines for a boundary layer directly comparable to those of Figs. 1 and 2. In fact, using Eq. 69 with Fig. 25 one obtains for the transmittance rate (compare Eq. 18):

$$|W_t|^y_\infty = \left| -u\,\tau\right|^y_\infty = \left|\phi\, u_o\, \mu\frac{du}{dy}\right|^\eta_\infty = \frac{\rho\, u^3{}_o}{x}\sqrt{\frac{x\,\nu}{u_o}}\left|\frac{d\phi}{d\eta}\phi\right|^\eta_\infty$$
$$= \frac{4}{\omega}(W_\delta)_x\left|\phi\frac{d\phi}{d\eta}\right|^\eta_\infty = 3.83\,(W_\delta)_x\,\mathrm{E}'_t \quad\ldots\ldots(85)$$

in which E'_t is a dimensionless multiplier:

$$\mathrm{E}'_t = \left|\phi\frac{d\phi}{d\eta}\right|^\eta_\infty \quad\ldots\ldots(86)$$

obtained from the data in Fig. 23, and traced as curve ATO in Fig. 26. Physically the quantity $|W_t|^y_\infty$ indicates the work performed on the block B, Fig. 25, by the overlying fluid prism A. The other cumulative quantities take the form:

$$|W_b|^y_\infty = \int_\infty^y w_b\, dy = \frac{\rho\, u^3{}_o}{x}\sqrt{\frac{x\,\nu}{u_o}}\int_\infty^\eta\left(-\phi\frac{d^2\phi}{d\eta^2}\right)d\eta$$
$$= \frac{4}{\omega}(W_\delta)_x\int_\infty^\eta \epsilon'_b\, d\eta = 3.83\,(W_\delta)_x\,\mathrm{E}'_b \quad\ldots\ldots(87a)$$

and

$$|W_s|^y_\infty = \int_\infty^y w_s\, dy = \frac{\rho\, u^3{}_o}{x}\sqrt{\frac{x\,\nu}{u_o}}\int_\infty^\eta\left(\frac{d\phi}{d\eta}\right)^2 d\eta$$
$$= \frac{4}{\omega}(W_\delta)_x\int_\infty^\eta \epsilon'_s\, d\eta = 3.83\,(W_\delta)_x\,\mathrm{E}'_s \quad\ldots\ldots(87b)$$

in which the generalized functions E'_b and E'_s are numerically equal to the areas under the curves AO and AS in Fig. 24, traced in Fig. 26 as AbB and AsB.

The Energy Balance Equation.—By analogy to Eq. 17, the local energy balance takes the generalized form

$$26.4\,(w_\delta)_x \left\{ \epsilon'_b \qquad = \epsilon'_s \qquad + \epsilon'_t \right\} \qquad (88)$$

that is,

$$\frac{\rho\, u^3_o}{x} \left\{ -\phi \frac{d^2\phi}{d\eta^2} = \left(\frac{d\phi}{d\eta} \right)^2 - \frac{d}{d\eta} \left(\phi \frac{d\phi}{d\eta} \right) \right\} \qquad (89)$$

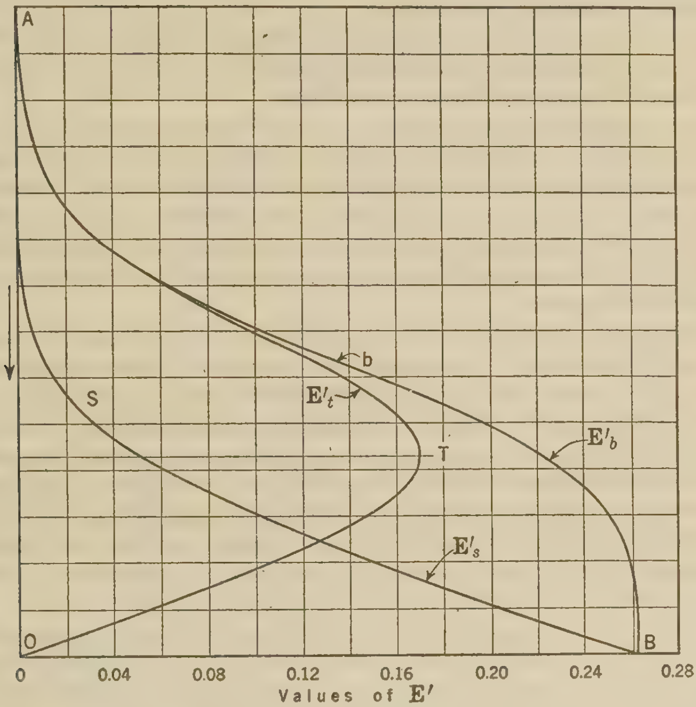

FIG. 26

The analogous symbolic formula expressing the relation between the cumulative energy quantities is:

$$3.83\,(W_\delta)_x \left\{ E'_b \qquad = E'_s \qquad + E'_t \right\} \qquad (90)$$

SUMMARY

1. In the mechanism of energy loss attendant to fluid friction, the "borrowing," or withdrawing of energy from the flow, and its spending do not coincide spacially. Spending is concentrated in the regions of high vorticity near the solid boundaries, whereas the borrowing takes effect principally in the central reaches of high local velocity.

2. The aforesaid implies an in-between "transmittance" function, by which surplus borrowed energy is transmitted to the wall region, where local borrowing is insufficient to cover the requirements for spending.

3. The aspects of the three consequent phases are determined by the internal stress structure and the concomitant velocity profile. The local energy balance equation

$$-\underset{(w_b)}{u\frac{d\tau}{dy}} = \underset{(w_s)}{\tau\frac{du}{dy}} + \underset{(w_t)}{\frac{d}{dy}(-\tau u)} \ldots\ldots\ldots\ldots\ldots\ldots (91)$$

applies equally to laminar and turbulent patterns, and serves as the basis for the reasoning presented in the paper.

4. For laminar patterns the energy exchange process can be presented wholly in analytical form. In turbulent patterns the application of the equations must rely on detailed experimental data.

5. The outstanding characteristic of all turbulent patterns is that only a small portion of the total energy is spent directly within the central turbulent zone, and that the larger part (80% to 85%) is concentrated by transmittance for spending in a narrow reach near the solid boundary. This "conversion" zone is ostensibly the seat of the pivotal process in the turbulent mechanism—namely, that of converting flow energy, concentrated by transmittance, into turbulent eddying form.

6. The function of conversion in itself reverts to viscous action. The larger part of the energy, concentrated for conversion, is ostensibly dissipated in loco in the viscous deformation of the laminar sub-layer and in the course of forming eddies in the adjacent turbulence generating zone. Thus only a smaller part of the spent flow energy reappears after conversion in the turbulent form which animates the familiar turbulent manifestations and activates the convective mass transfer instrumental in shaping the turbulent stress structure.

7. The final dissipation of the turbulent energy, continuously replenished from the generating boundary zone, is by way of viscous attrition in the course of the swarming motion of the eddies across the main flow.

8. The direct local loss of flow energy in the central transmittance zone can be accounted for dynamically by applying the Carnot impact law to the process of mass embedment in the course of transverse mixing. This loss, attributable to the convective action of the larger eddies which shape the stress structure, is concomitant to turbulent diffusion.

9. In relation to natural watercourses, the transmittance of energy and its spending in the boundary zone, shed possible light on the "working" of a river in regard to bed-load movement.

10. The analysis of boundary layer motion is extended to consider the energy aspects of the flow patterns. In the case of a laminar layer with $dp/dx = 0$, a complete account of the energy exchange mechanism is revealed in generalized dimensionless form.

Acknowledgments.—The subject matter covered by the paper forms part of a broader investigation on the "Nature of Hydraulic Resistances" conducted under the general auspices of the Society's Committee on Hydraulic Research. For the velocity profile used in Fig. 20 the writers are gratefully indebted to the Mississippi River Commission of Vicksburg, Miss.

DISCUSSION

GEORGE R. RICH,[32] M. AM. SOC. C. E.—In strikingly clear and compact form, the paper presents the spatial implications of the differential energy balance for representative problems in which the flow is directed by a wall or plate. This same differential expression also appears to afford a ready method of calculating similar energy exchanges for cases without confining walls, such as wakes behind obstacles and jets discharging into still fluid.

The requisite velocity distribution for the jet discharging into still fluid under the condition of laminar flow is given in Fig. 27 and the formula:[33]

$$u = 0.4543 \left(\frac{M^2}{\rho^2 \nu x} \right)^{1/3} \operatorname{sech}^2 \left[0.2752 \left(\frac{M}{\rho \nu^2} \right)^{1/3} \frac{y}{x^{2/3}} \right] \ldots\ldots (92)$$

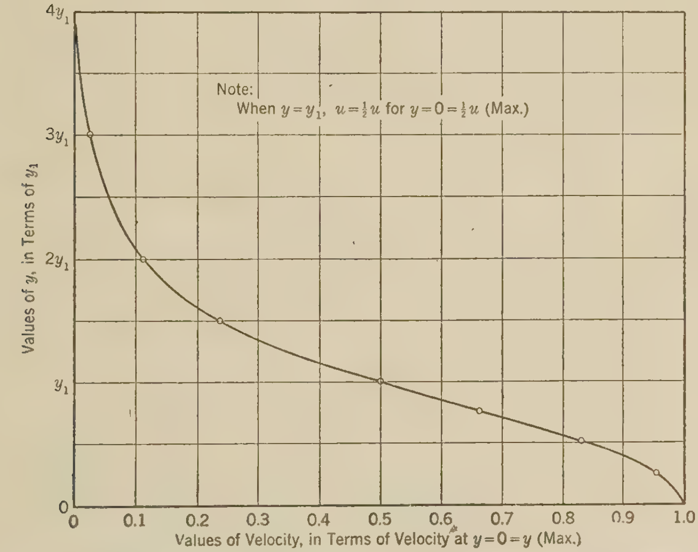

FIG. 27.—VELOCITY DISTRIBUTION FOR RECTANGULAR JET DISCHARGING INTO STILL WATER; LAMINAR FLOW

in which M, the total momentum flowing across a section, is the same for all sections x and is equal to

$$M = 2 \rho \int_0^\infty u^2 \, dy \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (93)$$

Since the present investigation is concerned only with changes in the variable y, Eq. 92 may be written:

$$u = C_1 \operatorname{sech}^2 (C_2 \, y) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (94)$$

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[32] Chf. Design Engr., TVA, Knoxville, Tenn.

[33] "Modern Developments in Fluid Dynamics," edited by S. Goldstein, Clarendon Press, Oxford, England, 1938, Vol. I, p. 145.

By direct substitution in Eqs. 11 and 12*a* and integrating with respect to y:

$$\left|W_s\right|_0^y = 4\,\mu\, C^2_1\, C_2 \int_0^y \operatorname{sech}^4 (C_2\, y) \tanh^2 (C_2\, y)\, C_2\, dy$$

$$= 2\,\mu\, C^2_1\, C_2 \left[\frac{64}{5\,(e^{2C_2y}+1)^5} - \frac{32}{(e^{2C_2y}+1)^4} + \frac{80}{3\,(e^{2C_2y}+1)^3} - \frac{8}{(e^{2C_2y}+1)^2}\right]_0^y \quad \text{. . (95}a\text{)}$$

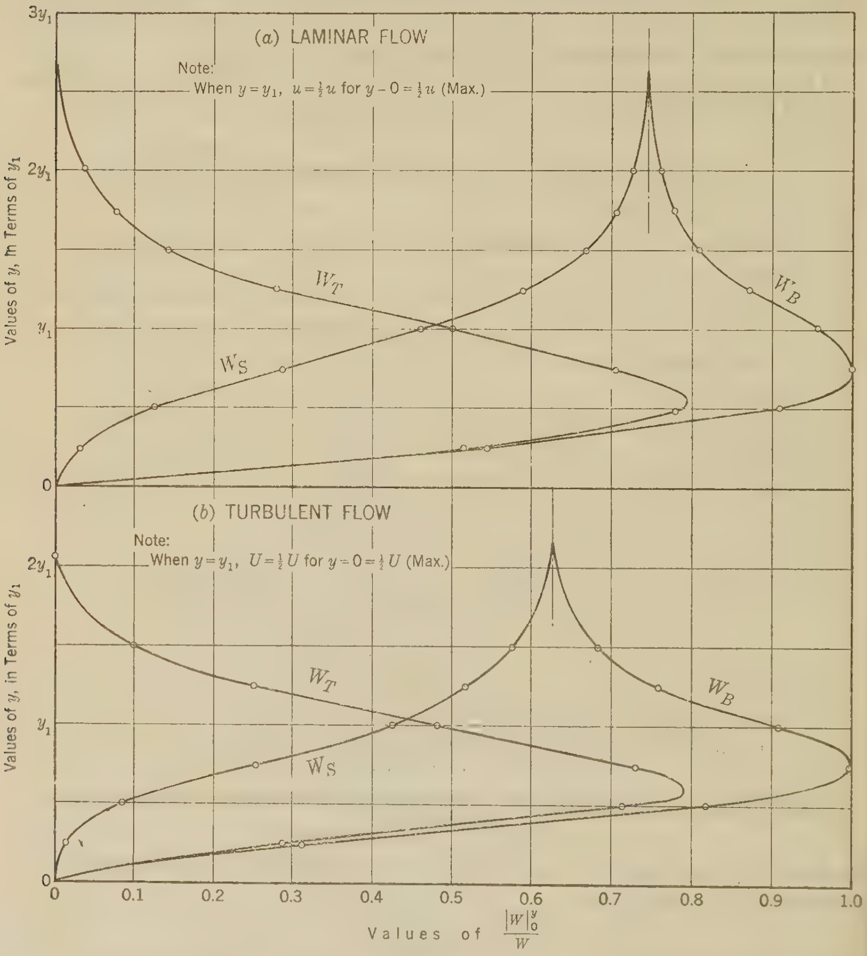

FIG. 28.—ENERGY EXCHANGES FOR RECTANGULAR JET DISCHARGING INTO STILL WATER

and

$$\left|W_B\right|_0^y = 2\,\mu\, C^2_1\, C_2 \int_0^y \left[\operatorname{sech}^6 (C_2\, y) - \operatorname{sech}^4 (C_2\, y) \tanh^2 (C_2\, y)\right] C_2\, dy$$

$$= 2\,\mu\, C^2_1\, C_2 \left[-\frac{96}{5\,(e^{2C_2y}+1)^5} + \frac{48}{(e^{2C_2y}+1)^4} - \frac{112}{3\,(e^{2C_2y}+1)^3} + \frac{8}{(e^{2C_2y}+1)^2}\right]_0^y \quad \text{. . . (95}b\text{)}$$

The resultant energy exchanges are given in Fig. 28(a), in which the characteristic shape of the borrowing curve W_B reflects the abstraction of kinetic energy from the central zone of the jet—not only for the subsequent work of viscous deformation at the particular section x, but also for imparting velocity to water at increasingly higher ordinates as the jet diverges progressively in the x-direction. This salient feature may be inferred from the curvature of the velocity distribution graph, Fig. 27; thus:

$$K = \frac{\dfrac{d^2y}{dx^2}}{\left[1 + \left(\dfrac{dy}{dx}\right)^2\right]^{3/2}} \qquad \text{(96)}$$

In the vicinity of the jet axis $y = 0$, the curvature is convex upward, changing to convex downward at a point of inflection somewhere about $y = 0.75\ y_1$. Thus, $\dfrac{d^2y}{dx^2}\left(\text{or } \dfrac{d\tau}{dy}, \text{ which measures the rate of borrowing}\right)$ must change algebraic sign near $y = 0.75\ y_1$; and, consequently, the summation of borrowed energy W_B must decrease, finally approaching, asymptotically with W_s, the value of 0.745.

The corresponding physical picture appears to be that, at low values of y, the shearing force on any element from the moving water below exceeds the opposing shear due to the inertia of the relatively slower moving water above, and the sign of the resultant increment of borrowed energy is positive. At higher values, the inertia shear from above exceeds the kinetic shear from below the element, and the algebraic sign of the increment becomes negative.

For the case of the rectangular jet discharging into still fluid under conditions of turbulent flow, the actual equation for the longitudinal velocity, although it involves only elementary functions, is exceedingly cumbersome.[34,35] Since the actual distribution of velocity is very much the same as that for the laminar condition, Fig. 27 and Eq. 92 may be assumed sufficiently accurate for present purposes in calculating energy exchanges under the turbulent condition.

Under turbulent flow let τ be the virtual viscous, or Reynolds' shearing, stress,[36] expressed in accordance with the momentum transfer theory as

$$\tau = \rho\, l^2 \left(\frac{\partial U}{\partial y}\right)^2 = \rho\, c^2\, x^2 \left(\frac{\partial U}{\partial y}\right)^2 \qquad \text{(97)}$$

in which the mixing length l is assumed to be $l = c\, x$, c being constant.

The symbol U denotes the longitudinal component of the mean velocity at ordinate y and abscissa x. Substituting in the appropriate equations of the

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[34] "Berechnung Turbulenter Ausbreitungsvorgänge," by W. Tollmien, *Zeitschrift für Angewandte Mathematik und Mechanik*, Vol. 6, 1926, pp. 468–478.

[35] "Modern Developments in Fluid Dynamics," edited by S. Goldstein, Clarendon Press, Oxford, England, 1938, Vol. II, p. 594.

[36] *Ibid.*, Eqs. 52 and 53, p. 582; and Eq. 97, p. 592.

energy balance:

$$\left|W_s\right|_0^y = 8\,C^3_1\,C^2_2\,c^2\,x^2\,\rho \int_0^y \operatorname{sech}^6(C_2\,y)\,\tanh^3(C_2\,y)\,C_2\,dy$$

$$= 8\,C^3_1\,C^2_2\,c^2\,x^2\,\rho \left[\frac{\operatorname{sech}^8(C_2\,y)}{8} - \frac{\operatorname{sech}^6(C_2\,y)}{6}\right]_0^y \ldots\ldots\ldots (98a)$$

and

$$\left|\mathrm{W}_B\right|_0^y = 8\,C^3_1\,C^2_2\,c^2\,x^2\,\rho \int_0^y$$

$$\times \left[\operatorname{sech}^8(C_2\,y)\,\tanh(C_2\,y) - 2\operatorname{sech}^6(C_2\,y)\,\tanh^3(C_2\,y)\right] C_2\,dy$$

$$= 8\,C^3_1\,C^2_2\,c^2\,x^2\,\rho \left[\frac{\operatorname{sech}^6(C_2\,y)}{3} - \frac{3\operatorname{sech}^8(C_2\,y)}{8}\right]_0^y \ldots\ldots\ldots (98b)$$

Fig. 28(*b*) shows the corresponding energy exchanges. In comparison with the laminar jet, the chief difference is that a relatively greater portion of kinetic energy is convected from the zone of the central axis $y = 0$ for use in imparting velocity at the higher ordinates in sections subsequent to x and also that the spending at section x is completed at a somewhat lower ordinate, indicating greater concentration of spending in the mixing zone, as would naturally be anticipated.

J. C. STEVENS,[37] PRESIDENT, AM. SOC. C. E.—Entirely novel concepts of the internal mechanism of fluid friction have been presented in this paper. In 1942 the senior author accepted an assignment, heading a research project on the Nature of Hydraulic Friction for the Society's Committee on Hydraulic Research, of which the writer was then chairman. The research committee felt that it was high time to leave the beaten path of empiricism and to step boldly into newer fields of inquiry in fundamental concepts if further advancement in fluid mechanics was to be realized. This paper, although largely a mathematical analysis of viscous forces and their resultant effects, will serve as a beacon to direct the trend of future experimental studies in laboratory and field. From this analysis and from the physical research that it will stimulate, new methods of evaluating flow characteristics of liquids and gases are certain to be evolved.

The suggestion that the principal difference between laminar and turbulent flow may be one of vorticity scale seems to conform with observed phenomena. In pure laminar flow, the vorticity induced by viscous shear is microscopic, even molecular, for which the conversion of energy into heat is definitely confined to the shear plane. As velocities increase, flow characteristics gradually undergo a transition into a state of turbulence. One might indicate the transitional stages by such words as "pure laminar," "laminar," "mildly turbulent," "turbulent," and "shooting." Each progressive stage is characterized by a corresponding increase in the size of the vortices formed by increasing stresses induced by increasing viscous attrition following each increment in the over-all velocity. At least, this is a comforting explanation of why the losses seem to be proportional to an increasing power of the velocity exponent, from 1 for pure laminar to 2 for shooting flow.

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[37] Cons. Hydr. Engr. (Stevens & Koon), Portland, Ore.

It is well to recognize that the analysis presented applies only to steady uniform flow, whether laminar or turbulent. The case of steady nonuniform flow offers a challenge that can be met only by experimentation. A. A. Kalinske,[38] Assoc. M. Am. Soc. C. E., has encountered this problem in the case of dilating tubes in which the flow is constant but in which velocities are being reduced, thus converting kinetic energy into potential energy. It is well known that, through reducers where velocities are increasing, losses from turbulence are quite low. In fact, such losses may be less than those for steady uniform flow through a corresponding length of an equivalent tube of constant diameter. Therein lies a neat little research problem. If the direction of flow is reversed, however, the losses become inordinately high. This is due to excessive turbulence in the vortex generating zone which, owing to receding boundaries, now occupies a much greater portion of the total cross-sectional area than is the case for uniform flow in a conduit of constant area.

In the example of the large river, the vertical velocity curve, Fig. 20(*b*), does not show the customary lessening of velocity at the surface. If the curve did show this typical characteristic, the cumulative curves of Fig. 20(*c*) and Fig. 21 would be slightly different. If the vertical velocity curve has a lesser surface velocity than occurs at a point below the surface, the effect is to make the "borrow" curve less than the "transfer" curve. This cannot be true for it would mean that more energy is transferred than is available.

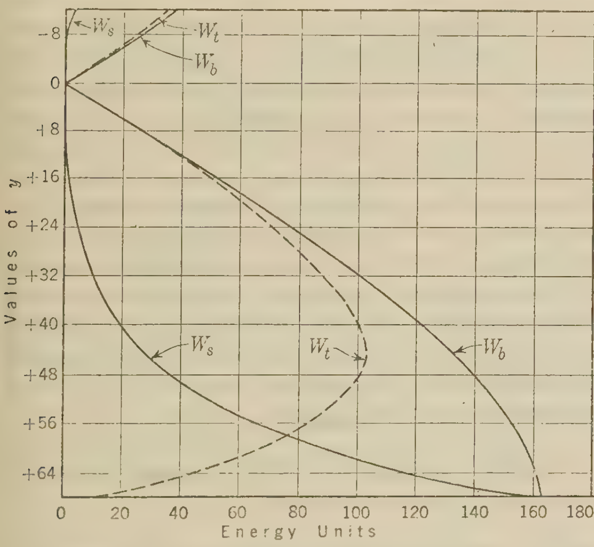

FIG. 29.—VERTICAL VELOCITY CURVE, STATION 395, AT THE DALLES, OREGON

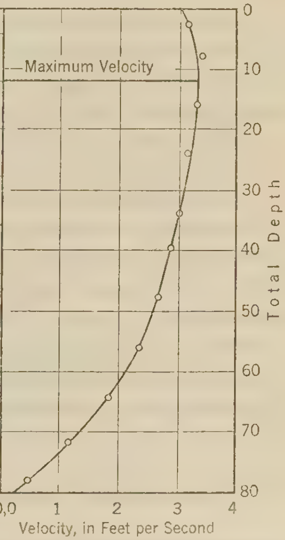

FIG. 30.—ADJUSTMENT OF NEUTRAL AXIS TO AVOID THE ANOMALY OF TRANSFERRING MORE ENERGY THAN IS AVAILABLE

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[38] "Conversion of Kinetic to Potential Energy in Flow Expansions," by A. A. Kalinske, *Transactions*, Am. Soc. C. E., Vol. III, 1946, p. 345.

Some vertical velocity curves do not show a lessening of velocity at the surface, but many curves do evidence such a lessening.[39] The writer was curious to know what effect this would have on the "borrow," "transfer," and "spending" curves. He therefore selected one of many vertical velocity curves taken by the U. S. Geological Survey on Columbia River at The Dalles, Oregon. The curve is shown in Fig. 29. The banks of the river are steep basalt cliffs with submerged talus. The stream bed is very rough and is strewed with large boulders. Depths are great and velocities low. The vertical velocity curve is quite typical of such curves taken at this site near midstream.

To avoid the anomaly of transferring more energy than is available, the expedient of fixing the neutral axis at the depth of maximum velocity was adopted. This is exactly the procedure for flow in a pipe. The resulting curves of "borrow," "transfer," and "spending" are shown in Fig. 30. The afore-mentioned anomaly is not in evidence, which seems to indicate that instead of the surface of an open stream being considered as the neutral axis for energy transfer, it would be better to use the thread of maximum velocity.

The writer hopes that this paper is only the beginning of a searching program of study and experimentation through which the fundamental nature of fluid resistance to flow will become better known and that practicable usage of the knowledge will speedily follow.

Leon Beskin,[40] Assoc. M. Am. Soc. C. E.—A thorough description of the well-known phenomena involved in fluid friction is contained in this paper. In the writer's opinion, however, the authors have failed to propose new or more simple physical or mathematical concepts of viscous flow, in a form that would be useful to the practicing engineer, who thus could rely less on "detailed experimental data" (see item 4 of the "Summary"), and more on theory.

The authors have not translated their analysis into general mathematical expressions, since their analytical expressions are given only in the case of a two-dimensional rectilinear permanent flow and (in a somewhat less complete form) in the case of a cylindrical rectilinear permanent flow—both with constant kinetic energy. For example, Eq. 91 which is offered as a complete picture of the "local energy balance" (see item 3 of "Summary"), actually applies only to two-dimensional flow. This "balance" equation is the well-known expression for the differentiation of a product:

$$d(\tau\, u) = \tau\, du + u\, d\tau \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (99)$$

If a cylindrical flow were considered, Eq. 91 would be:

$$\underset{(w_b)}{-\frac{u}{r}\frac{d(\tau\, r)}{dr}} = \underset{(w_s)}{\tau\frac{du}{dr}} + \underset{(w_t)}{\frac{d}{r\, dr}(-\, r\, \tau\, u)} \ldots\ldots\ldots\ldots\ldots (100)$$

Thus, the mathematical expressions of the new concepts are seen to be inadequate because their validity is limited to an extremely simple case of viscous flow. It is impossible to ascertain what the consequences would be if the authors had generalized their concepts for an arbitrary three-dimensional motion, and whether new information would thus be revealed as to the phenomenon of viscous motion. To the writer it seems likely that nothing much would be gained by generalizing these concepts because, although Stokes' dissipation function has an absolute nature, the authors' borrowing and transfer functions have only a relative nature. This fact can be demonstrated by a simple inspection of Eq. 91. Velocity u is measured in reference to some axes of coordinates. If the water is flowing in a conduit which is also moving, u can be defined either in reference to the conduit, or in reference to absolute axes. This does not change the expression of w_s, but modifies both w_b and w_t. Academic as it may seem, this fact has far-reaching consequences. For instance, assuming that the same flow pattern occurs in a stationary conduit, in a turbine, or in a centrifugal pump, both terms, w_b and w_t, have different values in each of these cases; but their difference is zero when viscosity is neglected (they are both equal to zero in this case) and has a definite value when viscosity is considered. It can also be shown that the reversal point has a purely relative nature. It moves toward the center of the conduit in the case of a pump, and reaches the boundaries in the case of a turbine. Instead of "reversal point" (see Section 11) it would be more correct to use consistently the term "reversal surface" (see Fig. 14), or at least the term "reversal curve" in a section.

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[39] "River Discharge," by J. C. Hoyt and N. C. Grover, John Wiley & Sons Inc., New York, N. Y., 1920, p. 52.

[40] Senior Stress Analyst, Eng. Dept., Consolidated Vultee Aircraft Corp., San Diego, Calif.

To summarize: The concepts introduced in this paper are defined in terms that are too narrow and should be generalized. The definitions have no absolute significance and change with every change in the system of coordinate axes. To justify the practical usefulness of the ideas presented in this paper, the authors need to offer a comprehensive series of illustrative examples.

HUNTER ROUSE,[41] M. AM. SOC. C. E., AND A. A. KALINSKE,[42] ASSOC. M. AM. SOC. C. E.—Energy dissipation in turbulent flow is at best an extremely complex phenomenon, and a successful investigation of the mechanics of such a process requires more ability, intuition, and courage on the part of the investigator than perhaps any other phase of fluid motion. It is for this reason that the writers have only commendation for the authors of the present paper in undertaking so broad and fundamental a treatment of the problem. No fault can be found with the mathematical derivations, and portions of the interpretive matter are considered well worthy of reading and rereading by every one interested in fluid mechanics. Section 10, for instance, which contains a detailed description of the turbulence mechanism, is particularly lucid and significant.

Because of the very complexity of fluid turbulence, a paper of this nature can be of value to only a limited number of readers if the treatment is not in itself a simple one, devoid of all ambiguous or superfluous material. The authors presumably sought to clarify the problem for more than such a limited group, through use of analogies, new concepts, and numerous charts; but the writers are inclined to question whether the goal of clarity has actually been achieved. Indeed, since the essential justification of the new concepts introduced by the authors was not properly noted, the application of these concepts to laminar and turbulent flow alike served to conceal rather than to emphasize the fundamental difference between the two types of motion.

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[42] Associate Director, Iowa Inst. of Hydr. Research, Univ. of Iowa, Iowa City, Iowa.

Perhaps the writers' thesis may best be explained by reverting to the authors' use of analogy, to emphasize the fact that analogies are, to say the least, frequently misleading in their oversimplification. Instead of the pulley system to illustrate the general flow process, one might more logically consider a uniform pile of lumber. If an equal force is applied to the end of each plank to simulate a constant pressure gradient, movement apparently comparable to that of laminar flow will result in the longitudinal direction. Thus, assuming a constant coefficient of friction (that is, "viscosity") between planks, the intensity of shear will increase linearly with depth of pile, just as—although for a different reason—the shear increases linearly with distance from the water surface or the center line of a pipe. The work done by the force acting on the end of the top plank, therefore, will evidently not be equal to the generation of thermal energy through friction with the next plank, since both planks are moving and a portion of the force will be transmitted via shear to the lumber below. However, to assume that the forces exerted upon the plank ends represent a form of potential energy (just as the pressure within a fluid is often improperly assumed to be potential energy of flow), which is then "transferred" by the shear process from one plank to another until finally dissipated in the form of heat, is hardly in accordance with accepted methods of mechanics. Moreover, it obviously would be impossible to extend either the plank analogy or the pulley analogy to the case of turbulent motion, since there is no conceivable intermixing process that the planks could undergo to simulate the well-recognized mechanism of turbulent diffusion. Neither analogy (nor, for that matter, any other) is thus capable of truly elucidating the problem under consideration.

In order to be rigorously acceptable in the mechanics sense, the authors' concept of energy borrowing and transmission must involve an actual mass exchange between neighboring zones of flow. Such an exchange is the very essence of turbulent motion, to be sure, but it appears at first glance to be no more characteristic of laminar motion than of the plank analogy. It must be recalled, nevertheless, that the large-scale or molar transfer of fluid matter produced by the mixing process of turbulence has its counterpart in the infinitesimal molecular mixing which takes place in turbulent and laminar flow alike. As a matter of fact, the kinematic molecular viscosity ν and the kinematic eddy viscosity ϵ are both products of mixing lengths and velocities, differing effectively only in scale. Just as the mixing process of turbulence is considered to produce a shear between neighboring zones of flow which is proportional to the lateral rate of momentum transfer, the viscous shear of laminar motion may be resolved into the lateral transfer of momentum through mixing on a molecular scale. Indeed, one frequently compares the rates of molecular and molar diffusion of salinity, or of heat, so that the molecular transfer of kinetic energy is actually just as real a process in laminar flow as the molar transfer is in turbulent flow.

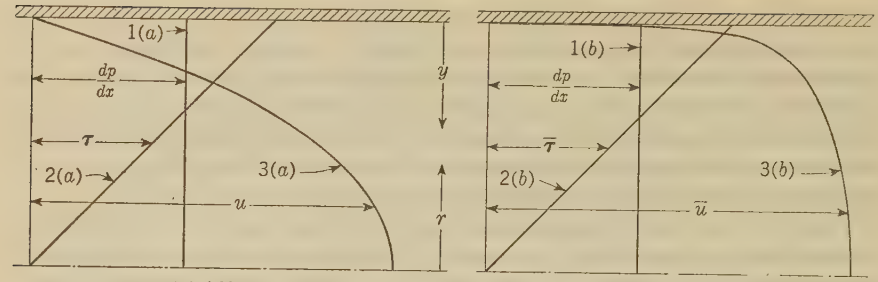
The authors' concepts of energy transmission in both laminar and turbulent flow are therefore fully justifiable, although for reasons not brought to the attention of the reader. In the light of this justification, however, the essential difference between the two types of flow still must be properly clarified. The writers seek to accomplish the necessary distinction by assembling certain of the curves presented by the authors, together with additional functions which they have discussed only casually, in the schematic chart of Fig. 31.

In the uppermost diagrams are included the primary characteristics of laminar and turbulent flow in pipes: (1) The pressure gradient, which is constant across both sections; (2) the intensity of shear, which increases linearly with the radius in both cases; and (3) the velocity, which varies parabolically in laminar flow, and approximately logarithmically in turbulent flow. Neither these nor the remaining curves, it must be emphasized, are drawn to a particular scale, since it is the form of each function, rather than its absolute location, which is essential to the argument.

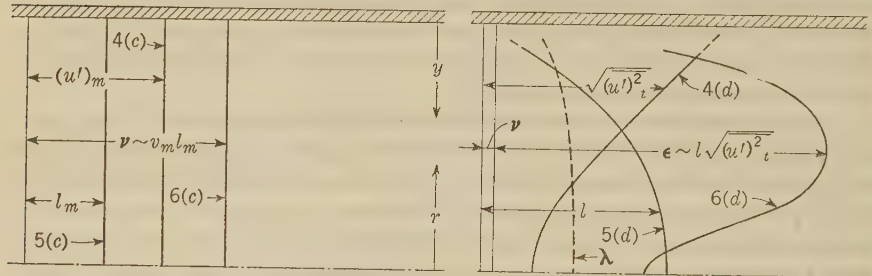
The diagrams next in order depict the mixing characteristics of the secondary motion—molecular in the laminar case and molar in the turbulent. Curve 4(*c*) represents the mean molecular velocity u'_m, and 4(*d*) the mean velocity $\sqrt{\overline{(u')^2}_t}$ of the turbulence (to which $\sqrt{\overline{(v')^2}}$ is assumed proportional); curve 4(*c*) is necessarily constant across the section, whereas curve 4(*d*) decreases markedly toward the center line. Curve 5(*c*) represents the mean free path l_m of the molecules, which is likewise constant across the section; but its molar counterpart curve 5(*d*), the mixing length or mean eddy size l, increases toward the center line. The remaining curves 6(*c*) and 6(*d*) for the mixing coefficients ν and ϵ are simply the products of the velocity and length characteristics of the respective mixing processes. Noteworthy is the fact that ν is constant across the section while ϵ reaches a maximum approximately midway between center line and wall. In either case, nevertheless, the mixing functions are related to the shear and velocity curves of the upper diagrams through the parallel expressions $\tau = \nu\,\rho\,\frac{du}{dy}$ and $\bar{\tau} = (\nu + \epsilon)\,\rho\,\frac{d\bar{u}}{dy}$.

The third pair of schematic diagrams portray the transfer and the spending of kinetic energy by the mean laminar and turbulent motion. The local rate of spending, rather than the cumulative emphasized by the authors, is used herein for the same reason that local conditions of velocity and shear rather than their cumulative effects are considered more significant in the present argument. Likewise, instead of the authors' cumulative transmission function W_t, the writers have chosen the ratio $W_t/2\,\pi\,r$, since it is the latter quantity which is directly proportional to the mixing coefficient and the velocity gradient of the foregoing diagrams. In other words, the rates of transfer of kinetic energy of the mean motion toward the walls are, in terms of the respective mixing processes: $\left(\frac{W_t}{2\,\pi\,r}\right)_{\text{lam}} = \nu\,\frac{d\left(\frac{\rho\,u^2}{2}\right)}{dy}$ and $\left(\frac{W_t}{2\,\pi\,r}\right)_{\text{turb}} = (\nu + \epsilon)\,\frac{d\left(\frac{\rho\,\bar{u}^2}{2}\right)}{dy}$.

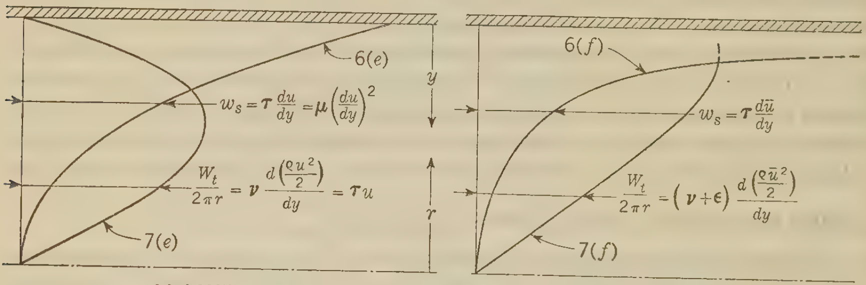
Although it is evident from these diagrams that both the transfer and the spending of the energy of the mean motion in turbulent flow are greater in the wall region, owing to the different nature of the mixing process, the true sig-


1(a)
$\frac{dp}{dx}$
τ
2(a)
3(a)
u
y
r
1(b)
$\frac{dp}{dx}$
$\bar{\tau}$
2(b)
3(b)
$\bar{u}$

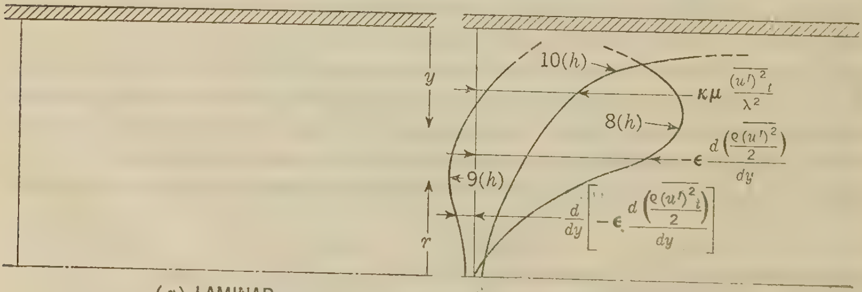
(a) LAMINAR (b) TURBULENT


4(c)
$(u')_m$
$\nu \sim v_m l_m$
l_m
5(c)
6(c)
y
r
ν
$\sqrt{\overline{(u')^2}_t}$
4(d)
$\epsilon \sim l\sqrt{\overline{(u')^2}_t}$
6(d)
l
5(d)
λ

(c) LAMINAR (d) TURBULENT


6(e)
$w_s = \tau \frac{du}{dy} = \mu \left(\frac{du}{dy}\right)^2$
$\frac{W_t}{2\pi r} = \nu \frac{d\left(\frac{\varrho u^2}{2}\right)}{dy} = \tau u$
7(e)
y
r
6(f)
$w_s = \tau \frac{d\bar{u}}{dy}$
$\frac{W_t}{2\pi r} = (\nu + \epsilon) \frac{d\left(\frac{\varrho \bar{u}^2}{2}\right)}{dy}$
7(f)

(e) LAMINAR (f) TURBULENT


y
r
10(h)
$\kappa\mu \frac{\overline{(u')^2}_t}{\lambda^2}$
8(h)
$-\epsilon \frac{d\left(\frac{\varrho \overline{(u')^2}}{2}\right)}{dy}$
9(h)
$\frac{d}{dy}\left[-\epsilon \frac{d\left(\frac{\varrho \overline{(u')^2}_t}{2}\right)}{dy}\right]$

(g) LAMINAR (h) TURBULENT

FIG. 31.—COMPARISON OF CHARACTERISTIC FUNCTIONS FOR LAMINAR AND TURBULENT FLOW IN PIPES

nificance of these curves becomes apparent only through their correlation with the fourth pair of diagrams. The spending curve 6(*e*) for laminar flow, it must be noted, represents essentially direct dissipation into heat. The corresponding spending curve 6(*f*) for turbulent flow, on the contrary, embodies not only direct dissipation but also the conversion of energy of the mean motion into energy of turbulence through the continuous generation of new eddies.[43] As these eddies are formed, they (and their kinetic energy) become subject to the same lateral transfer as has already been discussed for the mean motion. In the case of laminar flow, the constancy of both the mixing coefficient ν and the kinetic energy $\frac{\rho(u')^2{}_m}{2}$ of the molecular motion results in a zero rate of transfer, diagram (*g*) therefore remaining blank. The rate of transfer of the kinetic energy of the turbulence, however, will be both finite and variable across the section, since—as shown by curve 8(*h*)—it must equal the product of the mixing coefficient ϵ and the negative gradient of $\frac{\rho\overline{(u')^2{}_t}}{2}$. Such transfer, in contradistinction to that of the kinetic energy of the mean flow 7(*f*), will be directed toward the center line, since that is the direction of decreasing $\overline{(u')^2{}_t}$.

The gradient of this transfer function (that is, the difference between the rates of efflux and influx of energy for a unit volume), plotted as curve 9(*h*), represents the difference between the local rate of turbulence generation and the local rate of turbulence decay. Near the wall, where the curve is positive, the generation process evidently predominates, but over the major part of the central region negative values indicate preponderant decay. The local dissipation rate, in turn, may be evaluated independently in the approximate form $\frac{\kappa\,\mu\overline{(u')^2}}{\lambda^2}$ noted by the authors and plotted as curve 10(*h*). (The length λ, it must be remarked, differs from the mixing length or eddy size l in that it represents the average scale of the smaller eddies which control the dissipation process; its trend is indicated in diagram (*d*) by a broken line.) Comparison of the dissipation curve 10(*h*) with the spending curve 6(*f*) will now show that the two differ essentially by the local magnitude of curve 9(*h*), which is positive near the walls and negative near the center. In other words, if the comparatively small rate of dissipation by viscous action in the mean motion (not plotted herein) is ignored, the spending curve 6(*f*) will be seen to represent the algebraic sum of the turbulence-dissipation curve 10(*h*) and the generation-minus-decay curve 9(*h*), in accordance with previous statements. Thus, the ultimate difference between the laminar and the turbulent mixing processes lies in the fact that turbulence generated near the walls is continuously being transferred toward the center, so that the dissipation rate in this region actually exceeds the rate of spending.[44]

Despite the qualitative clarity which such an analysis should yield, the fact must be stressed that existing experimental data permit exact evaluation of only part of the functions shown schematically in the several diagrams. The laminar curves, of course, are accurately known. Completely lacking, however, are systematic measurements of $\overline{(u')^2_t}$, l, λ, and ϵ, for various boundary and flow conditions. Even the curve of velocity distribution in turbulent flow becomes indeterminate in the neighborhood of rough walls, as a result of which all curves for turbulent flow in Fig. 31 have advisedly been broken in the wall vicinity. Just what happens in the neighborhood of the wall, and at the wall itself, must therefore remain for the present a matter of complete conjecture. It is possible, however, to perform the following bulk evaluation of the relative parts played by the wall and the central regions, which may shed further light upon the dissipation problem.

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[43] "Statistical Theory of Turbulence," by Theodor von Kármán, *Journal of the Aeronautical Sciences*, Vol. 4, No. 4, 1937, p. 135.

[44] "Statistical Theory of Turbulence," by G. I. Taylor, *Proceedings*, Royal Soc. of London, Series A, Vol. CLI, 1935, pp. 462–464.

In the case of smooth pipes, of course, the method of analysis discussed in the foregoing pages can be carried to the wall itself, for reasonably precise information as to the velocity distribution from boundary to center line is at hand. If one were to assume, arbitrarily, that the nominal thickness of the laminar sublayer $\delta = \frac{11.6\,\nu}{\sqrt{\tau_o/\rho}}$ represents[45] the border between the central and the wall regions, it becomes possible at once to evaluate independently W_c, W_w, and W_o, the rates of energy spending per unit length of pipe in the central region, the wall region, and over the entire cross section, respectively. For example, since the velocity gradient in the sublayer is very nearly constant, as a close approximation (except at very low Reynolds numbers) the rate of dissipation in this wall zone will be $2\,\pi\,r\,\delta\,\mu\,(du/dy)^2$, which reduces to

$$W_w = 11.6\,\pi\,D\,\tau_o\,u_f \quad \text{(101a)}$$

while the total rate of energy expenditure for the pipe becomes, in similar units,

$$W_o = \pi\,D\,\tau_o\,u_f/\sqrt{f/8} \quad \text{(101b)}$$

The following ratio then results:

$$\frac{W_w}{W_o} = 11.6\sqrt{\frac{f}{8}} \quad \text{(102)}$$

Since the resistance coefficient f is a known function of the Reynolds number, a plot of W_w/W_o versus R may be made directly.

A diagram of a somewhat similar nature[46] is reproduced in Fig. 32. The over-all ordinate AD of this diagram represents the total rate of energy expenditure as given by Eq. 101*b*, and the ordinate AB the rate of dissipation within the laminar sublayer approximately in accordance with Eq. 101*a*. The spending rate in the central region, shown by the ordinate remainder BD, has been further subdivided into part BC, the rate of dissipation due to the mean motion, and part CD, the rate of dissipation within the superposed turbulence.

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[45] "Modern Conceptions of the Mechanics of Fluid Turbulence," by Hunter Rouse, *Transactions*, Am. Soc. C. E., Vol. 102 (1937), p. 498.

[46] "Sur la répartition entre le mouvement moyen et le mouvement d'agitation de l'énergie dissipée dans l'écoulement turbulent d'un fluide incompressible," by J. Kampé de Feriet and A. Martinot-Lagarde, *Comptes rendus des seances de l'Académie des Sciences*, December 13, 1937.

Immediately evident is the fact that the wall region plays a relatively large part in the total dissipation process, although the turbulence mechanism becomes more and more effective with increasing Reynolds number (that is, decreasing thickness of the laminar sublayer). Equally apparent is the very small rate of dissipation attributable to the mean motion in the central region, which likewise diminishes with increasing **R**.

Comparable representation of the relative dissipation rates for rough pipes would require a separate diagram for each successive degree of roughness. It would appear, moreover, that preparation of such diagrams must entail equally explicit information as to flow conditions near the boundary, which information is notably lacking in the case of boundary roughness. However, it will be recalled that the ratio W_w/W_o is equal to $1 - W_c/W_o$, which can be evaluated independently for all conditions. The authors, for instance, utilize this fact in interpreting the similar gradients of the velocity curves for different roughnesses in Fig. 16. That is, by assuming the same hydraulic slope for flow through pipes of the same diameter but different relative roughness, they show that the energy expenditure in the central region will be identical, whereas the total expenditure—and hence that in the wall region—will decrease with increasing roughness. The writers believe, however, that a far more general and significant interpretation may be made.

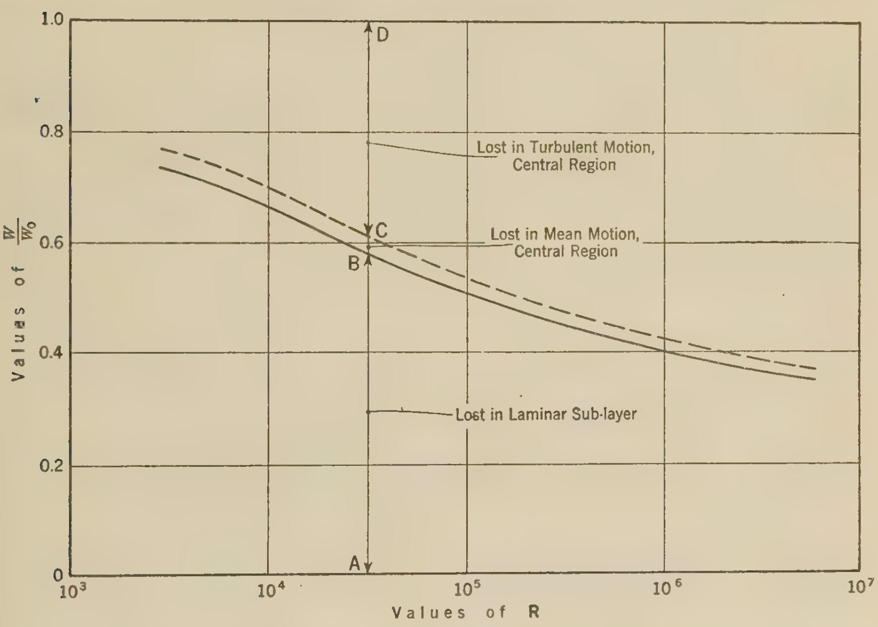

FIG. 32.—RELATIVE RATES OF ENERGY LOSS IN DIFFERENT ZONES OF FLOW FOR SMOOTH PIPES, AFTER KAMPÉ DE FERIET AND MARTINOT-LAGARDE

As must be concluded from Fig. 16, the dimensionless gradient $\frac{d(\bar{u}/u_f)}{d(r/r_o)}$ is the same for all velocity-distribution curves at any particular value of r/r_o in the central zone, regardless of the magnitude of the Reynolds number or the relative roughness. Therefore, the rate of energy expenditure in this central zone, up to that value of r/r_o at which the type of boundary begins to influence the given dimensionless gradient, may be expressed in completely general terms. In other words, letting $x = \bar{u}/u_f$ and $z = r/r_o$, it may be shown that

$$W_e = \pi\, D\, \tau_o\, u_f \int_0^{z'} z^2 \left(-\frac{dx}{dz}\right) dz \ldots\ldots\ldots\ldots\ldots (103)$$

in which z' is the limiting value of r/r_o for the central zone under discussion. The integral may readily be evaluated from the logarithmic velocity equation, for which $dx/dz = -\frac{1}{\kappa(1-z)}$, its magnitude evidently depending solely on the value of the limit z'. Upon division of Eq. 103 by Eq. 101*b*, and subtraction from unity, the desired ratio of the rates of wall and total energy expenditure is seen to become simply a function of z' and f:

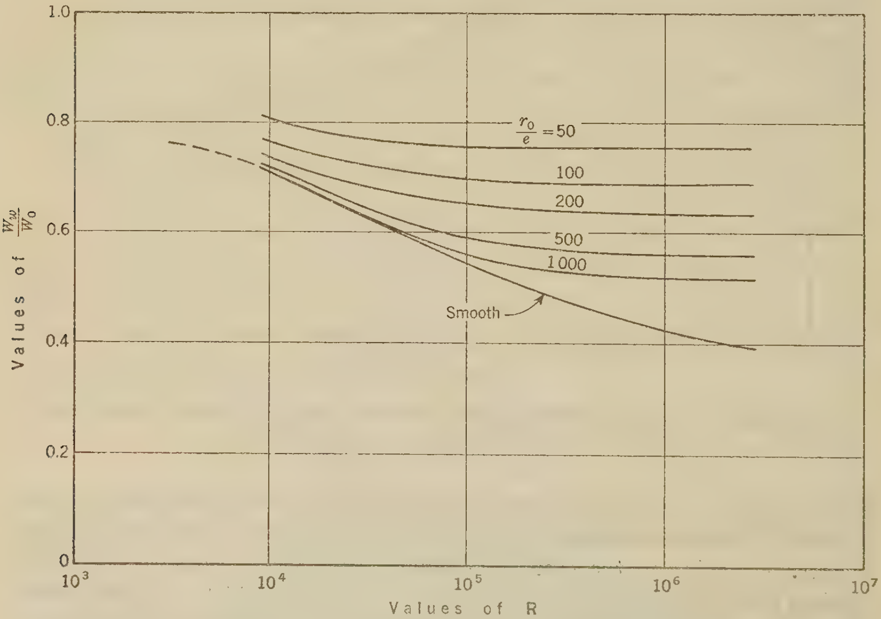

FIG. 33.—RELATIVE RATES OF ENERGY LOSS IN WALL REGION FOR SMOOTH AND ROUGH PIPES

$$\frac{W_w}{W_o} = 1 - \frac{W_c}{W_o} = 1 - \sqrt{\frac{f}{8}} \int_0^{z'} z^2 \left(-\frac{dx}{dz}\right) dz \ldots\ldots\ldots (104)$$

The question still must be answered as to what magnitude of z' to specify in determining values of W_w/W_o to plot against **R** and e/r_o. In the case of smooth pipes, of course, use of $z' = 1 - \frac{\delta}{r_o}$ will reduce the result to that of Eq. 102. In the case of rough pipes, it would then appear reasonable to seek a function of the roughness e which would involve a comparable portion of the pipe section; it would be desirable, moreover, to incorporate in this function the effect of viscosity in the practically important transition range from the smooth to the rough phase of the resistance phenomenon. Comparison of the intercepts on the smooth-pipe and the rough-pipe velocity-distribution curves indicates that a value of $3\,e$ is approximately the counterpart of the value δ used in Eq. 101a; and further consideration indicates that the relationship $1 - z' = \frac{\delta + 3\,e}{r_o}$ is the only simple function approaching the assumed limits of $\frac{\delta}{r_o}$ and $\frac{3\,e}{r_o}$.

Based upon this function, the writers have prepared from Eq. 104, together with a diagram of f versus **R** for smooth and rough commercial pipes,[47] the system of curves of W_w/W_o versus **R** shown in Fig. 33. Despite the admittedly arbitrary nature of the assumed relationship for z', it is believed that this diagram gives a particularly clear picture of the part played by the wall region in the process of energy expenditure, and of its variation with the relative roughness and the Reynolds number. In fact, only through a plot of this nature is it truly possible to judge the effect of roughness under comparable flow conditions. The similarity of Fig. 33 to the f : **R** diagram is simply an indication of the close relationship which exists between f and the ratio W_w/W_o in the several equations given.

If the writers' premise that Eq. 103 reduces the mechanism of energy transfer and expenditure in the central region to a truly common denominator is justified, it must be concluded from Fig. 33 that the rôle played by the wall zone increases markedly with the relative roughness under comparable flow conditions (that is, constant Reynolds number rather than constant hydraulic slope). It must be realized, of course, that the relative thickness of the wall zone considered by the writers is by no means fixed, since $1 - z'$ has been assumed proportional to both δ/r_o and e/r_o, which vary considerably in magnitude. On the other hand, since the velocity curve in the central region depends upon these same two variables, and hence is just as flexible a function, the writers' assumption appears in close accord with the probability that the zone of marked turbulence generation must also vary in radial extent as the Reynolds number and the relative roughness change.

Since, in the case of fully developed roughness action, there is no longer a laminar film at the wall in which direct dissipation of energy can occur, one is also led to conclude that the increase in the proportional rate of energy expenditure in the wall region must be due to the relative violence of the turbulence generated by the wall irregularities. It must not be assumed, however, that this large rate of expenditure represents an equally large rate of conversion into energy of turbulence which is subject to transfer toward the central region. On the contrary, as shown by the authors, a considerable portion of the actual energy dissipation probably occurs locally during the generation process. This is due to the extremely high instantaneous stresses produced within the eddies as they are formed. The energy required to maintain equilibrium of the turbulence structure, in other words, while eventually dissipated in its entirety, does not all pass through the intermediate stage of transferable energy of turbulence.

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[47] "Evaluation of Boundary Roughness," by Hunter Rouse, *Proceedings*, 2d Hydr. Conference, June 1–4, 1942, State Univ. of Iowa, Iowa City, *Bulletin No. 27*, Studies in Eng., 1943.

In this discussion the writers have consistently sought to add to, rather than detract from, the value of the authors' detailed presentation. As a new approach to a problem of vital importance is invariably subject to modification and improvement, the writers are hopeful that the interpretation presented herewith may likewise be altered and extended by the authors in their closure, and by those who later use the paper as a reference. Indeed, quite aside from the fundamental material which the paper has provided, it is probable that it will have lasting worth as a stimulus to creative thought on a very stubborn problem.

Boris A. Bakhmeteff,[48] Hon. M. Am. Soc. C. E., and William Allan,[49] Assoc. M. Am. Soc. C. E.—A step forward in treatment of "a very stubborn problem," as the reviewers express it, and a "stimulus to creative thought" is all that the writers could aspire to in summarizing the results of years of cogitation and study. The encouraging comment of a practical engineer like Mr. Stevens is especially appreciated. Mr. Rich has at once seized upon the possibilities by extending the new approach toward energy balance in "free discontinuity" or "separation" zones. This field is of special importance, as separation forms the basis of all resistances of the "local" or "form" type. For the laminar case, Mr. Rich's solutions are indisputable. Unfortunately in practice, most separation zones are in highly turbulent condition, and here the difficulties begin as the physics become very complex[50] and the factual knowledge most defective. The writers hope to return to the subject in some not too distant future.

Messrs. Rouse and Kalinske have generously added to the original paper. The remarks which follow, and which deal largely with their discussion, are offered in the same cooperative spirit of contributing toward a better understanding of a complex case. The writers realize only too well, how difficult it is for reviewers, especially if a subject lies in the field of their own interest, to adjust their viewpoint to that of the writers and to disengage themselves from the particular problems which preoccupy their own thoughts. Indeed, the statement that—

> "* * * the application of these concepts [the different phases in the energy exchange mechanism] to laminar and turbulent flow alike served to conceal rather than to emphasize the fundamental difference between the two types of motion."

—indicates that Messrs. Rouse and Kalinske have misinterpreted the purpose of the paper.

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[49] Associate Prof., Civ. Eng., School of Technology, College of the City of New York, New York, N. Y.

[50] *Transactions*, Am. Soc. C. E., Vol. III, 1946, p. 382.

The major objectives of the paper were to reveal what Mr. Rich so aptly describes as the "spatial implications of the energy balance," and to show that these implications, which embody the interrelations between the different phases of the process, are wholly determined (1) by the shape of the velocity profile, and (2) by the stress distribution. In fact, the same mathematical expressions describe the quantitative features of the process irrespective of the nature or the physical agencies that happen to shape the flow pattern. Therefore, it is the unity and not the difference of the fundamental premises which the writers tried to emphasize.

In approaching any natural phenomenon two questions arise, the "how" and the "why." The "how" relates to the functioning as observed, and it implies a comprehensive interpretation of factual data derived from experiment, without recourse to ambiguous or conjectural assumptions. Thus, the graphs of Figs. 2, 11, and 17 portray "how" the energy balance mechanism unfolds in the laminar and in the turbulent case, with the graphs typifying observed unquestionable facts. The human mind, on the other hand, is never satisfied with knowing only the "how." It naturally strives to penetrate into the mysteries of the universe and to unveil the internal working of things, in an endeavor to answer the "why." This realm is fascinating even if usually closely guarded; necessarily, it is a realm of hypothesis, conjecture, and guesswork.

The writers have nothing but commendation for the effort of the discussers to penetrate deeper into the "why" of the internal turbulence mechanism, but a word of warning might be inserted against the danger of placing too much emphasis on the alleged similar traits in the internal workings of "eddy kinematic viscosity" ϵ with the customary ν. The elegance of a single mathematical form for expressing stress in both instances, should not shield the essential differences in physical phenomenology. The important point, however, is that the comparative analysis presented by Messrs. Rouse and Kalinske relates principally to the stress-producing process, and bears only indirectly on the energy exchange as such. The pivotal point in the case is the concept of what the writers term the "transmittance function." Messrs. Rouse and Kalinske seem to visualize the latter as a bodily transfer of kinetic energy, inasmuch as they state:

> "In order to be rigorously acceptable in the mechanics sense, the authors' concept of energy borrowing and transmission must involve an actual mass exchange between neighboring zones of flow."

The writers brought out the new concept of the internal energy transmittance process as a physical fact, based upon direct observational data. Whatever the internal workings, the function is a dynamic reality because there is an extant velocity and stress distribution. The ostensible "how" is indisputable, independent of whatever hypothesis is used to disclose the secrets of the internal "why." In any event the internal process does not imply bodily transfer of kinetic energy. The concentration of borrowed energy near the wall zone is a cumulative process; and, if such accumulation were affected by a bodily transfer of kinetic energy from one layer to another, the transfer would necessarily require compensation by an appropriate reduction of the kinetic energy in the axial direction. Kinetic energy in uniform motion remains unchanged. In fact, all the borrowing and transmittance unfolds at the expense of the potential energy, the withdrawal of which is manifested in the axial pressure drop.

Work or energy is not a physical entity such as salinity, sediment, or heat, which are transferred in the course of diffusion. Work is a composite quantity involving force and displacement. A belt, or a sequence of gears, in a mechanical system conveys the force, whereas the kinematics of a mechanism conditions the displacements. In a moving fluid the extant velocities and stresses supply the constituents; they combine in shaping the transmittance process and the resulting spatial accumulation of borrowed energy. The writers feel that concentration on the idea of bodily energy transfer, instead of acceptance of the extant mechanical implications of the process prompted the rather severe attitude of Messrs. Rouse and Kalinske toward the mechanical analogy used by the writers. Perhaps it should be explained that, long before the paper took final shape, the writers on different occasions discussed the subject matter with members of the profession. There never was any difficulty about the borrowing and spending phase, but it was not always easy to "put across" the implications of the transmittance function; and just for this reason the writers took recourse in a mechanical analogy, with repeated and distinct success. Of course, analogies may be objected to on general principles. Indeed, the question has preoccupied the science of logic since the ancient days of Aristotle's "Organon" down to the present-day scientific epistemologists. Obviously an analogy is only a simile and therefore is not a proof; but, in the words of Raymond Poincaré ("Science and Hypothesis"), "a semblance of a proof" may be "so luminous."

The graphs of Fig. 31 deserve every possible commendation. There is only one reservation and that is in regard to curve 6(*f*) in Fig. 31 (*f*), which is termed a spending curve, with the concomitant curves 9(*h*) and 10(*h*) in Fig. 31(*h*). In the paper the writers systematically insisted on distinguishing between the process effecting loss of the main flow energy, and the events in the superimposed turbulent structure. In fact, the term "spending" was used exclusively to signify the disappearance of the main flow energy, whether caused by direct local dissipation or by conversion into eddying vortical form. The sequence of the different phases and their spatial distribution were presented schematically in Fig. 14. The concepts underlying this diagram constitute, in the view of the writers, the very essence of the over-all turbulent pattern.

The first question which arises in connection with the interpretation of curves 6(*f*) to 10(*h*) in Fig. 31 is whether or not local spending of the main flow energy in the central zone is accompanied by any appreciable local generation of turbulence. In this connection it should be noted that, in the light of recent knowledge, turbulence may properly be defined as a product of disintegration of "eddying patterns." Furthermore, it should be remembered that the formation of eddying patterns, exemplified by the rolling up of flow filaments under the influence of excessive strain, is accompanied by a highly concentrated absorption of main flow energy. The larger part of such absorbed main flow energy is dissipated in place through viscous attrition, and only a comparatively small part of the absorbed energy may survive in the form of disintegrated eddies, which vitalize and manifest turbulence. No single investigator has contributed more toward a proper quantitative understanding of these facts than has Professor Kalinske.[38]

Whether or not turbulence is produced in the course of local spending in the central zone depends on the intensity of the local viscous strains, engendered in the course of the swarming motion of the large turbulent eddies. At the present state of knowledge, scholars cannot answer this question conclusively in either one sense or another. They can state definitely, however, that, even if local turbulence happened to be generated, the added turbulent energy would be only a small fraction of the locally spent main flow energy and that, in turn, the local rate at which energy is spent in the central region is only a minor part of the total energy. Thus, under all circumstances the locally generated turbulence is insignificant in comparison with the bulk turbulence, that appears and diffuses from a region near the wall in the conversion zone. That is the reason why, when visualizing the over-all turbulent pattern as in Fig. 14, for all practical purposes, the turbulent energy added in the central zone may be neglected, and turbulence may simply be considered as a superimposed pattern, arising from the conversion of the main flow energy accumulated by transmittance in the near-wall zone, and spreading from the latter. Under all circumstances, to maintain the proper perspective in the succession of events, the process of local spending of the main flow energy and the dissipation of turbulent energy diffusing from the wall region should be kept separate. Embracing the two under a single spending concept, as done by the discussers, inevitably obscures the broad spatial implications of the turbulent flow mechanism.

The questions may be raised, of course, as to where the "central" zone ends and what delimits the inner region in which the eventual local generation of turbulence is held negligible. It stands to reason, furthermore, that one zone passes into the other gradually, and that the rate of turbulence generation increases continuously, so that the distinction between the central and wall conversion zone is at best a question of relative "quantity." The answer is that there are so many instances (as, for example, in interpreting Eq. 45) in which the predominance of one factor or the other lends ostensibly distinct over-all characteristics to a phenomenon, substantiating once more the dictum of Hegelian dialectics that at a certain degree "quantity changes into quality." A glance at Figs. 11 and 17 indicates a distinct and rather abrupt change in physical circumstances after the reversal point M is passed, especially in regard to the transmittance W_t-curve, which starts to decline rapidly, evidencing a greatly intensified absorption of energy. For this reason the writers broadly associated the wall "conversion" zone with the reach between the reversal surface M and the solid boundary. The writers would not be surprised if future research were to show that, actually, the region of active turbulence generation is much narrower (see last paragraph of Section 11, in the paper).

The discussers refer to G. I. Taylor's classical "Statistical Theory of Turbulence."[44] When that paper was written, the existence and the rôle of the transmittance phase were not realized, and it was only natural to present facts without taking this basic function into consideration. Hence, in Fig. 34, which appeared in Mr. Taylor's paper and was reproduced by subsequent commentators,[51,52] the local spending of main flow energy (curve 1) was set face to face with curve 2, portraying local dissipation of turbulent energy. Curve 1 corresponds to the accrual of the ordinate measuring the difference between OD and OC, Fig. 14, whereas curve 2, Fig. 34, exemplifies the rate of decay of the curve B'O, Fig. 14. Physically, the Taylor curves refer to phases, which are not interrelated in any direct fashion and therefore are not comparable.

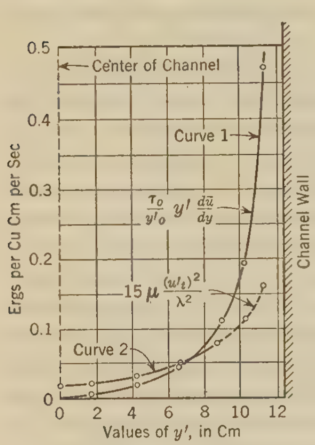

FIG. 34

A very interesting part of the Rouse and Kalinske discussion is related to Fig. 33, and attendant Eqs. 103 and 104. It is not clear whether Eq. 103, which, represents the cumulative spent energy W_s, was established by the discussers independently. However that may be, Eq. 103 follows directly from substituting Eqs. 43*a* and 43*b*, with the discussers' symbols, into Eq. 44, making

$$\left|W_s\right|_0^z = \pi D \tau_o u_f \left[\int_0^z 2\,z\,x\,dz - z^2 x\right] = -\pi D \tau_o u_f \int_0^z z^2 \frac{dx}{dz}\,dz \quad (105)$$

which, for the logarithmic velocity curve, leads to:

$$\frac{W_s}{W_o} = \frac{1}{\kappa}\sqrt{\frac{f}{8}}\int_0^z z^2 \frac{dz}{1-z} = -\frac{1}{\kappa}\sqrt{\frac{f}{8}}\left[\frac{z^2}{2} + z + \log_e (1-z)\right]_0^z \quad ..(106$$

In seeking what they claim to be a more general and significant interpretation than the writers' analysis (based initially on the premise of an identical hydraulic slope), the discussers merely demonstrate what is already presented by the curves in Fig. 17(*b*), which serve to account for smooth and rough boundaries of the entire cumulative exchange process for the range of high Reynolds' numbers, where fully developed rough patterns have been attained and where the shape of the velocity curve in terms of u/u_f remains practically unchanged. The aforementioned outlines are based on observed data, and the conclusions cannot be questioned. Inasmuch as the curves of Fig. 33 are intended to represent essentially the same facts as Fig. 17, both sets should point toward identical conclusions. The implications to be drawn from Fig. 33, however, are diametrically opposite. The fundamental idea revealed by Fig. 17 is that the spending curve for rough boundaries lies above the smooth outline—thus,

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[51] "Modern Developments in Fluid Dynamics," edited by S. Goldstein, Clarendon Press, Oxford, England, Vol. II, 1938, p. 396.

[52] "Turbulence and Energy Dissipation," by A. Kalinske, *Transactions*, A.S.M.E., Vol. 63, 1941, p. 41.

with an increase in roughness, a relatively larger part of the over-all energy is spent in the central region and therefore a smaller part is left for concentration through transfer and for ultimate conversion into eddies in the near-wall region. Contrary to the foregoing interpretation, in Fig. 33, the part of energy alleged to be spent in the wall zone is shown to increase with the boundary roughness. At first this might seem to be a fundamental discrepancy; but a glance at Eq. 106 (which was used by the discussers in preparing Fig. 33) indicates that, for the same relative distance $z = r/r_o$, the proportionate part of the spent energy increases with the friction coefficient f and that therefore within the reaches where the logarithmic velocity equation does apply, the general properties indicated by the curves of Fig. 17 are substantiated.

The reason for this inconsistency is to be found in the method used for calculating Fig. 33 and primarily in the manner of selecting the integration limits. As the discussers state, the present knowledge of flow circumstances in the proximity of rough boundaries is very meager, and guesses of "just what happens" remain "a matter of complete conjecture." The discussers proceeded by way of certain assumptions (which in all fairness were just as good as any other premises based on preconceived theoretical notions) without a solid foundation in actually known facts. On the other hand, as seen from Fig. 17, the W_s-curves near the wall rise very abruptly so that a slight difference in assumed transverse distances may lead to widely diverging results. The writers feel, sincerely, that the revealed discrepancy offers a pertinent example of the type of questions the further study of which is usefully stimulated by the discussion.

In general, space will not permit a thorough analysis of the question of what constitutes the limits of the "boundary zone." Moreover, as stated, for rough boundaries, additional factual knowledge is needed urgently. For smooth surfaces the question can be approached with some measure of certainty. Thus, it is known[53] that the numerical value of the so-called distance friction parameter $u_f = 11.6$, in Eqs. 101 and 102, actually refers to the intersection of the extended straight laminar sublayer line with the prolonged turbulent curve, and that therefore in reality the value 11.6 represents a distance from the wall, which is larger than the thickness of the purely laminar sublayer and considerably smaller than the total transverse dimension of the over-all boundary region beyond which a fully turbulent regimen is established. Usually the thickness of the latter is associated with $y_f \sim 30$. In this light the appraisal for the wall-zone spending, appearing in Eq. 102 and reflected in Fig. 33, are much too small and lead to an erroneous perspective. Equally misleading are the presentations of the French scientists (Fig. 32), who used the wholly unjustifiable parametric value of $y_f = 15$. Actually the energy lost in the laminar sublayer is only about one half of the indicated amount. Also the main flow energy spent in the turbulent central region is considerably smaller than suggested by Figs. 32 and 33. Indeed, in the case of smooth pipes the energy concentrated and used for conversion in the entire boundary region is of the order of 80% of the total; and, in so far as approximate appraisals are concerned, Eqs. 47 to 50 seem to represent events in truer fashion. Messrs. Rouse and Kalinske, as well as the French scientists, seem to disregard, entirely, the rôle and function of the transitory region between the laminar sublayer and the fully developed turbulent zone, clearly indicated by the Nikuradse experiments. The writers feel, that this "in-between" zone is the main seat of the turbulence generation process.

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[53] "The Mechanics of Turbulent Flow," by B. A. Bakhmeteff, Princeton Univ. Press, Princeton, N. J., 1936, pp. 78 and 82.

The writers could not close this paper without referring to the very vital questions raised by Mr. Stevens in regard to watercourses, in which the maximum velocity filament is below the surface. In the stream gaging manual of the U. S. Geological Survey,[54] a set of typical velocity curves shows the maximum velocity, in all instances, to occur at the surface of the stream; this is in contrast to earlier assumptions, by which the fastest filament was invariably placed below the surface. The reason for the "drowning" of the maximum velocity, which always occurs in comparatively narrow channels, has been a subject of lively discussion for many years. Generally, the phenomenon may be traced to secondary currents, always present in nonsymmetrical profiles;[55] perhaps the most plausible explanation was the one given by Ludwig Prandtl.[56] The writers dealt only with two-dimensional flow, in which case the maximum filament would be at the surface. For the present, available knowledge of the more complicated three-dimensional patterns is much too meager to permit any comprehensive energy-exchange analysis, except by way of broad qualitative generalizations.

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[54] "Stream-Gaging Procedure," by Don M. Corbett, *Water-Supply Paper No. 888*, U. S. Geological Survey, U. S. Printing Office, Washington, D. C., 1943, p. 36.

[55] "Untersuchungen über Turbulente Strömungen in nich Kreisförmigen Röhren," by J. Nikuradse, *Ingenieur Archiv.*, Vol. I, 1930, p. 306.

[56] "Über die ausgebildete Turbulenz," by L. Prandtl, *Proceedings*, 2d International Cong. for Applied Mechanics, Zurich, Switzerland, 1926, p. 70.

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Paper No. 2289

EFFECTS OF RADIANT HEAT ON REINFORCED-CONCRETE RIGID FRAMES

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Synopsis

The reinforced-concrete rigid frame is subjected to two types of temperature stresses: (1) Those resulting from long-range, or seasonal, temperature changes, and (2) those resulting from the radiant heat of the sun.

The method of determining stresses caused by seasonal temperature changes has been well established, although designers differ in opinion as to the importance of these changes. The purpose of this paper is to present an analytical method of finding the moments at the knee and crown of a reinforced-concrete rigid frame subjected to the second type of temperature change—namely, that caused by radiant heat from the sun. In order to present numerical computations for a structure of known cross section, a two-span rigid frame highway bridge, previously designed for H-20 loading, was used. A rigid frame was selected because of the increasing popularity of this type of structure for railroad and highway grade-separation projects.

Analysis shows that the radiant-heat moment at the crown of the specific frame used is equal to 35.5% of the maximum design moment. This moment is reduced somewhat because the deck is probably elongating due to the rise in seasonal temperature on the days when radiant heat causes the greatest effect. Main longitudinal steel in the crown for the purpose of resisting the effects of a seasonal temperature drop is also available for the radiant-heat stresses.

Introduction

In the future there should be a marked increase in the number of long-span rigid frames built as a feature of superhighway grade separation. Consequently, the secondary effects of temperature will become more important in

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their design, especially as the spans become longer. Many designers now include in their design of reinforced-concrete rigid frames the secondary effects of temperature change, shrinkage, and settlement of footings.

The present temperature specifications, whenever used in the design of indeterminate structures, refer to the axial strains caused by long-range or seasonal temperature variation. These unit strains, although not necessarily uniformly distributed over a cross section, will be symmetrical about the neutral axis as long as the difference in air temperature causes the strains. Hence, maximum axial deformations will occur roughly in half-year cycles.

Methods of analyzing the stresses due to seasonal temperature changes are well established, but the authorities responsible for design specifications among the various states differ in opinion as to the importance of these changes. (Questionnaire data collected by D. H. Pletta, and loaned to the writer, show this to be true among state highway departments.[2]) This difference of opinion is not surprising in view of the many variables present in temperature design.

For example, some designers rely on low allowable unit stress for the primary loads to provide additional safety from secondary effects. They also realize that any strains due to extreme temperature changes during the early period of the structure are modified by plastic flow. The data collected by Captain Pletta[2] showed that most of the states having mild climates, with a small mean temperature difference between January and July, either disregarded temperature changes or provided very little plus and minus temperature change from some mean value. As an exception to this, some of the states with mild climates provided nearly twice the plus and minus temperature change of states having much greater mean temperature difference between January and July.

Closely allied with seasonal temperature changes is the effect of radiant heat on rigid frames. This paper will be limited to the analysis of stresses caused by radiant heat not only to illustrate a method of analyzing a more complex type of temperature change, but to emphasize the need for more uniform seasonal temperature specifications in the milder states where radiant heat has its greatest effect.

Some work already has been done in Europe on variable temperature change in rigid frames,[3,4] but, as far as the writer knows, this is the first time such a method has been offered for publication in the United States. According to tests made by the Swiss Government on fifty reinforced-concrete rigid frames built between 1924 and 1927, variable temperature change has no damaging effects on frames properly designed and built, even though variable temperature change was neglected in the design.

THE EFFECT OF RADIANT HEAT

Differential temperature strain, most frequently caused by radiant heat from the sun, occurs because the temperature of the top surface of the deck is relatively higher than that of the bottom surface. An experiment conducted in 1941 and 1942 by the Minnesota State Highway Department[5] showed that the surface temperature of a concrete slab is sometimes raised 30° F above the air temperature because of radiant heat. Investigations conducted in states with warmer year-round climates would probably show even higher relative surface temperatures. This difference in temperature tends to elongate the upper fibers and to cause the deck to bulge upward. In the basic, or simply supported, beam this bowing effect causes angular rotations at the supports as shown in Fig. 1. In a rigid frame, where the ends are restrained, this end rotation is subdued by the resisting moments at the knees, causing fiber stresses in all the members. Furthermore, the bridge is subjected to these stresses in short cycles—once every clear, hot summer day.

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[2] "Notes on Existing Rigid Frames in the United States," by D. H. Pletta, *Journal*, A.C.I., March–April, 1938, p. 517.

[3] "Die Neuern Methoden der Festigkeitslehre und der Statik der Baukonstruktionen," by Heinrich Mueller-Breslau, 5 durchges aufl., A. Kröner, Leipzig, 1924.

[4] "Ueber das Elastische Verhaulten von Eisenbetonrahmen unter dem Einfluss Ungleicher Erwaermung," by Adolf Eppler, St.: Wittwer, 1932.

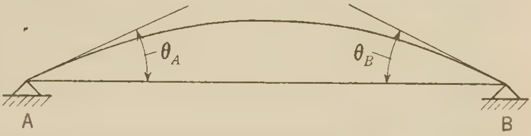

Fig. 1.—End Rotations in the Basic Beam Caused by the Heat of Radiation

Increase in the temperature of any surface above the air temperature depends on many factors—such as the intensity of the sun, the angle between the surface and the sun, the quantity of heat reflected back into the air, the wind velocity, etc. A theoretical calculation of the surface temperature rise necessarily must take into account so many assumptions and variables that it would be not only approximate but impracticable. Therefore, the surface temperatures used in the following analysis are taken from actual readings (original data) made on a comparable surface August 18 and 19, 1941, at Minneapolis, Minn.[6]

General Procedure and Tools of Analysis

Stresses due to strain in a statically indeterminate structure can be investigated only by the analysis of a specific structure. A rigid frame structure with two spans of 80 ft each, designed for an H-20 live loading, was selected. This particular selection was made because of the increasing popularity of this type of structure for grade-separation projects and because it was relatively long, since the magnitude of temperature stresses increases with length.

Details of the rigid frame are shown in Fig. 2. The deck has been divided into segments, each with a length Δs, for use in the angular rotation equation:

$$\theta = \sum \frac{M \, \Delta s}{E I} \qquad \text{(1)}$$

The general procedure for finding the moments at the knee and crown due to radiant heat is as follows:

1. Find the distribution of the differential temperature strain on a cross section of the deck (a method for doing this is presented under the heading, "Numerical Example");

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[5] "Temperature and Moisture Variations in Concrete Pavements," *Investigation 132*, Minnesota Dept. of Highways, Div. of Materials and Research, 1941–1942.

[6] "Heat Transmission Through Insulation as Affected by Orientation of Wall," by Frank B. Rowley and C. E. Lund, *Technical Paper No. 44*, Univ. of Minnesota Eng. Experiment Station, Minneapolis, Minn., 1943.

2. Compute the angle change caused by temperature strain for as many sections as necessary to develop an angle-change loading equation for the entire deck;

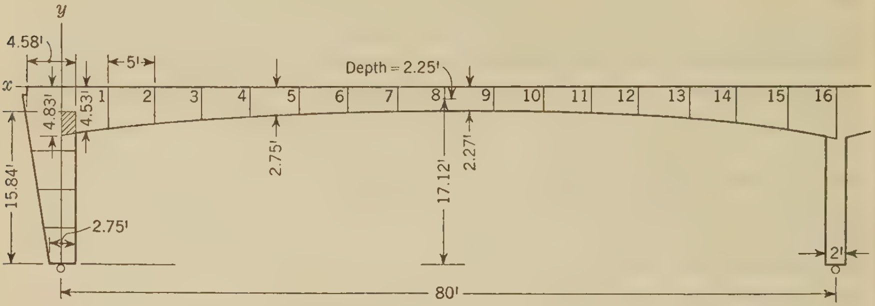

FIG. 2.—DETAILS OF THE RIGID FRAME USED IN THE NUMERICAL EXAMPLE

3. Load a 1-ft increment of deck with the angle change and compute the end rotations, assuming the deck to be simply supported;

4. Load a 1-ft increment of deck with the $\frac{M}{E\,I}$-diagram and find the end-rotation equation in terms of $M/E\,I$, assuming the deck to be simply supported;

5. Since there is no end rotation in a beam with fixed ends, equate steps 3 and 4 to find the fixed-end moment M; and

6. By the method of moment distribution, find the final or true moments in the rigid frame.

Notation.—The letter symbols used in this paper conform essentially to American Standard Letter Symbols for Heat and Thermodynamics Including Heat Flow (ASA—Z10.4—1943), American Standard Letter Symbols for Mechanics of Solid Bodies (ASA—Z10.3—1942), and American Standard Letter Symbols for Mechanics, Structural Engineering and Testing Materials (ASA—Z10a—1932)—each compiled by Committees of the American Standards Association, with Society representation.

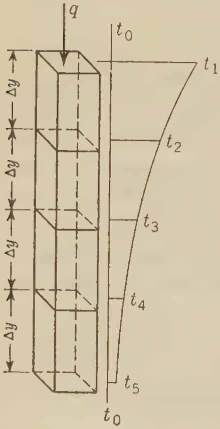

FIG. 3.—INDEPENDENT PRISM METHOD OF TRACING THE RADIANT TEMPERATURE RISE THROUGH A SLAB

NUMERICAL EXAMPLE

Step 1. Distribution of Temperature Strains on the Cross Section of the Rigid Frame Deck.—For this step, use is made of a method of tracing heat flow through a solid body presented in 1937 by R. W. Carlson,[7] Assoc. M. Am. Soc. C. E. Mr. Carlson divides the body into independent prisms of unit cross-sectional area as shown in Fig. 3. This method was modified by the writer especially to analyze the problem of tracing temperature throughout the depth of a slab if the quantity of heat q entering at the surface is known.[8] Only the general features of Mr. Carlson's method are explained in the first step, as the development of this method with subsequent numerical computations of the modified method is much too long to include in this paper.

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[7] "A Simple Method for the Computation of Temperatures in Concrete Structures," by Roy W. Carlson, *Journal*, A.C.I., November-December, 1937, pp. 89–102.

Let t denote temperature in degrees Fahrenheit, t_0 being the initial temperature at the top surface of the slab. In Fig. 3, the subscripts 1, 2, 3, 4, and 5 denote the readings at corresponding temperature stations (spaced at depths Δy) after the first hour of radiant heat applied at the surface. Each station is considered to be the center of a volume of concrete extending halfway to each adjacent station. Since the cross-sectional area of each prism is equal to unity, the volume of the concrete (or prism) between each station is numerically equal to the distance Δy between stations.

It is assumed that the heat flow is in one direction only and that the temperatures of the deck and the air are the same at the beginning of the first hour of heat flow computations. The temperature change per hour at each station is equal to the average quantity of heat entering, and the average quantity of heat leaving, a volume during each hour. The quantity of heat entering and leaving each station depends upon the heat capacity of each volume in British Thermal Units (Btu) and the conductivity of concrete in Btu per hour per degree Fahrenheit.

The thermal diffusivity, which is the relation between heat conduction and the rate of temperature change, is equal to $\frac{k}{c\,\rho}$, in which k is the conductivity of concrete in Btu per hour per degree Fahrenheit, c is the specific heat per unit of weight, and ρ is the density of the concrete. For the numerical computations of heat flow through the slab the term $\frac{k}{c\,\rho}$ was taken as 1.00 sq ft per day.

The temperature distribution was computed for sections 1, 5, and 9 (Fig. 2), the results for sections 1 and 9 being shown in Fig. 4. The hourly surface temperatures were taken from actual readings on a concrete surface.[6] The temperature flow was computed on the basis of the method and assumptions stated previously.

Step 2. Angle Change Caused by Temperature Strain Distribution.—The strain on each fiber of the cross section is taken as equal to

$$\epsilon = \alpha\, \Delta t_f \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

in which α is the coefficient of thermal expansion of concrete, and Δt_f is the temperature change of each fiber in degrees Fahrenheit. The temperature distribution shown in Fig. 4 will also represent the strain distribution multiplied by $1/\alpha$. It can be seen from these curves that the temperature or strain distribution does not vary as a straight line. Furthermore, the strains are present only in the first foot or so of the slab depth. Angle-change computations are simplified with slight error by approximating a straight-line strain distribution as shown by the heavy dashed line in Fig. 4.

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[8] "The Effect of Temperature Stresses in a Reinforced Rigid Concrete Frame," by Milan A. Johnston, thesis submitted to Univ. of Minnesota, in Minneapolis, in 1942 in partial fulfilment of the requirements for the degree of Master of Science, pp. 20–27.

Fig. 5(*a*) shows a single unrestrained section with temperature strains drawn in solid lines. The adjoining section is shown as a dashed line, having been rotated to touch the top and the bottom of the unrestrained section.

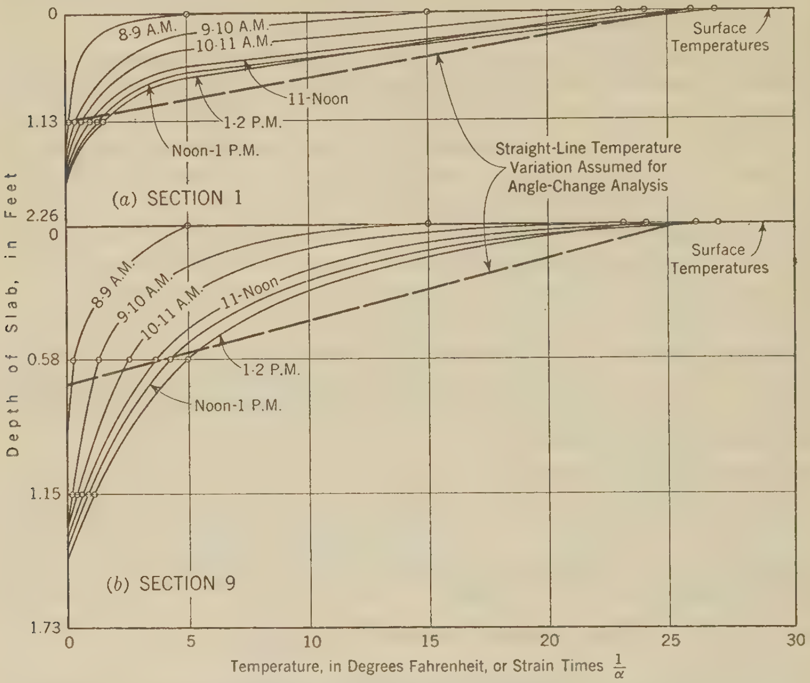


FIG. 4.—HOURLY TEMPERATURE RISE THROUGH GIVEN SECTIONS OF THE DECK, DUE TO THE HEAT OF RADIATION

Since the surfaces or adjoining sections must actually be in continuous contact, and assuming that "a plane before bending remains plane after bending," it is necessary to continue the movement of both surfaces until all the boundaries meet between top and bottom of the section. This has been done in Fig. 5(*b*).

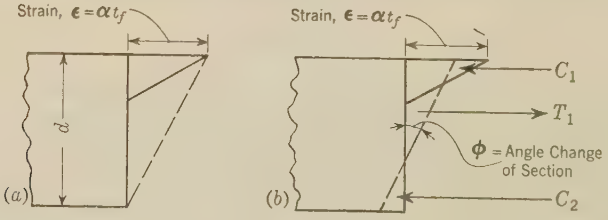
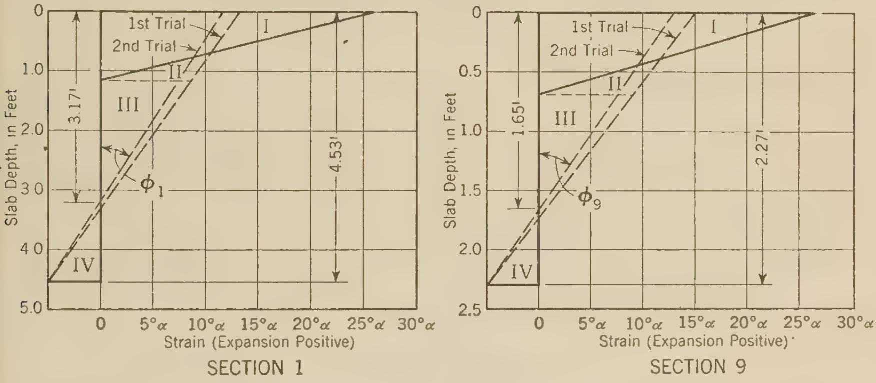

Strain, $\epsilon = \alpha t_f$
d
(*a*)
Strain, $\epsilon = \alpha t_f$
C_1
T_1
ϕ = Angle Change of Section
C_2
(*b*)

FIG. 5

The final position of the dashed line in Fig. 5(*b*) must be such that the summation of the horizontal forces and moments on the section is zero (ΣH and $\Sigma M = 0$; and also assuming that stress is proportional to strain). The movement of the surfaces to their final position (dashed line in Fig. 5(*b*)) will produce compressive forces C_1 and C_2 and a tensile force T_1. Since an algebraic solution for the final position of the section yields cumbersome equations, a semigraphic trial solution is used. This method is shown in Table 1, numerical computation being by slide rule.

TABLE 1.—SEMIGRAPHICAL TRIAL SOLUTION FOR ANGLE CHANGES AT A GIVEN SECTION OF DECK



SECTION 1 SECTION 9

Area	Section 1				Section 9			
	Computation	H	Arm	Moment	Computation	H	Arm	Moment
	First Trials							
I	13 × 0.69 × 0.5	−4.49	4.29	−19.25	11.8 × 0.40 × 0.5	−2.36	2.17	−5.13
II	8.5 × 0.47 × 0.5	2.00	3.52	7.04	9.1 × 0.30 × 0.5	1.36	1.70	2.31
III	8.5 × 2.10 × 0.5	8.92	2.69	24.00	9.1 × 1.04 × 0.5	4.73	1.25	5.91
IV	5.0 × 1.27 × 0.5	−3.18	0.42	−1.34	5.0 × 0.53 × 0.5	−1.33	0.19	−0.25
...	$\Sigma H = 10.92 - 7.67$	+3.25[a]	....	....	$\Sigma H = +6.09 - 3.69$	+2.40[a]	....	....
...	$\Sigma M = +31.04 - 20.59$	....	....	+10.45[a]	$\Sigma M = +8.22 - 5.38$	....	....	+2.84[a]
	Second Trials							
I	14.4 × 0.77 × 0.5	−5.54	4.27	−23.60	13.3 × 0.46 × 0.5	−3.06	2.14	−6.55
II	7.4 × 0.39 × 0.5	1.44	3.51	5.05	7.7 × 0.24 × 0.5	0.92	1.68	1.55
III	7.4 × 2.01 × 0.5	7.44	2.68	19.95	7.7 × 0.95 × 0.5	3.66	1.28	4.69
IV	5.0 × 1.36 × 0.5	−3.40	0.44	− 1.50	5.0 × 0.62 × 0.5	−1.55	0.217	−0.34
...	$\Sigma H = -8.94 + 8.88$	−0.06[a]	....	....	$\Sigma H = -4.61 + 4.58$	−0.03[a]	....	....
...	$\Sigma M = -25.10 + 25.00$	....	....	− 0.10[a]	$\Sigma M = -6.89 + 6.24$	....	....	−0.65[a]

[a] Unsatisfactory in the first trials but satisfactory in the second trials.

Assuming the coefficient of thermal expansion of concrete $\alpha = 0.0000065$, the angle change at the center line of section 1 of the rigid frame (see Fig. 2) is $\phi_1 = \dfrac{0.0000065 \times 11.6^\circ}{3.17 \text{ ft} \times 12 \text{ in.}} = 0.000002$ radian; and the angle change at section 9 is $\phi_9 = \dfrac{0.0000065 \times 13^\circ}{1.65 \text{ ft} \times 12 \text{ in.}} = 0.0000042$ radian. Similarly, the angle change at section 5 is found to be $\phi_5 = 0.0000035$ radian.

Inspection of these three sections shows that the angle change varies inversely as the depth of the section. For example, section 9 has an angle change approximately twice that of section 1 and its depth is one half of section 1. Similarly, section 5 has an angle change equal to 1.67 that of section 1, and its depth is equal to 0.606 times the depth of section 1.

Since the thickness of the deck (see Fig. 2) conforms with the parabolic equation,

$$y = \frac{(x - 40)^2}{2\,a} + 2.25 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

(in which a is a constant found from known boundary conditions = 310), it follows that the angle-change loading in this case is also parabolic. The angle-change loading for the deck is shown in Fig. 6.

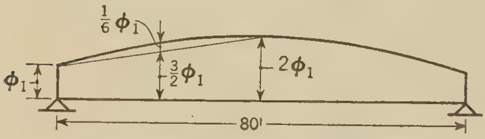


FIG. 6.—ANGLE-CHANGE LOADING OF DECK

Step 3. Computation of End Rotation in Basic Beam.—From Fig. 6, using the method of elastic weights,

$$R_e = \theta_A = \theta_B = \frac{\phi_1 + 2\,\phi_1}{2} \times \frac{L}{2} + \frac{2}{3} \times \frac{1}{6}\,\phi_1 \times \frac{L}{2}$$

$$= 40 \times 12 \left(\frac{3\,\phi_1}{2} + \frac{1}{9}\,\phi_1 \right) = 773\,\phi_1 \quad \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

in which R_e is the elastic reaction of the angle-change loading.

Step 4. Computation of End Rotation of Basic Beam Loaded with $\frac{M}{E\,I}$-Diagram.—Since in beams with fixed ends there can be no end rotation, it is necessary to apply moments at each end of the beam large enough to rotate the tangents back through the angles θ_A and θ_B. Since θ_A is equal to θ_B, the applied moments are equal. In this step, $\theta_A = \theta_B$ equals the elastic reactions of a beam loaded with the $\frac{M}{E\,I}$-diagram, or

$$\theta_A = \theta_B = \frac{M}{E} \sum_{0}^{40} \frac{\Delta s}{I_s} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

in which Δs = the length of each deck division for use in the summation = 60 in. The summation equation (Eq. 5) is necessary since the moment of inertia of the deck is not constant. The moment of inertia of each segment I_s is equal to

$$I_s = \frac{1}{12}\,b\,(d')^3 + A_s\,(n - 1)\,\frac{d_s^2}{4} \quad \ldots\ldots\ldots\ldots\ldots\ldots (6)$$

in which b is assumed to be a transverse 12-in. strip of the deck; d' is the total depth of the section; A_s is the cross-section area of the steel reinforcement; $n = \frac{E_s}{E_c}$ = the ratio of the moduli of elasticity; and d_s is the distance between compression and tension steel, assuming neutral axis of section halfway between steel bars. Values of the ratio $\Delta s/I_s$, for use in Eq. 5, are:

Segment No.	$\frac{\Delta s}{I} \times 1{,}000$
1	0.325
2	0.480
3	0.712
4	1.063
5	1.450
6	1.900
7	2.280
8	2.295
$\sum \frac{\Delta s}{I} \times 1{,}000$	10.505

Step 5. Equating Steps 3 and 4 to Find Fixed-End Moment.—Combining Eqs. 4 and 5, $\frac{M}{E} \times \frac{10.505}{1{,}000} = 773\ \phi_1$; and

$$M = \frac{773\ \phi_1 E}{0.0105} \qquad (7)$$

If $\phi_1 = 0.000002 = 2 \times 10^{-6}$ radians and $E = 3 \times 10^6$, the fixed moment for the deck is $M = \frac{773 \times 2 \times 10^{-6} \times 3 \times 10^6}{0.0105} = 450{,}000$ in-lb.

Step 6. Distribution of Fixed-End Moments.—In this step the Hardy Cross method of moment distribution is used to find the final moments at the ends of the deck of each span, the stiffness and carry-over factors for the beams of varying moments of inertia being as shown in Fig. 7.[9] Accordingly, the final resisting moments in span BC, Fig. 7 (symmetrical about the center line) are:

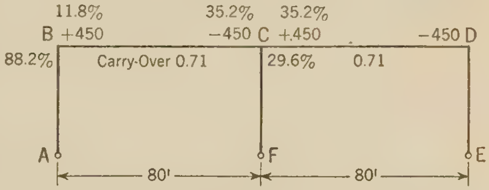


FIG. 7.—FIXED-END MOMENTS READY FOR DISTRIBUTION, SHOWING STIFFNESS FACTORS AND CARRY-OVER FACTORS

Description	End A	End B
Inch-kips	397.0	488.0
Foot-kips	33.1	40.6
Direction	Clockwise	Counterclockwise

The moment at the crown of span BC is the average of the end moments, or knee moments, plus the thrust at end B, multiplied by its arm to the center line of the deck at the crown: $\frac{33{,}100 + 40{,}600}{2} + \frac{33{,}100}{15.83} \times 1.28 = 36{,}850 + 2{,}670 = 39{,}520$ ft-lb.

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[9] "Analysis of Rigid Frame Concrete Bridges," Portland Cement Assn., 11th Ed., 1936.

INTERPRETATION OF RESULTS

The moments M_{BC} and M_{CB} at the ends of the deck, caused by radiant heat, act in a direction opposite to the primary dead-load moments and live-load moments. Therefore, radiant heat causes no serious effects at these points. At the crown, however, the radiant-heat moment of 39,520 ft-lb acts in the same direction as the primary moments. The design moment at the crown for this particular frame included the following moments:

Description	Foot-pounds
Dead load	37,120
Live load	25,340
Impact	6,200
Shrinkage and movement of footings	25,500
Temperature change (−45° F) from a mean value	17,160
Total design moment	111,320

For this frame the radiant-heat moment at the crown equals $\frac{39{,}520}{68{,}660} \times 100$ or 57.5% of the primary moments, and $\frac{39{,}520}{111{,}320} \times 100$ or 35.5% of the total design moment. This represents a moment of considerable magnitude, and one which would necessitate the provision of additional steel were it not for the compensating seasonal temperature changes. The following relationships exist between these seasonal changes and the theoretical radiant-heat moment: (*a*) The maximum design moment at the crown includes the effect of a 45° F drop in temperature; and (*b*) the remaining 56% of the radiant-heat moment and its corresponding fiber stress effect are further reduced by a seasonal temperature rise.

(*a*) *Effect of a 45° F Drop in Temperature.*—The maximum design moment at the crown includes the effect of a 45° F drop in temperature. Accordingly, in this design there is steel and compressive area to resist 17,160 ft-lb of moment which would not occur on the days when the effect of radiant heat is greatest. This material would be available for $\frac{17{,}160}{39{,}520} \times 100$ or 44% of the radiant-heat moment at the crown.

(*b*) *Effect of a Seasonal Temperature Rise.*—The remaining 56% of the radiant-heat moment and its corresponding fiber stress effect are further reduced by a seasonal temperature rise. On the days when the radiant heat causes the greatest strains, the deck is probably elongating due to the rise in seasonal temperature. In addition, the axial strains in the deck, due to the radiant heat itself, increase the length of the deck somewhat. The effects of radiant heat and axial elongation counteract each other. Exactly how much the remaining 56% or 22,000 ft-lb is reduced is difficult to evaluate since radiant-heat stresses occur in short cycles whereas seasonal temperature elongation is a long-range process. It is possible that the reduction could be from a small amount on certain days to an amount corresponding to the maximum seasonal temperature rise. In this particular design, if one of the radiant-heat cycles occurred on the same day that the full seasonal elongation was effective, the reduction would be 17,120 ft-lb. The axial elongation accompanying the radiant-heat strain distribution is also difficult to compute because there is both elongation and contraction of the cross section as shown in Fig. 5(*b*).

CONCLUSIONS

The most definite design procedure to provide for the effects of radiant heat in a rigid frame is to include in the maximum design moment at the crown the moment due to a sizable seasonal temperature drop (−45° F or more for frames comparable in length to the one just analyzed). This should be done in states with warm climates, where radiant heat is likely to be high, even though the actual temperature drop is moderate.

The assumptions made in computing the heat flow through the slab due to radiant heat, in this paper, are on the conservative side. For example, the heat was assumed to flow in one direction only—downward into the slab. The effect that the main longitudinal steel may have on the temperature distribution was not taken into account. Temperature and shrinkage steel at right angles to the main longitudinal steel would also probably absorb some of lateral heat strain which surely exists. Practical measures, such as expansion joints in the slab itself, would relieve some of the strain. However, the magnitude of the moments that were found with the assumptions made should indicate a need for further study and discussion of this type of temperature stress. Possibly more uniform seasonal temperature change specifications will result.

Six conclusions seem reasonable on the basis of this paper:

1. The radiant-heat moments at the knee sections of the rigid frame would not affect the maximum combination of design moments as they act in a direction opposite to the dead-load and live-load moments.

2. The radiant-heat moment at the crown of the particular frame analyzed in this paper is equal to 35.5% of the maximum design moment and acts in the same direction as the dead-load and live-load moments.

3. If the maximum design moment at the crown includes the moment caused by a substantial seasonal temperature drop (45° F or more from a mean value), steel will be available for the radiant-heat stresses, as this temperature drop will not coincide with maximum radiant heat. Material was present in this design to account for 44% of the radiant-heat moment, corresponding to a 45° F drop in seasonal temperature.

4. Axial strains due to seasonal temperature rise and radiant heat itself reduce the effect of the radiant-heat moment, although it is difficult to ascertain exactly how much on any given day or radiant-heat cycle.

5. The magnitude of radiant-heat moment at the crown should warrant serious consideration of seasonal temperature specifications, particularly temperature drop, in the design of long-span reinforced-concrete rigid frames.

ACKNOWLEDGMENT

Material from which this paper was prepared was taken from a thesis by the writer, entitled "The Effect of Temperature Stresses in a Reinforced Rigid Concrete Frame," presented in 1942 to the University of Minnesota, at Minneapolis, in partial fulfilment of the requirements for the degree of Master of Science. Constructive criticism and guidance in the preparation of the thesis were given by Prof. C. A. Hughes, formerly of the structural department at the University of Minnesota.

DISCUSSION

I. OESTERBLOM,[10] M. AM. SOC. C. E.—Had the author used a more restrictive title his paper would have had mathematical interest, limited, however, to the extent that it is based on the conventionally assumed (but approximate) relations between stress and strain. The paper does not seem to cover the subject of radiant heat as applied to frames.

Assuming that radiant heat comes from the sun, the investigator must begin with the basic radiation formula:

$$H = \sigma\, \epsilon_1\, \epsilon_2\, dA_1\, dA_2 \frac{\cos O_1 \cos O_2}{\pi L^2} (T^4_1 - T^4_2) \ldots\ldots\ldots\ldots (8)$$

Eq. 8 defines the quantity H that is radiated from dA_1 to dA_2 with radiating temperatures T_1 and T_2 (absolute), respectively. The symbols ϵ_1 and ϵ_2 represent the emissivities and σ represents the Stefan-Boltzmann constant. The angles of incidence (the symbols O_1 and O_2) may be set equal to zero for solar radiation, if a perpendicular plane is assumed at the earth.

With what is known about dimensions, distances, and temperatures, Eq. 8 gives a theoretical maximum value of the heat radiation on earth of about 400 Btu per sq ft per hr for any perpendicularly exposed area of black body emissivity—when no atmospheric interception occurs before the rays of the sun reach this area. Usually only 300 Btu is taken as the reduced maximum.

Some of this heat is absorbed, some is re-radiated into cosmic space to $-50°$ F, and some is re-radiated into the atmosphere. When the atmosphere is heavy only a part of the 300 Btu can reach the earth. Much of that which arrives is re-radiated to the heavily absorbent atmospheric moisture.

Then there is also the angle of incidence which, for construction planes, generally is not perpendicular to the rays of the sun. Here again the effect is very much reduced through the low value of O_2. In the middle of the day only roofs are heavily affected; and walls are very lightly affected. During the forenoons and afternoons even the walls are spared because the atmospheric penetration distance is so much extended. The over-all, all-day, picture thus is not a bad one.

The temperature of the air adjacent to any exposed area is now a function of the residual heat which is not re-radiated. It is not a function of that alone, however. The heat that has penetrated the atmosphere on its way from the sun must equal that which has been re-radiated into space, plus another portion which the construction is able to absorb and store or pass through to the interior. The second part must pass through the all-important surface film, a concentration of air molecules with definite thickness and temperature gradient which depends on the thermal balance and the convection velocity for its magnitude.

Even assuming that the residual heat is known, the establishment of a balance between the film and the interior is very difficult. Mass, and the shape of the construction, affect both heat storage and conduction, according to the Fourier laws, and the film is governed by the radiation and convection laws. Thus one must resort to guesses about film temperatures; but this can be done only if, afterward, one proves that the guesses were reasonably correct and that they actually apply to both film and construction.

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[10] Engr., Carbide & Carbon Chemicals Corp., South Charleston, W. Va.

The author has assumed surface temperatures as reported by Frank B. Rowley and C. E. Lund of the University of Minnesota,[6] Minneapolis, Minn. One would like to know how high those temperatures were, how they varied during the day, how the surface was located, and what mass and form were below the exposed surface. This information would be useful not only to follow the author's reasoning, but also to enable an engineer to repeat the method for other cases, if it seems good enough. It is quite possible that, in the light of the foregoing argument, the "comparable surface" (see heading, "The Effect of Radiant Heat") computed in the "Numerical Example" is far different from the one that the author asks his readers to accept on faith. The proposed "comparable surface" is quite insufficient to establish reasonable similarity.

In setting up his numerical example, the reason for selecting a slab is not clear. In a thermal sense a frame element is not a slab, and in Fig. 2 it is not shown as such. Reference to the work of Roy W. Carlson,[7] Assoc. M. Am. Soc. C. E., and his (no doubt) very specialized and seemingly graphical solution is also questionable. It would be almost impossible to extend any graphical solution to a two-way, or a three-way, flow of heat, and that is the type of problem encountered in a frame element protected by walls. For an exposed frame, involving a four-way flow of heat, the possibility of extending a graphical solution is even more remote.

Ever since J. B. Fourier[11] first raised the question of flow and absorption of heat this problem has been treated extensively by various authorities, generally physicists and mathematicians (Laplace, Poisson, Green, Lamé, Heine, Liouville, Thompson (Lord Kelvin), and Riemann) and more recently also by engineers (Gurney, Heilman, King, McAdams, McMillan, Newman, Preston, Schack, Schmidt, and others). However, Fourier's theory does not take into account heat used for evaporation of water—free, adsorbed, or crystallic—which is present in most construction materials. There is a very definite and well-established procedure, therefore, by which to compute the thermal changes in bodies of various shapes.

A homogeneous section must be assumed (the problem is exceedingly difficult even so) and, therefore, reinforcement cannot be included in the setup. However, this fact is not important. The conductivity of steel is so high in relation to the heat-storage capacity that the disturbing influence through a very high diffusivity acting on the small area of reinforcement is insignificant.

The Fourier equation for heat flow through a prism—with the use of conventional symbols and the coordinate axes at one of the corners—reads as follows:

$$\alpha \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) = \frac{\partial u}{\partial \tau} \qquad (9)$$

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[11] "Théorie du mouvement de la chaleur dans les corps solides," by J. B. Fourier, l'Académie Française, 1812.

in which the thermal diffusivity is given by α, the same as in the paper. Symbol τ denotes time, and

$$u = \frac{t_1 - t}{t_1 - t_o} \qquad (10)$$

is the temperature function, which expresses how far, in temperature increase, the point x,y has gone as compared to the ultimate possible change, when the uniform mass at temperature t_o has reached the surface temperature t_1 at that point.

The Fourier equation (Eq. 9) is for a two-way (or a four-way) flow of heat along the x-axis and the y-axis—in other words it applies to a rectangular prism only. By dropping one of the axes and making the equation apply to a slab, the solution becomes greatly simplified.

One can thus write, in trigonometric form:

$$S_x(\theta_x) = \frac{4}{\pi}\left[e^{-(0.5x)^2\theta} \sin\frac{\pi x}{2\,b_x} + \frac{1}{3}\,e^{-9(0.5x)^2\theta} \sin\frac{3\,\pi x}{2\,b_x} + \frac{1}{5}\,e^{-25(0.5x)^2\theta} \sin\frac{5\,\pi x}{2\,b_x} + \frac{1}{7}\,e^{-49(0.5x)^2\theta} \sin\frac{7\,\pi x}{2\,b_x} + \cdots\right] \qquad (11)$$

Values of $S_x(\theta_x)$ from Eq. 11 may be tabulated and charted so as to obtain a quick solution of specific problems. The symbol θ_x is defined as:

$$\theta_x = \frac{\alpha\,\tau}{b^2_x} \qquad (12)$$

in which b_x is the distance from the outer surface to the thermal center of the slab; and, as in Eq. 10:

$$S_x(\theta_x) = \frac{t_1 - t}{t_1 - t_o} \qquad (13)$$

The solution has been presented by many authorities following parallel methods. An outline derivation of Eq. 11 is offered by T. K. Sherwood and C. E. Reed[12] who in turn refer to H. S. Carslaw[13] for further details. The derivation of Eq. 11 has been presented by F. C. W. Olson and O. T. Schultz. All writers make a general acknowledgment to other helpful authorities.

For reintroducing the y-axis and finding the following simple solution for a two-way heat flow—

$$S_x(\theta_x)\,S_y(\theta_y) = u \qquad (14)$$

—credit is due especially to Albert B. Newman[14] who presented a proof that Eq. 14 is true. Thus, by taking two infinite slabs perpendicular to one another and finding $S(\theta)$ for each at a common point (x,y), the investigator is able to establish first the temperature function u (which is common to the two) and then the temperature for the point; and by solving Eq. 10 for the temperature t:

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[12] "Applied Mathematics in Chemical Engineering," by T. K. Sherwood and C. E. Reed, McGraw-Hill Book Co., Inc., New York, N. Y., 1939.

[13] "Mathematical Theory of the Conduction of Heat," by H. S. Carslaw, The Macmillan Co., London, 1921.

[14] "Heating and Cooling Rectangular and Cylindrical Solids," by Albert B. Newman, *Industrial and Engineering Chemistry*, May, 1936, p. 546.

$$t = t_1 - u\,(t_1 - t_o) \qquad (15)$$

Various graphs for the determination of $S(\theta)$ have been produced by H. P. Gurney and J. Lurey[15] although the logic supporting them has never been published. By the use of such special graphs the problem of a rectangular prism thus becomes quite simple: (1) Establish the relative location of the point x,y in terms of b_x and b_y; (2) indicate the diffusivity by $\alpha = \frac{k}{c\,\rho}$; (3) find the two values of θ by Eq. 12; (4) find the values of $S(\theta)$ by the charts; (5) combine the two values of $S(\theta)$ as shown by Eq. 14; and (6) find the temperature of the point by Eq. 15.

The complication introduced by variable surface temperatures now enters, and a problem already difficult (to find a rational solution such as Eq. 11) becomes almost impossible. The variable boundary conditions would make it impossible, and even if that were not so the variability cannot be expressed directly and rationally.

However, it is possible to assume that the average surface temperature is constant for the duration, and to compute the interior temperatures on the basis of this approximation. One would then be ready for the structural problem of stress, as soon as enough critical points had been computed. The effect of the thermal stress would be equivalent to a supplementary axial eccentric load.

For a section away from the walls one would have the top exposed and subject to high temperature. For a section next to a wall the top and one side would have higher temperatures than the inside faces. In all cases there would be set up an extra stress covering the entire area and not (as shown in Fig. 5) applied above the neutral axis only.

Fig. 5 is based on conventional approximations regarding stress and strain, and that is scarcely permissible when one ventures into a new field. The true answer must be ascertained first, and only then may one be allowed to look for reasonable simplifications. Serious consequences have followed opposite procedure. Did the London Bridge fail in 1832 because of the overextension of a method? Most likely it did not, because, when it was built, one did not know even the basic method; but the second Quebec Bridge fell—due to a fateful extrapolation of a limited formula—in 1916. Because of his misapplication of the basic thermal theory, as well as the original stress diagram due to load, the author should recompute his example so that comparable figures would be available as an illustration. Only the author can do this, on the basis of the assumed surface temperatures used in the example, but not stated in the paper. The stresses thus computed certainly would be substantially less than those reported in the paper. Indeed, if it were not for this probability there would have been many failures of many famous structures due to thermal stress.

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[15] *Industrial and Engineering Chemistry*, 1923, p. 1170.

The integration of stress into deflections and moments is elementary, and many methods might be used for this purpose. The revision proposed in these notes will cause a surprise, however. A frame next to an outer wall would show lateral deflection and, as a result of that deflection, torsional shears would occur next to the supports. A point that is often overlooked should be given thought in this connection: One cannot assume an average temperature, arbitrarily, as a basis for computing temperature stresses. The basic temperature of the frame (the atmospheric temperature plus something added for chemical heat) is the one at which concrete takes its initial set when poured. The chemical heat is a relatively small item on ordinary frames but a heavy item on heavy construction. Cold weather construction thus gets a heavy, extra load during the summer and hot weather construction is subject to a considerable reduction.

The writer would like to summarize his criticism as follows: "David" went out to slay a "Goliath," but he did not search first for a slingshot, and thus this "Goliath" is still very much alive. Nevertheless, the entire profession should be highly grateful for the paper, because the author has ventured into highly dangerous and difficult territory which should have been explored extensively but which all research engineers have feared to enter. He may indeed have stirred into action many thoughtful but hesitant engineers who have constructive ideas to present.

It is not solar radiation alone that should be considered; neither is the concrete frame the most important structure. There are other sources of heat; and there is also fire, accidental fire. When one gives thought to this vast field it comes to mind at once that so little has been done to produce a safe design. Why? Because so little is known, or expressed in rational form.

This is true even in the important field of fireproofing. Suppose that a 3-in. fireproofing cover is safe for a given steel column. The same covering may be applied to another section where similar heat intensity is expected; and the column fails nevertheless because of a fanciful curiosity of shape and texture. Why? Simply because the diffusivities of compounded materials in relation to the areas they cover, and the flow paths they present, offer unlimited combinations of time and temperature, for which no one has ever found a rational formula.

N. M. NEWMARK,[16] ASSOC. M. AM. SOC. C. E.—So much essential information has been omitted from this paper that it is difficult to follow the author's reasoning, particularly in those places where mistakes seem obvious.

The curves shown in Fig. 4 for the temperatures at various times in two different sections of the deck have the correct general shape, but the numerical values are grossly inconsistent. Since the temperature rise does not extend very far into the slab in either case, it seems unreasonable that the variation of temperature with depth should be different for the two sections. One should expect to have the same temperatures at equal depths, and, moreover, these temperatures should be the same as in an infinitely deep slab at the same depth from the surface. One suspects from the figure that the author used increments of depth of different lengths for the two sections, and obtained different curves by reason of inaccuracies in his numerical procedure. If this is the case, the lower set of curves should be more dependable, and should apply to all sections of the bridge.

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[16] Research Prof., Civ. Eng., Univ. of Illinois, Urbana, Ill.

The explanation of the computation of angle change in step 2 is not easy to follow. The writer does not understand why the relatively tedious computation of Table 1 is necessary when the angle changes can be written directly if one knows the area of the depth-temperature strain curve and the position of its centroid. Let the area of the depth-temperature strain curve be denoted by Q, and let the distance from the top of the slab to the centroid of the area be denoted by y_0. Then the angle change ϕ is given by the equation

$$\phi = \frac{Q\,(c - y_0)}{I} \quad \ldots\ldots\ldots\ldots\ldots\ldots (16)$$

in which c is the distance from the top surface to the neutral axis of the section.

In addition, the direct strain at the neutral axis, ϵ_c, from which the axial elongations may be determined, is given by the relation

$$\epsilon_c = \frac{Q}{A} \quad \ldots\ldots\ldots\ldots\ldots\ldots (17)$$

in which A is the area of the cross section. Values of Q and y_0 were estimated from the lower curve in Fig. 4(*b*) with some difficulty since the scale of the drawing is small. The results were: $Q = 8.9\ \alpha$ ft-degrees and $y_0 = 0.24$ ft. The straight-line distribution given in the figure as an equivalent strain distribution leads to slightly different results—namely, $Q = 8.0\ \alpha$ ft-degrees and $y_0 = 0.21$ ft.

The latter set of values is used in the further calculations below, where it is assumed that these quantities are the same for all sections, in contradistinction to the author's assumptions. Angle changes for sections 1, 5, and 9 are computed in Table 2. With $\alpha = 0.0000065$ ft per ft per degree, the value of Q becomes 0.000052 ft. It will be noted that only the angle change at section 9 checks the author's value, since the author used different strain-distribution curves for the other sections. Incidentally, the units for angle change as given in the paper should be radians per inch. The author's diagram for angle-change loading is changed somewhat. The final values for fixed-end moments at B and C, estimated from the physical constants tabulated, are approximately +32.5 ft-

TABLE 2.—ANGLE CHANGES FOR SECTIONS 1, 5, AND 9

Description	Section 9	Section 5	Section 1
Depth d, in ft	2.27	2.75	4.53
Moment of inertia I, in ft⁴ per ft, $= \frac{d^3}{12}$	0.98	1.74	7.8
Distance to neutral axis $c = \frac{d}{2}$	1.14	1.38	2.27
$c - y_0$	0.93	1.17	2.06
Angle change ϕ, in radians per ft (Eq. 16)	0.000049	0.000035	0.000014
Angle change ϕ, in radians per in.	0.0000041	0.0000029	0.0000011
Strain ϵ_c at the neutral axis, in ft per ft (Eq. 17)	0.000023	0.000019	0.000011

kips instead of +37.5 ft-kips as given by the author (using the designer's convention where positive moment indicates compression in the top fibers).

The effect of the elongations at the neutral axis may be estimated as follows: The average strain is approximately 0.000018. This corresponds to an elongation of the neutral axis of 0.0014 ft. In order to bring the footings back to their proper position, rotations of joints B and D (Fig. 7) are required, of magnitude $\frac{0.0014}{15.84} = 0.00009$ radian. These correspond to fixed-end moments at B and C of magnitudes −7.4 ft-kips and +5.3 ft-kips, respectively, using the designer's convention.

With the author's numerical constants, the final moments in span BC are computed, as shown in Table 3. The final values are considerably less than those given by the author.

In interpreting these results, one factor completely neglected by the author must be taken into account. This factor is the stress due to the internal distortions caused by the nonuniform temperature. For example, if the deck were completely unrestrained at the ends, there would still be a statically balanced stress distribution at every cross section, corresponding to the conditions shown in Fig. 5(*b*). At section 9 the compressive strain at the top fiber, even if no moment acted at the section, is determined from the quantities in Table 2, as follows: $\epsilon = 27\,\alpha - 0.000023 - 0.000049 \times 1.14 = 0.000176 - 0.000023 - 0.000056 = 0.000097$. The neglected compressive stress is, therefore, 291 lb per sq in. There is also a compressive strain at the bottom of the section, of magnitude 0.000079 in. per in., and tensile strains near the middle of the section. These stresses or strains are to be added to the others computed, and are of the same order of magnitude.

TABLE 3.—FINAL TEMPERATURE MOMENTS, IN FOOT-KIPS (PLUS INDICATES COMPRESSION AT TOP FIBER)

Description	Point B of BC	Crown of BC	Point C of BC
Neglecting axial elongation	+28.7	+34.3	+35.2
Due to axial elongation	−8.5	−2.6	+4.7
Total moments	+20.2	+31.7	+39.9

The net effect in this structure is almost, but not quite, the same as if the only stresses introduced by the radiant heat were the compressive stresses near the top of the deck required to eliminate the temperature strains near the top. This is not unreasonable because of the great restraint given the deck by the short stiff columns.

The foregoing comments are made in order to clarify the method of calculation which may perhaps find utility for other problems. However, the writer would like to emphasize that the stresses due to temperature should not be considered in the same light as stresses due to loads. Temperature strains are fixed by the temperature conditions, but the stresses produced thereby are reduced when the load stresses are high since the effective modulus of elasticity of the material is reduced for high stresses. In a structure made of concrete or of structural steel, temperature stresses are generally secondary stresses, and do not ordinarily greatly impair the capacity of the structure to carry overloads, although at working loads the stresses may be fairly high. The question raised by Mr. Johnston—whether the temperature effects can always be neglected—is an important one. Certainly in the structure he has considered, the radiant-heat effects do not appear to be serious.

L. C. Hollister,[17] Assoc. M. Am. Soc. C. E.—The effects of radiant heat on reinforced-concrete rigid frames, as outlined in the paper, are very interesting. Most engineers have been aware that, in some climates, the differential temperatures within a member of a concrete frame could be rather large. However, they have not bothered to attempt an analysis of such stresses; the paper, therefore, should be a valuable contribution.

Mr. Johnston finds, for the example used, that stresses due to radiant heat amount to 35.5% of the total design moment. This is astonishingly high, and indicates that designers should not completely ignore the possibility that such a condition exists.

The California State Highway Bridge Department has built many so-called rigid frame structures. It is the practice of this department to design for 35° temperature rise and 45° temperature drop in the colder parts of the state, and 30° rise and 40° drop in the more moderate areas. Stresses due to shrinkage, equivalent to about 30° drop in temperature, are also considered. Spans for the reinforced-concrete rigid frames built in California vary from moderate lengths to 130 ft. Some have quite wide roadways, and others have rather sharp skews, all of which tend to add to the intensity of temperature stresses. Some of these structures could be subjected to relatively severe radiant-heat stresses. For instance, the Sacramento and San Joaquin valleys of California have large daily temperature ranges throughout the summer months. The daily change in temperature is from 30° to 50° and a change as high as 62° has been noted in some parts of the state. With an accumulation of heat on the surface of the slab, to 25° or 30° above air temperature, it would not be unreasonable to expect the differential temperature from top of slab to bottom of girder to be around 60°. This is approximately twice that used in the "Numerical Example" of the paper. These structures are inspected frequently and no signs of distress that might be attributed to temperature have been noted.

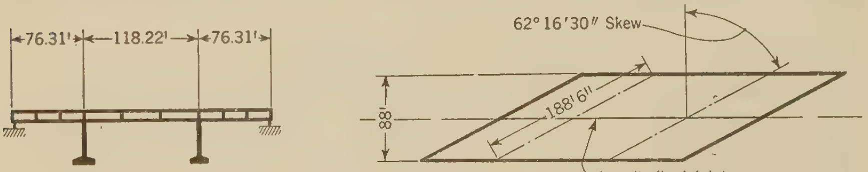

Fig. 8.—Bridge Across the Arroyo Seco Channel Near Hough Street, South Pasadena, Calif.

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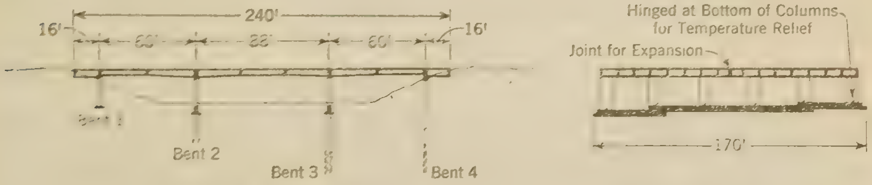
[17] Engr. of Design, State Bridge Dept., Sacramento, Calif.

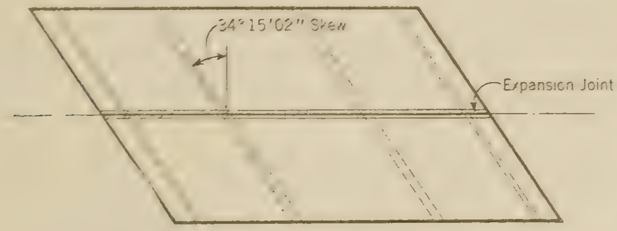
Some modern framed highway structures are becoming so wide and so severely skewed that stresses due to temperature are becoming increasingly important. For instance, several years ago the state constructed a reinforced-concrete frame structure with the dimensions shown in Fig. 8. The superstructure was provided with a longitudinal joint down the center; but the two main piers were one solid mass of concrete 4 ft 9 in. wide by 188 ft 6 in. long. Inspection reveals no distress in the superstructure that could be blamed on temperature or radiant heat. The piers, however, have cracks which are probably due to shrinkage. Other than this there appear to be no harmful effects of any kind.

Another example is afforded by Fig. 9. To relieve temperature stresses it was necessary to provide a longitudinal joint down the center of the superstructure, and also a joint in the cap of the pier at the center line. The bottoms of the columns are hinged to relieve temperature stresses.

It is regrettable that California does not have more specific data on the relative temperatures between top of slab and the interior parts of the girders, especially regarding some of the larger structures located in regions subject to high radiant heat.



(*a*) ELEVATION OF STRUCTURE (*b*) ELEVATION; BENTS 2 AND 3



(*c*) PLAN

Fig. 9.—Design of Undercrossing on Glendale Boulevard, Los Angeles, Calif.

The author has presented the paper in a clear, understandable manner, and outlines six well-defined steps necessary for the computation of radiant-heat stresses. The work of finding the angle changes at a given section would be somewhat more complicated for a T-beam or a box girder than for the example used; however, the same principle would apply.

Probably it will be a long time before designers are willing to go to the trouble of computing radiant-heat stresses as a regular practice. Nevertheless, it is well to have on record a method by which such stresses can be computed.

A. A. EREMIN,[18] ASSOC. M. AM. SOC. C. E.—That radiant heat has an important effect on the stresses in bridge frames, as stated by Mr. Johnston, is undeniable. However, the writer does not agree with conclusion 3, namely, that the radiant-heat effect on the maximum moment at the crown may be obtained by considering "a substantial seasonal temperature drop (45° F or more from a mean value)." The action of radiant heat is not the same as that of seasonal temperature variation. The primary effect of radiant heat is to rotate plane sections whereas the effect of seasonal temperature variation is to displace joints. Furthermore, the author's computation of the stresses resulting from radiant heat may be simplified so as to make unnecessary the substitution proposed by the author.

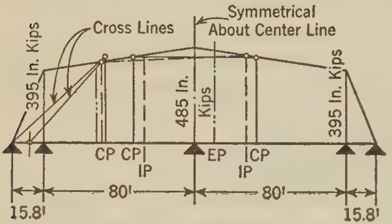


FIG. 10.—*M*-DIAGRAM

Radiant heat may be assumed to decrease uniformly along the slab section as shown in Fig. 5(*a*). Then, in the author's numerical example, the identical angular rotation of slab ($\phi = 2 \times 10^{-6}$ radians) may be obtained from Figs. 4 and 5(*a*) by assuming $t_f = 16.5°$. The end rotations of a beam may be expressed as

$$\theta_A = \theta_B = (e + e_0)\frac{L}{I} \dots\dots\dots\dots\dots\dots\dots (18)$$

in which the constants e and e_0 vary with the shape of member and are read from prepared diagrams.[19] For the slab in Fig. 2, $e = 0.168$ and $e_0 = 0.112$. Substituting the values of constants in Eq. 18, $\theta_A = \theta_B = 10{,}300$, which is very close to the value computed by the author.

Fixed-end moments induced by radiant heat may be distributed graphically by the use of "cross lines," as shown in Fig. 10. From the foregoing it is evident that the effect of radiant heat can be computed very quickly without the analogy proposed by the author. The best result is always obtained by a direct estimate of the cause.

MILAN A. JOHNSTON,[20] JUN. AM. SOC. C. E.—As Mr. Oesterblom states, the problem of flow and absorption of heat through solid bodies has been treated extensively by many authorities. Because of this fact and for the sake of brevity, the writer purposely omitted a detailed analysis of the equations for obtaining the heat flow of curves of Figs. 4(*a*) and 4(*b*) in the paper.

However, since Mr. Oesterblom is under the misapprehension that the temperature curves of Figs. 4(*a*) and 4(*b*) are the result of a graphical solution; and, since both he and Professor Newmark raise other pertinent questions regarding the assumptions made in determining the radiant-heat temperature flow, the writer welcomes this opportunity to discuss the subject more completely.

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[18] Associate Bridge Engr., Bridge Dept., Div. of Highways, State Dept. of Public Works, Sacramento, Calif.

[19] "Analysis of Continuous Frames by Graphical Distribution of Moments," by A. A. Eremin, Sacramento, Calif., 1943.

[20] Asst. Prof., Mech. Eng. Dept., Univ. of Minnesota, Minneapolis, Minn.

First of all, it did not occur to the writer that any reader would assume the paper was attempting to cover any other subject than radiant heat as applied to reinforced-concrete rigid frame highway bridges. The particular case illustrated in the paper was the "barrel type" concrete deck poured monolithically so as to act as an integral unit with the supporting legs. The width of the deck of the so-called rigid frame structure is so large in comparison with the thickness that it certainly can be regarded as a slab with the exception of the elements close to the edges and the ends. There will, of course, be heat loss through the outside vertical edge of the slab, but this local effect can be neglected for all practical purposes because of the aforementioned ratio of horizontal surface exposed to slab thickness. Mr. Oesterblom's objection to the use of a slab instead of the entire area of the rigid frame for the temperature load is pertinent to a treatise attempting to cover the subject of radiant heat as applied to a series of isolated or single rigid frames used for industrial structures, but seems to be irrelevant for the particular structure used in the numerical example of the paper.

Although Mr. Oesterblom's discussion of the variables and guesswork confronting the investigator attempting a theoretical computation of film and surface temperature rise due to radiant heat was interesting, it should be made clear that the writer was already aware of this fact. This was evidenced by the writer's statement that "A theoretical calculation of the surface temperature rise [of the deck] necessarily must take into account so many assumptions and variables that it would be not only approximate but impracticable."

In view of the aforementioned assumptions, experimental or actual readings of the hourly variations of surface temperature rise (above the air temperature) due to radiant heat were used as a known quantity in the heat transfer equations for tracing the flow of heat to the interior of the slab.

The surface readings were taken on a horizontal 6-in.-thick concrete slab, with top dimensions of approximately 4 ft by 6 ft. The slab was blocked up to waist height to allow free passage of air under it. Readings were taken by thermocouples embedded at the surface of the slab. The hourly rise of the surface temperature above the air temperature can be seen by inspection of Fig. 4 of the paper. Thus, at 9 a.m., the temperature of the concrete surface had risen 5° F above air temperature; at 10 a.m., 15° F; at 11 a.m., 23° F; etc. At 1 p.m., a maximum of 27° F above air temperature was recorded. These readings were the average of readings taken on two consecutive hot days.

Although it is not claimed that these readings are the same as would occur on actual rigid frame decks in the field, it does seem reasonable to assume that the readings are an indication of the magnitude of surface temperature variation which can occur. Hence, the statement of "comparable surface," to which Mr. Oesterblom voiced an objection.

The equations for tracing the heat flow into the interior of the slab, using the aforementioned experimental hourly surface values as known variables, apply to one-way heat flow through a slab. Their solution is an algebraic, not a graphical, one.

Professor Newmark is correct in assuming that the temperatures in Fig. 4 should be the same at equal depths for the several sections of the deck. A projection of the computed points (shown as small circles) at depth 1.13 ft, section 1, Fig. 4(*a*), down to the curves interposed between the computed points at depths 1.15 ft and 0.58 ft section 9, Fig. 4(*b*), shows the temperatures to be the same opposite depth 1.13 ft, section 9. Similarly, computed points at depth 0.58 ft, section 9, will intercept approximately the same temperatures on the curves of section 1 at 0.58 ft. Unfortunately, the ordinates or depths of the two sections were drawn to different scales in Fig. 4, causing other points on the interposed curves between the computed points to be somewhat in error.

The equations used to compute the temperatures at each station will give equal temperatures at the same depth for each section. The only limitation is that the same number of temperature stations must be used for comparison between sections of different depths.

The writer is also indebted to Professor Newmark for the method of finding the angle change ϕ by Eq. 16. This method eliminates the necessity of approximating a straight-line temperature-depth strain relation, and will simplify the writer's angle-change computations, providing a planimeter is used to find the area Q under the maximum temperature-depth strain curve. Integrating to find the area Q under the curve becomes involved because of the complexity of the equation for the maximum curve.

Mr. Eremin's objection to conclusion 3 would be valid if the conclusion read as he restates it: "* * * that the radiant-heat effect on the maximum moment at the crown may be obtained by considering 'a substantial seasonal temperature drop' * * *." However, Mr. Eremin has evidently misinterpreted the meaning of conclusion 3 which actually reads, "If the maximum design moment at the crown includes the moment caused by a substantial seasonal temperature drop * * * steel will be available for the radiant-heat stresses, as this temperature drop will not coincide with the maximum radiant heat."

Conclusion 3 states, in effect, that both radiant heat and seasonal temperature drop create moments, and subsequently fiber stresses, at the crown which necessitate the addition of steel (or extra cross section). It does not imply that the analysis for obtaining radiant-heat moments is the same as the analysis for obtaining seasonal temperature moments.

Mr. Eremin's use of prepared diagrams to compare the end rotations of the basic beam is interesting. Most design offices do make use of prepared diagrams as labor and time-saving devices.

The author was pleased to learn from Mr. Hollister that the California State Highway Bridge Department designs for a temperature drop of from 40° F to 45° F. As was stated previously, the resulting steel from this design temperature is available to resist about 50% of the radiant-heat moment. Safety factors on the primary dead and live load stresses and other practical measures, such as expansion joints, will also help to eliminate temperature cracks.

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CORRELATING FLOOD CONTROL AND WATER SUPPLY, LOS ANGELES COASTAL PLAIN

By Finley B. Laverty,[1] M. Am. Soc. C. E.

With Discussion by Messrs. A. L. Sonderegger, Warren N. Thayer, George B. Gleason, and Finley B. Laverty

Synopsis

Depletion of ground-water supplies by the heavy draft of new war industries has reemphasized the importance of water conservation in Los Angeles County, California, and other arid sections of western United States. The scope of this paper is limited to a description of the use of flood control reservoirs to supplement normal surface water supplies as well as underground storage; a description of development of, and a presentation of data obtained from, extensive off-stream flood water spreading grounds used to replenish ground water by percolation; and data gathered on the quality of flood waters.

Flood control works in Los Angeles County consist of three principal types—dams, channels, and water spreading grounds. Each of these takes a part in the water supply of the Los Angeles Coastal Plain. Since its inception as a State District in 1916, Los Angeles County Flood Control District has carried on a program of development of flood control and water conservation works. At present this system includes 18 dams and 18 debris basins, controlling 447 sq miles of mountain watershed; 226 miles of permanently or temporarily improved flood channels; about 2,500 check dams; and 7 water spreading grounds located adjacent to some of these channels.

It is unusual to operate reservoirs for both flood control and water conservation. Yet this dual function is necessary—first, to cope with the intense storms which frequently occur in the Los Angeles area making flood control of prime importance during the storm season; and second, to meet the demands of a high degree of agricultural development in a naturally arid region by conserving water which would otherwise waste to the ocean. Consequently, the reservoirs are operated primarily for flood regulation during the period of the greatest likelihood of severe storms, which is from December into March.

Note.—Published in June, 1945, *Proceedings*. Positions and titles are those in effect when the paper or discussion was received for publication.

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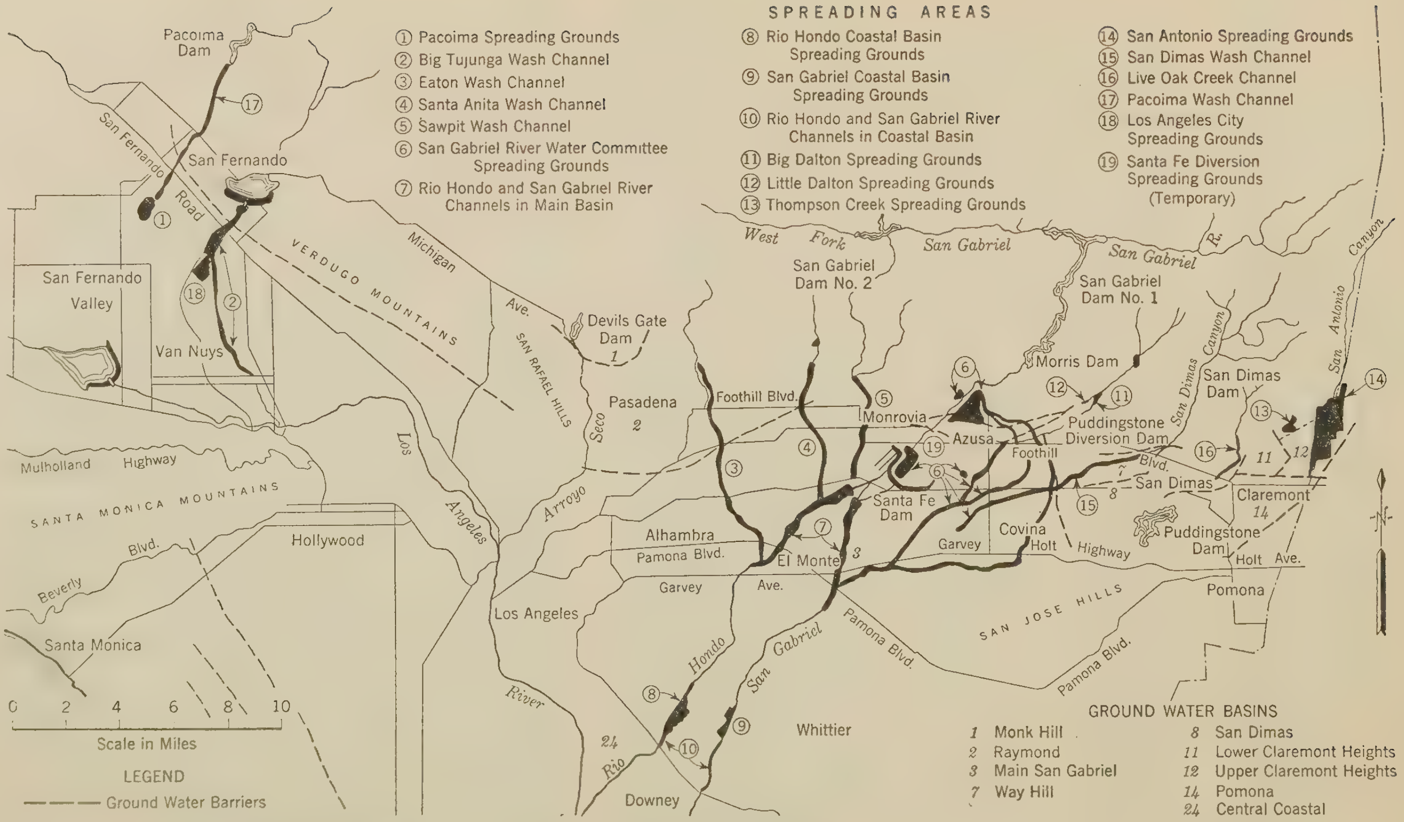

SPREADING AREAS
① Pacoima Spreading Grounds
② Big Tujunga Wash Channel
③ Eaton Wash Channel
④ Santa Anita Wash Channel
⑤ Sawpit Wash Channel
⑥ San Gabriel River Water Committee Spreading Grounds
⑦ Rio Hondo and San Gabriel River Channels in Main Basin
⑧ Rio Hondo Coastal Basin Spreading Grounds
⑨ San Gabriel Coastal Basin Spreading Grounds
⑩ Rio Hondo and San Gabriel River Channels in Coastal Basin
⑪ Big Dalton Spreading Grounds
⑫ Little Dalton Spreading Grounds
⑬ Thompson Creek Spreading Grounds
⑭ San Antonio Spreading Grounds
⑮ San Dimas Wash Channel
⑯ Live Oak Creek Channel
⑰ Pacoima Wash Channel
⑱ Los Angeles City Spreading Grounds
⑲ Santa Fe Diversion Spreading Grounds (Temporary)
GROUND WATER BASINS
1 Monk Hill
2 Raymond
3 Main San Gabriel
7 Way Hill
8 San Dimas
11 Lower Claremont Heights
12 Upper Claremont Heights
14 Pomona
24 Central Coastal
0 2 4 6 8 10
Scale in Miles
LEGEND
Ground Water Barriers
Pacoima Dam
San Fernando
San Fernando Road
San Fernando Valley
Van Nuys
VERDUGO MOUNTAINS
Michigan Ave.
SAN RAFAEL HILLS
Devils Gate Dam
Pasadena
Arroyo Seco
Mulholland Highway
SANTA MONICA MOUNTAINS
Beverly Blvd.
Hollywood
Santa Monica
Los Angeles River
Los Angeles
Alhambra
Pamona Blvd.
Garvey Ave.
Foothill Blvd.
West Fork San Gabriel
San Gabriel Dam No. 2
San Gabriel Dam No. 1
San Gabriel R.
Morris Dam
Monrovia
Azusa
Foothill Blvd.
Puddingstone Diversion Dam
San Dimas Canyon
San Dimas Dam
San Antonio Canyon
San Dimas
Claremont
Puddingstone Dam
Holt Ave.
Pomona
Santa Fe Dam
El Monte
Garvey
Covina
Holt
Highway
SAN JOSE HILLS
Pamona Blvd.
Pomona Blvd.
Rio Hondo
San Gabriel
Whittier
Downey

FIG. 1.—DAMS, GROUND-WATER BASINS, AND SPREADING GROUNDS, LOS ANGELES COUNTY FLOOD CONTROL DISTRICT, CALIFORNIA

During this period regulated releases often increase contributions to ground water from stream-bed percolation; and off-stream spreading grounds are operated whenever the quantity of water stored and the probable period between storms warrant constructing temporary diversion dams in natural river beds, or operating permanent diversion works in improved channels. After the middle of March, flood control regulating capacity is gradually encroached on until the middle of April when all available surplus stream flow is stored. This accumulated storage is used for direct domestic and agricultural supply. Any additional reservoir water is put into underground storage by percolation in channels and spreading grounds. In this manner the cycle is completed and the reservoirs are ready for flood regulation prior to the next storm season.

Pertinent data relative to the principal mountain reservoirs (Fig. 1), listed from west to east, are presented in Table 1. These data show that for years of plentiful water supply the storage capacity of the reservoirs is relatively small. This emphasizes the importance of utilizing the large underground reservoirs in the alluvial filled valleys below. The tabulation also shows that very little of the peak flows of severe storms can be detained for spreading.

TABLE 1.—MOUNTAIN RESERVOIRS OF THE LOS ANGELES COUNTY FLOOD CONTROL DISTRICT

Item No.	Dam (1)	Drainage area (sq miles)[a] (2)	Reservoir capacity to spillway (3)	ANNUAL RUNOFF			Maximum 24-hr runoff of record (7)
				Maximum (4)	Minimum (5)	Average[b] (6)	
1	Pacoima	27.8	4,840	38,000	630	8,090 (26)	6,120
2	Tujunga	81.4	4,240	76,400	800	19,600 (41)	24,870
3	Devils Gate	31.9	2,500	37,600[c]	80[c]	7,640[c] (29)	7,420
4	Eaton	9.5	630	14,000[c]	62[c]	1,930[c] (25)	1,590
5	Santa Anita	10.8	700	17,800	771	4,300 (26)	3,400
6	Sawpit	3.3	320	2,970	107	1,050 (12)	820
	San Gabriel—						
7	Dam No. 1	202.0	44,030	410,000	9,630	130,000 (48)	74,830
8	Dam No. 2	40.4	10,500	61,300	6,080	30,200 (8)	16,010
9	Big Dalton	4.5	950	3,270	41	1,000 (14)	890
10	San Dimas	16.2	1,070	12,500	485	3,680 (14)	3,400
11	Live Oak	2.3	230	818	0	325 (9)	310
12	Thompson Creek	3.7	610	1,100	0.3	228 (10)	570

[a] Except in Col. 2, all units are in acre-feet. [b] Averages are for the number of years shown in parentheses.
[c] Does not include the Pasadena Water Department diversion to direct use above the reservoir.

This paper will set forth: (1) Examples of direct use of flood waters conserved in the reservoirs; (2) water spreading practices in this area, and some of the problems encountered, together with percolation rates under various conditions; and (3) water analysis data based on samples taken from flood waters.

DIRECT USE OF STORED FLOOD WATERS

Practically all the reservoirs listed in Table 1 serve to increase the supply of water furnished by these streams for direct domestic or agricultural use. A few of these uses will be described (see also Fig. 1).

Pacoima reservoir, controlling a drainage area of 27.8 sq miles in the western portion of the Sierra Madre, lies above valuable agricultural and residential areas in the San Fernando Valley. Annually, 1,000 to 2,000 acre-ft of the stored flood waters are released to the stream and picked up by downstream diversions for irrigation of several hundred acres of citrus groves. Since the stream above the reservoir is dry in the late summer and fall months, this supply is of large value to the ranches which would otherwise have as their only local supply a very limited ground-water basin. Additional stored flood waters are percolated in the spreading areas downstream.

Devils Gate Dam controls a mountain drainage area of 31.9 sq miles including principally the Arroyo Seco northwest of Pasadena. Here the reservoir area is owned by the City of Pasadena which has constructed galleries under, and wells adjacent to, the reservoir bed for the purpose of extracting an urban water supply. The storage in the reservoir is the primary means of supplying the galleries. These galleries had functioned well for several decades prior to the forest fire and subsequent mud flows in the fall and winter of 1933–1934, but the organic matter carried by these mud flows so increased the carbon dioxide content that the reduced amount of water percolating to the galleries has since been useless. Recently the District has conducted sluicing operations, using stream flow to scour the overburden from the reservoir bed. Results indicate that further sluicing, particularly with the aid of improved facilities, will succeed in regaining this source of water supply.

San Dimas Dam, Puddingstone Diversion Dam, and Puddingstone reservoir offer the most complete conservation system so far constructed for direct water supply. Flood flows originating in the 19 sq miles of San Dimas Canyon are regulated by San Dimas reservoir and Puddingstone Diversion reservoir and diverted through a 2.7 mile concrete channel (capacity 3,500 cu ft per sec) into Puddingstone reservoir. This entire storage may be utilized for irrigating 2,700 acres which constitute one of the most productive citrus areas in Los Angeles County. This system made it possible to conserve the entire runoff of the heavy storm of March, 1938, amounting to 9,365 acre-ft, and also that of the storm of January, 1943, amounting to 6,835 acre-ft. This salvage created an asset worth at least $15 an acre-ft for irrigation use. Puddingstone reservoir has a capacity to spillway of 17,900 acre-ft. Subsequent use of this stored water in lieu of pumping has aided ground-water basins Nos. 7 and 8 (Fig. 1) to regain sufficient storage to cope with withdrawals during dry years. The rise in water table underlying this agricultural area, since 1938, has ranged from 65 to 175 ft.

San Gabriel Canyon flows are regulated by the Flood Control District's Dams No. 1 and No. 2, and by Morris Dam which creates a 36,000 acre-ft water conservation reservoir (owned by the Metropolitan Water District of Southern California). The flows from this and near-by tributary areas will eventually be controlled by the Santa Fe and Whittier Narrows reservoirs as part of the flood control program of the United States Engineer Department. The reservoir system in the San Gabriel Canyon mountain drainage area of 210 sq miles also furnishes the means of conserving the most important local source of water supply in Los Angeles County. The first 135 cu ft per sec of normal flow is allocated to direct diversion for irrigation of citrus lands. Of this flow 90 cu ft per sec also forms a steady winter-time source of power generation which seldom falls below 30 cu ft per sec during the summer months.

The remainder of the flows which can be salvaged (Table 1) are available (*a*) for storage as direct water supply, or (*b*) through the medium of pumping from underground basins after storing in these basins by means of channel and off-stream spreading ground percolation. Subsequent discussion will deal specifically with spreading on the San Gabriel River.

WATER CONSERVATION BY SPREADING

Extensive spreading ground development was first undertaken in Los Angeles County in 1906 at the mouth of San Antonio Canyon. Since then, the District has developed 1,300 acres, and other agencies 765 acres, of percolating area utilizing one or more methods.

General methods of water spreading practiced in Southern California are (1) the ditch and furrow method, (2) the basin method, and (3) percolation of regulated flows in natural stream beds. All three have modifications or variations, and often are used in conjunction with each other.

(*1*) *The Ditch and Furrow Method.*—This is generally used in areas of very rough or sloping terrain, where the water at times may have a heavy silt content. Canals and ditches are laid out roughly on contours with sufficient slope to prevent deposit of the suspended material, yet flat enough to prevent scour of the waterways. Due to variation of soils across the spreading area, the proper slope is often quite difficult to determine, yet may be corrected after construction by installation of check gates.

A modification of this method is to install main canals, as broad as possible and with shallow depth, from which smaller ditches at regular intervals lead the water out and spread it over the ground, much in the same manner as flood irrigation. Water not absorbed by the flooded area flows to the next lower main distribution canal by which it is again distributed to a series of smaller ditches and their flooding area.

Advantages of the ditch method and its variations are the low costs of maintenance and operation when the works are once correctly installed. When so constructed that the main channels may be flushed, this method will handle a greater silt content than any other type. An objection to this system is that the percentage of the total spreading grounds in direct contact with the water is very low, usually not more than 10% or 12% as compared with 75% or 80% for basins.

(*2*) *The Basin Method.*—In its simplest form this involves the construction of dikes or small dams at regular intervals across the abandoned natural channels on a debris cone or former river-bed areas outside of controlled channels. The water is led into the upper basin by means of a canal and from succeeding basins, as each fills, it spills to the next; or the canal may extend past and feed basins on each side. From the last basin the waste water is returned to the flood channel. The upper basins may serve as desilting ponds and prevent excessive silt from being deposited in the lower ponds. During recent years the basin method has been used predominantly due to its economic use of land.

(*3*) *Stream Channel Spreading.*—In some form this method is practiced on most major streams in Southern California. All these streams are subject

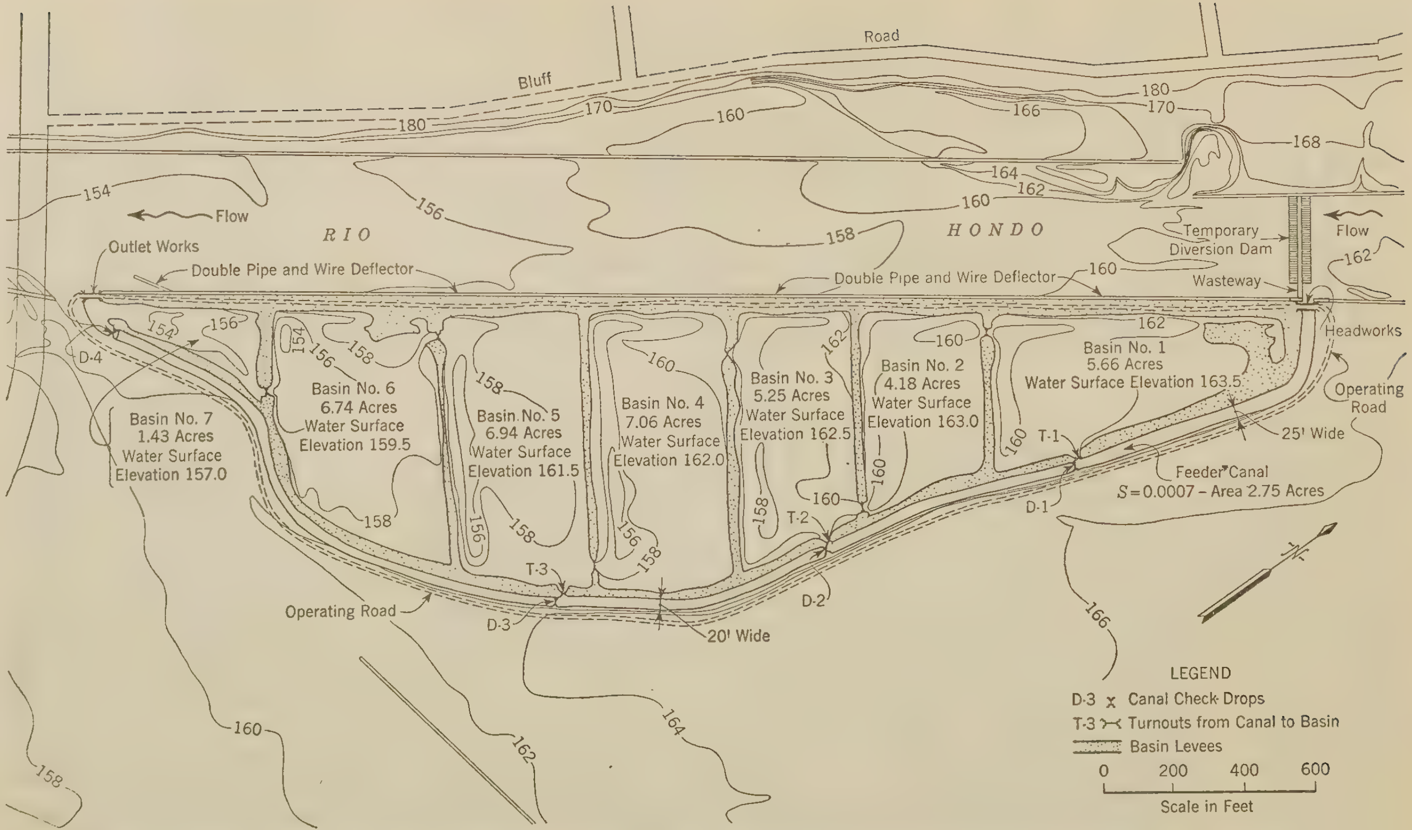

Road
Bluff
180
170
166
160
168
164
162
160
154
Flow
156
158
R I O
H O N D O
Temporary Diversion Dam
Flow
162
Outlet Works
Double Pipe and Wire Deflector
Double Pipe and Wire Deflector
160
Wasteway
Headworks
Operating Road
D-4
154
156
154
156
158
162
Basin No. 1
5.66 Acres
Water Surface Elevation 163.5
Basin No. 2
4.18 Acres
Water Surface
Elevation 163.0
Basin No. 3
5.25 Acres
Water Surface
Elevation 162.5
Basin No. 4
7.06 Acres
Water Surface
Elevation 162.0
Basin No. 5
6.94 Acres
Water Surface
Elevation 161.5
Basin No. 6
6.74 Acres
Water Surface
Elevation 159.5
Basin No. 7
1.43 Acres
Water Surface
Elevation 157.0
160
158
156
158
156
158
160
T-1
T-2
T-3
25' Wide
Feeder Canal
S=0.0007 – Area 2.75 Acres
D-1
D-2
D-3
20' Wide
Operating Road
166
164
162
160
158
LEGEND
D-3 x Canal Check Drops
T-3 ⤙ Turnouts from Canal to Basin
Basin Levees
0
200
400
600
Scale in Feet

FIG. 2.—INITIAL TEMPORARY DEVELOPMENT OF RIO HONDO COASTAL BASIN SPREADING GROUNDS UTILIZING BASIN AND CANAL SYSTEM

to flash flows of short duration, but of comparatively high rate. On the lower valley areas this condition results in a broad, flat, sandy bed which is dry during the summer months and during most winter months normally has a surface flow covering but a small portion of its area.

For several years conservation associations and county agencies have diverted the streams in these areas and spread them by various systems. Where the stream channels are composed of materials that tend to consolidate and seal off, the stream bed is broken up by tractors dragging cultivators or plows. In flat sandy stretches, the stream is diverted into plowed furrows. Temporary basins are also constructed by furrowing machines on these flood plains. In these ways water from a narrow meandering channel may be made to cover a considerable portion of a wide wash.

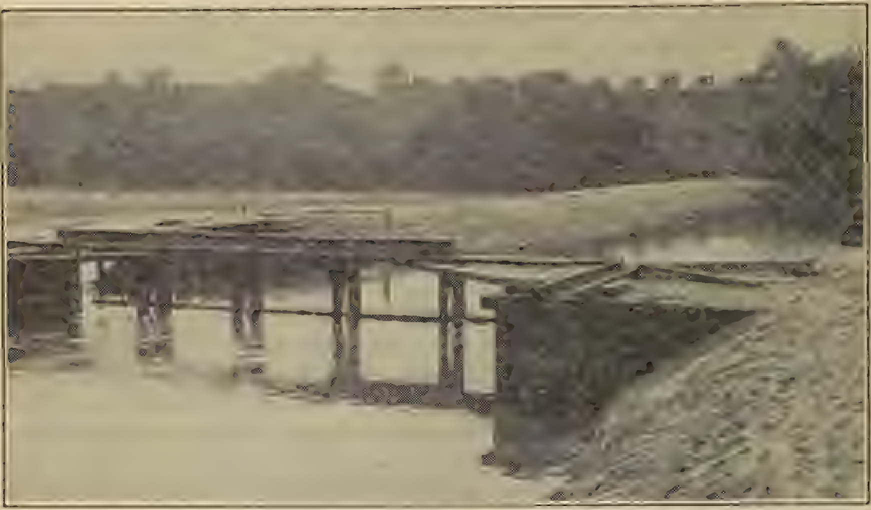
In Practice.—The general procedure followed in selecting spreading areas involves an initial study of ground-water use and supply. From these data and measurements of stream-flow percolation under existing conditions, the quantity of water which it would be desirable to handle in off-stream spreading grounds is determined. The possible locations of the spreading areas relative to geologic features of the ground-water basin concerned, ground-water level range, and general soil characteristics which would indicate the precolating capacity are studied. Promising areas are checked to depths of 30 ft or more by test holes to reveal the presence of clay lenses and other soil conditions which would have an important effect on the long-time spreading rate.

Following study of these data, if a local water supply of a few cubic feet per second is available, a small sample basin is often tested for a period of several weeks to obtain further data on percolation rates. When the most suitable set of conditions has been ascertained, the ground is purchased and developed on a temporary basis—that is, with wooden intake, canal and basin structures, and a temporary sand dam for diversion from the stream. This development is operated full scale until the data derived warrant revision or replacement of the structures.

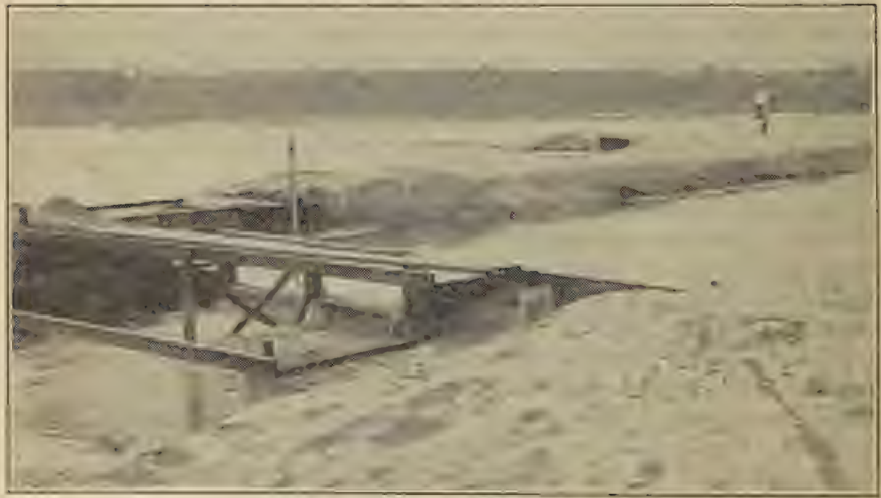
An initial 40-acre temporary development at the Rio Hondo Coastal Basin spreading grounds, constructed in 1938, is illustrated in Figs. 2 and 3. The canal is controlled by a 25 ft wide 5-bay flashboard intake faced with a 20 ft wide 4-bay flashboard sluiceway which governs the head available for the canal and enables discharge for river percolation as well as sluicing of sand from the forebay. At all drops or outlets, flow gages or recording gages are installed. The canal can handle a flow of 200 cu ft per sec (to care for additional development). The cost of developing this area was approximately $10,000; the water conserved is worth at least $10 an acre-ft in the ground.

It is estimated that the cost of spreading, including the wooden structures (retired on a 10-yr basis), interest, maintenance, and operation, amounts to less than $5 per acre-ft of water spread. With several hundred acres of coastal basin spreading grounds completely developed, the cost might be as little as $3 per acre-ft for conservation of the long-time average available flow which would otherwise waste to the ocean. (Cost of retirement of land was not included in these unit charges as the value is not impaired.) This average flow is calculated to amount to a minimum of 10,000 acre-ft annually.

Adjacent to these spreading grounds the Rio Hondo channel has been intensively developed (Fig. 4). This is a typical example of the coverage which may be obtained by the use of a large plow and occasional checks to redistribute the flow. Of course, it is only practical to set up this type of intensive river spreading for regulated flows to be released after the storm season. The work performed is also instrumental in spreading the next



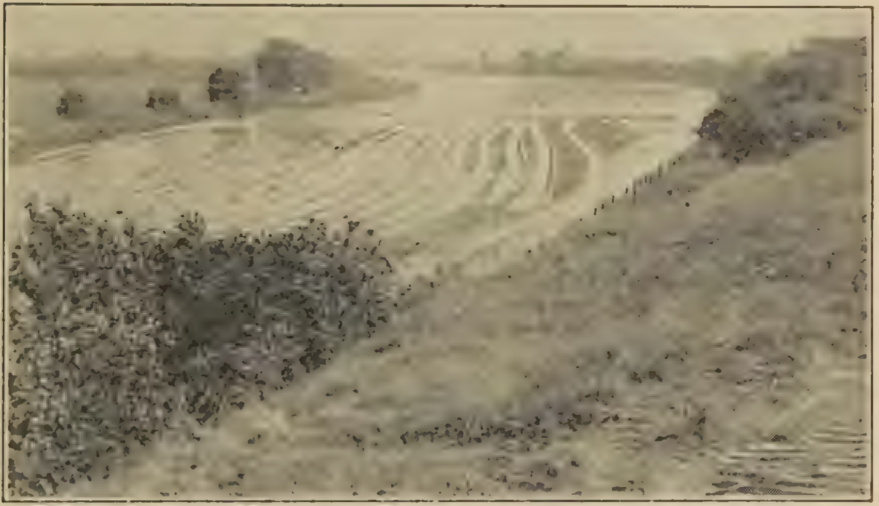
(*a*) Canal Intake, Sluiceway, and Temporary Diversion Dam



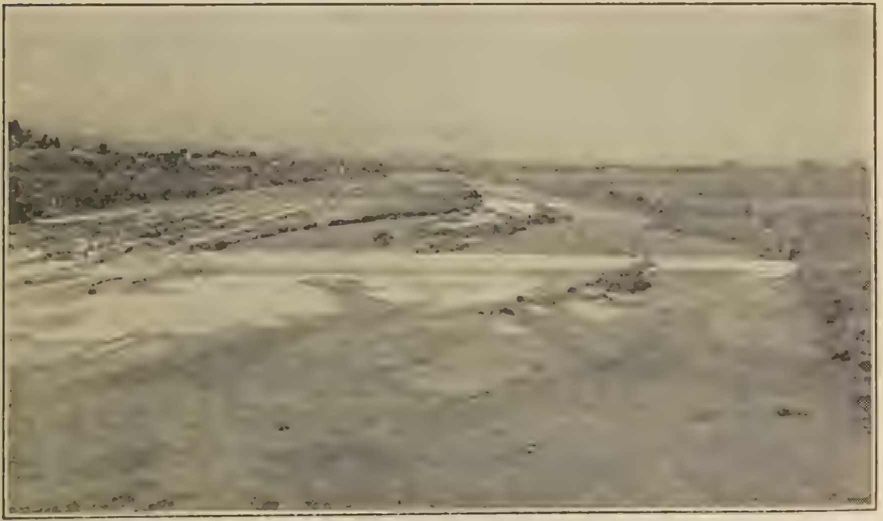
(*b*) Check-Drop No. 2, Turnout No. 2 in the Main Canal

FIG. 3.—SPREADING GROUNDS, RIO HONDO COASTAL BASIN, AUSUGT 3, 1938

season's storm flows until low water channels have been created again. The equipment used was a specially constructed plow capable of making a furrow from 2½ to 3 ft deep and was drawn by a 70- or 80-hp tractor. The check levees were constructed by bulldozer. One hundred ten acres developed in this manner percolated approximately 150 cu ft per sec, with rather high ground water.



(a) Plowed Ditches Designed to Give Complete Channel Coverage, July, 1940



(b) Check Barriers Erected by Bulldozers to Increase Channel Coverage, April, 1942

Fig. 4.—Rio Hondo Channel Adjacent to Spreading Grounds

Some Spreading Problems.—Intensive study has been given to both useful and economic means of conserving San Gabriel Canyon flows for use in the two principal ground-water basins over which the river and its distributary, the Rio Hondo, flow (see Fig. 1, basins 3 and 24). The upper, known as the Main Basin, including the area from the mouth of the canyon near Azusa on the north to the vicinity of Whittier and Montebello on the south, and from Alhambra on the west to Covina on the east, has a capacity between experienced ground-water levels of 615,000 acre-ft. It is overlain by 61,000 acres of irrigated land.

The lower basin, known as the Central Coastal Basin, is also largely agricultural, having an area of 99,000 irrigated acres. Its useful capacity is approximately 275,000 acre-ft. While the upper basin is of consistently porous material typical of upper portions of a detrital cone, the lower basin contains porous material in only the upper five miles along the courses of the rivers. The balance is overlain by clay strata which controls an intermittently artesian pressure supply.

Replenishment of the Main Basin is accomplished readily by percolation in the stream channels and the normal spreading which will automatically occur in Santa Fe reservoir. Percolation of flows in the Main Basin has supplied interesting data relative to the distribution of ground water dependent on the location of percolation.

Fig. 5 shows the course of the San Gabriel River and its distributary, the Rio Hondo, through the Main Basin. Prior to the construction of the Santa Fe Dam and reservoir, the 200% of normal rainfall during the season of 1940–1941 provided a large volume of canyon flow which traversed the channels. These channels split near the upper portion of the reservoir site, and the contribution of percolation from the channels to the ground-water body was very significant. The dashed ground-water contours indicate the distribution occurring principally from this stream-bed percolation. In comparison, the full line contours show conditions following the 160% normal rainfall season of 1942–1943 when the Rio Hondo was cut off from percolation and the principal source of contribution to ground water was from percolation in Santa Fe reservoir and flows released down the San Gabriel River. It is evident that the latter condition spread the ground water over the broad front of the cone making full use of available storage instead of concentrating ground-water ridges under the channels. Knowledge of such characteristics is particularly helpful in handling spreading works in relation to developed areas which are subject to high ground-water difficulties.

In contrast to the easily replenished Main Basin, the Coastal Basin has been depleted to such an extent that, in certain areas, sea-water intrusion threatens the quality of its supply. Hence, it is in this basin that the Flood Control District has developed its primary off-stream spreading works. The works now developed in the upper portion of this basin (Fig. 1, spreading areas 8, 9, and 10) consist of 240 acres adjacent to the San Gabriel River and Rio Hondo, a portion of which has been operated since 1938. Operations have included test runs to determine the characteristics of the spreading grounds as well as routine use between winter storms and percolation of flows stored in reservoirs during spring months.

Valuable data have been gained relative to percolation of water under varying ground-water conditions. The principal factors determined have been percolation rates, relation of shape of spreading grounds to outlying areas, effect of depth of water in basins, and effect of ground-water mound. Rates obtained have varied from initial values of 9 cu ft per sec per wetted acre to final values under high ground-water conditions of 0.9 cu ft per sec per wetted acre, the latter having held constant for many weeks of continuous spreading.

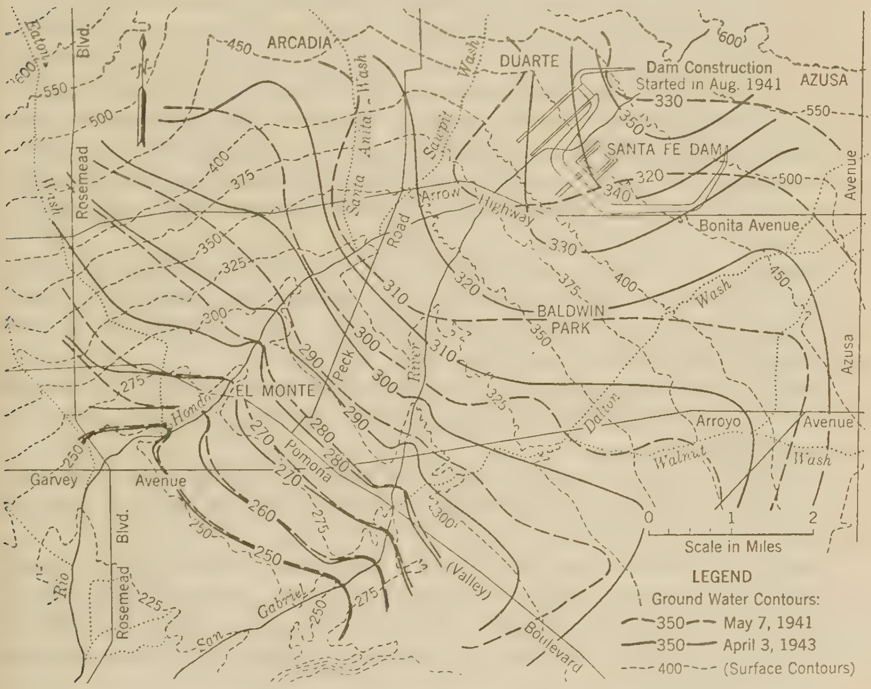

ARCADIA
DUARTE
AZUSA
Dam Construction
Started in Aug. 1941
SANTA FE DAM
BALDWIN PARK
EL MONTE
Eaton Wash
Rosemead Blvd.
Santa Anita Wash
Sawpit Wash
Arrow Highway
Peck Road
River
Bonita Avenue
Azusa Avenue
Arroyo Avenue
Walnut Wash
Dalton Wash
Garvey Avenue
Rio Hondo
Pomona (Valley) Boulevard
San Gabriel
600
550
500
450
400
375
350
340
330
325
320
310
300
290
280
275
270
260
250
225
0
1
2
Scale in Miles
LEGEND
Ground Water Contours:
350 May 7, 1941
350 April 3, 1943
400 (Surface Contours)

FIG. 5.—COMPARISON OF GROUND-WATER LEVELS AT SANTA FE DAM AND VICINITY

When the normal water table is less than 25 ft from the surface it has been found that spreading quickly builds a water mound up to the surface of the spreading basins. Consequently the water percolated in these basins, which would normally descend vertically to a low water table, can only move away from the spreading area at its periphery. This indicates the value of elongated spreading grounds. Fortunately, the best natural sites are the natural stream benches of these rivers. Consequently, the most inexpensive land is often the best shaped and situated area for spreading.

Tests for effect of various depths of impounded water in basins, relative to percolation rates, indicate that, when the percolating water has a free escape, surface depth has little effect. However, when ground water is in contact with the water in the basin, an increase in the depth from an average of 1 ft to an average of 2 ft may increase the percolation rate as much as 20%.

Ground-water mound—that is, the tendency of the underground water surface to build up to a higher elevation immediately under the stream bed or spreading area—is probably the most important factor governing percolation rate in porous soil. Tests along the San Gabriel River have indicated that, when the normal water table is less than 20 ft from the ground surface, continuous percolation will result in the mound reaching the stream surface within a period of a few days and thereafter will reduce the percolation rate. Hence, the economy of creating water supply in underground storage at any location may be affected largely by the normal depth to ground water.

Data on Percolation Rates.—Obviously, each spreading ground location offers benefits and problems which can be ascertained only from trial operation. Hence, a concise summary of percolation rates obtained at the several spreading grounds which the District has operated, together with pertinent comparative material at each location, should be useful. In Table 2, these are given for seven spreading grounds and three additional locations at which temporary tests were conducted. A further description of the areas in Fig. 1 and Table 2 is as follows:

Area 14.—This area is on both sides of San Antonio Wash below the mouth of the canyon. The soil (Col. 8) is a coarse sand containing a large proportion of gravel and stones. Subsurface exploration revealed gravel, and gravel containing boulders, as far as the water table.

Area 11.—This area is at the mouth of the Big Dalton Canyon. The soil (Col. 8) is a porous, gravelly, sandy loam containing a variable proportion of large and small stones.

Area 12.—This area is at the mouth of the Little Dalton Canyon. The acres (Col. 3) are only partly developed. The soil (Col. 8) is the same as in area 11.

Area 6.—This area is on the east side of the San Gabriel River channel below the mouth of the canyon. The soil (Col. 8) is a coarse micaceous sand, such as that found west and southwest of Azusa, containing large quantities of stones. Subsurface exploration revealed gravel and gravel mixed with boulders to a depth of 255 ft.

Area 9.—This area is on the west side of the San Gabriel River channel. In the gross area of 94 acres (Col. 3) only 73 acres have been developed. The soil (Col. 8) is a medium textured, friable, micaceous sand containing a proportion of fine sand and silt in the subsoil. Subsurface exploration in the upper area revealed: Sand from the surface to 10 ft, sand and gravel to 70 ft, clay to 80 ft, and sand and gravel to 120 ft; and in the lower area: Soil and sand from the surface to 7 ft, sand and gravel to 8.5 ft, fine silty sand to 12.5 ft, and silty sand to 14 ft.

Area 8.—This area is on the east side of the Rio Hondo channel. Of the 309 acres gross area (Col. 3), only 160 acres are developed. The maximum initial percolation rate at the apex of the ground-water mound (Col. 5) was 2.0 whereas the maximum initial rate for the area was 9.0. The soil (Col. 8) is a light-textured, micaceous, friable, fine, sandy soil containing stratified

TABLE 2.—DATA FROM DEVELOPED SPREADING GROUNDS AND TEST AREAS, LOS ANGELES COUNTY FLOOD CONTROL DISTRICT

Area No. (see Fig. 1)	Spreading grounds	Type[a] (B, D or O)	AREA (ACRES)		PERCOLATION RATE[b]		Average seasonal capacity (cu ft per sec)	Soil type (U.D.S.A. Classification)	DEPTH TO WATER TABLE, IN FEET	
			Gross	Wetted	Maximum	Minimum			From	To
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
14	San Antonio	B, D	771	137	4.5	1.5	300	Tujunga stony sand	100	270
11	Big Dalton	D	30	9	6.0	1.1	15	Hanford stony sandy loam	18	139
12	Little Dalton	B	16	2.0	5.4[c]	....	10	Hanford stony sandy loam	18	139
6	Canyon Basin	D	500	62	1.6[c]	....	100	Hanford fine sand	1	160
9	San Gabriel Coastal Basin:									
	Upper area	B	94	54	2.5[d]	1.0[d]	75	Hanford sand	20	73
	Lower area				4.0	1.8				
8	Rio Hondo Coastal Basin	B	309	150	2.0[d]	0.9[d]	164	Hanford fine sandy loam	12	72
1	Pacoima:									
	Silty flows	B, D	174.6	80	....	0.8	60	Hanford fine sandy loam	196	260
	Cultivated area, clear water					1.9	145			
19	San Gabriel Valley, Santa Fe Diversion[e]									
	San Gabriel Cone (Upstream Area):									
	(*a*) Basin in old overflow channel—									
	Before blasting with dynamite	B	....	0.92	1.5[f]	0.7[f]	....	River wash	124	144
	After blasting with dynamite	B	....	0.81	4.0[c]	....	....			
	(*b*) Off-channel contour basins	B	....	12.75	5.0	3.8	58.6			
19	San Gabriel Cone (brush land in downstream area)	O	....	1.07	2.4[c]	....	2.6	Tujunga stony sand	55	90
19	South side of Rio Hondo (still farther downstream)									
	Natural river bed (3 days)	B	....	0.51	3.8	2.7	1.6	Hanford sandy loam	45	49
	Cultivated surface (9 days)				7.5	3.8	3.1			

[a] B = basin type, D = ditch type, and O = shallow overflow type. [b] Maximum initial rates (Col. 5) and minimum rates after an extended run (Col. 6) are given in cubic feet per second per wetted acre (Col. 4). [c] Mean percolation rate for the area. [d] Apex of ground-water mound at the level of the basin beds. [e] Temporary testing grounds. [f] 1.5 in Col. 5 is the mean value for a month with clear-water flow and 0.7 in Col. 6 is the mean value for a month following silty storm flows.

deposits of sand, silt, and gravel in the subsoil. Subsurface exploration revealed alternate strata of sand and gravel with 2-ft layers of clay at depths of 58 ft and 115 ft.

Area 1.—This area is on both sides of Pacoima Wash. The percolation rates (Col. 6) and the capacities (Col. 7) are averages of 26 days of silty flows and 16 days of clear-water flows, the latter on cultivated basins. The soil (Col. 8) is the same as area 8 except in the south half of the east side. This part is Hanford gravelly sandy loam, a coarse, sandy loam with a large percentage of granitic gravel. Subsurface exploration revealed that underlying the recent alluvium, at depths of 10 ft to 20 ft, is a less permeable surface of weathered old alluvium. Beneath this material there is sand and gravel with a 20-ft layer of clay at a depth of 220 ft.

Area 19.—In the upstream part of the San Gabriel Cone, Area 19, the river wash (Col. 8) was composed of undifferentiated alluvial deposits. It was generally very coarse and stony soil, the finer material having been washed away. Subsurface explorations revealed sand, gravel, and boulders to a depth of 260 ft. To blast the overflow channel, thirty holes were driven 10 ft apart and 3 ft deep. Each hole was charged with six $\frac{7}{8}$-in. sticks of 40% dynamite which formed craters 3 ft deep. The off-channel contour basins were interconnected to permit the circulation of water. A shutdown of two hours resulted in a 15% reduction (to 85% of 5.0) in the percolation rate for these off-channel basins. The rate of 3.8 is the average for 72 days.

In the downstream part of the San Gabriel Cone, Area 19, the soil (Col. 8) differs from the upstream part in that there is a scarcity of gravel and stones on the surface. Subsurface exploration revealed a medium gravel to a depth of 200 ft.

In the farthest downstream area on the south side of Rio Hondo (Area 19), the soil (Col. 8) is a brown, friable, micaceous, sandy loam, open and porous. There is a 6-ft layer of such soil, underlain with sand and gravel.

The area data emphasize the economic use of land by the basin method. Spreading areas 6, 11, and 14 (Fig. 1) were set aside as spreading grounds when land was relatively cheap. Equipment for development was scarce and hence "ditch" spreading solved the problem of conserving the flows available at that time.

San Antonio spreading grounds (Fig. 1, spreading area 14) is an unusual layout. It lies at the apex of the cone immediately below the canyon of the same name. The tributary uncontrolled watershed of some 30 sq miles rises to an elevation of 10,000 ft. A large portion of the runoff results from melting snow and is therefore often prolonged into the late spring—a condition which makes spreading without reservoir control and storage quite practicable.

In order to cope with storm conditions, basins have been constructed on the east side of the stream, into which flows up to 400 cu ft per sec of sand and silt-laden water can be diverted, stored, partially desilted, and circulated for percolation. These are used but a short period for flash flows, after which the water characteristically clears rapidly. When the solid content has reduced to 0.5% or less, the water is turned into the ditch and basin system on the west side of the stream. These spreading grounds have absorbed practically all the dry season flows and a maximum of 26,630 acre-ft, which was 93% of the total runoff during the 200% of normal rainfall season of 1940–1941. They are practically the sole source of supply, other than direct rainfall, for ground-water basins 11, 12, and 14 (Fig. 1) which are covered by citrus ranches.

Table 2 shows that percolation rates in terms of cubic feet per second per wetted acre vary widely with location. Maximum initial rates under low ground-water conditions are quite high for all locations because these locations are on recent alluvial cones or in river beds.

Areas 8 and 9, already discussed in some detail, were recently developed where land available for spreading is limited and land costs are high. During 1933 Area 1 was developed as a model canal and ditch system. Later experience gained from test basins indicated the probability of better economic use of the area. Hence, half of the grounds have been completed recently as a canal-basin layout. The rates shown in Table 2 were obtained during the transition and despite the old alluvium noted at shallow depths, it appears that the average rate will be increased to 2 cu ft per sec per wetted acre for even slightly silty flows. The ditch system wetted area was 12% of the acreage while the basin system covers 80%.

Extended periods of spreading bring into play factors such as clay lenses, control of finer sands, spreading of silty flows, and rising ground water. A study of the surface soil type, log of subsurface materials, and depth to water table emphasizes these characteristics.

Under Area 19 data are tabulated for temporary test spreading grounds (1) on coarse porous stony soil; (2) on brush land containing considerable gravel; and (3) on an area in which sand predominates.

(1) On the stony soil the initial test was in a well-sealed overflow channel in the San Gabriel River. Sealing had been by deposit of silt between the rocks and by calciferous cementation. The blasting experiment was undertaken primarily to break up this sealing. Its effect was pronounced.

Later tests on the same type of stony soil were conducted on an off-channel area of approximately 15 acres. Many years previous, this area had been a portion of the river bed during major floods, but was not sealed by either silt or calciferous deposits. Clear water was diverted to contour basins which spilled from one to the next, thereby providing slow circulation through the basins. As long as the test continued uninterrupted, the water sank at the rate of 10 ft a day. This unusually high rate continued for several weeks until the shutdown of the regulating dam upstream dried up the basins temporarily, although the soil did not have a chance to dry out. When the basins were filled a few hours later, the percolation rate had reduced approximately 15%. A similar sequence has been noted on several other spreading areas having considerably finer soil.

(2) Tests on brush land were undertaken to compare with percolation rates on barren areas which have been scoured by stream flow. Water was sunk by sheet overflow and from still ponds as compared with the basins described under (1). Although the base soil was somewhat finer, it is believed that the

lack of circulation as well as the fines introduced by growth of vegetation were the principal factors in giving a reduced percolation rate. Experience on several soil types and with water almost clear or slightly silty has indicated a definite benefit from pond velocities as low as 0.05 ft per sec. In addition to carrying fine particles through the basins, even this slow movement discourages the concentration of algae and the scum it leaves on basin beds.

(3) Tests on a quite sandy soil in the flood area of a natural river bed gave an opportunity to check the relation of percolation through the natural crust formed by a small amount of silt and calciferous action, and that through the same ground after cultivation. Although the differential found was more pronounced than has been experienced in larger scale operation, nevertheless the trend was the same.

Comparative Water Analyses

It is pertinent to compare the qualities of the supply available from local mountain and valley storm waters with other sources of water supply available to the Los Angeles Basin. Table 3 presents such a comparison through the medium of samples from storm and after-storm flows, which were taken during the storm seasons from 1932 to 1934. The data were gathered as a cooperative project of the Sanitary Group of the Los Angeles Section of the American Society of Civil Engineers and the Flood Control District. Laboratory facilities arranged by R. F. Goudey, M. Am. Soc. C. E., were an important contribution in analyzing the flood waters.

Despite the fact that it has been several years since the data were obtained, it is believed that the values on other supplies are representative with the exception of the water which the Metropolitan Water District is furnishing the Coastal Plain from its Colorado River source. Although study of the data reveals many interesting comparisons, those that are perhaps the most significant will be mentioned.

The pH-value of the local mountain and valley flood water averages about 7.6, which is comparable to other supplies. Chlorides are all within the drinking water range and generally average the same as other available supplies. The total hardness is less or not greater than the average local supply.

Nitrites and nitrates are generally lower than free ammonia, showing that there is nitrogen that needs to be oxidized in order to have stable water. However, most samples show sufficient dissolved oxygen to accomplish this oxidation. Organic nitrogen was generally equal to or less than that in aqueduct water. The higher values were from the upper Los Angeles River—that is, valley water having its source in farming lands.

The comparison of dissolved oxygen and oxygen demand as shown by B.O.D. from 21 mountain water samples shows only 5 with excess B.O.D., and only 1 of the 5 was from an after-storm flow, indicating further evidence of the stability of the water. The majority of mountain water samples compare well with those from the High Sierras brought in by the Owens River aqueduct.

Color and turbidity, and suspended solids furnish the prime difference between storm waters and other supplies. The after-storm flows are generally better in respect to these factors and reservoir storage readily corrects the excess derived from storm flows.

The greater part of the total solids is comprised of suspended solids. The analysis divided suspended solids into the subdivisions of quickly falling solids and those remaining in suspension. The division was found to be at a size of approximately 25 microns. The data from this division indicate that more than 50% of the suspended solids might be classed as quickly falling—that is, those which would drop immediately upon contacting relatively still water.

TABLE 3.—COMPARISON OF LOCAL STORM WATERS WITH OTHER SOURCES OF WATER SUPPLY AVAILABLE TO THE LOS ANGELES BASIN

Description	(a) LOS ANGELES AREA[a]				(b) OTHER SOURCES					
	Mountain Water		Valley Water		Mono Basin snow water	Aqueduct	Los Angeles River Gallery (Crystal Springs)	Lomita Plant No. 2 (well)	Experiments on Sewage Reclamation[b]	
	During storm	After storm	During storm	After storm					Raw	Final
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Flow Range (Cu Ft per Sec)										
Minimum	65	50	35	40	....	....	....	....	....	....
Maximum	1,550	2,700	10,000	700	....	....	....	....	....	....
Potential of hydrogen (pH)	7.8[c]	7.9[c]	7.4[c]	....[d]	8.0	8.0	7.2	8.0	7.2	7.2
Parts per Million, by Weight:										
HCO_3	90	125	96	158	10	200	205	270	320	240
CO_3	0[c]	0[c]	0	0	0	10	0	0	0	0
SO_4	28	18	58	93	2	23	115	0	125	125
Cl	24	11	26	36	0	22	30	10	85	85
Ca	32[c]	37[c]	35[c]	57	5	24	74	27	86	86
Mg	0.2[c]	8.2[c]	9[c]	9.0	3	9	25	14	30	30
Total hardness	80[c]	127[c]	126[c]	177	25	96	285	124	340	340
NO_2	0.01	0.01	0.04	0.05	0	....	0	0	0	3
NO_3	0.50	0.75	0.48	0.82	0	....	4.0	0	0	4.7
NH_3	1.82	1.02	1.23	0.46	0	0	0.4	0.8	21	11
Organic nitrogen	2.80	1.50	3.82	0.68	0.4	4.0	....	2.1	28	4.0
Dissolved Oxygen (D.O.)	6.6	6.8	6.0	8.6	9.0	8.2	7.0	7.1	0	4.0
Biochemical oxygen demand (B.O.D.)	5.6	3.7	12.4	4.9	0.1	2.0	0.2	7.1	300	4.9
Color Range—										
Minimum	150	5	60	....[d]	5	20	0	70	....	33
Maximum	600	75	800	....[d]						
Turbidity Range—										
Minimum	320	25	410	50	0	5	0	0	....	10
Maximum	500	3,800	5,000	4,500						
Total Solids Range—										
Minimum	1,530	15	2,160	407	....	....	....	....	....	....
Maximum	17,594	2,039	30,985	10,015	....	....	....	....	....	....
Suspended Solids Range—										
Minimum	1,210	12	2,060	200	....	....	....	....	....	....
Maximum	17,420	1,794	30,550	9,500	....	....	....	....	....	....
Organic and Volatile Solids Range—										
Minimum	190	0	86	63	....	....	....	....	....	....
Maximum	697	260	1,885	840	....	....	....	....	....	....
Number of samples	9	12	29	8	....	....	....	....	....	....

[a] Average of total samples (see last line) except as otherwise noted. [b] Los Angeles experiments under the direction of R. F. Goudey. [c] Average computed from less than the total number of samples. [d] Not run by the laboratory.

Although the copious rainfall since 1936 has filled many of the ground-water basins in the Los Angeles area nearly to capacity, the importance of maintaining these supplies through artificial replenishment is emphasized by recent war-industry draft and the fact that approximately 75% of the water supply of the area is derived from this source and direct mountain stream flow.

It is hoped that this paper will stimulate interest in the presentation of further data on many phases of this work.

ACKNOWLEDGMENTS

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DISCUSSION

A. L. **Sonderegger**,[2] M. Am. Soc. C. E.—Clear statements of the principal methods applied in the utilization of flood waste by spreading and of the correlation of flood control and water conservation, as practiced by the Los Angeles County Flood Control District, are presented in this paper. Conservation by spreading is supplementary to, and an improvement on, the natural process of seepage from stream beds on the debris cone and also supplementary to the natural cyclic regulation of water supplies by underground storage which is peculiar to Southern California. It is this natural regulation which has made possible the development of this area and which has carried its population with immunity through a period of predominantly dry years which extended from 1922 to the latter part of the 1930's.

Referring first to the quality of conservation water, Table 3 shows what was to be expected—namely, that the storm runoff from the granitic watersheds of the Sierra Madre is stable, organically pure, and potable from the standpoint of total dissolved solids. The total hardness of 80 of Southern California storm water appears to be high, but compares favorably with the total hardness of 25 of the Mono Basin snow water and of aqueduct water of 96, which in part is fed from snow fields.

Cols. 2 and 3, Table 3, indicate that the comparatively short-time contact of the "after storm runoff" with the disintegrated soil cover of the watershed materially increases the percentage of dissolved solids and the hardness. This tends to prove that disintegration and decomposition of the granitic formation make some of the constituent components available for solution. This conclusion is further borne out by the fact that storage in ground-water basins and the long-period contact with alluvial deposits materially increase the dissolved solids. This is apparent from a comparison of the analysis of mountain water "during storm" (Col. 2, Table 3) and valley water "after storm" (Col. 5, Table 3), the latter being a mixture of "after storm" mountain runoff and rising ground water. The proportion of sulfur hydroxide increased from 28 ppm to 93 ppm, chloride from 24 ppm to 36 ppm, and total hardness from 80 ppm to 177 ppm. The relatively harmless increase in dissolved solids by underground storage is compensated by the preservation of the purity and potability of the water for indefinite periods.

Conditions, more severe, result from reservoir storage in the mountain watershed. Under natural conditions, the periodic scour of canyons by floods, small and great, removes decayed vegetable matter, ashes, and other impurities. With the interception of all debris, fine and coarse, in storage reservoirs, or behind debris barriers, this flushing process remains incomplete, with the result that silt and ashes accumulate above dams, decaying debris is piled up in stagnant pools, and a growth of a multiplicity of organic life may be induced, including Crenothrix. Nevertheless, the more important fact remains that water supplies are conserved by impounding, and if they are of inferior quality modern methods of purification are able to cope with them.

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[2] Cons. Engr., Los Angeles, Calif.

The volume of water that can be salvaged in wet years by spreading is substantial and is directly dependent upon available storage and the methods of operating flood-control reservoirs. The most complete plans for flood control and conservation are those for the San Gabriel River prepared jointly by the Los Angeles County Flood Control District, and the U. S. Engineer Department. Available storage as of 1945 is as follows:

Dam	Storage in acre-ft
San Gabriel No. 2	12,000
San Gabriel No. 1	47,000
Morris	36,000
Santa Fe	30,000
Whittier Narrows	....

Assuming the proportion of the total storage at Morris Dam available for flood control as 18,000 acre-ft, and the proportion of the combined total storage at Santa Fe Dam and Whittier Narrows Dam as 10,000 acre-ft, the total storage available for purposes of conservation will be 85,000 acre-ft. Under these conditions the Flood Control District estimates the net seasonal salvage available for the valley above the Narrows at 3,000 acre-ft and for the coastal plain at 12,700 acre-ft. The total is 15,700 acre-ft, which is 11.5% of the mean seasonal runoff of 136,000 acre-ft at the mouth of the canyon and 18½% of the conservation storage.

In 1924 an estimate of possible salvage of flood waste was made for a report on the water supply of the metropolitan area of Los Angeles by a board of engineers composed of the late J. B. Lippincott, Hon. M. Am. Soc. C. E., Louis C. Hill, Past-President, Am. Soc. C. E., and the writer. Under the assumption of an available storage of 200,000 acre-ft then planned at San Gabriel site No. 1, the estimated salvage for San Gabriel River watershed was 34,000 acre-ft per acre or 17% of the conservation storage, and this quantity was credited in the report to the ultimate local water supply available for the Los Angeles metropolitan area.

Mr. Laverty refers to the first spreading development in Los Angeles County, on San Antonio Creek cone above Pomona in 1906. An interesting story is connected with the initiation of spreading in that area. In the 1890's, there was a beeman by the name of Arenz, of whom it was said that he truly had "a bee in his bonnet." Arenz noticed that along the wet washes in the upper San Antonio Creek cone there was a more vigorous growth of brush and a more luxurious bloom than along the dry channels. More bloom meant more food for the bees, and more food for the bees meant more honey in the hives; so in order to promote the growth of brush over wider areas he conceived the ingenious idea of diverting and spreading the stream artificially. This practice continued over some years and as a matter of fact ripened into a right of appropriation. After the turn of the century, a great development of citrus began on the cones in that vicinity, after it had been proved that the debris deposits were an excellent medium for the propagation of citrus, particularly lemons. The ranchers, in need of more water supplies, realized the advantage of recharging the underflow by spreading the flood waters and a general movement for the appropriation of flood waste began. Some bright men realized that the beeman had acquired a priority to a substantial portion of the creek flow of the early spring months and they promptly bought him out. They continued the diversions, not for the propagation of brush however, but for ground-water recharge and in a subsequent action in the courts in 1912 successfully maintained their position and priority by proving that the spread water percolated to their wells and fed their ciénagas. The late F. E. Trask, M. Am. Soc. C. E., was the first engineer to practice water spreading in Southern California in the nineties.[3]

The question naturally arises as to the legal status of the spread water. A water right is established generally by diversion for beneficial use. The Los Angeles County Flood Control District is charged by Legislative Act with the conservation of flood water; but the District is not a water user and hence not an appropriator. After the water is diverted and spread, it changes its status from that of surface flow to that of percolating ground water and is then available for appropriation to overlying lands or for export. In most instances where conservation is practiced in this area, the diversions are made by incorporated conservation and protective associations who are not water users, but operate for their volunteer membership. Spread water mingling with natural seepage loses its identity, although not necessarily its status as appropriated water, provided the appropriator can trace his spread water to the point of abstraction. In the case of these associations, the member water users are not the original appropriators. Moreover, their wells may be many miles from the spreading grounds and widely distributed over the ground-water field. In closed basins, it is believed that the appropriator and impounder of floodwater may legally defend his right.

In the case of Pasadena vs. Alhambra,[4] relative to the adjudication of rights to the water supply of the Raymond ground-water basin in the Pasadena area, rights were established on the basis of pumping abstractions for beneficial use; no credit being awarded for the spreading of local supplies. As to possible rights to imported water, if spread in the basin, the situation is left open as indicated by the following quotation[5] from said judgment:

> "That nothing in this judgment contained shall be considered as a determination of the right of any party to water hereafter imported by it into the Raymond Basin Area and spread therein, with the intent to recapture the same."

After the destructive floods of 1914 and 1916, Los Angeles County undertook a program of flood control by the construction of reservoirs with the dual intent of regulating peak flow and conserving floodwaters. As a matter of fact, bond elections were carried on the issue of water conservation. A majority of the population had moved in after 1916 and were not familiar with local flood conditions; hence they did not grasp the necessity for flood control. However, they did realize the effects of dry years and the need for conserving flood waste.

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[3] "Water Conservation in Southern California," by F. E. Trask, *The Rural Californian*, 1903.

[4] No. Pasadena C-1323 in the Superior Court of Los Angeles County, California, decided December 23, 1944.

[5] *Ibid.*, paragraph XVIII, p. 23.

During the recent protracted period of drought, public opinion and pressure were sufficiently strong to give conservation precedence over flood control and reservoirs were operated accordingly. Since the flood of 1938 a more cautious attitude in operating detention reservoirs has been adopted.

It is concluded, in the disposal of flood waste, that preference be given to: (1) Flood control, (2) conservation, and (3) quality of water—in the order of their economic importance.

The writer's opinion as to the relative importance of flood control and conservation may be expressed by paraphrasing one of the famous sayings of the late William Mulholland, M. Am. Soc. C. E., who, when campaigning in 1906 for a bond issue of $23,000,000 to finance the Los Angeles (Owens Valley) Aqueduct, told his audience: "If you don't get the water, you won't need it." If Southern California does not get flood control, it will not need conservation.

Warren N. Thayer,[6] Esq.—Flood control and water conservation methods as practiced in Los Angeles County are described adequately and concisely in this paper. In recent months some additional data have been obtained on the growth and dissipation of the ground-water mounds which need to be recorded.

Mounds and ridges on a water table (also cones of depression and troughs) are well-known ground-water phenomena in alluvial aquifers. They do not, and cannot, occur in aquifers that do not have potentially direct hydraulic continuity with the surface upon which the water is absorbed.

The principles governing the growth and dissipation of a ground-water mound are described in most textbooks on hydrology, and they are simple. When a drop of water enters the surface soil (after field capacity has been satisfied), it descends along a path as nearly vertical as possible until it reaches the water table. Then, the physical laws governing influent seepage stop functioning; the laws governing percolation begin to apply; and the rate of lateral movement of that drop of water in any direction depends on the $\frac{h}{l}$-ratio of the water table, modified by an appropriate coefficient of permeability, k. This rate is generally so much slower than the rate of vertical descent that a mound starts to build up; and, depending on the depth of the water table and the permeability of the aquifer, it may eventually reach the surface.

In January and February, 1946, it was possible to make some careful observations on the growth and dissipation of a large ground-water mound at the spreading grounds of the Rio Hondo Coastal Basin. Conditions were ideal.

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[6] Geologist, Los Angeles County Flood Control Dist., Los Angeles, Calif.

The preceding year had been one of below normal rainfall, and the water table was falling. Spreading began on January 3 and ceased on January 23, after a continuous run. There was no rainfall during the period.

To determine what actually occurred beneath the surface, measurements of water-table elevations were made daily at about forty wells within a radius of about a mile, measured from the midpoint of the spreading grounds. Eight of the wells are on a practically straight line passing through the spreading grounds parallel to the channel (see Fig. 1); and four of them are on a practically straight line normal to the channel and passing through basin No. 4 (see Fig. 2).

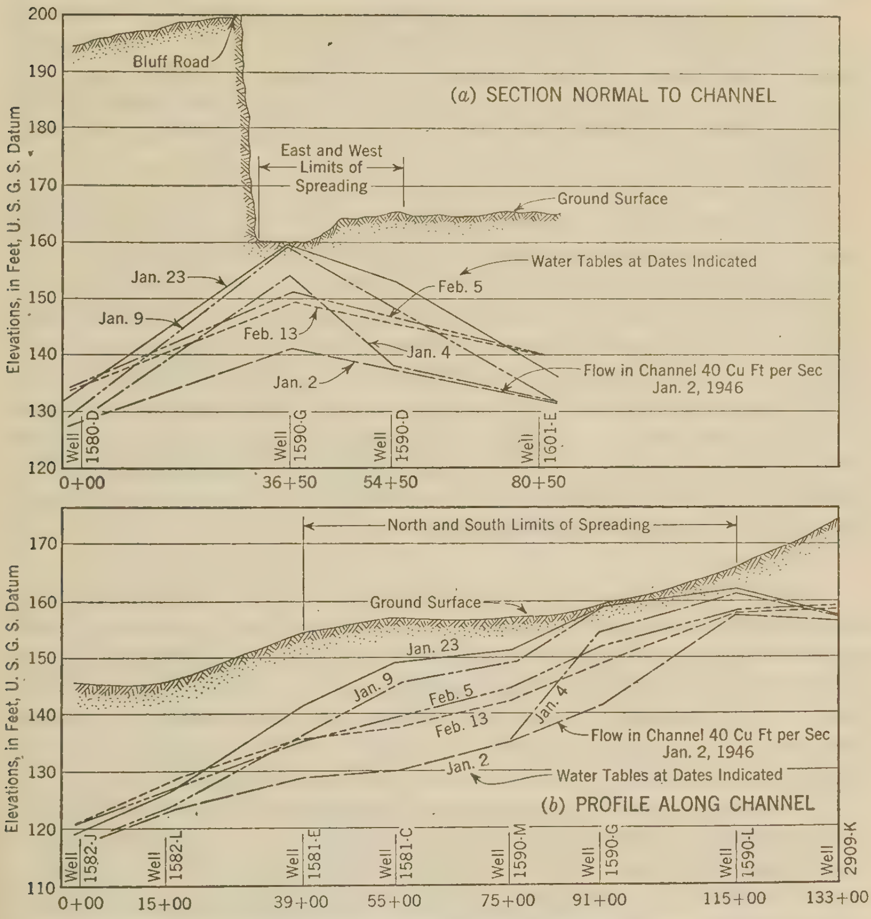

Fig. 6.—Study of Water-Table Elevations During a Spreading Period in January, 1946; Spreading Grounds, Rio Hondo Coastal Basin

Fig. 6 shows the water-table elevations along the two lines of wells on the day before spreading started (spreading started January 3, 1946), on three days during the spreading period, and on two days after spreading ceased (spreading ceased January 23, 1946). (Note the exaggeration of the vertical scales.) The day before spreading started the depth to water varied from 8 ft at the upstream end of the spreading grounds to more than 25 ft at the downstream end; and there was a low ground-water ridge under the spreading grounds (see Fig. 6(*a*), the section normal to the channel) because of a continuous flow of from 30 to 40 cu ft per sec in the channel for several months.

The day after spreading started (January 4), the water table had risen 4 ft at the upstream end of the grounds and 13 ft under basin No. 4. The spreading water had not yet reached the lower basins. Six days after spreading started (January 9) the ground water contacted the spreading water in basin No. 4; and the mound reached maximum height at that place, after rising 18 ft above the initial elevation.

About 3,500 ft downstream from basin No. 4 the mound also reached maximum height after rising 18 ft above the initial elevation, but the ground water did not meet the water in the spreading basins because the initial elevation had been farther below the ground surface. These facts verify the statements made in the paper that the normal depth to ground water is an important criterion in selecting spreading sites, and that the minimum depth for economic spreading in the coastal plain of Los Angeles County is from 20 ft to 25 ft.

After spreading ceased, the water table under the basins gradually dropped, but around the periphery of the mound it continued to rise for about three weeks as the water body sought its natural level. During the spreading period 4,800 acre-ft of water was absorbed in a wetted area of about 140 acres of spreading ground and adjacent channel.

GEORGE B. GLEASON,[7] ASSOC. M. AM. SOC. C. E.—Three beneficial purposes that may be served by spreading are indicated by the author: (1) The conservation of water which would otherwise waste; (2) the lessening of damage in areas of high water table through changing the location at which percolation occurs; and (3) the prevention of landward movement of sea water by increasing the elevation of the ground water near the coast. A fourth benefit might have been mentioned, namely, a possible reduction in the pumping lift.

In evaluating spreading, many factors are involved. Mr. Laverty has ably discussed those which fall within the scope of his paper. In the course of investigations conducted for many years by the California State Division of Water Resources in, and adjacent to, the area covered in the paper, others have been considered. When this is done it may in a particular instance be found among other things that: (1) Only a part of the spread water is actually conserved; (2) the intended purpose might be accomplished more effectively by other means; (3) the purpose might be accomplished equally well by other means at less cost; or (4) factors other than physical characteristics or economic considerations control. Within the area covered by the map (Fig. 1) are several basins that differ sufficiently in their significant characteristics to provide illustrative examples.

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[7] Senior Hydr. Engr., California State Div. of Water Resources, Los Angeles, Calif.

The author states that Main San Gabriel Basin is readily replenished by percolation in the stream channels and from the Santa Fe Reservoir. He also shows the effect of changing the place of percolation as related to the area of high water table in the lower portion of the basin.

Reference to Fig. 5 shows that the general water table and ground surface are almost coincident between El Monte and the southwest corner of the area there mapped. This condition is the result of a pronounced constriction of the cross-section area through which the oceanward moving ground water in Main San Gabriel Basin must pass in leaving that basin and entering the Central Coastal Plain. Along the incised channels of San Gabriel River and Rio Hondo the general water table is above the ground surface so a portion of the outflow is on the surface in these streams.

Since the cross section at the constriction (Whittier Narrows) is about full at all times and the water-table slope must closely approximate that of the ground surface, the underflow is virtually constant. The so-called rising water (that which originates in the ground water of the basin but flows out on the surface) varies, however, with the general elevation of the water table in the basin. Since 1923, the relationship between water-table elevation at a long record well near Baldwin Park and the measured quantity of rising water at the Narrows during each year has been quite well defined.

One characteristic of Southern California weather, quite significant from a water supply standpoint, is that periods of several years averaging materially wetter than the long-time average, alternate with periods about equally long which are correspondingly dry. Water-table elevations at the long record well reflect this "cyclic" variation as well as the variations in supply and extractions during each year. In January, 1905, the water table stood at 282 ft above sea level. By May, 1916, it had risen to 329 ft above, had dropped again to elevation 257 ft by November, 1931, and in June, 1944, it again stood at 325 ft above sea level. In 1931–1932, rising water totaled about 31,000 acre-ft and in 1942–1943 about 86,000 acre-ft. Based on the relationship between water-table elevation and rising water mentioned in the preceding paragraph, rising water outflow in 1915–1916 is estimated to have been about 119,000 acre-ft.

Despite this wide range in rising water outflow—from 31,000 acre-ft to 119,000 acre-ft—considerable regulation of the still more widely varying supply to Main San Gabriel Basin is afforded by the changes in storage in the basin. It is estimated that each foot of rise or fall of the water table at the key well represents a change in storage of about 4,000 acre-ft in the area affected by San Gabriel River. Thus, between May, 1916, and November, 1931, a total of 288,000 acre-ft of water came out of storage in this portion of the basin. Only a little less has gone back in during the subsequent wet period.

By far the greater part of the rising water is diverted and used directly on the Central Coastal Plain, or percolates in the channels below the Narrows, or is available for spreading in areas numbered 8 and 9 in Fig. 1. Thus Main San Gabriel Basin not only stores water for use on overlying lands, but serves as a partial regulator of the supply to the Central Coastal Plain (basin 24). If the interests of the latter area alone were considered, the utilization of all possible regulatory storage capacity in the main basin, through spreading or otherwise adding to the ground water a greater part of the flood flows, would be beneficial. Any material increase in the water so salvaged would result, however, in raising both the average and peak elevations of the water table throughout the basin and so increasing the area near the Narrows which is already adversely affected by a high water table. Under the circumstances, the practice followed by the Los Angeles County Flood Control District, in limiting so far as may be, the percolation in Main San Gabriel Basin and spreading as much as possible in the grounds below the Narrows, cannot be justly criticized.

In evaluating spreading, it is worthy of note that in a basin like Main San Gabriel Basin, where the outflow of rising water is large, and where it responds quickly to changes in water-table elevation, the greatest benefit so far as conservation is concerned accrues to the basin below and requires that a part of the water be spread again there. Until the water table is so drawn down as to virtually eliminate the rising water, there is always ample water for use on lands overlying the upper basin, so benefit there is limited to that resulting from change in location of percolation, mentioned before, and from reduction in pumping lift.

The Central Coastal Plain Basin (No. 24, Fig. 1) is characterized by a relatively small forebay area into which water percolates and within which storage changes occur, and a larger pressure area into which percolation from the surface to pumping aquifers is negligible because of virtually impervious intervening strata, and changes in storage are very small because the aquifers are confined and are substantially full of water at all times.

In the forebay, the water table dropped some 50 ft during the dry period preceding 1934; it has refilled during the subsequent wet period, with a considerable flow across the basin onto the pressure area and thence to the ocean at times when the forebay was full. At a well not far from the lower boundary of the pressure area near Long Beach, the pressure level decreased a little more than 80 ft between 1922 and 1937 and has not again increased since that time. In October, 1945, it there stood 36 ft below sea level.

Although other wells in the vicinity have recovered a few feet, the rise has in no case approached that in the forebay area, and if the intended purpose is to protect against the intrusion of sea water some procedure other than spreading alone seems to be indicated. Since it appears that the deficiency is due to the resistance to movement of water through the restricted cross section of the confined aquifers, rather than to a lack of sufficient water at the source in the forebay area, reduction of the heavy extractions near the lower boundary of the pressure area would provide a solution. This would require that a substitute supply be provided, either by importation, by the use of water from some other source such as reclaimed sewage, or by transferring the pumping to a point nearer the forebay area.

The regulation of the cyclically varying supply to the Coastal Plain requires that large storage capacity be available there. The fact that some water wasted recently because there was not room for it in the basin shows that extractions during the dry period prior to 1934 were insufficient to draw down the water table far enough to provide the capacity necessary to hold the excess which was available during the wet period. Had the supply for the lower portion of the pressure area been pumped in the forebay area throughout the cycle more storage capacity would have been available near the end of the cycle and there would have been less waste.

The measure of the water conserved by spreading or by any other means is the amount by which the waste is reduced thereby. In this case the spread water caused the basin to fill sooner than it otherwise would have with the result that some water which would have percolated naturally, was wasted. Thus, only that part of the spread water which was required in addition to natural percolation to just fill the basin was actually conserved.

This leads to the conclusion, that, in a basin that fills periodically, the amount that can be conserved is limited by the amount of the extractions until such time as average extractions exceed average supply, in which case the basin will no longer fill. In a basin of this type any water spread during a particular cycle in excess of requirements during that cycle is not conserved. However, in those cases where storage capacity will ultimately be available to conserve the percolation from larger spreading grounds, the difference between present and probable future costs of acquisition and construction is a factor to be considered. In the interim at least a portion of the area will benefit from decreased pumping costs during periods when the basin is nearly full. In such a case economy dictates when the works should be constructed.

Pasadena area of Raymond Basin (basin 2, Fig. 1) differs from Main San Gabriel Basin in that outflow changes very little with water-table elevation, and from the Central Coastal Plain in that demand is as great as total supply and the basin does not fill. For many years prior to July, 1944, at which time, in accordance with the terms of a court decree, extractions were limited to the safe yield, the basin was overdrawn and the water table was lowered far below the surface over a large part of the area. Compliance with the terms of the decree requires a large auxiliary supply if the demand is to be satisfied. Any water conserved, therefore, decreases the amount that must be imported or otherwise provided. Any water that is spread is virtually all conserved.

In a basin of this type, spreading or otherwise adding to the ground water an amount considerably in excess of present needs, solely for the purpose of decreasing pumping costs, might prove feasible. For purposes of discussion it may be assumed that a 25-ft increase in average elevation of the water table throughout a long period of time would produce a negligible change in outflow. To produce this increase in elevation a total of about 40,000 acre-ft in excess of demand is required. The safe yield of the basin is about 12,000 acre-ft per yr.

Assuming this to be the amount pumped and that the 25-ft reduction in lift reduces the cost of pumping 1 acre-ft by 50¢, a total importation of 40,000 acre-ft produces an annual saving of $6,000, or 15¢ per acre-ft. This justifies an expenditure of a little less than $3.00 per acre-ft of water spread, imported or otherwise supplied. In this basin, so long as the assumption that increased water-table elevation does not result in increased outflow holds true, it makes no difference whether the importation is all in one year or spread over a period of time. The total determines the benefit. In the case of Main San Gabriel Basin, where outflow responds quickly to changes in water-table elevation, this would not be true.

It is physically possible to accomplish the other beneficial purposes enumerated earlier in the discussion; namely, protection against saline intrusion and improvement of conditions in areas of too high water table through the use of imported water, of reclaimed sewage, or of water obtained by some other means. Such use, as a substitute for water now pumped in the Central Coastal Plain near Long Beach, has been mentioned as one means of raising pressure levels there. In some cases it might prove feasible to build up a protecting ridge against sea-water contamination, by forcing fresh water directly into the aquifers through wells.

In San Gabriel Basin, the area adversely affected by high water table increases and decreases cyclically with variations in the local water supply. If a greater part of the flood waters were wasted during the wet period of the cycle, with heavy extractions continued during that period but with water from some other source substituted during the dry period, it would be physically possible to reduce the fluctuation of the water table to any desired degree and thus hold the adversely affected area to a minimum at all times.

Comparative cost to the consumer is one of the factors that determines the choice as between spreading and each of the other sources that might be available. The cost of water imported from a single source may vary considerably under different conditions. That brought from the Colorado River by the Metropolitan Water District of Southern California is an interesting example. In portions of the area which, at some time, may be served by it, untreated water is available. The price, as fixed by the policy of the District, is $8.00 per acre-ft. In other portions only softened water can be obtained through present facilities. The price of this water is $15.00 per acre-ft. In neither case does this cover the total cost of the water, the remainder being covered by assessment.

The policy of the District requires (1) that except in special cases water be furnished only to member entities and (2) that all back assessments must be paid regardless of the time when the entity joins the District.

The total cost of the water per acre-foot thus depends in part upon the amount taken and the assessed valuation of property within the boundaries of the entity and, for a considerable period of time, might exceed $40.00 to certain member cities. It might eventually be less than $10.00, plus local distribution costs in portions of the agricultural area easterly of the Metropolitan Water

District treatment plant, where raw water is available and assessed valuations not so high. In the case of a basin where the only choice is between spreading or joining the District and the demand at any time in the future can be satisfied by spreading alone, this total cost of Colorado River water provides the basis for comparison. On the other hand, if membership in the District will ultimately be required, the \$8.00 or \$15.00 charge per acre-ft of water controls. It is important to note that where increase in supply is the objective the comparison is between cost per acre-foot available to the consumer rather than between cost per acre-foot delivered at some particular point in the area by the District and cost per acre-foot spread. Likewise it is the total cost of obtaining any of the stated objectives which controls rather than a comparison of cost per acre-foot.

Several factors other than cost have an important bearing on the choice of procedure. One of these is comparative quality. The author presents a comparison of local storm waters with supplies available from certain other sources. Chemical analyses of Colorado River water reaching the treatment plant and of softened water leaving the plant are now available. Averages, in parts per million, by weight, for year ending June 30, 1945, are as follows:

Chemical	Natural	Softened
HCO_3	139	110
CO_3	0	4
SO_4	365	365
Cl	96	102
Ca	92	25
Mg	32.5	10.5
Total dissolved solids	783	796
Total Hardness:	363	106
Carbonate	114	96
Noncarbonate	249	10

For many purposes both this water and the waters described by the author are entirely satisfactory. For other uses additional treatment may be required and one source or the other is preferable. The prejudice against the direct use of treated sewage for domestic purposes is difficult to overcome and must be recognized. As was stated early in the discussion, factors other than physical characteristics and economic considerations sometimes control. In an area like that tributary to San Gabriel River, where a high stage of cultural development has been attained, the interests of one part of an area might best be served by one procedure; those in another part by another. Benefits to one entity might well result in injury to another. An organization like the Los Angeles County Flood Control District, which is authorized to spread, but not to import, water, must serve these diverse interests as equitably as possible.

The Metropolitan Water District, which has provided facilities through which an ample supply can be brought in, has been mentioned as a possible source of imported water. This supply is not actually available, however, to a particular area until the voters of that area are convinced that their interests are best served by joining the District and then only if the area is one considered suitable by the District.

In Main San Gabriel Basin no entity is a member at present (1946). In the Central Coastal Plain, Long Beach and Compton are members but both are still heavy producers from the ground water in the lower portion of the area. Until such time as a greater part of the area has a flexible source of auxiliary supply available to it and means are provided by which the cost of improving the pattern of pumping on the Coastal Plain can be distributed equitably, it is improbable that the ideal procedure can be followed. Consideration of all the factors involved, in order that the best procedure possible under the existing and most probable future circumstances may be worked out, however, is justified.

In Raymond Basin the problem posed by diversity of interests was solved through court action. All large producers of water were brought into a suit; a decree finally was entered which fixed the pumping rights of each at an amount such that the total would not exceed the safe yield; a watermaster was appointed to enforce the terms of the decree. The court retained jurisdiction and provided for modification of decreed rights with changes in safe yield. Through an exchange agreement which was incorporated in the decree, each entity has available to it whatever water is required in excess of its decreed right. Under this arrangement the cost of importation and benefits derived from spread water are shared. In this basin spreading can be evaluated accurately on the basis of comparative costs for equivalent benefits.

FINLEY B. LAVERTY,[8] M. AM. SOC. C. E.—Inasmuch as the prime purpose of the paper was to present basic data for which there had been considerable demand because of the size and varying conditions of the developments outlined, the discussions have pertained primarily to further basic considerations applying to water conservation.

From his long experience in Southern California Mr. Sonderegger has enlarged on several statements in the paper. Since flood control comes first, possibly the allocations to conservation storage may be best interpreted as follows: During the winter months, 10,000 acre-ft in San Gabriel No. 1 and San Gabriel No. 2 reservoirs (consisting of the pool storage to prevent encroachment of debris on outlets); and, during the spring months, a gradually increasing storage to the totals shown for the reservoirs in Table 1, Col. 3, plus approximately 18,000 acre-ft of Morris Reservoir capacity, or a total of approximately 72,000 acre-ft. Since Santa Fe and Whittier Narrows reservoirs are being constructed by the United States Engineer Department purely for flood control, no permanent conservation allotment can be accorded to the storage available in these two reservoirs.

Mr. Gleason's discussion has provided several interesting comments on the water-right phases of the ground-water problem mentioned by Mr. Sonderegger.

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[8] Chief Hydr. Engr., Los Angeles County Flood Control Dist., Los Angeles, Calif.

The reservoir operating program of the Los Angeles County Flood Control District was the same prior to the 1938 flood as it has been since this flood—that is, the prevention of damage to persons and property adjacent to improved and unimproved valley channels by delaying and reducing flood flows until valley peaks have been dissipated through the use of storage as indicated in the first part of the paper. In storms of great magnitude, some of the reservoir capacities are too small for any regulation. This condition is emphasized by the fact that the majority of the data in Table 1, Col. 7, represent occurrences during the March 2, 1938, flood, which Mr. Sonderegger mentions. During this storm 24-hr runoffs, which did not exceed the reservoir capacity by more than from two to three times, had their peaks reduced and delayed very appreciably despite the fact that the storm of February 27 and 28 which immediately preceded the record storm was the largest which the majority of the reservoirs had experienced. Few engineers realize the meager warnings given of approach or termination of severe storms on the Pacific Coast, and particularly in Southern California. Until an adequate system of reports from weather ships and planes has been achieved, the detailed planning of flood control operation in these areas will often be but a matter of hours and, in some cases, even of minutes.

The additional information on the ground-water mound provided by Mr. Thayer furnishes an interesting check on the relation of the water table to spreading operations. Such a thorough network of wells, as that from which this information was gathered, is seldom available, and it would indeed be fortunate if other classic examples of ground-water phenomena could be reported to those interested in this field.

Mr. Gleason has presented a basic discussion of the relation of capacities of ground-water basins of various types to reduction of waste and also to economic considerations pertaining to other sources of water supply. In discussing the rising water derived from the Main San Gabriel Basin, he indicates a requirement that part of this water must be spread again in the Central Coastal Plain Basin; probably he intends to convey that this is natural channel spreading, for the channel capacities in the lower basin are normally adequate to percolate the rising water and therefore require no further operation expenditures.

Mr. Gleason also states that, despite several years of spreading operations in the Central Coastal Plain Basin, water levels have continued to drop, and sea water has continued to intrude in the Long Beach area near the lower boundary of the basin. This condition is attributed primarily to resistance to movement of water through the restricted cross section of confined aquifers. It would seem just as likely that the increased supply to this part of the basin has been absorbed by the extremely high demand of new water industries. No doubt the most immediate method of improving the situation is contained in one of Mr. Gleason's suggested substitute supplies—obtained by transferring the pumping point for principal users to a point nearer the location of water spreading in the forebay area. Such a solution would prevent further encroachment of ocean water and would much more nearly utilize the ground-water basin to salvage all water available for conservation.

The writer is indebted to Mr. Gleason for furnishing further data, as well as an example of the economic relation of local water conserved in underground basins to imported water, and also for presenting information on the adjudication of ground-water basins—a procedure which seems likely to become commonplace in areas where the safe yield of a single basin or related basins has been exceeded.

In addition to parts of the West where water spreading has been practiced for a considerable period, several new developments of this nature either are contemplated or have reached the stage of field tests. The soil types, percolation rates, and other pertinent conditions in these projects would undoubtedly add useful information to that which the writer has attempted to provide in this paper.

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Paper No. 2291

DESIGN DEVELOPMENTS—STRUCTURES OF THE TENNESSEE VALLEY AUTHORITY

A SYMPOSIUM

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Note.—Published in October, 1945, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

STRUCTURAL FEATURES OF HYDRAULIC STRUCTURES

By Ross M. Riegel,[1] M. Am. Soc. C. E.

Synopsis

Some of the more important or unusual features of structural design of the dams and other hydraulic structures constructed by the Tennessee Valley Authority (TVA) are treated in this paper. The design of power plants is discussed in the remaining two papers. The hydraulic performance of such structures is beyond the scope of the Symposium. This paper is confined to a few principles and methods governing the design of the structures themselves—and to only those which are considered to have some degree of novelty. Some evidence or argument supporting such procedures is presented. The principal hydraulic structures of the TVA, of course, are the dams. As a background for the treatment of this subject Table 1, showing the location and the more important features of these dams, is presented.

Concrete Dams

It is held to be a fundamental principle of dam design that an impervious surface should be presented to the impounded water and that provisions should be made for the escape to tailwater of any seepage or leakage that may pass the tight surface or diaphragm.

The two first dams of the TVA (Norris and Wheeler) were designed by the Bureau of Reclamation, U. S. Department of the Interior, as gravity dams and the original construction plant was assembled to be suitable for this type of structure. Efforts have been made by the TVA to transfer construction equipment from job to job. For this reason there were certain advantages in adhering to the same general types of structure from job to job. Under general conditions on these projects, form costs tended to be high, whereas the cost of materials for mass concrete tended to be low, and such comparison of costs as has been made in general showed little difference in cost between gravity dams and other types of structure. Structures of the mass type present less surface to the action of weathering agencies than do those of the reinforced type, for example. The massive TVA structures are made of excellent concrete and future maintenance expense on account of any surface deterioration of the concrete may be expected to be less for the mass structure than for the reinforced one. For these reasons the concrete dams of the TVA have been designed as gravity structures. In accordance with the principle enunciated in the preceding paragraph, the effort has been made to obtain a surface as tight as possible at the upstream face. In the larger dams the upstream face has been constructed with an enriched concrete. In the smaller structures, a richer concrete has been used throughout.

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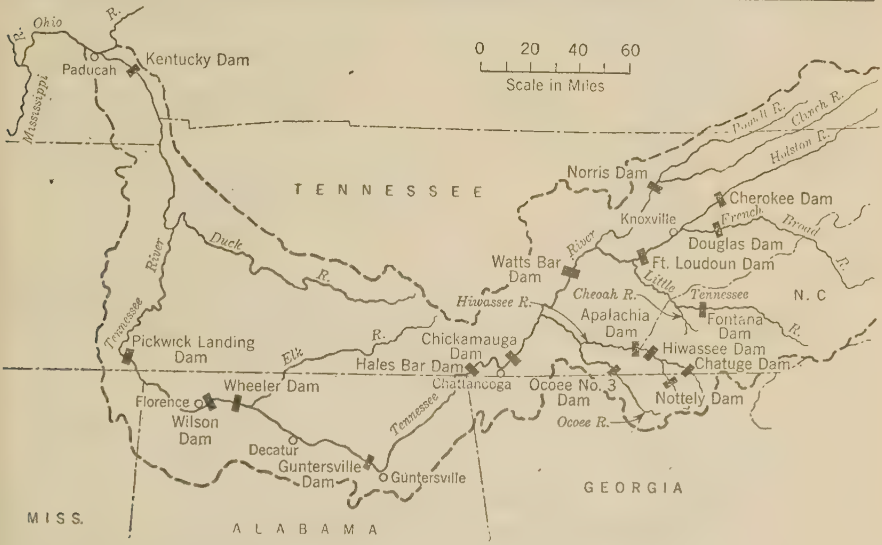
[1] Head Civ. Engr., Design Dept., TVA, Knoxville, Tenn.

Foundation Treatment and Uplift.—Considering the foundation as an essential part of the structure, elaborate cutoff grouting systems have been specified with a view to preventing seepage and uplift beneath the structures. Holes at a spacing as close as 2 ft have been drilled for this purpose. This grouting should be effective for many years and, in the course of time, it should become reinforced by the deposition of sediments in the reservoir bottom.

TABLE 1.—LOCATIONS AND PRINCIPAL DIMENSIONS (IN FEET) OF THE DAMS OF THE TENNESSEE VALLEY AUTHORITY


Ohio R.
R.
Mississippi
Paducah
Kentucky Dam
0 20 40 60
Scale in Miles
TENNESSEE
Powell R.
Clinch R.
Holston R.
Norris Dam
Cherokee Dam
Knoxville
French
Broad
Douglas Dam
R.
River
Watts Bar Dam
Ft. Loudoun Dam
Little
Tennessee
N. C
Duck
R.
River
Tennessee
Cheoah R.
Hiwassee R.
Apalachia Dam
Fontana Dam
R.
Hiwassee Dam
Chatuge Dam
Pickwick Landing Dam
R.
Elk
Chickamauga Dam
Hales Bar Dam
Chattanooga
Ocoee No. 3 Dam
Nottely Dam
Ocoee R.
Wheeler Dam
Florence
Wilson Dam
Tennessee
Decatur
Guntersville Dam
Guntersville
GEORGIA
MISS.
ALABAMA

Dam	Concrete Section (Including Powerhouse)			Embankment Section	
	Length	Height[a]	Power Head[a]	Length	Height[b]
Kentucky	1,823	206	57	6,800	60
Pickwick Landing	2,050	113	63	5,700	55
Wilson	4,862	137	96	....	....
Wheeler	6,342	72	53	....	....
Guntersville	1,384	95	45	2,600	46
Chickamauga	1,410	129	55	4,400	46
Watts Bar	1,739	97	60	1,200	57
Ft. Loudoun	1,285	122	80	2,600	55
Norris	1,570	265	194	290	50
Cherokee	1,697	175	148	5,100	60
Douglas	1,682	202	128	....	....
Fontana	2,330	480	430	....	....
Hiwassee	1,287	307	254	....	....
Apalachia	1,183	150	440[c]	....	....
Chatuge	....	....	....	2,850	140[a]
Nottely	....	....	....	2,300	184[a]
Ocoee No. 3	610	110	316[c]	....	....

[a] Maximum. [b] Average. [c] Total head by diversion tunnel.

On the main river dams the foundations generally have consisted of calcareous rocks with approximately horizontal bedding considered favorable to the accumulation of uplift pressures in the foundation itself. For this reason these dams, as well as those on tributaries, have been provided with a definite drainage system consisting of vertical holes 8 ft to 10 ft apart, drilled 30 ft to 40 ft into rock downstream from the grouted cutoff (see Fig. 1). The purpose of these drains is to prevent the accumulation of any considerable upward pressure in the foundation rock. In addition to the foregoing, liberal drainage has been provided under the aprons of the spillways, including the drilling of vertical holes to tap any horizontal openings that might exist in the foundation. Although these were provided primarily for the protection of the apron itself, they contribute to the maintenance of low uplift pressures under the main dam.

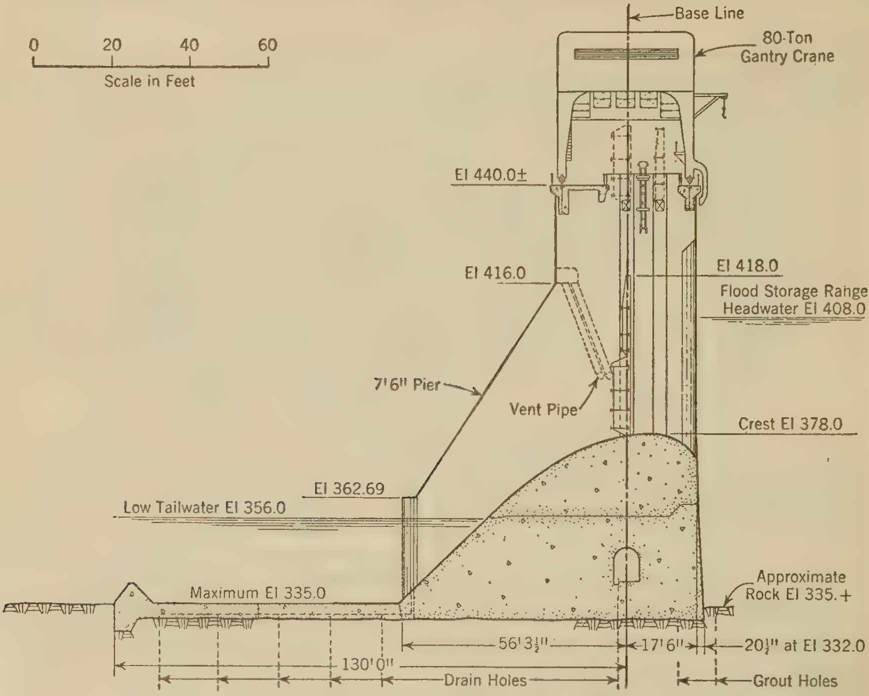

0
20
40
60
Scale in Feet
Base Line
80-Ton Gantry Crane
El 440.0±
El 416.0
El 418.0
Flood Storage Range Headwater El 408.0
7'6" Pier
Vent Pipe
Crest El 378.0
El 362.69
Low Tailwater El 356.0
Maximum El 335.0
Approximate Rock El 335.+
56'3½"
17'6"
20½" at El 332.0
130'0"
Drain Holes
Grout Holes

Fig. 1.—Section through Spillway, Pickwick Landing Dam

Having provided the drains, it was logical to capitalize upon their presence. Accordingly, in the main river dams, which are of moderate head, the uplift assumption in the foundation for design purposes was restricted to tailwater pressures downstream from the line of drains. Uplift pressures would increase uniformly from tailwater pressure at the drains to headwater pressure at the upstream face of the dam. Pressures were assumed to be applied over two thirds of the area of the base of the dam. In these cases, as elsewhere, such resistance to horizontal movement as the apron might offer was neglected.

For the higher structures such as Hiwassee, Douglas, Cherokee, and Fontana (Fig. 2), it was felt that the foregoing rule was not quite conservative enough even with the close drainage system provided. In these cases the uplift assumption was modified so as to assume one half of headwater pressure at the line of drains—the pressure varying in a straight line to headwater and tailwater, respectively. The assumed area of two thirds of the base was retained.

As to the uplift effects in the concrete itself, concerning which there is considerable uncertainty, TVA engineers have assumed that the open contraction joints, which have been used uniformly, are in themselves relief points for uplift pressures. Inspection galleries would also be relief points. However, in the higher dams where there is an excess of 100 ft of head, drainage wells were built in the dam itself, substantially on the same line as the foundation drains, to provide a more definite relief for possible uplift accumulation in the concrete than would be afforded by the joints and galleries alone. Pressure distribution is assumed the same as that for the foundation.

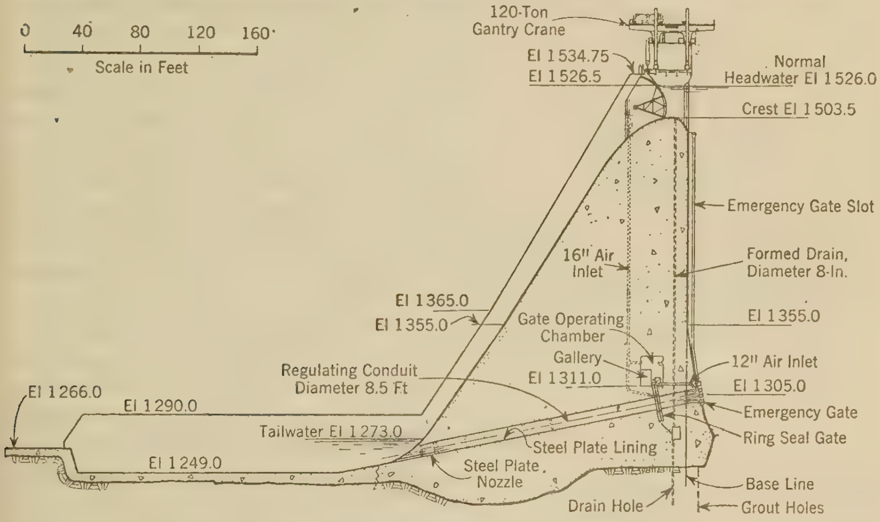

0 40 80 120 160
Scale in Feet
120-Ton Gantry Crane
El 1534.75
El 1526.5
Normal Headwater El 1526.0
Crest El 1503.5
Emergency Gate Slot
Formed Drain, Diameter 8-In.
16" Air Inlet
El 1365.0
El 1355.0
Gate Operating Chamber
El 1355.0
Gallery
12" Air Inlet
El 1311.0
El 1305.0
Regulating Conduit Diameter 8.5 Ft
El 1266.0
El 1290.0
Emergency Gate
Tailwater El 1273.0
Ring Seal Gate
Steel Plate Lining
El 1249.0
Steel Plate Nozzle
Base Line
Drain Hole
Grout Holes

FIG. 2.—SECTION THROUGH SPILLWAY, HIWASSEE DAM

Measurements of Uplift.—On Hiwassee, Douglas, Fontana, and Cherokee dams, a number of devices were installed to measure uplift in the foundation in the vicinity of the foundation contact. Fig. 3 shows a typical observation of this kind on Hiwassee Dam. Similar diagrams were prepared for three other stations in this dam as well as for similar stations in the other two dams, and they all have the same general character. The uplift pressure varies more or less uniformly from headwater to the drainage line where it is a minimum; between this drainage line and the toe of the dam there is a moderate rise and subsequent fall to the toe. Inspection of the drains and frequent measurements since construction have shown that this condition is stable and well within the design assumptions for the structures. Inspection also has shown that the drain outlets from the foundation continue to flow at a moderate rate. There is continuing evidence that the foundation drainage systems are functioning as intended.

The record of certain measurements of the pressure of pore water in the concrete at Hiwassee Dam, at various distances from the surface of the concrete, is presented in Fig. 4. Pressures were detected in 1943 at a depth of 12 ft. A gradual penetration of reservoir water is indicated. Of course, these observations indicate nothing as to the effective area of application of such pressures. (It is to be noted that this water passes an enriched surface layer of concrete.) The vertical drains, however, should limit the accumulation of pressure.

Contraction Joints.—The general practice of the TVA is to provide free contraction joints in its gravity dams, relying on duplicate metal seals for watertightness. With open joints, there is general freedom for individual blocks to adjust themselves, under dead load and water load, to the peculiarities of the foundation without throwing part of their individual burdens upon their neighbors. The open joint serves as a drainage outlet and a means of relief of uplift pressures in the concrete.

If the contraction joints in high dams are keyed and grouted, certain parts of the load at the highest part of the dam may be carried by shear-twist effects into the abutment. If the abutments are steep, this effect may be quite pronounced. It may be analyzed by a trial-load method, but it seems preferable to design the structures so that the shear-twist effect will be avoided and no allowance made for it. In general, therefore, the gravity dams of the TVA are designed with completely free transverse joints.

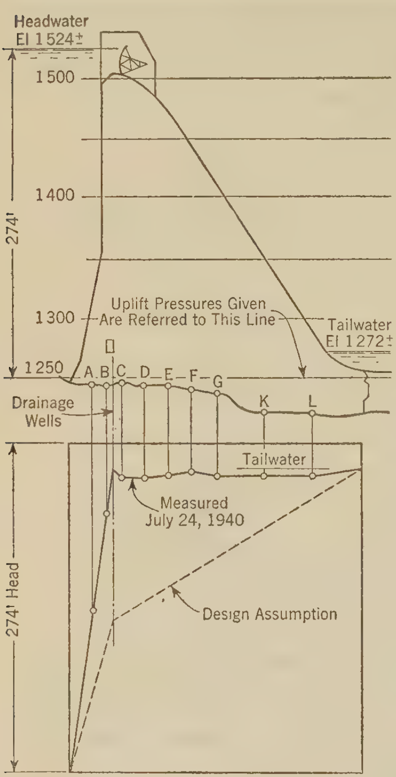


Fig. 3.—Uplift Pressure at the Base of Hiwassee Dam (Block No. 11)

In the two highest dams—Fontana and Hiwassee—this plan has been modified as follows: Keys and grouting have been provided in a limited portion of the transverse joints at the bottom of these dams. At Hiwassee Dam this amounts to about 25% of the height of the joint, and at Fontana Dam the height averages somewhat less than this proportion. This limited amount of grouting has been a concession to the belief of some members of the TVA staff in the practice of grouting. The most important justification in the writer's view is that the grouting of the joints at the bottom has a strong tendency to cure structural cracks in the concrete which tend to develop in the region of high restraint adjacent to the foundation. Although these cracks are subject to autogenous healing, if they are small, larger cracks should be penetrated by grout from the joints and be cured in this way. With limited grouting at the bottom of the joints, most of the dam section would still be free to deflect and the shear-twist effects would still be minimized.

Foundation Restraint.—Foundation restraint is taken seriously by TVA designers. Since concrete develops temperature in setting and then loses its temperature by radiation as setting is completed, volume must necessarily tend to change. At the foundation level these changes result in tensile stresses and the concrete tends to crack. It is general experience that a spacing of contraction joints of not much more than 50 ft will produce structures in which cracks transverse to the axis are not material. Nevertheless, cracks parallel to the axis tend to be frequent in the gravity dam and are considered deleterious.

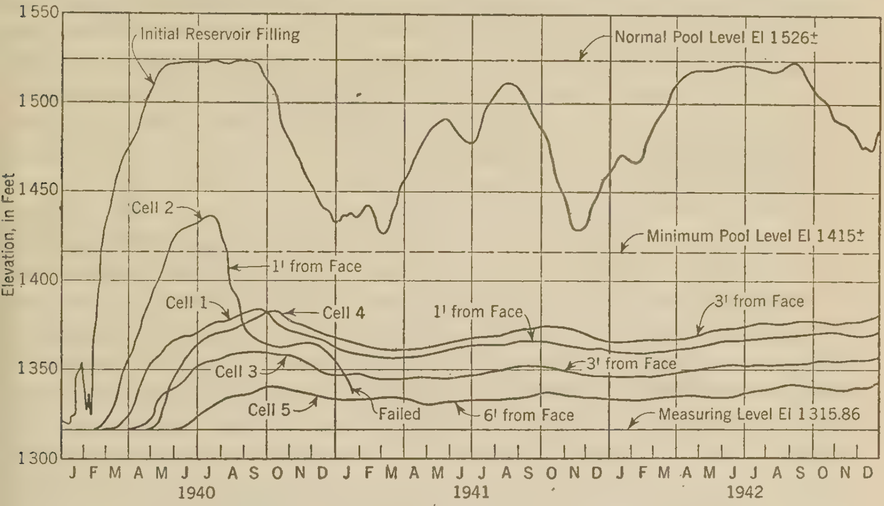


FIG. 4.—MEASURED PORE PRESSURE IN THE BODY OF HIWASSEE DAM

The most rational approach to the prevention of these cracks is to control the manufacture of concrete so as to reduce the maximum temperature of the concrete and the subsequent drop of temperature as much as practicable in the region of high restraint. Three methods to this end have been followed in the TVA organization: (1) The use of low-heat cement and a low cement content; (2) pouring concrete slowly to minimize the temperature rise in the region of restraint; and (3) circulating cold water through pipes to control the temperature rise.

(1) Lower maximum temperatures are obtained in the mass concrete when low-heat cement is used than would be realized under parallel conditions with standard or modified Portland cement. It is also the general practice of the TVA in major structures to use a maximum aggregate size of 6 in. with four grades of coarse aggregate and sometimes two grades of fine aggregate, all proportioned by weight. With such proportioning and a water-cement ratio of about 0.75 by weight, a very large amount of concrete has been made with about 0.8 bbl of cement per yard. This concrete is entirely adequate from the strength standpoint, whether low-heat cement or other grades have been used, and the low cement content causes low maximum temperatures, advantageous from the standpoint of restraint.

(2) Temperature rise in the region of restraint is minimized by slow pouring. By this is meant the placing of thin pours at a considerable time interval so as to permit the radiation of a material portion of the heat of hydration from the top of a pour before the succeeding pour is placed. This measure is of most importance in hot weather and near the foundation. It involves usually the placing of a pour, say 2.5 ft thick, with a time interval of two or three days before placing the next pour. This method is very effective in reducing the accumulated temperature rise and subsequent drop. Its necessity will vary considerably throughout the year. In the winter when the temperature of placing the concrete is likely to be below its final mean temperature, and the temperature changes are less in magnitude, this procedure is less valuable and usually can be modified materially, or eliminated.

(3) The rise of temperature can be limited by circulating cold water through pipes in the concrete to remove the setting heat and prevent the temperature from ever rising materially. This procedure was followed at Hiwassee and Douglas dams in certain local situations where the temperature conditions were particularly unfavorable, but on Fontana Dam it has been used systematically. In this case, where the dam is approximately 450 ft high and the concrete is subject to high stresses, extraordinary attention was felt to be justified in the effort to control temperature changes near the foundation and the consequent tendency to crack. In the first place, the effects of restraint were reduced by the introduction of longitudinal joints designed to be subsequently grouted. Low-heat cement could not be obtained for the Fontana Dam, but in the region of restraint adjacent to the foundation both the slow pouring method and that of artificial cooling were resorted to in the warmer weather. Concrete placed in this region in the hot months of the summer was cooled by refrigerated water at 40° F to 45° F, passing through pipes 2.5 ft, center to center. By these combined means the temperature rise was kept at about 15° F (see Fig. 5). In the region above the foundation, where restraint became unimportant, the concrete was allowed to rise in temperature and then was cooled by circulating water to approximately its final mean temperature so as to permit the effective grouting of the longitudinal joints.

All the foregoing discussion relates to mass concrete. Sometimes there are relatively thin structures, such as spillway piers, which are subject to restraint at the foundation or at the point of connection with larger structures. The tendency to crack is the same. However, in such cases it has been deemed preferable to control cracks by reinforcement rather than to endure the delay and complications caused by slow pouring and artificial cooling. Moreover, the durability requirements of such elements generally preclude a low cement content. Progressive observations on certain structures and office calculations finally led to a standard provision that, in the region adjacent to the restraint, about 0.75% of steel should be used to prevent or distribute cracks. In the thicker walls and piers, this was reduced to 0.50% of the outer 2 ft. As the height of the structure increases above the foundation, the restraint diminishes and the proportion of reinforcing steel is much reduced. Research and observation led to the conclusion that, at a height above the foundation equal to one half the length of the structural element in question, there is no longer any restraint, provided that the concrete is poured without any long interruption.

Stresses in Concrete Dams.—Some comment on stresses in gravity dams is pertinent. It has been the practice of the TVA to calculate stresses on the conventional straight-line basis. The weight of the concrete is determined as accurately as feasible and is used in calculations. Uplift assumptions have been made as outlined previously. The requirement has been made that no tension should exist at the upstream face under the conventional calculation and a somewhat variable margin over this requirement has been assumed. Some attention has been paid to the practices of other organizations as to the major dimensions so that the determination of such things as the ruling downstream slope of the dam has not been based exclusively on calculations. Fontana Dam was made somewhat heavier in section than Hiwassee Dam largely because it is higher.

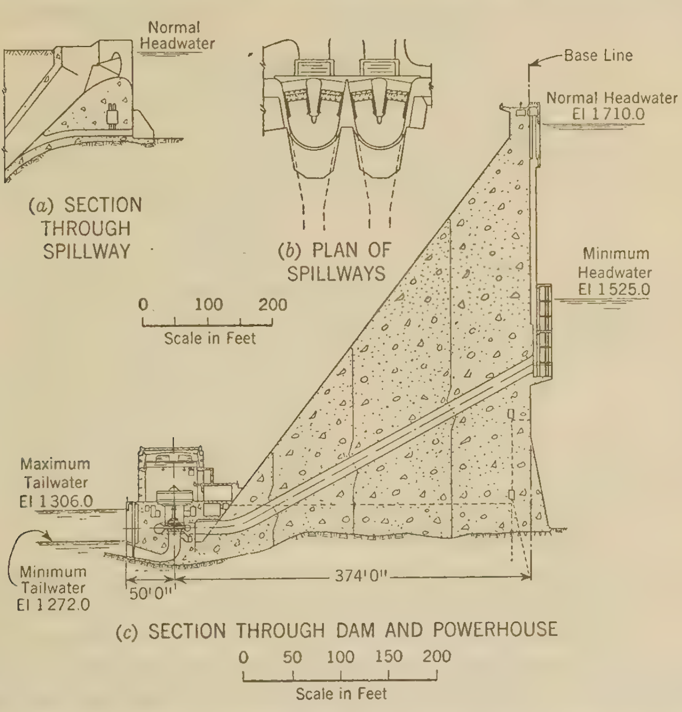

Normal Headwater
(*a*) SECTION THROUGH SPILLWAY
(*b*) PLAN OF SPILLWAYS
0 100 200
Scale in Feet
Base Line
Normal Headwater El 1710.0
Minimum Headwater El 1525.0
Maximum Tailwater El 1306.0
Minimum Tailwater El 1272.0
374'0"
50'0"
(*c*) SECTION THROUGH DAM AND POWERHOUSE
0 50 100 150 200
Scale in Feet

FIG. 5.—THE FONTANA PROJECT (TURBINES, 91,500 HP AT 325-FT HEAD; GENERATORS, 67,000 KW)

The TVA approach has been considerably influenced by the results of research conducted on Norris and Hiwassee dams for several years. In these structures a number of Carlson measuring instruments were installed for the purpose of measuring strains, stresses, joint openings, temperatures, etc. Fig. 6 shows some results at three measuring stations as reported by Douglas McHenry.[2] This diagram covers a period of four and one-half years, including the time of filling the reservoir, and of more than three years of subsequent fluctuation. The two most striking points about this diagram are the way in which the vertical strain persists near the upstream face of the dam with increasing water load and time while the strain at the midpoint diminishes. Strains near the downstream face generally rise with the increase of load and the passage of time, as might be expected.

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[2] "Measurements of the Structural Behavior of Norris Dam," by Douglas McHenry, *Technical Monograph No. 53*, TVA (mimeographed bulletin).

There appear to be two reasons for these marked departures from the conventional straight-line variation. During the period of observation the heat of hydration has been lost by radiation and the mean temperature of the dam has dropped to a point close to its final mean temperature. This drop has occurred mainly in the interior of the dam with consequent tendency to loss of volume. The tendency is then to throw an increasing proportion of the weight of the dam upon the concrete adjoining the faces of the structure and, therefore, to increase the vertical stress near the faces while diminishing the stress in the center of the dam. Mr. McHenry also points out that the lower part of the upstream face of Norris Dam has a considerable batter, namely 1 : 5, and that thereby a considerable vertical load is applied to the heel of the dam.[2] He believes that this water load is largely transmitted into the foundation directly, without ever being diffused in its effect over the structure as a whole. This theory seems plausible but is certainly not in accord with the conventional method of stability analysis. More recent analyses of similar conditions at Hiwassee suggest that the higher heel stress is more probably traceable to the penetration of the concrete by reservoir water, combined with the temperature load transfer.

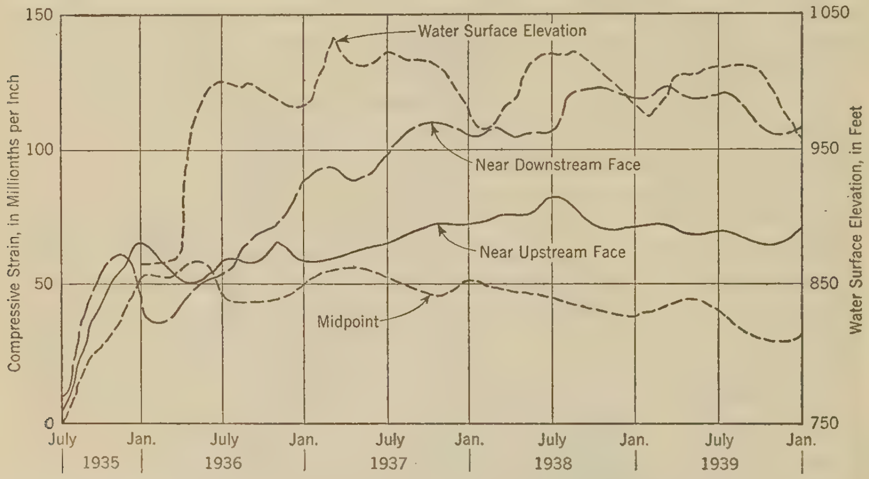

Fig. 6.—Measurements of Vertical Strain, Norris Dam

Fig. 7 records some tentative stress distributions at Hiwassee Dam, based, in this case, on data from five measuring stations. At each station strains are measured in five directions and correlated, and deformation of free cylinders under no stress is also measured. The effects of temperature change as well as load are reflected in the measurements. The phenomena are complex, as are the interpretations of them, and the diagram is tentative as to actual stress values.

The five sets were placed near the foundation in a block that was discontinued for several months, at a level a few feet above the instruments, while the river was diverted over it and cooled it. Then the pouring of the block was resumed and carried to full height. Fig. 7(*b*) shows the same high stress at the heel under water load that was found at Norris Dam. Later measurements show a drop in stress near the third points and an increase at the faces, particularly downstream as found at Norris Dam. The "hump" at the center is believed to be due partly to lack of uniformity in the foundation, and also to the manner in which temperature changes take place in the dam.

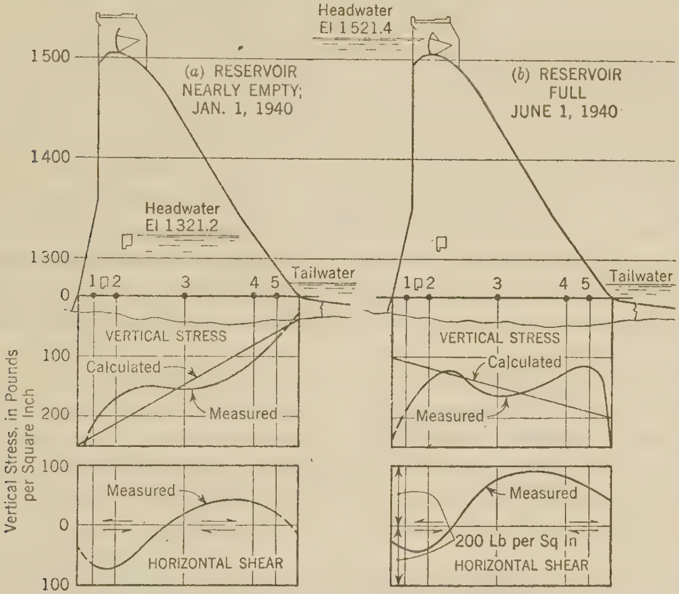

FIG. 7.—MEASURED STRESS DISTRIBUTION, HIWASSEE DAM

Attention is called to the shear diagrams. The horizontal shear diagram in Fig. 7(*a*) represents essentially temperature stresses, interpreted as being the effect of the expansion of the warm upper concrete upon the colder base. The change in the shear from that shown in Fig. 7(*a*) to that in Fig. 7(*b*) (reduced vertical scale) is roughly equivalent to the water load.

Measurements of deflection on Hiwassee and Norris dams have shown that the deflection of the top of the dam due to annual temperature variation is of the same order as that produced by water load. In certain parts of dams like Hiwassee and Norris, stresses due to temperature change are of the same order as those due to water load. When the magnitude of the temperature effects is considered, together with the indicated variation from straight-line stresses, the influence of foundation irregularity and the phenomena of plasticity and growth, one cannot rely too much on conventional calculations. Considerable skepticism is justifiable with regard to elaborate shear-twist calculations, for example.

In conclusion, there is evidence that high masonry dams actually have a greater margin of safety than is shown by the conventional calculations. So far as TVA structures are concerned, they are somewhat lighter, as a whole, than those sometimes developed by other organizations, but the evidence of the stress measurements is such as to produce confidence that the structures are entirely adequate.

Earthquake Effects on Concrete Dams.—One question confronting the TVA engineers was that of the possibility of earthquake effects on these structures. This was of particular importance in the lower valley. It is known that a very severe earthquake centering near New Madrid, Mo., on the Mississippi River had occurred in 1811, and there is some geological evidence that this quake had been preceded by another probably equal in intensity. The region near the confluence of the Ohio and Tennessee rivers appears to be seismically somewhat unstable and the underlying rocks in this locality are known to be much faulted. A special board of geologists reported that provision against earthquakes was advisable in dams on the lower river, but that it was not necessary for structures above Wheeler Dam. Consequently, no extra provision for earthquake effects was made in the storage dams on the tributaries or on the main river dams above Wheeler.

In the case of Kentucky Dam, however, much attention was given to this subject. Available literature and records were examined, geological advice was obtained, and some experimental work was done by Prof. Lydik S. Jacobsen at Stanford University, Stanford University, Calif., making use of a large shaking table. The main conclusions of this investigation (not published) were as follows:

1. When mass structures are founded upon rock at a reasonable distance from the center of seismic disturbance (at Kentucky Dam this may be 40 to 50 miles), the effects of the quake can be guarded against by applying, to the mass structure, a moderate horizontal acceleration. For Kentucky Dam an allowance of 5% of gravity was considered sufficient. To this acceleration effect is to be added the appropriate water-hammer effect as developed by the well-known theory of H. M. Westergaard, M. Am. Soc. C. E., if the structure is in contact with water.[3]

2. When slender structures founded upon rock are subjected to earthquake vibrations, a rational approach is to consider that a simple harmonic vibration is applied to the base of the structure and to trace the effects of this vibration throughout the structure. This includes determining the natural period of vibration of the structural element in question and then applying the assumed vibration. At Kentucky Dam this vibration was assumed to have a period of 1 sec and an amplitude of 0.25 in. (This corresponds to a maximum acceleration of 0.03 g.) By tracing this vibration through the structure, certain magnifications of the original impulse can be found, together with damping effects. Flexural effects on the elements will thus be determined against which suitable reinforcement must be provided.

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[3] "Water Pressures on Dams During Earthquakes," by H. M. Westergaard, *Transactions*, Am. Soc. C. E., Vol. 98 (1933), p. 418.

3. The investigation indicated that much greater effects were to be experienced in earth dams and in retaining walls supporting earth fills than would arise from the earthquake impulses in rock. This application seems to fall most logically in the subsequent discussion of earth dams.

EARTH DAMS

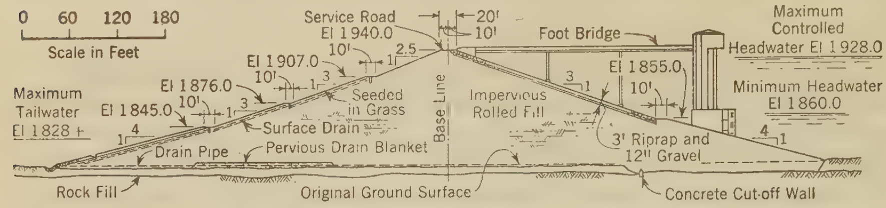
Earth dams constitute a considerable proportion of the hydraulic structures of the TVA. The design of these dams has depended largely upon the use of the materials available at the respective sites. In the case of the Nottely Dam (see Table 1), suitable rock was readily available and construction conditions were adverse to a full earth dam. Consequently, this dam is a rock-fill section with a rolled core. At Chatuge Dam rock was not readily available and plenty of earth could be had, so that this dam is entirely a rolled-fill section. At Cherokee Dam the earth dam had to be located on the top of a ridge and the use of a simple earth dam would have involved excessive costs by reason of flat slopes. In this case a composite of rock and earth fill was developed.

A few principles have governed the design of these fill dams. The soils laboratory determined the properties of available materials and, through field inspectors, supervised the selection and placing of the materials. The slopes of the dam sections have been generally checked by the slip circle method, using the constants determined by the soils laboratory. On the upstream face these calculations have been based on the reduced weight resulting from the sudden draw-down theory. Because this assumption is very severe, a rather moderate computed factor of safety (about 1.3) was considered sufficient. The slip circle method was applied to the downstream face also and in this case with larger factors of safety.

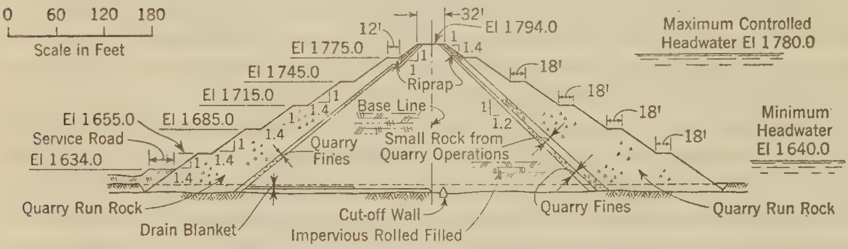
The primary principle enunciated in this paper applies here—namely, that the upstream face be tight and that such water as passes this diaphragm be permitted to escape readily. The minimum requirement, therefore, has included a continuous drain at the toe of the dam together with occasional wells which would drain the foundation into this toe drain. In the higher dams (Cherokee, Chatuge, and Nottely), a further drainage provision was made by actually placing a layer of porous material under a definite portion of the core or impervious element of the dam. This drainage is intended to hold down the saturation line, and to keep the downstream slope of the dam always dry, thus retaining its structural resistance. The application of these measures may be observed in the sections of Chatuge and Nottely dams (Figs. 8(*a*) and 8(*b*)).

Hydraulic fill has been used on TVA dams in only one instance—the south embankment at Pickwick Landing Dam containing about 1,000,000 cu yd. In this case materials were obtained from excavation for the lock. They consisted of clay soil gradually grading down through sand into gravel. They could have been utilized for a rolled fill, but only during the dry season. A hydraulic fill can be placed in almost any weather. For this reason, the hydraulic method was followed in this instance, with satisfactory results.

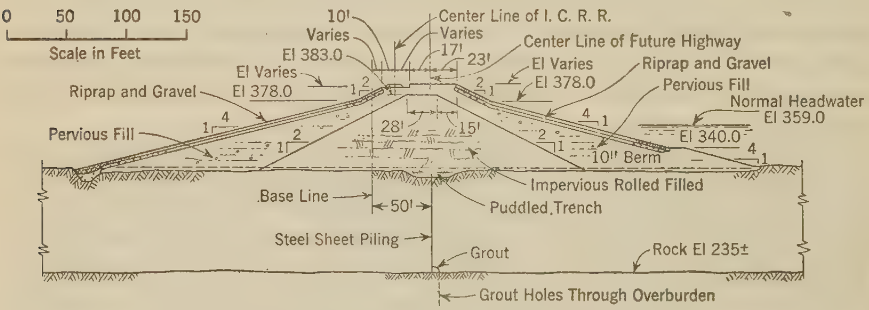
At Kentucky Dam, as indicated previously, it was necessary to design against earthquakes (see Fig. 8(*c*)). The basic design for the earth section of this dam, therefore, includes a core of rolled earth enclosed on both upstream and downstream slopes by additional fills of gravel and sand. The slopes (1 on 4) were built with high-friction material, and were decided to be sufficiently stable against the accelerations to be expected from the earthquake. If the


0 60 120 180
Scale in Feet
Maximum Tailwater El 1828
El 1845.0
El 1876.0
El 1907.0
Service Road El 1940.0
20'
10'
2.5
Foot Bridge
Maximum Controlled Headwater El 1928.0
Minimum Headwater El 1860.0
El 1855.0
Seeded in Grass
Surface Drain
Drain Pipe
Pervious Drain Blanket
Base Line
Impervious Rolled Fill
3' Riprap and 12" Gravel
Rock Fill
Original Ground Surface
Concrete Cut-off Wall

(*a*) CHATUGE PROJECT


0 60 120 180
Scale in Feet
12'
32'
El 1794.0
El 1775.0
El 1745.0
El 1715.0
El 1685.0
El 1655.0
Service Road El 1634.0
Maximum Controlled Headwater El 1780.0
Minimum Headwater El 1640.0
Riprap
Base Line
Quarry Fines
Small Rock from Quarry Operations
18'
Quarry Run Rock
Drain Blanket
Cut-off Wall
Impervious Rolled Filled
Quarry Fines
Quarry Run Rock

(*b*) NOTTELY PROJECT


0 50 100 150
Scale in Feet
10'
Varies
El 383.0
El Varies
El 378.0
Center Line of I. C. R. R.
Varies
17'
23'
Center Line of Future Highway
El Varies
El 378.0
Riprap and Gravel
Pervious Fill
Normal Headwater El 359.0
El 340.0
10' Berm
Riprap and Gravel
Pervious Fill
28'
15'
Base Line
50'
Impervious Rolled Filled
Puddled Trench
Steel Sheet Piling
Grout
Rock El 235±
Grout Holes Through Overburden

(*c*) KENTUCKY PROJECT

Fig. 8—Typical Sections, Earth and Rock-Fill Dams

rolled core should be cracked or sheared by the action of the quake, it was expected that the granular material of the shoulders would immediately fill the crack to such an extent that no material amount of water would pass.

A most important feature of design on this project is the west abutment or retaining wall which separates the spillway from the west earth dam.

The experiments of Professor Jacobsen, as well as certain experiments from Japanese sources, indicated that the center of pressure of the dynamic forces produced by the earthquake was at about two thirds of the height of the wall above the base. Furthermore, researches by the TVA staff indicated that accelerations in the ground, particularly in saturated ground, would be several times the accelerations in the foundation rock in the same quake. Accordingly, the following procedure was established for the design of retaining walls.

The horizontal acceleration in the ground is assumed at 18% of gravity. This is a judgment assumption based on examination of records, certain limitations of the Kentucky project, and the report of Professor Jacobsen. This acceleration, combined with that of gravity, is applied to the Coulomb sliding wedge and the pressure on the wall is determined. The static pressure is subtracted from this quantity and the difference is designated the dynamic increment. This increment is applied when the wall has a small batter at two thirds of the height above the base and is assumed to be the resultant of a pressure varying from zero at the base to a maximum at the top of the fill. When the wall has a large batter (flatter than 12 on 4) the assumptions as to height and distribution are a little less severe.

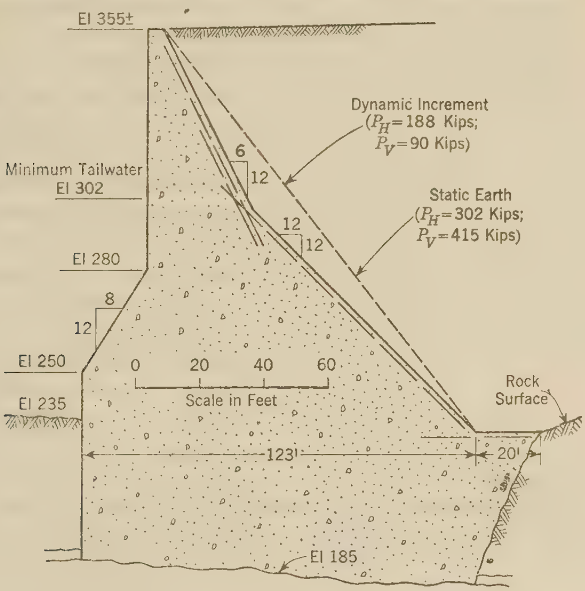


FIG. 9.—SECTION, WEST ABUTMENT OF KENTUCKY DAM

The application of this rule led to a maximum section on the west abutment as shown in Fig. 9. The base of this wall is extremely heavy whereas the top is heavily reinforced. The reinforced part is designed to withstand the dynamic increment at rather high stresses.

LOCKS

Kentucky lock is the only one on the Tennessee River designed by TVA, the others having been designed by U. S. Army Engineers. The hydraulic features of this lock have been discussed elsewhere[4] but it seems worth while to explain two unusual structural features. One is the river wall of the lock, as shown in Fig. 10. The usual drainage facilities were placed in the river wall of this lock and the wall was designed with earthquake provisions. The unusual section of the river wall is explained by the fact that the layout required that the lock be built with its river wall in the reservoir, the lower pintle being substantially on the axis of the dam. The inner face of the lock had to be a continuous vertical wall. The outer face was obliged to carry full reservoir head with the lock empty. After investigating numerous types, the design of the double wall was adopted. The chamber in the top of the wall is filled with gravel in order to secure the benefit of its weight.

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[4] "Basic Hydraulics of Water Storage Projects," by George R. Rich, *Civil Engineering*, August, 1944, p. 351.

The other feature is a floating guide structure at the upper end of the lock, built of reinforced concrete, although at one time the plan was to build it of steel. Allowance had to be made for an unusual fluctuation in lake level; and foundation conditions, particularly in view of the earthquake risk, were very unsatisfactory for any fixed structure. Accordingly this floating guide was developed. It is 440 ft long, 24 ft wide, and is designed to be submerged about 10 ft. In its connection to the lock it is provided with a ball-and-socket traveling anchorage and its end upstream is anchored by submerged cables leading to concrete anchor blocks in the lake bottom. Aside from these provisions, the design of the lock is along conventional lines.

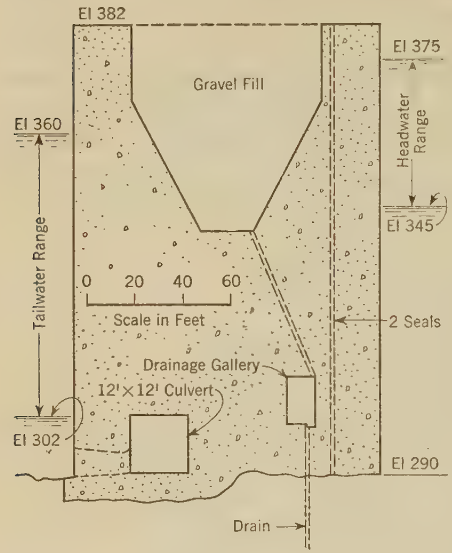

FIG. 10.—SECTION, RIVER WALL OF KENTUCKY LOCK

SUMMARY

This paper has described some of the most important features of design of TVA structures. Every project has its peculiar hydraulic and site requirements, and its design must be based upon them. However, the paper is intended to show the lines of thought governing the solution of some major problems from the structural standpoint. No mention has been made of various special designs such as the large surge tank at Apalachia Dam,[5] the several tunnels and their appurtenances, the chute spillways at Chatuge and Nottely dams,[6] and many other interesting structures that have been described adequately elsewhere.

ACKNOWLEDGMENT

The three papers of this Symposium represent activities of the design organization of the TVA. All engineering and construction work of the TVA is under the direction of C. E. Blee, M. Am. Soc. C. E., Chief Engineer. The design of structures and accessories, as well as the specification and inspection of equipment, has been the responsibility of the Design Department under the direction of George R. Rich, M. Am. Soc. C. E. until recently Chief Design Engineer.

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[5] *Civil Engineering*, October, 1943, p. 465.

[6] *Ibid.*, p. 418.

DESIGN OF HYDROELECTRIC STATIONS

By George P. Palo,[7] and Constant R. Marks, III,[8]
Assoc. Members, Am. Soc. C. E.

Synopsis

The title of this paper, if given its broadest interpretation, describes a subject of such scope as to require a treatise for its adequate discussion. Actually, this scope is limited to certain comments on general problems connected with station layouts and substructure design for large units. In discussing the layouts, emphasis will be placed upon designing the form of the structure most efficiently for the functions it must serve and upon the effect on the design of the interdependence of many of the factors involved. Illustrations will be drawn from the projects of the Tennessee Valley Authority (TVA), which include fourteen powerhouses and additions to one, for a total of forty-two units with a capacity of about a million and a half kilowatts. Discussion will be directed particularly to low-head, propeller type installations of which TVA has built seven, six with Kaplan runners. The locations of TVA projects are shown on the map in Table 1.

The Functional Layout

Securing a satisfactory and economical layout for the powerhouse should be one of the first concerns of the designer. A study to create a properly functional layout begins from the conception of a hydroelectric power station as a mechanism for converting the potential energy of water under head into electrical energy. The various features of the powerhouse may then be arranged in a manner suited to the several component functions necessary for accomplishment of this primary function.

Fig. 11 is a partly diagrammetric drawing of the Fort Loudoun powerhouse, planned to show its six functions and their arrangement: (1) Water flow; (2) flow of electricity; (3) mechanical control; (4) electrical control; (5) auxiliary electric equipment; and (6) auxiliary mechanical equipment. The most efficient layout is that in which each function is accommodated as directly as possible and without interference with other functions. The most economical structure is secured by compact arrangement of functions so as to avoid waste space in either initial or ultimate portions of the installation and by a plan in which as much as possible of the required space can be housed in relatively inexpensive types of construction. It is very difficult to state any general rules for layout, but the results of TVA experience may offer helpful hints for similar installations.

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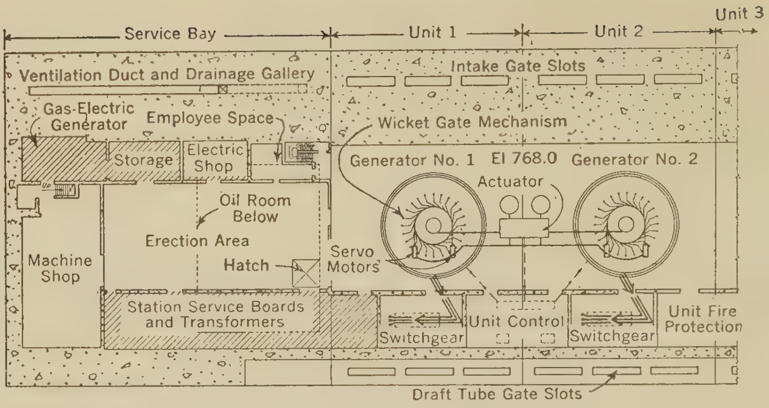
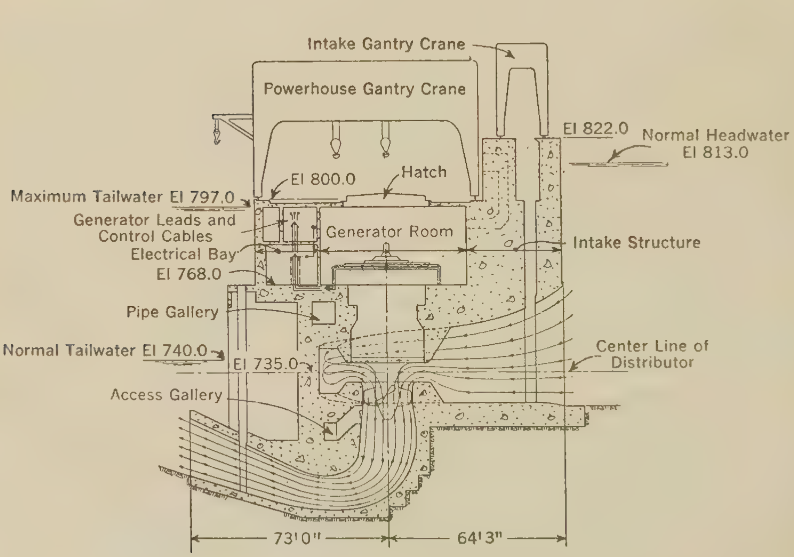
[7] Lt. (jg), U.S.N.R., Alexandria, Va.

[8] Senior Civ. Engr., TVA, Knoxville, Tenn.


Service Bay
Unit 1
Unit 2
Unit 3
Ventilation Duct and Drainage Gallery
Intake Gate Slots
Gas-Electric Generator
Employee Space
Wicket Gate Mechanism
Storage
Electric Shop
Generator No. 1
El 768.0
Generator No. 2
Actuator
Oil Room Below
Erection Area
Machine Shop
Servo Motors
Hatch
Station Service Boards and Transformers
Switchgear
Unit Control
Switchgear
Unit Fire Protection
Draft Tube Gate Slots

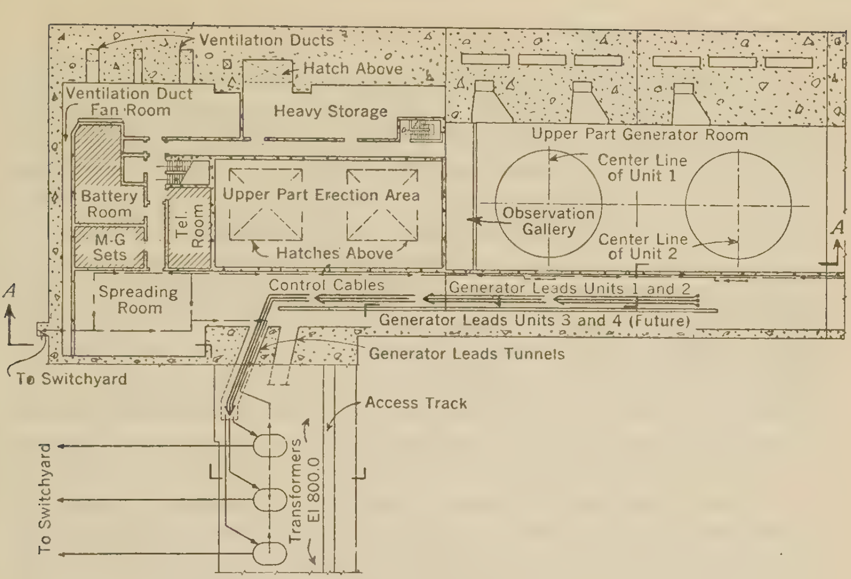
(a) PLAN OF GENERATOR ROOM FLOOR

LEGEND		
NO.	FUNCTION	SYMBOL
1	Water Passages	⟵ ⟵ ⟵
2	Electric Current	⟸
3	Mechanical Control	—
4	Electrical Control	—←—→——
5	Auxilary Electrical Equipment Space	▨
6	Auxilary Mechanical Equipment and Space for Employees and Visitors	▭

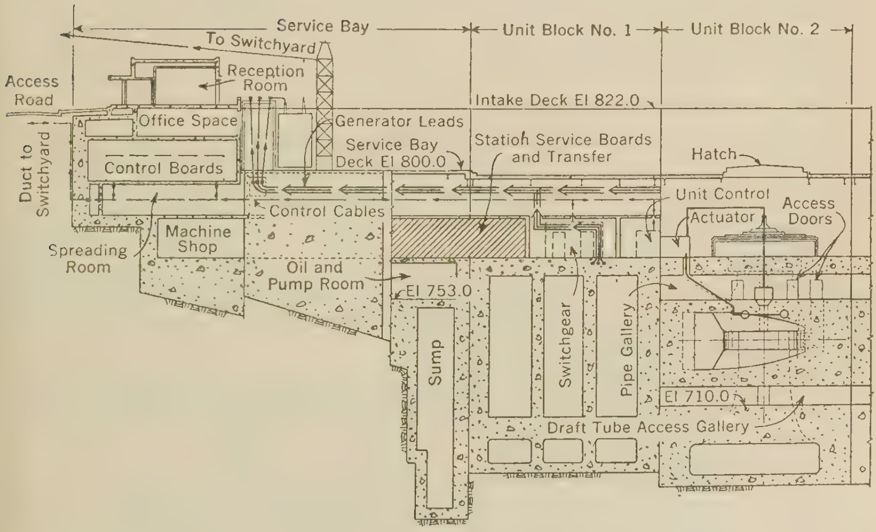

Intake Gantry Crane
Powerhouse Gantry Crane
El 822.0
Normal Headwater El 813.0
Hatch
El 800.0
Maximum Tailwater El 797.0
Generator Leads and Control Cables
Generator Room
Intake Structure
Electrical Bay
El 768.0
Pipe Gallery
Normal Tailwater El 740.0
El 735.0
Center Line of Distributor
Access Gallery
73'0"
64'3"

(b) TYPICAL SECTION ON CENTER LINE OF UNIT

FIG. 11.—FORT LOUDOUN POWER-


Ventilation Ducts
Hatch Above
Ventilation Duct
Fan Room
Heavy Storage
Upper Part Generator Room
Center Line
of Unit 1
Battery
Room
Tel.
Room
Upper Part Erection Area
Observation
Gallery
M-G
Sets
Hatches Above
Center Line
of Unit 2
A
A
Spreading
Room
Control Cables
Generator Leads Units 1 and 2
Generator Leads Units 3 and 4 (Future)
Generator Leads Tunnels
To Switchyard
Access Track
To Switchyard
Transformers
El 800.0

(c) PLAN AT EL 783.50


Service Bay
Unit Block No. 1
Unit Block No. 2
To Switchyard
Reception
Room
Access
Road
Intake Deck El 822.0
Office Space
Generator Leads
Duct to
Switchyard
Service Bay
Deck El 800.0
Station Service Boards
and Transfer
Hatch
Control Boards
Unit Control
Access
Doors
Control Cables
Actuator
Machine
Shop
Spreading
Room
Oil and
Pump Room
El 753.0
Sump
Switchgear
Pipe Gallery
El 710.0
Draft Tube Access Gallery

(*d*) SECTION *A-A*

HOUSE; FUNCTIONAL LAYOUT

Consider these functions in greater detail, as follows:

(1) The first function requires provision for the flow of the water through passages, where it imparts its energy to the turbine runner, and protection of the powerhouse against all expected conditions of headwater and tailwater. The waterways of this function, consisting of the scroll case and draft tube, are the basic factors determining the minimum outline for the powerhouse substructure. Waterway shapes have been simplified and standardized so that there is little difference between the requirements of the several turbine manufacturers. In fact, the designer may cooperate with manufacturers to obtain a single design in which any of them will install a turbine without qualifying his guarantee of its efficiency. This is particularly desirable if it is intended to complete, partly, the substructure to be used for future units. Further comment on this function will be made in the latter half of this paper, in which substructure design will be considered at some length.

(2) The second function involves provision for the flow of electric current, starting from the generator, through the conductors to the transformers, and thence to the switchyard and transmission lines. Layout for this function involves a consideration of the location of transformers on the intake deck, on the powerhouse roof, over the draft tube, or, at a greater distance from the generators, on the river bank near the powerhouse. Decision as to the best location depends on many factors, but in planning a layout the space requirement for the function should be understood clearly. In TVA practice, for example, transformers are usually on the bank, and low-tension electrical equipment is at a minimum. In the layout the engineer must plan for space for low-tension switchgear for each unit or pair of units and a clear run of the generator leads to the transformers, without interference with other features.

(3) The third function is the mechanical control of the turbine by means of the actuator, servomotors, and the gates or Kaplan mechanisms (see Fig. 11(*a*)). Layout for this function must envision the actuator cabinets and pressure tanks with piping to connect them to servomotors or Kaplan mechanisms. Economy of space and economy of direct connections are secured on many TVA layouts by placing actuator cabinets and pressure tanks on the generator-room floor, with the actuator cabinet planned, where possible, for the control of two units.

(4) Electrical control of the unit and the flowing current by means of unit controls and main control room linked with the necessary electric cables constitutes the fourth function.

(5) The fifth function concerns the auxiliary electrical equipment necessary for the operation of the main equipment and of the station itself.

(6) Finally, the sixth function constitutes the auxiliary mechanical facilities required for servicing the equipment, provision for access into and within the station, and space for use and convenience of employees and visitors.

The arrangement of the facilities serving functions (4) to (6) may vary greatly, depending on the number and size of the units, topography, type of superstructure, and other conditions. Design for economy and simplicity of layout can be aided by reclassifying these facilities, and the second and third functions as well, into unit service features and station service features. Unit service features include the mechanical control of the unit, the unit portion of the electrical control, and the provisions for physical access to the unit areas and access of leads, control cables, and pipes. Station service features include the main control area and nearly all of the auxiliary mechanical and electrical equipment as well as the employee and public space. The unit service areas are repetitive for each unit. The station service features, on the other hand, need be provided only once for the entire station. If the number of units is large and the cross section of a unit block is to be kept typical, structural economy can be secured only by limiting the space in the unit block to the minimum required for unit service features. This is particularly true if provision for future units with the same cross section is being made; for, in this case, station features must all be included in the initial construction and only additional unit features are needed when the structure for the future units is completed. Economy generally demands that station features be located in one or more service buildings separate from the unit blocks. An exception is an installation of from one to about three units where shortness of connections and operating convenience may favor location of station service features in a more central location.

The Fort Loudoun cross section is typical for several of the low-head stations of the TVA. The unit service area uses a portion of the generator-room floor for the actuator, a two-story electrical bay (one for unit control and switchgear, and the other for clear runs of generator leads and control cables), and galleries in the heavy concrete sections above and below the scroll case. Recesses from these galleries between units provide space for auxiliary mechanical unit service features which is conveniently segregated from other features. The resulting structure is a very compact, economical cross section.

The station service area is most commonly housed in a service bay at one end of the series of unit blocks. This structure usually provides an erection area at the level of the generator-room floor. As shown for Fort Loudoun, a machine shop at the same level, and space on a lower floor for station sump pumps and lubricating oil servicing, are desirable and may be planned readily in conjunction with the mechanical equipment at the same level in the galleries and recesses in the unit blocks. Other station service features should be located within the service bay so as to serve best the function of each feature and to fit them into an economical layout which coordinates with the main functions of the powerhouse. Space provided may be affected by the shape of service bay best suited to the structural problem of its design. If the structure is located in the river channel, it must be massive enough to be stable against high tailwater levels and uplift, against headwater if it forms a part of the dam, and possibly as a retaining wall if it is the transition between concrete and earth sections of the dam. If the river bank consists of suitable material for foundations and dam abutment, some of this massive structure may be saved by locating the service bay so close to the bank that a part of it may be founded at elevations well above the river bed, as was done at Fort Loudoun. Likewise, if space in the river channel is limited or if structures within it are expensive due to possibility of high tailwater, large savings may be made by a layout in which all station service facilities possible are moved to a separate structure located on the bank above maximum tailwater level. The Watts Bar project of the TVA is an example of such practice.

THE SUPERSTRUCTURE

The planning of the powerhouse layout requires early decision as to the use of indoor, outdoor, or semi-outdoor type of superstructure. A consideration of factors governing this choice will illustrate the interdependence of some of the several functions thus far discussed.

For maximum tailwater below the level of the generator-room floor, only indoor and outdoor types usually need be considered. Controlling factors are: Relative economy; operating convenience (involving alternate solutions to the problem of the mechanical service function); and appearance. There are many examples of each type of design, indicating the difficulty of evaluating these factors, and the choice of type must be considered largely a matter of organization preference and policy.

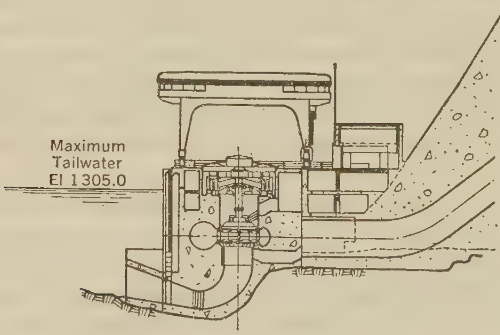
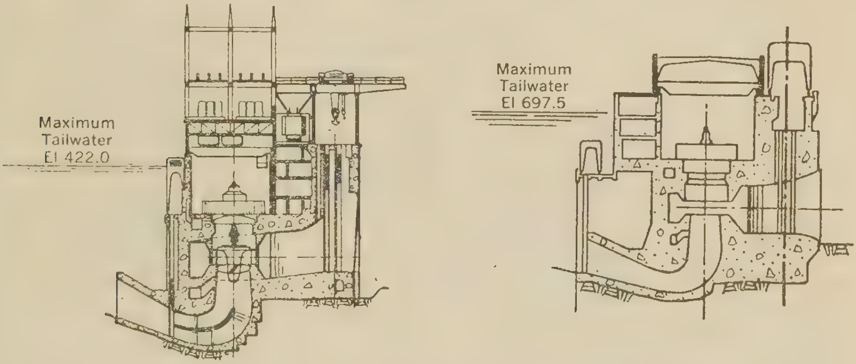


FIG. 12.—HIWASSEE POWERHOUSE

As larger streams with flatter gradients are developed and as flood studies and experience with actual developments point toward more conservative choice of maximum flood, many projects must provide for tailwater well above the generator-room floor. In this case layout requirements, particularly the one of access to the powerhouse, become of great importance in determining the proper type of superstructure.

For maximum tailwater a moderate distance above the generator-room floor level, the full outdoor type of design may be modified by building water-bearing walls to a height approximating that of the generator barrel and by roofing over the generator area. The resulting section may be like that for the Hiwassee project (Fig. 12). The space enclosed around the generator is useful, especially for electric services, and this type of outdoor station has sometimes been used for that reason alone. For high tailwater, access is at the level of the roof.

The indoor powerhouse can be adapted with minor modifications for tailwater a short distance above the generator-room floor in several ways. One solution is to locate the main entrance on a mezzanine which is above tailwater. Both size and elevation of this floor must be considered with care to minimize waste space either on or below the floor. Another solution, preferably only for small stations, may locate the entrance to the powerhouse at the generator-room floor level and reach it by a protected ramp which slopes from an entrance area above high tailwater; or, in rare circumstances, it may be considered reasonable to protect the powerhouse entrance with a temporary bulkhead set in place during floods. For conditions of tailwater more than about 15 ft above the generator floor, these solutions are rarely satisfactory. The indoor type of powerhouse can be adapted economically to very high tailwater levels only by providing access through a vertical hatch. In the Pickwick project (Fig. 13(*a*)), delivery of equipment is by barge and a crane transfers it from the barge to a hatch in the roof of the powerhouse immediately above the erection area. At the Chickamauga project (Fig. 13(*b*)), there is a railroad or truck access to the intake deck. The gantry crane used for moving intake gates can also handle incoming equipment and drop it through a hatch to the level of the generator-room floor. At this level it is moved downstream on a transfer car until within reach of the crane in the generator room. It will be noted that this layout creates waste space at the hatch opening and also prevents, for some 45 ft above the generator-room floor, any straight run of control cables or generator leads upstream from the generators through the length of the powerhouse. This is one of the factors requiring that the electrical bay for the project be on the downstream side of the generator room. At Pickwick, on the other hand, there were a number of factors pointing to an upstream electrical bay, and the need for a cable gallery necessitated the opening in the roof.

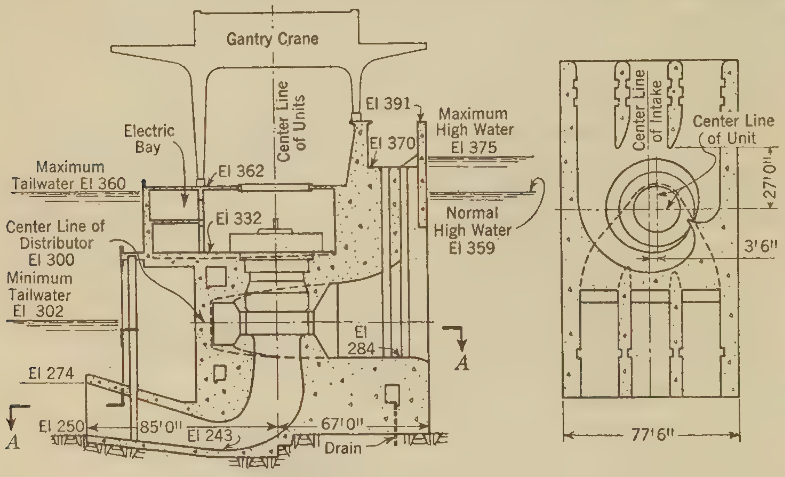


(*a*) PICKWICK LANDING (*b*) CHICKAMAUGA

FIG. 13.—INDOOR POWERHOUSES

The TVA has found the semi-outdoor type of powerhouse well adapted to many conditions. In some projects it has been the only reasonable solution. As illustrated by the powerhouse at Kentucky Dam (Fig. 14), the generator room is about 30 ft high, ample to clear the Kaplan head on the unit. The adjacent electrical bay is two floors high and the downstream and end walls provide protection against tailwater 28 ft above the generator-room floor. A gantry crane hoists equipment through hatches in the roof, and the main access road comes in at roof level. If the Kentucky section is compared to that for Hiwassee, it will be noted that the main difference is that the roof at Kentucky is at sufficient height to justify the design of a generator room, whereas at Hiwassee the lesser height leads to the adoption of a concrete barrel around the generator.

The TVA has used the semi-outdoor powerhouse at five projects and this design has proved to interfere less with other functions and to be more satisfactory than any other. At four of the projects (two Kaplan and two Francis wheel installations), tailwater may be 20 ft to 30 ft above the generator-room floor. At Watts Bar (the first project to use this design), maximum tailwater is only 13.5 ft above the generator-room floor. Here, however, the service bay is built against a precipitous abutment which precluded use of an end entrance. To have used a downstream entrance to a mezzanine would have interrupted cable runs in the downstream electrical bay which were necessary to permit use of what was considered the most economical cross section of intake and powerhouse.


Gantry Crane
Center Line of Units
Electric Bay
El 391
El 370
Maximum High Water El 375
Maximum Tailwater El 360
El 362
El 332
Normal High Water El 359
Center Line of Distributor El 300
Minimum Tailwater El 302
El 284
A
El 274
A
El 250
85'0"
El 243
67'0"
Drain
Center Line of Intake
Center Line of Unit
27'0"
3'6"
77'6"

TYPICAL SECTION SECTION *A-A*

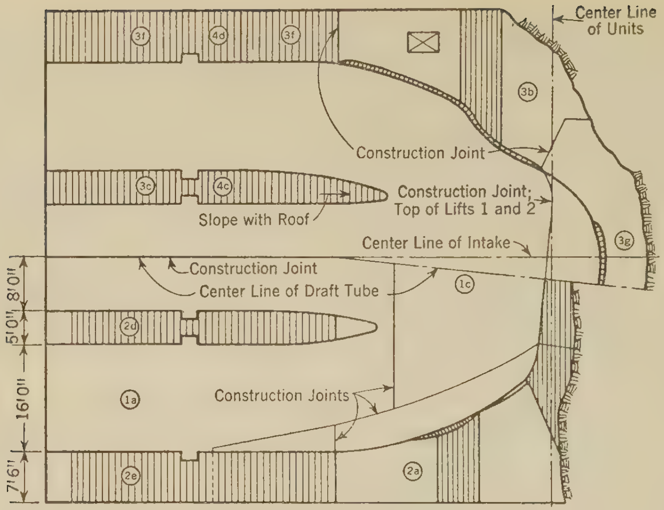
FIG. 14.—SEMI-OUTDOOR POWERHOUSE

SUBSTRUCTURES FOR LARGE PROPELLER UNITS

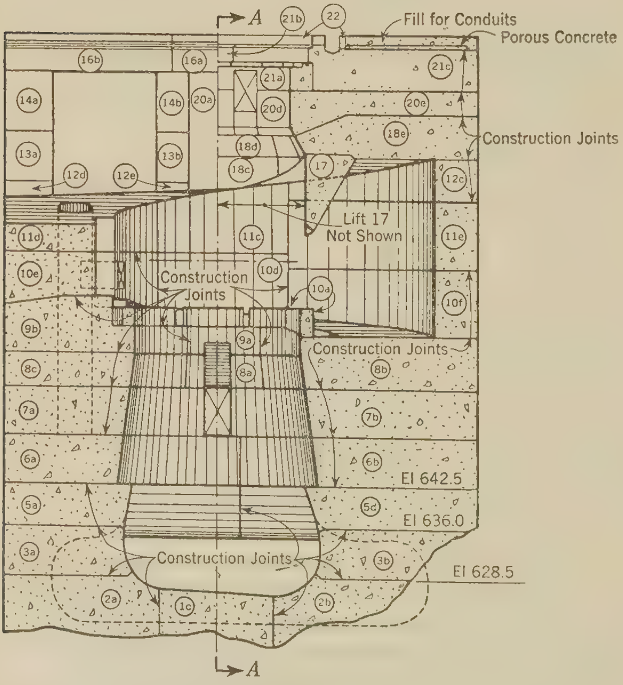
The shape of the substructure required for large propeller units prohibits calculation of anything more than a crude approximation of the stresses involved. Even structures with identical outline and identical water loads will have different stresses, depending upon the sequence, size, and shape of the concrete pours, the conditions of temperature encountered during the construction, and the properties of the concrete used. Indeed these factors may account for the major portion of the stress in the structure. The objectives of the structural design are (*a*) to obtain an economical structure safe from actual failure, and (*b*) to make the structure watertight. The fact that actual failures are almost unknown indicates universal success in the achievement of the safety, but not necessarily the economy, of objective (*a*). That objective (*b*) is obtained less frequently is evidenced by casual inspection of many power plants. The benefits of a watertight structure are: More satisfactory operating conditions; less depreciation and maintenance; and assurance of greater over-all strength in the structure. In addition, watertightness indicates reduction of movements in the structure, so that there is more certainty that the turbine and generator equipment will be erected and remain more closely to perfect alinement. These benefits are deemed to warrant the extra care and expense in the design and construction needed to obtain them.

Safety and Economy.—The cross section of the Fort Loudoun powerhouse is an example of structural safety, with economy, obtained by designing the intake and powerhouse to function as a unit. Two units were completed initially but the success of the section shown in Fig. 11 was dependent on its adaptation to initial construction for two future units. For these units the section is complete except for most of the scroll-case roof, the lower cone, and a recess sufficient to permit the placing of the draft-tube liner. Such construction requires extensive commitments relative to waterways in these future units. Equal commitments in plants built in the 1920's were found unsatisfactory within a few years because of development of the propeller wheel and the use of the elbow draft tube. There have been no major changes in turbines and waterway design since before 1935 and those commitments may now be made more freely for future units to be installed in a reasonable time. This is a fortunate condition because, in many large propeller installations, foundation conditions require a design that will buttress the inadequacy of the intake foundation when the draft tube is excavated to a greater depth than that required for the intake. This buttressing is obtained by completing a large part of future units. In addition, partial construction of future units is almost always included to permit unwatering the area of future construction by dropping stop logs or gates into slots in the partly completed draft tube.

The Roof of the Intake.—Perhaps the most critical design feature in a cross section for a future unit, as at Fort Loudoun, is the design of the roof of the intake. Water passages of the intake are protected by trashracks set against the upstream face and controlled by service gates moved by a gantry crane on the deck. For rare occasions requiring closure upstream from the service gate, trashracks are removed by an auxiliary boom on the gantry and replaced by stop logs. This simple method of control is a factor that aids the development of a short, economical intake. For the head on this project, however, the intake is not stable by itself and omission of the scroll-case roof requires that much of the reservoir water load be transferred by the intake roof from the intermediate to the end piers, which, with the scroll-case walls and draft tube, have the length necessary for stability of the structure. The framing of the intake roof with the end piers should be such as to insure spreading of the load so that the full length of the draft tube is actually effective as a gravity section. At Fort

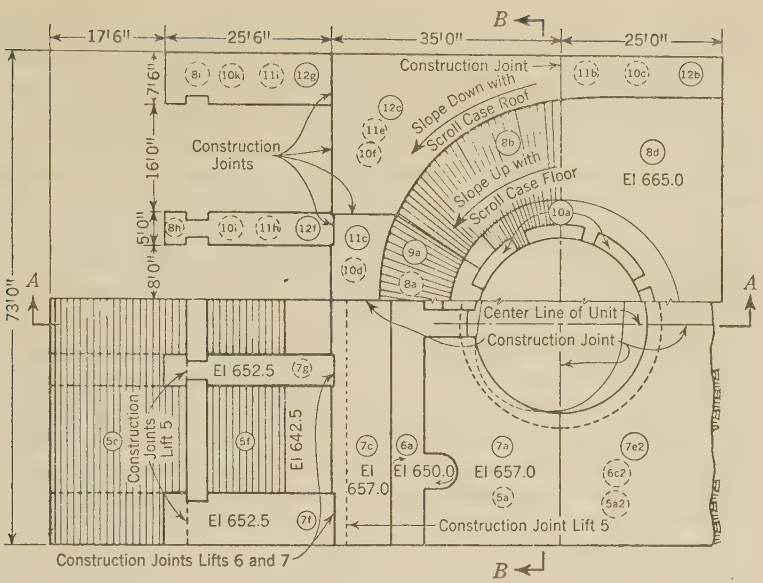

Center Line of Units
3f
4d
3f
3b
Construction Joint
3c
4c
Construction Joint; Top of Lifts 1 and 2
Slope with Roof
Center Line of Intake
3g
Construction Joint
Center Line of Draft Tube
1c
8'0"
5'0"
2d
16'0"
1a
Construction Joints
7'6"
2e
2a

(*a*) HALF PLAN, LIFTS 1 TO 4

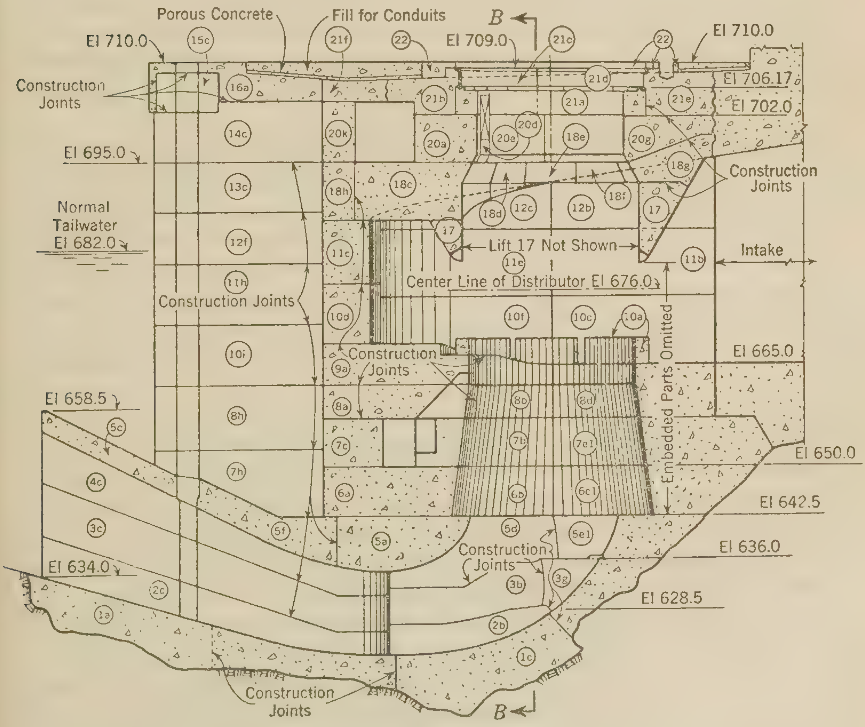

A
21b
22
Fill for Conduits
Porous Concrete
16b
16a
21c
21a
14a
14b
20a
20d
20e
13a
13b
18d
18e
Construction Joints
12d
12e
18c
17
12c
Lift 17 Not Shown
11d
11c
11e
10e
10d
Construction Joints
10a
10f
9b
9a
Construction Joints
8c
8a
8b
7a
7b
6a
6b
El 642.5
5a
5d
El 636.0
3a
Construction Joints
3b
El 628.5
2a
1c
2b
A

(*b*) SECTIONAL ELEVATION *B-B*

Fig. 15.—Layout of Construction


17'6"
25'6"
35'0"
25'0"
B
Construction Joint
11b
10c
12b
8i
10k
11i
12g
Slope Down with Scroll Case Roof
12c
11e
10f
8b
Slope Up with Scroll Case Floor
8d
El 665.0
Construction Joints
7'6"
16'0"
5'0"
8'0"
73'0"
A
8h
10i
11h
12f
11c
10d
9a
8a
10a
Center Line of Unit
Construction Joint
El 652.5
7g
El 642.5
Construction Joints Lift 5
5c
5f
7c
El 657.0
6a
El 650.0
7a
El 657.0
7e2
6c2
5a
5a2
El 652.5
7f
Construction Joint Lift 5
Construction Joints Lifts 6 and 7
B

(*c*) HALF PLAN, LIFTS 5 TO 12


Porous Concrete
Fill for Conduits
B
El 710.0
15c
21f
22
El 709.0
21c
22
El 710.0
Construction Joints
16a
21d
El 706.17
21b
21a
21e
El 702.0
El 695.0
14c
20k
20a
20d
20e
18e
20g
18g
Construction Joints
13c
18h
18c
18f
Normal Tailwater El 682.0
12f
18d
12c
12b
17
17
Lift 17 Not Shown
Intake
11c
11e
11b
11h
Construction Joints
Center Line of Distributor El 676.0
10d
10f
10c
10a
10i
Construction Joints
9a
Embedded Parts Omitted
El 665.0
8a
8b
8d
El 658.5
8h
5c
7c
7b
7e1
El 650.0
4c
7h
6a
6b
6c1
El 642.5
3c
5f
5a
5d
5e1
Construction Joints
El 636.0
El 634.0
2c
3b
3g
El 628.5
1a
2b
1c
Construction Joints
B

(*d*) SECTIONAL ELEVATION *A-A*

JOINTS, WATTS BAR POWERHOUSE

Loudoun this was accomplished by placing, initially, a part of the scroll-case roof, upstream from the cone, to function as a haunch on the intake roof beam.

Watertight Structure.—Throughout the design, the civil engineer must constantly consider the problems of volume change if he wishes to obtain a waterproof structure. Volume change as applied to a powerhouse substructure (most of which will be permanently submerged) arises largely from temperature changes in the concrete—initially as a result of the heat of hydration of the cement and later as a result of the annual temperature cycle. The magnitude of the change, and hence of its accompanying problem, will depend in part on the season of construction and the thermal coefficient of the concrete. These factors indicate the individuality of projects; yet certain general approaches to design are desirable even under most favorable conditions.

A problem encountered early in construction is the spacing of contraction joints in the powerhouse substructure. Units spaced 35 ft to 40 ft or more on centers should be built with contraction joints between units. Smaller unit spacings may be built with two units between pairs of contraction joints. For simplicity and clean design, in both substructure and superstructure, it is desirable that contraction joints be straight. The unit is always offset toward the small side of the scroll case and, if the draft tube is made symmetrical about the center line of unit, the end piers of the draft tube will not be of equal thickness. This lack of symmetry is undesirable in a structure planned for temperature changes. Best results are obtained if the draft tube is offset in the heavy elbow section so as to bring the center line of the downstream end of the draft tube on the same center line as that for the intake, and midway between the contraction joints.

With contraction joints established, consideration may turn to the individual unit. Fig. 15 affords some conception of the planning of the construction joints in the Watts Bar Hydro Station. Units are spaced 73 ft on centers, and the length from the construction joint at the intake to the downstream end of the draft tube is 100 ft. The design considers concrete outline, reinforcement, construction joints, and special concrete pouring sequence as an integrated problem with the objective of a strong, watertight substructure which can be obtained with a minimum of restriction on the construction forces.

Draft Tube.—In the draft tube the length of individual pours is held to a maximum of about 40 ft by construction joints near the center line of units and near the downstream wall of the powerhouse. These joints are made with a minimum of offsets at successive lifts so that any opening of the construction joint will not continue into monolithic concrete in another pour. Where stability requires that shears be transferred across vertical joints, joint surfaces are roughened by jackhammers before concrete is poured against them. The downstream end of the draft tube may be designed as a one-story frame. End piers for large units should be 6 ft to 8 ft thick. Intermediate piers must be 5 ft to 6 ft thick to provide for reasonable thickness at gate slots. As temperature change is considered in the frame design, the roof slab should be as light as consistent with reasonable reinforcing requirements. A thickness of 4 ft is recommended. One or two construction joints in its width also minimize temperature stress. Horizontal steel in the draft-tube piers is designed as $1\frac{1}{4}$-in. square bars, 12 in. on centers, so as to spread the cracks that will result as the concrete changes length over the restraining rock foundation.

The key to successful construction of the massive elbow of the draft tube is found in the proper concreting of the roof pours. In the entire substructure a time interval of three days between successive vertical lifts is specified; but, where effects of temperature are so critical that more complete dissipation of the heat of hydration is essential, longer intervals between successive lifts or adjacent pours are required. The elbow zone is particularly important as cracks will cause leaks into the draft-tube gallery with eventual large accumulations of laitance. The roof pour is made with one or two construction joints in the width of the unit, and a delay of seven days is specified between the placing of adjacent pours and between the first and second lifts. This gives the concrete time to dissipate a part of its heat of hydration and also to develop strength to carry the weight of succeeding pours. Roof reinforcement of $1\frac{1}{4}$-in. square bars 9 in. on center has proved satisfactory.

Additional features of interest in the design of the draft tube are the slope of the floor and the outline of the upstream nose of the intermediate piers. As the low point of the draft-tube excavation is usually well below the level of rock in the river bed, a steeply sloping draft-tube floor is usually desirable to reduce rock excavation not only in the draft tube but also in the tailrace channel. On TVA projects, floor slopes of 1 on 3 or even 1 on 2.5 have provided perfectly satisfactory tubes. Since most of the energy remaining in the flowing water is lost on emergence from the draft tube, it is necessary only to have the draft-tube outlet low enough to seal during the operation of the unit.

For hydraulic reasons manufacturers often request that the upstream end of the intermediate piers be rounded to a small radius and located as far downstream as possible. Structurally, these points are obviously subjected to considerable stress. On the Guntersville project an installation of Carlson strain and stress meters indicated stresses in the pier noses of about 400 lb per sq in. Stress analysis which considered concrete placing sequence and length change of piers under load had previously indicated this to be the probable stress although higher values had been considered possible. To allow for approximations in these data, pier noses on TVA projects are located about as shown for Watts Bar. They are rounded to a 6-in. radius, which is a good radius hydraulically, but also large enough to permit embedment of substantial, tied-column type of reinforcement.

Scroll Case.—The scroll case is usually built in two stages, the first of which permits the superstructure erection before the delivery of the parts of the turbine that are to be embedded. Such parts can then be handled by the powerhouse crane. The first stage consists of a U-shaped section formed by the downstream and side walls of the scroll-case passage. The downstream wall is carried to full height to provide support for the superstructure construction. The side walls may be carried to full height or may stop at the top of the scroll-case waterway. The design of the side walls of the scroll case requires that they be strong enough to carry the final water loads by beam action between the floor and the roof of the scroll case and that they be capable

Loudoun this was accomplished by placing, initially, a part of the scroll-case roof, upstream from the cone, to function as a haunch on the intake roof beam.

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A problem encountered early in construction is the spacing of contraction joints in the powerhouse substructure. Units spaced 35 ft to 40 ft or more on centers should be built with contraction joints between units. Smaller unit spacings may be built with two units between pairs of contraction joints. For simplicity and clean design, in both substructure and superstructure, it is desirable that contraction joints be straight. The unit is always offset toward the small side of the scroll case and, if the draft tube is made symmetrical about the center line of unit, the end piers of the draft tube will not be of equal thickness. This lack of symmetry is undesirable in a structure planned for temperature changes. Best results are obtained if the draft tube is offset in the heavy elbow section so as to bring the center line of the downstream end of the draft tube on the same center line as that for the intake, and midway between the contraction joints.

With contraction joints established, consideration may turn to the individual unit. Fig. 15 affords some conception of the planning of the construction joints in the Watts Bar Hydro Station. Units are spaced 73 ft on centers, and the length from the construction joint at the intake to the downstream end of the draft tube is 100 ft. The design considers concrete outline, reinforcement, construction joints, and special concrete pouring sequence as an integrated problem with the objective of a strong, watertight substructure which can be obtained with a minimum of restriction on the construction forces.

Draft Tube.—In the draft tube the length of individual pours is held to a maximum of about 40 ft by construction joints near the center line of units and near the downstream wall of the powerhouse. These joints are made with a minimum of offsets at successive lifts so that any opening of the construction joint will not continue into monolithic concrete in another pour. Where stability requires that shears be transferred across vertical joints, joint surfaces are roughened by jackhammers before concrete is poured against them. The downstream end of the draft tube may be designed as a one-story frame. End piers for large units should be 6 ft to 8 ft thick. Intermediate piers must be 5 ft to 6 ft thick to provide for reasonable thickness at gate slots. As temperature change is considered in the frame design, the roof slab should be as light as consistent with reasonable reinforcing requirements. A thickness of 4 ft is recommended. One or two construction joints in its width also minimize temperature stress. Horizontal steel in the draft-tube piers is designed as 1¼-in. square bars, 12 in. on centers, so as to spread the cracks that will result as the concrete changes length over the restraining rock foundation.

The key to successful construction of the massive elbow of the draft tube is found in the proper concreting of the roof pours. In the entire substructure a time interval of three days between successive vertical lifts is specified; but, where effects of temperature are so critical that more complete dissipation of the heat of hydration is essential, longer intervals between successive lifts or adjacent pours are required. The elbow zone is particularly important as cracks will cause leaks into the draft-tube gallery with eventual large accumulations of laitance. The roof pour is made with one or two construction joints in the width of the unit, and a delay of seven days is specified between the placing of adjacent pours and between the first and second lifts. This gives the concrete time to dissipate a part of its heat of hydration and also to develop strength to carry the weight of succeeding pours. Roof reinforcement of 1¼-in. square bars 9 in. on center has proved satisfactory.

Additional features of interest in the design of the draft tube are the slope of the floor and the outline of the upstream nose of the intermediate piers. As the low point of the draft-tube excavation is usually well below the level of rock in the river bed, a steeply sloping draft-tube floor is usually desirable to reduce rock excavation not only in the draft tube but also in the tailrace channel. On TVA projects, floor slopes of 1 on 3 or even 1 on 2.5 have provided perfectly satisfactory tubes. Since most of the energy remaining in the flowing water is lost on emergence from the draft tube, it is necessary only to have the draft-tube outlet low enough to seal during the operation of the unit.

For hydraulic reasons manufacturers often request that the upstream end of the intermediate piers be rounded to a small radius and located as far downstream as possible. Structurally, these points are obviously subjected to considerable stress. On the Guntersville project an installation of Carlson strain and stress meters indicated stresses in the pier noses of about 400 lb per sq in. Stress analysis which considered concrete placing sequence and length change of piers under load had previously indicated this to be the probable stress although higher values had been considered possible. To allow for approximations in these data, pier noses on TVA projects are located about as shown for Watts Bar. They are rounded to a 6-in. radius, which is a good radius hydraulically, but also large enough to permit embedment of substantial, tied-column type of reinforcement.

Scroll Case.—The scroll case is usually built in two stages, the first of which permits the superstructure erection before the delivery of the parts of the turbine that are to be embedded. Such parts can then be handled by the powerhouse crane. The first stage consists of a U-shaped section formed by the downstream and side walls of the scroll-case passage. The downstream wall is carried to full height to provide support for the superstructure construction. The side walls may be carried to full height or may stop at the top of the scroll-case waterway. The design of the side walls of the scroll case requires that they be strong enough to carry the final water loads by beam action between the floor and the roof of the scroll case and that they be capable

CIVIL ENGINEERING FEATURES OF STEAM PLANT AT WATTS BAR

By Adolf A. Meyer,[9] M. Am. Soc. C. E.

Synopsis

The civil design features of the Watts Bar Steam Station are described in this paper. The general station design and its functions within the system defined as the Tennessee Valley Authority (TVA) have been described elsewhere.[10] Only references to the general design necessary for the description of the civil engineering features are included herein.

The paper gives the reasons why the station was located on the high ground about 600 ft away from the river rather than at the river front, as is the more common solution. It outlines the characteristics of the foundation rock—a shale of very low structural strength—and the precautions used in the design of the substructure to avoid detrimental settlements of the building and to eliminate the formation of shrinkagé cracks.

The paper explains the conditions which led to the adoption of a gravity feed system for the condensing water supply, and it describes the main features and the functions of this system. It further describes the planning and design of the powerhouse structures, the yards for coal handling and coal storage, and the transportation facilities connecting these yards.

The Watts Bar Steam Station was authorized on July 31, 1940, shortly after the fall of Frânce. It was to increase the power capacity of the TVA system to insure adequate supply for the armament program then under consideration. The initial authorization was for a capacity of 120,000 kw. The ultimate station installation was to be 240,000 kw. The load center for the armament program was assumed to be in east Tennessee. As fuel costs in this area are favorable, the logical site for the new steam station was in this section at a location near a primary substation with ample transmission interconnections, good transportation facilities, and an adequate cooling water supply. These conditions were best met near the site of the Watts Bar hydroelectric development, on the Tennessee River halfway between Knoxville and Chattanooga, Tenn. At that time (1940) this project was in the course of construction. By combining the two projects, substantial advantages could be obtained from savings in construction plant and in construction supervision, in addition to the permanent operating advantages.

The paper will be limited to a treatment of the civil engineering features of the station. No sharp separation of these elements from the integrated plant can be made. However, civil engineering design predominates in the following features: Powerhouse substructure, powerhouse superstructure, condensing water supply, yards for coal storage and coal handling, railroad facilities, road facilities, and river dock facilities.

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[10] "The Watts Bar Steam Power Station of the TVA," by G. R. Rich and R. T. Mathews, *Mechanical Engineering*, November, 1941, p. 773.

At Watts Bar, the combined cost of these items represented approximately 20% of the total cost of the station. On first appearance it may be concluded that they did not affect the economy of the station to a marked degree. This impression, however, would be misleading. The largest items of cost are incurred in the main equipment, the purchase price of which is not very flexible for given specifications. Therefore, the cost increment involved by the civil engineering features is the most variable part when comparing different stations. For these reasons, it was considered a matter of great importance for the over-all economy of the station that a careful evaluation of the civil engineering features be the guiding rule in the selection of the site.

The Watts Bar Dam site is characterized by a steep bluff, approximately 150 ft high, on the right bank of the river. A railroad is accessible from this side only, which eliminated from consideration the wide flood plain on the left bank of the river. Sites upstream and downstream of the dam on the right bank were investigated. The costs at the upstream sites were high because an excessive amount of rock excavation was required. Approximately one-half mile downstream from the dam where the river bluff recedes, very favorable conditions for foundations and yard grading could be found. There the rock underlying the flood plain gradually rises away from the river. After a study of requirements for condensing water circulation and of the protective measures against water pressures during flood stages had established the optimum elevation of the building foundation, the specific site of the powerhouse could be chosen at a place where firm rock was at this optimum elevation. Railroad and highway facilities and a wharf area at the river front were all in immediate proximity to this area.

The generating station was built to house four units of 60,000 kw each. The plant is designed upon the unit principle; that is, all auxiliary and control equipment is arranged to allow each boiler-turbo-generator unit to operate as a self-contained plant (see Fig. 17).[11] At each such unit, coal is delivered from a bunker to four pulverizers which connect to the eight burners at the boiler. The boiler has a rated capacity of 600,000 lb of steam per hour. Steam, with a pressure of 900 lb per sq in. and a temperature of 900° F, is furnished to the 17-stage, single-cylinder turbine, which operates at a speed of 1,800 rpm. Current is generated at 13,800 v, which is stepped up to 154,000 v at an outdoor transformer yard adjacent to the turbine room. Because of the urgency of early completion of the station, all equipment was selected on the basis of established proven merit, with departures into new fields of design purposely ruled out.

The framing layout of the steel superstructure was developed hand-in-hand with the early equipment arrangements. The housing enclosure was to be kept to a minimum size and was to be built of members easily procurable, provided that these objectives did not prevent obtaining an accessible arrangement of all equipment.

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[11] "The Watts Bar Steam Power Station of the TVA," by G. R. Rich and R. T. Mathews, *Mechanical Engineering*, November, 1941, Fig. 4, p. 776.

No contraction joints were used in the framing for the 250-ft building. Bracing was introduced close to the sources of horizontal loads. The end bays, where wind and crane bumper loads act, are braced.

For reasons of economy and dependability of service, the coal bunkers are designed to contain a supply of coal sufficient for 1.5 days. They are large structures of 1,250-ton capacity each. The plates in the lower part are of special abrasion-resistant steel. All other bin plates are of copper-bearing steel to retard corrosion.

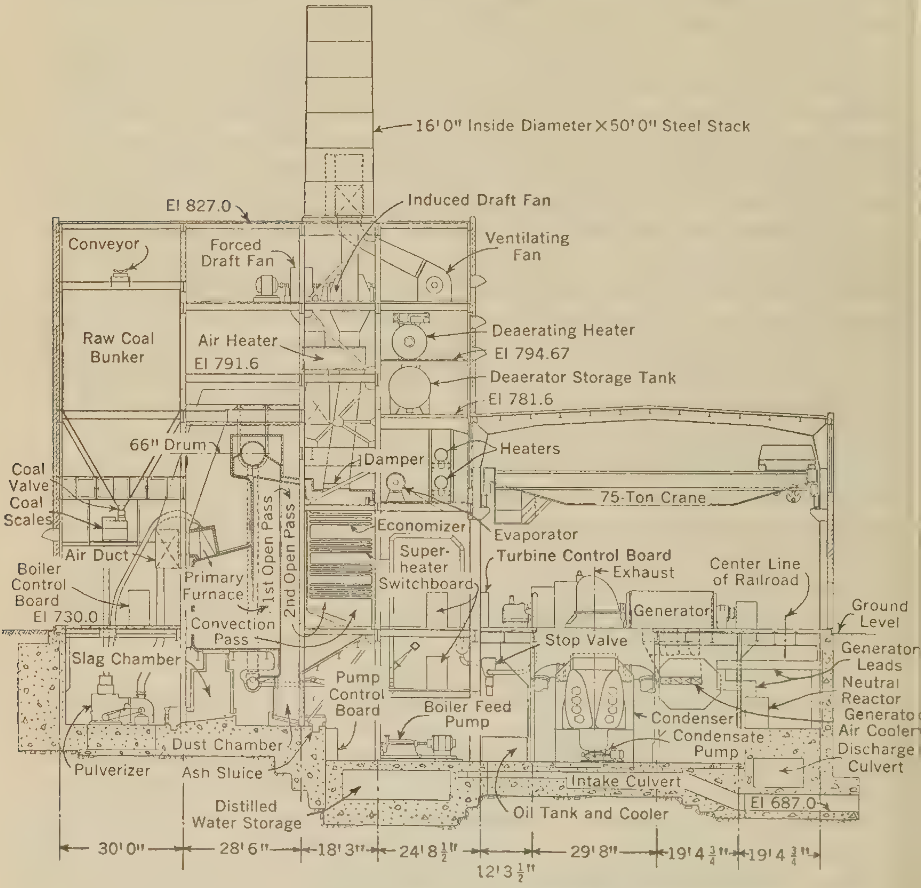
16'0" Inside Diameter × 50'0" Steel Stack
El 827.0
Induced Draft Fan
Conveyor
Forced Draft Fan
Ventilating Fan
Raw Coal Bunker
Air Heater
El 791.6
Deaerating Heater
El 794.67
Deaerator Storage Tank
El 781.6
66" Drum
Heaters
Damper
Coal Valve
Coal Scales
75-Ton Crane
Economizer
Evaporator
Super-heater
Switchboard
Turbine Control Board
Exhaust
Center Line of Railroad
Air Duct
Boiler Control Board
El 730.0
Primary Furnace
1st Open Pass
2nd Open Pass
Generator
Ground Level
Convection Pass
Stop Valve
Slag Chamber
Generator Leads
Neutral Reactor
Generato
Air Cooler
Pump Control Board
Boiler Feed Pump
Condenser
Condensate Pump
Discharge Culvert
Dust Chamber
Pulverizer
Ash Sluice
Intake Culvert
Distilled Water Storage
El 687.0
Oil Tank and Cooler
30'0"
28'6"
18'3"
24'8½"
12'3½"
29'8"
19'4¾"
19'4¾"

FIG. 17.—CROSS SECTION SHOWING ARRANGEMENT OF EQUIPMENT

The heavy boiler loads are suspended from heavy girders approximately 50 ft above the generator floor. For the generator-room framing, a welded frame fabricated from 33-in. wide-flange (WF) beams was used. Because this type of construction saved approximately 7 ft of building height, its cost compared favorably with the conventional truss layout. In appearance it is superior to any type of truss. The detail of the crane bracket itself is worth mentioning. A section of the column web, 3 ft long, was removed and the inside column flange was slotted. The 3-in.-thick bracket plate was then inserted and welded on all sides.

Because early delivery of the fabricated steel was essential, built-up sections and special shapes were generally avoided. Continuity of the framing helped a good deal in accomplishing this goal. The total weight of the steel framing of the completed four-unit powerhouse amounted to 4,200 tons.

The station location could be chosen at a place where firm rock was just slightly above the elevation of the boiler foundation. For the low part of the foundation under the condenser pit, the rock was removed easily (see Fig. 18).

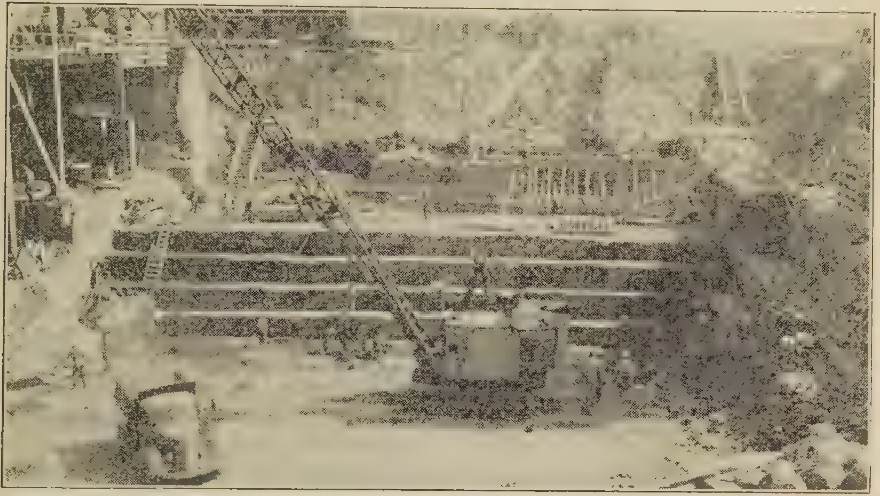

Fig. 18.—Excavation for the Powerhouse

However, this very favorable condition had one serious drawback. The structural strength of the foundation rock—a sandy fissile shale—was very low. Fortunately, considerable information on the characteristics on this rock was available because some of the structures of the hydroelectric plant already completed were built on closely related strata of shale. Many foundation test data and prototype settlement observations were on hand. Based on these data, the compressive stresses on the foundation were limited to 50 lb per sq in. In addition, careful consideration had to be given to uniform distribution of all loads in order to prevent unequal settlements of different parts. These requirements necessitated a heavy foundation slab over the full basement area, about 250 ft by 180 ft in size. Along the outside edges of the slab a continuous retaining wall supports the yard fill and building walls. The main yard level is 32 ft above the lower part of the basement. The assumed design flood reaches 19 ft above the basement floor level. Therefore, large earth pressure and uplift forces had to be taken into account. The total concrete yardage of the substructure amounts to 26,000 cu yd.

In the design of the substructure, many special conditions presented themselves. For instance, the south wall, because of space limitations, had to be designed as a gravity retaining wall. In order to prevent cracking of this wall, which was 17 ft thick, expansion joints were necessary, but no expansion provision was contemplated in the steel framing and the brick walls above. The procedure adopted was to build the retaining wall, with expansion joints at about 33-ft intervals, at a very early date. After they had cooled, a band of heavily reinforced concrete was poured as a continuous base for the brick wall above. In this bank 0.8% of reinforcement steel was used and it is free of cracks.

The foundations for the coal pulverizers consist of 6-ft slabs, 57 ft long. In order to localize vibration, these slabs are separated from the surrounding concrete at the sides and at the bottom by a 2-in. layer of a commercial cork insulating mat.

The part of the condenser pit adjacent to the boilers contains a storage tank for 40,000 gal of distilled water at each unit. To prevent any chemical reaction, the concrete surfaces are lined with carbon brick laid with a special commercially prepared mortar. To prevent any uplift under the bricks, the lining is placed over a layer of porous concrete connected by drain pipes to a special sump. Because of the high temperature to which these pits are exposed, all joints around these blocks have cork fillers to allow for expansion.

To prevent any ground water from entering the basement through these joints, two strips of copper seals were used at all contraction joints, and construction joints were sealed by a single strip.

Each of the 60,000-kw turbo-generator units is supported on a reinforced concrete frame which carries the load to a 7-ft mat for foundation load distribution. Resonant vibration and settlements are of major concern for such a machine foundation. In proportioning the foundation mat, an effort was made to obtain equal settlement under each frame column. To prevent the development of resonant vibrations, the special design recommendations of the turbine manufacturer in regard to loading assumptions, deflection limits, and unit stresses were followed closely. The structure designed on this basis is reinforced heavily. The steel amounts to 285 lb per cu yd of concrete. Because of the stipulation of the manufacturer to disregard shear strength of concrete altogether, columns and beams are crowded with stirrups. A definite sequence for placing the different groups of bars and the forms had to be established to facilitate construction of these members.

The results both as to balance of the rotating equipment and as to absence of resonant vibrations are very gratifying. From sensory inspection, vibrations are scarcely noticeable.

For the condenser water supply, special conditions led to the adoption of a somewhat unusual layout. With a site below the dam, the question arose immediately whether the system should provide circulation by pumping water from tailwater below the dam or by a gravity feed from headwater above the dam. On careful investigation, both systems proved feasible and their costs, with proper allowance for power for pumping and for the loss of generation at the hydro plant, respectively, were almost identical. Because of higher dependability, the gravity feed system was adopted.

The hydraulic conditions are illustrated in Fig. 19. A pressure conduit carries the water from the dam to the condenser. The maximum water requirement of each condenser, at full load with summer temperatures, is 156 cu ft per sec. With the station 800 ft away from the river in an area where foundation rock is substantially above normal river level, a discharge pool had to be provided at basement level. The available head above this pool, with the forebay at its lowest level, is just sufficient to provide the maximum flow. The hydraulic gradient for this condition is indicated in Fig. 19.

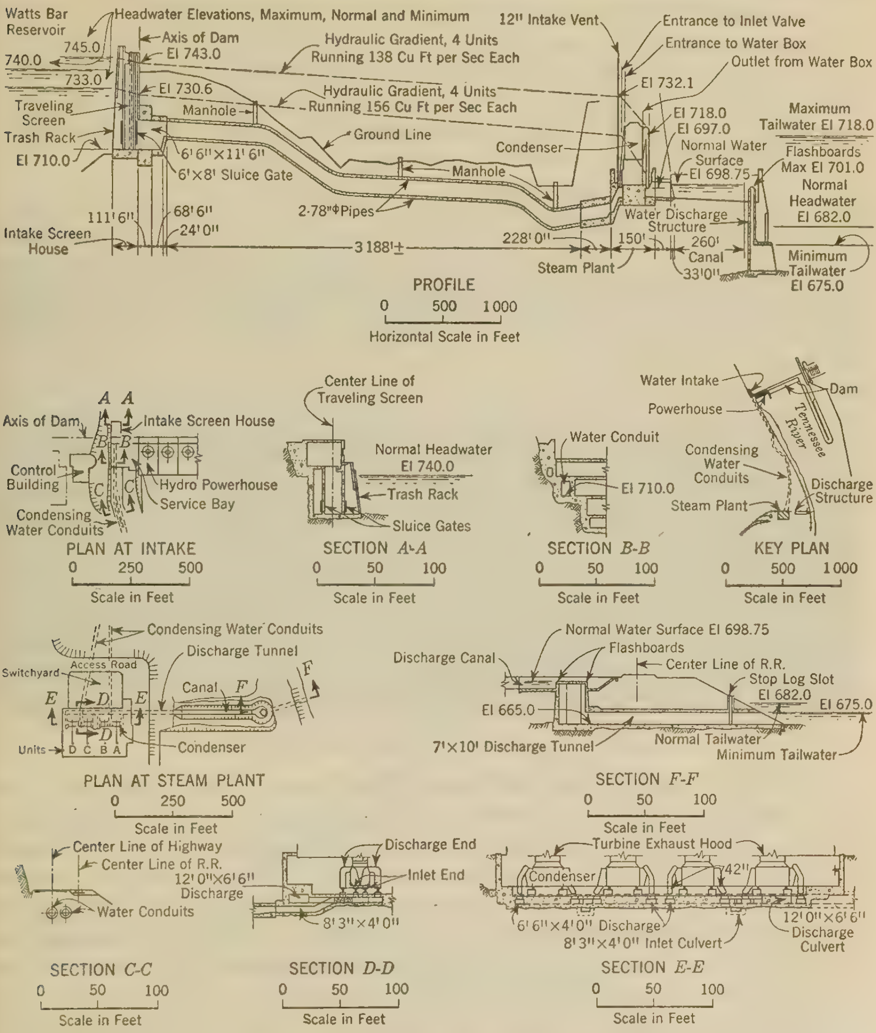
Watts Bar Reservoir
Headwater Elevations, Maximum, Normal and Minimum
745.0
740.0
733.0
Axis of Dam
El 743.0
Hydraulic Gradient, 4 Units Running 138 Cu Ft per Sec Each
12" Intake Vent
Entrance to Inlet Valve
Entrance to Water Box
Outlet from Water Box
El 730.6
Traveling Screen
Trash Rack
El 710.0
Manhole
Hydraulic Gradient, 4 Units Running 156 Cu Ft per Sec Each
El 732.1
El 718.0
El 697.0
Maximum Tailwater El 718.0
Ground Line
Condenser
Normal Water Surface El 698.75
Flashboards Max El 701.0
6'6"×11'6"
6'×8' Sluice Gate
Manhole
Normal Headwater El 682.0
111'6"
68'6"
24'0"
2-78"Φ Pipes
Water Discharge Structure
Intake Screen House
3 188'±
228'0"
150'
260' Canal
Minimum Tailwater El 675.0
Steam Plant
33'0"
PROFILE
0 500 1 000
Horizontal Scale in Feet
A A
Axis of Dam
Intake Screen House
B B
Control Building
C C
Hydro Powerhouse
Service Bay
Condensing Water Conduits
PLAN AT INTAKE
0 250 500
Scale in Feet
Center Line of Traveling Screen
Normal Headwater El 740.0
Trash Rack
Sluice Gates
SECTION A-A
0 50 100
Scale in Feet
Water Conduit
El 710.0
SECTION B-B
0 50 100
Scale in Feet
Water Intake
Dam
Powerhouse
Tennessee River
Condensing Water Conduits
Discharge Structure
Steam Plant
KEY PLAN
0 500 1 000
Scale in Feet
Condensing Water Conduits
Discharge Tunnel
Switchyard
Access Road
E D E
Canal F
F
D
Units D C B A
Condenser
PLAN AT STEAM PLANT
0 250 500
Scale in Feet
Normal Water Surface El 698.75
Flashboards
Discharge Canal
Center Line of R.R.
Stop Log Slot El 682.0
El 675.0
El 665.0
7'×10' Discharge Tunnel
Normal Tailwater
Minimum Tailwater
SECTION F-F
0 50 100
Scale in Feet
Center Line of Highway
Center Line of R.R.
12'0"×6'6" Discharge
Water Conduits
SECTION C-C
0 50 100
Scale in Feet
Discharge End
Inlet End
8'3"×4'0"
SECTION D-D
0 50 100
Scale in Feet
Turbine Exhaust Hood
Condenser
42"
6'6"×4'0" Discharge
8'3"×4'0" Inlet Culvert
12'0"×6'6" Discharge Culvert
SECTION E-E
0 50 100
Scale in Feet

Fig. 19.—Condensing Water Supply Systems

Also shown is a gradient for high tailwater conditions, caused by infrequent floods. The condition shown refers to the assumed design flood, with tailwater

20 ft above the discharge pool. In case of such a flood, the conduit capacity would be reduced to 138 cu ft per sec, but because such floods do not occur during the summer season, the required condenser vacuum could still be maintained.

The intake screen house is located upstream from the shore end of the hydroelectric plant; six screen chambers, with low intakes, connect to a common conduit which leads through the service bay substructure of the hydro plant. Once past the powerhouse area, the conduit divides into two sections, each consisting of a concrete pressure pipe 6.5 ft in diameter. These pipes were fabricated by the pipe manufacturers at the project site.

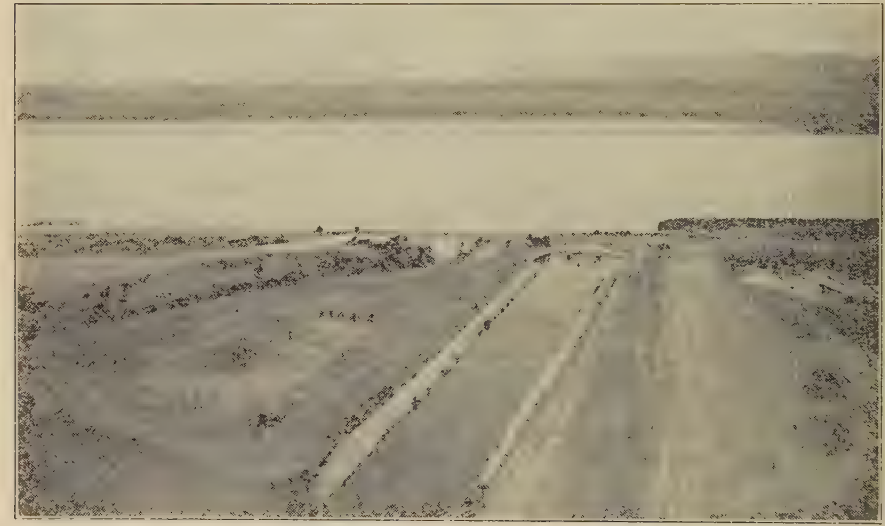

Fig. 20.—Discharge Canal and Morning-Glory Spillway

Where the conduits enter the heavy fill adjacent to the building, the pipe construction terminates and reinforced concrete boxes are used. These conduits continue as part of the substructure blocks until they reach the condenser entrance. The two entrances are each controlled by a 42-in. gate valve placed in a horizontal position and set for a slow 6-min closure, in order to minimize water hammer in the pressure line. At the outlet end of the condenser, 42-in. butterfly valves allow throttling for pressure regulation and closing against high tailwater. A branch conduit leads away from each condenser to the main discharge conduit which connects directly to the open canal in the flood plain between the station and the river.

To dissipate the energy of the drop between discharge pool and normal river level, a morning-glory spillway discharges into a box culvert built on rock under the access railroad to the hydro station. Fig. 20 is a view from the station roof overlooking the discharge canal from the outlet structure to the morning-glory spillway at the river end.

The service areas around the power station (Fig. 21) could be provided by a balanced earth grading job which involved moving approximately 400,000 cu yd of earth. Facilities for coal handling and for coal storage made the largest demand for yard grading. Coal can be delivered by rail, by truck, or by river barge. For rail delivery a large track loop is formed around the coal storage area with the tracks passing over the central unloading hopper. Ahead of the hopper, storage tracks accommodate one hundred loaded cars. At the hopper the main operating track is equipped with a rotary car dumper. Car pullers move the empty cars out of the hopper building to the head of an incline over which the cars move by gravity to storage tracks, having space for fifty cars. Trucks delivering coal enter from the access road and unload into the same main hopper.

For barge delivery a dock about 700 ft long was built along the river front adjacent to the coal handling area, using steel sheet piling salvaged from cofferdams. The dock has a face approximately 33 ft high above the river bottom. A stationary unloading crane unloads the barges, and dumps the coal into a receiving hopper which feeds a conveyer, approximately 1,250 ft long. This conveyer terminates at the hopper building. From the central hopper coal is delivered by conveyers to a crusher and from there to the bins of the power plant. Coal designated for storage is moved by conveyers and power scraper into and out of the 200,000-ton capacity storage pile.

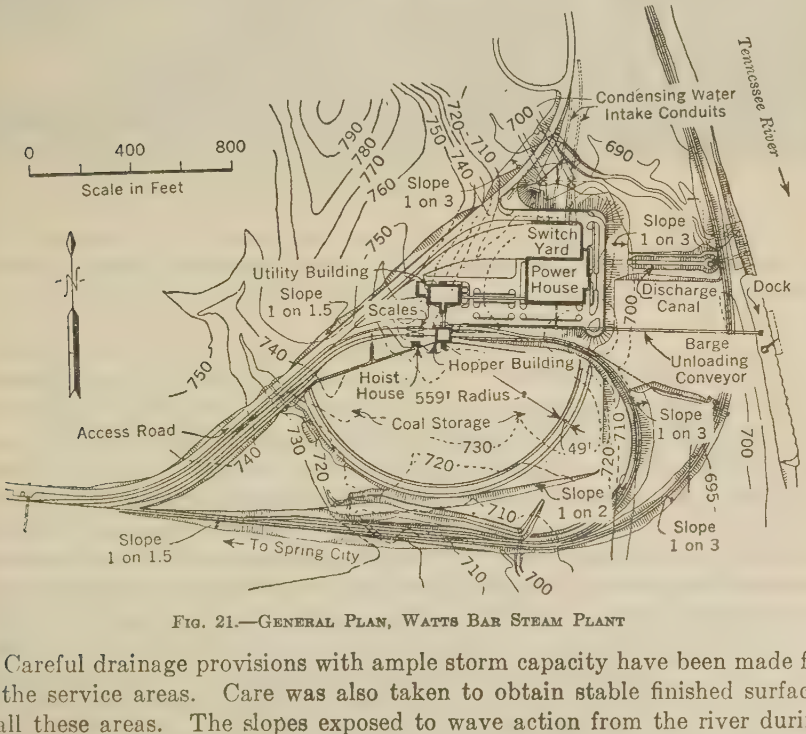
Condensing Water
Intake Conduits
Tennessee River
0
400
800
Scale in Feet
790
780
770
760
750
720
750
740
710
700
690
Slope
1 on 3
Switch
Yard
Power
House
Slope
1 on 3
Discharge
Canal
Dock
Utility Building
Slope
1 on 1.5
Scales
700
Barge
Unloading
Conveyor
740
750
Hoist
House
Hopper Building
559' Radius
Coal Storage
730
720
491'
Access Road
740
730
720
710
720
Slope
1 on 3
700
695
Slope
1 on 2
710
Slope
1 on 3
Slope
1 on 1.5
To Spring City
710
700

FIG. 21.—GENERAL PLAN, WATTS BAR STEAM PLANT

Careful drainage provisions with ample storm capacity have been made for all the service areas. Care was also taken to obtain stable finished surfaces in all these areas. The slopes exposed to wave action from the river during floods are well protected by riprap. Above the normal flood lines they are protected by grass. In the building area, roads, parking areas, tracks, and grass plots are all well defined by use of concrete curbs.

Like the main building, the yard buildings are all founded on rock. Superstructures were made of light steel frames with walls of asbestos cement boards. The same type of construction was used for the conveyer bridges. Fig. 22 shows the hopper building and its relation to the track loop and the coal storage pile. The rotary car dumper and a truck are shown unloading coal.

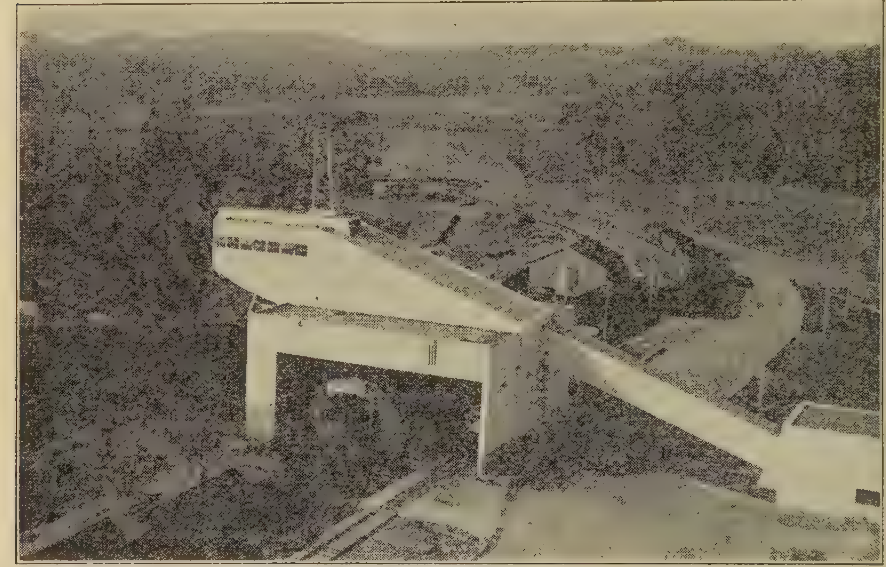

FIG. 22.—COAL STORAGE AREA

An appropriate and pleasing architectural appearance was obtained without any increase in building volume, which was established from functional requirements only and without any changes in the basic steel framing.

SUMMARY

The Watts Bar Steam Station was initiated as a war plant to furnish power for the armament program during World War II. It was ready at the time when the power was needed and until the summer of 1945 had generated more than 3.5 billion kw-hr of energy available for war production. When the war is over and normal criteria will be applied to measure its usefulness, it will stand up well to such an inquiry. Its capital expenditure is relatively low, the cost per kilowatt amounting to $76 exclusive of transformer yard, or $83 when the electrical tie-in to the switchyard is included. Its thermal efficiency is good; during average operation slightly less than 11,000 British thermal units (Btu) are required to produce one gross kilowatt hour. Coal and ash handling equipment are highly mechanized with resulting low operating cost. Dependability of service was a major endeavor in design and in equipment selection. Experience during three and one-half years of operation has shown very good results.

Now that the war task of the Watts Bar Steam Plant has ended, it can be expected with confidence that, based on the foregoing station characteristics, it will be an economical source of power in peacetime.

DISCUSSION

A. T. Larned,[12] M. Am. Soc. C. E.—Clearly and orderly, the second paper of the Symposium (by Messrs. Palo and Marks) describes the layout of a type of hydro station which has become quite standardized for large low-head, propeller type installations. The title of this paper is broad and, perhaps, should have been more explicit, as only a part of the total hydro plants is laid out with the station integral with the dam so that, for stability, advantage is taken of the mass of concrete in the dam itself. There are many other hydro developments already built and to be built in the future, with dams remote from the station and with tunnel, canal, and long penstocks utilizing water under medium or high heads, in which the design and layout of the station involve problems of a somewhat different nature than those outlined in the paper.

Functional Considerations of Design.—Quite appropriately, Messrs. Palo and Marks have emphasized the functional characteristics of power-plant layout, which enables one to visualize the over-all performance of each type of equipment and its interrelation with other types, so that each can be arranged to perform its assigned task in the most effective manner and with the minimum space requirement.

In power-plant design, the realization of directness and compactness, and the elimination of excess or waste space, assure that the lowest cost will be obtained in the construction of the plant. The designer sets the pattern and there is nothing the construction man can do later to reduce costs if the designer failed in his layout. In other words, "the money is already spent as soon as the design leaves the drafting table."

In regard to allotting space for future maintenance and repair requirements, it is of particular importance that heed should be given to the advice and counsel of the operating group who in the future must "live" with the plant, in order that all equipment be made easily accessible and the repair and tool rooms be arranged conveniently.

Simplification of Design.—The features of simplicity and economy of design have been stressed by Messrs. Palo and Marks, and it is evident that the outdoor and semi-outdoor types of station have been valuable contributing factors in the accomplishment of this objective. The writer is aware of no adverse experience resulting from this type of installation, and the elimination of expensive superstructure to house nothing more than a traveling crane has resulted in marked economy. Steam-plant designers have realized the advantages of the outdoor type of station and many of the newer steam installations utilize outdoor boilers, auxiliary equipment such as circulating fans, and, in some cases, outdoor turbine units.

Messrs. Palo and Marks make no mention of the rôle that the umbrella or overhung type of generator has played in the design of the outdoor type of hydro station. This arrangement of generating unit enables a substantial reduction in powerhouse costs through reduced head clearances. The advantages of the umbrella type of generator are that it reduces the guide bearings from three to two and materially shortens the span necessary for the bracket supporting the thrust bearing. This reduces the length of shaft necessary and also makes it possible to dismantle the generator for inspection and repairs without disturbing the thrust bearing, shaft, or turbine.

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[12] Chf. Civ. Engr., Ebasco Services Inc., New York, N. Y.

It is advantageous to restrict the control room to the minimum space possible, with due allowance for the addition of future electric controls. In most cases, for stations with one to three main generating units, the control room has been placed on the main operating floor between the units so that full visibility of the governors and generating equipment is possible. This avoids the need for a separate building and takes up floor space that would be wasted otherwise.

The advisability of locating the governors and actuating equipment on the generator floor to save space has been noted by Messrs. Palo and Marks. There is another reason for locating this apparatus and other auxiliaries on one floor in so far as possible, the number of men on a shift can be reduced to the minimum. This is an important consideration from the point of view of reduced operating costs and increasingly difficult labor conditions.

Model tests and tests of actual installations have shown that the entrance to the penstock or upstream side of the water passage need not be increased materially in size. A moderate increase in diameter by a properly shaped bellmouth is all that is necessary to keep the entrance losses low. This type of construction is economical because the size of the intake gate can be kept to a minimum.

Design as Related to Concrete Construction.—The discussion of the need for, and the method of, locating construction joints (see Fig. 15) is especially helpful and should serve to impress both the engineer and the construction man with the need for care in placing concrete. Experience has proved that it is advantageous to have the construction joints located by the engineer rather than by the construction man. The need for an early decision as to the location of these joints is well taken as the design will depend considerably upon their placement.

Although the Palo-Marks paper mentions only construction joints in the heavy substructure, it also would be well to call attention to the need for joints in the lighter concrete sections of the structure above the generator floor. Such joints are necessary for construction purposes and also to localize the cracks that would result if large areas of concrete walls were not protected by such construction.

In some of its recent designs for steam plants the writer's company has resorted to the use of special control joints in the reinforced-concrete walls and parapets of the buildings. These consisted of wood strips tacked to the forms to leave wedge-shaped joints in both the exterior and interior faces of the walls. These have been successful in eliminating the unsightly patterns of shrinkage or expansion cracks, and they control the location of the larger cracks that usually appear. The joints are filled with a concrete-colored calking compound which preseals any cracks that may occur.

A brief general statement as to the means adopted for securing watertight joints would be appreciated. The appearance of joints, after a few years, can either make or mar an otherwise perfect job.

The generation of heat in large blocks of concrete, and the damage resulting from subsequent shrinkage when the concrete is cool are well known in connection with the design and construction of dams. However, little has been published about this subject in connection with power-station structures, since most hydro developments have not been of the magnitude of those designed and built by the Tennessee Valley Authority (TVA) and other government agencies. The attention called to this factor, therefore, is timely and should help to make it easier for the engineer to sell to the construction organization the desirability of placing the concrete in the powerhouse substructure in the proper sequence and at the proper intervals of time.

Messrs. Palo and Marks have not mentioned form construction, particularly the construction of draft-tube forms. It has been found convenient to build these forms on an erection floor where wood-working machinery is convenient and working conditions neither cramped nor dangerous. The forms are built in large sections and transported to the powerhouse site by truck.

Large quantities of steel reinforcement usually are required in powerhouse substructures to carry and distribute the loads into the foundation. There is a tendency on the part of some construction men (as well as some engineers), not intimately connected with the design, to claim that the amount of reinforcement used is unnecessarily large. The Palo-Marks paper calls attention to the necessity and desirability of using considerable reinforcement both for structural strength and for resisting temperature and shrinkage stresses. In view of the large amount of reinforcing steel necessarily required, careful analysis should be made of the loads acting, or assumed to be acting, on the structure simultaneously so as not to make these loads larger than is absolutely necessary.

In the design and construction of the many hydroelectric plants with which the writer has been connected, it has not been considered necessary to provide a gate at or near the downstream end of the draft tube except where the runner is located below tailwater. Operating experience over a period of about thirty years has confirmed the opinion that the need for draft-tube unwatering is very slight, and that provision for the installation of stop logs or a single panel bulkhead that can be used for any one of the units should suffice. In many cases, the omission of such downstream gates has made it possible to put the transformers on a platform on the downstream side of the powerhouse, thus making the electrical connections from the unit to the transformers very short and simple. In most cases, this has been a considerable advantage over the practice of placing the transformers at a distance, especially where the plant is in a place where it is necessary to go a considerable distance to find a large enough level piece of ground.

The concentration of load that may occur at the upstream nose of the piers in draft tubes, and the difficulty of caring for these loads, have been mentioned by Messrs. Palo and Marks. On several occasions the writer's firm has placed a short, built-up steel column at this point, with spread base and capital to take the load concentration. This column has been of aid also in supporting the form work for the draft tube.

Attention has been called to the presence of cracks in the side walls of the volute casings of three of the stations described in the second paper. These cracks were about 10 ft upstream from the center line of the unit. The similarity of the cracks in the three cases leads to the conclusion that a definite set of stresses has been the cause for the pattern of cracking. By referring to the cross sections of the station in question it will be noted that this is the point where the deep excavation starts for the draft tube. This reentrant angle into the concrete, if considered as a casting, could account for the failure, particularly if the line of excavation changed abruptly from vertical to horizontal. The writer has always attempted to excavate so that there is a fillet in the concrete and so that the change in direction is gradual.

The authors of the second paper (to which all the writer's comments are limited) are to be commended. It is to be hoped that the paper will stimulate further study and discussion of the many problems which it has raised.

William P. Creager,[13] M. Am. Soc. C. E.—The painstaking care with which the authors and their colleagues have conducted research and tests on hydraulic structures is evident in these papers.

In Mr. Riegel's paper one test is missing—namely, that for the percentage of area of the base of dams subjected to uplift pressures. For several years, the Power Division of the Society has had a Subcommittee on Uplift in Masonry Dams, whose purpose was to devise experiments and report on its findings. The subcommittee has not reported, probably because it could devise no tests that would show conclusively whether only a part of the area or 100% of the area is subject to uplift. Notwithstanding the absence of such tests, the Tennessee Valley Authority (TVA) has assumed that only two thirds of the area is subjected to uplift pressure, after taking into consideration the maximum possible effect of the grouted cutoff and drainage system. With respect to this feature the writer presumes to take issue.

Uplift pressures under dams should be analyzed strictly according to two factors: (*a*) The hydraulic gradient factor and (*b*) the area-of-pressure factor.

(*a*) *Hydraulic Gradient Factor.*—Fig. 23 indicates a typical hydraulic gradient under a solid concrete dam. The piezometric pressures at the base vary from tailwater pressure h at the toe to tailwater pressure plus a percentage C of the head H on the dam at the drains and thence to full headwater pressure at the heel. The coefficient C is an indication of the effectiveness of the grouted cutoff and the drainage system; and w is the unit weight of water.

The hydraulic gradient factor, for the TVA main river dams, has been shown by Mr. Riegel to correspond to Fig. 23 with an adopted coefficient C equal to zero. The writer has no criticism of the hydraulic gradient factor adopted for the main river dams, since subsequent tests have shown the adopted factor to be correct. Incidentally, it corresponds to 100% efficiency of the drains. However, attention is called to the fact that similar tests[14] on many other dams with grouted cutoffs indicate much less than 100% efficiency of drains. Either the nature of the rock or the excellence of the engineering has resulted in high efficiency of the TVA drains. It is doubtful if such high efficiency can be assured for all future similar dams.

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[13] Cons. Engr., Buffalo, N. Y.

(*b*) *Area-of-Pressure Factor.*—The area-of-pressure factor for all TVA dams was assumed to be equal to two thirds of the area. The area-of-pressure factor has been the subject of much debate, ranging from about 50% to 100% of the area. The writer will not attempt to reopen arguments on this subject except to state that, by the very nature of so-called solid materials, if the area of pressure is less than 100%, the material must be capable of resisting tension.

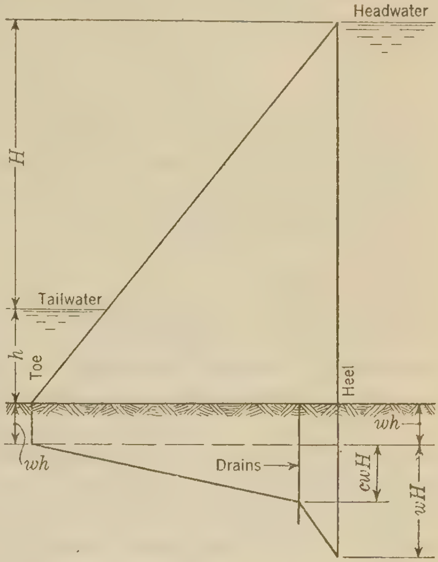

FIG. 23.—TYPICAL HYDRAULIC GRADIENT

Conversely, if the material, or any layer or bedding plane in the material, is of such a nature that it would not have appreciable tensile strength, then it should have an area-of-pressure factor of 100%.

The TVA main river dams were designed on the assumption that two thirds of the area of the base was subjected to uplift pressure. This area-of-pressure factor was adopted for "calcareous rocks with approximately horizontal bedding considered favorable to the accumulation of uplift pressures in the foundation itself." Thus, this factor is assumed to be advantageous in the accumulation of uplift pressures without a grout curtain and without drainage installation. Therefore, it is favorable to the accumulation of whatever uplift pressure remains after taking into account the full effect of the grout curtain and drainage on the hydraulic gradient.

For a river wall, supporting earth and with river depth and ground-water depth equal on each side of the wall, the use of less than 100% uplift would not be considered unless the foundation were capable of resisting tensile stresses; but this assumption is exactly what was made with respect to tailwater uplift in the TVA main river dams where the foundation is "favorable to the accumulation of uplift pressures."

The TVA criterion for uplift is not conservative when compared with what the writer believes to be common practice, and the writer cautions against adopting it without the advance knowledge that the drainage system will be permanently 100% effective and that the rock is of such a nature that 100% uplift will not occur over the area of the base.

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[14] "Uplift Pressure in Masonry Dams," by Ivan E. Houk, *Civil Engineering*, September, 1932, p. 578.

In the case of the higher dams, Mr. Riegel states that one half of the headwater pressure was assumed at the line of drains.

It might be mentioned that, as shown in Fig. 23, the pressure at the drains must be equal to tailwater pressure plus a percentage of the net head on the dam —because the effect of the grouted cutoff and the drains is not a function of the headwater pressure but is a function of the net head in the dam. Incidentally, Fig. 3 does not show one half of the headwater pressure at the drains, as Mr. Riegel states; but it does show the correct relation of tailwater pressure plus one half of the net head on the dam.

I. L. Tyler,[15] M. Am. Soc. C. E.—The field embraced by Mr. Riegel's excellent paper in this Symposium is such a large one that only a very brief treatment of many important features could be included. In the case of "foundation restraint" (see heading, "Concrete Dams: Foundation Restraint") some amplification of the text may be desirable.

In addition to the three major methods for controlling mass concrete temperatures discussed by Mr. Riegel another method was used at Hiwassee Dam. In this case, the mixing water for concrete was artificially cooled during periods of high placing temperature. Although the reduction in maximum temperature accomplished by this means was not much (perhaps 10% of the maximum rise), its possible effect on cracking cannot be ignored, since conditions favorable to the elimination of temperature cracks were actually closely approached and even small improvements in temperature conditions may have had considerable effect. Hiwassee Dam as finally completed was unusually free from cracks considering the very limited use of embedded cooling pipes. Perhaps some credit for this must be assigned to the benefits, even though small, which resulted at critical times from the cooling of the mixing water.

Mr. Riegel's discussion of the effect of the concrete placing rate on temperature rise of mass concrete without cooling pipes may not be sufficiently detailed to clarify all the variables involved. With a concrete placing rate averaging 1 ft per day the use of 2½-ft lifts at 2½-day intervals in place of the more common 5-ft lifts at 5-day intervals does permit some greater head loss to the air, as Mr. Riegel states, even with the low-heat cement and concrete of low-cement content. However, an even greater advantage, so far as cracking from foundation restraint is concerned, comes from the improved temperature distribution vertically through foundation and dam. The desirable condition, of course, is to avoid steep temperature gradients and for this the thin lifts are preferable as shown by R. W. Carlson,[16] Assoc. M. Am. Soc. C. E.

Mr. Riegel made no comments on the effectiveness of the measures taken to minimize temperature cracking in mass concrete. This may leave the reader with some doubts as to the feasibility of reducing cracking by such precautions. It is realized that many factors other than temperatures influence cracking in mass concrete, and that to cover them all would be out of the question. However, the writer suspects that Mr. Riegel has a considerable fund of information on the relation of temperatures and cracking on many of the TVA structures, and he respectfully urges that Mr. Riegel give the matter all consideration possible in his closure.

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[15] Mgr. of Field Research, Portland Cement Assn., Chicago, Ill.

[16] "Temperature and Stresses in Mass Concrete," by R. W. Carlson, *Journal*, A.C.I., March–April, 1938.

PHILIP SPORN,[17] M. AM. SOC. C. E.—As stated by Mr. Meyer, the civil engineering features of the design of any steam plant materially affect the ultimate cost of power generation. In the writer's experience, this has been amply demonstrated many times. However, the writer is of the further opinion that the design of both the electrical and mechanical systems, so far as arrangement of equipment and the selection of materials and methods for doing the work are concerned, are just as flexible and present just as many possibilities for savings in the initial cost of the development as do the civil engineering features. Therefore, all three phases should be considered at the same time and only when this is done and the selection of the best combination is made does the most economical design result.

Furthermore, the operation and maintenance of the plant must be given careful consideration, or the savings realized from some particular design may be quickly wiped out. This is particularly important when the present cost of labor and the trend that seems to be establishing itself in this field are considered.

The advantages of economical fuel supply, and the location with regard to load, must have been the main determinants of the location of the Watts Bar Steam Station because the other reasons given by Mr. Meyer for choosing this site are not very conclusive. The savings from combining the construction of the hydro plant and of the steam plant are small; and, from an operating standpoint, the plants are too far apart to reduce the size of the operating personnel appreciably. Possibly, if the hydro plant were electrically controlled from the steam plant, some saving in operating costs could be realized but Mr. Meyer does not mention if this was done.

The design developed by Mr. Meyer is well prepared and rationally developed. Rationalized as it is, however, some of the underlying assumptions seem open to question. At least, they run counter to the writer's experience. To illustrate: At Watts Bar, two stacks have been provided, each probably serving two units. The writer has found that the cost of this type of arrangement and the amount of critical materials employed are about the same as in the case where a separate stack is provided for each boiler. From an operating standpoint the individual stack for each boiler is preferable to the arrangement adopted at Watts Bar.

From a study of the plans, the width of the generator room seems to have been determined by turbine erection, considering that the generator rotor would be pulled parallel to the long axis of the machine. In pulling the rotor, if it were canted in a horizontal plane (as is frequently done), the width of the generator room could have been reduced. Furthermore, the writer has never found any need for carrying the track approximately the full length of the turbine room as has been done at Watts Bar. Generally, this increases the width of the generator room.

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[17] Executive Vice-Pres. and Chf. Engr., Am. Gas and Elec. Service Corp., New York, N.Y.

Furthermore, a wall seems to have been constructed between the generator room and the heater bay. The trend in modern design omits this wall. By opening the generator room to the heater bay and boiler room it is possible to obtain not only savings in structures but better ventilation. The operating personnel avoid the feeling that the boiler room is a place for burning coal and as a result must be dirty and does not deserve the same clean care as the generator room.

Since the main basement floor is only 19 ft below the level of maximum river stage, it is difficult to understand why the foundation slabs need to be as thick as indicated in Fig. 17. For example, at the Philo Plant of The Ohio Power Company, where the bottom of the condenser pit is about 56 ft (as compared to 32 ft at Watts Bar) below ground level, and 46 ft (as compared to 19 ft at Watts Bar) below flood level, the slab was about 4.5 ft thick as compared to a slab thickness of about 8 ft at Watts Bar. Furthermore, it has been the writer's experience that, wherever there is a deep basement, it usually has been possible to arrange columns and struts so that one wall of the pit was braced against the other. As a result, the pit side walls at the Philo plant are about 3.5 ft thick as compared to the apparent 11.5-ft south wall of the boiler room basement at Watts Bar. It would be interesting to learn more about the design basis behind these differences.

Mr. Meyer states that the slab beneath the main boiler columns was continuous for 250 ft and that the design of coal-bunker framing could not allow any movement of the foundations. Obviously, if the sandy shale foundation rock would only permit a loading as light as was applied, then some, perhaps substantial, saving could have been obtained by arranging the framing between the basement and main floors as a truss and reducing the thickness of the slabs. Was this considered and, if it was, and then rejected, what was the reason?

The handling of the circulating water problem and the adoption of the gravity system of supply is very interesting. From Fig. 19 the 3,500 ft of intake, which required such a deep excavation, appears quite costly, especially since the two pipes were not run together—apparently requiring 7,000 ft of excavation. Is such an intake required for the gravity system as economical and reliable as the use of circulating water pumps, considering that any settlement might break the lines? Was any procedure in addition to the tongue on each length adopted to prevent relative movement between adjacent pipes because of earth movements? Why was such a long canal used to connect the condenser outlet with the river? A long canal permits more sedimentation than would a short one, and this has an adverse effect on the operating expense, as well as on the initial investment. Again, if the length of the canal had been decreased, the power plant could have been nearer the river—thus decreasing the length of the conveyer between the barge-unloading dock and the hopper building, which would have resulted in a saving in the cost of constructing the coal-handling facilities. Could Mr. Meyer enlarge on this subject?

The facilities that have been provided in the coal-handling yard are unquestionably ample; and, although they may be too elaborate for present capacity, they appear adequate for the ultimate capacity. Nevertheless, one is inclined to ask the reason for the choice of a drag scraper with a traveling tail tower, instead of large tractor-scraper equipment. One is also inclined to ask some questions about the coal dock, although there is not enough description included in the paper to comment on its construction. The Cabin Creek Plant of Appalachian Electric Power Company, with an installed capacity of 325,000 kw, has a sheet-pile dock about 200 ft long supplemented by pile clusters for mooring downstream from the dock. This arrangement proved to be very economical and yet substantial.

Mr. Meyer and his architectural associates are to be complimented for the fine synthesis of structural function with architectural appearance that has been incorporated into the Watts Bar plant. The writer is acquainted with no modern power plant at which this has been done more effectively and where the two functions have been more perfectly wedded. Engineer and architect have not only succeeded in developing a structure that functions properly, but have also obtained one that looks well.

JOSEPH R. BOWMAN,[18] JUN. AM. SOC. C. E.—The title of the second paper of the Symposium is rather inclusive, whereas the paper itself is primarily concerned with but one of the several types of hydroelectric power developments designed by Tennessee Valley Authority (TVA). Messrs. Palo and Marks are to be commended, nevertheless, for their emphasis on the importance of functional layout to design. The relative importance of the functions of water flow, flow of electricity, and mechanical control is generally understood and accepted by those engaged in the design of hydroelectric power stations. On the other hand, considerable difficulty is experienced frequently through failure to understand the proper correlation of the electrical control, auxiliary electrical equipment, and auxiliary mechanical equipment functions. The most desirable layout of unit and station auxiliary services can best be obtained through appropriate evaluation of the interrelation of these functions. Evidence of the value of competent functional planning is reflected in the minimum of interference of unit and station service functions presented in many of the TVA hydro plants.

To simplify otherwise cumbersome sentence construction, the writer will use the term "semi-outdoor" with the implication that it includes the "modified outdoor" station of the type used at Hiwassee Dam, as well as the true "semi-outdoor" type of station adapted to the Cherokee, Douglas, Fort Loudoun, Watts Bar, and Kentucky projects.

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[18] Asst. Civ. Engr., Erik Floor and Associates, Chicago, Ill.

Notable among the accomplishments of the TVA program is the departure from the conventional indoor power station housing large units. Messrs. Palo and Marks cite relative economy, operating convenience, and appearance as the general bases for the selection of the various powerhouse types. In addition, they have advanced the conditions of (1) maximum tailwater above the generator floor and (2) access level above the generator floor as the criteria governing the selection of the semi-outdoor type of station. In the comparison of a number of the TVA powerhouses the writer has observed inconsistency and inadequacy in the application of these criteria. For example, these governing conditions, expected at six of the main-river, low-head developments, led to the selection of semi-outdoor hydro stations at Kentucky, Watts Bar, and Fort Loudoun; on the other hand, indoor stations were selected at the Pickwick Landing, Guntersville, and Chickamauga projects.

The appearance factor frequently exerts considerable influence on the selection of power stations employed in public power programs. Whereas some organizations direct attention to the powerhouse by the accentuation of vertical lines, the TVA has utilized accentuation of horizontal lines to direct attention from the powerhouse to other structures on the project. The effect of the latter policy is further promoted by the adoption of the semi-outdoor station.

Several factors are involved in the determination of the relative economy of the indoor and semi-outdoor stations. A considerable saving may often be obtained through the elimination of expensive superstructure walls. However, where very high tailwater conditions are expected, a weight of material equivalent to that eliminated must be added to the substructure to insure the stability of the structure. The net saving, therefore, will be determined by the unit price differential between superstructure and substructure types of construction. Another factor worthy of consideration is the replacement of the conventional roof structure by the deck-type roof used in semi-outdoor stations. The saving, if any, effected by the foregoing procedure may be offset by the increased cost of a massive outdoor gantry crane. Further economy may be obtained by modification of the crane to embrace the functions of handling intake and draft-tube gates (as adapted to the Kentucky station), thereby eliminating the need for separate cranes to perform those operations. From the viewpoint of operating economy, additional expense may be incurred in moving the massive outdoor crane, and in performing operations ordinarily not characteristic of indoor stations, such as the handling of unit hatch covers.

Floor operation (as distinguished from control room supervision) of as few as two or three large units has apparently become standard practice in many hydro plants of the TVA and other organizations. Efficient floor operation usually demands the location of unit auxiliary equipment in the immediate vicinity of the floor operator's station, which, in turn, is usually near the unit governor controls. Design for stability against extremely high tailwater results in very compact substructures in semi-outdoor stations. Consequently, a critical shortage of space for auxiliary equipment near the unit may result. As the physical size of auxiliary equipment is nearly constant over a considerable range in unit sizes, this shortage of space will become even more critical for smaller units, probably necessitating the location of auxiliary equipment some distance from the floor operation center. The compactness of the substructure of a semi-outdoor station frequently involves the elimination of a distinct turbine floor and necessitates the location of all governing equipment—actuator, sump, and pressure tank—in a compact group at the floor operation center. This is an important advantage in view of the requirements of efficient floor operation. Further operating economy is obtained by the utilization of the "twin" operation center, in which the auxiliary and governing equipment of two adjacent units is grouped into a common operation center. Indoor stations, except when modified by unusual local conditions, ordinarily utilize a distinct turbine floor. This practice frequently leads to the separation of the actuator from the governor sump and pressure tank, thereby increasing attendance requirements. The use of a separate turbine floor often creates unused or "nonproductive" space.

Protection of personnel from severe climatic conditions is frequently advanced as a basic factor governing the selection of indoor powerhouses. The semi-outdoor station appears to satisfy the normal requirements of personnel protection, and the writer believes that the relatively infrequent use of the powerhouse crane alone will not justify the use of an expensive superstructure building. Inasmuch as several outdoor stations are in operation in the northern part of the United States, it is apparent that this type of station has little respect for adverse weather conditions.

The two criteria advanced by Messrs. Palo and Marks for the selection of semi-outdoor stations are merely approximate indicators, and their application should be limited to preliminary studies. On further analysis, the basic factors of appearance, relative economy, and operating convenience will form the bases for final station selection.

As stated by Messrs. Palo and Marks, the rapid pace set in the design of hydroelectric power developments has slowed sufficiently to permit expansive discussion of this subject. The writer notes, however, that much of the technical literature published by the TVA itself has been of descriptive, rather than analytical, nature. The writer suggests that fewer limitations on the exchange of ideas could contribute substantially to the advancement of knowledge in the engineering profession.

PAUL GISIGER,[19] M. AM. SOC. C. E.—It is a pleasure to note that the commendable policy of the Tennessee Valley Authority (TVA) engineers to report on design and performance of their structures in Society publications has been continued with these highly interesting and instructive papers.

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[19] Structural Engr., Pennsylvania Water and Power Co. and Safe Harbor Water and Power Corp., Baltimore, Md.

First Symposium Paper.—The first paper by Mr. Riegel is especially valuable because of the data presented on various kinds of observations and measurements of completed structures. In this respect, it is an excellent complement to the Symposium on Dams which was presented at the meeting of the Society's Joint Committee on Masonry Dams in Chattanooga, Tenn., on April 20 and 21, 1939.

In the first part of his paper Mr. Riegel properly states that, in the choice between different types of concrete structures, the cost of future maintenance should be a factor and that established methods of construction may also have an influence on design. The writer knows from personal inspection that the concrete in all TVA structures presents an excellent appearance and that maintenance costs in that respect should be very low. It is only fair to record, however, that, in the Tennessee Valley, climatic wear and tear on the concrete is generally less than, say, in Pennsylvania or New England. There are fewer cycles of freezing and thawing, and the same composition of concrete, which in a more northern climate might show excessive water-line deterioration in a short time, may retain its original appearance where winters are less rigorous.

The treatment of uplift conforms to conservative assumptions. As TVA dams generally are standing on sedimentary rock formations with definite bedding and cleavage planes, the policy of drilling uplift relief holes from 20 ft to 40 ft into the foundation is well justified. It has been established elsewhere[20] that even in quite compact rock formations, without such holes, uplift pressures some distance below the foundation joint may be of greater magnitude than those in the joint itself.

Although measured uplift pressures afford a good corroboration of design assumptions with regard to magnitude, they still do not settle the question as to the fraction of foundation area on which they should be considered active. The TVA assumption in this respect, considering the general character of foundations, grouting, and drainage provisions, is on the conservative side. It is highly desirable, but admittedly difficult, to find a method for removing this particular point from the realm of speculation to that of measurable fact.

Mr. Riegel makes it plain that he does not believe in the practice of grouting transverse joints of gravity dams, and the writer fully agrees. Although the reasons given for grouting the lower parts of the joints of high dams are plausible, these questions may still be in order: (1) Will the desired results be obtained? (2) Do not the departure from design assumptions and change in stress conditions thereby produced result in too high a price for the possible benefits? (3) Could not the same benefits be obtained by using drilled holes for grouting, thereby avoiding the obliteration of the contraction joints?

The results of stress measurements at completed structures under load are highly interesting. There are two explanations offered for the discrepancy between measured and observed stresses near the upstream face of some dams. Might not a third (and very probable) one be that the wetted portion of concrete near the upstream face does not shrink, but rather does expand and that the shrinking of the interior portion puts it under compression? The question is also asked whether in Fig. 7(*b*) the vertical stress curve between point 5 and the extreme toe has any basis other than assumption.

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[20] "Uplift Measurements at Holtwood Dam," by Paul E. Gisiger, *Civil Engineering*, July, 1938, p. 449.

From these observational data, the conclusion is inescapable that much about actual stress conditions in dams under load is still to be learned and that the use of elaborate mathematics for the derivation of stresses is of questionable practical value.

The section on "Earth Dams," in Mr. Riegel's paper, gives some excellent illustrations of the principle that sound engineering consists in making the best use of the most readily available materials. It would be interesting to learn how the different materials and cross sections compare economically. Although the earth dams are somewhat overshadowed by the scope of the other structures of the TVA, they are important enough in number and size to justify the hope for a future paper to treat, in more detail, the topics of planning, construction, and performance.

Second Symposium Paper.—The general features of a modern low-head and medium-head hydroelectric unit have become fairly well standardized, and differences in powerhouse cross sections are generally limited to the following: (*a*) The powerhouse may be of the indoor or outdoor (or semi-outdoor) type; and (*b*) step-up transformers may be alongside the generating equipment, or they may be some distance from the powerhouse. Within these general types there may be other variations of a lesser order of significance, but with considerable influence on detail structural design. Such factors are: (1) Location of thrust bearing (above or below the generators); (2) location of transformers (on the upstream or downstream side of the powerhouse); (3) location of mechanical control equipment (all on the main floor, or part of it on an auxiliary lower floor); and (4) differences in complexity and space requirements of electrical apparatus in accordance with system operating necessities.

The irreducible minimum requirements of a hydro powerhouse may be assumed to be reached with a layout in which transformers are located in a separate switchyard, in which generator leads are taken directly away from the powerhouse without interposition of a low tension bus, in which all speed-control equipment is on the main floor, and in which no superstructure is provided. The Fort Loudoun powerhouse comes very close to this minimum layout. It is significant that earlier powerhouses of the TVA are more complex. In general, it will be easier to get along with a minimum of apparatus if a new development is only a fractional addition to an already existing large generating system. On the other hand, if a new development is a main component or large fraction of a power system, its layout tends to be more complicated and to require more space. Apart from the influence of operational and system requirements, personal judgment and organizational preferences may come into play, too. This may be the case to a considerable extent in the provision or omission of a generator-room superstructure. Although it has been stated sometimes that climate need not be a determining factor in deciding whether or not to provide a superstructure, it still appears significant that, to the writer's knowledge at least, no outdoor or semi-outdoor types of powerhouses are in existence in Canada or New England. In TVA territory real blizzards are not a regular feature of every winter and operational difficulties from the omission of the superstructure should not be serious. It will be a matter of great interest to learn how the operating personnel of the TVA will evaluate the relative merits of indoor and outdoor layouts. Local conditions and other features of the general layout may have a decisive influence on the saving possible from omission of superstructure. Comparison of the Chickamauga cross section with that of Hiwassee makes it obvious that in the former the omission of side walls and roof would have meant a relatively much less saving than in the case of Hiwassee.

It is to be noted that, on almost every TVA main-river development, maximum tailwater elevation is above the top of the generators, requiring massive downstream walls of great height. This necessity, although apparently typical of Tennessee River developments, is by no means general. It is quite apparent that, in case of the Pickwick and Chickamauga, this condition worked against the outdoor type of powerhouse.

A comparison of the Fort Loudoun and Chickamauga cross sections, which are otherwise very similar, suggests the question as to why, for Fort Loudoun, the semi-outdoor powerhouse was preferred.

In the section on "Substructures for Large Propeller Units," Messrs. Palo and Marks rightly stress the importance of transverse joints, the attention that should be given to the subdivision, size, and sequence of concrete pours, and the difficulties that may have to be solved to keep commitments for future units within certain limits. The writer fully agrees that, for unit spacings of 40 ft or more, there should be contraction joints between all units. Messrs. Palo and Marks mention the difficulty presented by the usual offset between center line of unit and center line of intake and the resulting nonsymmetry of the draft tube, if the contraction joints are to be in one plane and perpendicular to the powerhouse axis. If there is an electrical bay on the upstream side of the generator room (which means heavily loaded columns to be located over the ends of the intermediate intake piers, as at Pickwick Landing), there are additional difficulties from this source. The writer does not believe that serious disadvantages would result if, on account of these difficulties, transverse contraction joints should deviate from a straight line or a single plane provided that thereby their proper function (to allow volume changes in the direction of the longitudinal powerhouse axis) was not impaired.

Conflicting requirements of hydraulic streamlining and stress concentrations in the noses of intermediate draft tube as well as intake piers may be satisfied with the help of structural or cast-steel vertical members at these locations.

With regard to the complex matter of scroll-case design, the writer feels that, although the efforts of the designing engineers so far have produced acceptable (but by no means perfect) results, a real mathematical treatment never has been undertaken. Such a task is not too complex and should be accomplished some day. The conception of vertical beams between top and bottom slabs is obviously only an approximation of actual conditions. It follows from any assumption of vertical beam action that such action must be accompanied by circumferential stress; and this stress, rather than temperature, is responsible for vertical cracks in scroll-case walls. As a rule every scroll-case wall has one vertical plane of least cross section on its downstream side, approximately on the transverse center line of the unit. The writer has seen vertical cracks on a number of concrete scroll cases at this location and believes they are caused by circumferential stress from water load.

Mathematical investigation of this condition is in line with the conclusion by Messrs Palo and Marks (with which the writer fully agrees): That the field of powerhouse design is at a stage where the course of development has slowed down sufficiently to make desirable a review of present methods and practices, and application of refinements which were not feasible in the days when each new development was an advance in size and scope over preceding developments.

Messrs. Palo and Marks are to be congratulated for a capable and concise presentation of a complicated, and never fully described, phase of civil and hydraulic engineering.

Third Symposium Paper.—During the past ten years or fifteen years, the development of water power has attracted much more attention than the construction of steam power plants. Therefore, when an organization which made history in the design and construction of water power developments builds a large steam power plant, its accomplishment in this respect deserves careful study by all engineers interested in either steam or hydroelectric power plants.

The paper by Mr. Meyer, intentionally, does not include any discussion of the reasons why, even in a hydro power system like that of the TVA (which includes both a number of run-of-river, as well as high-head storage plants), the addition of a considerable block of modern steam capacity was desirable. The fact that this steam plant was built, however, indicates that a combination of hydro and steam power is very often desirable and economical and that the two kinds of power are not competitive (as is sometimes believed) but complementary.

Mr. Meyer rightly observes that civil engineering features in a steam power plant are quite important for the over-all economy of the station. It is possible that sometimes in the design of steam power plants these features have not received the attention that they should because they are usually overshadowed by the much greater cost of steam producing and generating equipment.

At first it may be somewhat surprising that the Watts Bar station is not directly on the riverbank; but convincing arguments are presented to show that the location is the most economical one after due consideration of all factors involved.

The general design of the station with the unit layout of all auxiliaries follows generally established practice in steam plant design. In view of the outdoor type of powerhouse used by the TVA at the adjacent Watts Bar hydro plant, as well as at several other hydro stations, it is of interest to note that the usual complete housing of boilers, generators, and associated equipment has been chosen for the steam plant. It is perhaps rather surprising that the superstructure of this station was made of brick, whereas the TVA, so far, has mostly used reinforced concrete for hydro station superstructures. The question may also be asked whether it would not have been preferable to put a constrction joint into the superstructure between each of the four units. It does not appear quite logical to have construction joints in the substructure but not in the superstructure, which is more subjected to dimensional changes induced by temperature variations.

The steel-frame design of the generator-room roof follows the precedents established by the hydro plants of the TVA and it is an improvement over the conventional truss layout in several ways. Mr. Meyer rightly calls attention to the ingenious design of the crane-rail brackets in which the possibilities of welded construction have been utilized fully and intelligently.

With regard to the turbo-generator foundations, the question is in order as to whether the use of reinforced concrete was dictated by wartime considerations of material priorities or whether the TVA engineers considered this type of foundation generally preferable to a structural steel foundation. It appears that, with the multitude of piping around the foundations of steam turbines and the relatively greater ease with which piping can be accommodated through and between steel foundations, and also because of the greater speed of erection, there would be considerable advantages in the use of structural steel. However, if (as Mr. Meyer states) vibrations of the completed structure in service are hardly noticeable, the designers are to be congratulated for the results achieved, considering the number of cases in which excessive vibration has occurred.

With regard to the circulating water system, it appears that the plan for taking water out of the upper pool and returning it to the tailrace of the adjacent hydro plant was the proper method for this location. In view of the situation at another steam plant alongside a hydro plant—where it was found advantageous to take circulating water from the hydro plant forebay, but to pump it back so as not to lose any hydro generation—the question may be asked whether the latter system would have been used if the steam plant could have been located closer to the hydro plant. It may be perfectly justifiable, economically, to take water away from the hydro plant for this purpose; but there is a certain suggestion of wastefulness in having to use an energy dissipator at the point where this water is returned to the river. The question also may be asked as to why two conduits were used, considering that one pipe of 9-ft diameter would have been more than equivalent to the two pipes of 6-ft diameter. Since only one intake has been provided and 99% of all troubles in condensing water systems occur either at the intake or at the pumps, it does not appear that the two conduits could be justified by any consideration of dependability.

A somewhat more extensive description of the coal unloading and handling facilities would have been welcome. Such features have a considerable influence on the operating cost of a steam power plant, but are seldom described or treated in literature.

Oren Reed,[21] M. Am. Soc. C. E.—Powerhouse design of the Tennessee Valley Authority (TVA) is presented in a very clear and comprehensive manner in the second paper of the Symposium. Most of the text and illustrations apply to the main-river plants of the TVA, where the powerhouse is an integral part of the dam. At the higher dams of the tributary storage installations, the powerhouse is at the toe of the dam but is usually separated from the dam by a longitudinal contraction joint.

The typical semi-outdoor powerhouse is an economically sound choice for the usual installation where an extremely high tailwater does not require consideration. The heavy, overhead gantry crane cannot reach anything except the generator, which is as disadvantageous for maintenance and repairs under operating conditions as it is initially during construction. It is sometimes necessary during construction to leave a part of the roof off the generator room temporarily to place certain apparatus properly. Such interferences with proper construction procedures are unsatisfactory but must often be countenanced because of late deliveries of materials.

Provisions must be made to take care of many visitors at power projects near large centers of population or on good roads. Where large numbers of visitors are expected, more attention must be paid to the architectural appearance of the powerhouse than might otherwise be justified. Both the interior and the exterior of an indoor-type powerhouse have better appearances than an outdoor or semi-outdoor powerhouse.

Powerhouse design of recent installations by the TVA has been well planned. There is no waste space but there is no evidence of overcrowding. The auxiliaries are conveniently arranged and all reasonable safety precautions have been taken. Facilities for visitors are ample and the general appearance of each installation is very satisfactory.

R. L. Smart,[22] M. Am. Soc. C. E.—The economic depression of the 1930's and World War II caused the development of hydroelectric projects virtually to stop except for the activity of certain government agencies such as the Tennessee Valley Authority (TVA). Engineers, interested in this field, have followed the developments of the TVA, and thus the paper on hydroelectric stations by Messrs. Palo and Marks is very timely. In a careful review of this paper, the advancement in ideas and methods that took place from the initial work at Pickwick Landing until the final work as represented by Fort Loudoun is described lucidly.

The various functions of a hydro station and their relation to each other are well presented. Far too many projects in the past indicate very clearly that the first function received all the attention and that the remainder became incidental—to be taken care of in almost any way. Simplicity of layout and the elimination of waste areas are of prime importance. Certainly one way to secure such a result would be to fit the installation into the topography of the site, with due consideration for the hydraulic conditions and the most logical access.

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[21] Constr. Engr., TVA, Fontana Dam, N. C.

[22] Project Mgr., Ambursen Eng. Corp., Cons. Engrs., Houston, Tex.

If such conditions indicate a distinct advantage for any one type of structure—namely, indoor, outdoor, or semi-outdoor—that type should be chosen. Many engineers have a distinct preference for some one type of structure and have allowed that preference to rule regardless of whether that type was most suitable to the conditions. The TVA most certainly took all conditions into consideration and produced layouts to suit those conditions. All functions of such layouts were properly related for simplicity and the requirements of the project.

The proposal developed by TVA, for the acceptance of uniform waterways by all turbine manufacturers, was a most noteworthy advance and one of particular importance to those engineers who are not in a position to purchase turbines previous either to the initiation of designs, or even to the construction of certain parts of the project itself. It is not at all difficult for turbine manufacturers to agree on a uniform waterway (exclusive of the scroll itself). That one item permits the installation of future draft-tube sections at the time of original construction. By so doing the later need of troublesome, and often very costly, cofferdams can be eliminated, as such provision can be incorporated into the initial construction as part of the ultimate structure.

Modern trends in architecture have produced structures for the TVA that are most in harmony with the entire project. The function of the architect should be emphasized to the fullest degree, not only for outward appearance, but for the development of the proper features within. Although they will not admit it, most engineers are poor architects, and many a powerhouse stands today an eyesore both within and without. Rigid frames, be they steel or concrete, modern welding developments, and streamline designing are willing and able tools for the architect. By virtue of economic conditions prevailing during much of its construction program the TVA was extremely fortunate in being able to assemble a group of engineers with many and diversified lines of thought. They combined to bring forth numerous advancements in design and construction methods that it is hoped will be made available to the engineering world in general. Not all were perfect, not all will be accepted, but there are none not worth knowing.

ABRAHAM STREIFF,[23] M. AM. SOC. C. E.—An excellent and well-arranged description of seven low-head hydroelectric plants built by the Tennessee Valley Authority (TVA) is contained in the second Symposium paper. Plants with single, direct-connected vertical units have been in use since about the beginning of the century, and their design was thoroughly established when the TVA was organized. Although the plants described contain many excellent features, the writer finds many points of difference with previously established practice. Such rules as are set up by Messrs. Palo and Marks should apply only to the plants discussed, as there are some radical departures which are not at all universally accepted by engineers engaged in the design of hydroelectric stations. Some of these concern: (1) The fluctuation of the headwater elevation; (2) the low elevation of the intake sill; (3) disadvantages inherent in the combinations of items (1) and (2); (4) the depths of draft tubes; and (5) straight vertical joints between units.

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[23] (Fargo Eng. Co.), Jackson, Mich.

(1) Headwater Elevation.—The most conspicuous point of difference with common practice in low-head design is the very large fluctuation in headwater elevation. At Chickamauga Dam, for instance, the headwater level varies 26 ft. The operating head is considerably less than the maximum head. This increases the cost greatly without corresponding return from power developed. All structures, as well as machinery, stability, runaway speed, and head gates, must be designed for a head much greater than the operating head. Therefore, although the machinery may have a high efficiency, efficiency of the plant as a power dam is low. The cost is greatly increased whereas the output is based on a lower head. This is a concession to "flood control," which the writer does not intend to discuss. The design, however, is based primarily on such large headwater fluctuation, or so-called "flood pool," and the question may well be asked as to what advantage is gained by the greatly increased cost. The areas subject to flooding are greater than before the dam was built;[24] and, over a length of 633 river miles, flood "protection" has been obtained by purchasing several hundred square miles of land subject to flooding and by removing the occupants! This result could have been achieved without building a single dam, which is precisely the method in vogue for the lower Mississippi River, the rivers of the Netherlands, the Tisza River in Hungary, and the Po River in Italy, where maximum flow lines mark the boundaries of the public domain reserved exclusively for use as floodways.

The low-head dam with varying water level, therefore, is "neither fish nor fowl nor good red herring." As a power dam its plant efficiency is low, and the same kind of flood protection for the 633 miles of the Tennessee Valley could have been achieved without building a single low-head dam.

(2) Elevation of the Intake Sill.—A second departure from usual practice in low-head design is the very low intake sill elevation. The intake sills are the lowest spot of the cross section of the river. It is well known that holes dug into the bed of a river tend to refill. In the case of a deep pool with low velocities, mud and "deadheads" will accumulate. The "deadheads" have a tendency to collect and pile up in the lowest places toward which the current is directed. The large discharge through the big Kaplan units will cause all waterlogged trash to gravitate and converge toward the lowest spot in the intake. With the large rack spacing provided, the penstocks will tend to become fouled and gate grooves may become obstructed. Gates must be designed to stand unnecessarily large pressures, and lifts must become greater. Therefore, raised sills were included in such early structures as the Keokuk (Iowa) power-

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[24] "The Chickamauga Project," *Technical Report No. 6*, TVA, 1942, p. 30, Fig. 8.

house (1913). The argument is advanced that a short and straight intake is hydraulically preferable. However, the loss in the intake is minimal; and reduction of the already minimal intake loss is obtained at the price of several and costly disadvantages. The hydraulic passages comprise an intake as well as a draft tube. Sharp bends are allowed in the draft tube, where the velocities are really high and losses really large. It seems inconsistent to introduce a further warp or twist in the draft tube, merely to obtain straight up-and-down construction joints between units, while at the same time trying to reduce a minimal intake loss by straightening the intake.

(*3*) *Disadvantages.*—The low intake sill combined with the fluctuating headwater has a number of disadvantages. The head gates must resist a greater head and must have a higher lift. The shape of the intake precludes the use of Tainter gates, which are low in cost and can be closed quickly. Presumably to reduce the cost, a traveling crane with lifting beams, dogging devices, etc., has been adopted. This arrangement is cumbersome and slow. Heretofore much importance has been attached to rapid closure of penstocks.[25] In this connection it is of interest to refer to the Ryburg plant on the Rhine River in Switzerland. This station, built prior to the seven TVA dams (1932), contains Kaplan wheels which are larger in diameter than the largest of the TVA Kaplan runners. It also contains the same kind of head-gate system as was installed in the TVA plants; but, in describing the station,[26] it is stated that the plant does not have a single head gate! In effect, the head gates of the TVA type are emergency gates and nothing more. They are too slow and cumbersome to serve as head gates.

To make up for the absence of head gates, the Ryburg plant has a special governor system, consisting of a separate governor with its own pump and tanks for the Kaplan runner as well as for the guide case. If the governor that operates the wicket gates should fail, the runner blades would still be operated by the independent governor. In addition, a third emergency governor, actuated by an overspeed relay, and having an emergency pump of its own, would close the runner blades to a $-5°$ position, so that the runner would then act as a brake. The great friction in the trunnion bearings of the movable blades at runaway speed (the centrifugal force acting on each blade is about 750 tons at runaway speed) requires that the emergency governor have a capacity of not less than 760,000 ft-lb to assure operation of the runner blades in such an emergency. For the TVA plants apparently no special precautions have been considered necessary, as they have a standard governor layout.

The disadvantage of such low intakes becomes apparent when the cost of the head-gate system is compared with a Tainter-gate installation of equal opening. For the Chickamauga Dam, the cost of two sets of three head gates, 38 ft by 17 ft 8 in., is given as $189,361.94—not including the hoist. Each set is equivalent in area to two Tainter gates, 40 ft by 25 ft. An installation of two Tainter gates in each penstock (including the hoists) as purchased the same year for the spillway of the Buchanan Dam in Texas (1937) would have cost

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[25] "Catalog on Quick-Closing Gates," J. M. Voith Co., Heidenheim, Germany, 1935.

[26] *Schweizerische Bauzeitung*, June 4, 1932, p. 295.

$68,022. This cost would cover the provision of permanent gates with fixed hoist for each of the three turbine units—not just for two sets of gates to serve three units and this without the hoist. Tainter gates can be closed quickly, are very reliable, and are used in many dams and at many hydroelectric stations. The sizes of the Tainter head gates, in feet, at various installations are shown by the following list:

Name of dam	Width	Height
Cooke	25	20
Alcona	25	20.5
Junction	30	22.5
Elkhart	22	17
Five Channels	26	20.5
Alpena	16	18
Loud	26	20
Foote	29	18.67
Hodenpyl	20	20
Wissota	26	24.5
Mio	29	18.67
Texas Hydro IV	24.67	20.5
Texas Hydro V	24.67	20.5
Texas Power IV	24.67	20.5

None of the foregoing head-gate installations have required the use of emergency gates, although the oldest was made in 1912 and the latest in 1932.

(*4*) *Depth of Draft Tubes.*—For plants of the size described by Messrs. Palo and Marks, the draft tubes are from 50 ft to 60 ft deep. The spillway discharge causes a great vortex in the tailrace, which carries gravel into the draft tube. A long separating wall is indicated in such cases, which can be extended as a so-called "dwarf wall."[27] Since the separating walls for the plants described seem too short, this will introduce difficulties in seating the draft-tube gates.

(*5*) *Vertical Joints.*—Much stress has been laid, by Messrs. Palo and Marks, on straight vertical joints between units. This requirement is definitely necessary only for plants on a rock foundation. For plants built on a soil foundation, no joint is needed for a length of as much as 114 ft. At the Junction Dam on the Muskegon River three concrete scroll cases form one unit 114 ft long, operating under a head of 60 ft. Cracks such as those described by the authors for the TVA scroll cases have not appeared, although the plant is thirty years old. Such unit construction results in economy as the piers do not require doubling.

When a dam is built on an unyielding rock foundation, there should be a joint between units. Why should the joints be straight up and down? Such a requirement only introduces another warp and twist in the draft tube; it complicates the form work on the draft-tube division walls and offers no visible advantage. Many plants have been built with offsets, with no indication that straight joints are desirable. The logical approach to a good design is to lay out scroll case and draft tube as they should be and then to place the joints wherever they come. In the Ryburg plant, the joint follows the outline of the scroll and draft tube.[28] The friction coefficient of concrete on concrete is very low for slow temperature changes,[29] and with proper reinforcement no difficulty is experienced in superstructures by offsetting substructure joints. The location of the joint should be decided according to hydraulic requirement. Since this has always been the case, the writer sees no reason to change the usual practice.

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[27] "The Irrigation Works of India," by R. B. Buckley, 2d Ed., London, 1905.

Ross M. Riegel,[30] M. Am. Soc. C. E.—In regard to the discussion by Mr. Creager, TVA engineers agree with the practice advocated by him of analyzing the uplift pressure under dams according to two factors—that is, the hydraulic gradient factor and the area-of-pressure factor. However, Mr. Creager apparently takes issue with the use of any area factor less than 100%. Before proceeding with discussion of this point, it is to be noted that Mr. Creager is correct in the definition of uplift intensity as applied to the higher dams—that the drainage coefficient applied to the difference between head and tailwater elevations has a value of 0.50.

The author is convinced that the difference between a rock foundation and a granular foundation should be reflected in a difference between the areas to which uplift pressures are assumed to be applied. In the case of granular materials there is no question as to the 100% factor. The author is one of several who has demonstrated this experimentally, but, when there is cohesion and continuity in the supporting structure, the relationships involved are somewhat different.

The most reasonable approach involves two assumptions:

(1) That the uplift pressure acts through and upon certain openings, pores, or channels, in such a way as to reduce the effective weight of the supported structure; and

(2) That the reduced weight must be supported by the solid part of the foundation, which will carry this load in compression.

The assumption that two thirds of the foundation area is subject to uplift means that the compressive loads are really carried upon one third of the gross area. In this case no tensile strength in the foundation need be assumed, at least with regard to principal stresses.

The use of the two-thirds ratio cannot be defended experimentally; it is justified chiefly by a conviction that the factor must be materially less than unity. A number of engineers have adhered to the two-thirds ratio as expressing their judgment; and, in the structures referred to, this value was used as representative of current conservative practice. Although the author has referred to a certain susceptibility of bedded rocks to the development of uplift, heavy excavation has been practiced on TVA dams to secure as sound foundations as the intrinsic character of rock permitted; and these have been very good foundations indeed.

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[28] *Schweizerische Bauzeitung*, May 7, 1932, p. 241, Fig. 14.

[29] "Sliding Joints Under Pressure," by F. Bolliger and W. Humm, *ibid.*, January 9, 1937, p. 15.

[30] Head Civ. Engr., Design Dept., TVA, Knoxville, Tenn.

Examination of the TVA main river dams indicates no cause for concern even if the assumed uplift intensities are applied over 100% of the base area. Although the margins assumed in the design would be somewhat reduced, they are still sufficient under any probable loading to avoid tensile effects at the heel. In the case of Fontana Dam, which has been made heavier because it was exceptionally high, calculations based on 100% area showed no tension at the heel under conventional calculations.

Mr. Gisiger feels satisfied with the adequacy of uplift assumptions, although he as well as others, including the author, are hopeful of sometime acquiring actual data as to effective areas and of taking this question out of the realm of assumption. The recently published investigations of Mr. Leliavsky may show the way.[31]

In regard to other questions raised by Mr. Gisiger, the extension of the stress curve in Fig. 7(*b*) from point 5 to the toe is supported by strain meter observations in the downstream face of the dam at depths, of 8 in., 20 in., and 36 in. but at 35 ft above the basic measuring section. The extrapolation of this stress curve is thus well supported, although that between point 1 and the heel is not so well supported.

It is the author's present belief that absorption of reservoir water, somewhat as suggested by Mr. Gisiger, by the concrete is largely responsible for the compressive stress observed near the upstream face of Hiwassee Dam. Certain aspects of this phenomenon are still unexplained, but it is hoped that later observations on Fontana Dam will be of value in this connection. A detailed study of the stresses in Hiwassee Dam is being prepared for publication.

Mr. Gisiger rather questions the placing of any grout whatever in transverse joints. In TVA dams the same view has generally prevailed. However, in the cases of Fontana and Hiwassee dams, the objective of curing potential cracks near the foundation level appear to warrant a departure from this policy. It is believed that no objectionable stresses arose from the limited grouting. The cracks could have been reached by drilling; but, even in this case, grout would be likely to escape into the contraction joints. As to Mr. Gisiger's hope for a further paper on earth dams, there are some prospects of publication on these lines.

Mr. Tyler asks about the relative effectiveness of various measures in the reduction of cracks. The control of actual temperatures in the concrete by cooling water circulated through pipes, as practiced at Fontana Dam, has probably been most effective. This measure may be supplemented by the use of chipped ice in the mixer during hot seasons or by otherwise cooling the mixing water so as to reduce the placing temperature. Fontana Dam is the only TVA dam in which a general program of forced cooling was conducted.

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[31] "Experiments on Effective Uplift Area in Gravity Dams," by S. Leliavsky, *Proceedings*, Am. Soc. C. E., December, 1945, p. 1474.

Aside from Fontana Dam, Hiwassee Dam represents the best results obtained with respect to cracking in TVA practice. In the writer's judgment the most important factor in this result was the basic program of slow pouring, set up and adhered to on this job. Next in importance he would rank the use of low-heat cement with its concrete of relatively high plasticity at the early ages, and the low cement content. The cooling of mixing water, as mentioned by Mr. Tyler, was certainly also a beneficial factor.

Construction under wartime pressure with nonavailability of certain materials and shortage of man power, coupled with urgent need for completion, sometimes has required some departure from the best methods. In closing, the author would remark that a certain deliberation is desirable in the placing of mass concrete.

George P. Palo,[32] and Constant R. Marks, III,[33] Assoc. Members, Am. Soc. C. E.—The second paper of this Symposium was presented in three main parts—namely, (1) "The Functional Layout," (2) "The Superstructure," and (3) "Substructures for Large Propeller Units." It is desirable to arrange this closing discussion in a similar way.

Functional Layout.—Several of the discussions, touching upon the problems of functional layout, have added appreciably to the thoughts presented in the paper. Mr. Larned and others have stated that the paper described the layout of a large low-head station integral with the dam. Such a station was used, as an example, because it involves nearly all the elements of equipment and space requirement commonly found in a powerhouse. However, many of the principles involved can be applied with some modification to other types of plants. To detail such application for the various types of plants would have required a much longer paper.

Mr. Streiff states that the headwater level at Chickamauga Dam varies 26 ft and that low-head developments for such a headwater variation are not sound. A detailed discussion of this subject is beyond the scope of this paper; however, it should be emphasized that the actual headwater variation at Chickamauga Dam for power operations is only 10 ft. An additional 16-ft headwater rise develops in the case of the maximum design flood, and at this time the tailwater is only 3.5 ft below the headwater level. Hence, for all practicable purposes, the headwater variation at Chickamauga Dam is only 10 ft. For a power plant with a reservoir and drainage area of the size existing at this site, variation of the headwater is quite necessary for efficient use of the stream flow. Comments on the economics of providing flood pools in conjunction with relatively low-head dams, as has been done on the Tennessee River, do not seem applicable to a paper that deals with station layout and substructure design. For this reason and because this subject has been treated at great length in other places, no attempt will be made to answer Mr. Streiff's questions on this subject.

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[32] Chf., Civ.-Architectural Design Div., TVA, Knoxville, Tenn.

[33] Chf. Design Engr., Ambursen Eng. Corp., Houston, Tex.

Mr. Streiff also refers to the low intake sill elevation as a departure from usual practice in low-head design. The cross section of intake waterways used by the Tennessee Valley Authority (TVA) on its low-head plants was not originated by that organization. The Bonneville, Wheeler (a TVA project designed by the Bureau of Reclamation), Rock Island, Safe Harbor, and even the smaller London and Marmet plants are all examples of propeller wheel installations built since 1925. All of them use a low intake sill, and all were designed by different organizations.

Mr. Streiff suggests the desirability of providing head gates suitable for rapid closure but then refers to the Ryburg plant on the Rhine River where no head gates are used but where the governor system adopts unusual protective measures. The Ryburg plant has four units, each of which is separately owned and each of which supplies its own distribution system through its own switchyard. Such conditions make Ryburg, in effect, four single-unit stations, each available for almost instant operation. The units in TVA stations, as parts of a large system, need only normal governor protection. In case of difficulty with a unit, or during periods when stream flow is insufficient for all units, a single head gate handled by a gantry crane offers adequate and amply rapid closure of the waterways.

Mr. Streiff's comment that the vortex in the tailrace caused by spillway discharge will result in deposit of gravel in the draft tube is correct provided the units are not operating. In the years of TVA experience, it has been found that river-bed gravel is deposited in draft tubes which have been completed and are ready for installation of units at some future date. No deposits have developed in draft tubes of completed units. When it has been necessary to unwater for the installation of the turbine and generator, the small additional cost necessary to clean out the gravel and set the draft-tube gates has been almost negligible as compared to the cost of extending the training wall between the spillway and the tailrace.

The writers are in full agreement with Mr. Streiff that head losses in an intake are so small that they do not justify great expense for small reductions that might be accomplished; and, in stressing the desirability of the short intake, it was intended only to emphasize that this procedure provided ample hydraulic efficiency at minimum cost.

Superstructure.—Almost all the discussers have commented on the choice of type of superstructure and have cited additional details that should be considered in making this choice. However, the items of cost, operating convenience, appearance, and access still remain the basic features to be considered in determining the type of superstructure. Mr. Bowman notes the apparent inconsistency of TVA projects in that indoor structures were used for Pickwick, Chickamauga, and Guntersville, whereas semi-outdoor structures were selected for Kentucky, Watts Bar, and Fort Loudoun. Mr. Gisiger offers the same suggestion in comparing Chickamauga and Fort Loudoun. These factors arise simply from the building schedules of the projects. The indoor projects were designed first, and not until Watts Bar and Kentucky were being considered was the concept of the semi-outdoor design fully developed. Within the writers' knowledge only one other plant in the United States, designed and built about 1923 by Mr. Larned's organization, has used the semi-outdoor scheme. TVA designs were originated without prior knowledge of this plant but did utilize knowledge of the Wetter plant in the Ruhr in Europe. Mr. Gisiger suggests that the outdoor plants must be limited primarily to warmer climates. The writers agree that in many organizations this factor will receive such consideration, but it is known that Mr. Larned's organization, which has been particularly active in the development of outdoor stations, has built such structures in Minnesota and Montana. Mr. Reed describes certain future obligations that should be considered whenever an outdoor or semi-outdoor powerhouse is under discussion. The inability of the overhead gantry crane to reach anything except the generator and interferences with construction procedures are definitely limitations to outdoor plants. It has been suggested that the entire roof sections of outdoor plants could be made removable. Such procedure would minimize these disadvantages.

In the next to last paragraph of his discussion, Mr. Bowman indicates his understanding that, in final choice of superstructure, access is a relatively minor factor as compared with economy, operating convenience, and appearance. Access can be regarded as a part of operating convenience, of course; but, whether it is a separate or a dependent factor, access in some plants must be considered the most vital single item in the choice of superstructure.

In view of the comments of the discussers, the writers would like to summarize the comparison between indoor and semi-outdoor types of powerhouse:

1. In designing for tailwater elevations more than 20 ft above the generator-room floor, the semi-outdoor type will usually offer great economy, since it avoids an excessively high superstructure, large watertight doors, or the use of a transfer crane for bringing equipment into the powerhouse;
2. For lower tailwater elevations marked economy will usually be possible only if the number of units is large so that the superstructure saving over several units can be balanced against the extra cost of a gantry crane;
3. Where the economy of eliminating superstructure is not demonstrably large, the factors of appearance, operating convenience, and more readily constructed watertight roof will generally lead to adoption of the conventional indoor powerhouse.

Substructures for Large Propeller Units.—

General.—Mr. Larned has requested a brief general statement as to the procedure adopted for securing watertight joints. TVA designs have attempted to secure watertightness, as has been emphasized in this paper, by a careful consideration of volume change so that cracks may be held to a minimum and then design of reinforcement so as to spread cracks that do form. Concrete is proportioned carefully. When the concrete is quite heavily reinforced and when $1\frac{1}{2}$ in. is the maximum size of coarse aggregate, cement content is about 1.5 bbl per cu yd. Special care is taken in the cleanup of horizontal construction joints, and steel strips are used as water stops. In vertical construction or contraction joints, copper strips are used. These strips may be a single line if they are intended as protection against leakage from occasional high tailwaters; or they form a double line if their purpose is protection against permanent headwater pressures. Where shears are transferred across construction joints, the surfaces of the joints are roughened by jackhammers, and the improved bond so obtained assists in obtaining watertight conditions.

Draft Tubes.—Mr. Streiff has objected to the offset in draft tubes to permit the use of a straight contraction joint between units. He feels that waterways should be determined according to hydraulic requirements and that the structure should be built around them. The writers, however, agree with Mr. Smart's comment that "Far too many projects in the past indicate very clearly that the first function [waterways] received all the attention and that the remainder became incidental * * *." Proper design must balance the cost of waterways together with the cost of structure and all other features in the building of the powerhouse. Actually, no turbine manufacturer has objected to the offset provided in the draft tube, and there has never been any indication of additional hydraulic losses from it. The offset does not complicate the forms beyond the fact that two or three draft-tube frames used in the elbow are not symmetrical—but this condition has no effect on the draft-tube construction. The benefits of a straight joint are considerable in the detailing not only of the substructure but of the superstructure; and these benefits far outweigh any minor complications needed to offset the draft tube. If the tried and relatively simple practice of extending the joints between unit blocks straight up through the superstructure by double columns and girders is to be followed, the problem of superstructure design presented by offset joints is very large. The writers cannot agree with Mr. Streiff that the friction coefficient of concrete on concrete in practicable structures is very low for slow temperature changes. Actual experience and difficulties of incorporating sliding seat details in field construction are such that these designs should be avoided wherever possible. The writers agree with Mr. Larned's comments that draft-tube forms should be built at a point separate from the actual construction and should be hauled by trucks to the place where they will be finally set. This has been standard TVA practice. They are also in accord with Mr. Larned's comments that draft-tube gates are almost never required where the turbine is set above low tailwater. However, experience with cavitation has tended toward lower settings of units than were prevalent some years ago. In addition, propeller wheels usually are set lower than Francis wheels. In almost all TVA projects the turbine is below low tailwater, and the draft-tube gates are used whenever the draft tube is unwatered. Both Mr. Larned and Mr. Gisiger have mentioned the use of cast-steel noses for the upstream end of the draft-tube piers. Many actual projects show the practicability of such procedure. TVA designers have preferred the use of concrete nosings, however, on the basis that plastic flow of the concrete must eventually create extremely high stresses in steel nosings with resultant high bearing pressure on the concrete at the top of the steel nosing and on the rock foundation at the bottom.

Scroll Case.—The writers have commented on the recurrent crack in the scroll-case walls approximately 10 ft upstream from the center line of the unit, and Mr. Larned has suggested that this condition may result from a sharp break in the excavation just below this point. As this crack has never appeared in the massive concrete floor of the scroll case, the writers are convinced that it is a result of temperature stress in the scroll-case walls, resulting from the restraint of the massive floor. However, Mr. Larned's warning about the danger of such reentrant angle as may be formed by a sharp break in foundation excavation is sound. Actually, TVA excavation lines have not been the sharp corners indicated on some of the drawings in the paper; and, even where a tendency for a sharp corner has existed, reinforcement has been used to control cracks. Furthermore, the steps in the excavation are not usually continuous across the unit but extend farther downstream near the contraction joints, where bedrock can be left in place between the upstream ends of adjacent draft-tube elbows. Mr. Gisiger suggests the desirability of a thorough scroll-case design instead of the admittedly approximate treatment that has been used. He states that the concept of vertical beams between top and bottom slabs must be accompanied by circumferential stress and that this stress, as well as temperature, may be responsible for vertical cracks in scroll-case walls. The writers agree fully with these comments, and all TVA scroll cases have included substantial horizontal steel to withstand circumferential stresses. The subject for a real mathematical design of scroll cases, however, involves many serious difficulties—among them the analysis of a structure of extremely complicated shape with stresses affected by many holes cut in the concrete. The temperature stresses must be analyzed properly for various possible conditions that will be met in construction. Even the usual methods for proportioning reinforcement in concrete must be reviewed, because in the heavy corner sections the stress conditions simply do not permit the proportioning of reinforcing steel on the basis that the concrete cracks to the neutral axis. The writers are convinced that such analysis must always assume a certain allowable tensile strength for the concrete. The desire for greater knowledge of stresses within scroll cases is no doubt shared by every designer. It is the writers' belief, however, that the best approach is the experimental one—probably by stress measuring devices embedded in actual scroll cases. They cannot emphasize enough, that measurement be made with fully as much attention paid to the history of stresses due to temperature changes as to those for the water loads. There are certainly many points in the scroll case where tension in the concrete due to shrinkage far exceeds that due to the bursting pressure of the water.

The writers appreciate the contributions of those who have discussed the paper and feel that they have added much to its original content.

Adolf A. Meyer,[34] M. Am. Soc. C. E.—The discussions have widened and enlarged the scope of the writer's paper; for, as Mr. Sporn states, an economical and well-performing station can be designed only when all elements—civil, mechanical, electrical, and operational—are taken under consideration at the same time and with equal weight. The design for the Watts Bar plant was developed under just such a procedure. Unfortunately, this well-integrated process of work could not be clearly reflected in the paper because, for apparently good reasons, it was agreed at the outset to limit the discussion to the civil engineering features. Based on his experience with this paper, the writer believes that a more useful and more substantial presentation could have been made by including the more important phases of mechanical and electrical design.

All the specific features of design mentioned in the discussions were the results of comparative studies, and the suggested alternatives were considered carefully during the early design stage. The most constructive procedure for this closure, therefore, will be to refer to these early studies and to state why, n each case, the adopted feature was found preferable to the alternatives mentioned in the discussion.

The station location, in its broad aspects, was governed by load requirements and fuel costs. These considerations placed it in the eastern part of Tennessee; but the specific site selection within this rather extensive area was based on advantages which were unique for the Watts Bar site: Ample transimission interconnections for the hydro plant; existing railroad siding and access road provided for the hydro plant; a construction organization; and construction facilities, such as warehouses, shops and utilities, camp facilities, and a village for operators. Together these items, which could be made usable for both plants with only small changes or additions, constitute a substantial expense. Not mentioned in the paper was the saving in operation obtained by placing the control of the main electrical switching for the steam station in a central control room adjacent to the main switchyard.

For the number of stacks used, it appears consistent with the unit design adopted for the station to provide one stack with each boiler, as suggested in Mr. Sporn's discussion. However, outages for stack maintenance are very rare, and therefore the unit principle is not essential in this regard. With one stack for two boilers, the framing of the stack base is far more economical in the narrow bay between boilers than is the framing for the two bases each over the wide boiler bay. In addition, at Watts Bar provision had to be made for the possible future addition of dust collectors. The adopted layout which provided one stack for two boilers lent itself readily to this provision whereas, for the layout with an individual stack for each boiler, this future installation would introduce considerable complications.

The width of the generator room was determined by requirements for turbine erection. Two ways of pulling the rotor were considered. One was canting in a horizontal plane as suggested by Mr. Sporn. The alternative provided a removable plate in the wall, through which the shaft could be extended beyond the generator room. This alternative required 2.5 ft less width and was therefore adopted. By adding 1 ft to the minimum clearance requirement (which was desirable from the point of view of erection, in any case) a track could be arranged parallel to the long axis of the generator room. By adopting this layout no track changes of any kind were necessitated in the yard during the three consequent construction stages.

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[34] Head Civ. Design Engr., TVA, Knoxville, Tenn.

In the initial construction of the steam station no wall was provided between generator room and heater bay. Only after the station had been in operation for some time was it considered desirable from reasons both of operation and of architectural appearance to add this wall.

The thickness of the basement-floor slabs is the direct result of limiting the allowable compressive stresses on the foundation to 50 lb per sq in. By adopting this severe assumption the loads had to be distributed uniformly over practically the full basement area, with the slab thickness proportioned to obtain the distribution with minimum material requirements. The load limitation was adopted after observing settlements on similar rock foundations in the vicinity, which amounted to $\frac{7}{8}$ in. under a load of less than 150 lb per sq in. This and other similar observations on the behavior of the foundation rock under load invited extreme caution. Unequal settlements which would be detrimental to equipment and structures had to be avoided. A uniform distribution of the load by a continuous foundation mat was the answer to these conditions. The observed results of these measures were very gratifying; no unequal settlements were experienced.

Mr. Gisiger states that the outdoor-type of powerhouse is a possible alternative to the adopted enclosed construction. As there was very little time available for design studies, the generally proved type of steam-station layout with a fully enclosed structure was adopted at the beginning. A brick enclosure was chosen because it is more economical and more durable than a concrete superstructure. Mr. Gisiger assumes this to be a departure from the TVA practice on hydro-plant design. During flood stages almost all TVA powerhouses have tailwater above generator-room level. They must be watertight; and, for this reason, concrete was adopted. Where this criterion does not exist, TVA, since about 1938, has preferred construction in tile with stone facing for the hydro powerhouses.

Since the substructure was built without any permanent contraction joints, it was logical to omit the joints in the superstructure. The foundations for the turbo-generator units were built in reinforced concrete because structural steel was critical war material; but, apart from the necessity imposed at a particular construction period, this type of foundation would have been preferred by the TVA designers in normal times. Clearance requirements can be satisfied with concrete construction; but this construction is much better adapted to the elimination of objectionable vibrations. Such reasoning agrees with the very definite recommendations of the turbine manufacturer.

As stated in the paper, the gravity-type of circulating water system was adopted because of its superior dependability after a careful analysis of all cost elements entering the problem had shown that the pump and the gravity systems were almost identical in cost. Whereas Mr. Gisiger supports the contention of superior dependability by his statement that "* * * 99% of all troubles in condensing water systems occur either at the intake or at the pumps * * *," Mr. Sporn seems to have doubts regarding the dependability of the pipe lines. They were all placed in natural ground and carefully backfilled, and the writer can perceive no cause for any break or damage of this system.

Of course, had the actual construction sequence been foreseen at the time the first two units were installed, the two pipe lines would have been combined into one conduit—with a resultant substantial saving. However, in 1940–1941, during construction of the first two units, the need for the third and fourth units appeared to be many years off. After the Japanese attack on Pearl Harbor, Hawaii, this condition changed very rapidly; but no one could have foreseen it.

The dissipator at the end of the canal, which suggests wastefulness to Mr. Gisiger, covers the range of the normal tailwater variations. The energy involved, of course, was included in the cost analysis. It is interesting to note that this same feeling of waste led to a study of the advantages of using a small hydro unit instead of a dissipator. However, the study showed that such an installation could not be justified economically.

Mr. Sporn requests an enlargement of the previous statements of the reason why the station was moved 600 ft away from the river. He notes the cost of the long discharge canal and the long coal conveyer connecting with the coal dock.

The cost of the canal—excavation and riprap—was $26 per ft. The coal conveyer cost $33 per ft. Assuming a possible shortening of these features by some 400 ft, a saving of about $24,000 could have been realized on these items. The rock foundation at this location for the plant would have been 50 ft lower and even the natural ground level would have been below the basement level, if the station were to be built with the same conservative protection against floods. For a station at this location, the substructure costs would have been increased many times the saving realized on conveyer and canal. In addition, the yard grading would have required an enormous quantity of borrow material. The freedom which the present layout gave in the arrangement of the yard structures all around the main station would have been lost; and a less convenient yard layout would have resulted. In fact, the advantages that could be obtained by the location away from the river impressed the writer so much that in the future he would not consider a location study for a steam station complete if it did not include investigations of sites away from the immediate river shore. Of course, different conditions may result in an answer quite different from the one obtained at Watts Bar, but the prevailing notion that under all conditions it is best for a steam station to be close to the river was severely shaken by the result of the Watts Bar site investigation.

As a liability of the long canal, Mr. Sporn stresses the maintenance cost for removing sedimentation from the canal. The conditions at Watts Bar are such that no sedimentation is to be expected. The water is taken out of a reservoir 70 miles long; only during extreme floods is silt carried to the dam.

The coal-storage yard, which was designed for 200,000-ton capacity, has shown its flexibility in handling the contingencies of wartime operation. As much as 300,000 tons of coal have been in storage. Both tractor-scraper and drag-scraper systems for handling coal in storage were originally investigated. The data on performance of large tractor scrapers were still meager, and a long investigation was out of question; therefore, the well-established drag-scraper system was adopted.

The question is raised as to why the coal dock was extended over a length of 700 ft. The conditions of the river in front of the dock are somewhat special. Whenever the spillway above discharges water, the velocities are rather high. Safe operation under these conditions requires that the barges be strung in single line along the shore. Experience has shown that the dock was too short or handling large shipments. In 1946, sheet-pile cells were added upstream and downstream from the dock, giving mooring facilities over a total length of 1,400 ft.

The main dock was built with secondhand sheet piling and backfilled with slag. The cost per linear foot was $94.

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Paper No. 2292

FUTURE OF LAKE MEAD AND ELEPHANT BUTTE RESERVOIR

By J. C. Stevens,[1] President, Am. Soc. C. E.

With Discussion by Messrs. D. C. Bondurant, John H. Bliss, Luna B. Leopold, Carl B. Brown, G. E. P. Smith, Albert E. Coldwell, Walter B. Langbein, C. S. Howard, Charles Kirby Fox, H. V. Peterson, L. C. Crawford and P. C. Benedict, Charles P. Berkey, Stafford C. Happ, Berard J. Witzig, Hugh Stevens Bell, E. W. Lane, F. E. Bonner, Joe W. Johnson, Harold H. Munger, William Mayo Venable, Edward Gerald Smith, Clarence S. Jarvis, L. Standish Hall, and J. C. Stevens.

Synopsis

The gates in the by-pass tunnels at Boulder Dam (Arizona-Nevada) were closed February 1, 1935, and the storing of water and sediment began. In ten years following that date some 1,500 million tons of sediment passed the Bright Angel silt-sampling and gaging station in the Grand Canyon, in Arizona. Virtually all of this sediment was arrested by Boulder Dam. There is also an added contribution from the 30,000 sq miles of drainage area between the Grand Canyon Station and Boulder Dam.

Storage in Elephant Butte Reservoir in New Mexico on the Rio Grande began January 6, 1915. Before the construction of the dam a topographic map was made of the reservoir. After its completion five surveys of sediments deposited in the reservoir were made. During this period records of suspended sediments have been kept on the Rio Grande at San Marcial, N. Mex., above the highest backwater from the dam. So far as known this is the only reservoir for which a comparison of sediment inflow and sediments deposited can be made[2] in order to determine some over-all value of the reservoir space occupied by sediment deposits.

An analysis of the sediment records for Elephant Butte Reservoir is presented. Utilizing these data an estimate of the useful lives of both Lake Mead and Elephant Butte Reservoir is made. A discussion of what feasible steps may be taken to prolong their usefulness is included.

Note.—Published in May, 1945, *Proceedings*. Positions and titles given are those in effect when the paper or discussion was received for publication.

[1] Cons. Hydr. Engr. (Stevens & Koon), Portland, Ore.

[2] "A Study of Methods Used in Measurement and Analysis of Sediment Loads in Streams," *Reports 1 to 9*, Iowa Inst. of Hydr. Research, Iowa City, Iowa, 1940–1943, *No. 9*, p. 19.

Although the writer has served as Consultant for the National Resources Planning Board and also for the Bureau of Reclamation, U. S. Department of the Interior, on matters pertaining to the Colorado River and Rio Grande basins, the conclusions and opinions here reflect his personal views and are in no sense to be construed as reflecting the opinions of those agencies.

General Considerations

The term sediment, as used herein, will be understood to include all clays, silt, sand, gravel, boulders—in fact, any transported material that will occupy space in a storage reservoir. These sediments are mostly water-borne, although considerable volumes are of aeolian origin, and others are talus from the confining shores of the reservoir. The principal contributions are in the form of deltaic deposits by the main river and by the tributaries that flow directly into the lake.

Deltas.—A delta forms at the mouth of each entering stream. Only deposits at the lowest stage of the reservoir have any chance of remaining in place. All deposits at higher stages are reworked by the streams as the reservoir falls and are thus intermittently carried farther and farther into the reservoir. As these deltaic deposits are reworked they are ground finer and transported farther into the reservoir at every freshet.

The sediments brought in by steep tributaries contain much coarse material. The effect of reworking is to flatten the slope at which they were originally deposited, but such coarse material never moves far into the reservoir.

Density Currents.—The finer sediments, clays, and colloidal material remain in suspension longer than the sands and are therefore deposited well into the reservoir. Frequently such fines are carried by the muddy stream supplying them through the reservoir along its thalweg while the water surrounding this muddy stream remains clear. Such silt flows are called "density currents." By them the fines are deposited against and immediately above the dam. If low outlets are open these mud flows will pass entirely through the reservoir with no visible evidence of their presence on the reservoir surface. By this means the so-called "dead-storage" space lying below outlets may become filled readily with fines.

Talus.—A large quantity of heavy material in the form of talus accumulates from, and settles at, the bases of the steeper shores and cliffs. When exposed above water the talus weathers down into finer sediments but normally does not move far from the base of the cliffs from which it originates.

Aeolian Deposits.—Any one who has driven between Zion National Park, in Utah, and the Grand Canyon can attest to the enormous wind-borne sediments of which many of the bluffs and canyon walls skirting the highway are composed. In arid sections such as these the volume of aeolian deposits is extremely large. Compared with water-borne deposits however their volume is relatively small, yet they cannot be ignored in the long period of years that it takes to fill a reservoir with sediment.

The Life of a Reservoir.—The great complexity in the process of sedimentation in reservoirs and in the inflowing stream channels, the uncertainties regarding the space occupied by sediments, lack of knowledge regarding the contributions by water and wind from tributary drainage area, the variability of sediment inflows year by year and in wet and dry cycles, the inability to evaluate accurately between suspended sediment and bed load, all militate against the drawing of invariable conclusions regarding the useful life of any reservoir that is subject to silting.

PART 1.

ELEPHANT BUTTE RESERVOIR

Elephant Butte Dam was completed in 1916 although impounding of water began on January 6, 1915. The drainage area at Elephant Butte Dam, exclusive of the closed basin in Colorado, is 25,923 sq miles; at the San Marcial gaging station it is 24,176 sq miles. Fig. 1 is a map of the Upper Rio Grande basin on which the alluvium and sedimentary deposits are distinguished from volcanics and metamorphics. The former are largely responsible for the sediments carried by the streams. The Rio Grande is perennially muddy downstream from the mouth of Rio Chama, N. Mex. Upstream from this point it is fairly clear except at times of freshets.

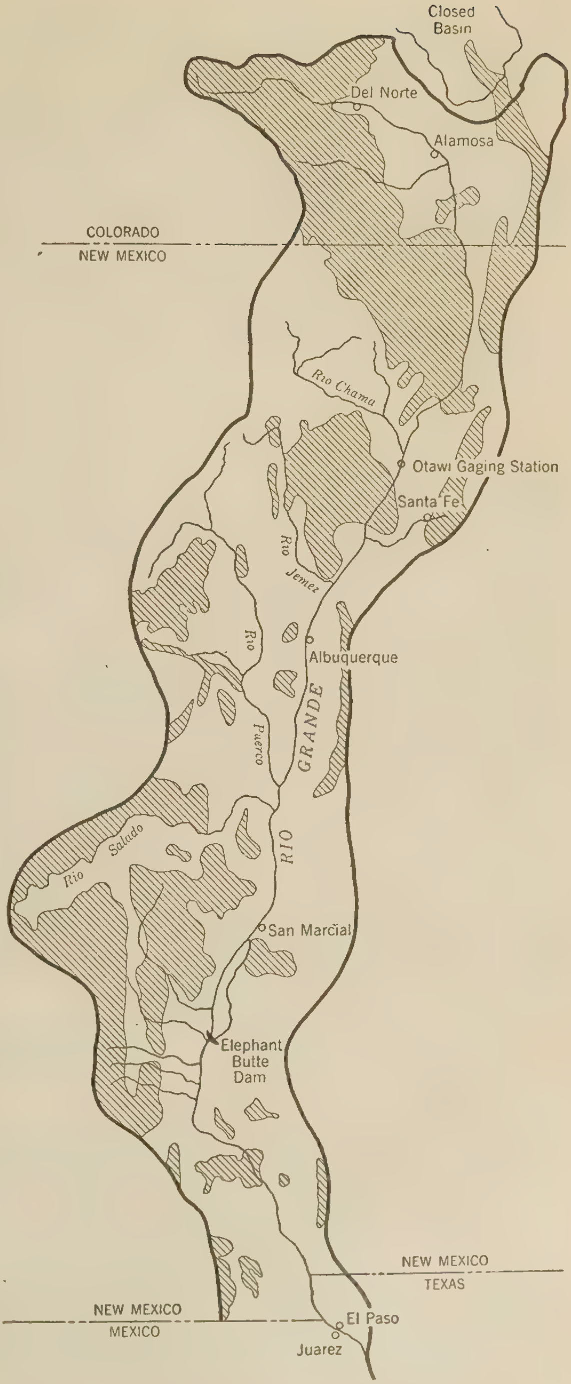
Closed Basin
Del Norte
Alamosa
COLORADO
NEW MEXICO
Rio Chama
Otawi Gaging Station
Santa Fe
Rio Jemez
Albuquerque
Rio Puerco
RIO GRANDE
Rio Salado
San Marcial
Elephant Butte Dam
NEW MEXICO
TEXAS
NEW MEXICO
MEXICO
El Paso
Juarez

FIG. 1.—UPPER RIO GRANDE BASIN (UNSHADED AREAS ARE SEDIMENTARY DEPOSITS)

SEDIMENTS DEPOSITED

Table 1 gives the results of surveys of sediments deposited in Elephant Butte Reservoir below spillway level.[3] It represents the deposits reported from all sources between the sill of the outlet gates and the spillway level. To compare this record with the measured sediments passing the San Marcial gaging station it is necessary to know the sediments deposited downstream from the San Marcial gaging station and above spillway level, as well as the deposits below the sill of the outlet gates. These quantities are not found in any publications but were ascertained by correspondence to be 13,929 acre-ft and 3,432 acre-ft, respectively. The latter value was changed to 3,218 by a recomputation of capacities made in 1940 (letter to the writer dated January 9, 1943, from Carl B. Brown, Assoc. M. Am. Soc. C. E., Head of the Sedimentation Section, Soil Conservation Service, Washington, D. C.).

TABLE 1.—SEDIMENTS DEPOSITED IN ELEPHANT BUTTE RESERVOIR IN NEW MEXICO

Period	Years	Cumulative years	DEPOSITED (THOUSANDS OF ACRE-FEET)		Capacity lost (percentage of original)
			Period	Cumulative	
Storage began January 6, 1915	....	0	0	0	0
January 1915 to December 1916	1.91	1.91	54.0	54.0	2.04
December 1916 to August 1920	3.67	5.58	86.0	140.0	5.30
August 1920 to August 1925	5.00	10.58	91.7	231.7	8.76
August 1925 to April 1935	9.67	20.25	133.5	365.2	13.8
April 1935 to September 1940	5.50	25.75	50.6	415.8	15.8

For the purposes of this paper it is necessary only to consider the first and last surveys, and to find the total sediment deposited from the beginning of storage in January, 1915, to September 30, 1940—a period of 25.75 years. In the surveys the deposits in the long arroyos of the west-side tributaries were not included. The volume of the excluded deposits is unknown but for the purposes of this paper 10,000 acre-ft have been added for this item. (An accuracy closer than the nearest 100 acre-ft is not justified by the nature of the surveys.) Following is the summary:

Where deposited	Thousand acre-ft
Above the outlet gate sill and below the spillway level	415.8
In arroyos	10.0
Below the outlet gate sill	3.2
Above the spillway level and below San Marcial gaging station	13.9
Total (19.29×10^9 cu ft)	442.9

SEDIMENT INFLOW

These deposits may now be compared with the sediments that have flowed into and through the reservoir during the same period. The records of suspended sediments passing San Marcial begin in 1897, but, since the present study is concerned with the period from 1915 to 1940, only the record for those years is given in Table 2. The data were compiled from the following sources:

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[3] "Silting of Reservoirs," by Henry M. Eakin, revised by Carl B. Brown, *Technical Bulletin No. 524*, U.S.D.A., Washington, D. C., 1939, p. 12.

TABLE 2.—SUSPENDED SEDIMENTS OF THE RIO GRANDE AT SAN MARCIAL, N. MEX.

Calendar year	No. of samples	Million tons	Calendar year	No. of samples	Million tons	Calendar year	No. of samples	Million tons
1915	....	25.0	1924	....	14.6	1933	59	25.7
1916	....	30.0	1925	....	6.8	1934	44	6.7
1917	....	9.4	1926	32	9.5	1935	66	22.5
1918	13	3.7	1927	28	30.6	1936	66	13.9
1919	82	45.8	1928	17	4.8	1937	322	15.1
1920	111	31.5	1929	49	60.0	1938	365	6.3
1921	81	42.9	1930	8	6.2	1939	365	3.6
1922	12	13.3	1931	41	15.0	1940[a]	274	4.5
1923	....	19.4	1932	45	23.3	Total	....	490.1

[a] Nine months, ending September 30, 1940.

The period from 1915 to 1925, inclusive, was obtained from information supplied by the project engineer, Rio Grande Project, Bureau of Reclamation. The data are based on sampling in 1919 and, for the other years, on monthly silt curves prepared in 1924 from the sampling in 1919 and in years prior to 1913 (824 determinations 1905–1912). The data were compiled in acre-feet based on a specific weight of 53 lb per cu ft as used by the late W. W. Follett,[4] M. Am. Soc. C. E. The sediment load was converted to tons by the factor 1 acre-ft = 1,153 tons when $\gamma = 53$ lb per cu ft. The data thus converted were presented by the writer in a former paper.[5] Suspended sediments (in tons) since 1926 are found in the Water Bulletins,[6] published by the International Boundary Commission at El Paso, Tex.

The records for 1915–1925, inclusive, are open to question. Silt curves have been found to be only fair for the coarser sediments but may be quite inaccurate for the fines. However, the errors are compensating and, over a period of years, the totals may not be too far awry. For the years since 1925 the sediment was determined from intermittent sampling throughout each year as shown in Table 2. For some years, notably 1928 and 1930, the number of samples was too few to form the basis of a reliable estimate for the entire year. However, these are all the data in existence for the period in question.

ADJUSTMENTS

In order to compare measured deposits with sediment inflow, certain adjustments in the record of suspended sediments at San Marcial must be made:

1. Samples were secured by lowering small necked bottles "into the water at one or more verticals in the stream section, being careful to approach but not to strike the bottom."[6] The coarser sediment load increases with depth

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[4] "Silt in the Rio Grande," by W. W. Follett, Dept. of State, International Boundary Commission, 1913.

[5] "The Silt Problem," by J. C. Stevens, *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 228.

[6] *Water Bulletins Nos. 1 to 12*, International Boundary Comm., El Paso, Tex., 1931–1942.

until there is a slurry of sands moving along the bottom that is scarcely touched by the sampling bottle.

Fig. 2 is believed to represent, more or less typically, the conditions in a sediment loaded stream. The data are the results of sampling at various depths in ten verticals of the central main canal of the Imperial Irrigation District in the Colorado River delta.[7] The concentration of the coarser sediments is seen to increase rapidly as the stream bed is approached and a reasonably good concentration curve can be drawn. For the fines, however, the concentration is quite uniform from the surface to the bed.

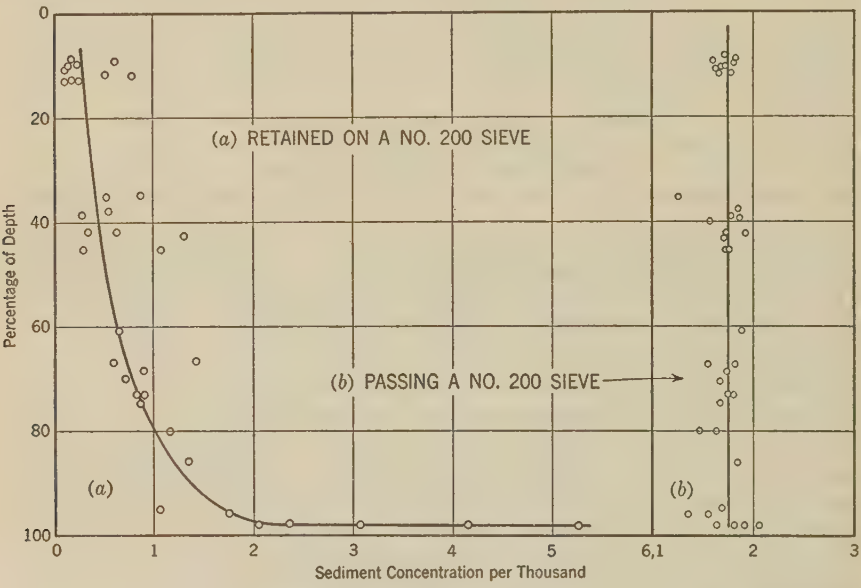

FIG. 2.—SUSPENDED SEDIMENT, IMPERIAL CANAL, CALIFORNIA

To the measured suspended load an addition of 15% was made in order to account for the unsampled bed load. The bed load would thus be 13% of the total sediment load.

2. There are 1,750 sq miles of drainage area tributary to Elephant Butte Reservoir below the San Marcial gaging station. From this arid area many tributaries flow into the lake and substantial wind-borne deposits accumulate in the reservoir (see Fig. 1).

The Soil Conservation Service has made a determination of the sediment contribution by the tributaries of Rio Grande in the valley above San Marcial[8] (see Table 3). This work has been continued since 1939 by the U. S. Engineer Department. Two methods were followed—one by an evaluation of sediment sources through mineralogical studies and the other by sampling suspended loads. The mineralogical studies were made by Gordon Rittenhause for the

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[7] "Silt in the Colorado River and Its Relation to Irrigation," by Samuel Fortier and Harry F. Blaney, *Technical Bulletin No. 67*, U.S.D.A., Washington, D. C., 1928, p. 47.

[8] "The Sedimentation in the Middle Rio Grande Valley, New Mexico," by Stafford C. Happ, SCS (unpublished), 1942, Tables 11 and 12.

Soil Conservation Service between 1937 and 1941. Composite samples were collected from the bed of the Rio Grande near the mouths of the six principal tributaries and also from farther up each tributary. These samples were sized mechanically. Samples were mounted on microscopic slides for identification and the number of grains in each species was determined. With the specific gravity known, the approximate weight of each species in each size grade was computed. The relative abundance of the different mineral species was used as an index of the relative contribution from each of the six tributaries.[9]

TABLE 3.—Acre-Feet of Sediment per Year Contributed by Tributaries of the Rio Grande

Method	Tributary (see Fig. 1)						Specific weights (7)	Sediment (thousand tons) (8)	Drainage areas sq. miles (9)	Sediments[a] (10)
	Santa Fe (1)	Galesteo (2)	Jemez (3)	Rio Puerco (4)	Rio Salado (5)	Totals (6)				
Drainage area (sq miles)...	228	697	1,009	6,092	1,434	9,460	....	....	....	....
Mineralogical analyses....	121	362	1,448	4,224	1,569	7,424	86.0	14,400	9,460	1,500
Suspended load sampling..	....	438	684	9,749	769	11,640	62.5	15,800	9,232	1,700

[a] Tons per square mile per year.

Although there are wide differences by the two methods among tributaries, the final values are close enough to justify the use of an average contribution of 1,600 tons per sq mile per yr for the area directly tributary to Elephant Butte Reservoir. The mineralogical analyses purport to show only the deposits on the valley floor; nevertheless they doubtless represent a major portion of the contribution by the tributaries.

3. The talus from the canyon walls that lodges below spillway level should be included. This represents a small volume that is very difficult to determine. Whatever it may be it is considered as included in that of adjustment 2.

4. Some sediment has passed entirely through the reservoir in the form of "density currents."[10] L. R. Fiock[11] estimated that to the end of 1934 a total of 5,000 acre-ft of fine silt had passed through the outlet gates at the dam. This estimate was based on samples some of which contained as much as 6% by weight of fine suspended material.[12] By correspondence it was ascertained that a specific weight of 53 lb was used to determine this volume. The weight, therefore, was 5.8 million tons. It was also learned from correspondence that another major silt flow occurred between August 9 and September 10, 1935, for which the writer estimated that 0.6 million tons of silt passed through the gates. The total passing out during the period of record is therefore 6.4 million tons.

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[9] "The Sedimentation in the Middle Rio Grand Valley, New Mexico," by Stafford C. Happ, SCS (unpublished), 1942, pp. 145–150.

[10] "The Passage of Turbid Water Through Lake Mead," by Nathan C. Grover and Charles S. Howard, *Transactions*, Am. Soc. C. E., Vol. 103 (1938), p. 720.

[11] "Records of Silt Carried by the Rio Grande and Its Accumulation in Elephant Butte Reservoir," by L. R. Fiock, *Transactions*, Am. Geophysical Union, 1934, p. 468.

[12] *Ibid.*, p. 472.

SPECIFIC WEIGHT OF DEPOSITS

For the 25.75-yr period from January 1, 1915, to September 30, 1940, the total net sediment inflow to the reservoir becomes:

Item	Source of sediments	Million tons
1	Passing San Marcial	490.1
2	Passing San Marcial as bed load (15% of item 1)	73.5
3	Contributed by 1,750 sq miles of tributary drainage area in 25.75 years at 1,600 tons per sq mile per yr	72.0
4	Total sediment inflow	635.6
5	Less sediment outflow	6.4
6	Net sediment inflow (equivalent to $1{,}259 \times 10^9$ lb)	629.2

For the same period the measured deposits were 442,900 acre-ft $= \gamma\, 19.29 \times 10^9$ lb, from which the average specific weight of the entire volume deposited is $\gamma = \frac{1{,}259}{19.29} = 65.3$ lb. An average value of 65 lb per cu ft will be used in this paper.

PROBABLE USEFUL LIFE

One undertakes a prediction of the useful life of such a reservoir with many misgivings. Nevertheless the best prediction that can be made from available data may be of considerable value even though subsequent years may prove it to have been in error.

The original capacity of Elephant Butte Reservoir was determined from a topographic map on a scale of 1,000 ft per in. with 10-ft contours, made from 1903 to 1908. The volume was recomputed in 1940 and found to be 2,634,800 acre-ft, of which 3,200 acre-ft are below the gate sill.[13]

The cumulative years and the percentage of the original capacity lost, given in Table 1, are shown in Fig. 3. If the average rate of sedimentation shown for the first 26 years should continue and all sediment should remain in the reservoir, it would appear that the reservoir would be full to the spillway level in a total of 158 years, leaving a remaining life of about 130 years more. The useful life, however, would be much shorter because the storage capacity becomes inadequate for the purposes for which it was constructed by the time half its original capacity is lost; at least its usefulness would then be greatly impaired. This would occur in about 80 years from its initial storage date, or about 1995.

These speculations, however, may not hold for the reason that the rate of depositions in the reservoir proper appears to have been decreasing materially as shown by the curved line of loss in Fig. 3. Deposits in the valley above the reservoir have been enormous so that a substantial portion of the water-borne sediments is arrested before it reaches the reservoir. The Soil Conservation Service has estimated that 59,000 acre-ft of sediments were deposited on the valley floor between Santa Fe Creek and San Marcial in the period from 1936 to 1941,[14] an average of more than 12,000 acre-ft per yr. That Service also reports that aggradation of the lower portion of this valley began eight years prior to the construction of Elephant Butte Reservoir (say, about 1907).

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[13] "Silting of Reservoirs," by Henry M. Eakin, revised by Carl B. Brown, *Technical Bulletin No. 524*, U.S.D.A., Washington, D. C., 1939, pp. 90–92.

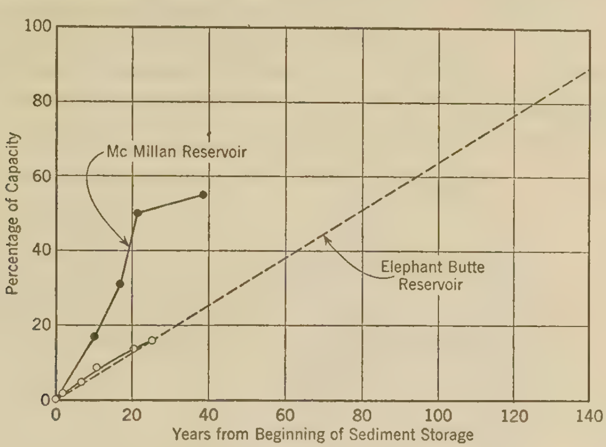

FIG. 3.—COMPARISON OF RATES OF SEDIMENT STORAGE

No very satisfactory explanation has been found for the aggradation of the valley floor. The writer believes it has been caused in part by backwater from the reservoir but mostly by the low runoff cycle the West has been experiencing since 1920. Floods have certain economic values in flushing the river channels of the sediment accumulations of previous low-water periods.

Fig. 4 demonstrates the presence of the low-water cycle. The levels of Great Salt Lake (Fig. 4(*b*))[15] are a good index of cyclic variations in the hydrologic features of the entire West. These levels show the culmination of a high-water cycle in the early 1870's and a continuous decline to 1905, a partial recovery until 1923, and a sharp decline since then.

For comparison, the 5-yr running averages of the flow of the Rio Grande at Otowi Bridge,[16] in percentages of the 52-yr mean, are also shown. The parallelism of the two curves is quite striking for the period of simultaneous records. If the Rio Grande records covered the same period as those of Great Salt Lake, a high-water cycle would undoubtedly have been disclosed between 1865 and 1880.

The Otowi record shows a gradual decline in water yield from the peak of 1909. Slight recoveries appear in both records from the all-time western recorded low of 1934.

If this is a major cause of valley filling, a succession of flood years in a cycle of greater rainfall would very likely move large volumes of the arrested sediments into the reservoir. Although the 25 years since 1920 show declining rates of sedimentation in the reservoir, a series of wet years may increase those rates to such values that the straight-line extension of Elephant Butte curve in Fig. 3 may not be amiss.

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[14] "The Sedimentation in the Middle Rio Grande Valley, New Mexico," by Stafford C. Happ, SCS (unpublished), 1942, Table 4.

[15] *Water-Supply Paper No. 880*, U. S. Geological Survey, Government Printing Office, Washington, D. C., 1939.

[16] "Regional Planning," Pt. 6 on "The Rio Grande Joint Investigation in the Upper Rio Grande Basin in Colorado, New Mexico, and Texas, 1936–1937," U. S. National Resources Board, Government Printing Office, Washington, D. C.

In Fig. 3 is also shown the rate of sedimentation of McMillan Reservoir on the Pecos River.[17] In 20 years the rate of filling diminished markedly. The explanation offered[18] is that a rank growth of tamarisk or salt cedar, that started about 1912, has covered the valley floor immediately above the reservoir and that this dense growth reduces velocity so much that the sediments are deposited in it. No doubt this explanation is correct and as long as the tamarisk thicket remains the sediments deposited will remain in place in spite of years of great floods.

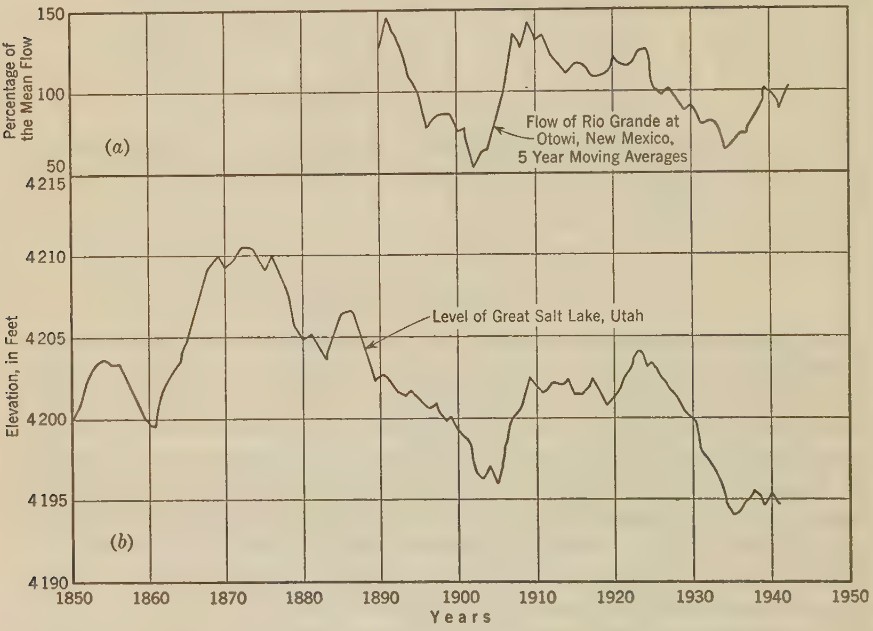

Fig. 4.—Study of Hydrologic Cycles

Such extensive growths are not prevalent on the Rio Grande, however; hence the sediments deposited in low-water cycles may very likely be moved out in high-water periods. One of two possibilities appears certain—either aggradation of the Rio Grande above Elephant Butte Reservoir will continue, or the river will again be degraded in subsequent wet cycles which will hasten the filling of the reservoir.

When the survey of Elephant Butte Reservoir was made in 1935 the lake level was so low that practically all the area above the Narrows (somewhat more than half) was mapped by plane table and stadia since the sediments were firm enough to support the men. Therefore, the water for this area had drained out of the upper layers. Without doubt, there was a zone of saturation below the exposed surface of the sediments.

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[17] "The Silt Problem," by J. C. Stevens, *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 211.

[18] "Silting of Reservoirs," by Henry M. Eakin, revised by Carl B. Brown, *Technical Bulletin No. 524*, U.S.D.A., Washington, D. C., 1939, p. 16.

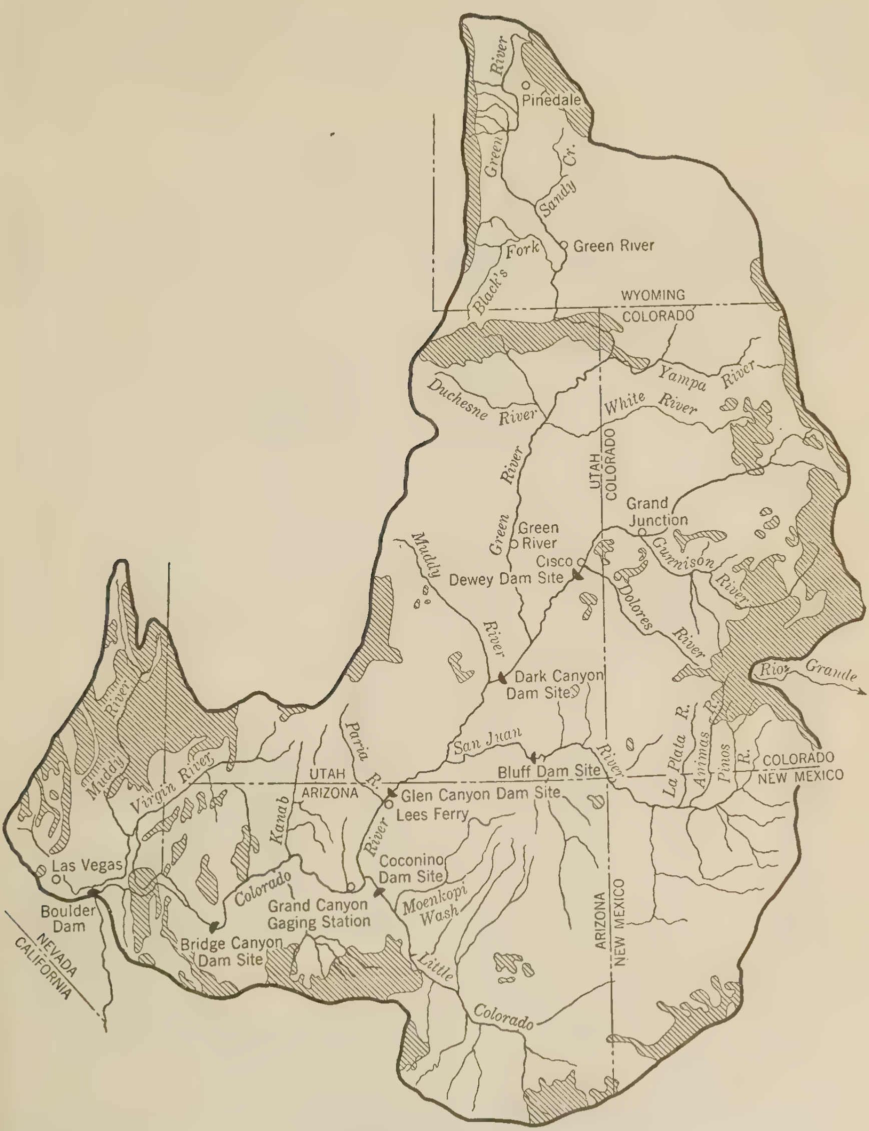
Green River
Pinedale
Sandy Cr.
Green River
Black's Fork
WYOMING
COLORADO
Yampa River
White River
Duchesne River
Green River
UTAH
COLORADO
Grand Junction
Green River
Cisco
Dewey Dam Site
Gunnison River
Dolores River
Muddy River
Rio Grande
Dark Canyon Dam Site
San Juan River
Bluff Dam Site
La Plata R.
Animas R.
Pinos R.
COLORADO
NEW MEXICO
Paria R.
UTAH
ARIZONA
Glen Canyon Dam Site
Lees Ferry
Kanab
River
Coconino Dam Site
Moenkopi Wash
Colorado
Grand Canyon Gaging Station
Bridge Canyon Dam Site
Little Colorado
ARIZONA
NEW MEXICO
Virgin River
Muddy River
Las Vegas
Boulder Dam
NEVADA
CALIFORNIA

Fig. 5.—Upper Colorado River Basin (Unshaded Area Represents Sedimentary Deposits)

No information is available as to the quantity of gravity water that flowed from the deposits, but whatever it was the loss of storage capacity given should be reduced by that volume. To take account of this ground water with so many other uncertainties, particularly with regard to valley deposits upstream, seems to be an unnecessary refinement.

A profile of the deposits in Elephant Butte Reservoir[19] shows that reservoir deposits are spread to a considerable extent over the entire bed of the reservoir. There is insufficient evidence to warrant the adoption of other than an average linear rate of filling. The straight line of Fig. 3, therefore, is offered as being a fair index of the useful life of Elephant Butte Reservoir according to present knowledge.

PART II.—LAKE MEAD

Lake Mead was created by Boulder Canyon Dam on the Colorado River, which forms the boundary between Nevada and Arizona in this reach. The gates in the outlet tunnels were closed on February 1, 1935, and the storage of water began on that date. Lake Mead is bordered by an arid plateau which is drained principally by the Virgin, Little Colorado, and San Juan rivers. It contributes to the Colorado River about 64% of its load of sediment but only 20% of its water.

TABLE 4.—Suspended Sediments in the Colorado River Basin
(Millions of Tons)

Years ending Sept. 30	Location of Sampling Stations (See Fig. 5)						
	Green-river, Utah (1)	Cisco, Utah (2)	Lees Ferry, Ariz. (3)	Grand Canyon, Ariz. (4)	Bluff, Utah (5)	Col. 4 minus Col. 3 (6)	Virgin, Utah (7)
River........	Green	Colorado	Colorado	Colorado	San Juan	Little Colorado	Virgin
Years of record.....	12	12	5	16	12	5	4
1926	....	....	....	225	....	....	....
1927	....	....	....	398	....	....	....
1928	....	....	....	172	....	....	....
1929	....	....	352	480	....	128	....
1930	34.5	18.6	195	235	58.3	40	....
1931	7.4	9.9	57	69	15.3	12	....
1932	36.1	26.4	214	261	75.3	47	....
1933	15.4	8.2	112	178	23.6	66	....
1934	1.8	2.7	....	50	21.2	....	....
1935	14.3	17.8	....	122	42.7	....	....
1936	33.8	17.2	....	158	32.6	....	3.85
1937	43.4	23.6	....	191	52.2	....	....
1938	38.2	35.7	....	233	52.1	....	....
1939	22.8	14.5	....	86	21.4	....	3.78
1940	8.9	9.9	....	76	26.3	....	2.47
1941	31.9	32.2	....	270	112.7	....	2.32
Total	288.5	216.7	930	3,204	533.7	293	12.4
Average	(24.0)	(18.1)	(186)	(200)	(44.5)	(58.6)	(3.1)

Colorado and Green rivers are perennially muddy because of the enormous areas of sedimentary deposits through which they flow. Fig. 5 is a map of the Colorado River basin above Boulder Dam showing the great extent of sedimentary deposits. These consist of shales, friable conglomerates, and sandstone from the Jurassic to recent alluviums. All streams draining such deposits carry heavy loads of sediment.

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[19] "Silting of Reservoirs," by Henry M. Eakin, revised by Carl B. Brown, *Technical Bulletin No. 524*, U.S.D.A., Washington, D. C., 1939, Fig. 19.

Record of Suspended Sediments

Table 4 gives the results of suspended sediment measurements of the major streams of the Colorado River basin above Boulder Dam.[20]

The water carried past each of the sampling stations and the sediment concentration for the period of record are given in Table 5. The results for Little Colorado River (line 6) include contributions from 29,800 sq miles as determined from the difference between the records of Lees Ferry and Grand Canyon, whereas the drainage area of the Little Colorado itself is 24,400 sq miles. The record of Virgin River at Virgin, Utah (line 7), is unpublished. It was obtained by correspondence with the Bureau of Reclamation.

TABLE 5.—Sediment Concentration, Colorado River Basin, for the Years Given in Table 4

Line	Sampling station	River	Sediment (million tons)	Water Yield		Concentration per 1,000
				Million acre-ft	Million tons	
1	Greenriver, Utah	Green	288.5	67.8	92,400	3.12
2	Cisco, Utah.........	Colorado	216.7	59.3	80,700	2.69
3	Lees Ferry, Ariz.	Colorado	930.0	63.7	86,700	10.7
4	Grand Canyon, Ariz..	Colorado	3,204.0	201.9	274,000	11.7
5	Bluff, Utah	San Juan	533.7	27.4	37,200	14.3
6	....	Little Colorado	293.0	1.8	2,450	119.6
7	Virgin, Utah........	Virgin	12.4	0.61	830	14.9

TABLE 6.—Water Yield and Sediment Records (in Millions) Expanded to a 16-Yr Period

No.	Sampling station (1)	Symbol (2)	Period (inclusive) (3)	Water Yield (Acre-Ft)		Sediment (tons) (6)
				Computed (4)	Observed (5)	
	Colorado River—					
1	Lees Ferry, Ariz.............	A_1	1929–1933	63.7	....	930
2	Grand Canyon, Ariz..........	B_1	1929–1933	65.5	....	1,223
3	Grand Canyon, Ariz..........	C	1926–1941	201.9	....	3,204
4	Lees Ferry, Ariz.; expanded....	$A_1\ C/B_1$	1926–1941	196.0	195	2,440
5	Cisco, Utah..................	A_2	1930–1941	59.3	....	217
6	Grand Canyon, Ariz..........	B_2	1930–1941	135.0	....	1,928
7	Cisco, Utah; expanded........	$A_2\ C/B_2$	1926–1941	88.9	90	360
	Green River—					
8	Greenriver, Utah............	A_3	1930–1941	42.5	....	288
9	Greenriver, Utah; expanded....	$A_3\ C/B_2$	1926–1941	63.6	64	479
	San Juan River—					
10	Bluff, Utah.................	A_4	1930–1941	22.5	....	534
11	Bluff, Utah; expanded.........	$A_4\ C/B_2$	1926–1941	33.7	32	888
	Virgin River—					
12	Virgin, Utah.................	....	1936–1939 1940–1941	0.61	....	12.4
13	Virgin, Utah; expanded........	....	1926–1941	2.66	2.31	48
14	Virgin and Muddy rivers......	....	1926–1941	5.0	No record	246

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[20] "Suspended Matter in the Colorado River 1925–1941," U. S. Geological Survey, unpublished report. (An extension of the data in Table 4 to include 1942, 1943, and 1944 is presented subsequently in Table 13.)

It will be expedient to expand the records to represent a 16-yr period to conform with that of Grand Canyon. This was done, as shown in Table 6, by using the Grand Canyon record as an index. The method is apparent from the letter symbols in Col. 2, Table 6. As an approximate check on the validity of this expedient, the water yield is expanded as well as the sediment records; then the actual volume of water, as independently measured and reported in Water-Supply Papers of the U. S. Geological Survey, for the 16-yr period is given. The agreement for the water flow is remarkably close for all stations.

The Virgin River and Muddy River expansion (item 14, Table 6) is estimated on the basis of the 4-yr record at Virgin and the 3-yr record of flow of Muddy River at St. Thomas, Nev., from 1913 to 1916 when it was a direct tributary of Virgin River, and taking into account the differences in drainage areas. The expansion for the Virgin and Muddy basins was estimated independently of the Grand Canyon records.

TABLE 7.—SEDIMENT FROM TRIBUTARY DRAINAGE; COLORADO RIVER

Item	Sampling station (1)	Drainage area (sq miles) (2)	Suspended sediment (10^6 tons) (3)	ANNUAL CONTRIBUTION	
				10^6 tons (4)	Tons per sq mile (5)
	Green River—				
1	Greenriver, Utah	40,600	479	30.0	740
	Colorado River—				
2	Cisco, Utah	24,100	360	22.5	935
3	Item 1 plus item 2	64,700	839	52.5	810
4	Lees Ferry, Ariz.	107,900	2,440	153.0	1,420
5	Item 4 minus item 3	43,200	1,601	100.0	2,310
6	Grand Canyon, Ariz.	137,800	3,204	200.0	1,450
7	Item 6 minus item 4	29,900	764	47.7	1,600
8	Item 6 minus item 3	73,100	2,365	148.0	2,020
	San Juan River—				
9	Bluff, Utah	23,900	888	55.5	2,320
	Virgin River—				
10	Virgin, Utah	934	48	4.0	4,300
11	Virgin and Muddy rivers	11,040	246	15.4	1,400

There are 30,000 sq miles of drainage area between the Grand Canyon sampling station and Boulder Dam, and it is important to know how much sediment this area is likely to contribute. For this purpose Table 7 has been prepared using the expanded records for the 16-yr period from Table 6.

SEDIMENT HELD IN LAKE MEAD

Storage of water and sediment in Lake Mead began February 1, 1935. It is of interest to know the rate at which sediments have been captured by Boulder Dam since closure of the gates. These consist of (*a*) measured suspended matter at Grand Canyon, (*b*) unmeasured bed load, (*c*) sediments contributed by the 30,000 sq miles of intervening drainage area, by both wind and water, and (*d*) talus from canyon walls. From the sum of items (*a*) to (*d*) must be deducted the sediments that passed through the dam in the form of density currents.

(*a*) *Suspended Matter Measured at Grand Canyon.*—The Bureau of Reclamation has prepared a breakdown of the suspended matter measured at Grand Canyon by months since 1935. From all records the following total is obtained:

Dates	Million tons
February 1 to September 30, 1935 (Bureau of Reclamation)....	117
October 1, 1935, to September 30, 1941 (Table 4)..............	1,014
October 1, 1941, to September 30, 1942 (Bureau of Reclamation)	232
Total passing Grand Canyon...........................	1,363

(*b*) *Unmeasured Bed Load.*—The method of sampling is described in a Water-Supply Paper[21] of the U. S. Geological Survey. The U. S. Geological Survey Colorado sampler[22] held a pint milk bottle. A small weight could be released to slide down the line to make a 5/8-in. hole in the paper bottle cap at any desired depth. Usually three samples were taken at each of three verticals at about one fourth, one half, and three fourths the distance across the stream.

An analysis of the accuracy of this sampler as well as of other types is found in a report of the Iowa Institute of Hydraulic Research.[23] For present purposes it is sufficient to state that the greater concentration (particularly of the coarser sediments) is in the 1.5 ft between the mouth of the bottle and the stream bed. For this reason an addition of at least 15% to the measured suspended load for unmeasured bed load is believed to be fully justified.

(*c*) *Sediments Contributed by Intervening Drainage Area.*—Of the 30,000 sq miles of drainage area between Grand Canyon and Boulder Dam 11,040 is in the Virgin and Muddy river basins. Before the filling of Lake Mead, Muddy River was a tributary of the Virgin River, but the lake has now submerged their junction so that each appears as a separate tributary to the lake.

In the 267 miles from the Grand Canyon sampling station to Boulder Dam there are a great many tributaries, the major ones being Bright Angel Creek, Kanab Creek, Havasu Creek, Diamond Creek, Grand Wash, Hualpai Wash, Virgin River, Muddy River, and Los Vegas Wash. All contribute substantial quantities of sediment which in time are discharged directly into Lake Mead or are carried into Lake Mead by way of the Colorado River.

An examination of Table 7 will justify the assumption of an average tributary sediment inflow of 1,600 tons per sq mile per yr, the same as was found for the area tributary to Elephant Butte. This may be conceded to include whatever wind-borne material is carried directly or indirectly into the lake, below the Grand Canyon sampling station.

(*d*) *Talus from Canyon Walls.*—There is no means of estimating the space in the reservoir occupied by the talus material that falls from the canyon walls. The length of the reservoir at high level is about 115 miles, most of which consists of high canyon walls. Since this contribution is a small portion of the total sediment inflow it may be considered as included in the foregoing estimate of tributary inflow.

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[21] *Water-Supply Paper No. 636B*, U. S. Geological Survey, Government Printing Office, Washington, D. C., 1929.

[22] "Field Practice and Equipment Used in Sampling Suspended Sediment," *Report No. 1* of "A Study of Methods Used in Measurement and Analysis of Sediment Loads in Streams," Iowa Inst. of Hydr. Research, Iowa City, Iowa, 1940–1943, p. 149.

[23] *Ibid.*, p. 102.

The Sum of Items (a) to (d).—The silt outflow from Lake Mead occurred after control of the flow began on February 1, 1935, and while the reservoir was filling until the diversion tunnel gates were finally closed May 1, 1936. N. C. Grover, M. Am. Soc. C. E., and Charles S. Howard proposed[10] the following estimate of silt discharged by these turbid outflows:

Silt outflow during:	Million tons
1935	6.0
1936	2.4
Total	8.4

Since closure of the diversion tunnels no turbid waters have passed through the control works. The silt brought in by the density currents is deposited in the lowest portions of the lake bed and against the dam.

Summarizing, the sediment flow during the 7.66-yr period from February 1, 1935, to September 30, 1942, is found in the third column of Table 8. A similar summary as shown in the fourth column of Table 8, may be made for the 17-yr period from October 1, 1925, to September 30, 1942, on the assumption that it is representative of future accumulations.

TABLE 8.—SEDIMENT DEPOSITED IN LAKE MEAD (MILLION TONS)

Item	Description	7.66 yr[a]	17 yr[b]
1	Suspended sediments passing Grand Canyon	1,363	3,436
2	Unmeasured bed load at Grand Canyon; 15% of item 1	204	515
3	Contributed by tributary drainage[c]	368	816
4	Total inflow	1,935	4,767
5	Outflow by density currents, 1935 and 1936	8	8
6	Net deposits	1,927	4,759
7	Average annual accumulation	252	280

[a] February 1, 1935, to September 30, 1942. [b] October 1, 1925, to September 30, 1942. [c] Contributed by 30,000 sq miles of tributary drainage in the years of record stated, at 1,600 tons per yr per sq mile.

SPACE OCCUPIED BY DEPOSITS

For the Lake Mead deposits no better estimate of the specific weight can be made than to use the composite value of $\gamma = 65$ lb found for Elephant Butte Reservoir. On that basis an average annual sediment deposit of 280 million tons per year would occupy a volume equivalent to 198,000 acre-ft.

Fig. 6 gives the storage capacities for Lake Mead. The sediments deposited have remained submerged except the upper portions of the deltas of the main and tributary streams.[24] Only in a cycle of dry years after virtually all water-consuming projects in the upstream parts of the basin have been completed is there likelihood of any substantial portion of the deposits becoming exposed, which would tend to consolidate the deposits.[25]

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[24] "Silt Problems," by W. E. Corfitzen, *Civil Engineering*, November, 1942, p. 616.

[25] See *Report No. 9* of "A Study of Methods Used in Measurement and Analysis of Sediment Loads in Streams," Iowa Inst. of Hydr. Research, Iowa City, Iowa, 1943.

PROBABLE LIFE OF LAKE MEAD

If the foregoing rate of deposition should continue there would be deposited a volume equivalent to the storage capacity at the crest of spillway (28,632,000 acre-ft) in a period of 144 years from the time storage began, or 135 years from 1944.

Sediments are not deposited to uniform depths within the reservoir. Deltas form at the mouth of every tributary and at the inlet of the main stream. These are reworked at varying lake levels; yet they maintain a sloping surface toward the thalweg of the reservoir.[26] Density currents carry fines to the low parts of the reservoir bed and against the dam. The reservoir walls are continually weathering down to form talus slopes at their bases, whereas their upper reaches are being sloped back to stable angles of repose.

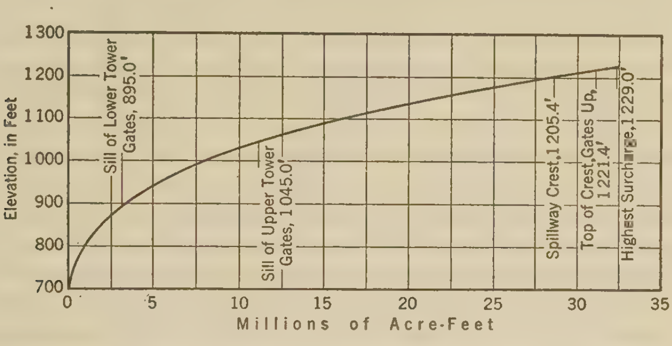

FIG. 6.—STORAGE CAPACITY FOR LAKE MEAD

The main stream has a tendency to build up its bed above the reservoir to maintain its original slope as it was before its deltaic deposits began to form. If the material is coarse the back sediment curve may extend a long way upstream, as demonstrated by the profile of Bear Creek in California.[27] If the stream joins the reservoir in a flat valley the entire valley floor will be built up to some grade commensurate with the hydrology of the region and the character of the sediments carried, such as is now occurring at the upper ends of Elephant Butte and McMillan reservoirs.

When the river joins the reservoir in a narrow canyon, there is little opportunity for the stream bed to aggrade itself because any deposits left at low and medium stages are carried away in subsequent freshets. Of such is the junction of Colorado River and Lake Mead, since the upper end of the reservoir lies generally in narrow gorges. The opportunity for extensive deposits above lake levels therefore is lacking, and practically all the sediments transported by the river as measured at the Bright Angel station in the Grand Canyon are carried into the lake and cause it to lose storage capacity.

It may be assumed that the climatic conditions over the life span of such a reservoir as Lake Mead will not change materially. The allowances that have been made for contributions by tributary areas and bed load do not seem out of place and the storage volume lost by sediment depositions in the lake is based on the best information available.

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[26] "Sedimentation in Reservoirs," by Berard J. Witzig, *Transactions*, Am. Soc. C. E., Vol. 109 (1944), p. 1057.

[27] "The Silt Problem," by J. C. Stevens, *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 219.

There are 3,200,000 acre-ft below the lowest gate sill; but water cannot be drawn down to that level. The dead storage, therefore, may be estimated at 4 million acre-ft. At the present rate of sedimentation (198,000 acre-ft per yr), 20 years will be required for the lake to fill the dead-storage space, although encroachment on the active storage has already begun. Half the remaining volume (11,300,000 acre-ft to the spillway crest) will be occupied in about 57 years more, which will see the usefulness of Lake Mead seriously impaired for hold-over storage.

A volume equivalent to the total capacity to spillway level probably will be caught in the reservoir in about 144 years from 1935 or, say, by the year 2080, whereupon it will have ceased to exist as a storage reservoir. However, it will still be useful as a run-of-the-river power project, since 2.5 million acre-ft of storage will remain, controlled by the crest gates to El. 1221.4.

This residual 2.5 million acre-ft may be encroached upon but will not be entirely lost because the sediments will be carried through the reservoir. It will serve as regulating storage for the power such a river-run plant can generate —possibly about half its ultimate installed capacity of 1,300,000 kw.

PROLONGING THE LIFE OF LAKE MEAD

Because of the great investment and the importance of this development to the entire southwestern part of the United States, it will be prudent to adopt whatever measures appear feasible to increase the period of usefulness of Lake Mead. The measures to be considered are:

(*a*) Land management on the watershed;
(*b*) Control of density currents;
(*c*) Upstream storage for sediments and other purposes; and
(*d*) Removal of sediments.

Land Management.—The artificial stimulus given to increasing vegetal cover on the watershed, even if it were 100% effective on the areas where it can be practiced, cannot affect the total volume of sediment flow greatly. The areas where such treatment can be used are on range land and forest areas.

If grazing areas are managed to produce their maximum forage yield the erosion prevention feature will have been provided for automatically; that is, the prime objective in management of range land should be to furnish as much food as practicable for livestock on a sustained yield basis.

In areas suitable for forest cover the prime objective should be to produce continuing yields of merchantable timber. The planting and care of forests solely to prevent erosion can rarely be considered a sound financial investment.

In the Colorado River basin, or in fact in any other arid section, natural erosional forces so outweigh those attributable to man's activities that the best that man can do in the way of land management can have only minor effects in reducing the sediments naturally carried by the streams. It is doubtful if the average of 280 million tons of sediment being carried annually into Lake Mead could be reduced as much as 10% by all the proved practicable methods of land management known today even though lavishly financed.

Density Currents.—Engineers are just beginning to learn something about the stratifications of the contents of reservoirs. Layers of silt-laden waters, as well as those of varying temperatures and of varying concentrations of dissolved solids, remain intact to a remarkable degree. A nontechnical discussion of this subject was published in 1942 by the Department of Agriculture.[28]

Knowledge of such currents was not available, or at least was not recognized to be of possible value, until after Boulder Dam was completed. Facilities for handling density currents and withdrawing water at will throughout the depth of the reservoir are lacking. The technique to take advantage of the current limited knowledge of this subject has not been developed as yet to a practical science. Although it offers some promise in the operation of some future reservoirs under certain favorable conditions, it is not likely to be of much practical benefit for the two reservoirs under discussion. There is also the question as to whether these fines, if deposited on irrigated land, might not have the effect of sealing the interstices of the soil and thus prevent proper air and water drainage.

The subject is worthy of extensive study. Projected reservoirs in arid sections where such density currents and stratifications are likely to exist might well be provided with outlet gates at varying levels in order to take advantage of future possibilities in passing on inflowing sediments.

Upstream Storage for Sediments.—This subject cannot be treated in generalities but will be confined to the specific case of Lake Mead. A number of reservoirs on the Colorado River and silt-laden tributaries have been proposed. The principal ones are Bridge Canyon, Glen Canyon, Dark Canyon, and Dewey Reservoir on the Colorado River; Bluff Reservoir on the San Juan River; and Coconino Reservoir on the Little Colorado. A reservoir for silt storage should also be found on the Virgin and Muddy rivers, but all studies so far made have been concerned with irrigation and flood control, and only incidental studies of the storage possibilities for sediments have been made.

Bridge Canyon Reservoir.—This project has been proposed for additional power. The storage capacity is 2,400,000 acre-ft, of which 1,600,000 acre-ft is active. The sediment inflow to the reservoir may be estimated as follows on the basis of the 17-yr record, 1926–1942.

Item	Description	Million tons
1	Suspended sediments passing Grand Canyon	3,436
2	Unmeasured bed load, 15% of item 1	515
3	Sediments contributed by 7,400 sq miles of tributary drainage in 17 years at 1,600 tons per sq mile per yr	204
4	Total	4,155
5	Average per year	244

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[28] "Stratified Flow in Reservoirs and Its Use in Prevention of Silting," by Hugh Stevens Bell, *Miscellaneous Publication No. 491*, U.S.D.A., Sedimentation Div., Office of Research, Washington, D.C., 1942.

On the basis of a specific weight, γ, of 65 lb, this would represent an average deposit in the reservoir of 172,000 acre-ft per yr. At this rate the dead storage would be filled in less than 5 years and the reservoir would be completely filled in 14 years. If any power equipment were installed more than that required for a river-run plant, the excess would become virtually useless in about 10 years, which is too short a time for amortization. Any power development therefore should be made with these facts in mind.

Bluff Reservoir.—This reservoir is near the mouth of the San Juan River. The suspended sediments passing this site (Table 6) are 55 million tons per year. The storage capacity has been placed at 1.5 million acre-ft. The sediments from this stream are believed to be coarser than those in the Colorado River. The loss of storage space in the Bluff Reservoir, therefore, may be placed at about 30,000 acre-ft per yr on a basis of $\gamma = 85$ lb requiring 50 years to fill this reservoir. During this period considerable power could be generated but the capacity to be installed should be considered in the light of the possibility that in 30 years the development would revert to a river-run plant.

Coconino Reservoir.—This reservoir would be located in the lower reaches of the Little Colorado River. The proposed storage capacity is 1,600,000 acre-ft. Table 7 shows the difference in the sediment flow between the Colorado River at Lees Ferry and at Grand Canyon to be about 48 million tons per year. Of this quantity 35 million tons may be conceded as coming from the Little Colorado River into this reservoir. The loss of storage space is placed at 23,000 acre-ft per annum on a basis of $\gamma = 70$ lb. Therefore, the reservoir would become filled in 70 years.

This stream would be of no value for a river-run power plant, and it would be unwise to attempt any commercial power development at this site.

Dewey Reservoir.—The dam for this reservoir is proposed on the Colorado River below the mouth of the Dolores River. Its storage capacity as projected is 6 million acre-ft, of which 5 million is active. The sediment passing the Cisco sampling station, from Table 6, was 22.5 million tons per year. If $\gamma = 75$ lb, this sediment inflow would cause the reservoir to lose 14,000 acre-ft of storage space per year. This is a relatively small volume for it would require more than 70 years to exhaust the dead storage alone and 430 years to fill the reservoir.

Glen Canyon Reservoir.—This is perhaps the largest projected artificial reservoir in the world, since it may admit of being developed for a storage capacity of 70 million acre-ft,[29] of which 60 million would be active. The dam would be just above Lees Ferry. It would be 800 ft high and would back water to the foot of the Dewey Dam, a distance of 350 miles.

If the dam were built to this height the reservoir would include the Dark Canyon Reservoir. The foundation and abutment would be in a Jurassic quartzose sandstone of uniform texture that stands in vertical cliffs 1,000 ft or more in height.[30] The crushing strength is from 2,000 to 6,000 lb per sq in.

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[29] "Water Power and Flood Control of Colorado River Below Green River, Utah," by E. C. LaRue, *Water-Supply Paper No. 556*, U. S. Geological Survey, Government Printing Office, Washington, D. C., 1925, p. 18.

[30] *Ibid.*, p. 20.

Whether this material would support the loads for a dam 800 ft high or not is a matter for careful consideration.

If Dark Canyon Dam is to be considered, Glen Canyon Dam could only be built to a height of 400 ft and the combined storage of both reservoirs would be only about 14 million acre-ft. Thus, 56 million acre-ft of storage would be sacrificed.

At its highest level the area of the larger lake would be 300,000 acres and at its lowest drawdown 75,000 acres. The annual evaporation from this lake would approximate a million acre-feet per year.

The evaporation from Lake Mead now averages about 800,000 acre-ft per yr. During its later years, on account of increased consumptive use upstream, Lake Mead cannot be held to the high levels possible now, which will result in reducing the annual evaporation over that which obtains at present.

Glen Canyon Reservoir should not be developed to its highest level to run parallel in time with Lake Mead, but should be reserved for a future generation when the usefulness of Lake Mead has become impaired.

Irrigation must be conceded as having a decided priority over power in this region. The loss of 1.75 million acre-ft of water by evaporation, if both reservoirs were in operation simultaneously, is too big a price in water to pay for the temporary advantage in power generation. This loss would irrigate more than a half million acres of land whose value to the nation is far greater than the power that its sacrifice would make possible. Power may be had from other sources but the Colorado River is the only source of water for irrigation in this region.

The sediment flow at the Glen Canyon Dam is that at Lees Ferry (Table 6); namely, 150 million tons per year. The sediments are doubtless somewhat coarser than those at Lake Mead; hence, if $\gamma = 69$ lb, the rate of filling becomes 100,000 acre-ft per yr. This would fill the low-level reservoir of 9 million acre-ft in 90 years. If the high-level reservoir were built it would take 700 years to fill it completely, although its usefulness as a regulator of the flow of the Colorado River would be lost in half that time or less.

The power possibilities at such a high dam and with such enormous storage back of it would be very great indeed. It would probably justify as much as 2,000,000 kw of installed capacity.

Dark Canyon Reservoir.—For the height of dam of 550 ft, this reservoir would have only 5 million acre-ft of storage capacity. It would back the water up the Colorado River to the Dewey Dam and up Green River to the City of Greenriver, Utah. Its normal area would be about 20,000 acres, from which the evaporation would be close to 100,000 acre-ft per yr.

There is some uncertainty as to whether the sedimentary rocks forming the foundations and abutments for the dam would be stable under the loading. If that were decided adversely the stretch would have to be broken up into smaller reservoirs or the project abandoned in favor of the Glen Canyon project.

The sediment that would be caught may be estimated from Table 7, item 5, to be 100 million tons, representing a loss of 61,000 acre-ft per yr on a basis of $\gamma = 75$ lb. This would fill the entire reservoir in 82 years.

Power development at this site would have a useful life of 45 years, after which the generating capacity would gradually be reduced from that of a storage plant to a river-run plant, but the cost of the equipment sacrificed could be amortized.

A Schedule of Development

In order to determine the effect of sediment storage upstream from Lake Mead a schedule of development was projected from which Table 9 has been prepared. The sedimentation rate is without regard to other upstream reservoirs. The capacity lost is contingent upon the reservoirs upstream.

TABLE 9.—Computation of Storage Capacities Lost in the Reservoirs of the Colorado River Basin (1,000 Acre-Ft Units)

Years (Inclusive)		Number of years	Reservoirs (A = Annual Loss, T = Total Loss, and C = Cumulative Total Loss):														
			Dewey		Glen Canyon		Bluff		Coconino		Bridge Canyon		Lake Mead		
From:	To:		A	T	A	T	A	T	A	T	A	T	A	T	C
River			Colorado		Colorado		San Juan		Little Colorado		Colorado		Colorado		....
Year completed			1950		1970		1960		1960		1955		1935		....
Storage capacity			6,000		9,000		1,500		1,600		2,400		28,632		....
Sedimentation per year			14		100		30		23		172		198		....
1935	1950	16	..	....	..	....	..	....	..	....	..	....	198	3,168	3,168
1951	1955	5	14	70	..	....	..	....	..	....	..	....	184	920	4,088
1956	1960	5	14	70	..	....	..	....	..	....	158	790	26	130	4,218
1961	1970	10	14	140	..	....	30	300	23	230	105	1,050	26	260	4,478
1971	1999	29	14	406	86	2,494	30	870	23	667	19	551	26	754	5,232
2000	2010	11	14	154	86	946	30	330	23	253	0	0	45	495	5,727
2011	2030	20	14	280	86	1,720	0	0	23	460	0	0	75	1,500	7,227
2031	2075	45	14	630	86	3,870	0	0	0	0	0	0	98	4,410	11,637
2076	2167	92	14	1,288	0	0	0	0	0	0	0	0	184	16,928	28,565
Totals		233	..	3,038	..	9,030	..	1,500	..	1,610	..	2,391	..	28,565	....

Obviously many such schedules of development could be forecast, and for each schedule data similar to those of Table 9 could be prepared. The one shown is more illustrative than suggestive of a plan of development. The Boulder Canyon Project Act authorizes the Secretary of the Interior to prepare a comprehensive plan of development for the Colorado River basin. The first draft of that plan has been completed by the Bureau of Reclamation and it is not the intent herein to usurp the prerogatives of that agency in this regard.

In Table 9 the sedimentation periods are recorded for even years only, which explains why the total capacities lost do not check exactly with the original reservoir capacities. The time required to exhaust the entire storage capacity of each reservoir may be found as follows:

Reservoir	Years (inclusive)	Years
Dewey	....	470
Glen Canyon	1971–2075	105
Bluff	1961–2010	50
Coconino	1961–2030	70
Bridge Canyon	1956–1999	44
Lake Mead	1935–2167	233

Removal of Sediments.—No practicable method of removing the enormous volume of sediments with which this paper deals has been found as yet. Once the sediments have lodged in the reservoir, there they will remain unless some mythical Paul Bunyon and his blue ox can be found to plow them out. Lake Mead, if filled to the crest of the dam, would contain 44 billion cubic yards of sediment. Even at 1¢ per cubic yard for removal the cost would be 440 million dollars—about four and one-half times the cost of the project.

CONCLUSIONS

The cumulative totals for Lake Mead from Table 9 are shown graphically in Fig. 7. On the same drawing is shown the capacity lost at the uniform rate of 198,000 acre-ft per yr which it is estimated would occur if the present rate of filling continues. The relatively moderate effect of the upstream storage scheduled on the useful life of Lake Mead is pointedly shown by these graphs. The operation of all upstream reservoirs that could serve to withhold sediments and that now are believed to be feasible will only prolong the total life of Lake Mead 89 years or 62%.

There is a question whether the high dam at Glen Canyon would be found feasible. If it were built, however, it would continue to withhold 100,000 acre-ft of sediment per year from Lake Mead after Bridge Canyon, Bluff, and Coconino reservoirs were filled. This would extend the life of Lake Mead 173 years beyond 2075 to the year 2248—a total life of 314 years from 1935.

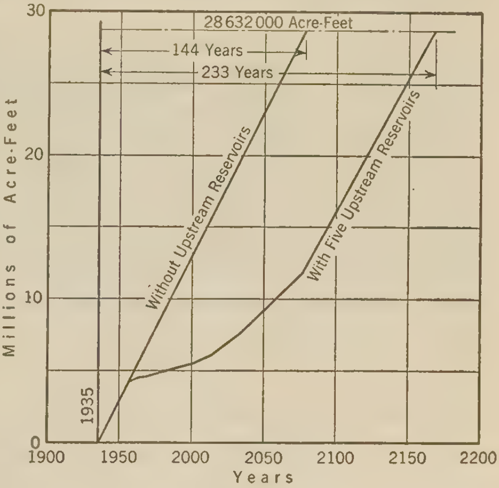

FIG. 7.—LIFE OF LAKE MEAD

If both Lake Mead and Glen Canyon high-level reservoir were operated in parallel, there would be a great loss of water by evaporation. Even if it were pronounced feasible, it is strongly recommended that the construction of Glen Canyon high dam be deferred until some future time when the storage facilities of Lake Mead shall have become seriously impaired.

Bridge Canyon Reservoir would have its life extended from 14 to 44 years by the upstream reservoirs projected, which is still a very short time in which to amortize large investments in power equipment in excess of that suitable for a river-run plant.

In this study Elephant Butte Reservoir was used primarily as an index of the average value of the specific weight of sediments in it, as well as in Lake Mead. The feasibility of constructing reservoirs upstream from Elephant Butte is doubtful. The Rio Grande Compact between Colorado, New Mexico, and Texas precludes any increase in consumptive use of the water in the Upper Rio Grande basin. Any all-year storage would add the evaporation from them to the present high consumptive use and could well be considered a violation of that compact.

The only course left, therefore, appears to be to continue the use of Elephant Butte Reservoir until its usefulness has become impaired enough to warrant the construction of a substitute reservoir. Even this might increase the present consumptive use. That, however, is a subject for future consideration.

In these studies the effect of land management in withholding sediments permanently from the reservoirs has not been taken into account because its efficacy is so closely bound up with administrative policies and congressional appropriations that its effect, even if known, lacks the essential element of assurance.

The western arid states depend upon irrigation for their very existence. Sedimentation constitutes a direct threat to that continued existence. Something must be done about it, but to do anything engineers must first have data on sedimentation in all of its phases. The available data are pitifully meager, and must be greatly enlarged before any constructive program for relief can be developed. For example, it appears that some 64% of the silt in the Colorado River is contributed by the sedimentary plateau drained by the San Juan, Little Colorado, and Virgin rivers, the area of which is only one tenth that of the Colorado River drainage basin. Here is an area where extensive research and consequent improvements would pay handsome dividends.

DISCUSSION

D. C. BONDURANT,[31] **ASSOC. M. AM. SOC. C. E.**—The irrigated regions of the United States have been developed to their present economy by reservoirs; and without such reservoirs these regions could not continue to exist in that economy. Since it may be assumed that existing reservoirs have been located at the most satisfactory sites, it follows that they could be replaced only at less satisfactory sites and that ultimately they could not be replaced at all. Unfortunately, reservoir sites are limited and the loss of an existing project may well destroy a community. It is fortunate, indeed, that a few far-sighted individuals have foreseen the seriousness of the problem and have undertaken to bring about its realization.

The writer, however, does not agree with the author that it is not feasible to construct reservoirs upstream from Elephant Butte, N. Mex. It is believed that such reservoirs are not only feasible but are necessary for the continued stability of a large area in the Middle Rio Grande Valley, in New Mexico, supporting about 17% of the population of the state. The problem confronting this region includes not only sedimentation in Elephant Butte but also aggradation of the Rio Grande which is now at or above the level of the valley floor in many places. Consequently, the water table is maintained at such high level that the productivity of the land is seriously impaired due to ineffectiveness of drainage ditches and resulting excessive alkali accumulation. Flood hazards have also increased commensurately. It is possible that aggradation of the Rio Grande is the result of a period of low flow, wherein the flows have been insufficient to carry the sediments brought in by tributary floods and that a later period of high flows may degrade the stream and carry the excess sediments to Elephant Butte Reservoir. It is believed more likely, however, that this condition is resultant from irrigation diversions reducing the flow, and the condition will continue as long as irrigation exists.

Reservoir sites are available at one of several points on the main stem in New Mexico, below the Colorado-New Mexico State line, and in the lower reaches of the Rio Chama, Jemez River, and Rio Puerco. These reservoirs could be utilized for flood control, sediment storage, flow regulation, and hydroelectric power. The sediment carried by the streams into these reservoirs would be retained and the clear water released would tend to degrade and stabilize the channel downstream, a premise which is substantiated by observed results below several operating projects. The Rio Grande below Elephant Butte Dam degraded appreciably from Elephant Butte to near El Paso, prior to the channel improvement made by the International Boundary Commission, and it is quite probable that the lack of a heavy sediment contribution from upstream has aided materially in maintaining this improvement. Recent surveys of the South Canadian River below Conchas Dam in New Mexico show that, since 1941, retrogression has been significant for a distance of more than 40 miles downstream. In the Middle Rio Grande Valley, such retrogression would notably improve the status of irrigated lands.

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Extensive investigations have been made to determine the feasibility of these projects—both singly and in combination. It has been determined that their construction and operation will in no way abrogate the Rio Grande Compact, but instead will provide a better water supply to the users. The problem of evaporation, as mentioned by the author, is more than compensated by the difference in water surface areas exposed and the difference in evaporation rates at the varied locations. At Elephant Butte evaporation is 5.364 ft per annum computed on a factor of 0.67 to reduce observed pan evaporation to probable reservoir evaporation. At Santa Fe, N. Mex., about 2,500 ft higher in elevation, the reservoir evaporation would be 3.567 ft. The water surface area of Elephant Butte, and consequently the evaporation volume, increases at an appreciably higher rate with storage exceeding 1,000,000 acre-ft than it does with storage less than that amount. During the months of high evaporation, the use of the upstream reservoirs to maintain lower pool elevations in Elephant Butte would result in a net saving of water.

The author's statement that Salt Cedar growth on the Rio Grande is not as extensive as on the Pecos River is correct. However, a dense growth of this plant exists at the head of Elephant Butte Reservoir and the incidence of Salt Cedar growth along the Rio Grande is increasing. This growth should aid in reducing sedimentation in Elephant Butte, although the quantity of water consumed and transpired by the plant is so great that the value of water lost in this manner may offset the efficiency of the plant as a sediment screen.

JOHN H. BLISS,[32] ASSOC. M. AM. SOC. C. E.—A problem common to practically all storage reservoirs—the reduction and eventual loss of their useful storage capacity due to siltation—is treated in this excellent paper by Mr. Stevens. This is a serious problem, which threatens the very existence of irrigation in the arid west.

In his paper, Mr. Stevens uses the silt data obtained at Elephant Butte Reservoir as the yardstick for determining the unit weight of sediments deposited in reservoirs. Elephant Butte is apparently the only reservoir at which a long-time record of the necessary base data has been kept. Using the 25.75-yr record from January, 1915, through September, 1940, the author derives 65.3 lb per cu ft as the average specific weight of the entire volume of silt deposited in Elephant Butte Reservoir up to the time of the 1940 resurvey. This specific weight is based (1) on the measurements of suspended silt past San Marcial station above the reservoir, adjusted for an estimated bed load and for unmeasured silt flows into the reservoir below the station; and (2) on surveys showing loss of storage space in the reservoir, the volume of silt deposited between San Marcial station and the reservoir, and the loss of dead storage in the reservoir itself. It seems as reasonable a value as may be derived in the light of present knowledge. Any specific weight used depends to a considerable extent upon the bed load that the given stream carries—or is assumed to carry—since no accurate measurements of bed loads are available for the streams in question. The author adjusted the suspended load measurements by increasing them 15% to account for the unsampled bed load. He also deduced, from measurements made on tributaries of the Rio Grande above San Marcial station, that tributary flow contributed an average of about 1,600 tons of silt per square mile of drainage area. Therefore, with 1,750 sq miles of drainage area, the unmeasured tributaries to Elephant Butte Reservoir below San Marcial were assumed to contribute an average of 2.8 million tons per year.

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[32] Asst. State Engr., State Engrs. Office, Santa Fe, N. Mex.

Since no silt measurements were kept for the first eleven years of Elephant Butte operation, the silt inflow past San Marcial for this period was estimated from silt curves. The accuracy of this is questionable. Incidentally, using the author's method of adjustment, the period 1926–1940, when actual silt determinations were taken, yields a value of about 75 lb per cu ft as the unit weight of the deposited sediments. The International Boundary Commission derived a specific weight of 70.2, based on recorded data.[33] Adjustment of this value gives a specific weight for the sediments of well above 80. Part of the discrepancy would seem to lie in the reduced tributary silt inflows which would lessen the bed load carried during this period of below normal runoff. If these latter values are reasonable, it may be that the volume of deposited sediments in reservoirs, particularly over long periods of time, may be somewhat less than that derived by the author. The theoretical life of the reservoirs, in that case, would be extended somewhat, although their eventual demise would be no less certain.

The writer was quite interested in the statement (see first paragraph of Part II) that, although the San Juan, Little Colorado, and Virgin rivers contribute only 20% of the discharge into Lake Mead, they transport 64% of the silt of the Colorado River system above the lake. Here indeed is "an area where extensive research and consequent improvements would pay handsome dividends."

In his "Conclusions" the author makes a statement regarding the future of Elephant Butte Reservoir which is erroneous and should be corrected. The tri-state Rio Grande Compact does not preclude "any increase in consumptive use of the water in the Upper Rio Grande basin." Rather, it is an agreement between the states that provides the means for making additional and better use of the available water supply by permitting upstream storage and use of water formerly spilled from Elephant Butte Project storage. This is accomplished by converting nonbeneficial uses along the stream to beneficial uses through drainage and reclamation, by permitting development of feasible power projects, etc. Congress has already authorized construction of major reservoirs on Rio Grande and Conejos River in Colorado. New Mexico is currently introducing in Congress a bill for a $40,000,000 project on the Rio Grande and tributaries involving considerable new storage for flood and silt control, power, and re-regulation of the flow through the Middle Valley for irrigation. This project is being integrated by the Bureau of Reclamation and the Corps of Engineers of the U. S. Army and complies, in all respects, with the provisions of the Rio Grande Compact. One of the important features of the proposed project is the detention and control of silt on tributary streams, thereby reducing the average rate of silting of Elephant Butte Reservoir from about 17,000 acre-ft to about 5,000 acre-ft annually. The project will not eliminate the silt problem of the basin, of course, since it merely retains the silt load in reservoirs upstream from Elephant Butte Reservoir. It will alleviate a bad situation for many years, however, during which time it is to be hoped more adequate control measures will become available.

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[33] *Water Bulletin No. 7*, International Boundary Comm., El Paso, Tex., 1937, p. 63.

LUNA B. LEOPOLD,[34] JUN. AM. SOC. C. E.—The Rio Grande and Colorado River watersheds differ somewhat from many other large basins in that normal erosion in the semiarid areas is more widespread and more obvious than in humid areas. Therefore, there is a tendency to minimize the importance of watershed management as a factor that might influence the rates of sediment production.

The author states (see heading, "Prolonging the Life of Lake Mead: Land Management") that in the semiarid areas natural erosional forces are relatively so important that the best land management can have only minor effects in reducing the sediments carried by streams. It should be emphasized, however, that continued mismanagement could increase the sediment available to streams. In a watershed such as the Rio Puerco, an important sediment-producing tributary to the Rio Grande, gullying has proceeded so far that land management alone could have but negligible effect on the lateral widening of the gully walls and the deepening of the existing arroyos in their upper reaches. In the Rio Grande and Colorado watersheds, however, there are myriad small valleys which are not yet gullied and where land management, as a preventive method, can be effective in keeping sediment out of the main channels. Flash flows tend to clog the main channel with debris that might be moved into the reservoirs by larger floods. The larger the area of watershed dissected by arroyos, the larger will be the percentage of storm rainfalls capable of producing flash flows in the tributary channels.

The history of arroyo cutting of the Rio Puerco will serve to illustrate the rapidity with which dissection of such alluvial valleys proceeds and the effect of recent erosion on the problem of sedimentation of reservoirs. The Rio Puerco comprises 23.6% of the total area above Elephant Butte Dam. The average annual runoff at its mouth[35] amounts to 62,000 acre-ft. Stream-flow records at San Acacia, just below the mouth of the Rio Salado, indicate that the Rio Puerco contributes 6.6% of the runoff and 60% of the suspended load passing that station.[36]

Prior to 1885 the Rio Puerco, an ephemeral stream, was subject to numerous small floods and occasional great ones, the water spreading over a wide and

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[34] 1st Lt., Army Air Forces Weather Service, Los Angeles, Calif.

[35] "Rio Puerco Watershed, New Mexico," U. S. D. A. Survey Report, Flood Control, 1941, p. 221 (unpublished).

[36] *Ibid.*, p. 307.

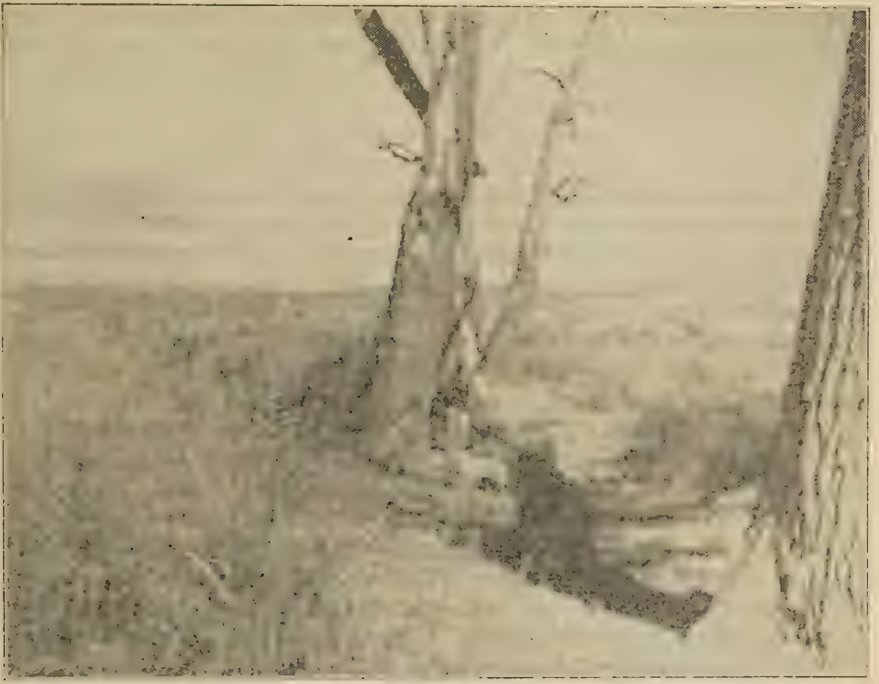

FIG. 8.—OLD STREAM CHANNEL OF THE RIO PUERCO, NEAR SAN LUIS, NEW MEXICO

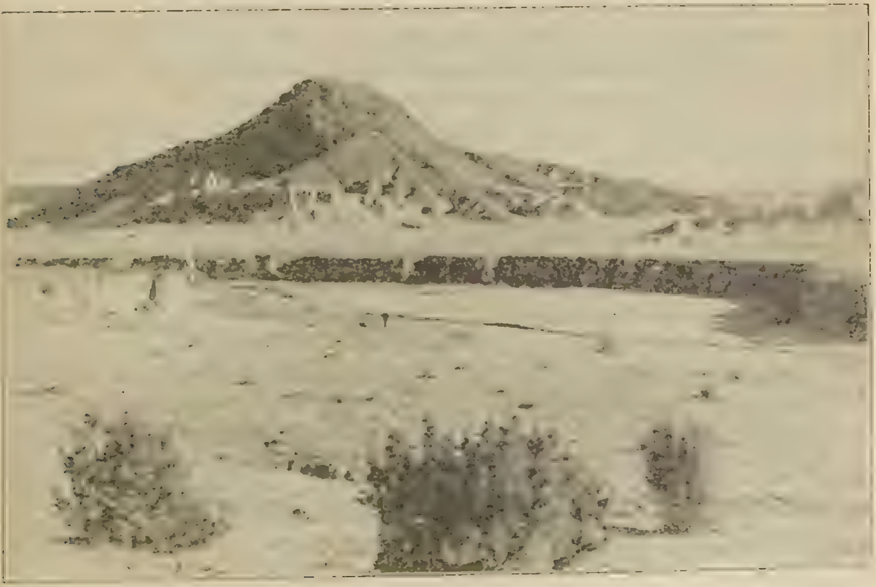

FIG. 9.—ARROYO OF THE RIO PUERCO IN 1940, ABOUT THREE MILES BELOW SAN LUIS, NEW MEXICO

grassy valley floor.[37] The flood plain was discontinuously channeled, a natural feature of such a watercourse. The continuous gully, now approximately 156 miles long, cut headward from the mouth, the greatest growth taking place in the period from 1885 to 1895.

In 1939 the writer talked to one of the original settlers of the village of San Luis, N. Mex., the late Miguel G. Dominguez, who came there in 1867 when his father settled in the Rio Puerco valley. The old man showed the writer the position of the Puerco channel as it was when he was a youth. As the present arroyo developed, many small remnants of the original channels have been left on the old valley floor. Fig. 8 was taken at the position indicated by Mr. Dominguez as the 1870 channel. As can be seen the watercourse was shallow and about 20 ft wide, lined with cottonwood trees. These trees died when the water table was lowered by the cutting of the continuous arroyo.

Mr. Dominguez said that in his youth there was a gully "head-cut" 4 ft or 5 ft deep about one mile below the village of San Luis. The gully was probably discontinuous at that time. Such evidence corroborates the picture of the valley constructed by Kirk Bryan and G. M. Post.[37]

In 1904, according to Mr. Dominguez, the arroyo at San Luis was 12 ft deep. It is now 30 ft to 40 ft deep. The magnitude of the present channel can be visualized from Fig. 9, a photograph taken by the writer in 1940.

No discussion of erosion problems of the Rio Grande would be complete without reference to the voluminous work of Kirk Bryan.[38] The classic report by Messrs. Bryan and Post[37] provides detailed data on southwestern erosion history. These men made a survey of the Rio Puerco channel in 1927 and another was made in 1939 by the Soil Conservation Service. Using Mr. Bryan's estimate of the volume of the channel in 1885 and the results of the 1939 survey, it is computed that approximately 250,000 acre-ft of material were eroded from the Puerco channel during the 54-yr period.[39]

Data on widths of the Rio Puerco channel recorded by land surveys in 1881, 1897, and 1900 were collected by Messrs. Bryan and Post and, together with later measurements by the Soil Conservation Service,[39] provide indications of the rate of widening of the arroyo. The width of the channel on six section lines in T 6 N, R 1 W where the Puerco forms the west boundary of the N. D. Chavez Grant were measured by early land surveys. The increase of width of channel, averaged for these six places, is shown in Fig. 10.

This résumé of the history of arroyo cutting in the Rio Puerco characterizes the development of recent erosion in most of the watersheds of the southwest. This discussion cannot possibly do justice to the large volume of data published on the subject; nor is this the place to discuss the relative importance of overgrazing and climatic change as causes of the present epicycle of erosion.

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[37] "Erosion and Control of Silt on the Rio Puerco, New Mexico," by Kirk Bryan and G. M. Post, Rept. to the Chf. Engr., Middle Rio Grande Conservancy Dist., October, 1927.

[38] "Historic Evidence on Changes in the Channel of the Rio Puerco," by Kirk Bryan, *Journal of Geology*, Vol. 36, 1928, pp. 265–282.

[39] "Rio Puerco Watershed, New Mexico," U. S. D. A. Survey Report, Flood Control, 1941, p. 218 (unpublished).

The fact remains that overuse of land has contributed to the timing and to the extent of the present erosion problem. Because there are many subtributaries in various parts of the Rio Grande and Colorado watersheds which are not yet gullied, continued mismanagement of land can still be a factor in producing large amounts of sediment which might further aggrade the main channels or be deposited in reservoirs.

The author states that the prime objective of range and forest management is to produce as much forage and merchantable timber as possible on a sustained yield basis ("Prolonging the Life of Lake Mead: Land Management"). The qualification might well be added that this assumes that the soil and vegetation will be ecologically stable under continued use. Such is the case in most areas where deterioration has not progressed too far.

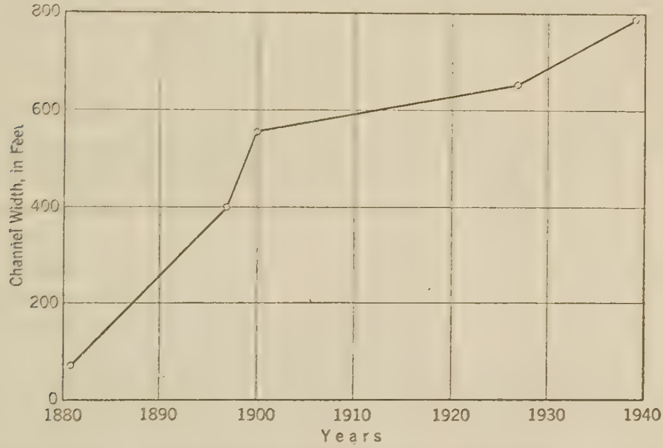

Fig. 10.—Increase in Width of Rio Puerco Channel, New Mexico, in T 6 N, R 1 W

Planting of forests solely to prevent erosion is seldom a sound financial investment, as Mr. Stevens states. This is certainly true in so far as private enterprise is concerned. From the standpoint of the nation as a whole, however, the time has already come when the value of cover on certain watersheds as relatively permanent protection for areas downstream far exceeds the actual cash value of the land itself. It is to be hoped that there will be an increasing realization of the value of watershed cover as protection for water supplies and as a factor promoting soil stability. These are separate values and no less important than the value of the merchantable produce.

The sedimentation problem brings the engineer face to face with many nonengineering aspects of land use. In many respects watershed control, including all the biological considerations of grazing, forestry, and farming, constitutes an engineering problem in that it is susceptible to engineering techniques of over-all analysis and planning. A growing number of engineers are extending themselves into biological fields, but there is still a need for practicing engineers to broaden the scope of their activities in order to apply engineering experience to the vast job of conserving the deteriorating land resource. Papers such as the one under discussion constitute a stimulus toward this end.

Carl B. Brown,[40] Assoc. M. Am. Soc. C. E.—In 1934, Mr. Stevens "focused attention on the part sedimentation will play in the future history of Western civilization." [41] In the present paper he analyzes the effects of sedimentation on the useful life of Lake Mead and Elephant Butte Reservoir. His conclusions strongly re-emphasize the urgency of the task lying ahead of engineers and conservationists to find some practicable means for conserving the limited, irreplaceable, and indispensable reservoir sites on major southwestern streams.

Lake Mead.—Previous estimates of the annual rate of silting in a reservoir on the Colorado River at Boulder Canyon have ranged[42,43] from 80,000 acre-ft to 150,000 acre-ft. A rate of 137,000 acre-ft originally estimated by Harry F. Blaney and the late Samuel Fortier, Members, Am. Soc. C. E., in 1928 [44] was quoted by the U. S. Bureau of Reclamation in 1936 with the qualification that "this amount will decrease with upstream development. It is estimated that the total silt deposits in the reservoir will not exceed 3,000,000 acre-feet at the end of 50 years." [45] However, the Bureau's Final Reports on the Boulder Canyon Project contain this statement: "Estimates indicate that before Boulder Dam was built the average rate of silt transportation through Black Canyon was about 100 thousand acre-feet per year, approximately 5 tons per second." [46]

On the basis of new data, Mr. Stevens has concluded that the average rate of silting in Lake Mead without upstream developments will be 198,000 acre-ft annually. There is considerable evidence to indicate that even this estimate is too low. The author's analysis is based primarily on the suspended load records obtained by the U. S. Geological Survey, at Grand Canyon, during the 16-yr period from October 1, 1925, to September 30, 1941 (see Table 4). His analysis contains no allowance, however, for the fact that this was a period of predominantly subnormal stream flow throughout the Southwest, although he mentions this fact several times in the text. Fig. 4 shows the rapidly declining level of Great Salt Lake and the declining 5-yr moving average of stream flow of the Rio Grande at Otowi, N. Mex., from 1925 through 1935. A cumulative curve of departure from average annual runoff of the Colorado River at Lees Ferry, presented by the writer,[47] shows that the declining trend at this station continued through 1940.

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[40] Head, Sedimentation Section, SCS, Washington, D. C.

[41] "The Silt Problem," by J. C. Stevens, *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 208.

[42] "Colorado River and Its Utilization," by E. C. LaRue, *Water Supply Paper No. 395*, U. S. Geological Survey, Government Printing Office, Washington, D. C., 1916, p. 222.

[43] "Handbook of Applied Hydraulics," by C. V. Davis, McGraw-Hill Book Co., Inc., New York, N. Y., 1942, p. 131.

[44] "Silt in the Colorado River and Its Relation to Irrigation," by Samuel Fortier and Harry F. Blaney, *Technical Bulletin No. 67*, U.S.D.A., Washington, D. C., 1928, p. 4.

[45] "Boulder Canyon Project—Questions and Answers," U. S. Bureau of Reclamation, Washington, D. C., May 1, 1936.

[46] "Boulder Canyon Project—Final Reports," *Bulletin No. 1*, Pt. 4, U. S. Bureau of Reclamation, Denver, Colo., 1941, p. 238.

If the quantity of suspended sediment carried by a stream is related to the annual discharge, the long-term mean sediment load would not be fairly represented by a period of exceptionally subnormal flow. The annual suspended load of the Colorado River does have a fairly well-defined exponential relationship to the annual runoff as shown by Fig. 11. As calculated by the method of least squares the curve on this graph was plotted from the equation:

$$S = 0.00295\, R^{1.52} \ldots\ldots\ldots\ldots (1)$$

in which S is the annual suspended load in tons; and R is the annual runoff in acre-feet. As the exponent is greater than 1, the suspended-sediment load is shown to increase at a greater rate than the runoff. For some other streams, not in the Southwest, similar calculation has shown an exponent less than 1.

The spread of points in Fig. 11 is caused mainly by understandable natural conditions. In a year of heavy snow-melt runoff without large summer storms in the silt-producing Colorado plateaus, points may fall well below the line. When the converse is true, points may fall above the line. Over a long period the sum of all measured values should be almost the same as the sum of values derived from the curve, assuming that existing data cover the full range of probable values.

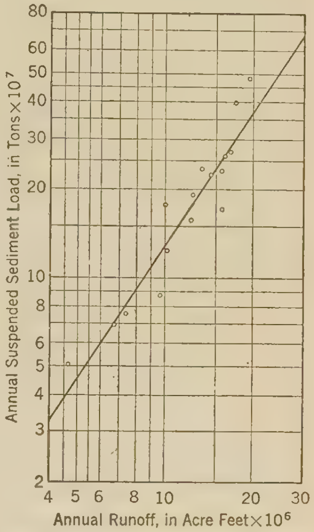

FIG. 11.—RELATION OF ANNUAL RUNOFF AND SUSPENDED SEDIMENT LOAD OF THE COLORADO RIVER NEAR GRAND CANYON, ARIZ., 1925–1941

In order to adjust the mean suspended load of the 16-yr period of predominantly subnormal runoff to a longer, more nearly average, period the following three steps were taken:

(1) An estimate was made of the probable flow at Grand Canyon for each year of the period from 1897 to 1922 by taking a value midway between values estimated by E. B. Debler,[48] M. Am. Soc. C. E., for these years at Boulder Canyon and at Lees Ferry. For a few years certain minor adjustments were made after comparison with estimates made for the same years by E. C. LaRue[49] and R. J. Tipton,[50] M. Am. Soc. C. E. It is considered that Mr. LaRue's

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[47] "Rates of Sediment Production in Southwestern United States," by Carl B. Brown, *SCS-TP-58*, SCS, U.S.D.A., January, 1945.

[48] "Stream Flow of Lower Colorado River and Its Tributaries," by E. B. Debler, U. S. Bureau of Reclamation, December, 1934, p. 61 (processed report).

[49] "Water Power and Flood Control of Colorado River Below Green River, Utah," by E. C. LaRue, *Water-Supply Paper No. 556*, U. S. Geological Survey, Government Printing Office, Washington, D. C., 1925, p. 121.

[50] "Report on Water Supply of Colorado River and Allied Matters," by R. J. Tipton, *Exhibit C, Report and Proceedings*, Fact Finding Committee of the Upper Colorado River Basin States, July, 1938 (unpublished).

estimates of flow at Lees Ferry from 1851 to 1897,[49] based on a correlation with levels in Great Salt Lake, are too uncertain to warrant use in this analysis. If they were used, the indicated mean annual flow would be about 1,000,000 acre-ft larger than that since 1897. Therefore, considering the period from 1897 through 1941 as average seems reasonably conservative by comparison with the 90-yr period since 1850.

(2) Using the estimated runoff at Grand Canyon for each year of the period from 1897 to 1922 in Eq. 1, the value of S for each year was obtained. For the years 1923 to 1925, inclusive, the measured flow was used. The sum of the annual suspended sediment loads thus derived is 8,053,500,000 tons. The measured load during the period from October 1, 1925, to September 30, 1941, was 3,203,900,000 tons, making a grand total of 11,257,400,000 tons.

(3) The measured suspended load during the period from October 1 to December 30, 1925 (approximately 22,000,000 tons, which is duplicated in the preceding addition), was subtracted, and the load for the same months in 1941 (approximately 84,400,000 tons), was added. The record thus derived extends over a period of 45 calendar years, and the mean annual suspended sediment load thus computed is 251,550,000 tons. The computed suspended load for only one year—522,000,000 tons in 1909—was in excess of the 480,000,000 tons actually measured in 1929.

Bed Load.—Considering the physical characteristics of the Grand Canyon gaging station, the sediment load that can be measured with existing sampling equipment probably represents at least 95% of the total load passing the station. However, the results of experimentally-controlled tests, and field comparisons of the milk-bottle sampling equipment used at Grand Canyon, with the scientifically designed equipment developed at Iowa Institute of Hydraulic Research,[51] at Iowa City, Iowa, indicate that Mr. Stevens' addition of at least 15% to the reported suspended load is justified to compensate for bias in sampling method plus unmeasured bed load. This percentage addition to the long-term mean value of suspended load, previously derived, would give a mean annual total load at Grand Canyon of 289,282,000 tons.

Sediment from Intervening Drainage Area.—The author's estimate of 1,600 tons per sq mile for the annual sediment contribution between the Grand Canyon gaging station and Boulder Dam can be fully substantiated as a long-term average [47] if applied to the area that actually contributes surface runoff. Approximately 9,000 sq miles of the 30,000-sq-mile area between Grand Canyon and Boulder Dam comprises the surface of Lake Mead and areas such as the White River, Upper Meadow Valley, and Upper Las Vegas Valley, which so rarely contribute any water to the Colorado River that they should be excluded as areas of sediment contribution. If the sediment production is estimated as coming from 21,000 sq miles at a rate of 1,600 tons per sq mile, this additional annual contribution of 33,600,000 tons would make the long-term mean annual sediment load entering Lake Mead, in round figures, 323,000,000 tons, rather than the 280,000,000 tons estimated by Mr. Stevens.

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[51] "Comparative Field Tests of Suspended Sediment Samplers," *Progress Report, Appendix A* of "A Study of Methods Used in Measurement and Analysis of Sediment Loads in Streams," Iowa Inst. of Hydr. Research, Iowa City, Iowa, December, 1944.

Space Occupied by Deposits.—The specific weight of deposits in Lake Mead needs to be studied exhaustively. The rate of compaction will affect future operation of the reservoir and should govern the timing of the construction of upstream developments. For example, a mean sediment inflow of 323,000,000 tons (assuming an average specific weight of 65 lb per cu ft) would cause an average annual capacity loss of 228,200 acre-ft. This is equivalent to the loss of the dead storage (3,207,000 acre-ft) plus half of the active storage capacity (12,742,000 acre-ft) to spillway crest (El. 1205.4) in 70 years (from 1935), by which time, according to the author, the usefulness of the reservoir for hold-over storage would be seriously impaired. However, if the final value were only 55 lb per cu ft the period would be 59 years, but at 75 lb per cu ft it would be 81 years.

The effective hold-over storage at any particular time depends on the average specific weight at that time. Practically no usable water can be obtained through compaction of sediment or effluent seepage from a reservoir like Lake Mead in any single seasonal drawdown.

TABLE 10.—RELATION OF SEDIMENT ACCUMULATION TO INFLOW IN ELEPHANT BUTTE RESERVOIR

Period	Years	Average annual inflow (thousands of acre-ft)	Sediment by volume (%)	Average annual sediment deposit since previous survey (acre-ft)
January 1915 to December 1916	1.91	1,574	1.80	28,270
December 1916 to August 1920	3.67	1,414	1.66	23,437
August 1920 to August 1925	5.0	1,130	1.62	18,345
August 1925 to April 1935	9.67	853	1.62	13,801
April 1935 to September 1940	5.5	936	0.98	9,200

Elephant Butte Reservoir.—Two factors account for the decline in the rate of sedimentation in Elephant Butte Reservoir. One is the smaller inflow for successive periods between the date of construction and 1935 as shown in Table 10.

Secondly, during the 1937 flood the river broke through its banks above San Marcial and subsequently has deposited huge quantities of sediment over its flood plain. As shown by repeated surveys along eleven ranges crossing the valley between the lower end of the Conservancy District levees and the original head of the reservoir (a distance of about 15 miles), the average annual rate of sedimentation in this area was 50% higher during the period from 1936 to 1941 than during the period from 1914 to 1926. Because of partial desilting of the river flow in this area the rate of deposition in the reservoir was materially lessened.

It is reasonable to expect that the rate of sedimentation in the reservoir will increase when the channel again becomes stabilized in the lower reaches of the Middle Valley and a series of years of heavier flow is experienced. The writer concurs in the opinion that a straight-line extension of the silting rate curve in Fig. 3 "may not be amiss" in the absence of new sediment and water-control structures in the drainage area above.

Under the heading, "Probable Useful Life," the author credits the Soil Conservation Service as reporting "that aggradation of the lower portion of this valley began eight years prior to the construction of Elephant Butte Reservoir." This should be amended to read "at least eight years."[52] Questionable data indicate that the river bed at San Marcial may have been rising since 1880. The Santa Fe Railroad bridge profile at San Marcial, surveyed in 1880, shows a channel 350 ft wide with an average bottom elevation of approximately 4,450 ft. A profile under the bridge in 1904 shows a channel 500 ft wide with an average elevation of 4,452.7 ft.

Aggradation of at least the lower part of the valley since 1906, and perhaps for a much longer period, makes it improbable that the low runoff cycle beginning in the middle 1920's can be the primary cause of this condition. Backwater effects from the reservoir and tamarisk, which the author will find in considerable abundance, have accelerated deposition in the lower reach of the valley, but did not initiate it; nor have they been responsible thus far for increased aggradation more than 25 to 30 miles above the original head of the reservoir.[53]

Although sedimentation in Elephant Butte Reservoir has decreased during successive periods since 1915, coincident with a general decrease in runoff, it does not follow that valley aggradation has resulted from lack of floods capable of flushing out the sediment accumulation of low-water periods.

Records and estimates of the discharge at San Marcial since 1890 show that:

(1) During seven months prior to, and during seven months since, January 1, 1920, the mean monthly discharge has exceeded 10,000 cu ft per sec; and

(2) In ten months prior to, and in ten months since, January 1, 1920, flood flows exceeding 15,000 cu ft per sec have been experienced. The years of largest floods in order of magnitude were 1904, 1929, 1937, and 1941.

In the writer's opinion, aggradation can be accounted for readily by the increase in volume of coarse sediment delivered to the valley by the Rio Puerco and many smaller arroyos that did not exist or were only discontinuous channels prior to 1880. Much of this sediment has come directly from trenching of the alluvial valleys; and, since their develpoment, the arroyos have served as direct flumes for transportation of sediment from gullying slopes into the Rio Grande.

Prolonging the Life of Southwestern Reservoirs.—The author's discussion of methods for controlling reservoir sedimentation leaves one with a rather hopeless feeling. The control of density currents has some potentialities, he concedes. Dredging is properly eliminated as economically impracticable. He dismisses land management in the Colorado River basin, and by implication in other arid sections, by expressing doubt (see heading, "Prolonging the Life of Lake Mead: Land Management") that "all the proved practicable methods of land management known today even though lavishly financed" would reduce the sediment load (entering Lake Mead) as much as 10%. By process of elimination, he concludes to spread the sedimentation loss through a series of major reservoirs, which would be constructed according to a schedule based partly on sedimentation rates and partly on the increasing need for stored water. In effect, this plan would throw the burden of sediment control on those very natural resources—the limited and irreplaceable major reservoir sites—that should be protected against, not exhausted by, rapid sedimentation.

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[52] "The Sedimentation in the Middle Rio Grande Valley, New Mexico," by Stafford C. Happ, SCS, 1942, p. 85 (unpublished).

[53] *Civil Engineering*, February, 1945, Fig. 2, p. 84.

One of the most common flaws in reasoning on sediment control in the Southwest results from an assumption that most of the current sediment production is coming (1) from rain-wash erosion of barren slopes that have never supported much, if any, vegetal cover, and (2) from corrasion of rock-bound channels by stream flow. This inference may be readily gained from a cursory look at the spectacular canyons and mesas of the region. All evidence that the writer has been able to collect, however, points toward a contrary conclusion. The present sources of the great bulk of sediment, in order of importance, are the alluvial valley fill, the alluvial fans, and the colluvial and residual soils of the land slopes. In the same order of importance, these were the most favorable sites for, and originally had the densest stands of, vegetation.

When white man first came to know the southwestern country, a cycle of valley and fan alluviation was in progress, except in the canyons of the Colorado River, the lower reaches of some of its tributaries, and in the high mountains. Historical records indicate that channels were lacking or discontinuous in the tributary drainage areas [54] that are now the primary sources of sediment load in the main streams (for example: Rio Puerco and Rio Salado, tributaries of the Rio Grande; Chaco Canyon, tributary of the San Juan; San Simon and San Pedro washes, tributaries of the Gila; Upper Kanab Creek, tributary of the Colorado; Polacca Wash, and other tributaries of the Little Colorado). Erosional debris from the land slopes was deposited mainly on alluvial fans at the edge of valleys or intermontane basins; and intermittent stretches of channel in the valleys terminated in broad areas of water spreading and deposition. At various times in different parts of the Southwest during the nineteenth century, a shift began in the physiographic cycle from predominant alluviation to predominant channeling. Arroyo development has now progressed to the point where most of the western alluvial valleys are cut by deep, steep-sided trenches for most of their length. Branching trenches have extended into many of the tributaries and thousands of gullies run from them on to the land slopes.

A survey of the Rio Puerco made by the Soil Conservation Service in 1939 showed that this arroyo had a volume of 267,000 acre-ft between the mouth and San Luis Dam, a distance of 117 miles. Kirk Bryan estimated in 1928 [55] that the original discontinuous Rio Puerco channels (prior to 1885) contained less than 5% by volume of the then-existing channels. The Soil Conservation Service estimated in 1939 that the current sediment production resulting from enlargement of the main channel of the Rio Puerco was 4,600 acre-ft per yr, or approximately 40% of the estimated average annual load of 11,500 acre-ft passing the mouth of the Puerco. An erosion survey of the entire drainage area gave a basis for estimating that 3,400 acre-ft per yr, or an additional 30% of the total load, was coming from enlargement and headward extension of tributary arroyos; and 3,500 acre-ft per yr, or about 30% of the total, was derived from sheet and gully erosion on the land slopes.

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[54] "Climate and Accelerated Erosion in the Arid and Semi-Arid Southwest, with Special Reference to the Polacca Wash Drainage Basin, Arizona," by C. Warren Thornthwaite, C. F. Stewart Sharpe, and Earl F. Dosch, *Technical Bulletin No. 808*, U.S.D.A., Washington, D. C., 1942, pp. 125–129.

[55] "Historical Evidence on Changes in the Channel of the Rio Puerco, a Tributary of the Rio Grande in New Mexico," by Kirk Bryan, *Journal of Geology*, Vol. 36, 1928, pp. 265–282.

The reduction or elimination of grazing has halted the head cutting of many smaller arroyos and gullies. Proper management of vegetation certainly can retard the acceleration of erosion. Where main drainage channels are already deeply cut, however, vegetation alone cannot control the major part of the sediment output from the watershed. Various types of water-control structures are also essential to sediment control. If the author would broaden his term "land management" to include "upstream engineering" a much different conclusion might be reached. Permanent-type minor structures throughout the watershed can (1) reverse the arroyo cycle now in progress by creating local base levels of erosion, thus forcing water to spread again over valley flats and fan slopes as it did during the cycle of alluviation which existed prior to the present arroyo cycle, and (2) utilize the enormous volume of existing arroyos for sediment storage.

The effectiveness of a barrier system of sediment control has been experimentally tested during a 9-yr period from 1935 to the end of 1943 at the 65,000-acre Navajo Experiment Station of the Soil Conservation Service, situated in the typical rolling plains and broken mesas of the Colorado plateau province, 20 miles north of Gallup, N. Mex.[56] Drainage from this area enters the San Juan River. The experiments were designed to determine (1) the effects of water spreading and sediment deposition on forage and crop yields, and (2) the effectiveness of channel barriers and water spreading in controlling arroyo cutting and reducing the sediment output from the drainage basin of a major southwestern reservoir. All of the experimental area is a direct contributor to sedimentation in Lake Mead.

In 1934 and 1935, fourteen earth-fill dams were constructed to plug the main arroyo channels. Earthen dikes were extended from the dams part way across the adjacent flood plains. The structures forced water out of the channels around the edge of the dikes and caused it to spread over a large area of flood plain. At places on the flood plains where scouring was expected or observed, wire-bound rock sausages were buried or spreader fences were erected.

The percentage of sediment by weight in the runoff from 9,470 acres above three of the fourteen dams, which were chosen for detailed studies, varied from less than 1% to more than 60%, the unweighted average being 13%. In the 9-yr period, approximately 500 acre-ft of sediment was deposited behind the three dams and on the flood plains over which water was spread. The runoff was completely dissipated on the flood plain and no sediment originating above these dams was carried out of the drainage area. The sediment deposits were equivalent to 3.75 acre-ft per sq mile per yr—a high rate of sediment production for even the Colorado plateau area. All of this sediment plus a large additional amount controlled by the other eleven dams would have been added to deposits in Lake Mead.

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[56] "Effects of Diverting Sediment-Laden Runoff Water to Crops and Range Lands," by D. S. Hubbell and J. L. Gardner, SCS, April, 1945 (unpublished).

Complete sediment detention and runoff dissipation can be accomplished by barriers only in small drainage areas. Barriers equipped with overflow spillways can be used effectively, however, in areas up to several thousand square miles. For example, if a series of some 72 such barriers, with spillway sills set at or just above the present valley floor level and with earthen dikes 5 ft to 20 ft high extending to the valley sides, were constructed on a fixed schedule over a 50-yr period on the Rio Puerco, it would be possible:

(a) To utilize most of the 267,000 acre-ft channel cavity for sediment storage;

(b) To reduce the sediment load of the stream at its mouth by most of the 4,600 acre-ft now coming annually from scour in 117 miles of main channel;

(c) To trap approximately 70% of the total sediment output of the drainage basin;

(d) To raise the water table in the valley and increase net forage yield; and

(e) To retard flood flows.

Such a program would be little, if any, more costly than the construction of a single large reservoir near the mouth of the Puerco to do an equivalent job of sediment control.

The Rio Puerco has an average but highly variable runoff of about 60,000 acre-ft, or less than 7% of the average annual inflow to Elephant Butte Reservoir; yet it contributes more than half of the sediment deposited below its mouth in the valley and reservoir. Loss of 7% of the total inflow to reduce sedimentation in Elephant Butte Reservoir by 50% would be an economic investment. Actually, the control of sedimentation by a watershed conservation program does not involve a sacrifice of water as does the net loss by evaporation from free water surfaces of storage reservoirs. Rather, it would shift the beneficial use of a small fraction of the total water supply of major watersheds (generally in the range of 5% to 15%) to upland slopes and headwater valleys. In compensation it would permit the orderly utilization of major upstream storage sites, such as those enumerated by the author, for beneficial purposes and not for sediment storage.

Mr. Stevens should not have too much misgiving about the efficacy of land management (in its broader sense including upstream engineering) (see "Conclusions") being so closely "bound up with administrative policies and congressional appropriations that its effect * * * lacks the essential element of assurance." It is only 10 years since the effects of land management on agricultural and range land began to be studied on a scale anywhere near commensurate with the scope of the problem. Although the subject is highly complex, enough already has been learned and demonstrated to point the way. Much more research is, indeed, needed to draw the blueprints for full-scale conservation of Western drainage basins. As rapidly as sound plans can be prepared, the afore-mentioned difficulties will doubtless resolve themselves. The will of the local people for conservation of soil and water resources is already expressing itself in the rapid organization under state laws of locally-controlled Soil Conservation Districts which thus far include, for example, 25% of the land in Colorado, 49% in New Mexico, and 68% in Utah.

G. E. P. SMITH,[57] M. AM. SOC. C. E.—The paper by President Stevens is of absorbing interest. It deserves most earnest consideration in the further development of the comprehensive plan for utilization of the Colorado River, and to a lesser extent of the plans for the Rio Grande.

The rate of silt accumulation in Elephant Butte Reservoir as determined by the author is not greatly different from the rate predicted by the designers at the time of the dedication; the rate is a little less but the difference is accounted for by the difference in the value of γ. The rate for Lake Mead is greater than was assumed prior to construction of Boulder Dam.

In the realm of history, two or three hundred years is a brief period. Structures and other normal investments can be amortized in a predetermined number of years, but a population settled on land with accessory rural institutions has never yet been amortized successfully. One can picture the farmers on the Colorado River projects clinging to their homes and hoping for exceptionally well-distributed river discharge to enable them to raise a few more crops. Perhaps the present civilization has been too uncharitable in its criticism of the civilization that once flourished in the valleys of the Tigris and Euphrates rivers. In some spots the present civilization is not too secure.

An important query is whether the runoff in quantity and character during those base periods of records available to the author are indeed representative of a far longer period. Since the records of short duration have been extrapolated six to twenty times, it is essential to inquire whether these periods are reliably representative. The writer has studied the rainfall records for Utah, Wyoming, and Colorado, the three states from which runoff at Lees Ferry is derived, from 1880 to 1945. The mean annual rainfall for the three states separately, obtained from the U. S. Weather Bureau, was averaged, weighting the Colorado rainfall double. The record was then smoothed moderately in 3-yr running averages, weighting the middle one double. The resulting record, plotted as percentage of the long-time mean, is shown in Fig. 12. It appears that the rainfall for the years from 1935 to 1942, inclusive, was 105% of the long-time mean. The record in Fig. 12 is not a safe index for the Elephant Butte studies, although a similar one could be made covering rainfall of southeastern Colorado and western New Mexico. It is recognized that runoff is not indicated precisely either in quantity or character by rainfall, but rainfall is the best index available and is relied on by hydrologists when missing runoff data must be supplied.

The author refers (see heading, "Probable Useful Life") to "the low runoff cycle the West has been experiencing since 1920." It is difficult to state just what significance is here attached to the much-abused word "cycle." There are wide variations in rainfall and runoff from year to year, but no regular periodic cycles, which repeat themselves over and over indefinitely, can be proved. Even the great irregularities apparent in the rainfall record of a basin, such as the record shown in Fig. 12, do not parallel very well the irregularities of rainfall in adjoining basins. The irregularities are, therefore, largely local; if there were any governing causes, at least they would not be uniform for broad regions like "the West." The vagaries of the air circulation over the earth and of the hairspring conditions that cause precipitation are so great that any truly cyclic tendencies are overshadowed by the sequence of fortuitous events.

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[57] Prof. of Agri. Eng., Univ. of Arizona, Tucson, Ariz.

As to the Colorado-Wyoming-Utah rainfall, it is not the drought in the thirties, but it is the generally high level of rainfall from 1905 to 1930 that is outstanding. Other sources of information confirm Fig. 12 and indicate that the long period of low rainfall began in 1870 instead of 1880. The inference is that the author's period of record for the Colorado River, although far too short, is reasonably representative, whereas the period used in the Rio Grande studies covered too many wet years. The importance of a few inches more or less of rainfall in a given year can be appreciated by plotting the quantities of sediment shown in Table 2 against either rainfall or runoff.

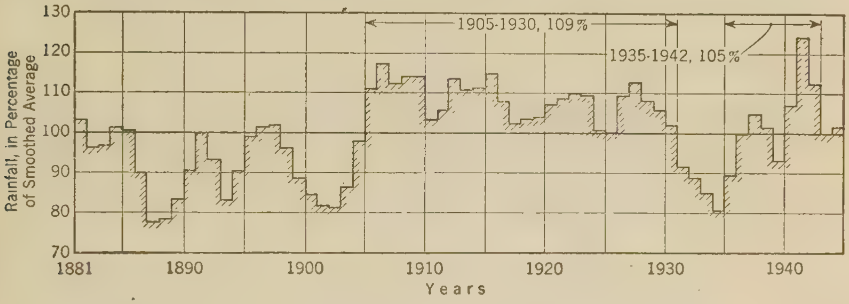

FIG. 12.—RAINFALL IN THE UPPER COLORADO RIVER BASIN STATES

The decreasing rate of siltation shown in Table 1 needs further study. A large proportion of the silt comes from the Rio Puerco. Has its bed been aggraded greatly? Also, settling and compaction of the deposited sediments in the reservoir would have the same effect on the rate as decreasing annual increments of silt. The value of γ varies within the same reservoir and tends to increase with time.

In the case of Lake Mead, various factors will operate to reduce the siltation rate. Utilization of the water above Lees Ferry, especially construction of projects to utilize the San Juan, will have that effect. Transmountain diversion of clear water, by reducing floods, will tend to aggrade some sections of the river system. The factors are not compensating.

The author proposes, under the heading, "A Schedule of Development," that the Dewey Dam be built first. This suggestion is a mark of real genius. It is desirable that the comprehensive plan for the Colorado River be revised and that the Dewey Dam be made the No. 1 project. That dam and the Bridge Canyon Dam should have been built in 1925 or before, followed by the Davis Dam, and then the Boulder Dam, and dams on the San Juan and Little Colorado rivers.

Two papers by the writer published in 1928 featured the wisdom of building the Dewey Dam first for many reasons. In the first paper[58] it was stated:

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[58] *Western Construction News*, February 25, 1928, p. 107.

> "Canal maintenance in Imperial Valley is made difficult by the deposition of silt. This condition will be greatly ameliorated when the Imperial Canal is extended upstream and connected to the Laguna Dam, as already provided for by agreement. The same methods which protect the Yuma canal system—skimming and sluicing—will be applied to Imperial Canal water at the intake. The silt problem will be greater in the future when storage capacities are threatened than it is now; the silt cannot be stored in reservoirs forever. Revised estimates of the quantity of silt transported annually by the Colorado River are about 50 percent higher than the published estimates."

In the second paper[59] it was stated:

> "Silt storage in the Colorado River reservoirs will suffice for less than two hundred years, and then what shall be done? Silt storage has always been considered a curse. During the next decades, before the water has been put to use and while most of the water will run to the sea, the effort should be to make it carry its load of silt with it, so as to conserve the storage space."

Silt storage as a means of eliminating the silt problem at diversion points, although favored by the farmers of today, won too ready acquiescence of engineers.

The U. S. Bureau of Reclamation is now building projects to irrigate the Yuma Mesa and Imperial East Side, where the soil, or rather the sand, much resembles a sieve. If the naturally silty Colorado River water could be used for 20 or 30 years those lands would be greatly improved; but, ironically, the silt is being stored in Lake Mead where it is not wanted.

The writer often has considered the feasibility of a long tunnel within the rock wall on one side of the canyon upstream from a high dam, at a low level, with inlet tunnels every 200 ft or 300 ft equipped with control gates, but with small permanent openings to bleed off some of the silt. The purpose, of course, is to create a long settling pool and reduce the siltiness of water in the penstocks and turbines. A floating barge with whirling vanes on vertical shafts to be operated seriatim in front of the inlet tunnels might be helpful also. Boulder Dam, more than all others, should have such a tunnel, perhaps one on each side of the canyon extending a mile upstream.

The author was confronted with inadequate and uncertain data, and has had to make the most of that situation. Records were too short, and silt sampling probably was done during high water or when convenient. He has exercised much license in extensions of data, and in various assumptions, including the various values of γ, and some factors have been neglected. It is a highly exploratory study; but the computations lead to two definite answers—158 years for Elephant Butte Reservoir and 144 years for Lake Mead.

These answers will be cited and bruited, not only in technical papers, but also in newspapers and in speech until many people accept them as irrefutably exact. In one instance the author refers to his studies as "speculations." It would have been wiser to state the answers as of the order of 158 years and of the order of 144 years, or to have predicted, roughly, about 150 years, and thus leave the impression of uncertainty in the reader's mind. A case in point is that of the preliminary synthetic estimates of the water supply of the Colorado River. It is known now that those estimates were too high; but they were generally accepted to the last significant figure and much damage and injustice have resulted.

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[59] "Progressive Arizona and the Great Southwest," by G. E. P. Smith, Vol. 9, Nos. 4, 5, and 6 (reprinted in the *Pan-American Geologist*, Vol. LIV, August, 1930).

ALBERT E. COLDWELL,[60] ASSOC. M. AM. SOC. C. E.—This paper furnishes a valuable and long-awaited sequel to the author's paper on "The Silt Problem."[61]

To many readers it may have been somewhat of a surprise or mild shock to learn that the estimated life of Lake Mead is less than that of Elephant Butte Reservoir. These two reservoirs, however, have such similar lengths of life that it seems desirable to examine whether that most useful tool in sedimentation studies, the capacity-watershed ratio, may be applied effectively to indicate that such similarity is reasonable. These ratios are:

Reservoir	Acre-ft per sq mile
Elephant Butte..............	$\frac{2,634,800}{25,923} = 102$
Lake Mead.................	$\frac{28,632,000}{167,000} = 171$

For reservoirs in Texas and Oklahoma, Carl B. Brown,[62] Assoc. M. Am. Soc. C. E., has shown that, on the average, the annual storage loss for reservoirs with capacity-watershed ratios of 102 and 171 may be expected to be about 0.88% and 0.85%, respectively, of the original storage capacity. Applied to the two reservoirs in question, the resultant values of 114 years for Elephant Butte and 118 years for Lake Mead are very similar; and, after due consideration is given to variation in location, climate, topography, and other conditions, these results appear to be in good agreement with the estimates of the author.

There is the possibility that either the capacity-watershed method or the linear method used by the author for estimating the lives of these reservoirs is too severe during the later portion of their lives. Large reservoirs in their early years have the ability to settle out virtually the entire inflowing sediment load, but with the passage of time the size of detention basin available becomes smaller and smaller until eventually a major portion of the inflowing sediment could be carried past the dam or deposited at elevations above the original pool level. McMillan Reservoir on the Pecos River seems to be a case in point. Much has been written, notably by the late R. F. Walter,[63] M. Am. Soc. C. E., about the effectiveness of the vegetative screen upstream from this lake, and since the writer has not had the opportunity to inspect closely these tamarisk thickets there is no desire on his part to minimize the value of this vegetation in preserving the capacity of McMillan Reservoir. The question is raised, however, as to what the condition of this reservoir would be in 1945 if the dense vegetation had not been present for the 30 years since 1915. This is an example of a small reservoir built on a large drainage area. Its capacity-watershed ratio has not at any time been more than 4 acre-ft per sq mile. Assuming an annual sediment inflow upstream from the vegetative screen of 600 tons per sq mile, approximately 183,000,000 tons of sediment (or, at 70 lb per cu ft, 120,000 acre-ft) had been deposited above lake level, in the lake, or had passed the dam in the 18-yr period from 1915 to 1932, inclusive. The capacity of the reservoir was depleted by only 5,000 acre-ft during this period. Why did not the portion of the remaining 115,000 acre-ft which passed by the vegetative screen become deposited permanently in the lake? Probably because the cross-sectional area of the remaining reservoir was so small that the velocities of sediment-laden inflows were not reduced any more by the lake than they had been by the vegetative screen. It has occurred to the writer that possibly if there were no vegetation above McMillan Reservoir its capacity would have now been additionally depleted only by an amount sufficient to insure that there would be sufficient velocity during high-water flows to carry all sizes of sediment past the dam. Floods on this river are often large enough to fill the remaining capacity of the reservoir in from one to two days, so it appears that, except for the storage of low flows, McMillan Reservoir virtually has become, so to speak, a run-of-the-river project, and possibly has been so for many years. It may be noted, as shown in Table 11,[64] that not during its life history has this lake had its capacity depleted by more than about 60%, even in its early years when the sediment problem appeared so severe. It would be interesting to know whether there are available profiles of the original and recent stream bed through this reservoir and upstream from it; and whether the volume of above-pool sediment deposits (which by now must be large) has ever been measured by valley cross sections or other methods.

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[60] Associate Engr. (Hydr.), Corps of Engrs., War Dept., Tulsa, Okla.

[61] "The Silt Problem," by J. C. Stevens, *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 207.

[62] "The Control of Reservoir Silting," by Carl B. Brown, *Miscellaneous Publication No. 521*, U. S. D. A., p. 26.

[63] Discussion by R. F. Walter of "Silting of the Lake at Austin, Texas," by T. U. Taylor, *Transactions*, Am. Soc. C. E., Vol. 93 (1929), p. 1719.

McMillan Reservoir has had its life history complicated by the construction of reservoir storage upstream, as eventually may be the case with all worthwhile reservoirs on heavily-laden sediment streams. Either as the result of acts of nature or man, it appears that the rate of sedimentation of reservoirs may not be a linear function, and that at some indeterminate time during their lives such a change in rate of sedimentation will act to their advantage. In the case of Elephant Butte Reservoir and Lake Mead this may occur too late to be useful in preserving any significant capacity for much additional time. However, at Lake Mead any storage thus accrued will be that much in addition

TABLE 11.—RELATION OF SEDIMENT DEPOSITS IN MCMILLAN RESERVOIR TO POTENTIAL ORIGINAL STORAGE CAPACITY AT VARIOUS ELEVATIONS

Date	Spillway crest stage (ft, on reservoir gage)	Capacity to spillway crest level, as if no sedimentation had occurred (acre-ft)	Sediment deposits below spillway crest level (acre-ft)	Ratio of sediment deposits to capacity (%)
1894	17.3	32,500	0	0
1904	17.3	32,500	16,000	49
1910	23.5	66,000	27,000	41
1915	27.0	90,000	48,500	54
1925	27.0	90,000	52,000	58
1932	27.0	90,000	53,500	60

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[64] From the discussion by R. F. Walter of "Silting of the Lake at Austin, Texas," by T. U. Taylor, *Transactions*, Am. Soc. C. E., Vol. 93 (1929), Table 8, p. 1722.

to the 2.5 million acre-ft of residual storage which the author has shown will remain available as regulating storage. The study of sedimentation of reservoirs has gained such momentum, in the ten years since 1935 (and the problem certainly deserves such study), that it may now be time for investigators to evaluate the possibility that they are swinging with the pendulum toward the tendency for predictions of too pessimistic a nature. For instance, suppose the curve of Fig. 3 were extended with only slightly decreasing rates from the last two points on the curve for Elephant Butte. How much longer a life this reservoir could then be expected to have!

It is gratifying to note that the author has treated the reservoir for sediment storage as a necessary part of river-basin development rather than as a nuisance that should be built only if storage for other purposes fails because of difficulties with sediment. There comes to mind a remark by an engineer concerning a project to the effect that "They provided enough space to store the water, but forgot about a place to store the silt." At that time the writer was not particularly concerned with sediment problems, but the cogency of this statement has been realized since then.

This paper effectively calls attention to the existence of problem areas in large river basins. Similar areas have been found in the Arkansas River basin in Oklahoma as the result of investigations by the U. S. Engineer Department. Of the total water and suspended sediment flow in the Arkansas River as it crosses the Oklahoma-Arkansas state boundary line, the Canadian River furnishes approximately 68% of the sediment but only 22% of the water; and the Cimarron River furnishes approximately 11% of the sediment but only 5% of the water. These streams drain the arid and semi-arid area consisting of northeastern New Mexico, the Texas and Oklahoma panhandles, southwestern Kansas, and northwestern Oklahoma, with only a small portion of their eastern watersheds being in subhumid territory. After entering the main stem of the Arkansas River the sediment-laden water from these rivers becomes diluted by the less turbid water of eastern tributaries, reducing the

TABLE 12.—RELATION BETWEEN SEDIMENT-CARRYING CHARACTERISTICS OF TRIBUTARIES AND MAIN STREAMS

Relative flow rates	Headwater streams[a]	Main streams[b]
(a) NUMBER OF DAYS PER YEAR REQUIRED FOR THE TRANSPORTATION OF EACH ONE THIRD OF THE TOTAL ANNUAL SUSPENDED SEDIMENT LOAD		
Highest flows	1	6
Intermediate flows	3	12
Lowest flows	361	347
(b) APPROXIMATE RANGE OF STREAM STAGE DURING WHICH EACH ONE THIRD OF THE TOTAL ANNUAL SUSPENDED SEDIMENT LOAD IS TRANSPORTED		
Highest flows	Above bankfull	Above bankfull to half bankfull
Intermediate flows	Above bankfull to half bankfull	Half bankfull to quarter bankfull
Lowest flows	Half bankfull and below	Quarter bankfull and below

[a] Before the entrance of tributaries. [b] After the entrance of tributaries.

sediment concentration but not the total load. In the course of the study of duration of heavy sediment flows a relationship between tributaries and main streams was found, which is generalized in Table 12.

The significance of this table is that in the larger streams the transportation of significant amounts of sediment is a more continuous process than in the small streams. In conducting investigations of the possibility of venting, by density currents or otherwise, some portion of the inflowing sediment through flood-control and multi-purpose dams, it is necessary to watch closely the relationship between capacity of outlet works, size of permanent and temporary pools, and the incoming rate of flow even at times when such inflow is well below bankfull capacity of the stream.

It seems wise that the author did not extend the estimated lives of Elephant Butte Reservoir and Lake Mead on the basis of land management. When put in terms of average areal soil loss, the 198,000 acre-ft of annual deposit in Lake Mead amounts to an equivalent thin layer of less than one fortieth of an inch over the entire drainage area. Every particle of soil saved at the headwaters should be gratefully appreciated by operators of downstream projects, but the latter should plan, for reasons of safety, as if conservation measures would not decrease the sediment problem. The major premise for this belief is the difference in design criteria between small agricultural structures and major river projects. Because of economic considerations, terraces, small stock ponds, and other water-spreading devices seem seldom to be expected to care for storms of greater intensity than 25-yr frequency without failure. The damage done at the time of failure may cause the loss of much soil which had been saved during previous years. It is to be noted that, when a terrace near the top of a watershed breaks, each succeeding lower terrace usually breaks. The concentration of water at the points of breaking tends to create a gully. The larger reservoirs downstream, designed to withstand much more severe conditions, must be prepared to accommodate sediment inflows caused by these intense upstream conditions.

Any reservoir that has a life of from three to twenty generations of mankind should be considered as a national asset if it provides efficiently the benefits for which it was designed during its life. Surely there are few citizens who would say that the nation would now be better off if Boulder Dam had not been built, just because it might be full of sediment 135 years from now! Is there a possibility of extending the length of usefulness of reservoirs by turning sediment deposits from liabilities into assets? This appears to be a challenge to research engineers of the future. Soil that has been washed from Oklahoma farm land should be more valuable if left in Oklahoma than if deposited in the Louisiana delta, which already has all the rich soil it needs. After extensive sediment deposits have occurred in reservoirs, is there a possibility of controlling floods on them by upstream diversion or storage and then raising crops on this fertile land? Many of the valuable bottom-land farms of this nation are the result of sedimentation caused by flooding, and on them crops are grown of such utility that the danger of flooding is worth risking. L. C. Gottschalk[65] has shown that river silt acted as an effective fertilizer when mixed with loamy sand from a cultivated field in significantly increasing the yield of a crop of millet. This experiment was made under greenhouse conditions where there were no worries about drainage, volunteer vegetation, selective deposition of various qualities of soil in layers, and other problems which would be met in trying to cultivate the delta areas of a lake. Would these deltas be good sites for fertilizer plants? Could a water-tolerant crop, such as rice, be grown successfully? Possibly the answers to such questions as these will be available by the time the students of the future have the opportunity to assess the validity of present-day prophecies as to the future of reservoirs.

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[65] "Results of an Experiment on the Improvement of Sandy Soil with River Silt," by L. C. Gottschalk, *Special Report No. 3*, Sedimentation Sect., SCS, U. S. D. A., Washington, D. C., November, 1942.

WALTER B. LANGBEIN,[66] ASSOC. M. AM. SOC. C. E.—In projecting the short-term measurements of sediment transportation in order to estimate the life of major reservoirs, consideration of the "arroyo problem" in the Southwest is relevant. The question the writer wishes to introduce is: How representative of long-term conditions are the sediment loads now being measured in the major streams? The ultimate source of the sediment would seem to be the minor washes and their laterals which in the Southwest are only wet-weather courses. Muddy floods accompany and follow rain, but throughout the remainder of the year the stream beds are dry. Regarding their present condition, Kirk Bryan states:[67]

> "These ephemeral streams flow in valleys floored with alluvium from twenty to one hundred feet thick. The alluvium has been accumulated by the building up of the grade of the streams which, before the year 1880, ran in discontinuous shallow channels and during floods spread widely over the valley floors. After 1880 and continuing to the present, stream after stream has cut a deep channel, or arroyo, twenty to fifty feet deep into its valley floor or flood-plain. The mud excavated from these channels or arroyos is carried into the main rivers, and as the arroyos are widened by each successive flood, more mud is carried down. Because of the main streams, the Colorado, the Gila, and the Rio Grande, great reservoirs have been built, the increased load of mud will, within a few generations of men, fill the reservoirs and render them valueless for storage. This is the great 'arroyo problem.' How and at whose expense are these reservoirs to be protected?"

The significant and reasonably well authenticated fact is that these spectacular arroyos did not exist before 1870 or 1880. Many "old-timers" still remember the grassy bottom lands where fall hay cutting was a common practice and indistinct tree-lined channels meandered through valleys where today there is a vertical-walled gulley trenching the valley headward mile after mile. It is not uncommon at present to find well-defined roads leading to the walls of an arroyo and appearing on the opposite bank, showing that not many years have passed since white men forded the stream with teams and wagons or even motor vehicles. The Rio Puerco, now a dirty tributary of the Rio Grande, was once such a stream. Rio Viejo, as the Rio Puerco is fondly recalled by the natives, is well marked in many places by lines of dead cottonwoods. The entrenched Rio Puerco reaches headward for a distance of about 140 miles.

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[66] Hydr. Engr., U. S. Geological Survey, Washington, D. C.

[67] "Erosion in the Valleys of the Southwest," by Kirk Bryan, *New Mexico Quarterly*, November, 1940, p. 228.

The present situation is plain, and there is sufficient evidence to discern the situation in Civil War days, but what induced the change?

A prominent theory advances the idea of a changing climate, suggesting that alternations from a more humid to a more arid climate in the Southwest have produced successive periods of sedimentation and erosion. Years of heavy precipitation, because of the increased growth of vegetation, are said to be times of minimum erosion by running water and consequently times of minimum silt transfer to the lower reaches of a stream. Existing conditions thereby are explained largely to have resulted from a change to a more arid climate. A widely accepted hypothesis, and certainly the most publicized, attributes the cause to man's abusive land practices—mainly overgrazing and the building of roads and trails. It is significant that both of these hypotheses attribute these abnormal gullies to the destruction of the vegetative cover, but differ as to the major cause of that destruction. Regional uplift causing rejuvenation of slopes is still another hypothesis that is offered to explain the change, although there is little to support and much to discredit such a theory.

It is a striking fact that the beginning of the arroyo cycle and the migration of cattle and sheep to the Southwest share a common date, and this circumstance has tended to encourage argument, one school arguing that the coincidence proves the guilt of man. Still another school argues that the cow is rather humble as a geologic agent and that, at worst, overgrazing was merely the trigger pull that timed the cutting rather than its cause. The engineer might well undertake as far as possible to retain a judicial view and establish any conclusions on documented evidence.

Physiographers point out that gullies are everywhere part of the landscape of arid regions and existed in the Southwest before 1870–1880. The gullies were more commonly discontinuous, as might be formed by local storms in eroding a small reach; the spreading of the water reduced the carrying capacity and the load was deposited no great distance beyond. Successive intensive storms served to carry sediments progressively downstream, but the movement was not continuous. There are indications that the present situation is quite different. Something has happened. The short discontinuous gullies in many basins have been integrated into a single gully system of spectacular dimensions.

Although fundamental, the matter has not been solved, mainly, the writer believes, because facts in the quantitative sense are few in number; and, indeed, the feeling is commonly expressed that conclusive evidence may never be established. A satisfactory working theory as to the cause would be valuable in framing a preventive and remedial policy and, more importantly, in avoiding measures that operate in opposition to natural adjustments.

Overgrazing can be indicted on its own account. The western range is a source of the major part of the nation's meat supply, but the use of the range must be adjusted to the growth of the forage if the yield is to be sustained. Thus the policy of government as expressed in the Taylor Act has been to discourage overgrazing through proper management of livestock in the public domain. The attitude here is not unlike that of a conservative landlord desirous of maintaining his equity.

Evaluation—through range surveys and mapping—of the grazing capacity of the several parts of the federal range, construction of stock tanks, drilling of wells, development of springs, and construction of fences to facilitate proper areal distribution of the grazing load in proportion to the capacity are all parts of modern range management. They can be justified in terms of meat production. The extent to which such management contributes to retardation of erosion cannot be estimated. Direct measures, such as the construction of check dams and similar structures intended to arrest the deepening or headward progression of gullies, have been widely attempted, with indifferent results in some places and an aggravation of the condition in other places. The situation has existed chiefly, it is believed, because (1) the introduction of such structures tends to upset the new balance that was being attained; and (2) the structures were built of local materials erosive by nature. Arresting the erosion of headwater gullies is attractive for its local benefits. In many places, notably Rio Puerco, streams have lowered their beds below the level of irrigable lands and subdrained the valley ground water. Any reduction of sedimentation in downstream reservoirs and elimination of other downstream damage effected are added, although highly significant, benefits. Nevertheless, no assured methods of arresting the erosion of deep gullies on widespread grazing lands of low unit value have yet been devised.

What bulk do the newly entrenched channels represent? No general surveys have been made to answer this question. Mr. Bryan and George M. Post in 1927[68] reported convincing evidence that the Rio Puerco had trenched its valley for a distance of approximately 140 miles since about 1885. They estimated that about one third of the heavy annual sediment loads delivered to the Elephant Butte Reservoir are being derived from the extension and widening of this arroyo system. In addition, the trench now serves as a great flume to carry sediment directly to the main valley. From channel cross sections they estimated that between 1885 and 1927 about 395,000 acre-ft of sediment, or an average of 9,400 acre-ft per yr, was produced by the trenching of the Rio Puerco and its tributaries.

In an effort to gain further factual information on the valley trenches, the Geological Survey, in 1934–1936, mapped three gullies near Shiprock, N. Mex., in detail. Repeat surveys in 1944 revealed only minor enlargements, the greater part of the gullies in that region showing signs of quiescence rather than aggression. Lack of activity may suggest nothing more than that few, if any, cloudbursts visited the area during the elapsed ten years. Other surveys have since been made in several areas where repeat surveys are also planned.

Most of the gullies are approaching the heads of their respective valleys (if they have not already reached there), and before long must derive all their sediment loads from side cutting. When this stage is reached there is prospect that a decrease in annual sediment load can be expected, the simple explanation being that, as the arroyos widen, there is less tendency for the flood water to impinge on the banks and hence less opportunity for the stream to load up. The lower parts of many gullies already have reached the state where there is little side cutting and the banks are showing evidence of a reduced rate of cutting. Possibly the peak annual sediment load of these streams has already passed. Unless man intervenes, the end point can be reached only when the valley sediments are removed to the level of the entrenched stream.

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[68] "Erosion and Control of Silt on the Rio Puerco, New Mexico," by Kirk Bryan and George M. Post, Rept. to the Chf. Engr., Middle Rio Grande Conservancy Dist., October, 1927 (unpublished).

What of the future? Should Mr. Stevens' extrapolation be straight lines, or convex upward or convex downward?

Field evidence, substantiated by artifacts, points to several periods of erosion and sedimentation. Mr. Bryan[69] places the period between 1100 A.D. and 1400 A.D. as another period of erosion, and the period between 1400 and 1870 as one of sedimentation. Evidently erosion, like rainfall, runoff, and other hydrologic factors, follows seemingly cyclic sequences. Field evidence cited suggests a slowing rate of removal of sediment in gullied valleys. However, there are many valleys as yet untrenched or where gullies are shallow and discontinuous. Aggression in these places may initiate a new and deeper erosion cycle, so the situation is not altogether clear.

However, to the extent that man has contributed to the cause or growth of valley trenching, the situation may in time be adjusted. The wider acceptance of controlled grazing should lead hopefully to some amelioration. To the extent that nature through climate is the cause, man's influence is problematical or impotent and forecasts are difficult. The writer believes that, to the extent that experience since the building of major reservoirs may have coincided with an aggressive phase of the erosion cycle, the long-term outlook may be more optimistic than other evidence suggests. Nevertheless, without quantitative knowledge of the effect of modifying factors one hardly can find serious fault with Mr. Stevens' straight-line extrapolation.

C. S. HOWARD,[70] AFFILIATE AM. SOC. C. E.—As reported by Mr. Stevens in Part II of this paper, a large proportion of the sediment carried into Lake Mead comes from the sedimentary plateau that contributes a relatively small proportion of the water (64% of the sediment and about 20% of the water). Other computations based on the data given in Table 6 indicate that the sediment loads at the Bluff, Cisco, and Green River stations amount to about 54% of the load measured at the Grand Canyon gaging station; but the runoff for the three upper stations amounts to about 92% of the runoff at Grand Canyon. It is apparent that high concentrations and large quantities of sediment are brought into the Colorado River from unmeasured sources. The Little Colorado, San Rafael, Fremont, and Escalante rivers are the principal tributaries above Grand Canyon for which, unfortunately, sediment records are not available. It is desirable that records for those streams, and also for the Virgin and Muddy rivers, should be obtained.

Mr. Stevens has made estimates of the rate of sediment deposition in the various proposed reservoirs, using different weights per unit volume for the different reservoirs. It is doubtful if the available information is adequate for the accurate selection of such weights, but studies on the rate of sedimentation in Lake Mead should provide valuable information on the density of reservoir deposits. Such studies should include the determination of the volumes of deposits that have been formed in various parts of the reservoir and determinations of the sizes of the particles in those deposits. Correlation of the data on particle size of the deposited material and of the material carried in suspension will make it possible to predict future rates of sedimentation and, to some extent, the places of deposition.

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[69] "Erosion in the Valleys of the Southwest," by Kirk Bryan, *New Mexico Quarterly*, November, 1940, p. 229.

[70] Dist. Chemist, Geological Survey, U. S. Dept. of the Interior, Albuquerque, N. Mex.

In his concluding paragraph Mr. Stevens states: "The available data are pitifully meager, and must be greatly enlarged before any constructive program for relief can be developed." The data are meager and inadequate, not only as to quantity but also as to quality; but in the past few years steps have been taken to obtain more adequate data on sediment concentrations and loads. The reports[71] to which Mr. Stevens refers describe studies that have been made through a cooperative agreement between the Office of Indian Affairs, Bureau of Reclamation, Tennessee Valley Authority, Corps of Engineers, Geological Survey, Department of Agriculture, and Iowa Institute of Hydraulic Research. Through the efforts of this group, studies were made of the existing methods of sampling and analysis of the samples. Thereafter, improved types of samplers were developed for the collection of depth-integrated and time-integrated samples. With these improved samplers it is possible to obtain accurate information on the average sediment concentration; also the sediment concentration at different points in the cross section. In addition, a study of methods of analysis[72] of particle size led to the development of a bottom-withdrawal tube method that gives promise of being a useful procedure for determining the particle size of suspended and deposited sediment.

TABLE 13.—SUSPENDED SEDIMENTS IN THE COLORADO RIVER BASIN

(Millions of Tons)

Years[a] ending Sept. 30	LOCATION OF SAMPLING STATIONS (SEE FIG. 5)						
	Green River, Utah (1)	Cisco, Utah (2)	Lees Ferry, Ariz. (3)	Grand Canyon, Ariz. (4)	Bluff, Utah (5)	Col. 4 minus Col. 3 (6)	Virgin, Utah (7)
River	Green	Colorado	Colorado	Colorado	San Juan	Little Colorado	Virgin
Years of record	15	15	7	19	15	7	4
1942	31.0	33.8	....	229.8	80.1	.	....
1943	12.8	7.7	72.6	89.9	11.1	17	....
1944	23.2	16.1	77.3	97.8	17.8	21	....
Total[a] Average[a]	355.5 (23.7)	274.3 (18.3)	1,085 (155)	3,626 (191)	631.7 (42.1)	331 (47)	12.4 (3.1)

[a] The total and average values include the years reported in Table 6.

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[71] "A Study of Methods Used in Measurement and Analysis of Sediment Loads in Streams," Reports Nos. 1 to 6, Iowa Inst. of Hydr. Research, Iowa City, Iowa, 1940–1943.

[72] "A Study of New Methods for Size Analysis of Suspended Sediment Samples," Report No. 7 of "A Study of Methods Used in Measurement and Analysis of Sediment Loads in Streams," Iowa Inst. of Hydr. Research, Iowa City, Iowa, 1940–1943.

Data to supplement Table 4, for three years following 1941, are presented in Table 13. The totals and averages given are the results of combining both tables.

Storage of water in a reservoir, as in Lake Mead, affects the chemical character of the water available for use below the reservoir. Studies of the quality of water above, below, and in Lake Mead show that the water released from the lake is more uniform in concentration than is the water flowing into the reservoir; the maximum concentration of the outflow from Lake Mead since storage began has been about one half the maximum concentration of the inflow. The average concentration of the outflow, however, has been somewhat higher than the average concentration of the inflow. Studies of the chemical character of the water in the inflow, outflow, and the lake have shown an increase in the concentration of certain constituents and a decrease in the concentration of other constituents. The increased concentrations are undoubtedly caused by the solution of the soluble constituents of deposits along the lake and the decreased concentrations represent chemical precipitation of certain constituents. The records indicate that, for the period from February 1, 1935, to September 30, 1944, 15,933,000 tons of solids were dissolved and 7,237,000 tons of solids were precipitated during storage in Lake Mead. It seems likely that the rate of solution will decrease as the soluble salts become less easily available, but the rate of chemical precipitation will remain constant and after a period of time there will be a small net decrease in the capacity of the lake because of these chemical processes. The volume of deposits formed by chemical precipitation will be a small percentage of the volume of deposits of suspended sediment from the inflow and it will be many years before the deposits from chemical precipitation will form an appreciable part of the deposits in the reservoir. There is, of course, stratification throughout the lake and the concentrations of dissolved solids in different parts of the lake vary from about 600 ppm to more than 900 ppm. Through regulation of releases of varying quantities of water from the upper and lower levels, it has been possible to maintain the average concentration of dissolved solids in the lake water at a nearly constant value.

Charles Kirby Fox,[73] M. Am. Soc. C. E.—The countless silt-filled reservoirs extending from the Mediterranean area across southern Asia bear testimony that silt has been the bane of irrigationists since before the dawn of history; hence Mr. Stevens is to be congratulated for starting the discussion of this important subject.

The estimates of the silt carried by the Colorado River are many and varied, beginning at about 50,000 acre-ft of silt annually; but the later estimates have shown a decided tendency to "climb." Also, the weights of silt vary from 100 lb per cu ft (specific gravity 1.59; 2,178 tons per acre-ft) to 65 lb per cu ft (specific gravity 1.05; 1,415.7 tons per acre-ft) which would make a large difference in the estimated volume of silt carried (see Table 14).

It is to be admitted, as Mr. Stevens states, that the lower basin of the Colorado is largely composed of soft easily eroded rocks, has many narrow steep canyons, high wind velocities, and is sadly overgrazed; but there is little apparent difference between the watersheds of the San Juan, Little Colorado, and Virgin rivers and the intervening plateau region or, for that matter, the Rio Grande, whose waters carry the highest percentage of silt and which, incidentally, is the only river whose silt load has been measured in a reservoir, except the great Grand Canyon, which some engineers call the world's greatest "silt mill."

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[73] Cons. Engr., Los Angeles, Calif.

TABLE 14.—COMPARISON OF ESTIMATES, SILT LOAD OF THE COLORADO RIVER

Item	Authority	Location	Period	Weight[a]	ANNUAL SILT LOAD (ACRE-FEET)		
					Suspended	Bed	Total
1	Dole and La Rue	Yuma, Ariz.	1895–1914	93	80,000	....	....
2	Mead, Schledcht, and Grunsky	Yuma, Ariz.	(Average)	100	90,000	12,000	102,000
3	Weymouth	Yuma, Ariz.	1909–1922	86	105,000	....	....
4	Davis	Yuma, Ariz.	(Average)	85	113,000	....	....
5	Davis	Boulder Dam	(Average)	85	88,000	....	....
6	Rothery	Yuma, Ariz.	1919–1921	86	94,800	....	....
7	Fortier and Blaney	Yuma, Ariz.	(Average)	....	119,000	19,000	138,000
8	Fortier and Blaney	Boulder Dam	(Average)	85	....	....	137,000
9	Grunsky	Yuma, Ariz.	1911–1927	100	94,500	....	....
10	Grunsky	Yuma, Ariz.	(Average)	100	....	25,000 to 30,000	....
11	Howard	Grand Canyon, Ariz.	1925–1928	86	152,700	....	....
12	Blaney and Howard	Yuma, Ariz.	....	....	94,000	....	....
13	Blaney and Howard	Topock, Ariz.	....	....	124,500	....	....
14	Blaney and Howard	Grand Canyon, Ariz.	....	....	154,500	....	....
15	Stevens	Lake Mead	....	65	....	....	198,000

[a] Dry silt, in pounds per cubic foot.

A series of four diagrams relating to water and silt runoff are presented in Fig. 13. Supporting descriptive data are given in Table 15. In each case the lines in Fig. 13 represent the range of the records quoted. The yearly points are very scattered and in some cases, such as the Little Colorado and Virgin rivers, so few points are available as to make it doubtful if the curves represent the conditions correctly. It will be noticed that the curves relating to the silt runoff in the Colorado River at Grand Canyon and the Rio Grande at San Marcial show a decided tendency to flatten during the periods of higher runoff. This is believed to be correct. In general, the water runoff during the silt-measured years is less than the water runoff during the other measured years.

The values in Table 4, Col. 6, are given as the difference between Col. 4 (Grand Canyon, drainage area 137,800 sq miles) and Col. 3 (Lees Ferry, drainage area 107,900 sq miles) indicating that the additional silt comes from the Little Colorado River. The difference in area is 29,900 sq miles. Mr. Stevens gives the drainage area of the Little Colorado River as 24,400 sq miles, whereas the Water Resources Branch of the U. S. Geological Survey gives it at 22,100 sq miles, a difference of 5,800 or 7,800 sq miles. Other data in the paper indicate that Mr. Stevens uses only the Little Colorado area in computing the silt, which varies from 7% to 14%, this range being outside the limits of the diagram in Fig. 13. It is immaterial which area is used as both stations are in the upper reaches of the Grand Canyon, with its steep grades, high water velocities in tremendous flood volumes, and steep canyon side slopes. It is very probable that the added silt is only that part which has been churned into suspension and that, for all practical purposes, the real bed silt is unmeasured.

TABLE 15.—Water and Silt Runoff; Colorado

No.	River	Place	Area (Sq Miles)		Annual Average				
					Fig. 13(a)				Fig.
					Years		Water Runoff		Years
			Total	Alluvial	Range	No.	Acre-ft	Unit[a]	Range
1	Colorado......	Yuma, Ariz.	244,800	....	1903–1939	37	14,100,000	57	....
2	Colorado......	Grand Canyon, Ariz.	137,800	120,000	1923–1941	19	12,800,000	94	1925–1941
3	Colorado......	Lees Ferry, Ariz.	107,900	95,000	1922–1941	20	13,300,000	123	1925–1941
4	Colorado......	Cisco, Utah	24,100	18,800	1914–1941	22	5,800,000	238	1926–1941
5	Green........	Greenriver, Utah	40,600	35,000	1895–1941	39	5,050,000	125	1926–1941
6	San Juan.....	Bluff, Utah	24,000	21,600	1916–1941	17	2,020,000	85	1927–1941
7	Little Colorado	....	22,100	18,800	1926–1941	16	242,000	11	1926–1941
8	Little Colorado	....	29,900	....	....	....	....	8	....
9	Virgin........	....	934	....	1911–1941	23	161,000	160	1926–1941
10	Rio Grande...	San Marcial, N. Mex.	27,700	14,700	1897–1941	45	1,130,000	41	1915–1941

[a] Runoff in acre-feet per square mile. [b] Ratio of silt runoff; total acre-feet to acre-feet per
[c] Percentage of silt to

It is doubtful if a reasonable estimate of the silt volume can be made from annual runoff data without reference to the ground, slope, and cover conditions and, above all, to the flood periods. It is not believed that bed silt can be measured except in reservoirs.

The late A. H. Koebig once expressed the opinion to the writer that the average silt content of the Colorado River waters was between 8% and 12% (by volume) and sometimes his measurements ran as high as 19% for hours and 41% for short periods. In view of the wide variations of the silt estimates and the practical impossibility of actually measuring the bed silt in a retarding basin, before the construction of the reservoir, the writer made a study of old maps, records of early explorers, logs of wells, subsidences, earthquakes, and other data which might throw light on the volume of the silt deposits and the length of time required for them to accumulate.

About 1936 some 128 maps of the head of the Gulf of California, dating from 1541, were obtained, traced, and carefully analyzed. Most of the maps were drawn to a small scale, but they were as good as other maps of a like period. The latitudes corresponding to Asia Minor (23°) and Spain (36°) were approximately correct but the intervening latitudes were often overstated by a degree or more on the old maps of all explorers.

Twenty-one maps follow the Friars' interpretation of the Indians' grunt they heard on the Iturbi expedition[74] in 1615 and show California as an island at the same time that Iturbi, Capt. Geronio Maquez, Franco Vasco, and others correctly reported that it extended to the vicinity of the present Indio. MacDougall[75] gives a map, by Sykes, of the northern part of the Salton Basin, clearly showing the old tilted shore line with elevations. Data relating to the southern area have been compiled from many sources; the shore line from the Castillo map of 1541 (Castillo spent thirty days surveying that area under instructions to show what he saw) clearly shows the mouths of the Gila River, Salton Basin, and the Laguna Salada.

RIVER AND RIO GRANDE BASINS (SEE FIG. 13)

WATER RUNOFF (ACRE-FT)							SILT RUNOFF (ACRE-FT)					No.
13(*b*)			Fig. 13(*d*)				Fig. 13(*c*)					
	Silt Runoff		Years		Acre-Ft per Sq Mile		Years		Silt			
No.	Acre-ft	Unit[b]	Range	No.	Water runoff	[d]	Range	No.	Acre-ft	Unit[c]	%[e]	
....	....	....	....	....	....	....	....	....	....	....	....	1
16	12,500,000	92	1925–1941	16	12,600,000	92	1925–1941	16	142,000	1.04	1.12	2
16	12,300,000	111	1929–1933	5	11,300,000	104	1929–1933	5	130,000	1.21	1.17	3
16	5,600,000	223	1930–1941	12	4,930,000	206	1930–1941	12	13,000	0.35	0.24	4
16	4,010,000	100	1930–1941	12	3,450,000	87	1930–1941	12	16,900	0.38	0.42	5
15	2,020,000	85	1930–1941	12	1,880,000	78	1930–1941	12	31,200	1.28	1.65	6
16	242,000	11	1929–1933	5	292,000	13	1929–1933	5	41,500	1.87	14.2	7
....	....	8	....	....	....	10	....	....	....	1.38	10.50	8
16	143,000	153	1936–1941	4	153,000	164	1936–1941	4	2,200	2.35	1.16	9
27	1,108,000	39	1915–1940	26	1,105,000	38	1915–1940	26	13,300	0.52	1.27	10

square mile. [c] Ratio of water runoff to silt runoff. [d] Silt runoff in acre-feet per square mile.
water in the runoff.

Four of these maps and a mass of other data distinctly show the Gulf of California as extending to the vicinity of Indio, Calif. Logs of some twenty-five Colorado Delta Wells were located and plotted to obtain an idea of the depths of the deposits. Roughly they show, from the top downward: 125 ft of silt and 40 ft of gravel; 75 ft of silt and 30 ft of gravel; 40 ft of silt and 30 ft of gravel; and 180 ft of silt and 50 ft of gravel—an average of 105 ft of silt and 37.5 ft of gravel. It is certain that every time a great gravel stratum occurs its formation was preceded by a subsidence. The increased grade of the river enabled it to transport the heavier gravels, after which the silt was moved.

Since the Salton Basin was full of water when the Spaniards arrived it is quite probable that the previous subsidence occurred about one century before, or five centuries ago. Mexican and Peruvian correspondents have

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[74] "Coleccion de Doucumentos de Inedious," by Joaquin Francisco Pacheco and Luis Torres de Mendoza, p. 30.

[75] "Salton Basin," by MacDougall, *Publication No. 193*, Carnegie Inst. of Tech., Pittsburgh, Pa.

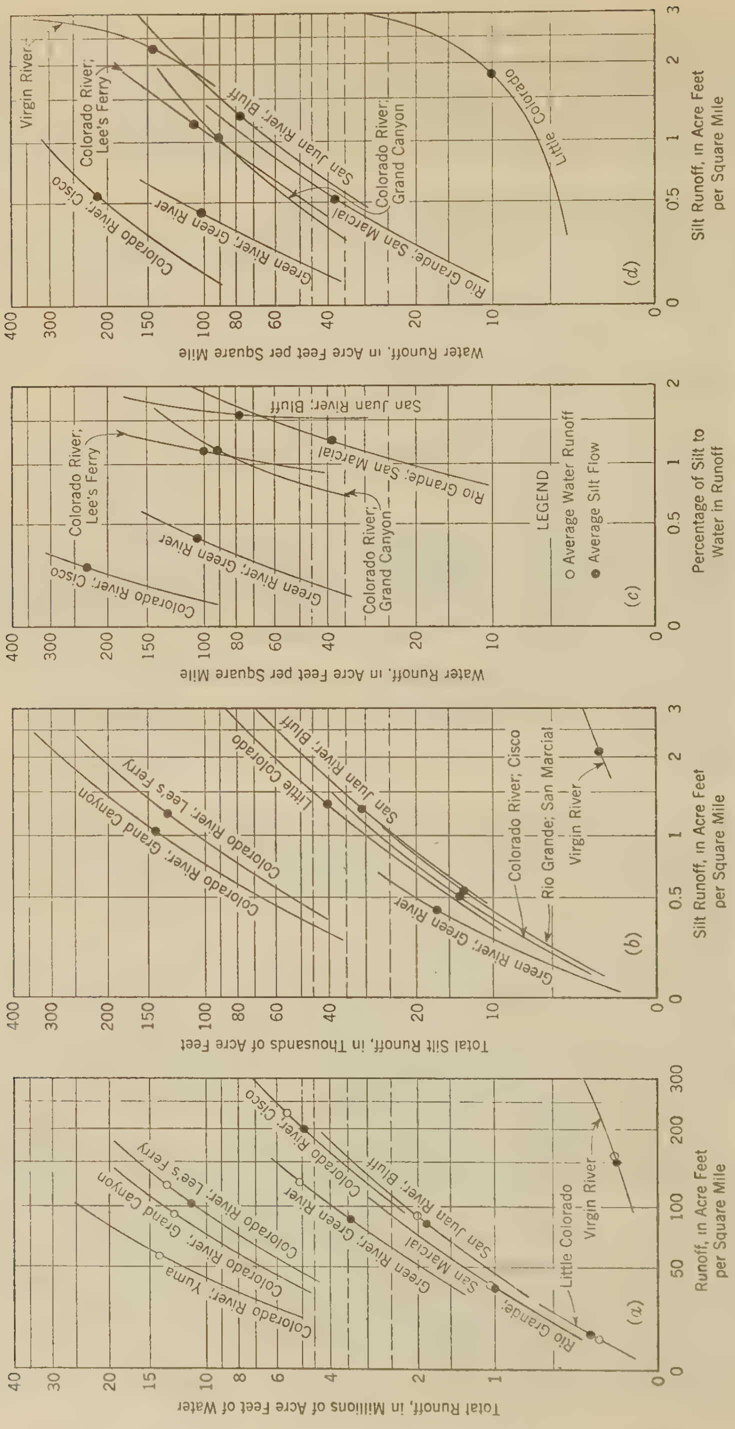

FIG. 13.—WATER AND SILT RUNOFF; COLORADO RIVER AND RIO GRANDE BASINS (SEE ALSO TABLE 14).

supplied the writer with Indian accounts of a very destructive earthquake occurring about this time. Also, in 1635, two earthquakes were recorded as being felt from Canada to Ecuador. It is quite probable that about this time the Salton Basin had lost enough water or weight to cause a readjustment which produced the tilted shore line.

As a check on the effects of the great changing weights some 5,000 to 6,000 earthquakes listed in the major catalogues were located and platted on a large map of the world. Then the centers of the great weight changing areas, with circles of radii—250, 500, 750, 1,000, and 1,250 miles—were platted. These circular arcs contained 21% of the world's land area, of which presumably one third is badly faulted and contained 93% of the plotted earthquakes. The changing volumes of some of the great lakes and the California debris (mining), with the earthquakes, were plotted with the same results.

From the aforementioned data, from a knowledge of approximate dates, from the depths of the silt and gravel deposits, and from an approximation of the areas, it is possible to approximate the average annual silt load of the Colorado River waters at something like a million acre-feet. Of course some silt has been carried to the Gulf and was not measured in the foregoing estimates. The estimate of 1,000,000 acre-ft may be a very rough approximation but is likely to prove more accurate than one which disregards or estimates the bed silt load which many geologists say amounts to from 100% to 400% of the suspended silt load, based on the measurements of many stream debris cones.

The Grand Canyon of the Colorado is about 250 miles long, narrow (the widest part being 12 miles), and deep (the deepest part being 1 mile), with a volume approximating 700 cu miles. The grade from Lees Ferry to the Bright Angel Trail, about 90 miles, averages 15.3 ft to the mile and the remainder of the canyon, 6.5 ft to the mile. These grades would cause very high velocities during floods, and the carrying power of water is supposed to vary with something like the sixth power of its velocity. Many geologists believe the canyon generally follows a series of faults. The Colorado River at Yuma usually digs 2 ft for each foot of rise in water levels.

The Elephant Butte Reservoir began to store water in 1915 and several silt surveys have been made of the reservoir site since then. Similar data should be available for Lake Mead. The actual data most probably will show that the bed silt load has been greatly underestimated due to the effects of the silt accumulated in the Grand Canyon.

Reservoirs on any of the branches or the main river above Lees Ferry will tend to regulate the flow and, if large enough, will eliminate the great floods in the Grand Canyon which are undoubtedly the greatest source of silt, thus greatly prolonging the useful life of Lake Mead.

H. V. Peterson,[76] Assoc. M. Am. Soc. C. E.—In furnishing an estimate of the probable useful life of these two reservoirs, the author has forcefully called attention to the insecure future of numerous western irrigation projects caused by the continuous reduction of reservoir storage capacity through silt deposition. Although the public in general and engineers in particular may claim just pride in the accomplishment of creating such magnificent structures as the two dams in question plus many others, an even greater task remains if the maximum possible benefits from such construction are to be attained.

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[76] Geologist, U. S. Geological Survey, Los Angeles, Calif.

Doubtless there will be disagreement with the author's estimates. An optimist might enumerate factors that would tend to show greater reservoir lives than the estimates given and, just as consistently, one of opposite attitude might find reasons for believing them to be less. That the reservoirs will be filled at some future period is patent to all and, whether the author's estimates are correct to within a year, a decade, or even a century, it is obvious that the usefulness of these great structures is being seriously imparied at an alarmingly rapid rate. What will perhaps startle many readers is the author's statement concerning the rather insignificant effect that man's efforts in land management can produce in prolonging the life of the reservoirs. It is with this phase of the subject that this discussion will deal.

During the past few years much thought has been given to the problem of erosion, and many discussions on the subject have appeared in both the popular and technical press. Doubtless, there is some reason for pride in the accomplishment during this period; but that only a beginner's step has been taken in solving the problem, particularly as it applies to the arid western region, is evident from the author's statement (see heading, "Prolonging the Life of Lake Mead"): "It is doubtful if * * * sediment being carried annually into Lake Mead could be reduced as much as 10% by all the proved practicable methods of land management known today even though lavishly financed." Since silt movement in the Colorado River is typical of much of the arid West, it is implied that similar management plans applied to other streams in regions would have as little effect. The statement casts a dark shadow over the future of western development and irrigation, and certainly leaves uncertain the value of the heritage bequeathed to coming generations; but, in the writer's opinion (much as he dislikes admitting it), there is little evidence at present to disprove or even temper the author's conclusion.

Is this an indictment on engineers and conservators for their failure to develop a practicable scheme for controlling erosion, at least such portions of it as are amenable to man-made controls, or does it mean (as the author implies elsewhere) that, in endeavoring to establish such controls, natural laws will be encountered because the forces of natural erosion so outweigh those attributable to man's activity that the best that man can do in the way of land management can have but minor effect on silt movement? An answer to the question is important. A possible extension of only 10% in the life of the reservoirs furnishes little incentive for investigation or for setting up any program looking toward future reduction of silt movement by the streams.

An examination of the alluvial fill found in the tributary valleys of the Colorado River and the Rio Grande (and, in fact, practically any of the valleys of the arid West) reveals that the basins have passed through alternate cycles of alluviation and degradation at various periods in their past history. Cycles of alluviation result where the products of weathering—including silt, sand, gravels, etc.—are delivered to the valley floor at rates faster than the transporting agencies—including wind and streams, but mainly streams—can remove them, and conversely degradation cycles occur where opposite conditions prevail. Evidence indicates that, although weathering under desert conditions is relatively slow, it is nevertheless constant; and therefore it may be assumed that the cycles resulted from changes affecting the transporting agencies.

The effectiveness of streams would be enhanced by increasing the stream gradient or by increasing the total volumn of flow, either through higher precipitation over the drainage basin or through decreasing infiltration (which increases the percentage of immediate runoff). Because streams are by far the most important transporting agency, any such conditions would be favorable for starting a degradation or erosion cycle.

According to numerous documents, reports, histories, and testimony of many persons still living, the West has passed from a cycle of alluviation to one of degradation since about 1870; or since white man's migration to this region. The resulting change in the landscape has been startling. Available evidence shows that, prior to white man's occupancy, most western valleys presented a smooth unbroken floor in which the stream at ordinary stages followed a meandering, shallow tree-lined or brush-lined channel. Flood flows that exceeded the capacity of the channel spread slowly over the valley depositing silt and doubtless loosing an appreciable percentage of its volume to ground-water percolation. The reports that many of the present barren valleys once carried permanent streams and supported large tracts of luxurious grass that yearly was cut as hay are not inconsistant with this picture, although the contrast with present conditions makes it almost unbelievable.

Today most of these same valleys are scarred by ugly gullies that cut through almost their entire length. Ground water, which once fed the permanent streams and nurtured the grassy flats, has been drained, leaving seared strips of barren waste adjacent to either bank of the gully. Flood flows now concentrate in the channels and instead of loosing silt, as they once did, they take on additional loads from the caving banks with the result that at times the discharge might appropriately be called watery mud rather than muddy water. It is reliably reported that 111 of the 115 major tributaries of the Colorado and Green rivers above Lees Ferry are in this condition. Below Lees Ferry the Virgin River, Kanab Creek, Johnson Wash, and practically every smaller tributary valley is likewise trenched. Tributary valleys of the Rio Grande are in similar condition, the valley of the Rio Puerco being traversed by one of the best known and most spectacular of all the gullies. It is the silt derived from these valleys that is destroying the two reservoirs under discussion and unless some means can be found to prevent the continued movement of this silt any future reservoir must also be destroyed.

It is not the purpose of this discussion to take sides in the arguments concerning the cause of this recent gullying. From the observation and experience gained through a lifetime residence in the West (the past several years of which have been spent in detailed investigations having to do with land and water utilization problems in many of the arid tributary valleys of both the Colorado River and the Rio Grande), the writer has concluded that no single cause can be held responsible for the abrupt change from one cycle to another. Land misuse, principally overgrazing, climatic changes resulting in an increase or a decrease in precipitation, and tectonic movements which bring about steepening of the stream gradient have each been advanced as adequate to institute the change; but the evidence supporting any of them is too indistinct and at times too contradictory to be conclusive. It would appear that a combination of all these processes, with possibly others not yet recognized, might more consistently be considered the basis for the change; and the engineer, in his attempts to prolong the life of the reservoirs, should recognize this fact and plan his treatment accordingly. Under these conditions, what avenues can be pursued and what results can be expected from his efforts?

Controlling silt movement by correcting land misuses is especially appealing to those who subscribe in whole to the theory that abuses instituted by man are entirely responsible for the destructive erosion now taking place. Overgrazing is cited as the most flagrant of these abuses and any one who has inspected large areas in both the Rio Grande and Colorado basins, as the writer has, must admit the seriousness of this accusation. What steps will be necessary to correct the condition and how soon and to what extent tangible results can be expected are questions of vital importance in considering the merits of a corrective program of this nature.

Extremists along these lines have advocated elimination of all grazing, contending that the value to livestock of the forage produced is far less than the protective value of the vegetative growth in stabilizing and holding the soil mantle in place. Such a stand, however, overlooks the political repercussions sure to result from such a violent disruption of the society and economy centered about the livestock industry within these basins just as it ignores many other practical aspects of the problem. As an example of what such a program would involve, the case of the Navajo Indian Reservation may be cited. This reservation is considered to be one of the worst silt producing tracts within the Colorado basin. Almost its entire area is underlain by easily eroded, friable, poorly-cemented sandstones and shales. Scores of washes and arroyos, including such well-known silt carriers as Moenkopi Wash, Polacca Wash, Oraibi Wash, Chinle Creek, Navajo Creek, and Chaco River, drain these unstable areas. Each is deeply gullied and together they contribute thousands of tons of silt annually to either the Colorado, the Little Colorado, or the San Juan rivers. Some 60,000 Indians live within the Reservation, the great majority of whom are dependent on the livestock industry for their living. Overgrazing in many parts of the reservation is apparent and doubtless is contributing in considerable measure to silt movement. To cure this ill by eliminating grazing would mean moving and rehabilitating this entire group—a task approaching in cost and probably exceeding in complexity the construction of another dam on the Colorado River. Such a step is obviously impracticable under present circumstances. Even a reduction in livestock numbers sufficient to correct the most aggravated areas of overgrazing must be accompanied by some provision for maintaining the present standards of living of the Indians; otherwise severe opposition can be expected not only from the Indians but from the public at large.

The problems of other graziers within the two basins must also be appreciated if a program of range improvement is to be successful. In general, livestock operators of the West are individualists of the most independent type. Through skill, daring, hard work, and dogged perseverance they have learned to wrest a living from these forbidding desert reaches, and it is the writer's impression, gained through a rather wide acquaintance among them, that they have no intention of allowing any one to interfere with their chosen method of obtaining a livelihood. They consider their prior interest in any program having to do with river developments equivalent to or superior to the interests of the irrigator, and any plan that seeks arbitrarily to reduce or eliminate grazing within the basins is doomed to failure unless this attitude is fully appreciated and adjustments made for it.

It should not be inferred from the foregoing that stockmen have taken a narrow attitude in regard to range management. The stand taken by most independent operators and owners, and by public officials engaged in administering grazing on the public domain, well agrees with the author when he states (see heading, "Prolonging the Life of Lake Mead: Land Management"):

> "If grazing areas are managed to produce their maximum forage yield the erosion prevention feature will have been provided for automatically; that is, the prime objective in management of range land should be to furnish as much food as practicable for livestock on a sustained yield basis."

These men are fully conscious of the over-all aspects of range utilization. They appreciate that an overgrazed scar is a potential source of debris, capable of destroying reservoirs as well as destroying forage; but they are not of the type who will "go overboard" to cure that scar by retiring from the livestock business. They will cooperate, and they are cooperating, to bring about better range conditions so far as practicable; but they refuse to be concerned over erosion cycles, or to assume responsibility in inaugurating them.

Criteria are lacking for judging the results that may be expected under best range management as it relates to silt movement. Optimists are inclined to point out certain ungrazed areas and experimental plots showing a minimum of erosion as good examples of what will obtain under perfect range management. However, range experience indicates that a long period of careful husbandry will be required before any appreciable reduction in silt movement from a wide area directly ascribable to this cause can be noted.

During the past few years the writer has had the opportunity to inspect a number of fenced plots in which grazing has been excluded for periods ranging up to 10 years. These areas indicate that, while improvement in vegetative cover may be noticeable within a relatively short period, the actual establishment of a growth sufficient to form a protective stabilizing soil cover will be a slow tedious process requiring years, perhaps decades, for its fulfilment. Actually vegetative growth within these desert areas is so dependent on available moisture that other factors are of distinct minor significance. For example, if a person unacquainted with desert vegetation had examined some of the major grazing areas in both the Colorado and Rio Grande basins during the extremely wet winter season of 1940-1941, he would have scoffed at the mention of overgrazing or of inadequate cover protection; but had he returned a few months later the grim effects of drought would have been fully apparent in the sparsity of grass and forage and in the large percentage of the land exposed to the erosive action of wind and rain. Any one familiar with the desert is fully aware that drought is a greater enemy of plant growth than overgrazing. Perhaps by careful husbandry a cover capable of withstanding these periodic and prolonged dry spells can be established over a substantial part of both river basins, but the proof of this remains to the future. To date the prospects look discouraging.

Even assuming that through careful husbandry and management the range will eventually be maintained in the best possible condition consistent with judicious grazing use, the question as to what effect this will have on silt reaching the main rivers still remains unanswered. Unfortunately, no data are available on which to base even an estimate of the effect. So far as the writer knows there is at present no demonstration or experimental plot of any appreciable size within a typical desert portion of either basin where an effort is being made to study and gather data on this phase of the erosion problem. A few preliminary and fragmentary records have been obtained from one or two experiment plots established for this purpose but unfortunately the stations were discontinued before any conclusive evidence could be gathered. Without such information the conservator and the engineer must work under a tremendous handicap.

In theory a combination of structural treatment and range management would appear to offer the most effective means for preventing erosion and silt movement on the valley floors. Structures of the proper type could be used temporarily to store the silt and form it into a suitable seed bed. Thereafter the plant cover would hold it in place and by proper manipulation this cover would continue to trap additional silt until the entire gully system was refilled and healed. Experience, however, does not confirm this theory; at least it has not in a number of rather elaborately treated eroded areas located on the floors of tributary valleys of both the Colorado River and Rio Grande, which have been examined by the writer during the past few years. Except where there has been a continuous effort to maintain, correct, and improve these structural installations, and the exceptions are few, the result obtained in the way of soil stabilization and increased vegetative growth has been highly disappointing. In some instances actual damage and increased erosion have resulted from the structural treatment. Poor construction, poor design, and in some cases poor planning must share the responsibility for the failure of such treatment, along with lack of maintenance and an improper appreciation of the forces involved in the erosion processes.

A successful program for controlling silt, holding it in the valleys and out of the reservoirs, will be found, but it obviously must be based on bolder, more imaginative lines than have heretofore been thought of. Myriads of problems in engineering, geology, botany, agriculture, and the arts of land and livestock management are involved, and available information with which to solve them is woefully meager. Information is needed on the type and design of structures that can be depended on for consistent performance under rigorous desert conditions. More should be known of what can be expected from a coordinated program of structural treatment and strict range management. Methods of range control and rehabilitation need to be studied and perfected. Regions and rock formations producing excessive amounts of silt need to be segregated and analyzed as prospects for special treatment. It has been suggested that erosional experiment stations should be established where treatment programs could be developed and tested on a small scale. The similarity of the erosion problem in so many of these valleys makes this proposal both consistent and desirable. In short, what is needed, as the author stresses, is more information on the entire erosion problem. With investment on the rivers, installed and in prospect, running into hundreds of millions of dollars, it would seem consistent to spend a small part of the development cost in measures aimed at protecting the investment by prolonging its life.

L. C. CRAWFORD,[77] M. AM. SOC. C. E., AND P. C. BENEDICT,[78] ASSOC. M. AM. SOC. C. E.—This comprehensive analysis of available sedimentation data in the Colorado River basin is timely because of the increasing recognition of the need for such studies and data throughout the United States. The general considerations and conclusions have attracted considerable interest, and the indicated paucity of data is of widespread concern not only in the Colorado River basin but elsewhere. In view of the measurements of suspended sediment loads and related field work done in Iowa, with improved sampling equipment, some information concerning this program may be of interest or value in connection with the discussion of this noteworthy paper.

Mr. Stevens refers to certain reports that have been prepared and published as a part of a project involving "A Study of Methods Used in Measurement and Analysis of Sediment Loads in Streams."[79] This project was inaugurated in 1939 under the sponsorship of an Interdepartmental Committee composed of representatives of the Tennessee Valley Authority, Corps of Engineers, Department of Agriculture, Geological Survey, Bureau of Reclamation, and Office of Indian Affairs in cooperation with the Iowa Institute of Hydraulic Research. Experimental models of depth-integrating and point-integrating sediment samplers, designated US D-43 and P-43 sediment samplers, respectively, were completed in 1943. Subsequently, both of the experimental models and several duplicates of the D-43 sampler have been subjected to tests under practical field conditions, alongside of, and in comparison with, other sediment samplers in current use, to determine the practicability of the new samplers, and to evaluate their sampling accuracy with respect to other samplers.[80] The comparative test data submitted in reports by thirteen field offices of various agencies have been analyzed, and a progress report[81] reproduced by the St. Paul Office of the U. S. Engineer Department. The development of a satisfactory bed-load sampler has not, as yet, been undertaken by the Committee.

The results of the interdepartmental investigation, for which the Corps of Engineers and the Geological Survey have been providing the active leadership in recent years, lead to the conviction that the needed extension and improvement of sediment data indicated by Mr. Stevens can be materially aided by the adoption of standard equipment, and methods of measurement somewhat comparable to the commonly accepted procedures[82] of stream gaging.

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[78] Hydr. Engr., U. S. Geological Survey, Iowa City, Iowa.

[79] "A Study of Methods Used in Measurement and Analysis of Sediment Loads in Streams," *Reports Nos. 1 to 5, 7, 9*, Iowa Inst. of Hydr. Research, Iowa City, Iowa, 1940–1943.

[80] "Preliminary Field Tests of the US Sediment-Sampling Equipment in the Colorado River Basin," by P. C. Benedict, mimeographed rept., U. S. Geological Survey, Iowa City, Iowa, April, 1944.

[81] "Comparative Field Tests on Suspended-Sediment Samplers," by M. E. Nelson and P. C. Benedict, mimeographed rept., U. S. Engr. Sub-Office, Hydr. Laboratory, Iowa City, Iowa, December, 1944.

[82] "Stream-Gaging Procedure," *Water-Supply Paper No. 888*, U. S. Geological Survey, Government Printing Office, Washington, D. C., 1943.

Therefore, and as a part of the program to standardize sampling equipment among the interested agencies participating in the joint investigation at the Hydraulics Laboratory in Iowa City, the D-43 sampler was selected for use in connection with the determination of daily suspended sediment loads in cooperation with the Iowa Geological Survey and the Rock Island Office of the U. S. Engineer Department at four stations on the Iowa, Cedar, and Des Moines rivers in Iowa. This field work afforded an opportunity to obtain further information on the field operation of the samplers. A summary of the preliminary reports of sediment discharge (in tons) for the water year 1943–1944 (ending September 30, 1944), at the Cedar Rapids, Iowa, and the Iowa City, Iowa, gaging stations, with drainage areas of 6,640 and 3,230 sq miles, respectively, is as follows:

Month	Cedar River at Cedar Rapids (tons)	Iowa River at Iowa City (tons)
1943—		
October	3,010	3,270
November	2,000	3,670
December	1,760	1,320
1944—		
January	2,950	39,700
February	26,900	115,400
March	104,500	282,100
April	61,100	328,500
May	262,400	1,121,300
June	441,200	561,800
July	68,000	131,700
August	17,600	26,100
September	8,410	4,790
Total	999,830	2,619,650

These quantitative records of suspended sediment loads are based on continuous and systematic field measurements made throughout the year, using the equipment and methods indicated in the reports prepared under the direction of the Interdepartmental Committee. The results show that the yearly suspended sediment load carried in the Cedar River at Cedar Rapids amounted to slightly less than 1,000,000 tons or 150 tons for each square mile of drainage area above Cedar Rapids; whereas, more than 2,600,000 tons, or 810 tons per sq mile for the year 1944, were determined at the gaging station on the Iowa River at Iowa City.

Studies made at the Hydraulics Laboratory of all available information on the density of deposited sediments (which included a review of the data for the Colorado River and the Rio Grande basins) indicate that the average weight of 1 cu ft of deposited sediment from the Iowa River may be taken as 60 lb.[83] On this basis, the suspended sediment measured at Iowa City during the year that included the flood waters of May and June, 1944, would occupy an equivalent space of about 2,000 acre-ft.

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[83] "Density of Sediments Deposited in Reservoirs," by E. W. Lane and Victor A. Koelzer, *Report No. 9* of "A Study of Methods Used in Measurement and Analysis of Sediment Loads in Streams," U. S. Engr. Sub-Office, Hydr. Laboratory, Iowa City, Iowa, 1943, p. 38.

It is interesting to note that, although the characteristics of the Iowa River basin are widely different from those found for rivers in the southwestern United States, an average suspended sediment discharge of 810 tons per sq mile per year was measured during a flood year in comparison with the average sediment inflow (suspended and bed load) of 1,600 and 1,350 tons per sq mile per year obtained by Mr. Stevens in his analysis of the average total load entering Elephant Butte Reservoir and Lake Mead, respectively.

During the floods that occurred in Iowa in 1944 and 1945, sediment samples were also collected with the 50-lb D-43 sampler at three to six stations in the water cross section at a number of regular gaging stations. The samples were taken, in general, at or near the crest after the completion of a current-meter discharge measurement. It is recognized, of course, that data thus provided do not adequately define the flood or sediment transportation potentialities of these or other streams in similar drainage basins. However, this type of field investigation does provide a convenient and inexpensive means of reconnaissance in connection with the location of desirable sediment sampling stations. For example, a sediment discharge rate of 2,030 tons per day per sq mile has been measured for a drainage area of 130 sq miles. The maximum average concentration of sediment by weight in a cross section was 23,600 ppm (2.4%) for a drainage area of 2,800 sq miles.

Such quantitative facts together with the daily suspended loads at regular stations, if obtained over a period of years, will be increasingly helpful with reference to the author's comment concerning the pitifully meager status of available sedimentation data for the proper consideration of a variety of problems relating to agriculture, conservation, land-use practices, and to the future of reservoirs. Furthermore, standardization of equipment and methods will assist in consolidating engineering effort (the Hydraulics Division of the Society has a joint committee on sedimentation in reservoirs) on all phases of sedimentation data, the need for which has been effectively demonstrated by Mr. Stevens.

CHARLES P. BERKEY,[84] HON. M. AM. SOC. C. E.—Although the principles involved in the paper by Mr. Stevens are well known, it is not certain that the implications are fully appreciated by many even in responsible relation to them. The factual data had been long known to the writer, but no statement before this one had brought so forcibly to mind their importance and bearing on long-range planning. This does not apply only to Elephant Butte or Boulder, but to the entire enormous undertaking of recovering lands by irrigation. The United States has virtually set up an empire on impounded and redistributed water. The nation is encouraging development, on a scale never before attempted, of lands that are almost worthless except for the waters that can be delivered to them by the works of man. There is building up, through settlement and new population, a line of industries foreign to the normal resources of the region and they appear to be doomed to a limited life.

The situation reminds one of the "finds" of archeologists indicating that civilizations have flourished while climatic conditions were favorable and have disappeared when the skies withheld rain. The factual accounts of R. Pumpelly [85] and E. Huntington [86, 87] in western Asia abundantly prove that even civilized peoples of some ingenuity could not withstand persistent drought. Eventually, of course, a cure was found and heralded as a new kind of salvation. Men had learned to impound such waters as there were and to carry them to their parched lands. On one occasion the writer found evidence of settlements along the streams which, in an earlier epoch, had been completely obliterated by steadily increasing aridity. Even fishing and hunting, which must have been the chief occupations of these people, had dwindled to the vanishing point while the winds drove drifting sands over their dwelling places until the last man gave up. Still later, in another age and in another valley nearer to the mountains, where mountain streams still carried waters far out into the adjacent desert, they learned to irrigate a little land; but the people of that valley also had to give up when their ditches ran dry. One could trace the faint lines of the distribution system to their garden plots where not a soul remained in this day to till them. In yet another valley its people built a dam to impound the dwindling water supply; but even that failed to save them and the surrounding structures built of granite lie around in ruins. The people who built them are gone.

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[84] Newberry Prof. Emeritus of Geology, Columbia Univ., New York, N. Y.

These instances, to be sure, are not quite parallel to the present effort to reclaim lands; but in capturing silt-laden rivers engineers are facing an equally devastating natural danger. So much silt is impounded with the water that it literally crowds out its assumed long-time promise. This danger is much more imminent than climatic change.

Effort to use water on desert lands is not a new adventure by any means; but a program involving development of a great region—inviting thereby a large new population under conditions that carry elements of certain future destructive encroachment in limited and computable time—that is new. Not only is it new, but in some of the implications it is fairly astonishing. As a public policy probably nothing like it has ever been faced—certainly not on such a scale. The nearest thing in that respect was the settlement of the western high plains in earlier days by people who believed that these dust-bowl lands could be farmed in the same manner as those they came from in the Central Mississippi Valley, and no voice was raised to warn them. That was to be a vast and prosperous empire, too.

For the first time, after reading this paper, the long-range significance of the suffocating effect produced by accumulating silt in all these reservoirs was borne down on the writer. He had been so much taken with the fine things being done that he had not fully appreciated the fact that the program carries elements of destruction sure to bring some kind of ending. It was always evident, of course, that there were severe limitations, but it was too easy to overlook or belittle this element of damage from within.

The experience of founding, in difficult surroundings, settlements which finally grew into influence and power is not new; and neither is their decline, and even their ending. In the past, however, none of them carried along, with the agents that built them up, such relentless elements of destruction as in the present reclamation of arid lands. The astonishing thing is that the life of these relief works promises to be so short. One could forget it if the time vista were indefinite, or if there were promise of a thousand years. In that time most human subsistence and economic lines take new turns and become adjusted; but in some of these projects, typical of the average more or less, the beginnings of decline loom already and will certainly grow into a serious problem in three or four generations: One wonders how many settlers gathering around these projects appreciate what it means.

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[85] "Explorations in Turkestan," by Raphael Pumpelly, Washington Carnegie Inst. of Washington, 1903–1905.

[86] "The Pulse of Asia," by Ellsworth Huntington, Houghton Mifflin Co., 1907 and 1919.

[87] "Civilization and Climate," by Ellsworth Huntington, Yale Univ. Press, 1915 and 1924.

Of course, if one is able to divorce his interest from the future, there is nothing to worry about. In this generation, and the next and the next, an upgrade can be maintained. One can claim (and it is true) that much has been added to the world; but the longer range view in this field, as in many others, is threatened by apparently incurable ailments and this one of slowly choking to death with silt is the most stubborn of all. There are no permanent cures. All are familiar with the simple elements of the matter; but the writer had faced all of that evidence without feeling greatly disturbed. The thing that did most to create a real shock was the figure on Bridge Canyon. The only time that site was visited, after mushing through thirty miles of silt-laden soupy river to get there, such close attention had to be given to the dam site itself that the silting of the proposed new reservoir was almost forgotten; but the paper by Mr. Stevens brings it out, and is a fair and timely reminder, that this is a problem of the first magnitude. Although there are ways of postponing the end stages of these developments, there is no permanent cure, for the final contest transcends engineering skill. It has all the elements of a last stand.

Stafford C. Happ,[88] Esq.—The approximate sediment loads of the Rio Grande and Colorado rivers are fairly well known from existing records; and there is no apparent reason to doubt the order of magnitude of Mr. Stevens' computations of past and probable near-future sedimentation rates. Projection of these rates so far into the future, however, seems questionable unless greater allowance is made for the normal effect of delta growth in reducing the rates of capacity loss. There is also need for clarification of the values given for rates of sediment delivery from major Rio Grande tributaries above Elephant Butte Reservoir.

Delta growth is one of the most important factors in reservoir sedimentation. The history of Lake McMillan is perhaps the best known example. Growth of salt cedar has often been cited as the cause of the remarkable decrease in the sedimentation rate in Lake McMillan, but this was secondary to the effect of delta development. The salt cedar has been very effective as a sediment screen on the top-set delta beds, because there the capacity of the Pecos River channel has been reduced by sedimentation and most of the flood waters are forced overbank through the salt cedar thickets. The rate of capacity loss would have been reduced by the delta development alone, although there is no doubt the salt cedar has been an added influence of considerable importance. Elephant Butte Reservoir had a much larger capacity relative to drainage area and inflow, so that its delta has been deposited more largely within the original reservoir basin, and the rate of capacity loss has been retarded less rapidly; but it is only a matter of time until the Elephant Butte delta will reach such size that its further growth will be largely outside the reservoir proper. Salt cedar was growing abundantly on the Elephant Butte delta in 1936 and 1941, although less conspicuously than at Lake McMillan because interspersed with larger proportions of willows and cottonwoods, and there undoubtedly will be sufficient vegetation to stabilize most of the Elephant Butte delta deposits.

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[88] Geologist, U. S. Engr. Office, Kansas City, Mo.

Surveys in 1941 showed that the effect of delta growth had extended approximately 15 miles upstream from the head of Elephant Butte Reservoir, to the lower end of the Middle Rio Grande Conservancy levees. From 1936 to 1941, the average annual deposition in this 15 miles of valley was about 12.5 million tons. Less complete, but apparently representative, survey data indicate that the 1936–1941 rate of sedimentation in the lower half of this area was about 2½ times as great as that from 1914 to 1936. The rate of increase in sedimentation was probably even greater in the upper half of the area, but data are not adequate for quantitative comparisons. The average rates of sedimentation in the reservoir, according to Mr. Stevens' paper, were about 27 million tons annually from 1914 to 1935, and about 13.5 million tons annually from 1935 to 1940. Suspended loads in the Rio Grande at San Marcial, approximately at the head of the reservoir, were about 11 million tons annually for both the 1935–1940 and 1936–1941 periods; so it is reasonable to compare the 1914–1935 and 1935–1940 rates of reservoir sedimentation with the 1914–1936 and 1936–1941 rates of sedimentation immediately above the reservoir. Such comparisons show that sedimentation in the area of delta influence, upstream from the reservoir, was about 48% of the combined total from 1936 to 1941, but only about 15% from 1914 to 1935.

The rate of sedimentation in the delta area was greatly increased by several channel avulsions between 1936 and 1941, but such phenomena are normal alluvial processes and are especially common on large deltas. They will not be equally effective every year in the future, but, in general, they will occur with increasing frequency and their effect will extend progressively farther upstream. Some of the sediment already deposited above the reservoir certainly will be moved downstream by floods from time to time; but, in general, the proportion of Rio Grande sediment deposited above the reservoir will certainly increase as more of the delta is built above reservoir pool elevation. Mr. Stevens suggests that increased runoff may counterbalance the tendency toward greater deposition above the reservoir, but there is no certainty either of increased runoff or of greater sediment production if the runoff does increase. Most of the Rio Grande water is derived from winter precipitation in the mountains of northern New Mexico and southern Colorado, which contribute relatively little sediment. The summer flash floods on intermittent tributaries in central New Mexico contribute most of the sediment, concentrations as high as 68% by weight having been reached in suspended load samples from the Rio Puerco in 1941; but these summer floods contribute very little of the total annual runoff. Under these conditions an increase in total runoff obviously might not involve any increase in sediment production, and might even be accompanied by a decrease in the total sediment load.

In contrast to such uncertain factors as future changes in runoff, and their effect on sediment transportation, the only question concerning delta growth is the magnitude of its effect. The data now available afford a reasonable basis for estimating this effect. If the 1914–1935 and the 1935–1940 rates of reservoir capacity loss are plotted as decreasing functions of the combined sedimentation in the reservoir and the 15 miles of valley affected by delta growth, and if a smooth curve is drawn through points representing the average values for these two periods, projection of this curve indicates that the "most useful life" of the reservoir as defined by Mr. Stevens, or the time until it will be half filled with sediment, will be about 90 years from the present, or nearly twice as long as that predicted by the straight-line projection. Further projection of the curve would indicate even greater increase in the total life of the reservoir, but projection so far into the future is of doubtful validity. The major importance of the factor of delta growth is fully emphasized by the indicated increase in the "most useful life."

Table 3 appears to show close agreement between suspended load records and the writer's estimates, based on surveys and mineralogical studies of the sediment contributions from five major Rio Grande tributaries above Elephant Butte Reservoir. The close agreement between these figures is fortuitous, however, and possibly misleading. The estimates of contribution to Rio Grande valley deposits should be nearly doubled if allowance is made for sediment carried into Elephant Butte Reservoir. The mineralogical data are not directly applicable to the sources of reservoir deposits, but reasonable extrapolations from available data suggest that the average rate of sediment contribution from these tributaries should be about 2,500 tons per sq mile per year, instead of 1,500 which includes only that part deposited above Elephant Butte Reservoir.

The average contribution computed from suspended load measurements also is probably too low. Apparently this is based on Soil Conservation Service sampling from 1937 to 1941, which was too infrequent and too irregular to provide more than rough comparisons between different streams. The summer flash floods, which obviously carry most of the sediment, were not sampled sufficiently to provide a reliable measure of the total annual load, except possibly for the Rio Puerco during 1941. An average of 1,700 tons per sq mile would indicate that these major sediment contributing areas produced only about four times as much sediment, per unit area, as all the remainder of the Rio Grande drainage area above Elephant Butte, most of which obviously produces negligible quantities of sediment. Such a relation appears unreasonable, in the light of all other evidence; but the suspended load records are not adequate to determine whether or not the rate should be as high as 2,500 tons per sq mile, as indicated by the surveys and mineralogical studies.

It does not necessarily follow that the contribution from direct tributaries to Elephant Butte Reservoir and Lake Mead should be estimated as high as 2,500 tons per sq mile, or that the average specific weight of 65 lb per cu ft for Elephant Butte deposits, as derived by Mr. Stevens, should be raised accordingly. The nature of the terrain suggests, on the contrary, that 1,600 tons per sq mile may be more reasonable for these areas than the true figure, whatever it may be, for the five major Rio Grande tributaries above Elephant Butte. However, there is a serious sediment problem in the Rio Grande valley above Elephant Butte Reservoir, and remedial measures are urgently needed there. It is important that the major sources of Rio Grande sediment be correctly evaluated, within reasonable limits, in order that remedial measures may be wisely concentrated where they will be most effective. Mr. Stevens has recognized that the rates based on surveys and mineralogical studies, as listed in Table 3, do not represent total sediment output, but he has not indicated the magnitude of variance. It should be understood that those rates are probably only about half the actual average rates of sediment contribution to the Rio Grande from the Rio Puerco, Rio Salado, Jemez Creek, and Galisteo Creek.

Data are not yet available for quantitative estimate of the effect of delta growth on the rate of capacity loss in Lake Mead; but there is no apparent reason why the same principles will not apply there as in Elephant Butte Reservoir, although the magnitude of effect may be less.

BERARD J. WITZIG,[89] ASSOC. M. AM. SOC. C. E.—Although the forecast for the future appears pessimistic, the paper presents a logical determination of the theoretical useful lives of two large reservoirs essential to the economic well-being of the Southwest. The lately emphasized concern of the engineering profession over the apparently inevitable decadence of the nation's river control resources is not new, but the mounting accumulation of records indicates that irreplaceable reservoir storage will be destroyed by sedimentation within a few generations.

Merely from a theoretical basis, the writer questions whether it is proper to assume a linear rate of loss of reservoir storage. If such factors as the method of reservoir operation, cyclic variations of rainfall and runoff, and changing erosion conditions are neglected, it seems reasonable to expect that the annual sedimentation rate would decrease with storage and with the effectiveness of the reservoir as a sediment trap. Available data for Elephant Butte Reservoir, plotted in Fig. 3, indicate that the rate of loss to 1940 was not linear. A more optimistic estimate of the life of the reservoir would be obtained if these data were extended by some exponential function, although the aforementioned omitted factors render such an extension highly academic. Linear extension undoubtedly results in the most conservative estimate of the probable useful life of the reservoir.

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[89] Engr. (Civ.), U. S. Engr. Office, Buffalo, N. Y.

The average specific weight of the sediment deposited in Elephant Butte Reservoir, estimated by Mr. Stevens at 65 lb per cu ft, is probably as correct a value as can be determined from available data. However, the validity of applying this value to Lake Mead is questionable in view of other data,[90] which indicate that Colorado River silts vary in specific weight from 37 lb per cu ft to more than 100 lb per cu ft.

Also, an estimate of the specific weight of sediment, based only on observations, probably errs on the low side, and the value thus computed is applicable only to the period of record and to the more or less immediate future. Additional sediment, to be deposited in the future, will impose a consolidating load on earlier deposits; and it is probable that the ultimate average specific weight of the deposits in their entirety will exceed the average for a shorter period during the earlier life of the reservoir. The older deposits will undoubtedly weigh more per unit volume than the newer deposits. Fluctuations of reservoir levels will also cause greater shrinkage in the sediments near the head and in the higher elevations of the basin than in those deposits which are continuously submerged.

Expectation of an increase in the specific weight is supported by the fact that, in 1913, the late W. W. Follett,[4] M. Am. Soc. C. E., used 53 lb per cu ft for silt in the Rio Grande; in 1935, the Soil Conservation Service[91] used 60 lb per cu ft; and Mr. Stevens adopts 65 lb per cu ft. The writer believes that within a few years the average specific weight of sediment in Elephant Butte Reservoir may exceed 70 lb per cu ft and may eventually approach 85 to 90 lb per cu ft. If data are available for the purpose, it would be of interest to learn the estimated ultimate specific weight based on the method proposed in 1943[25] by E. W. Lane, M. Am. Soc. C. E., and Victor A. Koelzer, Jun. Am. Soc. C. E.

The specific weight adopted for estimating future sedimentation rates may greatly influence the predicted useful life of a reservoir. Thus, if the average specific weight approximates 75 lb per cu ft at the end of the useful life, the reservoirs under discussion may actually have useful lives 15% longer than those estimated by the author.

However, differences of opinion on specific weights of sediments or on the estimated lives of the reservoirs do not reach the crux of the problem. The best and most economical reservoir sites have already been, or soon will be, utilized, and "a reservoir begins to die the day it is born."

Mr. Stevens suggests several measures to prolong the useful lives of Elephant Butte Reservoir and of Lake Mead. The writer agrees that land management is probably not a panacea for erosion ills, at least within economic limits, since it would probably be impracticable to reduce erosion rates below the natural norm prior to the influence of man-made conditions. Even if improvement in land management practices could decrease sedimentation rates by 50%, instead of by the 10% estimated by the author, the all-too-short estimated life of a reservoir would merely be doubled.

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[90] "The Silt Problem," by J. C. Stevens, *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 222.

[91] "Silting of Reservoirs," by Henry M. Eakin, revised by Carl B. Brown, *Technical Bulletin No. 524*, U. S. D. A., Washington, D. C., 1939, p. 99.

The possible use of density currents to void suspended silt before deposition through properly designed and located outlets in the dam has not yet been practicably demonstrated, and is generally not applicable to existing dams and reservoirs. Outlets in future dams, where silting is expected to be serious, should be provided to explore this possibility. Further field and laboratory research on density currents in reservoirs appears to be a professional imperative.

The writer does not consider the suggested provision of upstream silt storage as more than a temporary expedient. Such a measure would merely postpone the inevitable reckoning, and would only transfer the destruction of reservoir storage to other locations. The one recommendation for this expedient would be that it might decrease erosion rates and silt loads by decreasing the effective stream gradients. The plan for developing the upstream storage suggested by the author for the Colorado River would extend the useful life of Lake Mead from an estimated 80 years to about 233 years. Probably something like this will be done, but it should be recognized that the problem is thus merely being postponed and not solved.

Hugh Stevens Bell,[92] Esq.—This generation and the next almost certainly will develop every desirable large reservoir site west of the Mississippi River. In effect, this generation is in the midst of reaping a priceless harvest. The crop is not like grain or timber which can be grown again, but like gold or petroleum; and, as Carl Sauer of the University of California, at Berkeley, has stated, when the supply of a particular mineral is exhausted, the harvest of all geologic time has been reaped.[93] Among American engineers, Mr. Stevens is apparently the most thoroughly aware of the fact that this philosophy applies also to reservoir sites and that it places definite obligations upon the generations that harvest them.

Under the heading, "Space Occupied by Deposits," Mr. Stevens writes,

> "For the Lake Mead deposits no better estimate of the specific weight can be made than to use the composite value of $\gamma = 65$ lb found for Elephant Butte Reservoir."

By using this value to calculate the life of Elephant Butte, the author apparently makes the assumption that these sediments will not increase in density with time. By using it to estimate the life of Lake Mead he implies, at least, that it is the future value; and, judging by the assumptions made for Elephant Butte, the present value also.

How much space will be occupied by a given tonnage of sediment is, perhaps, the most perplexing question that must be faced in estimating the life of any reservoir. Consequently, it is unfortunate that no extensive sediment surveys, like those for Elephant Butte Reservoir, have been made at Lake Mead However, the Bureau of Reclamation and the U. S. Geological Survey have collected many pertinent data and there are certain others now available which, although they are not adequate for determining definitely the specific weight of sediment deposits at Lake Mead, furnish a basis for comparing these deposits with those at Elephant Butte. After studying these data the writer has come to the conclusions that: (1) At present, sediment deposits have a considerably lower specific weight at Lake Mead than at Elephant Butte, and (2) the appropriate value of γ for use in estimating the probable life of Lake Mead may be less than 65 lb, the value used by the author. A somewhat detailed analysis of the volume-weight problem and a discussion of sedimentation at Lake Mead during the ten years ending in 1945 are presented in the following paragraphs, accompanied by data and the writer's interpretation of them.

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[92] Soil Conservationist, Cooperative Hydraulics Laboratory, California Inst. of Technology, Pasadena, Calif.

[93] "The Relation of Man to Nature in the Southwest: A Conference," *The Huntington Library Quarterly*, Vol. VIII, No. 2, February, 1945, p. 123.

The Volume-Weight Problem.—There are several reasons why the present sediment deposits in Lake Mead should have a lower mean specific weight than do those at Elephant Butte. The manner in which a reservoir is operated, the conditions under which sediments are laid down, the size of the particles, the age and thickness of the deposits, and other variables, such as the degree of flocculation, influence the rate of compaction and, consequently, the unit weight the deposits will have attained at any particular time. Where obvious differences in these factors are found, they almost invariably indicate conditions favoring greater specific weight at Elephant Butte.

Reservoir Operation.—The extent to which a reservoir is drawn down during normal operation is an important factor in determining the specific weight of deposits. Regarding Elephant Butte, the late Henry M. Eakin wrote,

> "During low-water stages in the average year most of the reservoir basin above the Narrows is uncovered. Sediment deposits dropped here during floods are exposed to air and hardened."[94]

Approximately 40% of the basin lies above the Narrows, so it is probable that at least half of the reservoir bottom is exposed sometime during the average year. Frequently the reservoir is filled to less than 50% of capacity; occasionally it is filled to less than 25%.

The situation at Lake Mead is very different. Since June, 1938, there has never been a time when less than 73% of the capacity to spillway-crest elevation was occupied, or when less than 80% of the basin was submerged. As will be shown in discussing the conditions of sedimentation, this is of greater importance than the ratio of the exposed areas indicates.

Clay and colloidal particles, especially if they are flocculated, form deposits having very low specific weights. When sediments of this kind are brought to the dam by density currents, as they are at both reservoirs, they sometimes are discharged at Elephant Butte but they always are stored at Lake Mead. If the 6.4 million tons of fine material that passed through the outlet works during the period of record at Elephant Butte had been stored, they would have occupied enough space, at 35 lb per cu ft, to have reduced the specific weight calculated by the author from 65.3 lb to 64 lb—a change of 2%.

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[94] "Silting of Reservoirs," by Henry M. Eakin, revised by Carl B. Brown, *Technical Bulletin No. 524*, U. S. D. A., Washington, D. C., 1939, p. 94.

Conditions of Sedimentation.—The Rio Grande, upon entering the reservoir, often spreads out over broad areas covered with willow, tamarisk (salt cedar), and cottonwood. This encourages the deposition of even the finest particles at levels normally exposed to the atmosphere. Mr. Eakin states that the upper part of the basin is covered with above-crest deposits and that the surface sediment is characteristically very fine textured, containing a high percentage of true clay and colloidal matter.[94] Although drying makes little or no difference in the specific weight of deposits of sand, it makes important changes in that of the finer sediments. From data available to them, E. W. Lane, M. Am. Soc. C. E., and Victor A. Koelzer, Jun. Am. Soc. C. E., have estimated that fine sediment, deposited under conditions where it has ample opportunity to dry out, will reach the same specific weight in a short time that it would under less favorable conditions in a 1,000-yr period.[95] At Elephant Butte, fine deposits in the upper basin have ample opportunity to be compacted by drying and, although they may be wet at a later date, and may expand somewhat, they will not reach their former bulk. Furthermore, there is little likelihood that they will be reworked and redeposited at some later date in areas where they will remain permanently submerged and, consequently, have a comparatively low specific weight at any given time.

At Lake Mead, however, the situation now, and for many years to come, is entirely different. The upper 40 miles or so of the basin lie in Lower Granite Gorge down which (see Fig. 14) the Colorado River advanced its delta approximately 39 miles during the first 10 years of storage. Despite this great length, the mean width is somewhat less than 1,000 ft—a feature that encourages the rapid reworking of deltaic deposits and the washing out of a high percentage of the fine particles which are subsequently deposited in areas that are permanently submerged.

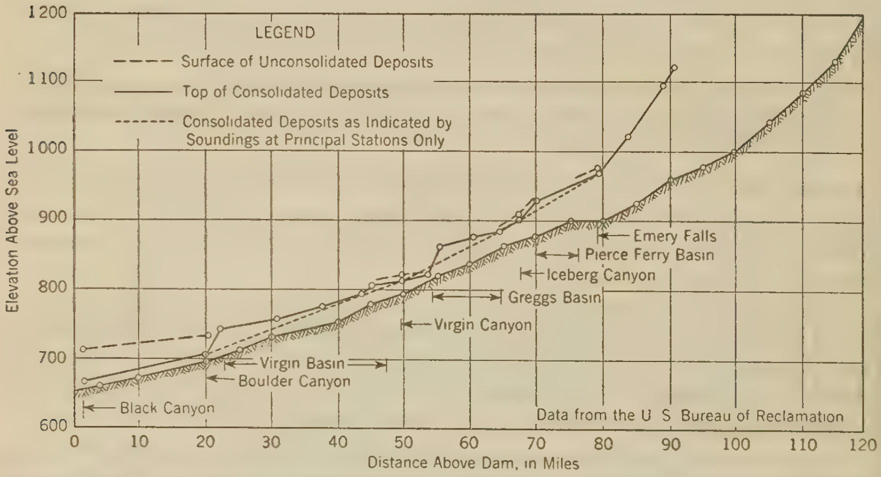

FIG. 14.—DEPTH OF SEDIMENT DEPOSITS AND LOCATION OF DELTA FRONT AT LAKE MEAD, 1938–1945

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[95] "Density of Sediments Deposited in Reservoirs," by E. W. Lane and Victor A. Koelzer, *Report No. 9* of "A Study of Methods Used in Measurement and Analysis of Sediment Loads in Streams," U. S. Engr. Sub-Office, Iowa Inst. of Hydr. Research, Iowa City, Iowa, 1943, p. 51.

In 1941 Thomas C. Mead of the Bureau of Reclamation sent the writer a sediment sample taken 2 ft above water level from the bank of a channel the river had cut through delta deposits about 100 miles above Boulder Dam. An analysis, made after the sediment had been dispersed with sodium oxalate, showed 60% sand ($> 50\ \mu$), 39% silt ($< 50 > 5\ \mu$), and 1% clay and colloidal material ($< 5\ \mu$). Upon drying out, a sediment of this composition experiences an 8% to 10% increase in specific weight, but delta deposits at Lake Mead tend to remain moist because, although the delta is perhaps 40 miles long, its area is only from 6 sq miles to 8 sq miles, including the channel of the river.

At Lake Mead, then, any increase in the specific weight of sediment deposits resulting from exposure to the atmosphere may be considered to be almost negligible for many years to come, whereas at Elephant Butte drying has been and, apparently always will be, an important factor.

Particle Size.—Analyses of more than 600 samples from each stream, taken under comparable circumstances, indicate a strong similarity in the sediment-size composition. The Rio Grande samples contained 22% sand, 37% silt, and 41% clay; and samples from the Colorado River, 28% sand, 69.2% silt, and 2.8% clay. Because the former were deflocculated before analysis and the latter were not, the data for silt and clay content are not comparable, but there are good reasons for assuming that actual differences are small. However, the samples are weighted neither for sediment concentration nor for stream discharge and, consequently, do not give a true picture of the actual composition of the total sediment load. Unpublished computations by Carl B. Brown, Assoc. M. Am. Soc. C. E., show that proper weighting of the Colorado River samples indicates the actual composition was 49.4% sand, 13.3% coarse silt, and 37.3% fine silt, clay, and colloids. This corresponds very closely with unpublished estimates by the U. S. Geological Survey which show that the suspended load passing the Grand Canyon gaging station in 1935 was composed of 45% sand, 15.2% coarse silt, and 39.8% finer material.

Mean Age of Deposits.—In 1945 the mean age of deposits in Elephant Butte Reservoir was about three times that of those in Lake Mead. Apparently time makes little or no difference in the specific weight of deposits of sand and coarser sediments, but becomes increasingly important as the size of the particles diminishes. On the basis of time alone, then, the deposits in Lake Mead should now be less dense than those at Elephant Butte; but, since the life of each reservoir, under present conditions, is estimated to be approximately a century and a half, the eventual difference from this cause is negligible.

The Thickness of Deposits.—Lake Mead is more than two and one-half times as deep as Elephant Butte Reservoir and, consequently, the possible increase in density due to the weight of the overlying sediments should be greater. It has been assumed that the sediments entering the two reservoirs are similar, but it has been shown that the venting of clay and colloidal material transported by density currents at Elephant Butte is a factor that tends to increase the specific weight of deposits there by about 2%.

Since the fine particles are highly flocculated at both reservoirs, it might seem that no difference in specific weight can be attributed to this factor. There is some evidence, however, that flocculation materially decreases the rate of compaction of submerged deposits. If so, it would tend to decrease the mean specific weight at Lake Mead more than at Elephant Butte because practically all fine sediment is retained and remains submerged in the former, but not at the latter.

Summarizing, a comparison of the factors involved indicates that the sediments deposited at Elephant Butte Reservoir should now have a mean specific weight greater than do those at Lake Mead. Eventually this difference will be reduced to an unknown degree by the mere passage of time and, perhaps, to a greater extent by the accumulation of much thicker deposits at Lake Mead. Although a value of $\gamma = 65$ lb per cu ft is now probably too high for Lake Mead, it may be approximately correct for the year 2075 and, therefore, is an acceptable value for use in calculating the life of the reservoir.

Nature and Extent of Sediment Deposits in Lake Mead.—Data on the depth of deposited sediment and on particle size have been gathered for several years at a number of stations in Lake Mead by the Bureau of Reclamation. Fig. 14 shows the depth of sediment deposited at these stations as of February, 1945, ten years after storage began. Fig. 15 is a map of the reservoir and shows the location of the principal sampling stations: Emery Falls, Iceberg Canyon, Virgin Canyon, Boulder Canyon, and near Boulder Dam. All stations are located at narrow places. Because the velocity of density currents tends to be somewhat higher at such locations, the quantity of deposition there is less than elsewhere, as is shown by Fig. 16.

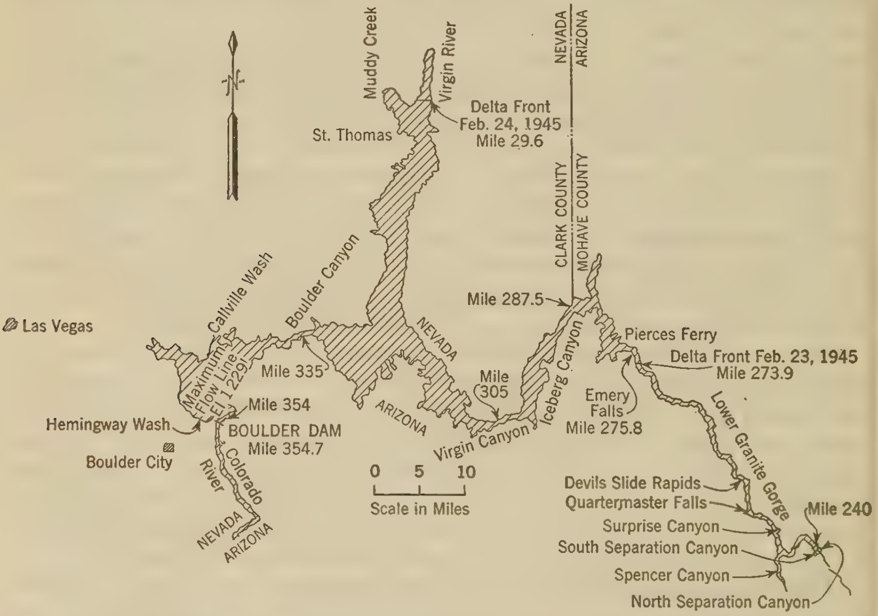
N
Muddy Creek
Virgin River
NEVADA
ARIZONA
Delta Front
Feb. 24, 1945
Mile 29.6
St. Thomas
CLARK COUNTY
MOHAVE COUNTY
Boulder Canyon
Callville Wash
Las Vegas
Mile 287.5
NEVADA
Pierces Ferry
Delta Front Feb. 23, 1945
Mile 273.9
Maximum Flow Line El 1229
Mile 335
Mile 305
Iceberg Canyon
Emery Falls
Mile 275.8
Mile 354
Hemingway Wash
BOULDER DAM
Mile 354.7
ARIZONA
Virgin Canyon
Lower Granite Gorge
Boulder City
Colorado River
0 5 10
Scale in Miles
Devils Slide Rapids
Quartermaster Falls
Mile 240
Surprise Canyon
South Separation Canyon
Spencer Canyon
North Separation Canyon
NEVADA
ARIZONA

Fig. 15.—Map of Lake Mead Showing Location of Principal Sampling Stations and Position of Delta Fronts on the Colorado and Virgin Rivers in February, 1945

By using these data, the contour maps of Lake Mead prepared by the Soil

Conservation Service, the author's estimate of the incoming sediment load, and various bits of pertinent data from the U. S. Geological Survey, a rather general picture can be constructed of sedimentation during the ten years ending in 1945. It also becomes possible to make rough estimates of the probable mean specific weight as of 1945.

Volume of Delta Deposits.—In February, 1945, the delta was approximately 40 miles long and had a cross-sectional area of slightly more than 5 acres at its lower end. A conservative estimate of the volume of the delta can be computed on the basis of a wedge as wide as the river at the upper end of the delta and with a base of 2.3 acres, and two pyramids each having a basal area of 1.35 acres—all with heights of 40 miles. These have a combined volume of 432,000 acre-ft. Irregularities in Lower Granite Gorge, and deposits laid down during high reservoir stages, could account for an additional 20%. The author has stated that the opportunity for extensive deposits above lake levels is lacking when the river joins the reservoir in a narrow canyon. Perhaps it should be emphasized that, although this is true at Lake Mead now, the advance of the delta into the various basins, and particularly into Virgin Basin, will afford a future opportunity which now is lacking. Sediment storage of this kind probably deserves consideration in any attempt to predict the life of Lake Mead.

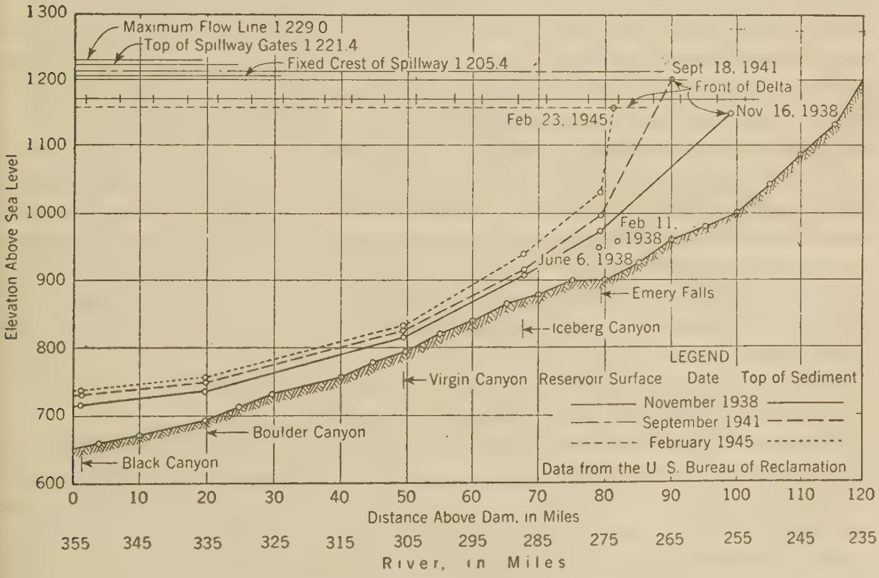

FIG. 16.—SEDIMENT DEPOSITS, LAKE MEAD, IN MAY, 1940

The writer is of the opinion that the delta contained at least 500,000 acre-ft in February, 1945. Although storage had been in progress for 10 years, the delta represented the accumulation of sediment during only 8.75 years. In February, 1945, the toe of the delta had reached a point at which the original bottom elevation was 900. On June 26, 1935, less than 5 months after storage began, the reservoir surface had reached that elevation and was at El. 905.2 as late as April 14, 1936. Never since June 11, 1936, has it been below El. 1000.

During the first 15 months of storage, therefore, sediment of the kind which subsequently formed the delta was being laid down as bottom deposits downstream from the point reached by the toe in February, 1945. On the basis of the estimated volume of the delta, this sediment should occupy approximately 70,000 acre-ft.

Mr. Stevens, in Table 8, estimates the average annual accumulation of sediment in Lake Mead for the period from February 1, 1935, to September 30, 1942, to be 252 million tons. Of this total, according to Table 7, the Virgin and Muddy rivers contribute 15.4 million tons. The contribution of other streams entering Lake Mead below the delta may be expected to increase this quantity to 25 million tons. Their deltas, and delta-like deposits laid down while Lake Mead was filling, are assumed to have a volume of 65,000 acre-ft, making a grand total of 635,000 acre-ft for such deposits during the first 10 years of storage.

Specific Weight of Delta Deposits.—From numerous specific-weight determinations, the late Samuel Fortier and Harry F. Blaney,[44] Members, Am. Soc. C. E., derived an average value of 84.5 lb per cu ft for Colorado River deposits near Yuma and Laguna Dam. Subsequently the U. S. Geological Survey made determinations (reported by Messrs. Lane and Koelzer) on 105 samples of Colorado River channel and flood-plain deposits from Lees Ferry to Yuma. These have an average specific weight of 87.5 lb per cu ft. Of the 105 samples, fifty six were taken between Grand Canyon and Parker Dam and of these forty three seem most likely to represent delta deposits under present conditions. Their average specific weight is 89.4 lb. All had been exposed to the atmosphere; but, since they contained an average of 73% sand, 24% silt, and 3% clay, and few were in locations that would have permitted thorough drying, their specific weight could have been increased only slightly by exposure. Any increase from this source might be more than offset in the delta deposits by compaction attributable to time and the weight of overlying sediment. The writer estimates the mean specific weight of delta deposits at approximately 90 lb per cu ft. At this value the estimated volume, 635,000 acre-ft, would weigh nearly 1.25 billion tons, which is slightly less than 50% of the 2.52 billion tons of sediment of all sizes which are estimated to have reached the reservoir during the first 10 years of storage.

Volume of Density-Current Deposits.—Sediment carried beyond the delta is transported by density currents and must be composed very largely of silt and clay because the velocities are too low, ordinarily, to keep sand particles in suspension. Before reaching Boulder Dam, these currents must pass through a series of basins in which they have an opportunity to spread out and greatly decrease their velocity. It would be surprising if much sand is carried beyond Pierce Ferry Basin. Apparently most of the silt is deposited in Greggs and Virgin basins (Fig. 15), because samples taken since 1942 at Boulder Canyon and Black Canyon indicate that less than 3% of the sediment in the lower 20 miles of the lake is coarser than 15 μ.

Estimating the volume of the deposits beyond the delta is made extremely difficult because the sampling stations are located in canyon sections where velocities are comparatively high and the opportunities for deposition correspondingly low. Almost no data are available for basin areas where deposits are broadest and deepest. Soundings were made in the basins during May, 1940, however, and the data have been used in Fig. 16 which illustrates nicely the difference in thickness of deposits at narrow and at wide sections. The basins actually contained much more sediment than was indicated by data from the regular stations.

The Soil Conservation Service has prepared topographic maps of the entire Lake Mead area on a scale of 1 : 12,000. By using these maps with data from the regular stations the writer calculated that, in February, 1945, deposits below the delta had a volume of 860,000 acre-ft. Because the data are from canyon sections only, the calculated volume is probably at least 50% too low; it should be increased to 1,290,000 acre-ft, of which 70,000 acre-ft are sandy bottom deposits already accounted for in discussing the delta.

Specific Weight of Density-Current Deposits.—Of the sediment that is assumed to have reached the reservoir during the first 10 years of storage, slightly more than 50%, or 1.27 billion tons, apparently has been transported by density currents. If this occupies 1,290,000 acre-ft, as estimated in the preceding paragraph, it has a specific weight equal to 47.5 lb, and the mean for all sediment in the reservoir is 62.5 lb per cu ft.

There is considerable evidence to indicate that the mean specific weight of density-current deposits is less than 47.5 lb. Of the large number of samples taken above Boulder Canyon by the Bureau of Reclamation prior to February, 1945, only two had greater concentrations, three others showed more than 45 lb, four had between 40 lb and 45 lb, and seventy three had less than 40 lb. Of the hundreds of samples taken from Boulder Canyon to the dam, only four had concentrations greater than 30 lb and the heaviest of these showed only 32.75 lb.

Fig. 17(*a*) shows generalized curves of sediment concentration with respect to time and elevation at Boulder Dam (mile 354.7) during the period from May, 1937, through December, 1943; and Fig. 17(*b*) shows similar curves for the station at Black Canyon (mile 353.5) from January, 1944, through June, 1945. Each small circle shows the elevation and date of collection of a sample that was analyzed for sediment content; the samples marked × (Fig. 17(*a*)) in the first few months represent known silty water but analyses are lacking. The sediment concentration, expressed as percentages by weight, was determined by dividing the weight of the sediment after drying by the weight of the original sample. For convenience the concentration is shown also in pounds per cubic foot.

The curves indicate an increasing specific weight at a lessening rate with time, and follow the general pattern that would be anticipated for clay compaction curves. The apparent inconsistency of the 35% curve during 1939 and 1940 probably reflects difficulties encountered in developing a sampler and a sampling technique that could be used effectively in concentrations greater than 30%. The 40% curve is wholly extrapolated, but it is reasonable to assume that such densities now exist. A specific weight of 30 lb per cu ft is approximately correct for 225,000 acre-ft of sediment deposited between Boulder Canyon and the dam prior to February, 1945.

The curve showing the top of the silty water is a succession of peaks rising sharply on the left when density currents reach the dam and subsiding during

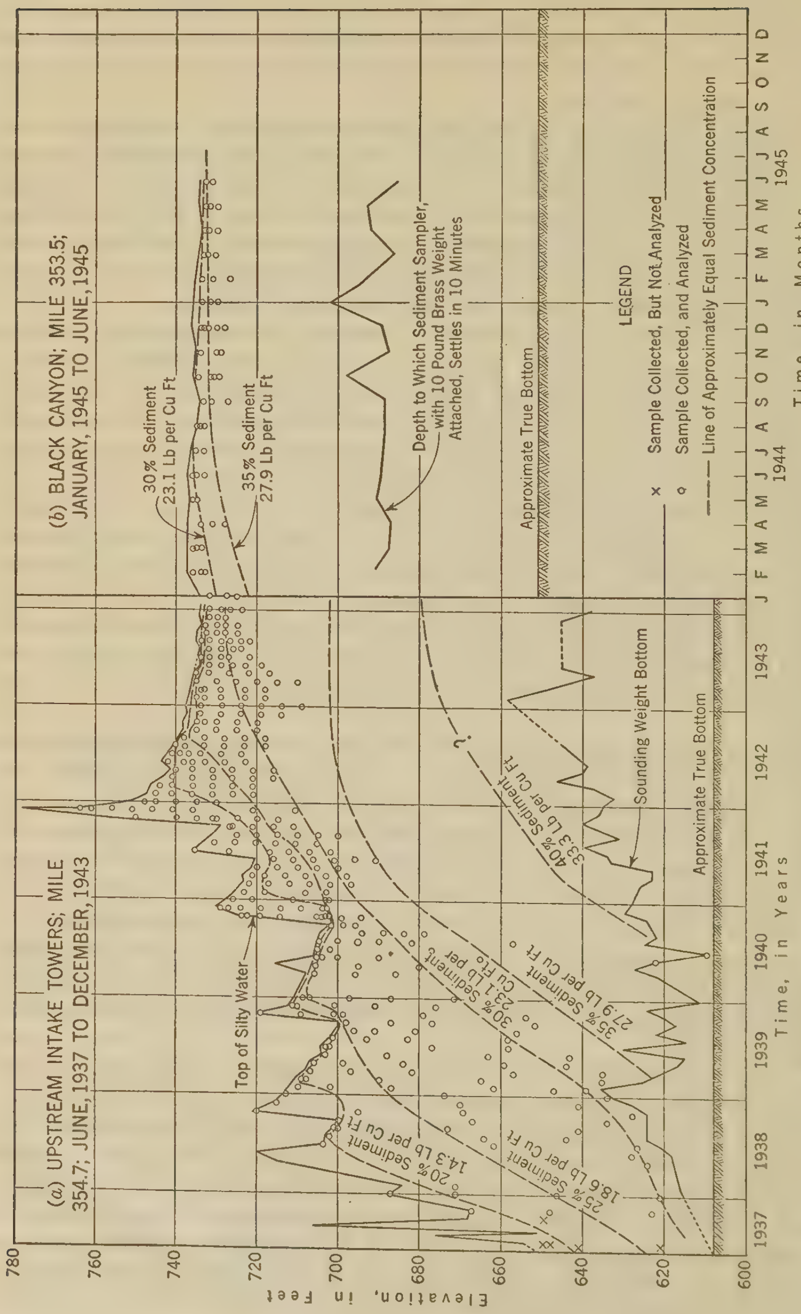

FIG. 17.—SEDIMENT CONCENTRATION IN LAKE MEAD WITH RESPECT TO TIME AND ELEVATION

months devoid of major density flows. The curve of bottom depths is erratic, probably because of differences in the types of sounding weights used and in the time allowed for settlement. Until 1944 "bottom" was the point to which a 10-lb weight would settle in 10 min. The weight usually was brass, but a lead one was used occasionally. Since 1944 the brass weight has been attached to a sampler and the pair has been allowed to settle as usual. This accounts for the greater elevation of the "bottom" as indicated in Fig. 17(*a*).

Since 1938 samples have been taken at several depths each time the Bureau of Reclamation has investigated the sediment concentration at the various stations from Boulder Canyon to the delta. Because of the inability of the sampler to penetrate to all depths, the densest sample taken on each occasion has been assumed to be most nearly representative of the mean specific weight for all sediment deposited at the station being investigated. An average value for each station has then been determined from these selected samples, and the results are shown in Table 16. The specific weight shown for the Boulder Canyon station is approximately 82% of the 30 lb which more complete data seem to indicate is probable for sediment of the same composition near the dam; and the average value increases with distance from the dam. Col. 5, Table 16, shows the average depth the sampler penetrated the deposits; and, since these values are small compared to the total depth, they justify the assumption that the samples are somewhat less dense than the mean value for the sediment at the location. Col. 4 indicates that sediment becomes coarser with distance from the dam; and Col. 5 shows that the firmness of the deposited sediment, as indicated by the depth the sampler penetrated, decreases as the particle size decreases. Deposits at Virgin Canyon apparently do not conform, possibly because that is the point of minimum deposition.

TABLE 16.—SPECIFIC WEIGHT γ OF DENSITY-CURRENT DEPOSITS (ESTIMATES BASED ON PUBLISHED AND UNPUBLISHED DATA FROM THE BUREAU OF RECLAMATION, U. S. DEPARTMENT OF THE INTERIOR)

					SPECIFIC WEIGHT γ (LB PER CU FT)					
Sampling station (Fig. 15) (1)	Location (river mile) (2)	No. of samples (3)	Description of sediment (4)	Average sampling depth (ft) (5)	Observed (6)	Adjusted (7)	$\frac{\text{Col. 6}}{\text{Col. 7}}$ (8)	Average; adjacent stations (9)	Between stations (%) (10)	Col. 9 × Col. 10 (11)
Emery Falls....	275.8	14	15% > 20 μ	4.1	35.5	59.2	0.60	53.6	20	10.7
Iceberg Canyon	287.5	10	10% > 20 μ	6.0	31.2	48.0	0.65	44.9	16	7.2
Virgin Canyon.	305.3	28	2% > 20 μ	1.8	28.9	41.2	0.70	35.6	37	13.2
Boulder Canyon	334.9	28	2% > 15 μ	8.7	24.6	30.0	0.82	30.0	27	8.1
Black Canyon..	353.5 354.7	....	3% > 15 μ	....	..	30.0	....			
Adjusted specific weight γ for all density-current deposits (lb per cu ft)...							....	....	....	39.2

Because firmness and coarseness give a measure of the specific weight, the writer has adjusted the values recorded in Col. 6 by assuming that these represent a percentage, shown in Col. 8, of the probable mean specific weight for the various stations. These adjusted values, which are admittedly only the writer's best guess, are shown in Col. 7, Table 16. The average of these for each pair of adjacent stations is taken as representative of the mean specific weight of the sediment deposited between those stations, as indicated in Col. 9. From his previous estimate of the volume of density-current deposits, the writer distributes them as follows: From the delta to Iceberg Canyon, 20%; from Iceberg Canyon to Virgin Canyon, 16%; from Virgin Canyon to Boulder Canyon, 37%; from Boulder Canyon to the dam, 27%. This distribution yields 39.2 lb per cu ft as the mean specific weight for density-current deposits and 54.4 lb for all deposits in the reservoir.

Summary.—This discussion, perhaps, has shown the hopelessness of the task of finding the correct or even a convincing answer for the volume-weight problem from the data now available for Lake Mead. The writer believes that the answer, for the present, lies somewhere between 54 lb per cu ft and 58 lb per cu ft, possibly rather close to the former. He believes also that 62.5 lb per cu ft, the value suggested by Mr. Blaney in discussing this problem in 1929,[96] might well be used as the basis for calculating the life of the reservoir.

E. W. Lane,[97] M. Am. Soc. C. E.—The future of Lake Mead and Elephant Butte Reservoir is of major importance to many people; and it is therefore very desirable that, as far as it is possible to do so, the future of these reservoirs be accurately predicted. The author should be commended for bringing this important subject up for discussion, and for collating the widely scattered data on which such a study must be based.

From the standpoint of sediment deposits, the future of these reservoirs depends principally upon three factors: (1) The amount of sediment brought down by the streams supplying the reservoirs; (2) the amount of sediment that will pass out of the reservoirs; and (3) the position in which the sediment will be deposited upstream from the dam. The author has collected many data on the amount of sediment which the streams have brought down in the past, and therefore may reasonably be assumed to bring down in the future. As far as the first factor is concerned, the paper seems to be quite complete. The second factor is covered by the assumption that all the sediment brought down will be deposited above the dam and that none of it will pass through to the river below. The third factor is also covered by an assumption—that all the material brought down will be deposited in the reservoir below the maximum flow line. These latter two assumptions are those ordinarily made in studies of the life of reservoirs, but the writer believes that, if a degree of accuracy is desired in the estimate of reservoir life comparable with that obtained in the estimated quantity of sediment inflow, a detailed examination of these two assumptions is necessary to be sure that errors of considerable magnitude are not introduced. The importance of an accurate estimate should justify such an examination, and the writer's discussion will be confined largely to this subject.

The assumption that all the sediment is deposited in the reservoirs is correct for the present conditions, but will become less so as the reservoirs are filled with sediment. As soon as Lake Mead is filled up to the bottom of the gate towers, sediment brought down by the underflows may begin flowing out through the turbines. In fact, this may occur even before the filling reaches this height, since the momentum of the movement of the underflows may cause the underflows to rise considerably upon reaching the dam, because the force of gravity acting on them is so nearly balanced by that acting on the surrounding clear water. As the reservoirs fill, the slope of the bottom will be increased, since the level at the outlet end will remain stationary and the length of the reservoir will be reduced. The velocity of the underflows will therefore be increased and more fine sediment will be transported to and through the dam. Since the underflows in Lake Mead under present conditions are known to carry very large quantities of sediment, the sediment outflow from this reservoir may eventually become a considerable part of the inflowing sediment.

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[96] *Transactions*, Am. Soc. C. E., Vol. 94 (1930), p. 1139.

[97] Eng. Bldg., Iowa City, Iowa.

The discharge of the fine sediment carried by the underflows tends to decrease the filling rate not only because a certain weight of sediment is eliminated but also because the unit weight of the part deposited in the reservoir is greater than the average. Mr. Stevens proposes a value of 65 lb per cu ft. Assuming that this value is correct for the average density of all the inflowing material, if considerable quantities of the fine sediment pass through the dam, the unit weight of the remainder will be somewhat greater than the average (since the unit weight of the fine material is considerably less than the average); and the space that the sediment will occupy will therefore be less than the use of the 65-lb weight would indicate.

The assumption covering the third factor—that the sediment will all be deposited below the level of the maximum flow line—is equivalent to the assumption that the sediment in the water will move as readily as the water alone. This is not the case, and such an assumption may introduce errors of considerable magnitude. The fact that the sediment upstream from a dam is not deposited on a level surface like the water surface of a reservoir, but is deposited on a slope, has been stated by the author. In the case of Bear Creek in California, the deposits above the level of the dam were of the order of 10,000% of the water storage capacity of the basin. Since, above water level, storages of such great magnitude can occur, it appears unsafe to dismiss this factor with a mere assumption. The author states (see heading, "Part I. Elephant Butte Reservoir: Probable Useful Life"):

> "Deposits in the valley above the [Elephant Butte] reservoir have been enormous so that a substantial portion of the water-borne sediments is arrested before it reaches the reservoir."

In the future, as the deposits in Elephant Butte Reservoir extend farther into the reservoir, the proportion of sediment that will be deposited above the water level will increase. This deposit not only will occur within the original area of the reservoir, but will eventually extend many miles upstream beyond this area. The valley above the reservoir is wide and is thickly covered with willows and brush. At times the river breaks into this densely covered area and deposits large volumes of sediment. Although tamarisk growth does not appear to have reached the density of that above the McMillan Reservoir, the growth is reported to be increasing rapidly along the Rio Grande, and before long may considerably retard the sediment inflow into the reservoir. The construction of flood control reservoirs upstream may reduce the sediment inflow further. For these reasons, the writer believes that (assuming that the years of record indicate an accurate average sediment flow) the straight line in Fig. 3, giving the percentage of the capacity of the reservoir occupied by sediment at various times is not justified; that the curve would probably be convex upward, as the portion based on actual records indicates; and that the life of the reservoir would be considerably longer than predicted by the author.

The writer is also unable to agree with the author's statement that, because the river joins the reservoir in a narrow canyon, the upper end of Lake Mead would not aggrade above the reservoir level. When the reservoir is filled, the river will flow the entire length of the reservoir in a channel through the sediment and will have a cross section of the same order of magnitude as it had before the reservoir was constructed. If the deposits are no higher at the upper end of the reservoir than at the dam, the river will have to flow the entire 115-mile length of the reservoir with no fall. Obviously, this will not occur. At a number of places along its course below the Grand Canyon, the Colorado River flows through canyons, similar to those at the upper end of Lake Mead, on a slope which does not differ greatly from the slope where the valley is wider. In times of flood, the bottom of the river may scour deeply but it refills as the flood recedes. When the reservoir is nearly filled with sediment, the slope of the deposits through the canyons at the head of Lake Mead should not differ greatly from that in the canyons below Lake Mead. Because the valleys in the upper end of the basin are narrow, the volume of the sediment deposits above reservoir level in this region will not be relatively as great as in the case of the Elephant Butte Reservoir; but, before the reservoir is filled, the sediment level in the upper end of the reservoir will probably be materially above the high-water level. In any event, there are large areas in Lake Mead, at considerable distances upstream from the dam, where the reservoir is wide and sediment will be deposited to a considerable height above the maximum water level before the reservoir is filled with sediment.

Some deposit of sediment above maximum water level in the reservoir is practically certain to occur eventually in any reservoir into which sediment-laden water flows. Whether or not deposits will build up rapidly enough to cause trouble in a sufficiently short time to be important depends upon the amount and size of the sediment brought in, the volume of storage space available, the condition of confinement of the stream, the fluctuation of water levels in the reservoir, and possibly other factors. Each reservoir is a special case and must be studied in detail to make certain what the result will be.

When such special studies were made, it may even be found that the effectiveness of the reservoir may be destroyed by less material than is necessary to fill the reservoir to the maximum water level, and this condition can result from the same cause as that which produces deposits above the maximum water level—namely, sediment will not move as readily as water. This action will take place to a certain extent in the Virgin-Muddy Creek branch of Lake Mead. The data given by the author indicate that about 7% of the total sediment coming into the reservoir is brought in by the Virgin-Muddy branch and rough computations indicate that about 20% of the capacity of the reservoir is in the Virgin-Muddy branch. Since comparatively little sediment brought down by the Colorado River will flow up into the Virgin-Muddy branch of the reservoir, the Colorado River branch and the reservoir below the junction of the two branches might be filled with Colorado River sediment before the Virgin-Muddy branch is filled. When this occurs, the useful life of the reservoir, from a storage standpoint, will end, since the Virgin-Muddy branch will become a lake, the surface of which cannot be fluctuated by varying the water level at the dam. Therefore, such a lake would have no useful storage. Rough computations indicate that in dry years the flow of the Virgin River may not even supply the evaporation on this lake and that if this occurred the lake would sometimes be entirely cut off from the Colorado River.

Without a detailed study, it is not possible to determine whether the magnitude of the deposits above the high-water level plus the discharge of sediment in the underflow passing through the turbines, will be greater than the space which will not be filled by sediment in the Virgin-Muddy branch. However, sufficient data are available to make a detailed study to determine with reasonable accuracy, not only what the useful life of Lake Mead will be under present conditions, but also what the conditions will be in the reservoir at any future date. The technique for such studies has already been largely developed in connection with investigations of other reservoirs. To explain it in detail would exceed the reasonable scope of discussion. Briefly, it consists of dividing the future record into periods and estimating the volume of the deposits in the various parts of the reservoir for the first period from available data on actual deposits or from the size composition of the sediments. A similar estimate is then made for the second period for the altered conditions of the reservoir brought about by the deposits in the first period. The process involves considerable computation; but, in view of the importance of the conclusions, it should be justified.

The fact that reservoir deposits tend to consolidate with time introduces an uncertain factor in reservoir life determinations. The density value of 65 lb per cu ft observed in the Elephant Butte Reservoir is for sediment with an average consolidation period of about 13 years. If the life of the reservoir is 158 years, as estimated by the author, the deposits would have an average consolidation period of 79 years and should, therefore, be somewhat more dense. Data for estimating this rate of consolidation are not readily available, but considerable new knowledge could no doubt be obtained by the use of the consolidation analysis methods developed in soil mechanics.

F. E. BONNER,[98] M. AM. SOC. C. E.—Another notable contribution to the subject of reservoir siltation in the Southwest is made by this paper. The comprehensive estimate of the silt yield of Colorado Basin from recently collected data is particularly interesting. Although the related forecast of storage depletion is less optimistic than earlier estimates, the difference is not enough to justify widespread alarm. Since ample space has been provided in the past for the writer to expound his views on this subject,[99,100] the present discussion is confined to only one phase which deserves more consideration.

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[98] Cons. Engr., Piedmont, Calif.

[99] *Transactions*, Am. Soc. C. E., Vol 100 (1935), p. 307.

[100] *Ibid.*, Vol. 101 (1936), p. 261.

As alluvial debris accumulates in Lake Mead, the space so occupied is not wholly lost for water storage. Even in that dim, distant day when the lake may be almost filled with gravel, sand, and silt, it will not have ceased to exist as a storage reservoir. Instead, it will have been transformed into a vast ground-water storage reservoir. True enough, the capacity will be reduced, but it will still be substantial. According to the author's estimate, the debris will be consolidated to a specific weight of 65 lb per cu ft. On such a basis, the void ratio will be 60% and therefore the actual water-holding capacity of the reservoir will never be reduced by more than 40% of the original capacity.

The specific yield of the deposit will depend on the composition, but it appears safe to guess that, for long-term withdrawals, it should be equal to at least half the porosity. Therefore, effective storage capacity of 30%, or about 7,500,000 acre-ft, plus several million acre-feet of open storage provided by the gates above spillway crest, will always be available.

This volume of storage combined with that of proposed reservoirs upstream will be fairly adequate, even assuming that all the reservoirs are filled and operating as ground-water reservoirs some 250 years hence—the runoff being diminished by that time (through Upper Basin consumptive use and diversions) to about 50% of the water supply currently available at Boulder Dam. When Boulder Dam and other surface reservoirs become ground-water reservoirs, the very large evaporation losses that now occur will be avoided. Such conservation may be considered very desirable practice in succeeding generations. In any event, practicable solutions of the silt problem will be available to engineers of the future, and present-day developments dependent on regulation of the Colorado River are not necessarily doomed to eventual extinction.

JOE W. JOHNSON,[101] ASSOC. M. AM. SOC. C. E.—In estimating the probable useful life of Lake Mead and Elephant Butte Reservoir, the author has utilized the results of rather extensive suspended-sediment observations. To arrive at a value of total sediment load, the bed load was assumed to be 15% of the suspended load. This estimate of the amount of bed load has proved of interest to the writer, not because of the magnitude of the value assumed, but because it appears that the observed suspended load at the Grand Canyon station may approximate closely the total load in the Colorado River at that point. Sufficient data are available, also, to make a computation of the annual bed load passing San Marcial, N. Mex., on the Rio Grande and thereby check the assumed value of 15% of the suspended load.

In a sufficiently long and uniform stretch of an alluvial stream there exists a relationship between the discharge and the rate of bed-load transportation, and this relationship can be determined by direct measurement or by application of a suitable formula. Except for relatively small streams, direct measurement is impractical; however, this method now appears unnecessary, even on small streams, since the development of the bed-load formula by Hans Einstein,[102,103] Assoc. M. Am. Soc. C. E. The Grand Canyon reach of the Colorado cannot be classed as an alluvial stream; it is a "rock-lined trough" in which velocities are probably high enough to transport (mostly by suspension) that sediment which is delivered to the canyon section by the upstream alluvial tributaries. During high discharges when the larger percentage of the annual load is transported, samples of suspended matter taken throughout the depth, down to as near the bottom as practicable, probably give a reasonably accurate measure of the total sediment load. Since the total sediment load of a stream is not a function of discharge, a continuous sampling program is the only method, known at present, for determining the sediment load entering Lake Mead.

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[102] "Formulas for the Transportation of Bed Load," by H. A. Einstein, *Transactions*, Am. Soc. C. E., Vol. 107 (1942), p. 561.

[103] "Bedload Transportation in Mountain Creek," by H. A. Einstein, *Technical Publication No. 55*, Soil Conservation Service, U.S.D.A., August, 1944.

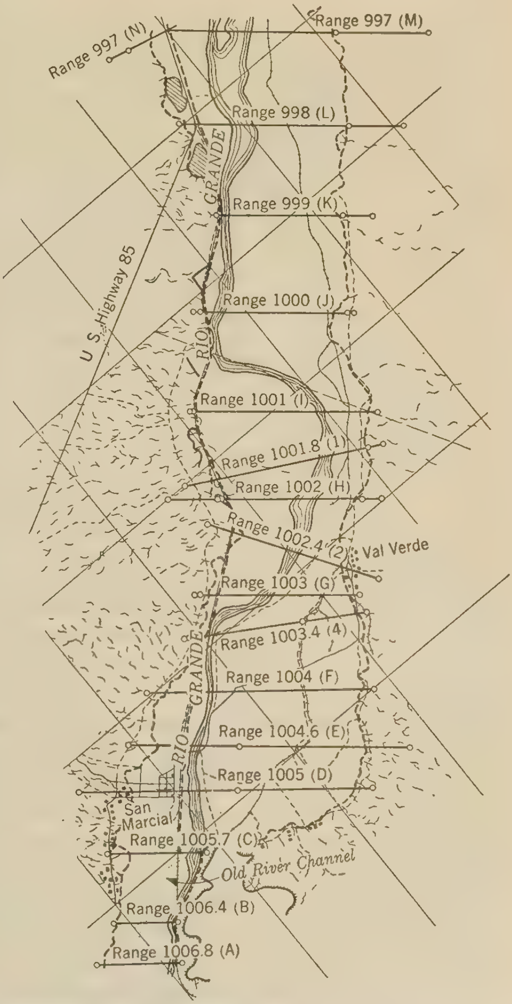
Range 997 (N)
Range 997 (M)
Range 998 (L)
Range 999 (K)
RIO GRANDE
U.S. Highway 85
Range 1000 (J)
Range 1001 (I)
Range 1001.8 (1)
Range 1002 (H)
Range 1002.4 (2)
Val Verde
Range 1003 (G)
Range 1003.4 (4)
Range 1004 (F)
Range 1004.6 (E)
Range 1005 (D)
San Marcial
Range 1005.7 (C)
Old River Channel
Range 1006.4 (B)
Range 1006.8 (A)

FIG. 18.—RIO GRANDE VALLEY AND RIVER CHANNEL IN THE VICINITY OF SAN MARCIAL, N. MEX.

The Middle Rio Grande, on the other hand, is an alluvial stream in which the average annual bed-load transportation can be estimated by application of the Einstein bed-load formula. Computations by this formula require a reasonably straight reach of the river in which the channel dimensions, bottom slope, the mechanical composition of the bed material, and a flow-duration curve are known. A reach in the vicinity of San Marcial appears ideal for such a computation because of the proximity of the San Marcial gaging station from whose records a flow-duration curve can be constructed. Fig. 18 shows a map of the Rio Grande Valley near San Marcial with the sedimentation ranges used in the surveys of S. C. Happ. Bottom profiles plotted from Mr. Happ's report[104] are shown in Fig. 19 for the reach of the valley between ranges 999 (K) to 1006.8 (A). Inspection of Fig. 18 indicates that the reach of the river from just below range 1003 (G) to 1005.7 (C) is relatively straight and uniform in width. For this same reach Fig. 19 indicates that the average bed slope is fairly constant despite the yearly variation in the absolute position of the bed. Cross sections of the various ranges are available to obtain data on channel dimensions. Numerous bed samples were taken in connection with the sedimentation survey and have been analyzed for mechanical composition. The mechanics of computing the average annual bed-load transportation from these available data has been described in detail elsewhere.[105,106] The method can be used to compute the total volume of bed load transported during the period discussed by the author (1915 to 1940), as well as for periods in the future. As in the case of the Colorado, future estimates of suspended load transported by the Rio Grande can be made only by continuous sampling.

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[104] "The Sedimentation in the Middle Rio Grande Valley, New Mexico," by Stafford C. Happ, SCS (unpublished), 1942, Table 3.

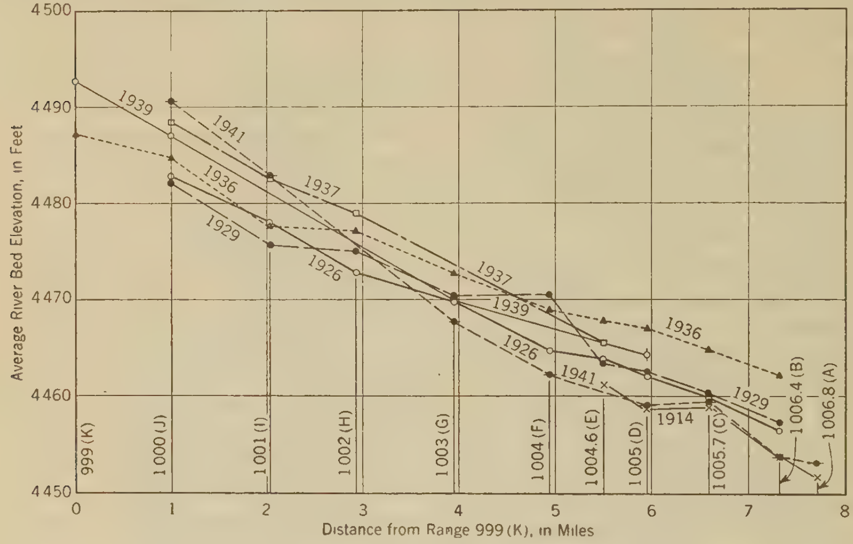

Fig. 19.—River Bed Profiles of Rio Grande at San Marcial, N. Mex.

The foregoing remarks refer to a factor which the author assumes to affect his result by approximately 15%. This is an accuracy which, if approximately correct, is well within the limits of error to be expected in an investigation of this type and probably is on the conservative side, especially for Lake Mead. For the Rio Grande, a computation of the annual bed-load transportation might be desirable to determine the general magnitude of this factor.

It is of interest to note that it is the bed load which is the primary source of the sediments which are deposited in the delta and in the stream channel (above the spillway level) within the limits of the backwater curve.[107] As long as the water level in the reservoir remains near the spillway crest, these deposits of relatively coarse sediments extend farther upstream as time progresses. The existence of a vegetative screen is merely to accelerate the process by trapping some of the finer sediment that otherwise would be carried into the

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[105] "Bedload Transportation in Mountain Creek," by H. A. Einstein, *Technical Publication No. 55*, Soil Conservation Service, U.S.D.A., August, 1944, p. 30.

[106] *Transactions*, Am. Soc. C. E., Vol. 109 (1944), p. 1074.

[107] "Movement and Deposition of Sediment in the Vicinity of Debris Barriers," by J. W. Johnson and W. L. Minaker, *Transactions*, Am. Geophysical Union, 1944, p. 901.

reservoir as suspended load. Large variations of the reservoir level, however, cause readjustments of these deposits. For instance, a flood occurring during a time of low-water level in the reservoir will cause an abnormal quantity of the above-spillway deposits to be moved to below the spillway elevation. Comparison of the bed profiles for the years during the period from 1914 to 1941 (Fig. 19) indicate the nature of the readjustments of these deposits of relatively coarse material.

HAROLD H. MUNGER,[108] ASSOC. M. AM. SOC. C. E.—A vast collection of valuable data, for those who must assume the responsibility for measures to be adopted in the endless battle for the preservation of the irrigation empire of the southwestern states, has been assembled by the publication of this paper. However, a number of factors have been ignored, or treated lightly, which may well prove to be the most significant elements in the long-time handling of the problem. The purpose of this discussion is to call attention to four factors that may seem trivial but that will loom larger and larger as the passing decades continue the inevitable process of reservoir silting.

1. The problem treated in this paper involves many elements that transcend economics. From time immemorial governments have expended huge quantities of man power and material to defend even minor provinces against human enemies or destructive natural forces. There is no sound reason to doubt that history will repeat itself in the Colorado and Rio Grande valleys and that generation after generation of engineers will devote itself to the struggle with only the most perfunctory regard for economics.

2. The storage capacity of a reservoir is by no means "destroyed" even when the basin is "entirely filled" by silt, if suitable details have been provided in the construction of the dam. The voids in silts are seldom less than 20% and often more than 30%. This space is still available for water storage. No great imagination is required to visualize a concrete dam containing in its base an infiltration gallery similar to those occasionally used in water works practice. Horizontal well points, inserted through suitable fittings in the upstream face of the dam, would tap the supply of ground water in the filled reservoir and provide a uniform year-round flow in many streams that, under present conditions, are alternately dry arroyos and sediment-laden torrents.

3. The silt-storage capacity of a reservoir is much greater than the water-storage capacity. Intermittent, silt-laden streams will aggrade their channels to a slope of 1:100, or steeper, above the spillway of a dam. The formation of the delta above Elephant Butte Reservoir illustrates the early stages of this process. Eventually, a lake 20 miles long may be expected to fill to a level of 1,000 ft above spillway elevation at the head of the lake, and the slope will finally extend up the valley until it intersects the stream bed. The available silt and water storage in a properly designed system of reservoirs on the major silt-producing tributaries of a river like the Colorado or the Rio Grande is practically unlimited.

4. It must be remembered that the cost of a system such as the author proposes will be spread over a period measured in centuries rather than in years,

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or even in decades, and that each succeeding generation will measure the cost of construction in its time against the total accumulated value of the empire received from predecessors.

William Mayo Venable,[109] M. Am. Soc. C. E.—The general problem of the disposal of silt created by erosion is involved in this paper; and the problem is not limited to the preservation of storage in reservoirs, although that is important.

Apparently, the only permanent method of preventing silting in a reservoir would be to by-pass the silt so that it cannot settle in the reservoir, because in a reservoir it is impossible to keep the silt in suspension. In most cases, the silt carried by a rapidly running stream is much greater in proportion to the quantity of water flowing during flood seasons than when the water flow is small; in fact, in the case of many large rivers that have their source in mountainous districts (such as the Ohio River), the water in the lower parts of the river is particularly free of suspended undissolved matter in the summertime during low water. Therefore, it is possible to keep silt from accumulating in upstream reservoirs within certain limits by suitable engineering works constructed to by-pass the water only when it is heavily silt laden; and by creating channels that will maintain the silt in suspension until it has passed the main reservoir. Whether this has been suggested, or whether it would be applicable to the Colorado River, is not known. Of course, in some reservoirs, at high elevations where the water is used for power purposes, the outlet from the water to the turbines is so low that silt could not accumulate above the outlet level; and the efficiency for the reservoir, for power purposes, is not destroyed with such accumulation as does occur within the reservoir.

Where the water is to be used for industrial purposes, for water supply, or for irrigation, the removal of the silt from the used water is necessary both because of the industrial purpose to which the water is put, and because the presence of excessive silt in irrigation ditches results in impairing the irrigation system. Thus, even if the silt be by-passed from the main reservoir, it must cause increased sedimentation in the valley below, which would not take long to become manifest in an alluvial valley.

Thus the problem under discussion is intimately connected with such problems as the control of the Mississippi River. All who are familiar with the Mississippi River problems know that, since the settlement of the Mississippi Valley and the placing of levees to prevent overflow of the river in adjacent lands (which are effective except during times of excessive floods and disaster), the silt is carried to the Gulf of Mexico; and the length of the channels from New Orleans to the outlets in the delta have increased many miles during the period of observation. The Gulf of Mexico, a few miles beyond the outlets of the Mississippi, is very deep and the rate at which the delta is being extended is said to be diminishing; but it is being extended, and the valley floor is not being built up. It would appear that, in rivers such as the Mississippi, the lower reaches at all times carry all the silt in suspension that they are capable of carrying; and this is probably true of all other rivers that flow some miles from their outlets through valleys, the floors of which have been produced by the river's own overflow—including the Nile, the Euphrates, and possibly the Colorado. The utilization of the water of the Colorado River at Lake Mead, therefore, must have serious effects upon the valley below where the maximum velocity flow in that vicinity, before the reservoir was constructed, was relatively high—sufficiently high to carry, at all times, all the silt that was washed into the river. By-passing the silt without excessive water would affect the valley and irrigation works.

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If the point in the ocean where the silt finally comes to rest is not extremely deep, it would be difficult to prevent, permanently, the building up of the valley floor by means of levees or other artificial works. Some proposals made in the past are intended for the benefit of future generations rather than for the immediate comfort of the present one. Similarly, it would seem that the effect of providing means for accumulating sediment in one part of the valley and raising the valley floor for long periods of years, and doing the same thing in other portions of the valley, during other periods, would promise to be of sufficient benefit to future generations in various parts of the world to justify such projects being considered seriously by national and international action. As suggested by Herbert Hoover, Hon. Am. Soc. C. E., former President of the United States, agriculture could be restored in the valleys of Mesopotamia if the silt problem were solved, and irrigation reestablished.

Edward Gerald Smith,[110] Assoc. M. Am. Soc. C. E.—The analyses of the futures of two reservoirs, where the sedimentation problem is notably important and on which some of the most complete data on the subjects of sedimentation and closely related retrogression have been recorded, is a thought-provoking treatise, timely because of the proposals pending approval by Congress for complete development of several watersheds where similar problems are faced.

A learned analysis was made by the author to arrive at the conclusions on the probable life of Elephant Butte Reservoir. A historical reference is made to a news article published in 1916,[111] in which it was predicted that Elephant Butte Reservoir would have a service life of 233 years. The records used were computed by the late W. W. Follett, M. Am. Soc. C. E., covering the period from 1897 to 1912. Weight of sediment per cubic foot was determined by sampling started in March, 1916, of deposits formed by the high water of June and July, 1915. Beds for sampling were chosen at various locations to represent average conditions. The results from seventeen samples showed dry weights per cubic foot ranging from a maximum of 101.18 lb to a minimum of 87.90 lb. The average weight was 92.34 lb per cu ft. From these data was determined an average annual inflow into the reservoir of 11,336 acre-ft of sediment. Mr. Stevens' predictions, arrived at in a different fashion, indicate a reservoir life of 158 years, an annual rate of sediment inflow of 17,200 acre-ft, and an average specific weight of 65 lb per cu ft.

This is not the best type of comparison but will serve to introduce the next thought concerning the accuracy of basic data Sediment loads in rivers, as determined by past field methods, are of doubtful accuracy on all heavy sediment-laden rivers, primarily because such methods have been deficient in supplying data to obtain the true mean concentration of sediment in the flow cross section. It is doubtful if there are comprehensive data on any stream showing the distribution of sediment in the portion of flow near the stream bed which would include the saltation and bed loads. This information is especially lacking for the highest 20% of flows when 75% of the suspended sediment load is carried on large rivers increasing to more than 90% for smaller tributaries. Because of the tractive force required to begin bed-load movement the percentage of time and bed-load disparity is even more evident. This lack of information is due largely to field problems. Experiments made in late 1944 and 1945 on the Arkansas River at Van Buren, Ark., and Little Rock, Ark., with experimental devices for attempting to measure the bed load contradicted some important theories of bed-load movement and denied the practicality of using bed-load formulas. However, the experimental nature of the information precludes formation of definite opinions until further data have been obtained and studied. However, there is proof to indicate that significant quantities of gravel as large as from 1 in. to 2½ in. are moved several feet above the stream bed even at moderate stages. With the Texas type sampler, which may sample much less than the true load at high concentrations, many results have been obtained showing sediment concentrations one foot above the bottom five times that at 8/10 depth and higher in the vertical. Similar data have been obtained by other departments on other rivers. These data emphasize the concentrated loads carried near the stream bed. To obtain accuracy comparable to that obtained in measuring river discharge, it is necessary to use procedures more involved and expansive than those required for obtaining discharge, since sediment load is a function of discharge and many other variables.

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[111] "Elephant Butte Reservoir to Serve 233 Years," by R. R. Coghlan and V. E. Lieb, *Engineering Record*, September 16, 1916, p. 349.

Sediment measurements, which are usually taken too infrequently for obtaining reasonably accurate definition of sediment hydrographs, and, concomitantly, sediment loads, must be taken much oftener than required for discharge measurements because of the greater inconsistencies in the character of sediment flow and because of the sharp irregular changes in sediment hydrographs in general.

Test data[112] show that none of the types of samplers tested is without serious deficiencies. Results showed sampling ratios of samplers compared to the depth and point integrations varying from 0.31 to 2.50. Errors are particularly large during periods of high flow and heavy concentration.

The writer draws the conclusion that, in addition to the possibility that field determinations of bed and saltation loads may some day require large additions to past records of sediment determinations to determine total sediment loads, development of accurate samplers and improved field practices also may demand substantial increases to past records. All these findings suggest the possibility that adjustments for basic data being deficient in quantity load could result in a weight per cubic foot of dry sediment in the range of values determined by the 1916 field data.

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[112] "A Study of Methods Used in Measurement and Analysis of Sediment Loads in Streams, Progress Report, Comparative Field Tests on Suspended Sediment Samplers-December, 1944," planned and conducted jointly by TVA, Corps of Engrs., Dept. of Agriculture, U. S. Geological Survey, Bureau of Reclamation, Indian Service, and Iowa Inst. of Hydr. Research, published at St. Paul Dist. (Minn.) Sub-Office and Hydr. Laboratory, State Univ. of Iowa, Iowa City, Iowa.

Prolonging the Life of Reservoirs.—It is an obvious necessity that all feasible measures be adopted for prolonging the life of each project. The need for these measures is emphasized by noting that, in most reservoirs, the greatest quantity of sediment is deposited in the higher pools, especially in the first few generations, where the loss in storage causes a direct and immediate economic loss. In many reservoirs the amortization period may elapse before considerable deposits appear in the lowest portions of the storage. Reviewing the measures proposed for lengthening the life of Lake Mead with a view to applying lessons learned to other developments exposes great weaknesses in all proposals. Of the four measures proposed for prolonging the life of Lake Mead, three have been eliminated as possibilities for significant contribution to lengthening the useful life of a large reservoir. Removal of sediments after deposition is economically infeasible. It is unlikely that density currents in large reservoirs will ever result in carrying significant quantities of sediment. Then too, the material carried may be of the finer silts and clays which would deposit as "bottomset" beds in lower parts of reservoirs and handicap use of reservoir storage less. A large reservoir may be operated for two or more conflicting purposes. To attempt to operate the reservoir for an additional variable in order to utilize density currents for removal of sediment may be infeasible. Land management assumes a position of doubtful value when it appears that the sediment load could be reduced by no more than 10% by most careful applications of the principle. Then too, drought, an uncontrollable force of nature, may exercise more effective control over vegetation than a plan of land management can be expected to accomplish. In addition to the upstream storage, which will be discussed later, a program for bank stabilization would aid materially in preserving downstream reservoir storage. "Notes on Experiments on Meandering and Bed Load Movement"[113] emphasizes the importance of caving banks in conbributing to sediment loads in streams, especially of the larger grain-size sediments which deposit readily in pools. Stabilization of banks is especially necessary in the meandering reaches of river developments.

Upstream storage, as a reservoir storage protection measure, has many ramifications, possibilities, and flaws. In a longer range view, the construction of an upstream sediment-storage basin will lengthen the life of a particular reservoir. In a much longer range view both basins will be full of sediment and all resources in both areas permanently disabled. The point of view depends upon whether the engineer looks into the future in terms of decades, centuries, or larger units of time. In each valley development engineering studies for providing upstream storage involve difficulties in obtaining satisfactory sites close enough to the reservoir to have considerable effect, so that intermediate sediment runoff and retrogression of beds by clear-water releases will not eliminate much of the effect of a sediment-storage project. Clear-water scouring channels will move the bed material, composed of larger grain-size material, which deposits readily in pools. It is necessary to evaluate the actual contribution to the life of the protected reservoir. In most instances, the sediment-storage project must be located so far upstream that benefits become small. Furthermore, the capacity of sediment-storage pools necessary to perform the required quality of sediment removal performance by detention of flows has yet to be determined within reasonable dimensions.

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[113] "Notes on Experiments on Meandering and Bed Load Movement," Mississippi River Commission, U. S. Waterways Experiment Station, Vicksburg, Miss., May 1, 1945.

Benefits below sediment-storage pools are provided by retrogression of channels below dams providing greater channel capacity, and stabilization of channels accomplished by eliminating the supply of material, the latter being pointed out in model tests.[113] However, even these benefits are offset by increased flood and other land damages upstream from the pools caused by aggradation of sediments upstream from the deltas that are formed at the heads of reservoir pools.

CLARENCE S. JARVIS,[114] M. AM. SOC. C. E.—One of the essentials in solving such momentous problems as those now facing the profession is to adopt a realistic attitude concerning reservoirs, instead of maintaining the complacency so prevalent only a few decades ago. For example, the residents of San Marcial, N. Mex., only a few feet above the proposed maximum water level of Elephant Butte Reservoir, and only a few thousand feet upstream, were calmly advised, and seemingly assured, that no harm would come to them from flooding as a result of the storage enterprise. The subsequent aggradation of the river channel necessitated repeated revisions of railroad and bridge elevations, and the eventual displacement of the town from the alluvial plain to the higher mesa—within a decade after the beginning of storage. Likewise, although seemingly authoritative estimates had consigned the dead storage of Lake Mead to the accumulation of sediment for the first 200 years of its normal life, conclusive evidence is presented that the useful span of its existence may be less than two centuries unless countermeasures are adopted promptly and executed with vigor.

Obviously, the bed-load contribution has upset the early estimates for both Lake Mead and Elephant Butte Reservoir. It is well known that the coarser materials constituting bed load may contain such high percentages of voids that one third or even one half of their volumes may still represent usable storage space. Only when finer sediments intrude to form a matrix for the coarser grains or pebbles, or to seal the coarse deposits in lenslike bodies, are such voids rendered useless for further storage. Inasmuch as the fine sediments may be transported readily by density currents, and may thus be withdrawn from the reservoir basin and delivered by canal systems to irrigated land areas, there to increase the fertility and to improve the soil tilth, the outlook is not entirely hopeless. Some of the views expressed in the writer's discussion of "The Silt Problem of the Zuni Reservoir," by H. F. Robinson, are readily applicable today in the light of later experience. For example:[115]

> "Proper manipulation of the outlet gates, * * * together with judicious use of hydraulic pumps and jets from hydraulic giants mounted on a barge for use around the edges of the reservoir * * * reducing the larger deposits of silt and maintaining a turbid outflow, could be adjusted so as to keep

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[114] Cons. Engr., Salt Lake City, Utah.

[115] *Transactions*, Am. Soc. C. E., Vol. LXXXIII (1919–1920), p. 883.

pace with and correct the silting process. The alluvial silt thus distributed upon the irrigated land would be of immense value as a fertilizer; the canal system would have less percolation losses, and there would be less likelihood of the canal breaking than where clear water is used."

Costs and Values.—Regardless of the low cost of the land acquired for a highway, a parking space, or for other construction, it is not uncommon these days to see great mounds of earth representing the topsoil that has been stripped from the surface, pending later distribution on selected areas where fertility and dense vegetal cover are more important than incidental costs. It is quite conceivable that the successive handling of such topsoil may cost hundreds of dollars per acre-foot, far exceeding the original cost of the land involved; yet the values may warrant the high outlay. Likewise, the investment of a few cents (or even much more) per cubic yard in fine silt to be removed by sluicing and by density currents through Lake Mead or other storage reservoirs—even though the original volumes for such storage were provided at the low cost of some fraction of a cent per cubic yard—might prove advisable and profitable in terms of fertility, storage values, and assessed valuations downstream.

Could it be that the author has dismissed the idea of "Removal of Sediments" without due consideration? By way of contrast with this view, most of his comments and conclusions are beyond challenge. The writer has witnessed sediment removal from stock-watering ponds and other small reservoirs by ordinary scrapers, or even by wagon haulage, to choice garden spots, without much regard to cost per acre-foot. The main concern seemed to be the value per cubic yard, which was generally more than a dollar when wage scales were but a fraction of those that now prevail. The usual types of desert soil, with sand as a prominent ingredient, seem to need the fine sediments that are readily conveyed throughout the distributing canal systems. The coarse granules associated with bed load would ordinarily be restricted to construction projects, after screening and washing. Fortunately, these heavy deposits are not often encountered far from the canal intakes; they are not likely to move by natural processes through an extensive reservoir basin. If only the coarser sands, gravels, and cobbles should lodge in the shallow upper reaches of the reservoir, and should provide usable storage space representing some 40% of the original volume (exempt from the usual evaporation losses because of the reduced exposure), it might be feasible to retain some 50% of the original storage volume indefinitely, even though eventually the reservoir might be filled to spillway crest. One of the prime essentials would be the exclusion of fine sediments and ordinary mud from the retained filling. These materials not only would impair the normal void spaces in sand and gravel bars, but also would retard the inflow and outflow of "bank storage." The probable maximum yearly inflow to bank material in Lake Mead is apparently more than 640,000 acre-ft; at Elephant Butte Reservoir, some 370,000 acre-ft; and at Coolidge Reservoir on the Gila River, 144,000 acre-ft—or, compared with total original usable storage, some 2.5%, 17%, and 12%, respectively. Computed on same bases, the maximum yearly withdrawals from bank storage for those same reservoirs (derived or estimated from official records, yet incapable of exact determination at this time) were found to be about 390,000, 190,000,

TABLE 17.—LAKE MEAD,[a] COLORADO RIVER; SUMMARY OF BASIC INFLOW, OUTFLOW, RESERVOIR STORAGE CHANGES, AND APPARENT YEARLY LOSSES IN DESIGNATED CHANNELS AND RESERVOIRS

(All Discharge and Storage Quantities Are in Thousands of Acre-Feet)

Calendar year[b]	Colorado River above Bright Angel, Ariz.[a] (1)	Virgin River above Littlefield, Ariz.[a] (2)	Ungaged drainage[d] (3)	Total inflow to Lake Mead[a] Σ(Cols. 1, 2, 3) (4)	OUTFLOW, LAKE MEAD[c]			INFLOW −(OUTFLOW ±STORAGE CHANGE)		
					Boulder Dam[c] (5)	Storage added (+) or subtracted (−) (6)	Total Σ(Col. 5 plus or minus Col. 6) (7)	Col. 4 minus Col. 7 (8)	Components[d]	
									Evaporation (9)	Storage[e] (10)
(*a*) CALENDAR YEARS, ENDING DECEMBER 31[b]										
1925	12,800	140[d]	140	13,080	12,600	....	....	480	480	− 00
1926	13,600	170[d]	170	13,940	13,300	....	....	540	440	+100
1927	18,200	176[d]	176	18,552	18,100	....	....	452	402	+ 50
1928	15,000	104[d]	104	15,208	14,600	....	....	608	408	+200
1929	19,900	125[d]	125	20,150	19,400	....	....	750	450	+300
1930	12,800	214	214	13,228	12,600	....	....	628	400	+228
1931	6,600	106	96	6,802	6,610	....	....	192	362	−170
1932	15,800	384	184	16,368	16,000	....	....	368	400	− 32
1933	10,000	116	84	10,200	10,200	....	....	000	350	−350
1934	4,186	77	53	4,316	4,316	....	....	000	300	−300
1935	10,616	162	100	10,878	6,270	+3,673	9,943	935	535	+400
1936	12,470	144	130	12,744	5,960	+5,673	11,633	1,111	570	+541
1937	12,540	234	150	12,924	6,018	+5,909	11,927	997	600	+397
1938	15,860	282	180	16,322	6,764	+8,146	14,910	1,412	771	+641
1939	9,090	149	70	9,309	8,645	+ 168	8,813	496	700	−204
1940	8,035	179	80	8,294	7,973	− 240	7,733	561	690	−129
1941	18,790	427	550	19,767	14,890	+3,770	18,660	1,107	600	+507
1942	14,920	187	180	15,287	15,763	−1,050	14,713	574	670	− 96
1943	11,320	179	150	11,649	12,692	−1,370	11,322	327	640	−313
1944	13,380	180	145	13,705	14,625	−1,360	13,265	440	600	−160
1945	12,245	178	135	12,558	12,493	− 582	11,911	647	590	+ 57
(*b*) SUMMARY OF THE CALENDAR YEARS,[b] 1925–1945 (21 YEARS) AND 1935–1945 (11 YEARS)										
21-year:										
Σ	268,152	3,913	3,224	275,281	239,819	....	....	12,625	10,958	....
Mean	12,769	186	154	13,109	11,420	....	....	601	522	....
11-year:										
Σ	139,266	2,301	1,870	143,437	112,093	....	133,830	8,607	6,966	....
Mean	12,661	209	170	13,040	10,190	....	12,166	782	633	....
(*c*) SUMMARY OF THE WATER YEARS,[b] 1925–1945 (21 YEARS) AND 1935–1945 (11 YEARS)										
21-year:										
Σ	268,091	3,889	3,219	275,199	238,307	....	....	12,091	10,767	....
Mean	12,766	185	153	13,105	11,348	....	....	576	513	....
11-year:										
Σ	138,815	2,284	1,840	142,939	109,867	....	134,668	8,271	6,835	....
Mean	12,618	208	167	12,994	9,988	....	12,243	752	621	....

[a] The area occupied by Lake Mead was not so named until February, 1935. The drainage areas are: Colorado River above Bright Angel, Ariz. (Col. 11), 137,800 sq miles; Virgin River above Littlefield, Ariz.—drained (Col. 2), 4,440 sq miles and ungaged (Col. 3), 25,560 sq miles; and (the total) Lake Mead above Boulder Dam (Col. 5), 167,800 sq miles. [b] Calendar years end December 31 and water years three months earlier, or September 30. Records for individual water years are omitted. [c] At Boulder Dam, or the next river station below the dam site. [d] Probable components of Col. 8. These quantities are estimated or derived. [e] Bank storage added (+) or withdrawn (−).

TABLE 18.—CARLOS RESERVOIR ON THE GILA RIVER ABOVE COOLIDGE DAM; SUMMARY OF BASIC INFLOW, OUTFLOW, RESERVOIR STORAGE CHANGES, AND APPARENT YEARLY LOSSES IN DESIGNATED CHANNELS AND RESERVOIRS

(All Discharge and Storage Quantities Are in Thousands of Acre-Feet)

Calendar year[a]	Gila River above Calva, Ariz.[b]	San Carlos River above Peridot, Ariz.[b]	Ungaged drainage[d]	Total inflow above Coolidge Dam site Σ(Cols. 1, 2, 3)	OUTFLOW, COOLIDGE DAM[c]			INFLOW −(OUTFLOW ±STORAGE CHANGE)		
					Drainage area[b] (12,890 sq miles)	Storage added (+) or subtracted (−)	Total Σ(Cols. 5 and 6)	Col. 4 minus Col. 7	Components of Col. 8[d]	
									Evaporation	Storage[e]
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
(a) CALENDAR YEARS, ENDING DECEMBER 31[a]										
1925	180[d]	30[d]	7	217[d]	185	....	185	32[d]	8	+ 24
1926	250[d]	42[d]	8	300[d]	266	....	266	34[d]	9	+ 25
1927	200[d]	35[d]	7	242[d]	211	....	211	31[d]	8	+ 23
1928	115[d]	21[d]	4	140[d]	88	+ 17[d]	105	35[d]	18	+ 17
1929	192[d]	26[d]	5	223[d]	28	+118	146	77[d]	26	+ 51
1930	190	37	7	234	220	− 19	201	33	34	− 1
1931	352	47	9	408	214	+ 97	311	97	40	+ 57
1932	384	42	8	434	279	+ 64	343	91	45	+ 46
1933	166	17	3	186	340	−174	166	20	44	− 24
1934	132	13	2	147	169	− 42	127	20	40	− 20
1935	158	88	18	264	191	+ 37	228	36	36	− 00
1936	149	45	9	203	236	− 75	161	42	44	− 2
1937	321	46	9	376	298	+ 31	329	47	40	+ 7
1938	94	15	3	112	152	− 50	102	10	33	− 23
1939	115	19	4	138	123	+ 2	125	13	33	− 20
1940	220	52	10	282	143	+107	250	32	38	− 6
1941	900	172	34	1,106	240	+662	902	204	60	+144
1942	141	22	4	167	373	−245	128	39	60	− 21
1943	(103)[a]	(30)[a]	(6)[a]	(139)[a]	(365)[a]	(−246)[a]	(119)[a]	(20)[a]	(50)[a]	−(30)[a]
(b) SUMMARY OF THE CALENDAR YEARS,[a] 1925–1943 (19 YEARS) AND 1935–1943 (9 YEARS)										
19-year:										
Σ	4,362	799	157	5,318	4,121	....	4,405	913	666	....
Mean	230	42	8	280	217	....	232	48	35	....
9-year:										
Σ	2,201	489	97	2,787	2,121	....	2,344	443	394	....
Mean	245	54	11	310	236	....	260	49	44	....
(c) SUMMARY OF THE WATER YEARS,[a] 1925–1943 (19 YEARS) AND 1935–1943 (9 YEARS)										
19-year:										
Σ	4,376	792	157	5,325	4,064	....	4,403	922	656	....
Mean	230	42	8	280	214	....	232	49	35	....
9-year:										
Σ	2,195	484	96	2,775	2,079	....	2,339	436	384	....
Mean	244	54	11	308	231	....	260	48	43	....

[a] Calendar years end December 31 and water years three months earlier, or September 30. Records of individual water years are omitted, except the substitution in 1943 which is for the water year ending September 30, 1943. [b] The corresponding drainage areas are: Gila River, above Calva, Ariz., 11,490 sq miles; San Carlos River above Peridot Ariz.—drained, 1,040 sq miles and ungaged, 360 sq miles; and Coolidge Dam, drainage area, 12,890 sq miles. [c] Before 1928, the discharge downstream from the present dam site. [d] These quantities are estimated or derived. [e] Bank storage added (+) or withdrawn (−).

and 37,000 acre-ft, respectively—or, in percentages of original total usable storage, 1.5%, 9%, and 3%. Naturally, the yield from bank storage may continue at nearly the maximum rate for two or more years in succession, keeping pace with the drawdown of reservoir surface. Tables 17 and 18 contain basic hydrologic data to support this discussion.

Taking into account the miles of drillings that must accompany the development of other storage projects to replace those destined to lose most of

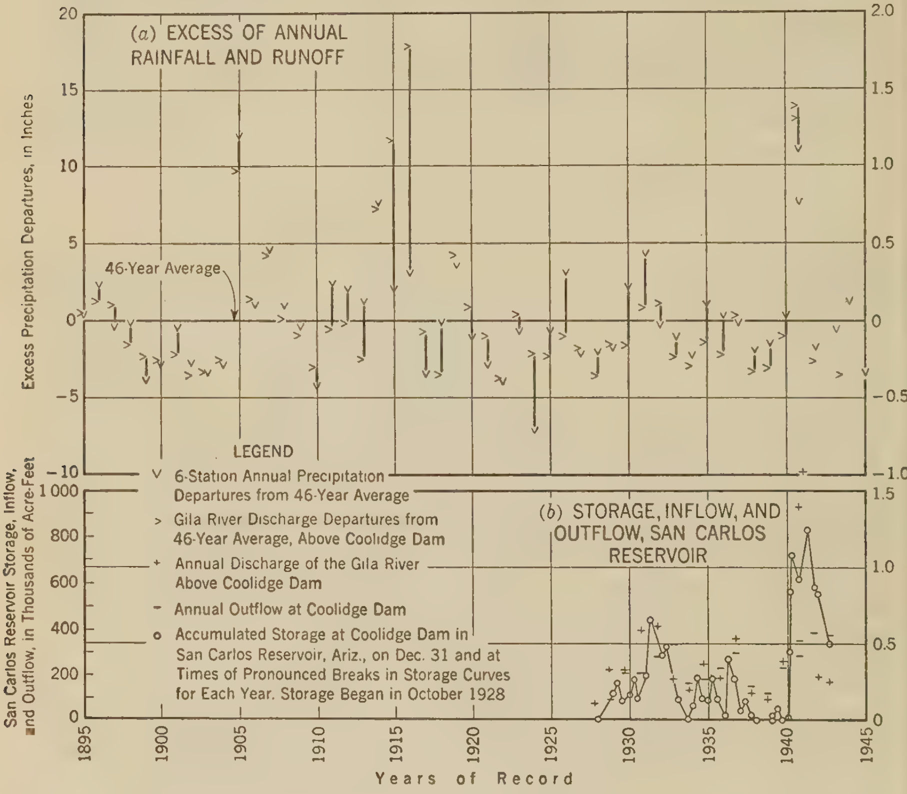

FIG. 20

their capacity within a few decades, engineers could well begin part of the drilling in an effort both to locate and develop lateral underground storage and to supplement the known "bank storage" now functioning. Geologists might thus find dikes or other impervious barriers, beyond which are structures capable of receiving and then releasing great volumes of stored water, if suitable access could be provided by blasting the formations at appropriate depths—in accordance with practice in both petroleum, and other deep-well, drilling. These studies would also locate the critical areas to be protected against sedimentary deposits from the suspended fine materials within the reservoirs (in case the program should be restricted to selected and limited areas) such as the intake or recharge channels leading to the most promising underground storage adjacent to the open water surface of the reservoirs. Too little attention has been accorded to "bank storage" as supplementing the measured capacities of reservoirs.

Probably no precise determination of either reservoir losses by evaporation or diversion to bank storage is possible at major reservoir projects. However, comparison of total inflow and outflow, together with the annual change in storage volumes, leads readily to quantities representing apparent loss, which is actually the evaporation plus the addition to bank storage or minus the volume withdrawn from bank storage. A tentative allocation of these separate factors by the writer shows that, for the 31-yr period of storage record at the Elephant Butte Reservoir, the sum of annual accretions to bank storage was nearly 1,200,000 acre-ft, or slightly more than half of the original storage capacity below spillway crest. On the other hand, the total withdrawals from bank storage were 1,125,000 acre-ft. The apparent annual losses ranged from a maximum in 1920 of 648,400 acre-ft to an actual gain of 47,800 acre-ft, representing the excess of bank storage withdrawal over annual evaporation in 1940. To the mean apparent annual loss of 200,300 acre-ft should be added

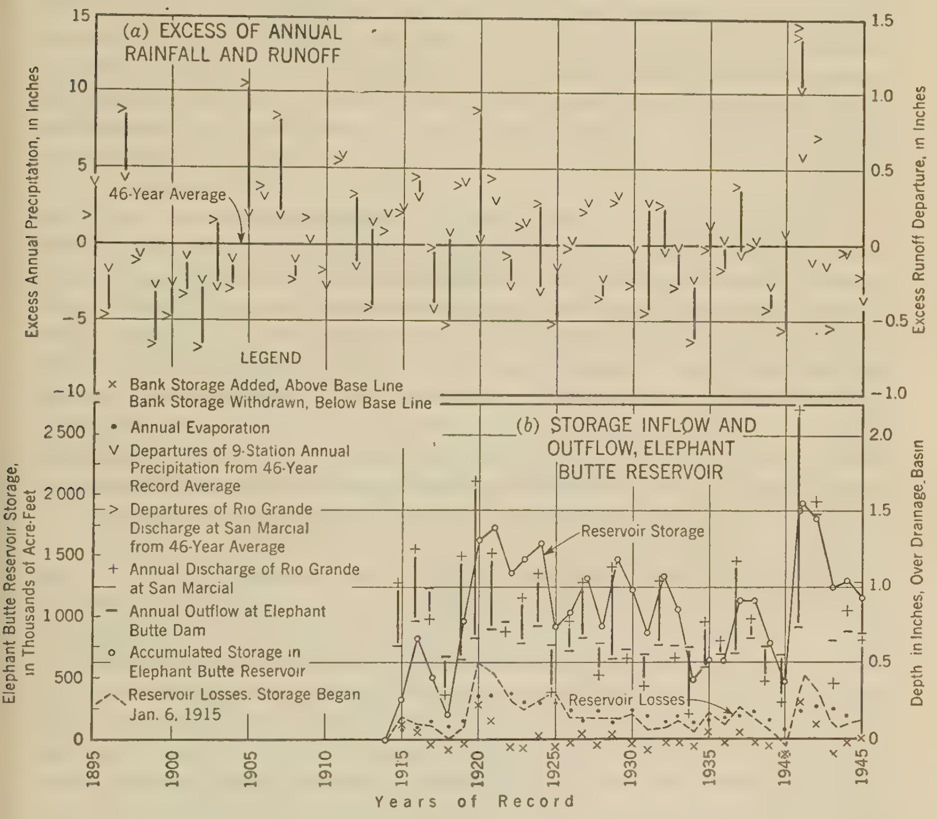

FIG. 21

the mean annual yield from the ungaged 1,750 sq miles of drainage area between San Marcial and the dam—or nearly 7% of the total. If the runoff yield should prove to be as low as 1% of the total, or about 11,000 acre-ft per yr from the ungaged 7%, the mean annual evaporation from the partly filled reservoir would be about 211,000 acre-ft, which conforms very closely with the values plotted in Figs. 20 and 21.

Similar computations showed mean annual evaporation of 633,000 acre-ft for the 11-yr period of storage ending with 1945 for Lake Mead—or fully 400,000 acre-ft per yr for the previous decade. Likewise at Coolidge Dam, the computations showed some 39,000 acre-ft yearly for the sixteen years of record storage, from 1928 to 1943, but only 9,000 acre-ft yearly for the three years immediately preceding the record storage. The volume of bank storage and release was naturally limited to lesser quantities for channels before storage began in the reservoirs.

If the yield from the ungaged 25,560 sq miles of drainage area about Lake Mead had been neglected, instead of being estimated at about 80% of that recorded for the Virgin River at Littlefield, Ariz., the apparent annual losses from river channels between Bright Angel station and Boulder dam site would have been more nearly 250,000 acre-ft for the decade just prior to 1935. Even with that reduction, it appears that the inherent difficulties of stream gaging under such natural conditions as prevail along the Colorado River may account for a part of this factor. It is difficult to conceive more than 100,000 acre-ft actually being lost from the river channel as it was before the existence of Lake Mead; but annual bank storage along the natural channel usually appears to have been from 100,000 to 200,000 acre-ft or more, assuming a high order of accuracy for the river gagings.

Continuing the same assumption and observing the successive annual additions to bank storage for Lake Mead, Elephant Butte, and San Carlos Reservoir back of Coolidge Dam, 1,940,000, 500,000, and 170,000 acre-ft capacities of bank storage seem to have been utilized at the respective sites, representing about 7%, 22%, and 14% of the measured original usable storage volumes. Conservation and possibly extension of those bank-storage capacities, together with from 30% to 40% of the original measured reservoir volumes within the coarser sedimentary deposits of a nearly filled reservoir (assuming that utmost use of density currents and other practicable methods for removing the finer sediments shall be developed forthwith, as outlined herein) might readily insure retention of at least 50% of the originally gaged usable storage volumes indefinitely. To the capacities below spillway crests may be added the storage volumes above the existing crests or above the crests that may later be safely developed—including the accompanying lateral bank storage.

L. Standish Hall,[116] M. Am. Soc. C. E.—This timely paper by Mr. Stevens has rendered a great service to the engineering profession by focusing attention on the relatively short life of reservoirs. The author has perhaps taken a more pessimistic viewpoint than is warranted in order to arouse engineers and conservationists to the need for intensive studies of the problem of sedimentation to the end that a solution may be found for the future of irrigation in the arid West. With the progressive depletion of water-storage resources, it is none too soon to begin the collection of data that will ultimately lead to a better understanding of the problems involved.

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[116] Prin. Hydr. Engr., East Bay Municipal Utility Dist., Oakland, Calif.

The sediments produced by erosion from the Rio Grande and Colorado watersheds are not so extremely large when compared with the size of the respective drainage areas. Many streams in the United States produce, annually, sediment loads per square mile of drainage area from two to five times as great as those produced by the Rio Grande and the Colorado rivers. However, if the sediment load is reduced to terms of annual runoff, probably the quantities produced on these two streams would rank well up on the list.

The action of running water as a modeling agent in changing the earth's surface has long been recognized in geology. It must be expected that in time all reservoirs ultimately will be filled with sediments, whether the time involved is 100 or 10,000 years. A survey of many of the reservoirs within the United States made by the Soil Conservation Service, and reported by Carl B. Brown,[117] indicates that generally only one half of the nation's reservoirs have a life of more than 100 years. In the case of Lake Mead and Elephant Butte reservoirs, the quantities are more impressive due to the size of the reservoirs and the extent of the drainage areas involved. If the economic development based on the irrigation of land in the arid western states were to be terminated within a none too distant span of time, measured in terms of the anticipated economic life of the West, the future of this area would indeed be dark. However, the problem is not quite so serious as it might appear, and it is believed that by proper control and conservation methods the production of irrigation water can be extended into an almost indefinite future time.

Production of Sediment by Erosion.—All streams erode and thus transport great quantities of weathered rock fragments and deposit them elsewhere along their courses. This is a natural geologic process. The most that man can hope to do is to control the process with a view of maintaining normal erosion at a minimum, and not by his works and activities to accelerate the process. Under natural conditions, eroded debris transported by streams is deposited into lakes, dry basins, or the ocean. When man creates a reservoir by interposing a dam in a stream channel, he at once creates a trap for the collection of water and wind-borne sediments. Streams loosen material along their banks by the impact of the water against relatively weak rocks, soil, and rock mantle. The heavier particles are rolled or bumped along the stream bed, while the lighter ones are swept along by the eddies and currents of the stream and are carried suspended for long distances until deposited in the relatively still water of the reservoir. Abrasion of the stream bed by the larger particles, carried along with the flowing water as part of the bed load, increases the total sediment carried by the stream. The particles themselves are reduced in size by collision with each other and with the bedrock. Thus, the erosive effect of the sediment is cumulative, increasing as the flow proceeds downstream until a saturation of the sediment load is reached.

The transporting capacity of the stream, or the size of particles that can be carried, depends on the velocity of the water at a given point along its course.

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[117] "The Control of Reservoir Silting," by Carl B. Brown, *Miscellaneous Publications No. 521*, U. S. D. A., 1943, p. 20.

In general, the maximum size of a particle that can be picked up from a position of rest by streams at a flood stage depends on the square of the velocity; but, once in motion, the transporting power of the stream on particles varies with the sixth power of the velocity. The rate of abrasion also varies as the square of the velocity, as a general rule. Thus, the movement of sediment is greatly increased when the stream is at flood stage and moving at its maximum velocity. On major streams and large tributaries having flat slopes, the sediment carried into the main channel by steep tributaries will be dropped near the junction of the two streams, resulting in aggradation of the main channel. A sedimentation of downstream reservoirs in such cases is caused by a reworking of the sediments carried into the stream channel. Because of the difference between the abrasive and transporting power of a stream, it is difficult to move all the deposited material once aggradation of the channel has started, as in the case of the Rio Grande. However, when the slope of the main channel is steep, the sediment carried into it by the tributaries will be carried on by the velocity of the current, and the dynamic action of the water will cause excessive bank and bed erosion by abrasive action. These two classifications of sediment transportation require different treatment in their control.

The picking up of material from the stream bed depends on the drag, which varies with the depth of the water and the slope of the stream bed. Hence, it may be inferred that the usefulness of a reservoir as a silt trap is not terminated with the filling of its water-storage space with debris. The increased width of the aggraded channel decreases the average depth of water, and thereby reduces the drag. Furthermore, if the size of the particles of sediment is such that the gradient of the stream is reduced below its original slope after the filling of the reservoir, the drag is further diminished. This points to the conclusion that the building of several reservoirs on a watershed, such as is suggested by the author as a possible development on the Colorado River, should have a progressive effect in diminishing the rate of erosion. From this, it may be concluded that the life of the reservoirs built to reduce the rate of sedimentation of Lake Mead may extend the life of the total reservoir system for a very appreciable period of time beyond that estimated by the author.

The effect of aggradation in reducing the sediment deposits can be compared from the records given in the paper for Elephant Butte Reservoir and Lake Mead. The author has estimated an annual sediment deposit from the Colorado River of 198,000 acre-ft from a drainage area of 167,800 sq miles above the Boulder Dam. This is equal to an annual sediment deposit of 118 acre-ft per hundred square miles of drainage area. The record of silt deposition in Elephant Butte Reservoir during a period of 25.75 years shows a total deposit of 442,900 acre-ft from 25,923 sq miles of drainage area. This is equal to an annual sediment deposit of 66 acre ft per hundred square miles of drainage area. These two adjacent drainage areas, having rock formations of a similar character and climatic conditions practically equivalent, show a large difference in the rate of reservoir sedimentation. The reason for the lower rate of deposition in Elephant Butte Reservoir has been explained by the author. He states (see heading, "Probable Useful Life"): "Deposits in the valley above the reservoir have been enormous so that a substantial portion of the water-borne sediments is arrested before it reaches the reservoir." It is further stated that the aggradation of the lower part of the valley above San Marcial, N. Mex., started prior to the construction of Elephant Butte Reservoir. The Soil Conservation Service has estimated that the deposits on the valley floor between Santa Fe Creek and San Marcial have averaged 12,000 acre-ft per yr. Assuming that the rate of aggradation of the valley has been uniform during the period from January 1, 1915, to September 30, 1940, while sediment was being deposited in Elephant Butte Reservoir, a total of 310,000 acre-ft of sediment must be added to the deposits in the reservoir to bring the gross erosion from the drainage area to a total of 753,000 acre-ft. This is equal to an annual sediment deposition of 113 acre-ft per hundred square miles of drainage area, or approximately the same as that estimated by the author for the Colorado River.

At many places along the inner gorge of the Grand Canyon of the Colorado, the walls are so steep that blocks of rock falling from the cliffs tumble into the water to be slowly swept downstream and ground up. Even where rocks do not fall directly into the river, either in the Grand Canyon or along the precipitous walls of other canyon sections of the main river or its tributaries, they fall on talus slopes extending to the river's edge. These slopes are gradually eroded and the material ground up and moved downstream by the waters of the Colorado. The building of dams for the storage of sediment along the river and its main tributaries would not stop the movement of sediment from the smaller tributaries into the main channels; but it would have the effect of keeping material originating in the main canyons in place and prevent its movement downstream. Hence, it is very probable that the construction above Boulder Dam of storage for sediments on the Colorado River will result in an aggradation of the bed of the river canyon between these various dam sites so that the movement of sediment originating in the canyon will be greatly reduced, with the result that the life of Lake Mead may be increased at to least twice the author's estimate.

In addition to the stream channel erosion, there is also sheet and gully erosion from those portions of the land surface carrying water only during rainstorms. The principal damage to the surface of the ground in arid regions due to erosion comes from unusually heavy falls of rain, or from the sudden melting of snow and ice caused by a rapid rise of temperature. Then, very little water sinks below the ground or is evaporated, and large quantities flow over the surface of the ground in a sheet flow. The velocity of the flow depends upon the quantity of water and the steepness of the slope. Since the vegetation is too sparse and scattered to break the force of the flood, the erosion damage depends upon the resistance of the materials over which the sheetflood travels. Weak sedimentary rocks and unconsolidated alluvium, especially if the latter is fine-textured and of little coherence, are quickly eroded by sheetfloods. Since large areas of the watersheds of the Colorado and Rio Grande are of this type of material, the potentiality for sheet and gully erosion is very high.

The sediment from these sources may be carried into streams, into dry valley bottoms, or it may be deposited over gently sloping areas adjacent to the steep slopes from which the material originated. Sheet flow rapidly collects into rills or furrows which will soon enlarge into gullies entrenched as narrow V-shaped cuts on steep slopes or which broaden out with flat bottoms on flatter grades. Roads may be converted into gullies by flash floods resulting from summer thundershowers. In many instances, excessive grazing of the native grasses and the succession of dry seasons since 1925, with the consequent damage of plant cover, are responsible for accelerated erosion following the return of normal rainfall.

In a recent survey of western range lands, more than 80% of the entire range areas, or some 728 million acres, was found to exhibit evidence of more rapid erosion than in the past. Overgrazing and drought have wrought pronounced changes in the range cover. On extensive areas, the better range grasses have been displaced by inferior nonforage plants. The encroaching plants are also of inferior soil-holding species, and the total vegetation cover is usually much less dense. A survey made by the Soil Conservation Service on the basis of grazing capacities revealed that on the average about one half of the range lands have been depleted, the extent of the damage varying from nothing in some regions to full destruction in other areas.

If the natural cover on the range lands is restored so that sheet erosion is reduced to a minimum, the channel erosion is likewise automatically reduced. As previously pointed out, the abrasive effect of sediment is cumulative after it reaches the stream channel. Hence, if water originating from the range lands enters the streams in a fairly clear condition, the erosion of the stream channels themselves is greatly reduced.

Deposition of Sediments in Reservoirs.—The value assigned for the specific weight of the deposits may result in a large variation in the estimated space occupied by sediment reaching a reservoir. Elephant Butte Reservoir affords a unique opportunity for comparison of measurements of inflowing sediments, measured for many years at San Marcial, to the surveys of the volume occupied by the sediments deposited in the reservoir. After making some well-considered adjustments to the records of samples of suspended sediments, the author determined that the average specific weight of deposits in Elephant Butte Reservoir is 65.3 lb per acre-ft. There is no reason to question the result of these computations as applied to the present deposits in Elephant Butte Reservoir; but a question does arise whether these results can be applied to sediment deposits in other reservoirs. Tests of dried sediments made by various observers have shown that the weight varies from 30 lb to 110 lb per cu ft. The lightest specific weight applies to fine clay that has not been exposed to air, whereas the heaviest weight is applicable only to delta deposits consisting of fine sand and gravel that have been dried out in the air and well compacted. The specific weights usually adopted for reservoir deposits range from 55 lb to 90 lb per cu ft. Even with this range, which does not represent the extremes, the volume of space occupied by sediments in a reservoir can be increased by 60%, if a specific weight of 55 lb per cu ft should be applicable rather than the higher value of 90 lb per cu ft. Furthermore, it seems reasonable to assume, particularly in the case of deep reservoirs such as Lake Mead and Elephant Butte reservoirs, that compaction of the lower deposits by the overlying beds will result in a gradual increase in the average specific weight of the materials. The weight per unit volume that may be stored within the reservoirs may ultimately be appreciably larger than would be found during the early stages of the deposition.

Even after the reservoir has been completely filled with sediments, there is still available space for the storage of water within the void spaces of the deposits. Assuming the higher values for the specific weight, there would probably be at least 50% voids remaining in the deposits which would be filled with water. The specific retention of the deposits would prevent the recovery of all this water, but it might be safely assumed that, with sand and gravel, 50% could be recovered by drainage, or 25% of the original capacity of the reservoir. The proposed construction of the five upstream reservoirs for storage of sediment to prolong the life of Lake Mead would create a total of 49 million acre-ft of storage on the lower Colorado River. In addition, the ultimate construction of the Glen Canyon Reservoir would furnish 60 million acre-ft of additional storage, bringing the total available storage capacity to approximately 110 million acre-ft. With all this storage filled with sediment, if it could be assumed that 25% of the volume would still be available from interstitial storage between the voids of the deposits after allowance for specific retention, there would remain a total available storage capacity on the Colorado River of 27,500,000 acre-ft, or a volume practically equivalent to the initial water storage in Lake Mead. Additional reservoir sites are available on the Rio Grande River; by a suitable program of reservoir construction on that river, it would be possible, ultimately, to provide sufficient underground storage equivalent to the initial storage capacity in the Elephant Butte Reservoir.

The author's statement of the ultimate deposition of sediment in reservoirs above the spillway level excludes many possibilities. The slope taken by the sediment deposit along the stream profile depends entirely on the grading of the materials carried with the water. If these are predominantly of fine sizes, the deposition will not extend very far upstream from the flow line of the reservoir, particularly if the volume of inflow is relatively large. If coarse material constitutes a large proportion of the load, there will be a tendency for the stream to build up its new bed very nearly parallel to the original stream gradient. Under these circumstances, with favorable gradient, the deposits may extend many miles upstream from the reservoir flow line; and in such instances the silt-storage capacity will greatly exceed the water-storage capacity. In some cases, with the proper configuration of the reservoir, a pond may be left permanently above the dam that will remain clear of sediment deposition. In other instances, sediment may be deposited in the reservoir to the maximum level of flood flow over the spillway, through which the low flows of the stream will cut a narrow channel on a gradient starting with the elevation of the spillway lip. Thus, it cannot be assured that much of the 2,500,000 acre-ft of storage above the crest gates at Boulder Dam will ultimately remain to serve as regulating storage for a river-run power plant. The foregoing conclusions were reached by the writer from a study of deposits in a number of reservoirs and erosion control structures; but an exhaustive study of the profile and cross sections of sediment deposits in and above completely filled reservoirs is highly desirable and will yield much more information on this phenomenon.

The availability of some small part of the water storage may exist for a considerable period of time beyond that indicated by the assumption of a linear rate of storage loss. This conclusion is reached, since the "trap efficiency" of a reservoir decreases as the total storage available in relation to the size of the drainage area decreases. Probably a more accurate relation of the "trap efficiency" of a reservoir could be reached by relating it to the mean annual runoff of the basin. In any event, the results in delaying the final sedimentation of a reservoir are appreciable. However, as the author states, the value of the reservoir for storage purposes would be seriously impaired after one half of the initial storage had been exhausted. Thus the extension of the time for the final filling of the reservoir with silt is of minor importance, as the "trap efficiency" of a reservoir is not materially impaired until approximately three fourths of the capacity is occupied by sediments.

Measures of Prolonging the Life of Reservoirs.—Even if underground storage can be made available after the complete sedimentation of existing reservoirs, it is highly desirable to adopt economically feasible means for prolonging their useful life for the storage of water due to the great investment involved. Under this heading, the author has considered as possible measurements: (*a*) Land management on the watershed, (*b*) control of density currents, (*c*) upstream storage for sediment and for other purposes, and (*d*) removal of sediments.

(*a*) Land Management.—Land management of watersheds must contemplate more than merely the conservation of soil and water. The conservation of soils and their more effective use must be planned so that food, forage, and timber may be produced continuously. Conservation is an aid in flood and erosion control and in drought alleviation for the purpose of protecting people and the works of man from damage and destruction. Extensive changes in farming and tillage practices, although of value, are insufficient to check destructive runoff and to control, fully, the erosion that occurs in desert regions. Reliance must be placed mainly on structural works. In cases where the benefits of erosion control to the general public exceed those accruing to the owner of the land, the use of public funds may be justified in meeting the cost of erosion and flood control measures.

The author states that the adoption of all practical methods of land management on the Colorado watershed, regardless of the expense involved, would not reduce the annual sediment load carried into Lake Mead by as much as 10%. The writer agrees that land-management practices would not reduce the soil erosion from the range land by more than 10% of the present total silt volume carried by the Colorado and the Rio Grande rivers; but, because of the principles underlying channel erosion and abrasion, it is the writer's opinion that by control at the source the total volume of sediment would be reduced by many times the 10% reduction resulting from the control of sheet and gully erosion. If the water reaching the stream channel from the fields and ranges is reasonably free from silt, the abrasive action in the main stream is greatly reduced, with the result that the total silt load farther downstream is diminished to an extent far in excess of the quantity of silt retained by the upstream erosion control measures. Obviously, once the sediment has reached the main river channel through sheet and gully erosion, the reduction of further erosion in the stream channel must depend to a large degree on structural works.

The types of erosion-control measures can be classified as vegetative and structural. Under vegetative control may be listed the following:

(1) Replanting of deforested areas and other places where shrubs and grass will thrive;

(2) Removal from cultivation of grass or marginal and mediocre lands subject to excessive erosion;

(3) Ploughing and cultivating along contours of land instead of up and down slopes;

(4) Terracing and furrowing hillside slopes along contours and planting of rows of strip crops; and

(5) Planting of willows or other suitable vegetation along eroded gullies and stream banks;

Under the heading of structural erosion-control measures may be listed the following:

(6) Dams or drops in gullies or stream channels to reduce the velocity of floods and initiate deposits behind them that ultimately reduce the stream gradient and afford support to the channel banks;

(7) Building spreader ridges or dikes which will divide flood waters and prevent their concentration in depressions with the ultimate formation of gullies; and

(8) Placing dikes, jetties, revetments, or tetrahedrons along streams to control stream meandering and bank erosion.

Sheet and gully erosion can be controlled by vegetative methods, whereas stream channel erosion lends itself in most cases most effectively to structural control. However, stream channel erosion on most watersheds greatly exceeds sheet and gully erosion. From what has been stated previously, this does not infer that vegetative control can be neglected; but these methods lie in the field of forestry and agronomy, whereas structural control is in the engineering province. Once the sediments have reached the stream channel, the control of erosion is primarily an engineering problem.

(*b*) Density Currents.—The control of sedimentation of reservoirs through the discharge of silt-laden waters through outlet valves is predicated upon the assumption that an appreciable part of the sediment is of sufficient fineness to remain in suspension during the time of its travel from the upper end of the reservoir to the discharge outlets. In the Pardee Reservoir on the Mokelumne River in California, where the discharge of fine sediments through sluice valves is reasonably effective, it is estimated that only 10% of the total sediment load has been disposed of by this means.[118] However, the removal of these fine sediments is important if, after complete sedimentation, the reservoir is to be used as a source of ground-water storage. The fine sediments are usually deposited near the dam and might seriously affect the drainage toward the outlets near the dam of the more permeable coarse materials at the head of the reservoir. If this drainage were seriously impaired by fine sediments, it might become necessary to install wells in the coarse deposits and to pump the discharge over the spillway of the dam. In some reservoirs, the proportion of coarse deposits may be so small that the specific yield would be much below the average of 25% previously assumed. In this phase of the subject, future research would point the way to the best solution. The wasting of a large proportion of the water yield of the stream, when heavily silt laden to avoid the deposition of fine sediment in the reservoir, may be indicated.

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[118] *Transactions*, Am. Soc. C. E., Vol. 109 (1944), p. 1097.

In Lake Mead, most of the sediments originating on the Colorado drainage area above Lees Ferry are so heavy that they form delta deposits at the head of the reservoir. Frequently, no turbid waters were discharged through the outlets during the deposition of such sediments, particularly during the snow runoff season with a preponderance of coarse materials in the stream. On the other hand, summer flood inflows resulting from thunderstorms originating on the Little Colorado, Virgin, and Muddy rivers carrying heavy loads of fine silt, were observed to pass through the outlets.[119]

Further study of density currents in reservoirs may lead to techniques that will increase the percentage of sediment that can be discharged through gates and valves set at properly determined operating levels. The writer is fully in accord with the author as to the need for exhaustive studies along these lines.

(*c*) Upstream Storage of Sediments.—The benefit of upstream storage of sediments has already been discussed by the writer and appears to be a very effective means of prolonging the life of the main reservoir, not only by providing additional storage for sediments, but also by reducing the bank erosion in the stream channel.

(*d*) Removal of Sediments.—As stated by the author, there appears to be no practical method to remove sediments once they have been deposited in the reservoir. Even if it were feasible to remove the reservoir from operation and to allow the stream to wash the collected sediments through a large discharge tunnel, it would be necessary to assist the stream by hydraulicking the deposits into the main channel. Past experience has indicated that normally an attempt to remove sediments by the action of the stream will result in the cutting of a narrow channel through the alluvial material leaving the major part of the sediments undisturbed. Furthermore, assuming that removal of sediments from a reservoir by this means is feasible, the discharging of huge quantities of debris into the river below a dam would impose tremendous problems in maintaining channel capacity so that overflow of valley lands downstream would not result.

Conclusion.—The future of reservoirs is not so dark as painted by Mr. Stevens, but the writer is in accord with his conclusion that extensive research and consequent improvements are necessary to provide an ultimate solution to the problems. Many of these have been only barely touched upon to permit the confining of the discussion within reasonable limits. Mr. Stevens has pointed the way to a proper approach to the problem of reservoir sedimentation; namely, (1) analysis of suspended sediment records to locate the most active sources of erosion and then (2) outline of a program for controlling the erosion both at its source and in the stream channel.

J. C. Stevens,[120] Past-President, Am. Soc. C. E.—The object of writing this paper was not so much to present factual data as to "rivet" attention on a problem that affects every one—a problem that must be faced and solved if western civilization is to endure. Twenty-four readers submitted discussions that have covered nearly every phase of the subject. Most of the discussers accept the paper in the spirit in which it was intended and agree in the main with the conclusions. Space will permit rejoinders to only a few of the discussions.

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[119] "The Passage of Turbid Water Through Lake Mead," by Nathan C. Grover and Charles S. Howard, *Transactions*, Am. Soc. C. E., Vol. 103 (1938), p. 720.

[120] Cons. Hydr. Engr. (Stevens & Koon), Portland, Ore.

Consumptive Use.—Messrs. Bondurant and Bliss take exception to the statement about the Rio Grande Compact precluding any increase in consumptive use of water. That preclusion is implied in the compact, although not definitely stated. Only in low runoff periods does unwarranted consumptive use become critical. Elephant Butte Reservoir now evaporates nearly 100,000 acre-ft of water per yr—enough to irrigate 25,000 acres of land. Even after this reservoir is filled with silt and its usefulness for storage entirely gone, the evaporation from its swampy surface will not diminish—it may even increase. Texas, therefore, may look with deep concern upon any plan to build another reservoir with holdover capacity in the Upper Rio Grande basin without compensating importation of water from the Colorado River basin. Elephant Butte will continue to take its toll of needed water indefinitely.

Land Management.—A number of discussers have "raised their eyebrows" at the writer's statements that even the best land management program that man can maintain will not make important reductions in the sediments carried by the streams; that in the Colorado River basin such management could not reduce the sediments more than 10%; and that land management of range land for sustained livestock feed, and of forest lands for sustained lumber yields, will automatically take care of the erosion-prevention feature. Emphasis should be placed on feed for livestock and on lumber yield; and any expenditures beyond such requirements can have no economical justification. The discussion on this phase of the subject has only confirmed the writer in the foregoing views.

Mr. Leopold has cited the work of Messrs. Kirk Bryan and G. M. Post[37] on the Rio Puerco. There is no doubt of the extent of erosion in recent years on this and other southwestern streams. The question is: What caused it? There is abundant geological evidence that these streams have passed through recurring cycles of erosion and sedimentation. Fig. 8 presented by Mr. Leopold shows a photograph of an ancient channel of the Rio Puerco.

A most illuminating story of these recurring cycles has been told by Mr. Bryan:[121]

> "Most of the minor streams of the Southwest are mere wet weather water courses. Muddy floods accompany and follow rain, but throughout the remainder of the year, the stream beds are dry. These ephemeral streams flow in valleys floored with alluvium from twenty to one hundred feet thick. The alluvium has been accumulated by the building up of the grade of the streams which, before the year 1880, ran in discontinuous shallow channels and during floods spread widely over the valley floors. After 1880 and continuing to the present, stream after stream has cut a deep channel, or arroyo, twenty to fifty feet deep into its valley floor or flood-plain. The mud excavated from these channels or arroyos is carried into the main rivers, and as the arroyos are widened by each successive flood, more mud is carried down. Because on the main streams, the Colorado, the Gila, and the Rio Grande, great reservoirs have been built, the increased load of mud will, within a few generations of men, fill the reservoirs and render them

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[121] "Erosion in the Valleys of the Southwest," by Kirk Bryan, *The New Mexico Quarterly*, November, 1940, pp. 228–229.

valueless for storage. This is the great 'arroyo problem.' How and at whose expense are these reservoirs to be protected?

"The western migration of cattle and cattle men began in the late seventies, and at the same time bands of sheep migrated from New Mexico. Both the cattle and sheep businesses were well developed by 1880. As the cutting of arroyos was begun at this time, some hold that the stock business is the malefactor. The claim is made that the reduction of the native vegetation, by the use or misuse of the grazing, has increased the flood run-off, and thus caused the cutting of arroyos. There is no question that our hesitant land policy, or lack of policy, has encouraged the misuse of grazing land and that overgrazing has resulted in many localities. Nor can one question the general coincidence in time between the cutting of arroyos and the development of the stock business. However, geologic studies of the valleys of the Southwest have demonstrated that the present arroyos had predecessors cut and filled up again before the coming of white men and the introduction of his domestic animals. Investigations of many stream valleys have been made, and the results are brought together in the accompanying table [see Table 19].

TABLE 19.—EROSION AND SEDIMENTATION IN THE FLOOD PLAINS OF SOUTHWESTERN VALLEYS

Alternate process	Evidence on the ground	Comment
Erosion.........	The existing arroyo	Cut from flood plains since 1880 and still being eroded.
Sedimentation...	Late channel fill	Contains, in places, potsherds and other human relics; began to be deposited from about 1300 to 1400.
Erosion.........	Ancient arroyo	Similar in size to existing arroyo; in places, known to have been cut in the period from 1100 to 1400.
Sedimentation...	Intermediate fill	Of Puebloan or earlier date; may be twofold.
Erosion.........	Earliest arroyo	Much larger than later arroyos; associated with strong wind action.
Sedimentation...	Early fill (Neville, Jeddito, and other formations)	Cemented and weathered; in some localities, contains extinct animals; absent in some valleys.

"There have been at least three periods of deposition of alluvium, and three periods of arroyo cutting of which the present arroyo is one. The dates at which these events took place have been determined mostly by interpretation of the relics of man found in the alluvium through the generous cooperation of many archaeologists. In this field the two sciences merge and the broken pieces of Pueblo pottery and other relics of man become fossils recording geologic events. The chronology of Southwestern Pueblo cultures is now in a highly developed state, so that types of pottery are correlated with the tree ring chronology invented by Douglass, and dates on the absolute time scale are possible. The third period of deposition which was brought to a close by the present arroyo cutting is quite well established as to date in these widely scattered areas and apparently occurred between 1200 and 1400 A. D. The period of arroyo cutting next previous is also fairly well established and occurred between 1100 and 1300 A. D.

* * *

"So far as the cause of the arroyo cutting which began after the year 1880 is concerned, the dates set forth in the table are conclusive that arroyos similar to and even larger than the recent arroyos were cut in past time. As these ancient episodes of erosion antedate the introduction of grazing animals, they must be independent of that cause. Each interval of erosion apparently occurred synchronously over the Southwest.

* * *

> "If, therefore, previous to 1880 A. D., alluvium was deposited under moister climates than the present and arroyos were cut under drier conditions, it seems reasonable to believe that the present arroyo is essentially climatic in origin. The introduction of grazing animals handled by optimistic owners may have reduced the already impoverished vegetation, and precipitated the event. Overgrazing thus becomes merely the trigger pull which timed the arroyo cutting in the thirty years following 1880."

In another paper,[122] Mr. Bryan correlates the rise and decline of Indian Pueblo culture with these periodic erosion and sedimentation cycles, and gives the following conclusions:

> "That such a cycle of erosion took place in prehistoric time and was succeeded by a period of alluviation that continued down to about 1880 is supported by an increasing body of fact. A buried channel representing such a cycle of erosion and alluviation has been found at Chaco Canyon and at other localities, notably on Rio Galisteo.[123] The channel formed and filled at about the end of the Great Pueblo period. The collection of data on this episode is still going on, but it appears that an adequate physical cause for the decline in Pueblo culture from its peak and the shifts in population at about that time is in sight. Such an explanation rests on the postulate that flood-water farming was the principal support of the prehistoric people."

If, therefore, the Pueblo indians lived mainly by floodwater farming their source of water disappeared when a cycle of arroyo cutting set in and they had to move to other localities.

Mr. Bryan's conclusion that the cause of these cycles is climatic seems to have some substantiation in Fig. 4 showing the levels of Great Salt Lake in Utah. The peak of 1865–1880 coincides with the end of the sedimentation period, mentioned by Mr. Bryan, and the subsequent rapid decline in levels with the erosion period. Man's activities loom large and important on the political horizon but in the cosmic sense they are quite puny and futile.

Lake Mead.—The validity of the estimates of the useful life of Lake Mead depends on a number of factors fully recognized and treated in some of the discussions:

1. "Blowing up" a 17-yr record of sediment inflow to a period of from two hundred to three hundred years;

2. Adjusting the record for unmeasured bed load and for contributions from drainage areas not included above the sediment measuring stations;

3. Appraising the climatology of the years of record as a "yardstick" of the entire period;

4. Determining the space occupied by the sediments; and

5. Deciding the accuracy of sediment flow determinations.

Mr. Brown argues that the period of record is not adequate as an index of the long period. By extending the record at Grand Canyon back to 1897, making a 45-yr record, he shows a mean annual sediment flow at Grand Canyon of nearly 252,000,000 tons as against the 200,000,000 tons reported in Table 7—an increase of 25% in sedimentation with an approximately corresponding reduction in useful life. Mr. Fox has estimated the sediment flow into the Gulf of California at 1,000,000 acre-ft per yr.

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[122] "Flood-Water Farming," by Kirk Bryan, *The Geographic Review*, July, 1929, p. 444.

[123] "Recent Deposits of Chaco Canyon, New Mexico, in Relation to the Life of the Prehistoric Peoples of Pueblo Bonito" (abstract), by Kirk Bryan, *Journal*, Washington Academy of Science, Vol. 16, 1926, pp. 75–76.

Messrs. Howard and Crawford and Benedict have shown the inaccuracies inherent in sediment sampling during past years. A new and more accurate approach to this problem is being initiated from which it is expected data will be obtained to correct the old records. Mr. Howard has also added 3 years to the sediment record in the Colorado River basin which were not available to the writer at the time the paper was being prepared. Thus, the sediment flow at Grand Canyon is reduced from 200,000,000 to 191,000,000 tons per yr; and that at Lees Ferry, from 186,000,000 to 155,000,000 tons per yr. This region is still in a low runoff period.

Mr. Bell has presented very pertinent data on the space occupied by sediments. His conclusions are that the Elephant Butte sediments should have a greater specific weight than those of Lake Mead; that this difference will disappear with the passage of time; and that the value of 65 lb per cu ft is too high for Lake Mead now but may be about right in 2075. The effect of these changes would be to lengthen the estimated life of Elephant Butte Reservoir and to shorten that of Lake Mead.

The writer is not aware of the news article published in 1916, mentioned by Edward Gerald Smith, in which a life for Elephant Butte Reservoir of 233 years was predicted; but he is certain that the late W. W. Follett, M. Am. Soc. C. E., used a specific weight of 53 lb per cu ft in his calculations.[111] Using 53 lb instead of the 92.34 lb given in the article cited, the 233-yr life reduces to 134 years which is nearer the estimated life of 158 years given by the writer.

Conclusions.—The writer can only repeat from the last paragraph of his paper that:

> "The available data are pitifully meager, and must be greatly enlarged before any constructive program for relief can be developed."

Regardless of the technical aspects of the problem and of the accuracy of the basic data and the estimates of useful lives of the reservoirs deduced therefrom one must agree with Professor Berkey that:

> "Although the principles involved in the paper * * * are well known, it is not certain that the implications are fully appreciated by many even in responsible relation to them."

Professor Berkey adds:

> "For the first time, after reading this paper, the long-range significance of the suffocating effect produced by accumulating silt in all these reservoirs was borne down on the writer."

Through the Hydraulics Division, the Society has appointed a Joint Committee on Sedimentation in Reservoirs on which not only the Hydraulics Division but the Irrigation, Power, Sanitary, and Waterways divisions have representation. The declared objective of this committee is "to study and report on problems connected with the depositing of sediment in reservoirs, its prevention and reduction." A well-rounded program of research sponsored by, and divided among, federal, state, and private agencies must now be formulated to obtain essential basic data before effective plans for ameliorating the silt menace can be formulated.

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Principles of Depreciation

FINAL REPORT OF THE SPECIAL COMMITTEE AUTHORIZED BY THE BOARD OF DIRECTION TO ANALYZE AND DISCUSS THE 1943 REPORT OF THE NATIONAL ASSOCIATION OF RAILROAD AND UTILITIES COMMISSIONERS' COMMITTEE ON DEPRECIATION

COMMITTEE

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September 30, 1944

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PRINCIPLES OF DEPRECIATION

FINAL REPORT OF THE SPECIAL COMMITTEE AUTHORIZED BY THE BOARD OF DIRECTION TO ANALYZE AND DISCUSS THE 1943 REPORT OF THE NATIONAL ASSOCIATION OF RAILROAD AND UTILITIES COMMISSIONERS' COMMITTEE ON DEPRECIATION

I.—Introduction

The Special Committee of the Society on Depreciation respectfully submits this, its final report on the subject of "Depreciation," for the purpose of expanding the discussion of important phases of the subject which could only be suggested, and the committee's opinion stated, in the Progress Report submitted[1] in April, 1944. The limited time then available to the committee made it impossible to secure records of past experience of any of the different utilities which have built up reserves for "depreciation" under regulated accounting, or for the members of the committee to discuss thoroughly the many different and complex phases of the subject so as to be in accord on an extended discussion of this most important subject.

The Report of the Committee on Depreciation of the National Association of Railroad and Utilities Commissioners (hereinafter referred to as the NARUC committee) is devoted primarily to a discussion of methods to be used in a revision of the Accounting Classification to be required. By far the largest part of that report is devoted to the purely accounting phase of the subject and to detailed discussions of various methods—straight-line, sinking fund, compound interest, and other methods, problems that belong primarily to the Accounting Profession. There are many things in that report with which the Society's committee is in agreement. The Society's committee in this, as in the Progress Report, has limited its discussion to the fundamental differences of opinion which have developed from a study of the NARUC committee's report.

That report, when analyzed, recommends that annual depreciation expense be calculated by the straight-line method of distribution, using as a basis for the distribution the average lives of groups or units of property. Also, it recommends that the straight-line method be used for the computation of the depreciation reserve and that such reserve, when so computed, shall measure the actual depreciation in the property.

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[1] *Proceedings*, Am. Soc. C. E., June, 1944, p. 887.

Even when the most complete and accurate data on average life are available, the straight-line method of distribution is still open to the objection that it assumes that a unit of service rendered today and one to be rendered some years in the future are of equal value, which obviously they are not.

The Society's committee, while fully recognizing the usefulness of the straight-line method in accounting for unregulated industrial property and possibly for certain utility properties, does not believe that its use is proper in the case of the great majority of utility properties; or that the reserve thus computed represents the "actual depreciation"; or that the depreciation reserve accumulated under regulated accounting should be "corrected" or readjusted to make it conform to the amount computed by the "straight-line" method by transfers of capital.

Each property, large or small, offers its own distinct problems of regulation and accounting. Its own management and engineering and accounting officials best know its problems and its needs, and, subject to the approval of the Regulatory Commission having jurisdiction, should be able to estimate the requirements of the property as to proper charges to operating expenses for "depreciation" and proper limits to reserve balances in a far more satisfactory manner than if compelled to use an arbitrary formula which not only deprives the management of control of this important matter, but places restriction on the exercise of informed judgment by the Commission in matters of regulation.

The Society's committee considers that the accounting plan adopted for the creation of reserves for retirement or "depreciation" in any particular case, whether "straight-line" or one of the interest methods, is immaterial, so long as it is founded upon sound economic and legal principles and creates adequate but not excessive reserves. It is the committee's opinion that for most utility properties the results of the application of one of the interest methods, preferably the present worth method, most nearly parallel the depreciation requirements.

The best proof of the results that follow by the use of any method is found in the statistics of properties which have used that method for a long period of years. Regulated accounting has been in operation for a quarter of a century and more statistics are becoming available. The committee has used certain of these statistics to give point to the argument which it presents in support of its views.

The Society's committee believes it proper to state here that all of its members are advocates of the regulation of public utilities; that they fully recognize the great amount of good that has resulted from this form of regulation. They believe that this regulation should be for the benefit not only of the patrons of the utility—the consumers—but also for stockholders and security holders as well; to insure fair and reasonable rates which will permit the utility to maintain its plant, adequately compensate its employees, furnish reliable service, permit a fair return to investors, and maintain the integrity of the investment. The management of a Public Service Corporation has three responsibilities—(1) to furnish the best of service to its patrons at the lowest rates consistent with, (2) the payment of good salaries and wages together with old-age retirement allowances to skilled employees many of whom devote a lifetime to the service, and (3) a return to the stockholders sufficient to permit the Corporation to attract new capital when needed for expansion.

II.—DEFINITIONS

Among the reasons why the general subject which has come to be called "depreciation" is so confusing and difficult of understanding are: (1) the use of the word depreciation to refer to the amount of physical deterioration or loss of service value to be found and deducted in valuations for the establishment of a rate or taxation base; (2) the use of the same word to refer to an accounting reserve created to provide for the retirement of plant through charges to operating expenses; (3) the frequent use of the word, sometimes expressed as "annual depreciation," to describe the amount of money annually charged to operating expenses and credited to the reserve; (4) the frequent use of the word to describe the credit balance in the reserve, sometimes using the term accrued depreciation for the same purpose; (5) the fact that in the voluminous literature of the subject published in the period, 1912 to 1925, and later, many attempts were made to coin terms to express certain distinctions, such as "physical depreciation" and "functional depreciation," which have come into general use and are well understood; and a number of others used to express some particular shade of meaning which the author was trying to express, and which have not come into general use.

In Chapter II of the NARUC report the nature of depreciation is discussed at length and the present-day concepts are outlined in considerable detail. A number of the definitions current in the literature and in the regulations of various public agencies are presented. It is quite apparent that the term "depreciation" is employed in all of the quoted definitions in a qualitative sense, which is correct because the term means literally the state of being of less worth. However, in valuation and accounting, the objective is quantitative and the long custom of employing the term in both a qualitative and quantitative sense has been in part responsible for the misunderstandings that characterize discussion of the subject.

The Society's committee in its meetings and discussions of the subject has found this confusion of terms to be the cause of much misunderstanding. In order to attempt to make clear to readers of this report the exact meaning attached by the Society's committee to the words used, certain definitions, with comments on each one, are given. It believes that there should be an attempt, on the part of leaders of the engineering and accounting professions, and regulatory bodies as represented by the NARUC, to clarify the subject by the adoption of an approved terminology.

1. Depreciation.—

"Broadly speaking depreciation is the loss, not restored by current maintenance, which is due to all the factors causing the ultimate retirement of the property. These factors embrace wear and tear, decay, inadequacy and obsolescence. Annual depreciation is the loss which takes place in a year."

Comments: The above is accepted as the best and most authoritative short definition. It is that of Mr. Chief Justice Hughes in the Lindheimer Case (292 U. S. 151–167). It clearly and definitely refers to the engineering concept—depreciation which is deductible in finding a rate base. It is shorter than the definition in the present NARUC Uniform System of Accounts which reads as follows:

> " 'Depreciation,' as applied to depreciable telephone plant, means loss in service value not restored by current maintenance, incurred in connection with the consumption or prospective retirement of telephone plant in the course of service from causes which are known to be in current operation, against which the company is not protected by insurance * * *. Among the causes to be given consideration are wear and tear, decay, action of the elements, * * * changes in demand and requirements of public authorities * * *."

The definitions of the Interstate Commerce Commission (177 I.C.C. 351–422), 1931; the Federal Communications Commission (Uniform System of Accounts, Telephone Co., p. 4), 1934; the Federal Power Commission (Uniform System of Accounts, p. 5); are practically identical with that of the NARUC. All of them use the term "service value" and all of them exclude all property protected by insurance. These omissions mark the major difference between these definitions and that of Chief Justice Hughes. The definitions of the commissions specify utility property. That of the Chief Justice is a good definition of depreciation of any property, regulated or otherwise. The Chief Justice, instead of using the term "service value," uses the expression "loss * * * due to all factors causing ultimate retirement," a much broader definition.

The definitions of depreciation used by the regulatory commissions have been in use for ten years or more and have controlled in matters of utility regulation. The Society's committee sees no particular objection to any of them, but would call attention to the fact that accounting for the adjustment of insured losses can be handled through the retirement reserve, as well as otherwise, and therefore places no emphasis on that omission from its adopted definition, which is clear cut and otherwise all inclusive.

The NARUC committee recommends the abandonment of all of them and after five pages of argument in its report recommends a new definition which would substitute expiration of "service life or utility of property" for "loss not restored by maintenance" in Chief Justice Hughes' definition, or "loss in service value not restored by current maintenance" in others.

2. Amortization.—

The liquidation or extinguishment of the investment in a property by means of a series of partial payments is prorated to extend over the period during which the property will exist, or over the period during which it is anticipated that benefit will be realized.

Comments: The NARUC committee on page xiv limits amortization to recouping the "cost of intangibles, such as patents, copyrights, leaseholds, etc." Depreciation and Amortization are not synonymous. The definition of Encyclopedia Britannica is as follows: "Amortization is the liquidation or extinguishment of an asset by means of a series of partial payments prorated to extend over the period during which the assets will exist." Amortization is in effect a return to the investors of money expended for wasting assets or for physical plant which is reaching the end of its useful life with no prospect of renewal. This is illustrated by the practice of writing off the investment in coal mining property, oil or natural gas property and the physical plant connected therewith as the coal, oil, or gas supply is depleted. Such utilities as pipe lines for natural gas or oil should create reserves for amortization. In past years, railroads in the lumbering districts built hundreds of miles of branch lines, which were torn up after the lumbering operations were finished. Reserves for amortization of the entire investment, every element of cost, would have been proper, but in many if not most cases were not established.

The Society's committee believes that amortization reserves should be entirely separate from reserves for the retirement of units of property on a continuing utility such as a water works or telephone property. In continuing properties the funds represented by the reserves are not returned to the investor. The object of the reserve is to maintain the integrity of the original investment.

The effect of the forced adoption of the straight-line method in the manner recommended by the NARUC committee is to provide for amortization of an unlimited life property rather than to provide a reserve for depreciation or retirement.

3. Annual Depreciation.—As included in the recommended definition of depreciation, annual depreciation is the loss that takes place in a year. The Society's committee believes that long custom has established this term to apply to the estimated annual charge to operating expenses which is credited to the reserve.

Comments: In the body of this report there is a full discussion of the difficulties involved in the exact determination of the amount of depreciation occurring in any one year. One of the fundamental differences between the NARUC report and that of the Society's committee is that the NARUC committee determines annual allowances for depreciation and assumes that the accrued depreciation follows, whereas the Society's committee believes that accrued depreciation should be determined and from accrued depreciation an annual allowance can be set up.

4. Accrued Depreciation is the total depreciation that has taken place in a unit of property or an entire existing property from the date when it was first placed in service up to the date of inquiry.

Comments: This term, accrued depreciation, is understood to be synonomous with "actual" depreciation as used by the NARUC committee. Retirements that have actually been made are not reflected in accrued depreciation.

5. Reserve for Depreciation or Retirements (hereinafter referred to as "The Reserve").—The accounting reserve is established by a series of annual charges to operating expenses and credits to a reserve, against which the cost of units of property, less net salvage, are charged upon retirement from service for any cause. The object of the reserve is to maintain the integrity of the investment in plant "so that the original investment remains as it was in the beginning."

Comments: The only reason for the reserve that has been approved by a long line of United States Supreme Court decisions is to restore to the property through this accounting device the full cost, less net salvage, of all units of plant, during the lifetime of the units. The accounting profession in the early history of this accounting device used the term "depreciation reserve" and through the years developed the theory that it should cover all theoretical expired service value or service life of all units, so that the credit balance at the end of any year would include all of the theoretical loss of value of all units in service, many of which would not be retired for many years or decades.

6. *The Credit Balance in the Reserve* is the amount of the book figure after all credits have been added and all retirements and other debits deducted.

Comments: The credit balance in the reserve should be large enough not only to cover all loss in service value presently known but also a reasonable amount to cover sudden retirements caused by storms and other unforeseen casualties.

The Society's committee considers it highly improper to call this credit balance "Accrued Depreciation," a term which should be applied only to total actual impairment to service value—a physical fact.

III.—PRINCIPAL CONCLUSIONS

The Society's committee, as a result of the study of actual statistics of regulated accounting for so-called "depreciation" over a period of thirty years, and of a review of the U. S. Supreme Court opinions bearing on questions of valuation and depreciation from the early years of this century, has reached certain conclusions, some of which are indicated in the text itself.

1. The summary that is included in the NARUC report states, "In summary statements it is obviously not possible to capture all the refinements of meaning or all the particular qualifications set forth in the main text. Therefore these statements are to be taken as condensed expressions of the ideas and principles that are developed more fully in the report." The report should have either chapter summaries or a general summary of conclusions to which the committee is unequivocally committed.

2. Depreciation is factual and quantitative, capable of reasonably accurate determination by an engineering study of plant and records.

3. The NARUC committee report discusses the application of the straight-line method with judgment in the determination of lives of property units, or groups of units, from mortality tables. The facts are that on large proportions of public utility property there are no reliable mortality data and on many properties of considerable magnitude there may be no mortality statistics at all. The Society's committee believes that the application of general data regarding mortality and the adoption of average lives for various units of utility property are to be strongly condemned. Such a use of any method cannot lead to other than inaccurate and inequitable results.

4. The straight-line method advocated by NARUC committee assumes that a unit of service to be rendered today and one to be rendered some day in the future are of equal value, which obviously they are not. The only theoretically correct method of distributing the depreciation of a unit of property over its

service life is on the basis of a method by which the time value of money is recognized.

5. The Society's committee is unable to see how allowances for depreciation expense can be made to the owners unless it is known with reasonable accuracy at all times just how much of the usefulness of the property has been consumed. It is believed that the NARUC committee should reconsider the statements of approved practice which it recommends to bring them in line with the fundamental purpose of depreciation accountancy, the clear implications of a long line of court decisions, and the plain common sense view that the allowance of depreciation expense is only the method of compensating the owners for property consumed and should in no wise be amortization of principal.

6. The cost of plant and equipment worn out or consumed in producing goods or rendering services is part of the cost of goods or service and should be included in the selling price of the products during the lifetime of the various units of the plant and equipment.

7. Reserves should be created and maintained to offset depreciation. This creation of reserves was first approved in the Knoxville Water Works Case (212 U. S. 1), in which the scope of the reserve was limited to "maintaining the integrity of the investment"; restoring to the property through charges to operating expenses the dollars which represented the cost of equipment retired from service. That principle has been recognized and approved in numerous cases since that time.

8. The Society's committee is of the opinion that, in establishing regulations for the accounting of public utility properties in the various utilities, it would be most unwise to attempt to apply any one formula or rule for universal application in accounting for depreciation. There are such wide differences in the factors causing depreciation in the properties of the different utility industries that the use of any accounting method will revert to estimates, and deal with many uncertainties. In any industry the great differences between small and large plants, or between static and rapidly growing ones, call for individual treatment. The present classification of the NARUC leaves latitude for the exercise of judgment by the various commissions and by the management of the corporations and permits the commissions to base their approval on records of past experience, as has been emphasized in a number of U. S. Supreme Court opinions. The change proposed by the NARUC committee substitutes a particular formula in the application of which adequate data are frequently lacking, and which is complicated by the question of salvage at some remote future date, and takes away in very large measure the opportunity for the exercise of sound judgment based on experience of the commission and the management.

9. The use of straight-line depreciation for thirty or forty years by two great utility industries has been discussed in this report at some length and actual data of its results have been shown. Analysis of these data has convinced the Society's committee that the enforced application of the straight-line method to the majority of public utility properties is not in the public interest.

10. The tables and charts in this report show very definitely that the telephone and railroad utilities, using the straight-line method, have built up credit

balances in the reserve that are vastly in excess of their requirements and are approximately double those of other utilities which have operated under the present NARUC accounting classification.

11. The Society's committee believes that, in cases where adequate mortality data are available, it is possible for an engineer, experienced in design, construction and operation of properties similar to those being studied, to combine the use of such data with his examination of property, and, by the application of one of the interest methods, preferably the present worth method, arrive at proper figures for annual allowances, which will result in reserves that are in fairly close relationship to actual requirements. It is also believed that the application of the same process will give reasonably close estimates of accrued depreciation, but that the major increment in such determination must be the exercise of judgment. The use of mortality statistics largely incidental should not be controlling.

12. The "reserves" in general do not have money or securities in them; they reflect money that has been charged to operating expenses and expended for new plant or other corporate uses. None of this has gone to the stockholders direct. Where reserves are excessive over and above needs for retirement and where the excess has been expended for new plant, the value of their equity has been increased. Analysis of these reserves as made in the pages of this report indicates that in the past the reserves have made very substantial contributions to the capital additions.

13. The reserves, to the extent that they exceed the needs for depreciation, reflect dollars that have been advanced by ratepayers on which the corporation has paid no interest, but which have been expended for new plant. The saving of interest to the corporations through the use of the money collected for depreciation, which otherwise would have been financed by issues of stocks and bonds, is in some cases very large.

The Society's committee believes that this condition resulting from the use of the straight-line method calls for some modified plan of reserve accounting that will prevent such occurrences in the future.

14. Equity requires that the part of the credit balance in the reserve, which is in excess of current requirements, should bear interest.

15. The Society's committee is wholly in disagreement with the conclusion of the NARUC committee that the credit balance in the reserve represents "actual" depreciation and should be deducted in a valuation for rate making. This is incorrect except in those very rare instances where the credit balance in the reserve and the actual requirements are in balance and where a depreciated rate base is appropriate. The NARUC recommendation disregards the purpose for which reserve accounting was established and approved by the Courts, namely, to keep the investment intact and to restore to the corporation the cost of all property retired from service from any cause.

16. In the Progress Report the Society's committee stated[1] its conclusions in reference to the proposed retroactive adjustment of existing retirement reserves to bring the current reserve up to the so-called "depreciation reserve" computed on the straight-line basis through charges to surplus or a special section of the income account. Those conclusions are here repeated:

> "* * * The Society's committee feels that in view of the fact that practically all utilities have been subject to governmental regulation over a period of two to four decades and the reserves currently carried on the books of the utilities have been accumulated, credited and charged in compliance with the regulation of such bodies, the adjustment of the current reserve upward so as to equal those computed on the straight-line basis by withdrawal from surplus or otherwise is not only generally financially impracticable but is also ethically unjust.
>
> "Present reserves should, and usually do, reflect rates charged and collected from consumers. To arbitrarily increase the reserves at this late date by withdrawal from surplus is to require the utility rather than its consumers to make good the cost of property used up in their service. This procedure is in conflict with the basic principle of depreciation as well as plain justice.
>
> "The reserve requirements should be measured by the actual needs of the utility and not by an arbitrary straight-line determination which in the case of many of the best utilities in the United States would require reserves substantially double those which a long background of experience under regulation has determined as adequate."

17. The Society's committee is of the opinion, as stated in its preliminary report, "that the actual depreciation should control the size of the reserve, rather than that the reserve should determine the actual accrued depreciation."

18. That method of accounting for depreciation and for the establishment of retirement reserves to provide for future retirements is best which most closely harmonizes with the facts found for each individual property.

19. Uniformity of treatment and administrative convenience in dealing with depreciation are desirable when equitable; but neither uniformity nor administrative convenience should control over the facts of a particular situation. The NARUC report under discussion would make incorrect theories, rigid uniformity, and administrative convenience controlling. Actual depreciation existing in the physical property is a fact. It is the kind of fact that can be determined with reasonable accuracy only by study of the property by qualified, trained engineers or operators with engineering training who follow approved methods.

20. The state regulatory commissions in the United States must be credited with contributing greatly to the abolition of serious evils that once beset the utility industry. The decisions of the commissions have generally been sound and wise. The commissions have had freedom to exercise their own judgment, rather than being bound by arbitrary rules and regulations. The Society's committee believes that accounting regulations which have left much to the wisdom and discretion of the commissions in the matter of depreciation is abundantly proven by the results.

21. Generally speaking, the Society committee's views regarding the proper procedure in determining annual depreciation allowances are as follows:

1. The preparation of an engineering determination of accrued depreciation.
2. The determination of an annual depreciation allowance which had it been in effect during the life of the property would have provided for all retirements and left a credit balance in the reserve approximately equal to the actual depreciation.

IV.—Historical Development of the Conception of Depreciation

A. Valuation in the Nineteenth Century.—

Valuations which were made to determine the price to be paid for property, when the issue was the purchase of a water works by a municipality on the expiration of the franchise; a valuation to establish a basis for taxation on the *ad valorem* method, as was the case in the Michigan valuation of railroads, or to determine a rate base as was true in other states, were the pioneer undertakings in what has come to be a large and important field of engineering practice. In all of them the early work was along practically the same general lines.

The engineers secured from all available sources data as to the units which composed the property—the inventory; they found the actual cost if data were available; they estimated what it would cost to build an identical property at the time of the investigation, thus establishing the cost of reproduction at then prevailing prices; from which was deducted accrued depreciation to arrive at a figure designated as "Cost of Reproduction Less Depreciation."

While this report deals with the problem of depreciation, a brief review of the history of valuation, of which the determination of depreciation is an important element, is essential to an understanding of the origin of some of the theories now prevailing.

There are very few references to the subject of depreciation in the literature prior to 1900. A few old reports are to be found in the libraries, none of which gives any details, but all of which indicate that the subject of depreciation in the sense of loss of value was recognized.

One paragraph in the annual report of John T. Clark, State Engineer of New York, for the year 1855, is worth quoting:

> "It is contended by many skillful managers that if the works and rolling stock are kept in thorough repair, they are in as good condition at the end of each year as they were at the end of the preceding year, and therefore that there can be no depreciation. The difficulty lies in determining the fact whether the repairs have been kept up in this thorough manner."

This indicates that in the very early days of the railroad industry the managers advanced exactly the same arguments that were used in hearings before the Interstate Commerce Commission seventy years later.

In the "Nebraska Rate Cases" which reached the U. S. Supreme Court as the famous Smyth vs. Ames (169 U.S. 466, decided March 7, 1898), there was much testimony as to the cost of building the roads at that date, 1894–1895, and the state based its testimony as to value on the cost of duplication theory at the period of the very lowest price levels recorded since the Civil War. In the often quoted "rule" laid down by the Court in Smyth vs. Ames there is no mention of depreciation. After naming various matters for consideration, the Court added a sentence: "We do not say that there may not be other matters to be regarded in estimating the value of the property." Depreciation has developed as the most important of those "other matters."

In 1897 the first paper on the subject of valuation in *Transactions* of the American Society of Civil Engineers, by the late Wynkoop Kiersted, M. Am.

Soc. C. E., entitled "Valuation of Water-Works Property"[2] discussed Measures of Value. Four members discussed the paper. The author and the four who discussed agreed that "cost of duplication less depreciation" was entitled to the greatest weight.

There are brief references to the accounting for loss of value by the establishment of "sinking funds" by the late Emil Kuichling,[3] M. Am. Soc. C. E., and by the late Alex Dow, Hon. M. Am. Soc. C. E., in 1898 in a paper presented to the National Electric Light Association which give recognition to the practice of private industry in accounting and to the need of some provision by the utilities to care for depreciation, but no general use of reserves was made by the railroads or early utilities. The reason for this was undoubtedly the opinion of the Supreme Court in the earliest recorded attempt to create reserves for replacement.

The Kansas Pacific Case (United States vs. Kansas Pacific Railway Co., 99 U.S. 455, decided October, 1878) dealt with the subject of "depreciation" reserves. The Auditor of the railroad undertook to establish a reserve for the replacement of rails, assuming a 20-yr life, and contended that it was proper to reserve from earnings each year an amount representing loss of life chargeable that year.

The Court said:

> "First. 'Depreciation account or expense not charged up.' This is explained to be the amount necessary to put the road in proper repair, but which was not actually expended for the purpose. We are clearly of the opinion that it is not a proper charge. Only such expenditures as are actually made can with any propriety be claimed as a deduction from earnings."

That opinion clearly disapproved the creation of reserves and was the controlling case for the rest of the century. There was at the end of the century no regulation of accounting of any utility. The Interstate Commerce Commission had been created in 1887, but was not given authority over accounting until 1907; and not until after the first decade of this century were other utilities than the carriers brought under regulation.

Valuation of properties for sale, control of security issues, rates and taxation commenced in a small way in the 1890's. The acquisition of water works plants by municipalities on the expiration of franchises accounted for most of it. A few valuations of railroads under the Texas "Stock and Bond Law" were completed before 1900. The Michigan Railroad Appraisal, begun in 1900, was the first state-wide appraisal in the United States. It was the first to develop rules and uniform methods, and to adopt cost of reproduction less depreciation as the measure of a taxation base. Depreciation was considered as a condition below the maximum service value at which property could be maintained and was based on inspection and the personal opinion of the senior engineers, all experienced railroad men.

This work was followed by state-wide valuations in Wisconsin, Minnesota, Washington, and several other states prior to the beginning of the Federal Valuation of Railroads in 1913. In all of the states the problem dealt with depreciation—a condition of plant to be found and deducted from value. None of them dealt with problems of reserves or of accounting for the purpose of recovering the cost of worn out or retired property through charges to operating expenses. The first extensive consideration of that subject was by the Bell Telephone Utility, discussed at length later in this report.

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[2] *Transactions*, Am. Soc. C. E., Vol. XXXVIII (December, 1897), p. 115.

[3] "The Financial Management of Water-Works," by E. Kuichling, *ibid.*, p. 1.

B. The Period of the State Valuations.—

The Texas valuations under the Stock and Bond Law recognized the existence of depreciation, but made no estimate or deduction for depreciation, assuming that the property was maintained in normal operating condition and that reasonableness of security issues was the chief problem to be solved.[4] Existing depreciation became a major consideration of the State Railroad Valuations, 1900 to 1912. The Michigan, Wisconsin, and Minnesota valuations were completed or under way when the Interstate Commerce Commission was first given authority to regulate railroad accounting in 1907. Depreciation was determined by the senior engineers by inspection and represented the personal judgment of experienced railroad men as to the extent of falling below a maximum condition of operating efficiency. A handbook issued by the Master Car Builders Association containing rules for determination of depreciation was used as to all cars.

Measured by standards of valuation practice of later decades, these early valuations were primitive. In most of them the total figure of depreciation was a composite of the personal judgment of many engineers. The Washington State Appraisal was unique in that the field determination of depreciation was done away with. The description of the method used in Washington is quoted from the first extensive paper on the subject of railroad valuation published[5] and is as follows:

> "A percentage of depreciation was not placed in the field, but was determined by 'mortality tables,' or by ascertaining the probable years of structure life, then determining from the age of the particular structure under consideration its percentage of depreciation, a method by no means new."

This is what is now called the "straight-line" method, a term not used in this 167-page paper. It is of further interest to note that all that is contained in the paper on the subject of depreciation are the foregoing paragraph, half a dozen scattered sentences, and the following statements:[6]

> "(*d*) *Depreciation.*—Thus far, this discussion has not dealt at length with the subject of depreciation, and it is not considered essential to the purposes of this paper that it be done. The State appraisals have raised a question as to the propriety of using mortality or life tables as compared with personal inspection and the placing of a percentage based on individual judgment. * * * It is certainly desirable to secure the opinion of the man who inspects a bridge, or building, or locomotive, as to its physical condition. It may be desirable to use the check secured by the fact that the age of the building is known and also the average life of structures of its class."

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[4] See "Method Used by the Railroad Commission, of Texas, Under the Stock and Bond Law, in Valuing Railroad Properties," by R. A. Thompson, *Transactions*, Am. Soc. C. E., Vol. LII (June, 1904), p. 328.

[5] "The Valuation of Public Service Corporation Property," by Henry Earle Riggs, *Transactions*, Am. Soc. C. E., Vol. LXXII (June, 1911), pp. 1-91.

[6] *Ibid.*, p. 141

These references to this old paper presented in January, 1911, are quoted to direct attention to the fact that the problems now under discussion are not new problems, but have been recognized for many decades. That paper was discussed by twenty-five prominent engineers and economists and constitutes a symposium on the subject just before the beginning of the Federal Valuation. One more quotation from the discussion is of interest. It has been noted that the Washington State Appraisal used "life tables," that is to say, the straight-line method, in finding "depreciation." That appraisal was made under the direction of Halbert P. Gillette, M. Am. Soc. C. E. In his discussion[7] Mr. Gillette said:

> "Of almost as great moment as the item of development expense is the question of depreciation. The author [referring to the author of the paper under discussion], in common with most engineers, holds that depreciation should be deducted. This is a consequence of regarding a public service plant as if it were a machine bought in a second-hand store. A public service plant is a device which is intended to perform a given service forever. It is true that its parts are subject to wear, and must be renewed from time to time; but the plant as a whole is everlasting, or practically so. Managers of public service corporations, perceiving this fundamental truth, have rarely established sinking funds for the redemption of any considerable part of the plant. In a great railway system the renewal of a freight car is not a proportionately larger item of expense than is the renewal of a tooth in a steam shovel bucket owned by a contractor. This fact, coupled with the permanence of the railway plant as a whole, has led railway owners to make no provision for a return of the money lost in depreciation. Railway ties in a large railway system inevitably reach a condition such that their average age is exactly half the life of the average tie. Shall a sinking fund be provided for ties? If not, where does logic place a line of demarcation? When does an element of the railway plant attain a condition of sufficient importance to warrant 'writing off' some of its value from the capital account? The facts are that railway managers have not 'written off' anything worthy of mention for depreciation, and, in the writer's opinion, they have been perfectly logical. * * * In other words, if depreciation is to be deducted from the cost of reproduction, it must be added to the development expense ascertained from the accounting records; so that, in the final analysis, depreciation should be ignored entirely in any appraisal of a public service corporation where the object is either rate-making or purchase of the corporation by the public. One qualification to this statement is needed, however, and that is that the depreciation shall not have gone far enough to result in an average age of plant less than half the life of the plant—that being the ultimate normal operating condition."

It is of special interest to note from the above quotation that the only man who used the straight-line method for estimating depreciation in a state valuation of railroads opposed its deduction from value.

The foregoing discussion is designed to throw light on the views of the profession prior to the beginning of the Federal Valuation of Railroads.

C. The Period of the Federal Valuations.—

A Committee was appointed by the American Society of Civil Engineers in September, 1911, to Formulate Principles and Methods for the Valuation of

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[7] *Transactions*, Am. Soc. C. E., Vol. LXXII (June, 1911), pp. 278–279.

Railroad Property and Other Public Utilities. It submitted a Progress Report in January, 1914. The Final Report was completed in the autumn of 1916 and submitted at the Annual Meeting, January 17, 1917,[8] a few weeks before the United States declared war on Germany. As a consequence of war only four members of the Society discussed this very significant report.

The Federal Valuation of Railroads was provided for in 1912; the Board of Valuation Engineers of the Interstate Commerce Commission was created and organized in 1913. The Texas Midland Opinion of the Interstate Commerce Commission (75 I.C.C. pp. 1–48) fully outlined the methods, rules, and theories as used in the primary valuations of this Commission.

This great undertaking was pushed through to the completion of the primary valuation in the middle 1920's; railroad records were made, such as had never existed before; the most elaborate and accurate studies of costs were completed dealing with prices of the period 1910–1914, which are admittedly as accurate as it is possible to make them, and valuations priced as of date June 30, 1914, were completed for all carriers. Depreciation was deducted on the age-life or straight-line method, the percentages of deduction varying on different carriers but for most of them ranging from 18% to 25%, some running as high as 30% or more.

The only case bearing on the Primary Valuation which reached the Supreme Court was the Los Angeles and Salt Lake Case in which the Court held that the valuation was merely the gathering of data for the files of the Commission which might be used some day in litigation, but the collection of which did not injure the carrier.

The "recapture valuations" which commenced in the middle 1920's made use of the Primary Valuation in the St. Louis and O'Fallon Case (279 U.S. 461). There the Court held that the use of 1914 prices in valuations of 1920 to 1922 and later was improper, and that weight must be given to the price levels of the years of valuation. The Bureau then adopted new methods to translate the primary valuations into the terms of values as of later years. The methods used were radically different from those of the primary valuation.

Many recapture valuations were completed, but no case reached the Supreme Court before the repeal of the recapture provision of the Transportation Act in 1933. The Federal Valuation of Railroads practically ended with that repeal, leaving as its most valuable result the record of the Primary Valuation.

D. Accrued Depreciation Is a Determinable Fact.—

The vast majority of public utility properties must be considered as having an indefinite, continuing life, a life that will last as long as there will be a demand for the service being rendered; therefore there is no necessity for amortizing the entire investment. Each property is a composite of many separate, and different, kinds of units, each having terminable life of varying length, each unit being made up of a large number of items which must be kept in a maximum condition of service efficiency and which must be replaced when it ceases to function in order that the unit be always in maximum service condition. Repair and replacement of items of property is distinctly maintenance, a straight operating expense. The unit of property, when retired, is written off the fixed capital. The claim of the railroads throughout the entire period of valuation was that the maximum or 100% condition of the unit was not "new" as claimed by the Bureau of Valuation, but maximum condition of service efficiency, with its component items in all stages of life from newly installed to imminent replacement. The Bureau of Valuation formula assumed that such items of the track or road units as ties, or of telephone units as poles, were 100% when newly installed and zero when replaced, thus reducing value of these millions of items to around 50%.

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[8] *Transactions*, Am. Soc. C. E., Vol. LXXXI (December, 1917), pp. 1311–1620.

A study of the Primary Valuation reports discloses not only very high percentages deducted from value but some interesting discrepancies due to the assumptions of "life" and the failure to draw a clear line between units, such as major sections of "road" or "track," composed of the items of grading, ballast, ties, rails, etc. As a result carriers with a large investment in right of way and grading show very much less depreciation than those in level territory with little grading.

One of the "best" carriers in the country, with approximately only 10% deduction for "depreciation," was found to be the Bingham and Garfield Railroad, an ore-carrying road a few miles long running from the Great Copper Mines in Bingham Canyon, Utah, to the Smelter on Great Salt Lake. This is a mountain road with very heavy cuts and fills which "do not depreciate" and with heavy steel bridges and trestles of long life.

If there is a better example in America of a single purpose road, with a life limited to the life of the mine, and with a real need for creating reserves for amortization of the entire property, it is hard to imagine. Yet less than half of the "depreciation" was found here than for such carriers as its only connection—the Union Pacific, or the New York Central, Norfolk and Western, Chesapeake and Ohio and other heavy duty highly maintained properties.

The assumption that such land as railroad, power, telephone and other rights of way, and grading, do not "depreciate" is hard to sustain. There are several thousand miles of abandoned right of way and grading of railroads that have been torn up. It is hard to imagine anything more worthless. The grade of a railroad is an element or item of the track or road unit, such as a main-line division or branch line, which calls for constant maintenance to overcome erosion, repair washouts, and similar expense.

Accrued depreciation, deductible in a valuation on any property, is only in part evident on inspection. The full amount of it is a matter of estimate based on studies of past maintenance, study of the history of the property, determination of the amounts of money that should be spent to restore the plant to full condition of operating efficiency, and thorough study of the obsolescent units to determine whether there is justification for their imminent retirement. Engineering valuation has been carried on for nearly half a century. Theories and methods have been developed and have been in constant use for all of that time that are at wide variance with those used by the Interstate Commerce Commission's Bureau of Valuation. They have not been based upon the age-life straight-line estimates. They have been approved by the Courts as later quotations in this report will indicate.

The railroad contention was that accrued or deductible depreciation in any utility property at any time is an existing condition below the maximum service condition which can and should be continually maintained, a physical fact bearing little relation if any to the theoretical elapsed life of the property. Its only relation to any reserve that may be established for the retirement of the units of property or for the amortization of the whole property in an expiring plant is that it should have been provided for, and that the reserve should be enough larger to fully provide for coming retirements.

This form of depreciation has been recognized by the engineering profession, by the owners of utility property, and by the Courts. There is no need here to quote from the many opinions, beginning with that in the Knoxville Water Case (212 U.S. 1). The Society's committee fully recognizes the fact that existing depreciation must be found and deducted in valuation. It believes that it is a matter of fact, not of theory. It cannot be determined with mathematical accuracy by formula, but is subject to determination within reasonably close limits by men experienced in the design, construction, and operation of utility properties. It is basically an engineering determination. It includes not only inspection of the plant, but detailed examination of conditions not evident to the eye. It calls for studies of past and probable future growth, types of plant and equipment, their functional features, and all matters that would affect their probable future usefulness. Such a study establishes a factual basis for appropriate financial and accounting procedure.

The Society's committee contends that accrued depreciation can only be ascertained accurately by this method.

V.—Historical Development of Accounting Reserve for Retirement or Depreciation

The two subjects, accrued or deductible depreciation in valuation, and accounting reserves for retirement of plant, which are both generally spoken of as "depreciation," are of vital importance in regulation both to the utilities and to the public. They are not well enough understood by engineers and accountants by whom the amounts of money involved must be determined, by the managements of the corporations or officials of the Regulatory Commissions in charge, or by the patrons of the utilities.

Practically all of the discussions of "depreciation" in papers in technical publications or society proceedings or in books by accounting authorities present the academic theories of the authors without using illustrative material drawn from the records of actual experience of the public utilities and railroads, all of which have been under regulation of accounting for twenty-five or thirty years or more. In the following discussion of accounting reserves the Society's committee has chosen its illustrative material from the voluminous public records of data of existing corporations, records actually made during the 30-yr period of regulated accounting.

The subject known as retirement or "depreciation accounting" deals with vast sums of money charged to operating expenses and credited to reserves which have been miscalled "reserves for accrued depreciation." The only purpose for which these reserves have been approved by the Supreme Court is to reimburse the corporations for the cost of property that has been retired from service or is being consumed in service and will be retired. These reserves were originally called Reserves for Retirement by the National Association of Railroad and Utilities Commissioners—a correct statement of what they really are.

The last data available to the Society's committee drawn from reports of regulatory commissions indicate that in 1942 the credit balances in these corporation reserves were well in excess of eight billion dollars after charging all previous retirements of plant to them. These figures include the credit balances of the major telephone, electric, gas, and railway equipment reserves. This vast sum of money is stated at this point in the discussion in order to emphasize the fact that the subject is of major financial importance not only to the corporations but to the patrons who furnish the money through payment of rates for service and to the commissions charged with regulation. The reserves should be ample for the purpose of "maintaining the integrity of the investment," a term frequently used in opinions of the U. S. Supreme Court, but as has also been made clear from those opinions, they must not be excessive.

The two subjects which are called "depreciation" have been referred to. There must be a clear distinction in any discussion of the subject between (1) accrued depreciation, loss of service value, which must be found and deducted in valuation for the determination of a rate base, a fact which can be determined with reasonable accuracy by inspection of physical plant and study of maintenance records; and (2) the accounting provision for a return to the company of the cost of property retired and for maintaining a reasonable reserve balance to provide for future retirements of property on which depreciation is now operative, and for casualties of flood, storm, or other disaster. The word "depreciation" has been used to describe this reserve and also to describe the annual charge to operating expense, thus using the one word to describe three different things, two of them having little real connection with true depreciation, which is loss of service value.

Origin of Reserve Accounting.—

For more than a hundred years there has been recognition of the sound principle that the owners of any manufacturing, mercantile, or utility business selling goods or services to the public are entitled to charge, as an essential part of the cost of production, all of the legitimate costs of management, all operating expenses including the cost of maintenance and repairs to the plant, and the original cost of all major units of plant and equipment worn out in service and retired. This latter item can best be spread with reasonable uniformity during the useful life of major units by means of yearly accruals to a reserve.

The earliest references to the methods adopted for spreading the cost of plant and equipment over its service life found by the Society's committee dealt with "sinking fund" and "annuities" for marine boilers and hulls in the period, 1810 to 1815.

Between 1845 and 1855 there was considerable discussion of the subject of replacement of railway plant and equipment in England, Holland, and United

States. In 1850 a volume entitled "Railway Economy" by Dionysius Lardner, published by Harper and Brothers in New York, contained the statement:

> "Estimating as nearly as possible the major limit of the cost of relaying, at the end of twenty years, the amount of an annuity (to use a term well understood) must be calculated which at the termination of twenty years would be equal to the cost of laying the rails; and an equivalent sum must be taken annually from revenue and invested at compound interest to meet the exigency."

This volume contains the earliest extensive discussion of the subjects of depreciation and of accounting for the cost of retirement of utility property found by the Society's committee.

Recognition by the Courts of the Necessity for Retirement Reserves.—

Not until 1909 did the Courts recognize the character of depreciation and provide, not only for the deduction of accrued depreciation, but also for making provision for maintenance of the integrity of the investment. In Knoxville vs. Knoxville Water Company (212 U.S. 1), Mr. Justice Moody said, on January 4, 1909:

> "Before coming to the question of profit at all, the Company is entitled to earn a sufficient sum annually to provide, not only for current repairs, but for making good the depreciation and replacing the parts of the property when they come to the end of their life. The Company is not bound to see its property gradually waste, without making provision out of earnings for its replacement. It is entitled to see that from earnings the value of the property invested is kept unimpaired, so that at the end of any given term of years the original investment remains as it was in the beginning. It is not only the right of the Company to make such a provision, but it is its duty to its bond and stockholders, and, in case of a public service corporation at least its plain duty to the public."

In the following month, February, 1909 (in R. R. Com. of Louisiana vs. Cumberland Tel. and Tel. Co., 212 U.S. 414), the Supreme Court said:

> "That it was right to raise more money for depreciation than was actually disbursed for the particular year there can be no doubt, for a reserve is necessary in any business of the kind, and so it might accumulate, but to raise more than enough money for the purpose and place the balance to the credit of capital upon which to pay dividends cannot be proper treatment."

These two cases are quoted because the opinions were handed down before the public utilities came under general regulation and only two years after the first classification of Railroad Accounts was issued by the Interstate Commerce Commission. Thus, right at the beginning of regulated accounting in the United States, there was clear recognition by the U. S. Supreme Court of the propriety of the creation of reserves to maintain the integrity of the investment by restoration to the corporation of the cost of all units of property worn out and equally clear warning against the accumulation of excessive reserves.

On May 21, 1934, Columbus Gas and Fuel Company vs. Public Utility Commission (292 U.S. 398), the opinion was handed down by Mr. Justice Cardozo. This case, and a companion case (Dayton Power & Light Co. vs. P.

U. Commission 292 U.S. 290), decided on April 30, 1934, involved large allowances for depletion in natural gas fields. The Court said:

> "To withhold from a public utility the privilege of including a depletion allowance among its operating expenses, while confining it to a return of 6½% upon the value of its wasting assets, is to take its property away without due process of law, at least where the waste is inevitable and rapid. * * * Plainly the State must either surrender the power to limit the return or else to concede to the business a compensating privilege to keep its capital intact. * * * It is idle to argue that a company using up its capital in the operations of the year will have received the same return as one that at the end of the years has its capital intact and interest besides."

These cases, and three telephone cases, later quoted in this discussion (the New York Telephone Case, 271 U.S. 23, April, 1926; Smith vs. Illinois Bell Telephone Co., 282 U.S. 133, December; 1930, and Lindheimer vs. Illinois Bell Telephone Co., 292 U.S. 151, April, 1934), are among the many cases which sustain the proposition that the cost of retiring units of property no longer useful in the manufacture of goods or the rendering of public service is part of the cost of service and is properly chargeable to operating expenses and thus paid for by the consumer of the goods or services. The creation of reserves for this purpose and the attempt to spread the cost over the life of the units of property with some reasonable degree of uniformity is fully sustained by a long line of U. S. Supreme Court opinions.

Accounting for Retirement.—

As already stated, the creation of reserves to collect the cost of retirement of units of plant during the life of these units has been in use for over a century. It obviously had its beginning in the accounting of manufacturing, mercantile and marine property, and other privately owned property.

Apparently during the nineteenth century some form of interest method was in quite general if not universal use. The early references which the Society's committee has found all speak of "sinking funds" or "annuities." In 1897 Mr. Kuichling[3] said "As the gradual formation of a fund by a series of annual payments is generally understood only in a vague manner outside of financial circles, a few words on the subject may be permitted." In that same decade the published papers of Mr. Dow make it evident that the electric power industry, then in its infancy, recognized the need of reserves but had no definite plans for their creation. Mr. Dow was an advocate of reserves as early as 1895.

VI.—THE STRAIGHT-LINE METHOD

The straight-line method was defined, or described, by the late Joseph B. Eastman, on November 2, 1926 (118 I.C.C. 353), as follows:

> "First of all the total depreciation expense is determined by estimating the service value of the property in question. This requires not only a ledger value corresponding as nearly as may be to original cost, but also an estimate of net salvage value at the time of prospective retirement and a similar estimate of service life. Finally the annual depreciation charge is determined by dividing the estimated service value by the estimated service life. The purpose of this method of determining depreciation charges is to spread the entire expense evenly over the property in question."

The accounting profession recognizes the straight-line method as a simple method of spreading the cost of a unit of property over the estimated life of that unit by collecting each year an equal part of the estimated cost less estimated salvage without the use of any interest accrual. In the case of a machine shop, printing office, or other privately owned plant, where the cost of all machines is known, where the sole object is to charge enough for the product to cover the cost of wearing out of the machines which make it, and where "conservatism" (that is, possibly excessive charges to depreciation) is not legally forbidden, the plan undoubtedly has great merit. It is evident that small isolated municipally owned utilities can make good use of reserves collected by this method and undoubtedly there are some special instances of privately owned utility properties which could use the straight-line method to advantage and without serious grounds for criticism.

The "Comments of the Committee of the American Institute of Accountants on the NARUC Committee Report" said of the straight-line method:

> "This Institute has for many years been engaged in a campaign in which it has sought to stress equally strongly the indispensability of accounts and the inescapable limitations on their significance. Straight-line depreciation is a crude process of allocation based on working hypotheses that must be constantly reviewed, and on incomplete and in some cases irrelevant statistics which must be constantly supplemented. To apply to such raw material highly refined mathematical processes would be to create an even more delusive suggestion of certitude than that to which Mr. Justice Jackson referred."

The reference to Mr. Justice Jackson in the above quotation relates to his dissenting opinion in the Hope Natural Gas Case which is in part as follows:

> "To make a fetish of mere accounting is to shield from examination the deeper causes, forces, movement and conditions which should govern rates. Even as a recording of current transactions, bookkeeping is hardly an exact science. As a representation of the condition and trend of a business, it uses symbols of certainty to express values that actually are in constant flux. * * * However our quest for certitude is so ardent that we pay an irrational reverence to a technique which uses symbols of certainty, even though experience again and again warns that they are delusive."

Mr. Justice Brandeis, who had an exceptionally clear understanding of accounting problems, in his dissenting opinion in United Railways and Electric Company vs. West (280 U.S. 234, 276–277, January, 1930), discussed the subject of accounting reserves in part as follows:

> "For an annual depreciation charge is not a measure of the actual consumption of plant during the year. No such measure has yet been invented. There is no regularity in the development of depreciation. It does not proceed in accordance with any mathematical law. There is nothing in business experience, or in the training of experts, which enables man to say to what extent service life will be impaired by the operations of a single year, or of a series of years less than the service life.[13]"

Footnote 13 of the opinion reads as follows:

> "Depreciation of physical units used in connection with public utilities, or, indeed, with any other industries, does not proceed in accordance with

> any mathematical law. * * * There is no regularity in the development of the increasing need for repairs; there is no regularity in the progress of depreciation; but, in order to devise a reasonable plan for laying aside allowances from year to year to make good the depreciation as it accrues, and to provide for the accumulation of a sum equivalent to the cost less salvage of a unit by the time it is retired, some theory of depreciation progress must be assumed on which such allowances may be based. 81 Am. Soc. Civil Eng. Transactions (1917) 1311–1462–3."

This quotation by Mr. Justice Brandeis is from the report of the Society's Committee on Valuation hereinbefore mentioned,[8] of which the late Frederic P. Stearns, Past-President, Am. Soc. C. E., was Chairman. This long dissenting opinion of Mr. Justice Brandeis, supplemented by that of Mr. Justice Stone who agreed on the subject of depreciation, constitutes one of the most complete legal discussions on the subject extant. Only a few more sentences of this opinion of Justice Brandeis need be quoted:

> "The main purpose of this charge (to operating expenses for 'depreciation') is that irrespective of the rate of depreciation there shall be produced, through annual contributions, by the end of the service life of the depreciable plant, an amount equal to the total net expense of its retirement. To that end it is necessary only that some reasonable plan of distribution be adopted. Since it is impossible to ascertain what percentage of the service life is consumed in any year it is either assumed that depreciation proceeds at some average rate * * * or the annual charge is fixed without any regard to the rate of depreciation. * * * It is a bookkeeping device introduced in the exercise of practical judgment to serve three purposes. It preserves the integrity of the investment. * * * It serves to distribute equitably throughout the several years of service life the only expense of plant retirement which is capable of reasonable ascertainment—the known cost less the estimated salvage value. And it enables those interested * * * to ascertain as nearly as possible the actual financial results of the years operations."

Every time $1,000 is charged to operating expenses for "depreciation" and credited to the "reserve," $1,000 is held back from net revenue, from the stockholders, and left as cash for the management to spend for new plant or for other legitimate corporate uses. There are a number of corporations which invest part of the money in securities until needed for construction but the general practice on expanding properties is to invest it in new plant. All retirements, less salvage, are charged to the reserve. The resulting credit balance is a bookkeeping entry representing all prior charges to operating expense, less all retirement. It is usually not maintained as a cash reserve. It is never available to the investor except in case of partial or complete liquidation of the property.

The expenditures for new plant to the extent of past retirements assist in fulfilling the function of "maintaining the integrity of the investment." It is new property replacing retired units of property and to that extent the method should not be a matter of controversy. That, however, does not relate to the credit balance in the reserve as upon the retirement of plant the resulting charge reduces the credit balance. Property previously retired is not represented in the credit balance of the reserve which is available for unmatured depreciation.

Some form of a reserve is essential to provide for future retirements. There can be no legitimate objection to a reserve, or surplus, of reasonable size to provide for future retirements, some of which can be foreseen with reasonable certainty a few years in advance, and some of which are the result of flood, sleet and ice storms, lightning or other disasters which frequently take a heavy toll of railroad, power, telegraph and telephone property. Such reserves have been approved again and again by the U. S. Supreme Court, but the Courts have been interested in the size of the reserve rather than the method of accrual. The Courts' warnings against excessive reserves, the obviously excessive nature of "straight-line" accumulations, and the fact that in many litigated cases accrued depreciation has been found to be much less than the straight-line reserve, all point to much lower provisions for retirement than have been used in the past by many corporations. The U. S. Supreme Court's decisions in a number of cases refer to experience as the proper guide to the establishment of reserves.

It is very clear, in the light of many U. S. Supreme Court decisions, that there is no legal justification for making provision for anything more than reserves sufficient to reimburse the corporation for the cost of all plant retired for any reason and for maintaining a balance sufficient to provide for depreciation in progress but not yet reflected in retirements. Any theory or any method of accounting that builds up reserves in excess of a reasonable balance or surplus for that purpose is almost certain to lead to controversy and litigation.

It is also true that, at the present time, the law is clear that credit balances that have been accumulated in the past under regulated accounting belong to the company. That fact, however, does not justify continuing charges to operating expenses at rates that are in excess of the actual provable requirements.

In the New York Telephone Case (271 U.S. 23–32), in 1926, the Court said:

> "It may be assumed * * * that in prior years the company charged excessive amounts to depreciation expense and so created in the reserve account balances greater than required adequately to maintain the property. * * * The revenue paid by the customers for service belongs to the company. The amount, if any, remaining after paying taxes and operating expenses, including the expense of depreciation, is the company's compensation for the use of its property * * * customers pay for service, not for the property used to render it. * * * Property paid for out of moneys received for service belongs to the Company, just as does that purchased out of proceeds of its bonds and stock. * * *"

This case was approvingly quoted by Chief Justice Hughes in Smith vs. Illinois Bell Telephone Company (282 U.S. 133–158) in December, 1930:

> "While it has been held by this court that property paid for out of moneys received for past services belongs to the company, and that the property represented by the credit balance in the reserve for depreciation cannot be used for imposition of a confiscatory rate (citing 271 U.S. 23) it is evident that past experience is an indication of the company's requirement for the future. The recognition of the Ownership of the property represented in the reserve does not make it necessary to allow similar accumulations to go on if they are excessive."

It is significant that the court suggests a change in the annual allowance as contrasted to the NARUC Retroactive Adjustment of the reserve.

The inevitable conclusion to be reached from a study of the opinions of the

U. S. Supreme Court is that any method that is shown by experience to provide a sufficient reserve is legal. "Some average rate * * * or the annual charge is fixed without any regard to the rate of depreciation" would satisfy Mr. Justice Brandeis' requirement. All of the opinions limit the amount of money to be raised to the amount of the retirements, to "maintaining the integrity of the investment." The capital of the corporation must be furnished by stock and bondholders—not by ratepayers. Capital consumed by the wearing out of plant in furnishing service is to be restored to the property by the consumers through charges to operating expenses. It must again be emphasized that this is in no sense a return of money to the investors whose money built the plant. It is money that is to remain in the plant to restore the loss in service value caused by depreciation.

The essential difference between the thoughts expressed by the two committees is that the report of the NARUC committee says in substance that the reserve, when properly computed, is the best measure of accrued depreciation, whereas the Society's committee states that the accrued depreciation is factual in nature and that the amount of such accrued depreciation should be included in the amount of the reserve. No member of the Society's committee would take exception to the use of the reserve when properly computed as a measure of accrued depreciation, but the NARUC committee goes further and recommends (Conclusion 26) "The straight-line method is generally recommended for public utility accounting and financial purposes and also for the computation of both depreciation expenses [annual allowances] and accrued depreciation for purposes of rate making." Such a method, in the opinion of the Society's committee, is not a proper one, if the reserve is to be used as a measure of accrued depreciation. Objection is taken to the use of a reserve as a measure of accrued depreciation, unless the reserve is in reasonable conformity with the actual depreciation.

Accrued depreciation, which manifestly involves many variables, should be determined on its own merits, and the reserve accumulated to cover it, and adjusted to it, by means of changes in the annual allowances. It would seem that such a plan would be the only means available for checking the propriety of the annual allowances. The basic thing sought is the determination of actual depreciation, and the provision of a reserve that is at least equal in amount to it, which reserve is accumulated through annual allowances for depreciation. To fix the accrued depreciation by the reserve set up by allowances, determined by any method, is fixing an item factual in nature, and possible of determination within reasonable limits, by an item that should be subject to frequent changes as conditions change. The object of the reserve is to provide for the actual depreciation and not to have the actual depreciation (a factual matter which cannot be changed) equal to the reserve. It is axiomatic that accrued depreciation which automatically integrates all the time variables can be more accurately estimated than the annual allowance. It therefore should be the starting point.

The NARUC committee recommends that the actual depreciation should be determined by application of the straight-line method, which entails the assumption of probable average useful lives for the different units or groups of units involved, and that the loss in service value would occur in equal annual amounts. The facts are that most adherents to the straight-line theory, of necessity, use tables or probable useful lives prepared by different authorities, mostly general in nature, and having no connection with the property being studied. The term, "of necessity," was used above as very few properties have mortality statistics that can be considered more than roughly indicative, and usually those that have such data do not have them for anything other than mass property. Most mortality statistics relating to mass property are derived from dollar ages, the only ages readily available, rather than property ages. With widely fluctuating prices over the years, the difference may be very substantial. It is probable that no public utility property can ever have mortality statistics on any large percentage of its plant account. Such statistics are of little conclusive value unless they are in the nature of mass data. No property will have mass data on retirements of such items as buildings, boilers, turbines, water purification plants, pumping equipment, and the like. It is probable that the average utility plant, with the possible exception of water works and gas properties having a majority of their investment in pipe lines, would never be able to have conclusive mass data on more than a small percentage of its property.

Even when mortality statistics are available, they cover, generally speaking, units that are not the same as those upon which the amount of accrued depreciation is being determined. Also they cover units retired during a different period and under different conditions. They also cover premature retirements due to casualty, such as storms, fire, automobile collisions and the like. It would seem illogical to say, for instance, that a new pole line had accrued depreciation existing at the start of its use, due to the fact that it might be wrecked by storm during the first year of its life. Saying it another way, the service value of the pole line at any one time would be less because some day the line might be wrecked by storm. That is exactly what takes place when age-life methods are used with useful life figures based upon mortality statistics that include such casualties.

There are many reasons why mortality statistics covering units of property in public utility systems should be used with care, as a basis of determining useful lives for use in age-life calculations. The main reason is that such statistics do not now, and probably never will, show any consistency of "dispersal pattern." The NARUC report likens mortality statistics obtained from utility retirement records to vital statistics upon which life insurance premiums are based, and states that such mortality statistics can be used as a safe means of useful lives for utility property. Vital statistics used by life insurance companies do show a fairly constant "dispersal pattern," but even those statistics are not infallible. This fallibility was recognized by Mr. Justice Brandeis in his dissenting opinion in Pacific Gas and Electric Company vs. San Francisco and in which Justice Holmes concurred, as follows:

> "A depreciation charge resembles a life insurance premium. The depreciation reserve, to which it is credited, supplies insurance for the plant against its inevitable decadence, as the life insurance reserve supplies the fund to meet the agreed value of the lost human life. To determine what

> the amount of the annual life insurance premium should be is a much simpler task than to determine the proper depreciation charge. * * * Moreover, for human lives, mortality tables have been constructed which embody the results of large experience and long study. By their use the required premium may be fixed with an approximation to accuracy. But, despite the relative simplicity of the problem, it was found that the variables leave so wide a margin for error that premiums fixed in accordance with mortality tables work serious injustice either to the insurer or to the insured. Although the purpose was to charge only the appropriate premium, the transaction resulted sometimes in bankruptcy of the insurer; sometimes in his securing profits which seemed extortionate; and rarely, in his receiving only the intended fair compensation for the service rendered. Because every attempt to approximate more nearly the amount of required premium proved futile, justice was sought by another route. Ultimately, strictly mutual insurance was adopted. * * *"

Even though it were possible to forecast accurately the life of a property or unit, the parallel between insurance and straight-line depreciation of utility property ends there, for insurance premiums are based on interest accumulating annual payments while straight-line depreciation neglects the interest factor.

It is apparent that very little conclusive mortality data exist so far as utility property is concerned, and that many of the available data are not properly usable as the basis of useful lives for a determination of accrued depreciation, although they may be valuable in its determination. This means that the selection of useful lives for use in age-life methods must be largely a matter of judgment.

One great difficulty in making the straight-line method seem practical to those experienced in the operation of utility property is the fact that, when it is applied to most properties, it results in many absurdities. There will usually be a number of items, actually in use, that should not be operating if the application of the straight-line method using customary useful life figures is accepted as gospel. The usual useful life placed on power plant buildings is 50 years. To illustrate by an actual case, a building from which, at the time it was 50 years old, the old equipment was removed, a new turbine installed, the building given minor repairs, from every known standpoint could be said to have as much value as a new building starting on a new life cycle. If the usual application of the straight-line method had been applied, this building would have been entirely written off about the time it received its new lease on life. The result of such accounting would have been that the ratepayers contributed the entire cost of building through annual allowances for depreciation, during what appears to be perhaps half its useful life. There are other examples, such as turbines that are topped, meters and transformers that are rebuilt, etc., which make the accurate estimating of useful lives very difficult.

When applied to most utility properties, the application of the straight-line method of determining accrued depreciation produces reserves that are so greatly in excess of the actual depreciation that the answer is manifestly wrong. The fault probably lies partly in the improper determination of useful lives in the particular property being studied, and is principally due to the fact that in actual practice depreciation does not take place in a straight line. The use of any age-life method of computing accrued depreciation, whether straight line, sinking fund, or present worth, is subject to the same inaccuracies so far as selection of useful lives is concerned. It is believed, however, that the interest methods more closely parallel the accrual of actual depreciation.

Perhaps the strongest argument against the use of any mathematically theoretical method of computing accrued depreciation is the fact that actual retirements in seasoned utility properties indicate the complete lack of a sound basis for them. The straight-line method is used quite extensively in industrial properties, and properly so in most cases, as the useful lives of the units making up a large part of the investment are short and amortization as well as depreciation is involved. In the case of utility properties, the reverse is true, as a majority of the units involved have relatively long lives, the property as a whole is continuing, amortization is not a factor in most cases, and equity to the consumers requires that the depreciation reserve be accumulated only as fast as the property is consumed in their service.

It is believed that the statement in the NARUC report to the effect that the use of the straight-line method of figuring both annual allowances for depreciation and accrued depreciation is in the ascendancy expresses a wish rather than a fact. It is true that certain governmental regulatory agencies favor such a plan, but it is certain that, in the field of the public utilities, there is no such ascendancy for determining accrued depreciation. In fact, as depreciation accounting becomes better understood, quite the reverse is true.

As stated previously, the Society's committee takes the stand that accrued depreciation is factual in nature and that accounting should fit the fact rather than let some uniform accounting rule determine what the facts are. This committee further states that, while accrued depreciation is a fact, its determination cannot be made by any mathematical formula or calculation, but must be based upon the consideration of all conditions existing, by someone thoroughly familiar with the design, construction, and operation of the property involved.

VII.—The Straight-Line Method was Developed by the Bell Telephone Companies

The earliest extended discussion in the present century of accounting reserves is to be found in the Annual Reports of the American Telephone and Telegraph Company. These were all signed by Theodore N. Vail, President.

Beginning with the report of 1907, under the captions, Maintenance and Reconstruction, it is pointed out that from 1903 to 1907 about $150,000,000 was spent on maintenance and reconstruction on plant which in 1907 had a value a little under $500,000,000.

In 1908 the report shows an expenditure of about $40,000,000 for these purposes, discusses "Permanency of Plant," and, under the caption "Depreciation," reports that studies were made during the year "to determine the actual rate of depreciation that should be provided for."

Beginning on page 5 of the report of 1910, under the caption "Depreciation," there is a long discussion on the subject of reserves from which the following quotations are taken:

> "The question of depreciation has been considered very critically and analytically during the past year, by commissions and other bodies, in connection with studies on the rate question. While a depreciation reserve was generally favored, there seemed to be a disposition to apply experience and theories, gleaned from other lines of business, to the telephone business.

* * *

> "The present policy of the Bell System is to provide against every probable contingency and to base the amount and extent of such provision on past experience—not on future expectations.

* * *

> "All of this has been made possible through the unremitting study and research of the staff of the Engineering and Experimental Departments of the Company, who by close attention, observation and study, anticipate and provide for all such contingencies and conditions as can possibly be anticipated or provided for in advance."

The next four or five paragraphs discuss the rapid development of the industry. Mr. Vail cited the need for improved construction in advance of hand-to-mouth needs of the day and referred to work then in progress which was estimated to cost $180,000,000, which, if it were to be built piecemeal, the portion then urgently needed would cost $112,000,000 at the time, but the whole job which was then under construction would total $250,000,000. Thus immediate construction of the whole job resulted in an estimated saving of $70,000,000. The subject is then closed with the following paragraph:

> "The capital for this advance construction must be provided by and at the cost of the present, as was the advance construction of the past provided by and at the cost of the past. To the extent that advance construction reduces the cost of necessary plant and anticipates reconstruction and replacement, to that extent the capital charge to be borne by present and future is reduced and to that extent it immediately puts the depreciation reserve to its intended use. The criticism that any excess of reserve is at the cost of the present for the benefit of the future is true, but only to the extent that it may be found eventually in excess of actual retirements. In any case it would be no more than might rightly be considered on insurance against obsolescence which cannot be foreseen."

In the 1911 Annual Report of the American Telephone and Telegraph Company, on pages 5 to 11, Mr. Vail went to the extreme in his argument for reserves created by charges to operating expenses. Space is available in this discussion for only one quotation of the many which the Society committee would like to use in presenting the argument made at that time for straight-line reserves.

> "There are however, in the conduct of business many conditions and possibilities which can not be met out of current net revenue and should not be met out of capital, but which if not provided for in some way would put all industrial companies on a speculative basis.
>
> "There is that obsolescence which comes from revolutionary improvements necessitating wholesale replacement of obsolete apparatus or plant, such as the replacement of overhead systems by underground systems, or such as took place when the present method of operating was introduced. There are those fluctuations in net revenue caused by business depression which cannot be overcome by immediate reduction of fixed charges, overhead expense or operating costs. There is that constant tendency to increase in

> wages and cost of material, that tendency to increase in operating expenses and capital charges caused by constant demand for increased efficiency of service, that demand for extensions, productive and unproductive, and that call for improvements in plant, equipment and apparatus. There is that increase in costs of operating in greater ratio than the increase in business, peculiar to some branches of the telephone service.
>
> "These and many other possibilities always confronting industrial and public service undertakings must be provided for. They are not the subject of capital expenditures, and can only be provided for by an accumulating surplus and reserves invested in productive plant or securities. If these are not provided for, trouble if not disaster or destruction is inevitable."

In the report for the year 1913, on page 10, is the following paragraph:

> "All of the present surplus and reserves aggregating $174,000,000 is invested in tangible and productive property the revenue from which enables the companies to maintain their efficiency, without paying capital charges on this amount."

The previously quoted paragraph from the Louisiana Telephone Case in 1909 seems to indicate that the Telephone Companies had reserves as early as that which were large enough to create the impression that this form of accounting (straight line) was creating excessive accumulations of money which were being invested in new plant.

The studies made by the Bell Telephone officials in the earlier years, 1905 to 1909, resulted in the development of the Accounting Classification of the Bell Telephone Company about 1909 or 1910. The classifications of the New York Public Service Commission and of the Interstate Commerce Commission issued in 1911 or 1912 were practically verbatim copies of the Telephone Company Classification with some slight rearrangement of paragraphs.

The Company has what are probably the most complete records of the life history of property retired over a long period to be found in any utility. Life estimates are based on these studies. Equally fine are the studies of retirements, salvage, cost of removal, etc. The many reports to state and federal commissions, exhibits in the many litigated cases of record, and other published statistics of the telephone utility, make available a great volume of statistical data. There are probably no more complete and accurate records extant than those of the Bell Telephone Companies.

VIII.—The Experience of the Telephone Utility with Accounting by the Straight-Line Method

In view of the great mass of statistical information available to the state and federal regulatory commissions and other governmental agencies it is a matter of considerable surprise that the NARUC committee in its study, extending over four years, did not investigate the actual results of the accounting methods in use by the telephone, railroad and electric utilities instead of setting up hypothetical computations.

The data drawn from various unimpeachable sources as to the actual experience over a long period of years furnish the best proof of the results of any particular method of accounting. Whether the results are adequate for the purpose or excessive, whether the hidden benefits, such as saving of interest to a corporation by the use of money represented by the credit balance in the reserve is substantial or not, can only be determined by an analysis of the record.

The Society's committee has made a partial study of the Bell telephone records available to its members. In order to obviate the use of many tables, and avoid great masses of dollar figures running into hundreds of millions or billions of dollars; and to reduce the figures to terms that will permit comparison of the results secured by the use of different accounting methods by different utilities, the tables showing charges to operating expenses, retirements, and growth of credit balances are generally stated in terms of percentage of fixed capital or depreciable plant. To indicate the magnitude of the amounts of money involved, the summaries of the conclusions of the committee are stated in dollar figures.

The telephone utility had its first beginnings in the late 1870's or early 1880's. During the last decade of the past century and the first decade of this one was the period of promotion when scores or hundreds of local telephone companies were organized in many cities and small areas; when such long-distance lines as existed were almost wholly in the hands of the Bell System and were in the process of expansion and development of equipment, and when many of the smaller companies and local systems were being absorbed by the Bell and other large companies.

During the next three decades the process of consolidation of companies, retirement of duplicate plant and of old and obsolete properties, and of invention and development went on. The annual reports of the Federal Communications Commission show that in 1930 there were 136 reporting corporations, whereas in 1938 there were only 73. In 1938 the Commission reported for these 73 companies the values listed as Item 1, Table 1. For the same year, Item 2 gives the statistics of the Bell System, which would indicate, for all other properties, the values given as Item 3, Table 1. The total credit balance in Item 2 is 27.84% of the total plant investment, or 30.1% of the depreciable plant. Obviously either the small properties have been unable to build up the reserves as rapidly as the Bell System or they began reserve accounting at a later date.

TABLE 1.—DEPRECIATION RESERVE DATA FOR 1938; REPORTS OF THE FEDERAL COMMUNICATIONS COMMISSION

No.	Description	Total investment in plant (dollars)	Depreciation Reserve	
			Dollars	Percentage of total
1	73 telephone companies	4,783,082,000	1,316,368,000	27.52
2	Bell Telephone System	4,489,078,000	1,249,563,000	27.84
3	All other properties	294,004,000	66,805,000	22.72

The statistics of the one system which has 93.85% of the total telephone investment in the country furnish an excellent illustration of the operation of the straight-line method.

From the early annual reports of the American Telephone and Telegraph Corporation we find that in 1907 the total investment in plant was a little under

$500,000,000. On December 31, 1913, this had increased to $820,885,000 of which 90% was classified as depreciable. The investment in plant steadily increased until it reached $4,183,236,000 in 1931 and for four years was practically stationary or slightly lower. In 1935 it was $4,187,790,000 since which it steadily increased to $5,387,790,000 on December 31, 1943.

During the period from December 31, 1913, to December 31, 1943, the credit balance in the depreciation reserve increased from $107,081,000 to $1,713,175,000. Thus it will be noted that while plant investment increased 556⅓% during the 30-yr period the credit balance in the reserve increased 1500%.

A careful study of the growth of the credit balance reveals the fact that, while its growth, in terms of percentage of depreciable plant, was negligible between 1917 when it was 20.55% of plant value, and 1933, when it stood at 22.63%, the lowest intermediate figure being 20.39%, the highest 25.39%, in terms of percentage of depreciable property the credit balance has increased, in dollars, every year since 1933 and at the end of 1943 stood at 34.34%, or more than one third of depreciable plant.

When the increase of the credit balance in dollars is considered, however, it is to be noted that there has been an increase every year in the terms of tens to hundreds of millions of dollars.

Col. 1, Table 2, shows the growth, in dollars, of the depreciation reserve as above analyzed, and Col. 2 shows the dollars of increase in each year. The total increase in the credit balance was $1,605,994,000 in the 30-yr period of which $914,923,000 came in the 11-yr period from December 31, 1932, to December 31, 1943, during which the investment in depreciable plant increased $1,138,467,000. Thus the money reflected by the increased credit balance in the reserve would have been sufficient to pay for more than 80% of the plant growth.

If Table 2 be again referred to it will be noted that from 1913 to 1926 the annual charges to operating expenses for "depreciation" (Col. 4) were in excess of the total retirements (Col. 5) in all years except one, while from 1927 to 1943 total retirements were substantially above the annual "depreciation" charge in all but the years 1935 and 1943. This fact apparently furnishes justification for the substantial percentages charged to operating expenses, but on the other hand is in conflict with the rapid growth of the credit balance. This calls for analysis.

Study of the statistics shows that plant retired over the entire period cost $4,184,461,000 while total charges to operating expenses amounted to $3,833,435,000, or $351,026,000 less than total retirements, whereas as already noted the credit balance increased more than one billion six hundred million dollars. This directs attention to salvage. The straight-line formula calls for an estimate of life of property and for an estimate of salvage. On actual retirement cost of retired property less net salvage is charged to the credit balance. Data for the analysis of the actual salvage of this more than four billion dollars worth of retired property are not available to the Society's committee, nor are they essential. Accounting accuracy has been almost a religion with the telephone staff for a third of a century and the results of that accounting may be accepted as correct. The point which the Society's committee desires to bring out is the extreme difficulty which will be encountered when an attempt is made to check salvage records kept under the straight-line method, or to order changes in that method, on a great far-flung property with an investment of hundeds of millions of dollars and with an accounting system less complete than that of the Bell Telephone Companies.

TABLE 2.—TELEPHONE RESERVE ACCOUNTING, ALL COMPANIES FORMING THE BELL SYSTEM

(Percentages in Terms of Depreciable Plant)

Year	Credit balance in depreciation reserve (thousands of dollars) (1)	Annual increase in credit balance (thousands of dollars) (2)	Credit balance in percentage of depreciable plant (3)	Charge to operating expense for depreciation (percentage of depreciable plant) (4)	Total retirements (percentage of depreciable plant) (5)
1913	107,181	....	14.55	5.01	5.10
14	122,338	15,157	15.58	5.15	5.07
15	142,307	19,969	17.38	5.38	4.60
16	168,044	25,737	19.25	5.63	4.55
17	201,090	33,046	20.55	5.35	4.26
1918	235,395	34,305	22.32	4.96	3.70
19	276,304	40,909	24.52	5.64	3.80
20	303,076	26,772	24.91	5.37	3.59
21	343,331	40,255	25.06	5.14	3.60
22	387,148	43,817	25.39	5.23	4.00
1923	434,129	46,981	24.78	4.97	4.06
24	476,374	42,245	23.07	4.87	4.37
25	519,023	42,649	22.33	5.01	4.80
26	564,541	45,518	21.86	4.98	4.93
27	584,762	20,221	20.93	5.05	5.51
1928	634,030	49,268	20.90	5.03	5.72
29	681,611	47,581	20.39	4.88	6.15
30	720,690	39,079	19.68	4.92	6.63
31	768,817	48,127	20.14	4.96	6.42
32	798,252	29,435	20.73	4.63	6.69
1933	876,373	78,121	22.63	4.34	4.96
34	950,628	74,255	24.46	3.87	4.22
35	1,042,011	91,383	26.65	4.31	4.20
36	1,124,809	82,798	28.38	4.06	4.30
37	1,195,269	70,460	29.43	3.98	4.66
1938	1,249,563	54,294	30.10	3.79	4.50
39	1,300,660	51,097	30.72	3.78	4.97
40	1,357,091	56,431	31.04	3.74	5.08
41	1,440,224	93,133	31.12	3.74	4.73
42	1,556,095	115,871	31.85	3.82	4.11
1943	1,713,175	157,080	34.34	3.95	2.86

Table 3 is derived from a table introduced by the New York Telephone Company and marked for identification in connection with the cross examination of a nationally known authority on accounting at Albany, N. Y., April 15, 1930. It is a summary of gross retirements of that company for the period 1910–1929 inclusive, sales excluded, showing the average percentage that each class of property is of total property retired during that 20-yr period together with the portion of original cost recoverable through salvage for each of these classes of property.

This 20-yr average percentage of net salvage on the retirement of nearly 193 million dollars does not, however, furnish any clue to the average annual net salvage from retirements during the 20-yr period. During these years the lowest percentage was 24.62%, the highest 59.02%. The average percentages for five years were less than 40% and five above 50%. This discussion of actual salvage experience is introduced to make it perfectly clear that the estimates of the "life" of plant or of "salvage" are not the simple matters that are indicated by the advocates of the straight-line method when the accounting deals with property of hundreds of millions of dollars.

TABLE 3.—TOTALS OF PLANT OF VARIOUS KINDS WITH NET SALVAGE THEREFROM RETIRED DURING PERIOD 1910–1929, INCLUSIVE

Acct. No.	Portion of property	Original cost, sales excluded	NET SALVAGE; SALES EXCLUDED	
			Dollars	Percentage of original cost
(1)	(2)	(3)	(4)	(5)
207	Rights of way	932,166	11,433	1.2
212	Buildings	6,253,816	1,020,156	16.3
221	Central office (C.O.) telephone equipment	31,717,472	4,416,055	13.9
222	Other equipment of central office	3,453,943	1,647,833	47.7
231	Station apparatus	54,691,545	40,384,344	73.8
234	Private branch exchange (P.B.X.) switchboards	31,473,557	21,313,744	67.7
234	Booths and special fittings	3,815,608	3,091,958	81.0
241	Exchange pole lines	8,935,427	1,909,547[a]	21.4[a]
242	Exchange aerial cable	7,858,225	1,547,781	19.7
251	Toll pole lines	4,390,289	176,142[a]	4.0[a]
252	Toll aerial cable	726,134	345,036	47.5
253	Toll aerial wire	3,073,548	1,285,938	41.8
244–14	Exchange underground (U.G.) conduit main	302,179	8,655[a]	2.9[a]
244–24	Exchange underground conduit subsidiary	123,459	4,673	3.8
245–15	Exchange underground cable main	9,543,735	5,477,389	57.4
245–25	Exchange underground cable subsidiary	7,289,682	165,782	2.3
246	Exchange submarine cable	270,556	52,672	19.5
254	Toll underground conduit	13,591	5,385	39.6
255	Toll underground cable	2,992,005	1,866,188	62.4
256	Toll submarine cable	329,493	105,750	32.1
261	Office furniture and fixtures	2,186,961	610,185	27.9
262	General shop equipment	67,734	29,492	43.5
263	General store equipment	206,897	74,400	36.0
264	General stable and garage equipment	3,547,095	563,462	15.9
265	General tools and implements	7,818,702	4,646,990	59.4
268	Interest during construction	920,730	....	....
274	Miscellaneous construction expense[b]	....	....	....
	Grand total	192,934,549	86,572,304	44.87

[a] Negative net salvage; that is, cost of removing exceeds gross salvage. [b] In view of special character of this small class of property, and of meager amount of available actual experience data relative thereto, quantities for this item have been omitted.

Table 3 brings out clearly the high percentages of salvage of telephone properties under the accounting plan of the telephone companies, higher probably than percentages to be found in any other utility industry due to the use, after restoration of many items of plant. This exhibit clearly shows the difficulty of the checking of accounts by people not familiar with these great properties.

Interest on the Credit Balance.—The credit balance in the reserve for retirement or so-called "depreciation" reflects money which has been collected from the ratepayers. Any excess over the total amount of retirements is usually invested in property additions. Mr. Vail stated in the 1913 American Telephone and Telegraph Company report that "all the present surplus and reserves is invested in * * * property * * * without paying capital charges on this amount." At that time the credit balance was well in excess of 107 million dollars and saving of interest was in terms of millions of dollars per year whatever rate be used for the computation. If simple interest be computed on all the credit balances over the 30-yr period shown in Table 2, Col. 1, it will be found to be the equivalent of one year's interest on more than twenty-one billion dollars. No matter what rate of interest be used the resultant saving to the corporation of interest on money used for legitimate corporate uses, such as construction of new plant, is found to amount to hundreds of millions of dollars.

Apparently this question of interest saved to the owning corporation by the use of money reflected by the credit balance has not been made an issue in any of the telephone cases involving rates and valuation in the past, although it was suggested by the company in the Lindheimer Case "that an adjustment might be made by giving credit in favor of the telephone users in an amount equal to 3½% upon the difference between the depreciation reserve and the amount deducted from the valuation for existing depreciation." No matter what method is adopted in accounting the fact is inescapable that the credit balance in the reserve reflects money that has been advanced by ratepayers through charges to operating expenses in excess of needs; that its use in construction of plant obviates the securing of capital on which interest or dividends would be paid; and that excessive reserves not only constitute what Prof. H. W. Hatfield in his books on accounting terms "hidden surplus" in themselves, but secure to the corporation a substantial return in the form of saving of interest. This subject is again referred to later in this report. No emphasis is placed on this important matter in the NARUC report.

Early in this discussion of the straight-line reserves of the telephone company attention was directed to the increase of the credit balance in terms of percentage of fixed capital. From December 31, 1930, to December 31, 1940, during which time the credit balance increased from 19.68% of depreciable plant to 31.04%, the increase of the credit balance amounted to $636,131,000. In that same period the total investment in plant showed an increase of $706,437,000. Thus it appears that during these years of depression when all American business was seriously affected, the equivalent of 90% of all the money for extension of plant was derived from operating expenses through the depreciation reserve.

The Bell telephone industry developed the straight-line method in its accounting before railroad accounting came under regulation and before any regulation of other utilities. It fully and frankly discussed its system of accounting in its annual reports, pointing out matters that were open to criticism. The method has done all that Mr. Vail predicted for it and the results probably exceed any early expectations of the officers.

The company has defended its position in many lawsuits. It has been sustained in the perfectly sound position that money collected through rates belongs to the company and that property which reflects reserves created by money paid for rates, under commission approval, also belongs to the company. The Smith and Lindheimer cases, however, have held that excessive contributions to reserves should not be permitted to continue and the Lindheimer Case places the burden of proof on the company to show that they are not excessive. As the law has been established by the opinions of the Court, the reserves are higher than needed for retirement and a reasonable balance to care for coming retirements. In the past they have served to build up an industry that has completely revolutionized the business and social communication system of the world, and through thorough research in this and other fields has perfected instrumentalities of service worth far more to the nation than any excess that has been paid by the patrons of the service.

The Bell Telephone Utility initiated and perfected its system of accounting just as the Congress approved public regulation of Railroad Accounting. As case after case involving utility rates and accounting practice has been decided by the Supreme Court a body of law has been built up which definitely limits the objectives of reserve accounting, and makes it very clear that the method used by the telephone and railroad utilities results in charges to operating expenses and the accumulation of reserve balances that are substantially in excess of permissible amounts.

IX.—The Experience of the Railroads with Accounting by the Straight-Line Method

The railroads of the United States have been under the regulation of the Interstate Commerce Commission since 1887, but the authority of the Commission was not extended to the regulation of Accounting until 1907. At that time the late Dr. Henry C. Adams was "In charge of statistics and accounts" for the Commission. Professor Adams had been statistician for the Commission from its organization and was familiar with conditions on the carriers. He was associated as a consultant on the Michigan Valuations of Railroads in 1900 and 1903. He was much concerned at the very large deductions for depreciation which were made by this and other later State Valuations on a number of decadent little railroads and on the then dying lumber roads and branch lines of the large systems into the timber areas of the states of Michigan, Wisconsin, and Minnesota. The introduction of depreciation of equipment as an operating expense account in the first classification was largely due to Dr. Adams' concern regarding the failure of many companies to make any provision in accounting for recouping capital losses due to retirement of road and equipment. Many of the larger roads made a practice of charging replacements, including many large additions or betterments, to operating expenses. The regulation of accounting resulted in the abolition of many very serious evils. The weaker roads, however, were unable to set aside reserves to maintain value even though the state valuations disclosed serious amounts of actual depreciation or loss of service value.

Depreciation of equipment was provided for in the first classification of accounts. The second classification in 1914 provided for depreciation of way and structures, but owing to the strenuous opposition of the carriers the creation of reserves was made mandatory for equipment and optional for way and structures. In the third classification of 1936 depreciation for way and structures was again left optional with the carrier.

The straight-line method is specified in railroad accounting. On pages 88 and 89 of the 1936 classification the following excerpts are to be found:

> "These charges for currently accruing depreciation shall be computed at such percentage rate of the ledger value of the equipment that the service value may be distributed under the straight-line method in equal annual charges to operating expenses and other accounts during the estimated service life of the equipment.
>
> * * *
>
> "(e) The carrier shall keep such records of equipment retirements as will reflect the service lives and value of salvage of each class of equipment; shall maintain in convenient and accessible form engineering and other data bearing on prospective service lives; and shall be prepared at any time upon direction of this commission to compute and submit for its approval, new percentage rates to take the place of those based on service lives or value of salvage found to be inaccurate."

The annual statistical reports of the Interstate Commerce Commission are among the most complete of all government statistics and furnish a wonderful record of railroad statistics and accounts for more than half a century but unfortunately do not permit a complete analysis of the accounting for depreciation until very recent years. Prior to 1934 the summaries for all class 1 carriers do not separate, on the balance sheet, either investment or depreciation, as between road and structures and equipment. As many carriers do not charge depreciation to operating expenses for road and structures at all, and those that do make only very limited charges, the resulting percentage that the credit balance is of the investment furnishes no clue to the effect of the required straight-line accounting for equipment.

Table 4, derived from the Commission's Table 155, "Development of Steam Railways—1890–1942," shows the total investment in road and equipment for all United States railroads and total reserves for depreciation. This shows the fairly rapid growth of investment in railroads from 1911 to 1930 and the complete lack of growth from 1930 to 1942, the latter year showing less total investment than 1930. During these years the growth of the total credit balance from 1.35% in 1911 to 13.78% in 1942 gives no indication of the effect of the straight-line method adopted in accounting for equipment and the resulting percentages of credit balance to investment place the total reserves in the class of those of the electric companies, but do not represent the effect of any "method."

An analysis of the 1942 statistics shows that of a total of $3,232,242,347 in the credit balance of reserves of class I carriers only $171,249,504, or 5.3% of this total, was accrued for way and structures, and $3,060,992,843, or 94.7%, was accrued for depreciation of equipment.

Not until 1934 did the statistics of the Interstate Commerce Commission show in the summarized balance sheets for all class I carriers the separation of the investment on the asset side and of the reserve for depreciation on the liability side as between way and structures, and equipment. To make this separation and prepare a table showing the growth of the reserve for equipment would be a difficult task which the Society's committee would not be justified in undertaking, if it were possible to do so.

In 1926, in the consideration of Docket 15100, the Interstate Commerce

Commission had a study made of the depreciation of equipment from the reports of carriers which made the separation between road and equipment in their returns. This was a smaller group than that analyzed in Table 4, having 81.3% of the investment and 81.1% of the amount of money in the credit balance. This study furnishes a basis for showing the rate of growth of the credit balance from 1914 to 1923 (118 I.C.C. p. 335). The Interstate Commerce Commission statistics give the data for all class I carriers from 1934 to 1942.

TABLE 4.—GROWTH OF DEPRECIATION RESERVES OF RAILROADS
(From Table 155, Interstate Commerce Commission Reports on Depreciation of Road and Equipment)

Year ended	Investment in road and equipment book value (thousands of dollars)	DEPRECIATION RESERVE; ROAD AND EQUIPMENT	
		Thousands of dollars	Percentage of investment
	(1)	(2)	(3)
June 30:			
1911	15,612,379	210,466	1.35
1912	16,004,745	259,661	1.62
1913	16,588,603	327,847	1.98
1914	17,153,786	435,835	2.54
1915	17,441,420	511,452	2.93
1916	17,689,425	571,360	3.23
Dec. 31:			
1916	17,842,777	628,934	3.52
1917	18,574,298	796,395	4.29
1918	18,984,756	936,979	4.93
1919	19,300,121	1,009,322	5.23
1920	19,849,320	1,081,342	5.44
1921	20,329,224	1,237,030	6.08
1922	20,580,001	1,335,453	6.50
1923	21,372,858	1,408,461	6.59
1924	22,182,267	1,547,969	6.98
1925	23,217,209	1,681,291	7.24
1926	23,880,740	1,811,002	7.58
1927	24,453,871	1,946,798	7.96
1928	24,875,984	2,043,976	8.22
1929	25,465,036	2,169,736	8.52
1930	26,051,000	2,360,767	9.06
1931	26,094,899	2,520,738	9.66
1932	26,086,991	2,632,922	10.09
1933	25,901,962	2,707,942	10.45
1934	25,681,608	2,764,726	10.75
1935	25,500,465	2,771,404	10.86
1936	25,432,388	2,809,063	11.45
1937	25,636,082	2,950,848	11.53
1938	25,595,739	3,044,972	11.89
1939	25,538,157	3,102,779	12.18
1940	25,646,014	3,095,237	12.10
1941	25,668,984	3,240,145	12.62
1942	25,838,351	3,561,570	13.78

From these sources Tables 5 and 6 have been prepared. The missing years 1924 to 1933, inclusive, in Table 5, would undoubtedly show a much more rapid increase in the credit balance after 1929 as from that date to 1936 the investment in equipment was practically stationary.

Analysis of Table 5 shows that investment in railway equipment has increased more than one billion dollars from 1936 to 1942 while during the same period the credit balance increased $686,197,000, or enough to pay for two thirds of the increase.

Some advocates of the straight-line method contend that as a result of its use the reserve will ultimately reach 50% of the investment and remain substantially stationary. Table 5 shows that this condition has nearly been reached with railway equipment. The statistics of the Interstate Commerce Commission, if thorough analysis were to be made, would hardly sustain the contention that the 49.03% credit balance in 1942 represents the physical condition of the equipment at that time. It is noted from Table 155 in the Interstate Commerce Commission statistics that in every year since 1911 there has been an increase in tractive power of locomotives and in capacity of freight cars, the present-day locomotive having 86% more power and the freight car

TABLE 5.—GROWTH OF DEPRECIATION RESERVES OF RAILROADS[a]; DEPRECIATION OF EQUIPMENT

Date[b]	Plant cost (thousands of dollars)	CREDIT BALANCE	
		Thousands of dollars	Percentage of plant cost
June 30:			
1914	1,475,589	243,589	16.48
1915	1,750,092	306,917	17.54
Dec. 31:			
1916	2,188,356	406,076	18.56
1917	2,462,635	539,074	21.89
1918	2,644,334	568,071	21.48
1919	2,749,519	631,647	22.97
1920	3,000,245	760,086	25.33
1921	3,076,335	874,659	28.43
1922	3,266,814	934,886	28.62
1923	3,967,271	1,074,892	27.09
....	....	....	....
....	....	....	....
....	....	....	....
1934	5,383,002	2,362,003	43.70
1935	5,269,885	2,369,273	44.96
1936	5,228,117	2,374,796	45.42
1937	5,483,350	2,504,131	45.67
1938	5,498,124	2,605,773	47.40
1939	5,475,784	2,661,811	48.61
1940	5,608,202	2,737,689	48.80
1941	5,869,545	2,866,570	48.84
1942	6,243,271	3,060,993	49.03

[a] Roads that report depreciation of equipment. [b] Years 1914 to 1923, inclusive, from 118 I.C.C. pp. 334 and 335. Years 1934 to 1942, inclusive, from Annual Statistics of Railroads, Interstate Commerce Commission.

TABLE 6.—CREDITS AND DEBITS; EQUIPMENT RESERVES FOR DEPRECIATION
(From I.C.C. Annual Statistical Reports)

Year	Total credits to reserve	Percentage of investment	Total debits to reserve	Percentage of investment
1938	$199,893,533	3.63	$ 98,250,949	1.80
1939	183,451,275	3.35	126,829,566	2.31
1940	187,680,918	3.34	111,802,662	1.99
1941	197,276,722	3.36	81,658,354	1.39
1942	262,501,070	4.20	60,022,716	0.96

37% greater capacity. The old all-wooden cars have largely disappeared and steel or steel under frame cars have taken their place. It is further noted that the highest average traffic densities on all class I carriers between 1910 and 1940 were in the years 1926 to 1929, the highest being 1,732,295 ton miles per mile of road in 1926. Densities of 1,950,166 in 1941 and 2,638,067 in 1942 do not indicate a physical condition of equipment corresponding to the percentage shown for the reserve balance.

Examination of individual reports of carriers shows great differences in the percentages of the reserve for "depreciation" in the road accounts, ranging from nothing at all to 4% or 5%, but in the required accounting for equipment all of them show high percentages ranging from 45% to 49% and in a few cases of small roads with a limited amount of equipment the reserve exceeds 50%.

The most noticeable result of an analysis of these "straight-line" reserves is that, in periods of depression or other economic disturbance, the charges to operating expenses greatly exceed the retirements and actual accrued depreciation which would be deducted in valuation, and result in credit balances which reflect the collection of money from this source sufficient to defray a very large part of the cost of new construction. That is not the purpose for which depreciation reserves are set up.

To illustrate: (1) The reports of the Federal Communications Commission show that, from 1930 to 1938, the credit balance in the "depreciation" reserves of class A telephone companies increased $554,022,000, or 72.6%, while in the same period the total investment in plant of these companies increased $565,372,000 or 13.4%. In other words, the increase in the reserves provided more than 97% of the money needed for plant extension during this period of economic depression. (2) The reports of the Interstate Commerce Commission show that from 1930 to 1942, a period four years longer, the railroad plant account for all class I carriers decreased $212,650,000, or 0.81%, while the credit balance increased $1,200,803,000 or 50.8%. During the period there were many changes and great improvements in the equipment of the railroads of the nation.

The Society's committee points out these facts as they are reported by the commissions for the purpose of showing the results of straight-line accounting. Undoubtedly the money reflected in the reserve balances of the railroads went far to put them in condition to handle the greatest traffic in their history in the years immediately following this 11-yr period. However, financing plant additions is not the purpose for which depreciation charges are made.

The statistics that have been cited as to the two great industries, one of which may be said to have originated the straight-line method as a means of creating reserves for retirement, the other of which was compelled by the Interstate Commerce Commission to use it, are presented to substantiate this committee's belief that the straight-line method accrues substantially more than is a proper reserve for retirement of plant, and that it is not a simple but a very involved method which has been, and is, difficult and costly for commissions to regulate.

X.—The Experience of Industries Which do not Use Straight-Line Method

Water Works.—

The Committee on Depreciation of the American Water Works Association (A.W.W.A.), in a study of 181 privately owned water works properties, reports for the year 1943 the following figures:

Total depreciable property	$282,082,402
Retirement reserves	38,092,121 (13.4%)
Annual depreciation allowance	2,820,540 (1.0%)

Similar figures for 107 municipally owned water works are reported in the August 1944 Journal of the A.W.W.A. as follows: Total depreciable property $1,047,000,000, depreciation reserves equal to 27% of depreciable property and annual depreciation allowances of 1.77%. Publicly owned utilities commonly accrue excessive reserves as a deterrent to the diversion of profits to other channels and to provide funds required for new construction without the necessity of bond issues.

Gas Companies.—

The Committee on Depreciation of the American Gas Association has recently made a study of 72 privately owned properties.

The committee's study reveals the following facts:

Total plant accounts	$6,650,836,000
Retirement reserves exclusive of depletion	1,018,701,000 (15.3% of plant cost)

The Committee of the American Gas Association states that "The essential purpose of a retirement reserve is to provide a reserve adequate for the retirement of major units of property, not taken care of by maintenance, and to equalize large retirement losses over periods of years." This is an excellent statement of what is actually being done by most of the properties which do not use the straight-line method.

Natural Gas, Pipe Lines, Etc.—

The Society's committee has made no study of the practice of these utilities in the matter of reserves for depreciation or amortization. Gas reserves and in some cases the pipe lines from them are wasting assets. The problem with them is amortization rather than depreciation so far as the wasting assets are concerned.

The Electric Power Utility.—

The Electric Power Utility has used for many years the classification of the NARUC and, since 1937, the almost exactly similar classification of the Federal Power Commission. The NARUC Classification reads as follows:

> "Each utility shall record as at the end of each month the estimated amount of depreciation or amortization and depletion accrued during that month on utility plant subject to charges for depreciation, amortization and depletion."

The Federal Power Commission classification differs only by one word; the word "utility" plant is changed to read "electric" plant; and, on page 89, the classification provides for six subdivisions: Steam, Hydro and Internal Combustion Plants, and Transmission, Distribution, and General Plant.

This leaves the estimating of a proper and reasonable charge to the management of each company, without any restrictions as to the method of making

TABLE 7.—FEDERAL POWER COMMISSION DATA COVERING THE ENTIRE INDUSTRY

Year	Total utility; book cost (thousands of dollars)	RESERVE FOR DEPRECIATION AND AMORTIZATION OF UTILITY		CHARGE TO EXPENSE FOR DEPRECIATION AND AMORTIZATION	
		Thousands of dollars	Percentage of total plant cost	Thousands of dollars	Percentage of total plant cost
1937	13,851,583	1,495,250	10.80	212,865	1.53
1938	14,048,019	1,629,182	11.59	223,540	1.58
1939	14,114,695	1,762,386	12.48	243,178	1.70
1940	14,406,987	1,912,974	13.28	256,396	1.80
1941	14,736,793	2,096,412	14.22	274,913	1.87
1942	14,848,281	2,306,145	15.53	285,809	1.94
Increase, 1942 over 1937:					
Dollars	996,698	810,895	....	....	....
Percentage	7.2	54.2	....	....	....

TABLE 8.—RETIREMENT RESERVES AS REPORTED BY SEVENTEEN COMPANIES

Year	Plant total (thousands of dollars)	CREDIT TO RESERVE		CHARGES TO RESERVE		RESERVE BALANCE	
		Total (thousands of dollars)	Percentage of plant	Total (thousands of dollars)	Percentage of plant	Total (thousands of dollars)	Percentage of plant
1913	220,447	4,456	2.02	2,418	1.10	9,364	4.25
1914	245,547	4,595	1.87	1,369	0.56	12,696	5.17
1915	269,924	6,212	2.30	1,889	0.70	17,016	6.30
1916	293,649	6,636	2.27	3,196	1.09	20,903	7.12
1917	317,696	8,376	2.64	2,627	0.83	28,091	8.84
1918	449,952	7,961	1.77	4,216	0.94	34,395	7.64
1919	478,697	8,732	1.82	4,029	0.84	40,532	8.47
1920	522,686	11,257	2.15	5,374	1.03	47,327	9.05
1921	630,842	13,528	2.14	8,271	1.31	59,120	9.37
1922	713,638	16,084	2.25	8,972	1.26	63,752	8.93
1923	793,324	18,883	2.38	9,928	1.25	73,246	9.23
1924	917,109	20,846	2.27	11,603	1.27	85,285	9.30
1925	1,087,043	25,278	2.33	14,872	1.37	98,599	9.07
1926	1,232,047	29,825	2.42	17,444	1.42	112,804	9.16
1927	1,364,055	33,966	2.49	20,651	1.51	126,963	9.31
1928	1,534,366	38,403	2.50	22,861	1.49	143,393	9.35
1929	1,672,044	38,992	2.33	24,559	1.47	159,328	9.53
1930	1,855,641	37,802	2.04	28,697	1.55	170,587	9.19
1931	1,955,849	35,064	1.79	28,460	1.46	179,231	9.16
1932	2,017,912	35,962	1.78	18,391	0.91	200,460	9.93
1933	2,073,801	37,511	1.81	26,575	1.28	212,180	10.23
1934	2,084,583	41,029	1.97	28,859	1.38	226,176	10.85
1935	2,082,882	38,683	1.86	22,711	1.09	245,676	11.80
1936	2,103,320	49,682	2.36	22,491	1.05	275,456	13.10
1937	2,143,567	46,789	2.18	21,946	1.02	303,379	14.15
1938	2,231,943	53,007	2.37	23,485	1.05	341,220	15.29
1939	2,324,684	55,319	2.38	30,926	1.33	374,969	16.13
1940	2,358,328	57,745	2.45	33,805	1.43	400,555	16.98
1941	2,430,894	62,059	2.55	35,023	1.44	430,500	17.70
1942	2,493,766	66,052	2.65	25,253	1.01	472,953	18.97
1943	2,493,233	66,584	2.67	20,837	0.84	499,732	20.04

the estimate, but subject to the approval of the commission in control. This is a wise provision as there are great differences in the requirements of different companies in any industry, and also between industries. This provision is primarily a matter for determination by the management subject to the orders of the regulatory body. There now exist records under regulated accounting of from 25 to 30 years of actual retirements, salvage and other matters upon which most companies can base their estimates of their annual requirements which will maintain an adequate reserve. Since the year 1937 estimates have been subdivided as prescribed by the Federal Power Commission.

TABLE 9.—A MIDWESTERN POWER COMPANY; STATE-WIDE SERVICE
(All Figures Stated in Terms of Percentage of Book Value of Plant as of December 31st)

Year[a] (1)	Charges to operating expense for depreciation amortization (2)	Total credits to reserve (3)	Total debits to reserve (4)	Excess of credits over debits (5)	Credit balance in reserve (6)
1918	1.31	1.44	0.24	1.20	2.10
19	1.28	1.35	0.17	1.18	3.23
1920	1.15	1.23	0.57	0.66	3.57
21	1.65	1.75	1.25	0.51	3.92
22	1.43	2.12	0.60	1.51	4.56
23	1.43	1.55	1.06	0.49	4.60
24	1.48	1.64	1.43	0.22	4.24
1925	1.35	1.63	0.95	0.69	4.34
26	1.37	1.42	0.63	0.80	4.81
27	1.15	1.54	0.91	0.62	4.64
28	1.37	1.44	0.70	0.74	5.00
29	1.42	1.58	1.13	0.45	4.95
1930	1.48	1.95	1.17	0.78	5.06
31	1.41	1.51	1.11	0.40	5.19
32	1.38	1.44	0.70	0.73	5.84
33	1.37	1.60	1.13	0.47	6.26
34	1.36	2.62	1.70	0.92	6.98
1935	1.33	1.39	0.98	0.40	7.23
36	1.43	2.14	1.14	1.00	8.04
37	1.70	1.90	0.91	0.99	8.77
38	1.68	1.80	0.89	0.91	9.22
39	1.86	2.12	1.57	0.55	9.37
1940	1.99	2.14	1.60	0.54	9.61
41	2.44	2.60	2.45	0.16	9.63
42	2.82	3.06	1.70	1.36	10.90

[a] During this period, book value increased from approximately $50,000,000 to approximately $289,000,000, or 477%. The major part of this growth, 330%, to $215,000,000, was from 1918 to 1931; hence the more rapid growth of the reserve from 1931 to 1942. The 1937 cost was $247,200,000. Credit balances were as follows:

Year Balance
1918 = $ 1,050,000
1931 = 11,158,500
1937 = 21,679,000
1942 = 31,501,000

The Financial Record of the Electric Utility Industry 1937–1942, issued in December, 1943, by the Federal Power Commission, to the industry as a whole, gives the data presented in Table 7.

It will be noted from Table 7 that while the book cost was increasing 7.2% the credit balance increased 54.2%, again reflecting the rapid increase in the reserve balance in periods of slow growth of plant.

In order to get comparable figures with those of the straight-line method as used by the telephone industry for the whole period 1918 to 1942, the Society's committee has used the percentage figures developed in a study by the Edison Electric Institute of seventeen companies with a plant total of $2,493,233,000,

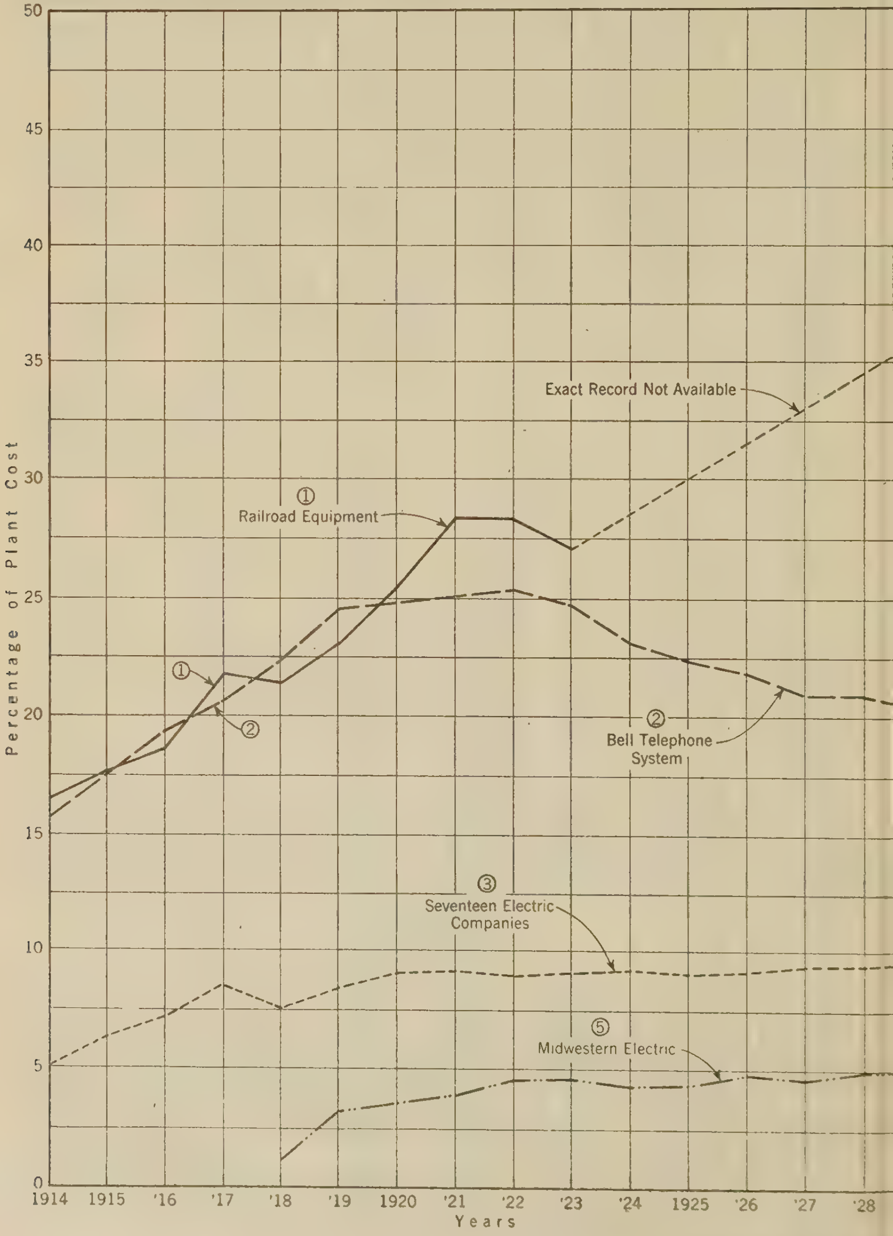
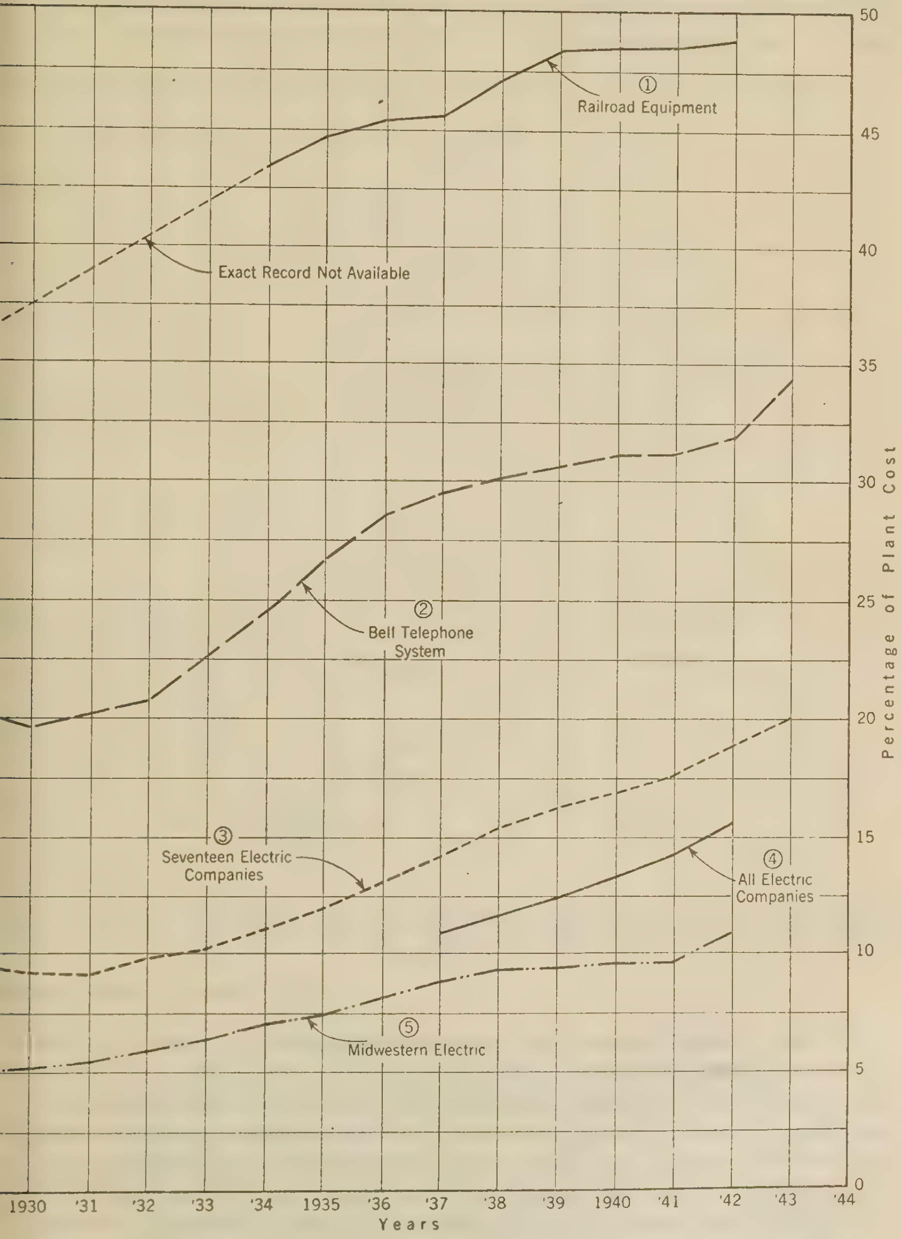


Fig. 1.—Growth of Credit Bala

and also a study of a state-wide midwestern electric utility in the $300,000,000 class. These figures appear in Tables 8 and 9.

Table 8 shows the consolidated figures, in dollars and percentages of plant cost, the credits to the reserve (which in all cases are principally made up of



RESERVES FOR RETIREMENT

charges to operating expense for depreciation); debits to the reserve (net retirements); and credit balance in the reserve. It will be noted that in every year of the period credits exceeded debits in both dollars and percentage of total value of plant; consequently the credit balance in the reserve steadily increased from \$9,364,000, or 4.25% of the book value of plant in 1913, to \$499,732,000, or 20.04% of the immensely larger value of the plant thirty years later.

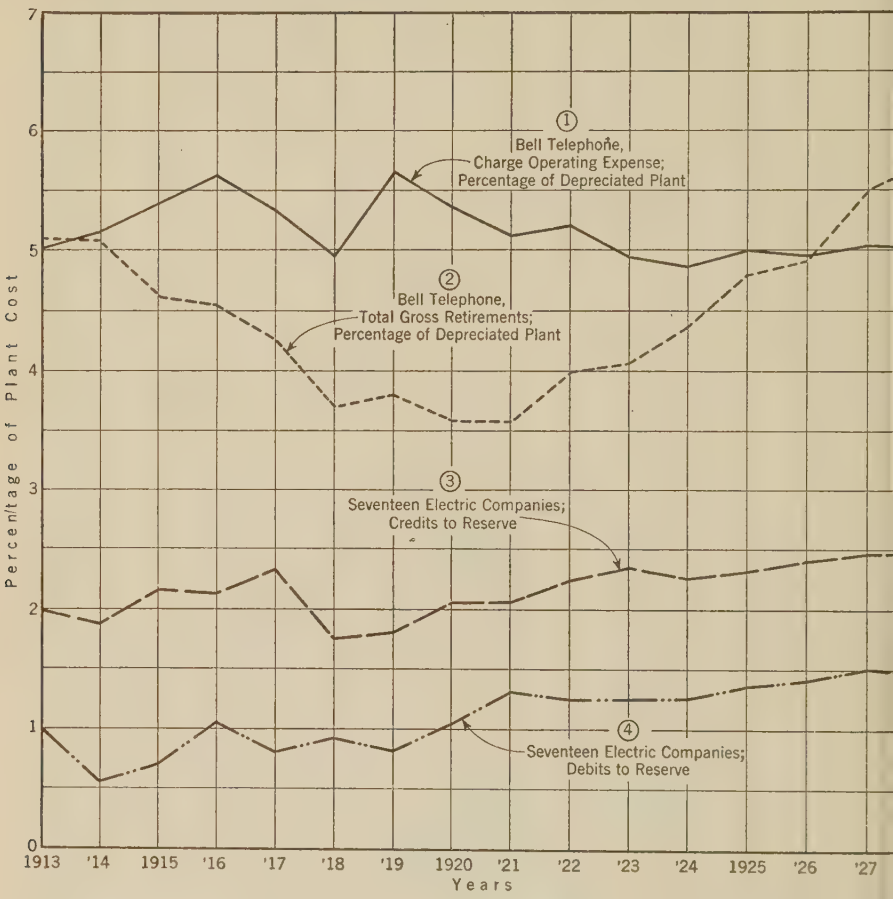
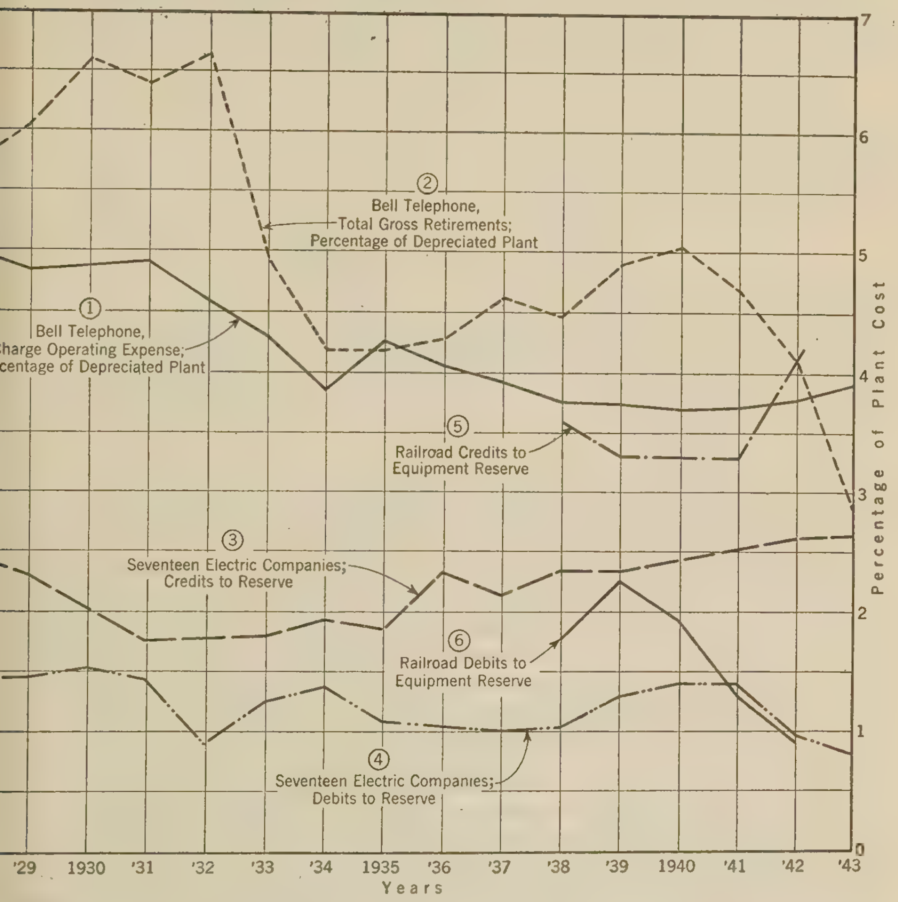


FIG. 2.—CREDITS AND DEBITS TO RES

In view of such a showing, or the similar showing of the data of the midwestern single plant (Table 9), or the Federal Power Commission data for five years covering the whole industry (Table 7), the Society's committee cannot agree that, speaking of the industry as a whole, there is any necessity for more ample reserves; that there is any possible justification for recomputing retroactively on the straight-line method; or that there is any justification for the deduction for "depreciation" of any amount computed on a straight-line basis.

It is believed that these records may be accepted as fairly typical of the results in the power industry of the method provided for in the present accounting classification. When the results from this far from complete study of existing records of the telephone and railroad equipment reserves treated by the use of the straight-line method and those of the Electric Corporation reserves, built up under the present NARUC accounting requirements, are compared in Figs. 1 and 2, it is perfectly evident that the straight-line method as used by the Telephone Utility and for railroad equipment results in credit balances very greatly in excess of those where judgment based on experience is the guide.



ERMS OF PERCENTAGE OF PLANT COST

These studies indicate that, as an average, the utilities covered maintain adequate balances in the reserve, although some individual companies may have balances which are either too large or too small. When a method has been in use for 25 or 30 years and in not one year of the period have retirements and other debits to the reserve been sufficient to reduce the dollars in the credit balance—but on the other hand the credit balance has steadily grown year after year, and unless it can be shown that the reserve is less than the amount of actual accrued depreciation—it cannot be said that the company is not making adequate provision "to keep the investment intact."

Careful Analysis of Statistics Necessary.—

The committee is fully convinced that Retirement Accounting, as now used by the electric, gas and water works industries, is in general providing credit balances in the reserves that are ample. Table 10 has been prepared to summarize and compare some of the figures already given.

TABLE 10.—SUMMARY AND COMPARISON OF DATA

Year	Utility	Investment in plant	Credit Balance in Reserve		Method of computing depreciation
			Dollars	Percentage of investment	
1943	Telephone...........	4,990,000,000	1,713,175,000	34.34	Straight line
1941	Railroad (equipment).	5,752,904,000	2,467,735,000	42.9	Straight line
1943	Gas	6,650,836,000	1,018,701,000	15.3	Other than straight line
1942	Electric.............	14,848,000,000	2,306,000,000	15.5	Other than straight line
1943	Water works (private).	....	....	13.4	Other than straight line
1943	Water works (public) .	....	....	27.0	Predominantly straight line

The total amount of money that has been collected from ratepayers over and above past retirements that have been charged to "depreciation," now probably well in excess of eight billion dollars, for all these groups of utilities, directs attention to the magnitude and importance of this form of accounting. Only the railroads, the largest in investment of all the utilities (and which in general apply it only on equipment), and many of the publicly owned water works use the straight-line formula. These figures are doubtless somewhat in excess of the total amount needed for retirement reserves. It must be clearly understood, however, that a very large part of these balances are proper and fulfill the purpose for which they were created and approved by the Courts.

The Society's committee does not condemn the straight-line formula or contend that it should not be used in accounting. It does, however, believe that on large, far-flung utilities it results in the accumulation of credit balances that are substantially above any needed requirement, and that, due to the fact that its accumulations have no relation to actual depreciation, and that the use of it tends to cause managements to consider the reserve a mere matter of formula and not to give this important subject sufficient attention, its use should not be required.

Those who advocate the use of the straight-line method usually adopt average lives in its application rather than those derived from a special study of the mortality of units on the particular property, and without any check of the results of its application in the relationship between the size of the reserve and the actual depreciation in the property. This may and often does result in wide variation from the truth, and in gross inequity.

XI.—RECONSIDERATION OF "DEPRECIATION" IS ESSENTIAL NOW

There is unquestionably a present need for a full and complete discussion of the subject of accounting methods for creating reserves for plant retirement which is the maximum permissible size of those reserves, and for the final determination of the relationship of such reserves, actual depreciation, and the amount to be deducted in the determination of a rate base.

In connection with his discussion of Dockets 14700 and 15100 which dealt with proposed changes in telephone and railroad accounting, Mr. Eastman said, in November, 1926 (118 I.C.C. on p. 356):

> "It is essential to bear in mind, however, that there is an inseparable connection between the straight-line method and the principle that accrued depreciation represented by the depreciation reserve must be deducted in ascertaining the rate base value. If any other principle should hereafter be adopted by the courts a reconsideration of the entire question of depreciation would at once become necessary."

It is quite probable that this statement of eighteen years ago was prompted by the opinion of the Supreme Court in the New Jersey Telephone Case (271 U.S. 23) decided earlier in that year, which held squarely that the property which was represented by the reserve belonged to the company, just as much so as if it had been acquired through the sale of stocks and bonds. Obviously the doctrine stated by the Court in that case and later reaffirmed by Chief Justice Hughes in Smith vs. Illinois Bell Telephone Company (282 U.S. 133) is in direct contradiction of the theory stated by Commissioner Eastman in the statement just quoted. These Supreme Court opinions clearly prohibit the deduction of reserves which have been built up under regulated accounting. The subsequent opinion in the Lindheimer Case (292 U.S. 151), handed down in 1934 by Chief Justice Hughes, discussed the straight-line method at length. A study of this long opinion, which was a unanimous opinion of the Court, expanded somewhat in a concurring opinion of Mr. Justice Butler, makes it very clear that it is in no sense an approval of the straight-line method.

The Society's committee believes that the evident meaning of the Court is not to be found in any single paragraph and therefore includes a rather extended quotation from this most important opinion:

> "Broadly speaking, depreciation is the loss, not restored by current maintenance, which is due to all the factors causing the ultimate retirement of the property. These factors embrace wear and tear, decay, inadequacy and obsolescence. Annual depreciation is the loss that takes place in a year. In determining reasonable rates for supplying public service, it is proper to include in the operating expenses, that is in the cost of producing the service, an allowance for the consumption of capital in order to maintain the integrity of the investment in the service rendered. The amount necessary to be provided annually for this purpose is the subject of estimate and computation. In this instance, the Company has used the 'straight-line' method of computation, a method approved by the Interstate Commerce Commission. 177 I.C.C. pp. 408, 413. By this method the annual depreciation charge is obtained by dividing the estimated service value by the number of years of estimated service life. The method is designed to spread evenly over the service life of the property the loss which is realized when the property is ultimately retired from service.

"As the allowances for depreciation, credited to the depreciation reserve account, are charged to operating expenses, the depreciation reserve invested in the property thus represents, at a given time, the amount of the investment which has been made out of the proceeds of telephone rates for the ostensible purpose of replacing capital consumed. If the predictions of service life were entirely accurate and retirements were made when and as these predictions were precisely fulfilled, the depreciation reserve would represent the consumption of capital, on a cost basis, according to the method which spreads that loss over the respective service periods. But if the amounts charged to operating expenses and credited to the account for depreciation reserve are excessive, to that extent subscribers for the telephone service are required to provide, in effect, capital contributions, not to make good losses incurred by the utility in the service rendered and thus to keep its investment unimpaired, but to secure additional plant and equipment upon which the utility expects a return.

* * *

"The calculations are mathematical but the predictions underlying them are essentially matters of opinion. They proceed from studies of the 'behavior of large groups' of items. These studies are beset with a host of perplexing problems. Their determination involves the examination of many variable elements and opportunities for excessive allowances, even under a correct system of accounting, are always present. The necessity of checking the results is not questioned. The predictions must meet the controlling test of experience.

* * *

"In this instance, the evidence of expert computations of the amounts required for annual allowances does not stand alone. In striking contrast is the proof of the actual condition of the plant as maintained—proof which the Company strongly emphasizes as complete and indisputable in its sharp criticism of the amount of accrued depreciation found by the District Court in valuing the property. The Company insists that 'the existing depreciation in the property physical and functional, does not exceed 9 per cent in the years 1923 to 1928 and 8 per cent thereafter.' The existing depreciation as thus asserted by the Company, and the amounts it shows as the depreciation reserve allocated to the intrastate business in Chicago (taking in each case the average amounts per year) are as follows:

Years	Existing depreciation	Depreciation reserve
1923	$11,992,000	$26,797,000
1924	12,865,000	29,316,000
1925	13,775,000	32,155,000
1926	14,621,000	35,572,000
1927	15,360,000	39,352,000
1928	16,241,000	42,769,000
1929	15,300,000	44,515,000
1930	15,863,000	45,829,000
1931	15,828,000	48,362,000

* * *

"Giving full weight to these considerations, we are not persuaded that they are adequate to explain the great disparity which the evidence reveals. As the Company's Counsel says: 'The reserve balance and the actual depreciation at any time can be compared only after examining the property to ascertain its condition; the depreciation, physical and functional, thus found can be measured in dollars and the amount compared with the reserve.' Here we are dealing not simply with a particular year but with a period of many years—a fairly long range of experience—and with careful

> and detailed examinations made both at the beginning and near the end of that period. The showing of the condition of the property, and of the way in which it has been maintained, put the matter in a strong light."
>
> * * *
>
> "This condition, kept at a nearly constant level, directs attention to the amounts expended for current maintenance. In the process of current maintenance, 'new parts' are 'installed to replace old parts' in units of property not retired. Such 'substitutions or 'repair'' are separate from the amounts which figure in the depreciation reserve. The distinction between expenses for current maintenance and depreciation is theoretically clear. Depreciation is defined as the expense occasioned by the using up of physical property employed as fixed capita; current maintenance, as the expense occasioned in keeping the physical property in the condition required for continued use during its service life. But it is evident that the distinction is a difficult one to observe in practice with scientific precision, and that outlays for maintenance charged to current expenses may involve many substitutions of new for old parts which tend to keep down the accrued depreciation."
>
> * * *
>
> "In the light of the evidence as to the expenditures for current maintenance and the proved condition of the property—in the face of the disparity between the actual extent of depreciation, as ascertained according to the comprehensive standards used by the Company's witnesses, and the amount of the depreciation reserve—it cannot be said that the Company has established that the reserve merely represents the consumption of capital in the service rendered. Rather it appears that the depreciation reserve to a large extent represents provision for capital additions, over and above the amount required to cover capital consumption. This excess in the balance of the reserve account has been built up by excessive annual allowances for depreciation charged to operating expenses.
>
> "* * * The point is as to the necessity for the annual charges for depreciation, as made or claimed by the Company, in order to avoid confiscation through the rates in suit. On that point the Company has the burden of proof. We find that this burden has not been sustained. Nor is the result changed by figuring the allowances at the somewhat reduced amounts fixed by the court below.
>
> "We find this point to be a critical one. The questionable amounts annually charged to operating expenses for depreciation are large enough to destroy any basis for holding that it has been convincingly shown that the reduction in income through the rates in suit would produce confiscation."

The Supreme Court reversed the Court below, dissolved the injunction, ordered a refunding of the amounts charged by the Company in excess of the rates in suit, and dismissed the bill of complaint.

As the Society's committee interprets this opinion it squarely holds that the straight-line reserves were excessive; that the straight-line method is "beset with a host of perplexing problems * * * and opportunities for excessive allowances * * * are always present"; that "the predictions must meet the controlling test of experience"; that the engineering determination of actual depreciation in the case was most convincing.

In a concurring opinion Mr. Justice Butler said:

> "The Company's properties constitute a complex and highly developed instrumentality containing many classes of items that require renewal from time to time. But, taken as a whole, the plant must be deemed to be permanent. It never was intended to be new in all its parts. It would be impossible to make it so. Expenditures in an attempt to accomplish that

> would be wasteful. Amounts sufficient to create a reserve balance that is the same percentage of total cost of depreciable items as their age is of their total service life cannot be accepted as legitimate additions to operating expenses. In the absence of proof definitely establishing what annual deductions from revenues were necessary for adequate maintenance of the property, the company is not entitled to have the rate order set aside as confiscatory."

The Society's committee is unable to read into the opinion in the Lindheimer Case, or any of the preceding cases back to 1909, any suggestion or inference that a reserve designed to return to the company, through charges to operating expenses, the money to make good losses due to retirement, or even to cover the theoretical consumption of capital, is to be deducted from value. The Lindheimer opinion of Chief Justice Hughes, and the concurring opinion of Mr. Justice Butler, emphasize the fact that the straight-line method results in reserves grossly exceeding actual depreciation.

To what extent the opinion of the Hope Natural Gas Case, decided January 4, 1944, will affect utility regulation or prescribed accounting remains to be seen. That case dealt primarily with the extent of authority of the Commission under the Natural Gas Act, where a 40-yr old company was being brought under regulation for the first time. The facts of the case as stated in Mr. Justice Douglas' opinion clearly appear to justify the conclusions of the Commission, which found the "actual legitimate cost" of the property to be approximately \$52,000,000 and deducted as depreciation \$22,328,000, or 43%, in the process of determining a rate base. The company had accumulated a credit balance in the reserve of about \$46,000,000, and in the presentation of its own case deducted about \$30,000,000 as depreciation, apparently based on a present value computation. It should be noted that these figures involve both depletion of Natural Gas facilities a wasting resource (an amortization problem) as well as depreciation of physical property, two distinctly separate things.

Mr. Justice Douglas said:

> "In determining the amount of accrued depletion and depreciation the Commission, following Lindheimer v. Illinois Bell Tel. Co. 292 U. S. 151, * * * based its computation on 'actual legitimate cost.' It found that Hope during the years when its business was not under regulation did not observe 'sound depreciation and depletion practices' but 'actually accumulated an excessive reserve of about \$46,000,000.' * * * One member of the Commission thought that the entire amount of the reserve should be deducted from 'actual legitimate cost' in determining the rate base. The majority of the Commission concluded, however, that where, as here, a business is brought under regulation for the first time and where incorrect depreciation and depletion practices have prevailed, the deduction of the reserve requirement (actual existing depreciation and depletion) rather than the excessive reserve should be made so as to lay a sound basis for future regulation and control of rates."

It is very clear on a study of this case that it is wholly unlike any that would be likely to occur where a property had been under regulation for a quarter of a century.

The deduction was an amount less than half of the credit balance in the reserve, less in dollars than the depreciation estimated by the Company, and certainly affords no precedent for the deduction of a credit balance established under regulated accounting. It does however emphasize in a most positive manner the warning of the Lindheimer and other cases against excessive reserves.

"To keep the investment unimpaired" is not to provide a reserve to be deducted from value in a rate base. That would be just plain confiscation. But any reserve that accumulates more than enough to provide for return to the company cost of plant that has been and is to be retired is taking from the consumers money to be invested in capital which should be furnished by stockholders or through the issue of bonds. It is very difficult to understand why the NARUC committee in its report recommends the straight-line method which results in the accumulation of vast and obviously excessive reserves at the expense of the consumers.

Referring once more to Commissioner Eastman's statement at the beginning of this section of the report, it seems to be evident that the adoption of the straight-line method was, in his mind, inseparably connected with the deduction of the accumulated credit balances in the "Depreciation" reserve as accrued depreciation in ascertaining a rate base value. He called for a reconsideration of the "entire question of depreciation" if the Court decided otherwise. In the same month in which Mr. Eastman wrote the statement quoted, which was probably prompted by the opinion in the New York Telephone Case (271 U.S. 23), John W. McCardle et al. vs. Indianapolis Water Company (272 U.S. 400) was decided, the opinion being handed down by Mr. Justice Butler on November 23, 1926. In this opinion are the following statements:

> "There is deducted approximately 25% of estimated cost new to cover accrued depreciation. The deduction was not based on an inspection of the property. It was the result of a straight-line calculation based on age and the estimated or assumed useful life of the perishable elements. The Commission's report indicates that property is well planned, well maintained and efficient."

* * *

> "The testimony of competent valuation engineers who examined the property and made estimates in respect of its condition is to be preferred to mere calculations based on averages and assumed probabilities. The deduction made in the City's estimate cannot be approved. Pacific Gas Co. v. San Francisco, 265 U.S. 403–406; Standard Oil Co. v. Southern Pacific, Supra 159; Landon v. Court of Industrial Relations, 269 Fed. 433–445; City of Winona v. Wisconsin Minnesota Light and Power Co. 276 Fed., 996–1004; New York Telephone Co. v. Prendergast, 300 Fed., 822, 826; Southern Bell Tel. & Tel. Co. vs. Railroad Commission, 5 Fed. 2nd 77–95."

This opinion makes it perfectly evident that there was a long line of precedent approving the deduction of observed depreciation as found on an engineering study of property. That case apparently should have caused abandonment of the belief that the straight-line computation of accounting reserves for retirement was a proper figure for accrued depreciation to be deducted in a rate procedure.

In the eighteen years that have elapsed since then the concept has been clung to by some of the men in regulatory bodies. Ten years ago Mr. Chief Justice Hughes made clear the fact that the purpose of reserve accounting is to maintain the integrity of the investment, not to destroy it, and discussed the straight-line method at length, pointing out its weaknesses, comparing the credit balance in the reserve with the accrued depreciation found by engineering inspection and definitely approving the inspection method.

In spite of his apparently definite opinion in the Lindheimer Case, which was approvingly cited in the Hope Natural Gas Case in 1944 (subsequent to the NARUC committee's report), we find, on page XIX of the NARUC committee's report, in No. 40, the following statement: "In fixing public utility rates adequate depreciation expense should be allowed accordingly to the service life basis and, in principle, the reserve requirement determined on the same basis should be deducted in determining the rate base."

With this principle the Society's committee is in complete disagreement for the reasons stated at length in this section of this report.

XII.—THE ENGINEERING DETERMINATION OF ACCRUED DEPRECIATION

The engineering determination of accrued depreciation referred to and recommended in the Progress Report of the Society's committee[1] is a determination made by competent and reliable individuals, who, after a study of the property involved, determine what the actual effect of depreciation has been up to some specific date. Such a determination includes the consideration of many factual data, upon which an opinion as to the amount of accrued depreciation is finally based. As stated in the progress report, this method has in the past been referred to as the "Observation" method, and the results styled as "Observed Depreciation." Opponents of this method have capitalized on the word observed, and have tried to belittle it by inferring that the determination was made by simply observing or looking over the property. Such may have been and probably was the case in some instances. However, there have been undoubtedly just as many instances where the straight-line method has been used, where the only basis was a life table in some textbook applied wholly without consideration of the property whose depreciation was being determined.

A proper engineering determination of accrued depreciation includes the consideration of all data available in regard to the unit of property, and is the sort of determination made by most responsible and experienced men engaged in valuation work.

The process of making an engineering determination of accrued depreciation varies somewhat from property to property and among engineers who are experienced in appraisal work. Essentially, it includes the following steps, not necessarily in this chronological order.

The books and records of the property are examined with the assistance of employees of the company charged with the responsibility of keeping records, and who are familiar with them. From these records there is prepared a list of property units, or groups of like units, and, if available, the dates when they were installed, their cost new, installed, ready for service, including appropriate overhead costs (engineering, general, and interest lost during construction) and the expenses for maintenance and repairs during the period from installation to date.

The engineer then makes an inspection of the units, accompanied by such assistants as may be required, examining each unit of property, observing its conditions during operation, determining its operation behavior from inquiry from operators, and ascertaining what repairs may be imminent, and any other factors bearing upon the character and extent of deferred maintenance. In many cases machines are partially dismantled to permit examination of working parts. In the case of static property, such as pipe, wire, poles, transformers and the like, the examination includes a notation of corrosion, decay, abnormal characteristics developed during service, etc.

Included in the study is an analysis of past and probable future growth of the system and the city as affecting the relationship of the size of unit and adaptability for continued future service. The character of the community is studied with respect to its type; that is, whether residential or industrial, etc., and any indication of impending changes in the character of the load. Load curves of the system are also helpful in the consideration of the adaptability of the unit in question, to serve with other units, the size of which also may have a bearing upon the probable place of the particular unit in the future operating schedule.

It will also be of assistance if mortality statistics for the property are assembled from the records of retirements whenever records permit. These data may be arranged in the form of curves or diagrams. Other pertinent mortality data are examined and where applicable to the local conditions are assembled in the form of mortality curves if sufficient applicable data are available to permit doing so. A study of type mortality curves and the selection of those adapted to the local conditions and the units of property will be helpful in interpreting such data as may be otherwise available and will assist in arriving at a final judgment as to future service by the unit.

It will frequently, perhaps usually, be found that no pertinent mortality data are available on the particular property or type of unit being studied and the experienced engineer should be very careful to use only such types of mortality curves as he feels certain are applicable.

From such a study it is practicable for the engineer to reach a conclusion as to the probable future usefulness of the unit. In the light of all the data relative to the unit, its operations, capacity for future service, probable future usefulness, the consideration of whatever mortality statistics may be applicable, and the use of any mortality curves applicable to the particular unit, the engineer can then arrive at the percentage condition of the property and the total accrued depreciation which is the ultimate end desired.

The Society's committee believes that an engineering determination of accrued depreciation, made as outlined above, is the only way the actual effect of depreciation can be obtained, and that accounting practice should be so planned that a reserve for depreciation sufficient to meet the requirements indicated by such a determination will result. This belief is in marked contrast to that expressed in the recommendations of the NARUC committee, that annual allowances for depreciation should be calculated by the straight-line method and that the reserve created by such allowances becomes the measure of actual depreciation.

It is believed that if those connected with public utility regulation would accept the fact that the effect of depreciation on utility property has been, is now, and probably always will be subject to many variables, differing in each property, and that it cannot be determined by any theoretical uniform system of accounting a lot of the fog that has been thrown around the subject would burn off. It is proper to use theoretical methods in the accounting for annual allowances, and to create a reserve on that basis, but such a reserve would seldom, if ever, constitute a fair measure of accrued depreciation. The annual allowances should be adjusted to meet the requirements by periodic determinations of what the accrued depreciation actually is. The statement of Mr. Justice Jackson in the Hope Natural Gas Case is well worth repeating—it expresses the danger in strict adherence to fixed accounting conventions as follows:

> "To make a fetish of mere accounting is to shield from examination the deeper causes, forces, movements, and conditions which should govern rates. Even as a recording of current transactions, bookkeeping is hardly an exact science. As a representation of the conditions and trend of a business, it uses symbols of certainty to express values that actually are in constant flux. It may be said that in commercial or investment banking or any business extending credit success depends on knowing what not to believe in accounting. Few concerns go into bankruptcy or reorganization whose books do not show them solvent and often even profitable. If one cannot rely on accountancy accurately to disclose past or current conditions of a business, the fallacy of using it as a sole guide to future price policy ought to be apparent. However, our quest for certitude is so ardent that we pay an irrational reverence to a technique which uses symbols of certainty, even though experience again and again warns that they are delusive."

XIII.—Retroactive Adjustment of the Depreciation Reserve

The NARUC Committee on Depreciation in its report recommends that "The reserve should be deducted * * * in arriving at the rate base for the reserve alone measures actual depreciation." The NARUC Depreciation Committee also recommends that, where the depreciation reserve as computed by the straight-line method materially exceeds the reserve currently in the property, an adjustment shall be made in the amount necessary to bring the current reserve up to that computed on the straight-line basis; this adjustment to be made through charge to surplus or special section of the income account.

Before proceeding to discuss these recommendations it is well to review the fundamental concept of depreciation. Depreciation is an operating cost. Even the NARUC committee recognizes this in their statement: "Depreciation results in a cost of service." Mr. Justice Moody, in the Knoxville water case, stated it admirably as follows:

> "Before coming to the question of profit at all, the company is entitled to earn a sufficient sum annually to provide, not only for current repairs, but for making good the depreciation and replacing the parts of the property when they come to the end of their life. * * * The Company is entitled to see that from earnings the value of the property invested is kept unimpaired, so that at the end of any given term of years the original investment remains as it was in the beginning."

It is apparent from this excerpt from the Knoxville decision which has been the primary judicial basis for depreciation allowances for nearly forty years, that the investor is entitled to see that the value of the property is kept unimpaired. How can the value be kept unimpaired if, as the NARUC committee now recommends where the credit balance in the depreciation reserve is less than the amount computed on a straight-line, age-life basis, the difference be withdrawn from surplus or capital? If that subtraction is made, as recommended by the NARUC committee, the capital is directly impaired. By that method an unearned operating expense becomes a deduction from capital.

The unfairness of this recommendation is further shown when it is recalled that practically all utilities have now been under supervision of public regulatory bodies for the past twenty to forty years. During that period their depreciation allowances have either been determined or approved by these regulatory bodies. To now arbitrarily require capital adjustments to bring the results of public regulatory policies up to the measurements of straight-line age-life depreciation, and to reflect this adjustment in the valuation or rate base cannot, in the opinion of the Society's committee, be viewed in any other way than the taking of property without the process of law.

In the Columbus Gas and Fuel Company vs. Public Utilities Commission of Ohio case decided May 21, 1934, in an opinion delivered by Mr. Justice Cardozo, the following statement appears:

> "To withhold from a public utility the privilege of including a depletion allowance among its operating expenses, while confining it to a return of 6½% upon the value of its wasting assets, is to take its property away from it without due process of law, at least where the waste is inevitable and rapid * * *. Plainly the state must either surrender the power to limit the return or else concede to the business a compensating privilege to preserve its capital intact."

Justice Cardozo then quotes from the Knoxville decision:

> "Before coming to the question of profit at all the company is entitled to earn a sufficient sum annually to provide not only for current repairs, but for making good the depreciation and replacing parts of the property when they come to the end of their life."

He then continued:

> "It is idle to argue that a company using up its capital in the operations of the year will have received the same return as one that at the end of the year has its capital intact and interest besides."

This quotation is identical with the statement appearing in the Dayton Power and Light case decided April 30, 1934.

The telephone utilities and railroads stand alone in their exclusive use of the straight-line method of figuring annual depreciation. As a result they have accumulated in many instances reserves materially larger than the actual depreciation in the property.

Practically all other utilities have depreciation reserves which currently amount to approximately one half to one third of the reserves that would be computed on a straight-line basis. It is with this great number of properties that the retroactive adjustment of the depreciation reserve is so serious. It is, however, most significant that in these telephone cases of excessive reserves the Court reduced the annual allowance rather than declaring the excessive reserve to be "actual" depreciation.

The NARUC committee report realizes that the retroactive adjustment is a serious problem but it nevertheless makes it even more serious when it states:

> "The depreciation reserve should not be readjusted gradually through modification of the annual depreciation rates when the difference between the book reserves and proper reserve (straight-line) is substantial."

With most utilities this difference is substantial, probably averaging 100% or more of book reserves. The stipulation above quoted closes the door to the only means of collecting depreciation reserves from the consumers whose service used up the property. Since the utility is precluded from a gradual readjustment through collections from the consumers, the only alternative necessarily results in impairment of its property. To require the utilities to deduct from capital amounts which they have allowed to collect through rates under regulation is contrary to all precedent and ethical procedure.

It is of interest to note that the recommendation of the NARUC committee on retroactive accounting is not new. In 1926, in the original order in Docket No. 15100, the Interstate Commerce Commission, Commissioner Eastman handing down the opinion, said (118 I.C.C. on p. 384):

> "Past depreciation unprovided for. In the case of steam railroads, and perhaps to some extent in the case of telephone companies, a difficulty which will be encountered is that as a rule depreciation accounting has not in the past been observed with respect to fixed property. In such cases there is no depreciation reserve which now covers that part of the service value of existing units which has already been consumed. The theoretically correct way of meeting this situation would be to establish the amount of past accrued depreciation which has not been provided for, and concurrently credit this amount to the depreciation reserve and charge it to profit and loss. It is the latter account which has profited in the past from the failure or partial failure to accrue depreciation charges. In view of the fact that such procedure might, in some and perhaps in many instances, result in converting a profit and loss surplus into a deficit with a consequent impairment of credit, it seems desirable that for the present the amount so credited to the depreciation reserve should be concurrently charged to a suspense account on the assets side of the balance sheet, instead of to profit and loss. This suspense account could then be gradually extinguished by charges to profit and loss over a maximum period of years or in such shorter time as the carrier might elect."

This ruling, which deducted approximately 20% of the total cost of reproduction found by the Commission from the basic valuation which was to be used in establishing the new accounting, was criticized by the railroads at the subsequent hearings. There were many objections but the principal one was that, in many instances, the effect of disregarding former accounting and setting up the "past accrued depreciation," as developed by the formula of the Commission, would be to seriously decrease or wipe out the profit and loss surplus, and to seriously impair the credit of all railroads. The device of a suspense account did not in any way relieve the railroad from the burden of liquidating these enormous sums through profit and loss and was merely a makeshift.

In the Proposed Report, dated August 15, 1929, the Commission substantially changed its position and in all the succeeding years has never placed retroactive depreciation accounting into effect.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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STEWARDSHIP OF PROFESSIONALISM GIVEN HIGH PRIORITY

ADDRESS AT THE ANNUAL CONVENTION, SPOKANE, WASHINGTON, JULY 17, 1946

By W. W. Horner,[1] President, ASCE

This is our first Summer Convention in three years. During those years we, together with all other Americans, have been concentrating on the war effort. A great many of our members have been in the armed services, and many others were engaged in activities essential to the successful and early conclusion of the conflict. It is a source of deep satisfaction to know that members of our Society, together with other men of science, played an important rôle in this most scientific of all wars. I am confident our pride in our part of the atomic bomb project, which contributed so much to hastening the cessation of hostilities, will be judged a pardonable one.

Now it is time to face the problem of peace with the same singleness of purpose and in the same spirit of unity which characterized our war effort. I am certain that the members of our Society are not unmindful of the innumerable problems which are vexing and perplexing the country in its uphill reconversion efforts, and although I could dwell at great length on what stake we, as engineers, have in the prompt and proper solutions of those problems, I shall not do so.

For this, it seems to me, is an occasion for discussion of Society affairs and of Society aims and efforts. Those of you who have been members of your Section membership committees at one time or another, and who have endeavored to explain to prospective members, the advantages of belonging to ASCE, will readily understand what I mean when I say, and heartily concur in my opinion that, the most intangible, though perhaps the most valuable, of all our aims is the one we call "professional improvements" of our members.

In choosing that phase of our activities for discussion here today I take particular cognizance of the increasing attention our Society has been devoting to the subject of its younger members, and, with the permission of you older members, I should like to direct my ensuing remarks principally to my junior colleagues.

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Not long ago, there came my way an opportunity for a man-to-man talk with members of a Student Chapter of the Society. It was a heart-warming and revealing experience, and one which stimulated my thinking on the subject of the younger members of our Society.

Most of these fine young men had returned from service in the armed forces to resume their war-interrupted education. Many had seen action in foreign lands. They were, I am happy to report, much more maturely serious than students I had known previously, and were keenly interested in discussing their future. They had definite ideas as to what they felt they ought to do.

I asked them if, in the light of recent events and developments, they had felt at all inclined to reconsider their decision to enter the engineering profession. Had they, I inquired, ever thought that there might be a greater reward in a craft under the protection of organized labor?

The replies I received were most definite. Yes, there had been a good deal of serious thinking devoted to the all-important subject of their future, and the glittering and immediate beneficial results held out to them by organized labor had received the attention such seeming prizes would naturally attract, especially from youth. But their decision to become members of a profession, rather than of a trade or craft, was declared emphatically. Definitely, they were in for the longer and harder pull. They expected to be free to develop their careers along constructive lines, and could not see the leveling process of union rules, which they felt would restrict their activity, as fitting into their picture of what they wanted for themselves in an America which they had fought to keep free. Theirs was no philosophy of doubt or compromise, in which "higher wages now" were to compensate for inhibition of initiative. They were concerned with fundamentals— opportunities for the future, salaries commensurate with professional status, and a satisfactory scale of living to reward them for their years of study, learning, and development into the careers of their choice.

And they were equally forthright in their inquiries as to what assistance they could look for from the older and established members of the engineering profession. What had the profession done to help them? What was it doing to assure them a fair chance? What were its plans for future aid to the younger fellows?

It was obvious that they weren't too well satisfied with what they had heard and that there was grave doubt in their minds that we elders had done all that it was possible for us to do. They inferred that possibly we had been sticking too close to our personal knitting and, by implication, asked whether we had not failed to develop a professional feeling to a reasonable degree.

Under the cross-examination of these straight-thinking young men, my derelictions were beginning to appear in the record, and I found myself feeling apologetic and definitely on the defensive. It became apparent that I, and not they, needed to reflect and reconsider.

I have done this at some length and have concluded, for myself and my contemporaries in the profession, that perhaps it is best that we plead guilty and ask the clemency of the new generation, who are so sure of themselves and so ready and willing to take over.

In throwing ourselves on the mercy of the court, so to speak, I think it just, however, that we be permitted to offer an explanation—hardly an excuse—for our conduct of our professional affairs to date. I think a review of such extenuating and mitigating circumstances as may be found to exist is in order at such a time, and I think, also, that I can best present it in terms of my own experience. This I shall proceed to do, presenting it (again with the leave of my older colleagues) in the form of remarks directed at our Junior Members and to the mature and earnest Student Chapter Members who will soon join them in our professional ranks.

When I entered the Junior grade of membership in ASCE, America was emerging from the gay nineties and plunging into the ensuing railroad panic of 1907. Immigration was still at high tide. Labor was plentiful and practically unorganized. Business and industry were enjoying freedoms they long since have lost—were, in fact, abusing their privileges in a manner so ruthless that it could lead nowhere but to the curtailment and abrogation of those privileges. It is significant that the loss of those freedoms and privileges, upon which we now look back as something which occurred overnight, was not, in fact, any single event which took place in anything even approaching so short a time. Deprivation of privilege came about gradually, as the people grew increasingly incensed over the abuses. Corrections were instituted piecemeal, as has been the case with remedial measures throughout history. Controls were instituted not only as the need for them arose, but in most instances only after the abuses were so flagrant as to cause the people to wonder if the means of abolishing them were to be forthcoming in time to avert national calamity. In short, just as today we are inclined to despair over abuses at the pendulum's other extreme, so did the people of the time of my youth fret and worry and fume and wrangle over the injustices and the inequities which menaced the opportunities which they sought and which they deemed it their right to have.

In those days, government was law and order and taxes, and had not yet entered business to an appreciable degree. Public works were largely municipal utilities and federal postoffices, and the taxes for their construction were so relatively slight that the average man did not have to concern himself about them. Certainly, a newcomer to the professional ranks of our Society, starting out to earn his livelihood as an engineer, was immune from them. Comparatively small though it was, his paycheck was all his own when he received it, and he was not afflicted with that relatively new malady, the "deducts," as are we today.

The American Society of Civil Engineers then was to the young man an order of eminent mystics who met primarily to compare notes on great achievements. Then the Junior was received into the Society after careful scrutiny and was thereafter seen, but for some time, not heard. Then, as now, the Junior needed a friend and friendly help, but the devil took the hindmost.

We Juniors did have impressed upon us sound ideas of professional integrity and an understanding that the results of our work must be always in the public interest. But the notion that we should devote some time to social and economic improvement of the membership was not advanced in any manner whatsoever. One got a job and, if fortunate, a chance to work under an accomplished elder. By hard work, he could rise in his profession. Individual engineers carried through classic projects and made great contributions to technological advance. Many members of the Society rendered conspicuous public service, and the profession profited and grew in the public esteem.

From World War I, we emerged with an increased prestige, for even the crude tanks and the handful of airplanes of that conflict, captured the public imagination to the extent that many called it "an engineers' war." From the war, we gained a broader professional solidarity, but our energy soon became diffused through a number of specialized societies. The ASCE began to modify its objectives in the direction of professional welfare with a faint sense of the beginning of the social revolution which we since have been undergoing. Technically and economically, we began to become closely related to industry, business, and public works, and we increasingly assumed major responsibility for technological advancement and for management and direction, as well as for our earlier endeavors in the realm of planning.

It is evident that we had little training in the service of the profession. The important social and economic side was at best a spare-time sideline for comparatively few members of the Society. Wiser heads recognized the need for enhancing the status of the profession, and occasionally voices sought to be heard on the subject. But the membership was busy, and the voices were heard but briefly and ineffectually. We were immersed so deeply in our individual improvement problems that for a long time we did not recognize the benefits or the essentiality of improving our position as a profession.

When it did come, our awakening was a gradual process. We created the Engineers' Council for Professional Development and supported it after a fashion. We saw the light on the need for legal recognition of the profession in the public interest, and got together to put through state registration. Our committee work began to point out somewhat more strongly the need for such things as professional recognition and adequate salaries and fees.

We were not well prepared to meet the bulge in the revolutionary tide that accompanied the economic depression and the New Deal. In that era, we did perform important service and the profession suffered comparatively little. While we had some economic distress of our own, we did not organize to produce widespread leadership, for which there was then great opportunity. A better conception of our responsibilities and a strong presentation of engineering ideals and ideas might have damped the swing to the left somewhat and might have placed your younger generation in a stronger starting position for the work you have to do.

In summary, we gave ourselves wholeheartedly to our engineering work; we made major contributions to developing the United States into the greatest and most capable nation of the world, a condition which was to stand her, and us, in good stead when World War II required of us that we be the arsenal of democracy. So we had little time to organize ourselves as a profession. Yet some members—a small part of our Society, and all too few—worked in that direction. In the past decade particularly some orderly procedure has been developing. The membership of ASCE and of the other major engineering societies has been increasing rapidly, and a greater part of those memberships has displayed a growing interest in professional welfare.

The need for unity in the profession as a whole has resulted in the federation of Society Sections with other engineering bodies in the metropolitan areas and in some of the states. Most recently, Engineers Joint Council has met at least in part, the need for unified action on the national level and has proved to its five constituent societies that they have much in common, can work together in harmony, and in doing so can render public service of a higher order than has heretofore been possible.

World War II has left us with a still greater recommendation of our capacity for service. Out of the war has come the realization by the public of the value of research and technological development. It took the atom bomb to do it, but the country now looks to scientists and engineers to make secure the future of the nation, as well as to enhance its mode of living. Engineers are directly concerned with scientific research. Basic and applied research in engineering provide the tools with which we practicing engineers earn our livelihoods. We bear the same relationship to fundamental research in the science of engineering as does the practicing physician to the basic investigations of the medical scientist in medical and biological fields. It rests with us to make use of the fruit of scientific engineering research and to turn it to the practical service of our fellowmen. It is our duty to accept the challenge offered by the scientific developments of the war and to strive for the same degree of progress in peace as we made during the war.

But thorns, as well as roses, adorn our postwar path. Collective bargaining on the one hand and a growing regimentation of the men in public service into "bigger," if not always "better," bureaucratic agencies on the other, have focused the spotlight of self-searching analysis on our present weaknesses.

We have in our Society, and with our affiliations, created the skeleton of good professional organization. At the present time, that organization is undermanned; too few of us are actively engaged in the relatively new task of molding firm flesh onto that skeleton. Too many of the older members retain their earlier individualistic tendencies, and too many of all ages are too busy with private affairs, thus bidding fair to repeat the errors of our earlier ways to which I am now, as I stated, pleading guilty.

That is why I am especially pleased to see you younger men come into the picture. Your awareness of the social and political scene, your concentration on the intrinsic value of the professional attitude, your determination to see that matters you consider essential are acted upon, and the natural zest of youth which you bring with you, provide the new light and the new energy which should transform our slower progress into accelerated achievement.

Today, the organization of the American Society of Civil Engineers is well distributed over the country. Its foundations and its strength are in its Local

Sections. Your real service to the profession must necessarily be applied there and at that level it can be adjusted best to the circumstances of your job and your environment. As you come into the Local Section membership, you will invariably find that these member groups are fulfilling a first essential task, that of extending the acquaintanceship between professional men into friendships and sympathetic understanding of the other fellow's problems and performance. Many of the members quickly will find time to become interested in you and to be helpful as advisers. In some of the Sections, you will find an arranged elder brother system, which assures you of welcome, personal guidance, and opportunity for service in the directions indicated by your interests and talents. You will find generally that your Section has both social and technical programs, and that you can participate in them and in arranging for them to good advantage. If, in some instances, they have not been well organized, there is a constructive service waiting for you.

Many of the Sections are member bodies of local engineering councils or federations generally interested in public questions and in the development of intelligent engineering opinion and expression with respect to them. Volunteer to serve on their special committees. You will come to recognize the varying viewpoints of the different branches of the profession and how they may be reconciled. You also will be introduced to local civic and political problems, and will find an opportunity for direct public service—a field in which we engineers have been all too inactive.

Either in your Section programs or through your personal work you may find encouragement to enter civic or business organizations, with the further opportunity of broadening your viewpoint while acting constructively for the public. Separately, there is opportunity here to demonstrate to the citizenry at large that the engineer has more than a one-track, technical mind. To those who may be inclined to question the fitness of professional men injecting themselves into community affairs, rather than waiting to be consulted, I would point out that our professional brothers, the doctors and lawyers, have followed this pattern, not only have they avoided impairment of their professional status, but they actually have enhanced their community positions by manifesting an interest in nonprofessional affairs.

Should you find that your job brings you into the sphere of employer-labor relations discussions that may involve you in collective bargaining procedure, your Section officers can obtain from the Executive Secretary pertinent, up-to-date information as to what you had best do to protect your professional status. The Society's past activities on this score are too well known to you, I am certain, to require repetition here. What I do want to pause long enough to do, however, is to give reassurance to any who may need it that the Society's position on this all-important matter is unchanged and that we propose, within the restricting limitations which the laws of the country impose on us, to continue our efforts to insure that professional employees shall be guaranteed the right to determine directly by their own votes whether or not they wish to bargain collectively; and, where they desire, to do so through representatives of their own choosing in a bargaining unit of their own choice. We shall continue to oppose regimentation of professional men into heterogeneous labor organizations in which they are outnumbered by nonprofessional men and where they have no mutuality of interests with the majority of the members.

In the past, Local Section activity has not been closely coordinated with matters of interest to the membership in the national field or of national legislation having a possible effect on the membership of the profession and its welfare. New procedure in this direction is now being developed. Your Board of Direction and its committees have been improving their work in this field and approaching the limits of what can be done by central bodies. To be fully effective, further development must be carried out by the entire membership, wherever it may be situated. Local Sections will find need to form committees on national affairs and to work very closely with the new National Affairs Committee of the Society. This latter is developing action procedure intended to give force to, and to seek results under, the broader policies which the Society is, from time to time, approving. It is not at all unlikely that some of the more forward-looking of our younger men may be called upon to join panels which will present the engineers' viewpoint to legislators and administrators. This would seem to be particularly indicated where the problem is one of improving the economic status of the younger man.

I trust that in this account of the older engineer's stewardship of the welfare of our profession you find cause for agreement that, while we have given less than the desired attention to that subject, this field of endeavor has not been wholly neglected by us, and that it is now given a high priority on the program of your Board. Further, we have developed a deep and sincere conviction of your ability to complete the job in a way that will be satisfactory to you, and to that end we are prepared to stay in the game even more effectively than heretofore. Full success requires your leadership.

It is my fond hope that as you ascend to that leadership, and as we turn it over to you, we both may profit by the experiences which we older men have had.

An excellent example of the advances that have been made toward obtaining for the engineering profession the recognition we all desire for it, and one which bids fair to open new avenues for recognized public service, is provided by the recent report on industrial disarmament of Japan which was sponsored by Engineers Joint Council. This council, as I am sure you know, is comprised of the presidents, junior past-presidents, and secretaries of the American Society of Civil Engineers, American Institute of Mining and Metallurgical Engineers, American Society of Mechanical Engineers, American Institute of Electrical Engineers, and American Institute of Chemical Engineers.

I am proud to recount that two ASCE members played important rôles in the preparation of this report and of the report for the disarmament of Germany, which preceded it, one as chairman of the National Engineers Committee, which prepared the report and the other as chairman of the Engineers Joint Council Executive Committee.

I am sure we are all sufficiently familiar with the nature of the report to recognize it as a major contribution of the engineering profession toward what we all hope will be lasting peace. Its transmittal to the State, War, and Navy departments attracted even more attention in the press than did its predecessor report on the disarmament of Germany,[2] when it was made public by the National Engineers Committee of Engineers Joint Council last September.

What strikes me as particularly significant, however, and as a milestone in our progress toward the kind of public recognition that will extend the engineers' scope of usefulness to the public incalculably, is an editorial that appeared in *The New York Times* on June 15, within a day or two after the Japanese report was made public.

Under the heading, "A National Resource," *The Times* stated editorially:

> "The National Engineers Committee was appointed in February 1945, by representatives of the five national engineering societies at the request of the then Secretary of State, Mr. Stettinius, to bring the best American engineering thought to bear on the problem of the industrial disarmament of aggressor states. Last September it completed a report on Germany, the recommendations of which were substantially all included in the directive of the Allied Control Council of last March 29 for the limitation and control of German industry. Now its report on Japan has been made public in Washington, with the hearty endorsement of the State, War and Navy departments, as well as that of General MacArthur. It thus clearly foreshadows long-range American policy toward our defeated enemy in the Pacific.
>
> "The basic controls put forward for Japan are at variance with those proposed for Germany. That report based the broad primary control on the allocation of coal, since 85 per cent of Germany's electric power is generated by steam. In Japan, on the other hand, 73 per cent of her electrical energy at the wartime peak came from hydro-plants, which were both numerous and widely scattered. This necessitated a different approach to the problem. The engineers committee has recommended the regulation of the mineral and metals industries, the heavy chemical industries, the fuel industry and the construction industry, supplemented by the absolute prohibition of scientific research for military application, especially with regard to all forms of nuclear energy.
>
> "One of the most striking things about the present report is nowhere to be found in it, able document though it is. This is the availability for national purposes of such organized entities as the engineering profession, whose technical competence is matched by their freedom from any suspicion of political motivation. General Clay has acknowledged that the report on Germany has been his most effective instrument in dealing with both the British and the Russians. The latter, in particular, trust American technologists, whereas they look with suspicion upon our diplomats. Our own Government might do well to utilize even more fully such a national resource."

With such recognition, in its infant stage though it is, as a beginning, to what limitless heights may we not aspire for our profession as a whole? If the nontechnical segment of our population is commencing to visualize the engineering profession as "a national resource," must we not find inspirational hope for our profession in such stirrings of acceptance and recognition? As engineers, we have known for a long time, of course, that there are many additional services we can render. This latest and so clearly stated recognition of our potential by nontechnical men, coming as it does at a time when our younger men are so ready and able to assume their position in the affairs of our

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[2] *Civil Engineering*, November, 1945, p. 524.

Society, seems to me to make for an ideal situation and to provide added stimulus to the ambitions of our younger men.

I am convinced that the time has come when the work in the field of professional improvement can no longer be that of a few committees, but must be the business of the entire membership, with much of the work done at the Local Section level, and by younger members. There you young men will find full opportunity to render service to the public, to the profession, and particularly to your generation of engineers.

Today you need not worry about a job. For the next several years there will be a scarcity of men to perform the great volume of engineering work waiting to be done. You will have a better opportunity than we had to select the line of work which most interests you. You will certainly find ready at hand obvious opportunity to assist in the improvement of the profession and of its social and economic status. If you put into that work the serious interest and the ability I see in you, the engineers of the succeeding generation can take pride in their place in society without reservation, and will call you blessed.

MEMOIRS OF DECEASED MEMBERS

GEORGE TILLEY SEABURY, M. ASCE[1]

Died May 25, 1945

When George T. Seabury died on May 25, 1945, he had completed a few months more than twenty years of service as Secretary of the Society. Having also recently reached the retirement age, he was ready to relinquish his work; his successor had been appointed; his official affairs were in perfect order. It might be stated then that his work for the Society was complete, his work for George T. Seabury, about to begin. That he was thus able to finish his task was a great boon to the organization for which he gave his all, during the best years of his life. No record can adequately give a true appreciation of this accomplishment.

The Seabury family was of English origin, first appearing in American annals during early Massachusetts days in the person of a sea captain who settled in Duxbury in 1639. Perhaps the most famous name in the succession was that of Bishop Samuel Seabury. One branch of the family moved to Tiverton, R. I., and part of this group, still later, moved further to near-by Newport. There T. Mumford Seabury, father of George T. Seabury, established a boot shop in 1840, where through many years the old residents as well as the summer "Four Hundred" kept their favorite shoe lasts on file and could replenish their supply by custom manufacture, to their own specifications. To T. Mumford and his second wife, Mary S. (Tilley) Seabury, was born on April 12, 1880, George Tilley Seabury. His father was then fifty nine and he was not only the youngest son, but the youngest by twenty-two years. Thus, each of his three half brothers was old enough to be his father.

When George was a boy, his father was a dignified and rather old-fashioned person, wearing a long circular broadcloth cape and, on dress occasions, a high hat. Among George's cherished memories was his recollection of several trips made with his father. The first of these was abroad, when he was only twelve years old. The family did not tell George about this trip until the night beforehand. It was then he first knew that he, rather than his mother, was to accompany his father. This trip of a young lad not yet in his teens with a man who was then a septuagenarian must have been somewhat unique.

George promised his mother to keep a diary of his experiences and for the purpose bought himself a five-cent notebook. This seems to have disappeared, but the story of his trip the next year with his father to Niagara Falls, N. Y., to the Chicago (Ill.) World's Fair, and to Denver, Colo., in 1893 still lives vividly in a five-cent notebook with an energetic looking skyrocket on its cover. Inside a meticulous record written in ink relates many thrilling impressions, punctuated faithfully with comments on the quality of the meals. Once, "While at dinner the waiter expecting to be tipped called me 'My Lord' but he didn't get it." "Next we went to the Cave Dwellers exhibit—it cost 25 cents to get in but it was worth it." Also, "We saw a man with trained lions—it was the same show we saw in the Crystal Palace in London but with three wild boars and two elephants they did not have in London. Dinner was very good—what there was of it."

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[1] Memoir prepared by Henry E. Riggs, Past-President and Hon. M. ASCE, and Van Tuyl Boughton and Sydney Wilmot, Members, ASCE.

Another five-cent notebook is entitled "European Tour—1896," in which he describes how he and his father "left Newport [R. I.] on the New York [N. Y.] boat *Puritan* at 9.15 P.M. Bright and Pleasant" and the next day saw Trinity Church, the Stock Exchange ("We went into one place to look down into the 'Pit' "), Central Park ("Had a dinner for 25 cts. each and then went to the Park"), and the Eden Musee. Then a faithful account of days at sea and landing in England at Liverpool. Not a trick could have been missed in seeing dear old Victorian London, and an especially festive season it was, for

> "The Princess Maude was married to Prince Charles of Denmark. * * * She was rather pretty and well-dressed of course, and had a guard of about twenty-eight fine horse-men, all on black horses. * * * We could only see the top part of the coach-men—they were covered with gold lace. We got back to lunch late because the streets were crowded."

Then in Windsor,

> "We saw the Queen herself go by in her carriage with us standing on the side-walk. She was going to London to attend the wedding of one of her grand-daughters. We got home at 6.30 P.M. a little late for dinner—but we had seen the Queen."

Then he went to Paris, France, and later back to London and to Edinburgh, Scotland.

This diary was finished in pencil, but was faithful in all details, ending with a list of fellow travelers and an artistic flourish with "FINIS" in large firm letters. On the last page of the diary appeared the following:

> "My trip to Europe is ended. It has been a very pleasant trip and I have met some very pleasant people yet I am not sorry to get home again. When we started Papa wished me to take it in as much as possible and get as much good out of this trip as possible. I have tried to. And I have derived a good deal of pleasure and I think some profit. He did his best to please me and refused me nothing reasonable, for which I thank him with all my heart."
>
> Aug. 15th, 1896.
> George T. Seabury.

Meanwhile he had been giving considerable thought to his future profession. With an engineering ideal in view and some latitude of choice in colleges available, that section of New England offered only one unquestioned answer to his quest for training—"Boston Tech." George Seabury's college record was good, even outstanding; he worked hard and was graduated with his class in 1902. Throughout his life he held a warm regard for the Massachusetts Institute of Technology.

In the next few years, like many another ambitious young man, he was connected with a variety of heavy construction projects. These were in or near New York City, of which the most prominent were the Grand Central Terminal and Riverside Drive.

About that time, one of the notable projects of engineering history was getting into full swing—the Catskill Water Supply for New York City. Its possibilities and opportunities for association with other graduates of "Tech" attracted George Seabury. Hence, he took the civil service examination, and was appointed to this great work in 1906 continuing until 1914. His first two years with the Board of Water Supply were spent in the Northern Aqueduct Department, as first assistant to the division engineer on preliminary field work for the Ashokan Dam and Reservoir in the hills back of Kingston, N. Y., on the west side of the Hudson River.

This was the key structure of the supply, from which a large one hundred forty-mile aqueduct led south; then under the river and again south to Kensico Dam near White Plains, just north of New York City (Southern Aqueduct Department); and then again south to and under the city (City Aqueduct Department) as far as Silver Lake reservoir on Staten Island. The whole construction totaled about $175,000,000. The details are of interest because Mr. Seabury had varied contacts with different parts. Perhaps most interesting, as he later recalled, was the two years, directly under his beloved "Chief," J. Waldo Smith, Hon. M. ASCE, in which he was one of two special "trouble shooters" on organization problems involving an engineering force totaling one thousand two hundred engineers. One phase of this work comprised trips and reports on the proper value of water power belonging to private enterprise, before condemnation and absorption by the city. His success in these various dealings with his fellows was perhaps a training for his later greater success with larger numbers spread over the United States.

Equally significant for his future were the five years he spent in the Southern Aqueduct Department, again as first assistant to a division engineer, Wilson Fitch Smith, M. ASCE. This work covered all the varied phases of aqueduct, gatehouse, aerator, and venturi-meter construction, as well as Kensico Dam and Reservoir. Here Frank E. Winsor, M. ASCE, division and later department engineer, had a chance to size up his young assistant. The two were intimately thrown together for the succeeding thirty years in warm friendship and professional association.

Graphic evidence of Mr. Seabury's intense efforts on this fascinating construction is his technical paper entitled "A Study of the Internal Temperatures of a Large Masonry Dam." Details of measurement installations throughout Kensico Dam, of months of temperature readings, and of broad interpretations are given with his usual care, completeness, and clarity. The paper was so long that the Society was unable to publish it in its entirety; and he did not find time to shorten it. Found in his effects after his death, the manuscript has been deposited in the Engineering Societies Library.

The account of Mr. Seabury's work with the Board of Water Supply would not be complete without reference to the men he met there, who appear to have had great influence on him personally and on his success in Society activities: J. Waldo Smith, Col. Merritt H. Smith, Alfred D. Flinn, George H. Honness, Robert Ridgway, Thaddeus Merriman, Walter H. Spear, and Frank E. Winsor. His associates also included such men as John P. Hogan, W. W. Peabody, and Lazarus White. All these men were loyal members or officers of the Society, many of them during the tenure of Mr. Seabury's secretaryship.

In 1912 he went into partnership with his friend, Albert A. Northrop, M. ASCE, with whom he had previously been associated in the New York Board of Water Supply work. Under the title of Seabury and Northrop, consulting engineers, they planned to do such work as landscape construction for private estates, retaining walls, and sea walls. However, the growing war clouds made a very uncertain future for such work, and it was considered best to close out the new firm.

When Frank Winsor completed his work on the New York Board, he became chief engineer of the proposed water supply development for Providence, R. I. In casting about for key assistants to himself and deputy chief Peabody, he naturally turned to George Seabury. This prospect was appealing. He was given field charge of preliminary studies, of surveys, and of actually beginning construction on the new Scituate supply.

Because of World War I, the work was perforce slow, and after two years, in April, 1918, his employment was finally interrupted. At that time Mr. Seabury resigned to become a Major in the Construction Division of the Quartermaster Corps. Until June, 1918, he served as supervising construction quartermaster in charge of maintenance at Camps Devens, Upton, Mills Merritt, Dix, Meade, and Lee—all near the north Atlantic seaboard.

When he came out of the Army in June, 1919, he was anxious to enter a field of activity that had long appealed to him—heavy contracting; and he took it with his eyes open. He had too long observed the hazards of such work, in peacetime and wartime pursuits, to be ignorant of its difficulties and its dangers.

In executive ability, financial sense, and good judgment he was wel equipped, but he was not trained to shoddy performance in cutting corners He experienced, as in a sizable road contract in Massachusetts, difficulty with the constantly rising cost of labor. Materials in the postwar period were als serious handicaps. Nevertheless, he persisted in this work for four years an fulfilled his contracts to the owners' satisfaction if not to his own.

It would be gratifying to record that this work was a most profitabl success. Fortunately for the Society, it was not. Otherwise he might hav gone down in engineering history as another increasingly successful contractor instead of an outstanding Society official. He did have the satisfaction, however, of closing this disappointing experience in a manner worth of respect.

For almost two years, beginning in March, 1923, he served as manager c the Providence Safety Council. There he succeeded in establishing rotar traffic—a new concept for a conservative city—and in promoting all th interests of public and private safety. He found it possible to use his engineering to a limited degree; but, more than this, he could develop his knack of dealing with and directing people. The training was definitely of great benefit. He left with warm friendships and a beautiful gift watch, which he treasured as a reminder of his successful work and his pleasure in accomplishment.

Meanwhile the American Society of Civil Engineers had also been going through successive periods of development and preparation. Following its firm financial and ethical strengthening under Charles Warren Hunt's long term of office, it was ready for expansion in all directions, in line with a basic report by its Committee on Development. After brief tenures by Col. H. S. Crocker and E. M. Chandler, the Board of Direction in 1922 called John H. Dunlap as its Secretary. Success seemed assured—every indication pointed to continued advance under this young progressive and idealistic leader.

However, an unkind fate intervened when Mr. Dunlap was fatally injured in a train wreck, returning from a Society meeting on the west coast in July, 1924. Immediately a search for a permanent secretary was actively instituted. Promising young men from all over the United States were considered in detail, among them George Seabury.

He had no national reputation to commend him—in fact, he was relatively unknown in this wider sense. His endorsers were perhaps few but they had this to commend them—they knew him intimately and had firm faith in his capabilities. His prime recommendation came from the then Directors Frank E. Winsor and Thaddeus Merriman, who because of their intense and loyal service to the Society carried great weight with the Board. They continued to support him magnificently during their lifetimes. It was no idle gesture on his part that their photos, framed together, held a prominent place in the Secretary's office through all those years.

It was at the December 1, 1924, meeting of the Board that George T. Seabury was elected Secretary. He took office on January 1, 1925, and with him began a new era in Society history. Be it said in all fairness that his predecessors had laid the foundations well. Its internal struggle resolved, the organization was a united force. A few years of engineering prosperity were at hand, and the outlying sections were more than ever ready and anxious to get under the load and help forward the corporate activities.

So it might be said the time was propitious; but it was also true that the new secretary himself was more than ready for his great lifework. His earlier jobs have been mentioned rather briefly rather than covered in detail—not that they were unimportant, but that they were simply preliminary and preparatory to a far greater opportunity and to a more glorious service.

What did he bring to his new work? First of all, a thorough training, an excellent education, faithfully pursued. Then there was variety of experience—in the consulting field; in civil service on a number of large public projects; in contracting, both as an employee and as manager; and, furthermore, in the field of public relations. Beyond these Mr. Seabury possessed in generous measure those inestimable values of heart and mind, a judicial attitude, an ingrained habit of honest thinking, and a natural executive ability. These in themselves were enough to guarantee that the venture would be a success; but, in addition, he brought to the new work that priceless quality—a deep and passionate love for the organization.

It might be thought that, with these many and favorable auspices, success was inevitable. To think so is to ignore the travail and midnight oil, the unending journeying and conferences, the utter selflessness of being a servant to twenty thousand engineers. He succeeded—yes; but at terrific cost. He gave his all and finally himself. The history of George T. Seabury for the following twenty years is the story of the American Society of Civil Engineers itself.

A few—and a few only—of his problems and more significant accomplishments can be mentioned, and those only in skeleton form. Many difficulties faced Mr. Seabury when he took over the administrative responsibility for the Society. In the closing years of the secretaryship of his eminent predecessor, Charles Warren Hunt, the organization had been deeply stirred by conflicting views as to what its future policies should be. Out of this controversy had come a new Constitution calling for the decentralization of control of the Society through division of the United States into Zones and Districts with locally designated Vice-Presidents and Directors, establishment of local sections and creation of Technical Divisions.

The Constitution was adopted in 1921, the year that Mr. Hunt retired. In the years intervening before Mr. Seabury took over the office a series of unfortunate circumstances, which culminated in the tragic death of Mr. Seabury's immediate predecessor, Secretary John H. Dunlap, had resulted in four men serving as secretary or as acting secretary in the brief four-year period. That chain of events, through no fault of the men who loyally carried on until a permanent appointment could be made, resulted in an accumulation of issues to be settled by the new Secretary and many matters of policy to be laid before the Board of Direction by him.

George Seabury took over the handling of these problems with the skil and diplomacy that characterized all his years of service to the Society. He did much to bring about the smooth working of the new Constitution and to eliminate causes of dissatisfaction which had been in existence in earlie years. He served in the period of transition of the Society from a primaril eastern seaboard organization to a truly national society. He contributed very greatly to the change that took place over the years in the more friendl attitude of many members and in the increased loyalty to the Society on th part of all members.

He was an exceptionally fine executive officer and a good administrator Fully informed at all times of the business affairs of the Society, he neve brought matters of minor detail to the Executive Committee or the Board o Direction, but if there was any serious trouble that called for advice he kep the President and Executive Committee fully informed. He laid facts a he saw them before the Board, but never took the lead in pressing for actio

As a result of these characteristics, his relations with the Board wer uniformly harmonious and cooperative. Few men could view a difficul problem as objectively as he could, and few had his quiet tenacity of purpose in seeking a correct solution once he had made up his mind what the correct solution should be. As a result, few of the controversial issues that arose were settled in a way that was unsatisfactory to him.

If George Seabury had one outstanding characteristic, it was ability to attain his objectives without appearing to take the lead. Superficially, the Board of Direction or the Executive Committee appeared to be shaping Society policy on important matters, but if Mr. Seabury had had an opportunity to think them through to a conclusion beforehand, as was generally the case, it was he who provided the initiative by a suggestion here or a quizzical inquiry there. The groundwork for many a Society policy was laid in a quiet discussion between him and one or two of the Directors around the morning cup of coffee or during an evening tête-à-tête.

In the Board meetings that followed, proposals to set up a new Society policy came from the Directors; Mr. Seabury seldom volunteered a suggestion, but spoke his mind freely if asked for an opinion. When he volunteered a suggestion, it generally meant that he saw the Board heading for trouble.

Outstanding among the changes in policy that originated with Mr. Seabury was the Society's acceptance of leadership in a movement to increase the compensation of engineering employees. This was one of the rare instances in which he found it necessary to take the initiative himself. When he became Secretary, the subject of better compensation for employed engineers was being actively agitated; but little progress was being made. Immediately he was deeply interested in the matter and came to the conclusion that the ASCE should take an active part, using its prestige to support the weaker and less well-known organizations then carrying the ball.

An opportunity to act came some three years later. Engineers in the employ of New York City were then endeavoring to get better pay, but the success of their efforts was in doubt—strong support was needed. Laying the matter before the Board of Direction, Mr. Seabury sought and received its permission to appear personally before the Board of Estimate of the City of New York to urge that engineers in city employ be paid at least as much as the mechanics they supervised. His making such an appeal was approved, and ultimately the increases were authorized.

This but marked the beginning of Mr. Seabury's active interest in bettering the working conditions and compensation of employed engineers—an interest that never lagged. Much of the success of the later activity of the Society in developing and approving schedules of salaries for engineers and engineering employees was due to his continued and unfailing support.

Getting successive Boards of Direction to give more than lip service to the idea that the Society should sponsor such salary schedules was one of his major accomplishments. When he had done that, he set out to see whether the Board's ideas could not be made the pattern for similar action by the other major engineering societies. In this, however, his efforts failed; the boards of other Founder Societies were unwilling to undertake similar studies in their fields.

When the financial collapse of 1929 brought loss of jobs to hundreds of engineers, Mr. Seabury was among the first to urge the Founder Societies to set up some system of aiding destitute men. That move resulted in the establishment of the Professional Engineers Committee on Unemployment, which provided financial aid and assisted many men in finding new jobs. He also took an active part in getting the federal government to expand its survey work as a means for creating jobs for engineers. To this end he made numerous trips to Washington, D. C. The ensuing government employment gave much needed help to more than twelve thousand engineers in their emergency.

When it became evident that the federal government would have to create work as a means for checking the spread of unemployment, the Society took the lead in advocating the desirability of constructing worthwhile public works as "pump priming." In that effort Mr. Seabury took an active part, especially in the selection of men to head the regional offices of the federal relief agencies. Subsequently, he was tireless in his efforts to raise the pay of engineers working for relief agencies to the level of the scale for skilled mechanics.

Improvement of interprofessional cooperation was another activity to which Mr. Seabury gave much of his time. When the Society finally joined the American Engineering Council (A.E.C.) after a heated controversy, Mr. Seabury worked arduously to make A.E.C. a success. Later, when he became convinced that its usefulness was limited and that in some respects it served as an actual check on attainment of the Society objectives, he had the courage to recommend that the Society withdraw its support, despite sharp criticism from influential members who wanted A.E.C. continued.

Ultimate abandonment of A.E.C. as an agency to represent engineers on national matters did not change Mr. Seabury's views as to the desirability of close cooperation between the Founder Societies. Consequently, he turned his attention to development of the Joint Conference Committee, which ultimately became the Engineers' Joint Council shortly before his death.

His acumen in dealing with financial matters which affected the organization is nowhere better illustrated than in the handling of the Society's property at 220 West 57th Street in New York. His partner in this activity was Col. Lincoln Bush, who also had a deep interest in Society affairs and an uncanny grasp of real estate matters. Which of the two was primarily responsible for the splendid solution of this problem is unimportant. Suffice it to emphasize that each supplemented the other most effectively and that a division of the credit still leaves ample for each.

This project was first active about 1921. Through the succeeding years the Board of Direction repeatedly discussed the matter, feeling that the Society should not continue in the real estate business and that its resources could be better cared for if invested in securities; at one time it even authorized the sale of the Society house and property at a figure which actually proved to be but a mere fraction of its value. This latter fact quickly became evident to Mr. Seabury, and it was shortly confirmed when offers for lease as well as for sale disclosed that the current value was far in excess of the Board estimate. More detailed investigations revealed that the Society investment in this particular district had been most fortunate and that the whole area was enhancing rapidly in value and offered rosy prospects for the future.

The Board, therefore, accepted Mr. Seabury's advice that the title be retained but that the property itself be subject to lease. The first tenant, for a few years only, was an automobile tire manufacturer. Meanwhile extended studies were under way, with negotiations for a more permanent tenant. In due course final arrangements were consummated with a permanent lessee, an old established New York restaurant of outstanding reliability. The lease was for twenty-one years, subject to revision at the end of that period and to two more extensions of like period, with possible increase but not reduction in rental. Part of the income was earmarked for amortization of the mortgage, and at the same time the maintenance and taxes were made the responsibility of the lessee.

As a result the rental was net profit to the Society, in such amounts that the mortgage was soon extinguished and the property made free of encumbrance. The idea of building up a reserve by annual allotments from this annual income was Mr. Seabury's. As a result of that plan, the Society now can modernize the building at any time without drawing on its other assets. The entire arrangement is indicative of the business foresight in the original purchase of the 57th Street property; but this value could only have been certified to the present membership by the outstanding wisdom displayed by Colonel Bush and George Seabury, leaving the Society forever in their debt.

Of those Society activities in which he had a major part and in which he took obvious pride none surpassed the establishment and operation of *Civil Engineering*. He was not the only one who had seen a vision of such a publication, but he was the one destined to bring it to realization. It was not an easy job, because the conservatism of the Society was still a hurdle.

To try out the idea in a preliminary way he set up a four-page illustrated supplement to *Proceedings,* designated under the innocuous title, Part II. It dealt with Society news, as well as with professional activities and personalities, both current and historical. It ran from the middle of 1926 to the fall of 1930. The membership obviously liked it—also the Board of Direction.

However, when the idea was extended to the next obvious step, a full-blown monthly publication, the Society fathers were hesitant, in spite of the fact that theirs was the only one of the Founder Societies without such a magazine. The question of commercial advertising was only one of the stumbling blocks. By Mr. Seabury's quiet persuasiveness, by conferences, committee meetings, and reports, the Board was finally won over and the publication begun in the fall of 1930.

Perhaps no worse time to inaugurate such a venture could have been found, as the full effect of the financial collapse of 1929 was only then beginning to be felt. It is therefore all the more to the credit of the administration that the effort was pursued without let-up and it is especially to

Mr. Seabury's glory that it was successful. His were the basic ideas and his the control of policies. His satisfaction in the outcome was not heralded but it was acknowledged openly to friends. It was all the more real because he knew it was justified.

Early in the depression that followed the financial collapse of 1929, Mr. Seabury saw with concern but no panic the spread of unionization among engineers employed by the relief agencies as well as in some of the engineering organizations in the civil engineering field. Immediately, he got the Board to appoint a small committee to study the subject and to report on its implications for the Society. He even went so far as to attend a union mass meeting in an endeavor to learn what the union strategy was to be.

Then when passage of the National Labor Relations Act made it obvious that collective bargaining was here to stay and might ultimately include many engineers, Mr. Seabury convinced the Board that there should be a man on the staff who would devote most of his time to questions relating to collective bargaining and related matters. About the same time the Society's Committee on Unionization was replaced by a large committee with the broader assignment of the economic status of engineering employees.

Out of the new committee came the unprecedented proposal that the Society sponsor the formation of collective bargaining groups among engineering employees. None saw the implications of that proposal more clearly than Mr. Seabury. Nevertheless, he was so convinced it was a long step in the right direction that he was willing to see the Society face the risks clearly indicated in such a movement.

Because he was convinced that the young engineering employees needed some such agency as the Society proposed to set up, he was willing to see the Board take an arbitrary attitude in refusing to submit the whole matter to a referendum until the emotional excitement had passed and the membership at large had had time to think the subject through. It was characteristic of Mr. Seabury that he could laugh off the criticism aimed at him as the instigator of that arbitrary action; but, despite his superficial lack of concern, the long controversy was a trying experience that sapped his strength more than he or anyone else realized. It proved to be the last major Society activity in which he was to play a leading part.

It is impossible to evaluate fully his services to the Society or his influence on the membership. His fine personality, his excellent memory for names and faces, his nationwide acquaintance due to his many contacts with members at Society headquarters and his extensive travel, not only to Society meetings, but to many Local Section meetings and district conferences all over the United States, probably brought him into personal contact with more members than any other officer the Society has ever had.

In dealing with these thousands of members he was most unusual in his sympathetic handling of the many personal problems all through the difficult years of the depression. Some of them were hard to solve. Many of them, if handled differently, would have left only sore spots, but none among the hundreds who were necessarily dropped from membership for financial reasons failed to keep the highest regard for the way Mr. Seabury handled the case. Hundreds of these took the first opportunity to resume their memberships.

His relations with the staff were also uniformly successful. He was intensely loyal to his associates and knew the strength and shortcomings of every employee. During the difficult years of the depression, from 1932 to 1939, when there were times, in the preparation of the budget, that it appeared probable that there would not be sufficient income to meet current expenses and that reduction of staff or salaries would be necessary to balance the budget, Mr. Seabury was willing to, and did, accept cuts in his own salary, but strenuously opposed reduction of pay of members of his staff. With extreme reluctance he finally acceded to the plan whereby a number of department heads shared with him in the necessary reductions.

George Seabury possessed in very large measure all the qualities which made him the model administrative officer of a great professional society—the personality which endeared him to the thousands of members, young and old, whom he met in his twenty years of service; the wisdom that kept him free from any taint of politics or clique; the executive ability that enabled him to conduct a great business well and smoothly without bringing details to his Executive Committee or Board of Direction; and the loyalty to his subordinates and to the department heads which resulted in a maximum quality of service from all.

To aver that George Seabury was a great Secretary of the Society is to give but a trite expression to a deep truth. He was so recognized everywhere —not only in the organization but in all gatherings of engineers. As early as 1930 no less an authority than the Italian Senator Luigi Luiggi, Hon. M. ASCE, spoke of him as "the ideal secretary." This is significant in that he then had been in office only five years. Certainly he lost nothing in capacity or in stature in the remaining fifteen years.

Acknowledging the fact, it still is not easy to analyze the basis of great accomplishment. Certainly it was not in a single tremendous personal asset, but rather in the unusual possession of many important but lesser attributes, which combined, went into the making of an extraordinary character. If there was nothing in him transcendentally brilliant, it is nevertheless true that there was no one facet of his make-up that was not perceptibly above average. It was this unusual combination that created the remarkable fact of George T. Seabury.

First and foremost, he was an industrious worker—too much so for his own good. He was a seven-day week type of Secretary. Night work and Sunday work were not strangers to him. He never gave a mere casual glance at a Society problem; instead, he was too apt to go into it exhaustively. As a result his associates tried to save him from himself by clearing up mutual matters at brief conferences, and avoiding reference to anything except basic questions. However, no one could relieve him of Board and policy matters —these he did carefully, faithfully, and minutely. The Board appreciated this loyalty and at the same time its dangers. In 1937 it instructed him to take a three-month leave for recuperation; a sea trip to Europe worked wonders, since it isolated him effectively from Society problems.

During World War II, conditions were particularly difficult. One after another of the staff were called into service and he acquiesced loyally, although recognizing the added burdens entailed. In each instance he tried to shoulder some of the transferred load. There was no complaint, yet there was no blinking at the increased work. Only his associates and Society officers realized the tax on his strength. In sober truth he, too, was a casualty of the war.

The amount of work Mr. Seabury did was prodigious, especially in connection with the many Society committees. Few committee chairmen failed to avail themselves of his counsel. As a result, the impress of his own judgment was felt throughout.

In statistical or financial matters he was at his best, as witness his handling of the 57th Street property. Every important file would contain somewhere a correlation or tabulation, perhaps on cross-section paper, revealing a summary of cogent data. There was also a methodical quality to his work, whether it was an important appointment for Society service, the remembrance of some employee's long service, or the preparation of his list of hundreds of Christmas cards. The Society was always the gainer in that no critical hazard was permitted to develop personal friction, simply because he foresaw and prevented that possibility. His financial sense was also uncanny, although probably his intense grasp of budgetary matters resulted more from midnight oil study than from any intuitive hunch. Certainly the Society's social as well as business affairs were most capably handled. When he left his office that last spring evening, not realizing he would never return, he left his Society and his personal affairs in perfect order. There was no "unfinished business."

In all Society matters he was a perfectionist. Questions of textual expression had to meet his very critical judgment. Of two ways to handle a problem only the better satisfied him, even if it happened to cost more. He was very jealous of the Society's prestige—the best hotels, the best printing, were not too good. He loved to recall the comment from a prominent member of another engineering group: "I notice that whatever your Society undertakes, it does a most creditable job." This was not meant as a personal compliment, but actually it was. The fact of doing things well was not just an accident. He had not only a passion for the best but the necessary drive to achieve it. In a personal letter he once confided, "I could not live without enthusiasm." This was characteristic of his whole life.

One might differ with Mr. Seabury's conclusions, but no one could successfully take issue with the fundamental soundness of his approach and the unimpeachable probity of his attitude. Thus it was that his official actions took on the strength of a moral quality, and thus seldom were they successfully contested. At times he had to delay execution on some borderline case unti the opinions of others caught up with his, but, in the end, nine times out of ten, it would be his judgment that was vindicated.

At such times he had a quiet method of approach, of rationalizing a difficulty. He could see the other person's point of view but he would quietly, patiently develop his own side until the weight of his arguments would usually win. Naturally, he was exceedingly diplomatic, which only added to his effectiveness. On a survey for water supply development—this happened during his Providence work—he came across an irate and obdurate farmer, who declared no survey party would touch foot on his land; and he had a shotgun to emphasize this argument. By gentleness and firmness combined, George Seabury secured a conference and within a half hour, in the man's own parlor, won him over. The man capitulated, but only with the proviso that George Seabury conduct the survey.

This consideration and courtesy in dealing with others paid handsome dividends. It inspired the loyalty not only of Board and Society members, but of the staff. All who ever worked with him, even if they disagreed, held him in utmost respect, often in affection. This was largely because they recognized his happy faculty of doing things, even unpleasant things with a gracious thoughtfulness of others. He curbed his feelings because he felt that such restraint was due the Society he represented.

No account of George Seabury's remarkable services would be complete without acknowledgment of the way in which Mrs. Seabury shared in those services. Her wide acquaintance, her interest in, and cordial relations with, the wives of Society officers and members were second only to his with respect to the men. As a wife her work complemented his work. She was always thoughtful of others, and to make them happy made her happy. The wives of Board members will long remember the enjoyable evening party held in the Seabury apartment on Washington Square each year at the time of the Annual Meeting.

It was Mrs. Seabury's custom to give a dinner to the wives of the members of the Board of Direction and officers of the Society, so that the newcomers might become pleasantly acquainted with those already accustomed to Society gatherings. Long years of warm friendships among the ladies who attended with their husbands have made very delightful the quarterly meetings held in all parts of the United States, and this has added materially to the unity and effectiveness of the Board.

Relatively few members of the Society had the opportunity to know the real George Seabury in a social atmosphere. His official manner was one of seriousness and dignity, but he could unbend, and successfully. He dearly loved those informal moments—as witness the whimsical response he gave at the Illinois Section's dinner on the occasion of his last Board Meeting in April, 1945. He loved most of all to gather around the piano in friendly singing; he had a pleasing, fairly strong, baritone voice, an ear for harmony, and much enjoyment in simple group singing. During his years in Rhode Island he was a loyal and enthusiastic member of the University Glee Club of Providence, which ever remained a pleasant memory in his busy life.

Members who were so privileged will best remember him at his office desk, always ready to defer some pressing work for professional or individual matters. Normally his decision would not be a snap judgment. Slow in reaching a conclusion, he was all the more sure of his ground and seldom had recourse to revoking it. That his judgment and advice were uncommonly good is evidenced by the results. Under his guidance, the organization grew in moral stature.

It also grew in size during his twenty years of service from about 11,200 in 1925 to about 20,700 when he died. The most notable aspect of this growth was in the number of Juniors which showed the remarkable increase from 736 to more than 6,500 during his tenure. Partly this was due to the operation of the new Constitution, but much was also attributable to Mr. Seabury's intense interest in, and appeal to, the younger men. Even through a great depression the Society kept a remarkable hold on its membership, at the same time ministering in varied ways to engineers both in and out of the Society. The relation was but a reciprocal one to Mr. Seabury's own attitude —the Society was his life, beyond which no personal consideration was permitted even a hint of precedence.

Of the work which he did to bring about better understanding and a closer relationship between the different branches of the engineering profession but little is generally known. This, however, was a very real service which he did well. His intimate relationship with the secretaries of the other great engineering societies did much to bring these separate groups together as one united profession in the solution of problems created by the war. His associations with the Engineering Institute of Canada were especially close and congenial, both as to the organization itself and as to countless individual members who were proud to claim personal friendship.

In ways other than length of tenure he was dean of American engineering secretaries. That his monumental work had impact far beyond the Society alone was evident in the spontaneous eulogies at his death. One of the clearest and most sympathetic of these appeared as the leading editorial of the July, 1945, *Mechanical Engineering*, organ of the American Society of Mechanical Engineers. It was from the pen of the editor, George A. Stetson, friend and confidant of Mr. Seabury during the full twenty years of Society work, and gave the following glowing appreciation:

> "What manner of man was George T. Seabury * * * for twenty years secretary of the American Society of Civil Engineers and for much of that time dean of engineering-society secretaries in the United States? Short, slender, with thinning hair, level steady eyes behind rimless spectacles, fair, clean-shaven, with a straight mouth that curled up on the right side when he smiled, he was a man of quiet dignity whose conservative dress was never awry. His voice lacked volume; was persuasive rather than commanding; at times, animated and buoyant, at others, unemotional and businesslike. For in him nature contrived to combine the contradictions of New England reticence and austerity with a warm friendliness and generous human impulses. The frankness of his speech was tempered by an astute diplomacy. His pleasures were not in athletics but in the quiet pursuits of simple living. He enjoyed the company of his friends. In convivial surroundings he would relax, tell stories, and throw his head back in wholehearted laughter, or punctuate an apt phrase with a snap of the fingers. When his less restrained constituents in the

sections laid care aside, he entered into the spirit of their horseplay and good-naturedly let them make him the butt of their jokes. This thoroughgoing humanness won for him affection without loss of respect.

"Energy without noisy and tiring bustle, enthusiasm and sincerity without affectation, intellectual and physical courage without braggadocio, frankness without malice, precision without pedantry in expression, were the qualities he brought to the conduct of his affairs. Wisdom, seasoned with experience, guided his judgments and his counsels.

"Seabury possessed the engineering characteristic of realistic planning. He could think objectively about himself and his job. One sensed that he viewed retirement from the secretaryship as a matter of obligation and wise policy and not as an opportunity for greater leisure. In a letter to the Board of Direction asking to be relieved of the secretaryship on his sixty-fifth birthday he wrote: 'I have given all that was in me to give . . It has been an arduous job and I have thoroughly enjoyed it, but . . . I should give none other than my best years to it.' It was apparent to his friends that he faced an emotional conflict over the problem of giving what aid and counsel he could to his successor without standing in the other's light or giving the impression of wishing to influence actions or decisions. It was apparent also that the warmhearted friendly Seabury was in conflict with the impersonal objective alter ego with its selfless sense of obligation to the greater good. This dilemma had been happily resolved by the action of the Board of Direction which was loath to lose Seabury's experience and wisdom and hence had created for him the post of assistant to the president. Scarcely more than a week before Seabury's successor took up the responsibilities of the secretaryship, Death itself intervened.

"The twenty years during which Seabury served as secretary of the American Society of Civil Engineers were critical ones for that society and for the Nation. They included the reconversion period which followed World War I, the great depression of the 1930's in which engineers experienced greater hardship and unemployment than ever before in their history, the era of made work and extensive public works projects, the fiasco of the N.R.A., the intense activity in construction that accompanied preparedness and national defense programs, and the global war in which we are still engaged.

"In the affairs of engineering societies these same years saw the rise and fall of the American Engineering Council, the organization of the Engineers' Council for Professional Development, the formation of the Joint Conference Committee of presidents and secretaries of the Founder Societies (now known as the Engineers' Joint Council), the almost complete acceptance of engineering licensure, comprehensive studies of the economic status of engineers, vigorous action for the relief of unemployed engineers, incorporation and extension of the Engineering Societies Personnel Service, Inc., constant struggles to upgrade engineering fees and salaries, and vexatious developments in collective bargaining for engineering employees. Those two decades from 1925 to 1945 were also marked by steady and gratifying growth of membership of the American Society of Civil Engineers, stability of its financial structure, the attainment of harmony and a progressive co-operative attitude in its management, and the initiation and successful development of a monthly magazine, *Civil Engineering,* to supplement the technical publications, *Proceedings* and *Transactions* issued by the society.

"In these and other events of the years covered by his secretaryship, Seabury played an important part, and although he scrupulously maintained the relationship of 'servant' to the Board of Direction and the committees of the Society, persons who knew him well must confess that his wisdom, intelligence, experience, and knowledge of men and affairs were powerful influences in the formation as well as the administration of society policies. Nor was his influence confined to the society he served as secretary; it was felt and gratefully acknowledged by sister societies and their officers and by other organizations in which members of the engineering profession hold common interests.

"A profound sense of personal loss, deep affection, and grateful appreciation of wise counsel frequently asked and never denied make it difficult for the writer of this appraisal and tribute to treat the subject with the restraint and objectivity which Seabury himself would demand. To one who was close to Seabury during the twenty years of his secretaryship yet sufficiently remote from the immediate environment in which his work was done, the greatest factor in Seabury's life was his powerful influence, frequently exerted in the face of criticism, in the welfare of civil engineers. One could not always agree with him and would sometimes be astonished at views and opinions that disclosed the conflicting emotions of realism and idealism in his nature, but one never questioned his courage, his integrity, and his sincerity. For oneself, for Seabury, and for his friends and family, one regrets his sudden death which robbed him of the satisfactions of an old age with its less strenuous tasks and its less onerous responsibilities. One regrets also that the suddenness of his going forestalled the high tribute in perpetuity to his memory that honorary membership in the society to which he gave so much of himself would have been.

"At the height of his career, with his prestige as a faithful servant and leader unimpaired, Seabury had the wisdom and courage to step down, unwilling to give any but the best years of his life to the task he performed so well. Twenty fruitful years of the history of the American Society of Civil Engineers will constitute his monument."

Many besides Mr. Stetson greatly mourned his death. Expressions of sympathy and eulogy were numbered by the hundreds, including those from personal or family friends. Reference will be made here only to a few typical ones among those having Society or other professional connotation:

From the Seattle Section—

"WHEREAS the Giver of all good and perfect gifts, the Designer and Builder of the Universe, has so ordered the lives of men that they know not what may be their span of life, and

"WHEREAS it is given to each according to his ability to weave a pattern or build a single part in the great structures of earth, and

"WHEREAS there comes a time in the affairs of every man when he is called upon to lay aside his responsibilities of life and go to a far country from whose bourne no one returns, and

"WHEREAS there has gone from our midst, a fine leader and Secretary of the American Society of Civil Engineers, George T. Seabury, and

"WHEREAS, we, of the membership of the Society who have known and loved him for his sterling character, shall miss his wide leadership and his friendly counsel;

"*Now, Therefore,* the Seattle Section of the Society wishes to record the highest admiration which we have had for Mr. Seabury, whose memory we will always cherish both as Secretary of the Society and as a friend."

From the Philadelphia Section, to Mrs. Seabury—

"Words are wholly inadequate to express the feeling of loss which we share with the entire engineering profession. For us, in the Philadelphia Section, the feeling is particularly keen because we had the privilege of knowing him personally, of having him and yourself with us from time to time, and of benefiting more directly than many from his wise guidance and counsel.

"With the engineering profession, we have lost and outstanding leader and one who worked long and earnestly in our behalf. More than this, however, we have lost a warm and kindly friend."

From a former member of the Board who has—

" * * * watched with the greatest admiration the way in which he has handled the affairs of the ASCE. The reason for his unmatched success was that we all loved him as well as trusted him. Te was gentle in all his dealings yet never yielded a principle."

From an English member—

"I never had the pleasure of meeting him but I corresponded with him as a member of the Society. I somehow felt that he went out of his way to be helpful to many of us recent members during the war. In particular he has been very kind to me personally and I had hoped to thank him in a personal way when I could have come to the States."

From one of the oldest members (New England)—

"I had great respect for him. He gave all he had to the Society. In its effect that was much. As Secretary he was a model of propriety and discretion. I have not met his equal."

From an officer of the Engineering Institute of Canada—

"Not long ago, a group of seven past-presidents of the Institute and a few other prominent members [met] in Montreal. * * * The three members of the Institute who are honourary members of the American Society of Civil Engineers were all present, and [one] spoke for them as well as for everyone else, when he recounted some of the things that George had done for the cause of engineering and for our Institute. Among other things, [he] said, 'He was a lovable and beloved friend of us all. He always looked for the good qualities in a person, rather than their faults. The high standing of The Engineering Institute of Canada with its sister societies in the United States is due almost entirely to his interest in us and his friendliness toward us.'

"In my own capacity as a friend, I want to tell you of the great loss which I have experienced. Almost from my first day of office with the Institute, George has been chief guide, mentor, and counsellor, in all matters relating to the profession and to professional organizations. Very early in my term of office, * * * he took me in hand, gave me a great deal of information and advice, and a warm friendly reception, which has been helpful and inspirational to me ever since.

"I never visited New York, no matter what my business may have been, without calling on him, and no matter how busy he was, he always took out time to see me. Although I am describing only my own case, I know there are dozens of others who have had the same experience with him. He was indeed a tower of strength to a great many of us."

From a retired staff member—

"Those of us who had the privilege of working with Mr. Seabury * * * we of the staff—past and present—will never forget him, as leader and friend."

From a member in far-off New Zealand—

"One who was so helpful and kind to us when in America, and one whom it was an honour and privilege to know—a true gentleman who was happy in furthering the interests of the profession and of the Society. Mr. Seabury's personal interest in all the members and, particularly for me, his very real interest and kindness to those, like myself, far from Headquarters and outside the United States, has always been for me a very pleasant knowledge, and we do very sincerely feel that in his passing we have lost a real friend.

"I often remember, and I mention it now as one little instance of my visit. In the Hotel Seneca in Rochester, N. Y., I was sitting alone—although due to the Society, I could not claim to be a stranger in a strange land—just before the Fall Meeting in 1938. Mr. Seabury came over to me and said 'Should I not know you?' His interest was in every Member of the Society. I believe every one of the 20,000 members of the Society knew Mr. Seabury as a friend, and for the very fine gentleman he was."

From a Canadian member—

"Mr. Seabury was my ideal of the perfect secretary, kind, thoughtful, considerate of everyone around him, and, with all that, very clear headed. He gave me the impression of knowing very well, all the time, just what he was about, yet never appeared as a sycophant; a hard fighter for what he deemed necessary, but a kind hearted man; and one full of interest in all he met. May the world be blessed with many more like him."

From the past-president of a sister professional society—

"In working with him for many years on interprofessional committees of various kinds I found him that rare thing—an efficient worker who was first and last high-minded, idealistic and yet practical!"

From a staff member—

> "For fifteen years it was my privilege to call Mr. Seabury my Chief. His inherent kindness, his understanding nature, his generous helpfulness and his wise counsel endeared him to me. He was the finest man with whom I have ever worked."

From a college dean, regarding association—

> "* * * during the years we served on the Board of the Society.
>
> "The joy of it all was the fact that no matter what differences of opinion or viewpoint might turn up in our formal meetings, our informal gatherings were always—well, I was about to say, like those of a happy group of grown up children. George was always so much a part, the essential part, both of our labors and our happiest moments, that it cannot be the same American Society of Civil Engineers without him."

From one of the women members—

> "When a life has been lived so richly and so fully, right up to the hilt, as Mr. Seabury's was, it can accept its Divine rest and Eternal Peace with the comfort that the job is well done."

From a long-time associate in another society—

> "There was not a small thing about George and the great contribution he made to his profession will last long."

It was but natural that he should receive the major credit for Society advances during his long and successful term of office. He cared little for credit and indeed he instructed that his name be avoided or minimized in any Society publicity. Nevertheless it was impossible thus to sidestep the acknowledgment so richly deserved. He received ample credit, in spite of himself, and inwardly it gratified him. So it was he was proud of the various tokens of esteem, represented in numerous social, professional and honorary organizations. Besides his Society pin, he always wore his Chi Epsilon and Tau Beta Pi keys.

In chronological order his engineering affiliations included:

Organization	Date
Engineers Club of New York	1925
Society for Promotion of Engineering Education	1926
Society of American Military Engineers	About 1927
American Water Works Association	1930
Ritual of the Calling of an Engineer (honorary)	March 27, 1931
National Society of Professional Engineers	January 1, 1935
New York State Society of Professional Engineers	1935
Royal Institution of Netherlands Engineers (honorary)	August 2, 1937
Chi Epsilon (national honorary)	March 8, 1939
Engineering Institute of Canada	August 14, 1943
Tau Beta Pi	May 3, 1944

George Seabury asked the Board of Direction at the end of its January, 1945, meeting to permit him to retire as soon as a successor could be found. He saw more clearly than his associates that he could no longer carry the full load; and, as he expressed it, he was determined to "give none other than my best years" to the service of the Society.

In April, 1945, William N. Carey, M. ASCE, was chosen by the Board to succeed Mr. Seabury and Mr. Seabury was appointed to the newly created position of Assistant to the President. By this arrangement it was hoped that he could be relieved of all administrative routine and yet would be available to represent the president on intersociety boards where his knowledge of Society affairs would be most useful—but death intervened to bring that plan to naught.

Following a coronary thrombus in the early morning of May 23, 1945, he rallied for a day or two but succumbed in the Post-Graduate Hospital in New York City on May 25. He is survived by Mrs. Seabury (who was Margaret Howard Knight of Providence before their marriage in 1904); by a daughter, Mary Knight (Mrs. Medley P. Ray) of New York City; and by a small granddaughter, Margaret Ray. All his brothers had predeceased him.

Services were held in the Church of Ascension on Fifth Avenue, New York, on May 28. The Society office was closed for one day, as well as many an engineer's office. Past officers came in great numbers; officials from other national societies; Local Section representatives from as far as the Middle West; Army and Navy officers; retired and active staff members; and relatives—including little Peggy who came to leave with her granddaddy as her last tribute a choice toy that they had played with so often. Later, that afternoon the body was laid to rest in the family lot, on a wooded hillside at Swan Point Cemetery, Providence, beside the two children, Howard Knight and Thomas Mumford, III, who had both died in boyhood.

Thus ended a strong and a successful career, devoted to intense activity and wholesome service to his generation—its last twenty years given over without reserve to the ideal of making the Society a more useful instrument in American life.

George T. Seabury was elected as Junior on March 31, 1903, elevated to Associate Member on December 1, 1908, and transferred to Member on May 7, 1913. He was Secretary of the Society from January 1, 1925, until his death on May 25, 1945.

WILLIAM WESLEY AMBURN, M. ASCE[1]

DIED FEBRUARY 27, 1944

William Wesley Amburn was born in Nevada, Iowa, on January 18, 1865, the son of Allen Amburn and Mary Ellen (Utterback) Amburn. Both parents were descendants of early Colonial families.

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[1] Memoir prepared by C. F. Thomas, M. ASCE.

Mr. Amburn attended elementary schools in Iowa and South Dakota, and entered Sioux Falls University, in Sioux Falls, S. Dak., in 1883 where he was a student until 1888.

He started on a career, largely associated with railroad location and construction, in 1889, as a rodman for the Burlington and Missouri River Railway (later a part of the Burlington System). From 1890 to 1893, he was employed by the Northern Pacific Railway on location and construction work, between Chehalis, Wash., and the Pacific Coast, at South Bend, Wash. He advanced through the usual grades from chainman to chief of party and then to bridge inspector and engineer in the Bridge Department.

In 1893, the City of Portland, Ore., decided to bring a pure water supply from Bull Run Lake, near Mt. Hood, some forty miles distant. The natural resources of the Northwest were an attraction to Mr. Amburn. Therefore, he joined the staff at the headworks in the Cascade Range forest area. He had charge of placing the upper ten miles of the forty-two-inch pipe line, and later he had charge of the placing of a similar line across the Willamette River at Portland.

Much of Mr. Amburn's engineering work, between 1897 and 1924, was directly associated with railroad location and construction on all the large railroads of the Northwest.

After completing work on the Portland water supply line, he was assistant engineer in charge of ten miles of construction for the Oregon Railroad and Navigation Company (later a part of the Union Pacific System). He also had charge of twelve miles of construction for the Northwestern Railway, along the Snake River in Oregon.

Mr. Amburn was then employed by the Great Northern Railway and made reconnoissance and location surveys in Montana, Idaho, Washington, and British Columbia. An especially difficult location survey was made for the Vancouver, Victoria and Eastern Railway, from the summit of the Cascade Range to Princeton, B. C., Canada, and from the summit to Hope, B. C.

For the Chicago, Milwaukee and St. Paul Railroad, Mr. Amburn worked as location engineer on lines in Montana, Idaho, and Washington, and he was in charge of main line revisions from the Bitter Root Mountains in Idaho to the Columbia River in Washington.

Mr. Amburn's work was not continuous on main line railroad location and construction. From February, 1904, to August, 1905, he was principal assistant engineer in charge of topographical surveys for the Deschutes Irrigation and Power Company. In this capacity he made locations for, and was in charge of the construction of, the Pilot Butte and Central Oregon canals and laterals.

Mr. Amburn did extensive work in connection with the location and construction of standard gage logging lines in western Oregon and Washington, involving tunnels and construction of unusual timber bridges, one of which was one hundred and eighty feet high.

After 1924, Mr. Amburn worked on planning and property development. The era of new railroad construction was over. In 1935, in charge of a Civil Works Administration (C.W.A.) project, he made a terminal and transportation survey for Portland, involving a coordination of railroad, street car, and vehicular traffic. This plan was known as the "Amburn Project." World War II interrupted the completion of this work.

Mr. Amburn's personal life was simple and quiet. He was particularly fond of camping and was an ardent fisherman. Most of his active life was spent in the great outdoors. Mr. Amburn was a member of Patrons of Husbandry.

Mr. Amburn was elected an Associate Member of the American Society of Civil Engineers on September 2, 1908, and a Member on June 4, 1913.

HORACE ANDREWS, M. ASCE[1]

DIED FEBRUARY 21, 1945

Horace Andrews' active life in engineering covered a period of approximately forty years, nearly all of which were spent in public service. In this work Mr. Andrews constantly fulfilled his passion for careful and accurate accomplishment, of which an early associate said: "He always did any piece of work a little bit better than it was necessary to do it." A scientific and systematic turn of mind was natural to him, for it lay in his inheritance and training; his grandfather, Ethan A. Andrews, having shown it in his work on Latin vocabulary and grammar; a great-uncle and a brother—the latter, Prof. E. A. Andrews of Johns Hopkins University, Baltimore, Md.—having shown it in the realm of biology.

Horace Andrews' background was New England, Dutch, and Scotch by origin. He was born on March 19, 1852, at the home of his maternal grandfather, William Johnson, at York Square, New Haven, Conn. His parents, Horace and Julia Russell (Johnson) Andrews, were residents of New York, N. Y., although of Connecticut origin. His ancestor, the Rev. Thomas Hooker, was the founder of the Connecticut colony.

After preparing for college, in New York and New Haven, he entered the then small and little-known Sheffield Scientific School of Yale at New Haven, in the class of 1871, which comprised twenty-five members. At graduation he received the Bachelor of Philosophy degree; and after a year of graduate work and teaching at Yale he received the degree of Civil Engineer (being one of the first to be awarded this degree).

From 1872 until 1876 Mr. Andrews was employed by the City of New Haven, and then by the U. S. Coast and Geodetic Survey, in field work and drafting for triangulation, hydrography, and plane-table work in and about New Haven Harbor, Connecticut, for the commissioners who were determining the harbor line. Following a summer of triangulation and retracing lines for the Horicon Iron Company on and near Lake George, New York, he spent the winter and spring of 1877–1878 in hydrographic and triangulation work for the U. S. Coast and Geodetic Survey. This work was done on the Indian and Banana rivers, along the east coast of Florida, and was, of course, long before the development of the resorts in that area, and when only scattered orange groves indicated the settlements of today.

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[1] Memoir prepared by the Rev. Theodore Andrews, St. Peter's Church, Mountain Lakes, N. J.

In 1878 Mr. Andrews worked with the New York State Trigonometrical Survey (the State Survey), which took him to many parts of the state on trigonometric and topographic work. These surveys entailed extensive trips by horse and buggy through the rural areas of central New York—regions where conditions then were very primitive.

He established his residence in Albany in 1879 where he brought his bride, Adeleine Louise Downer, of Hamden, Conn., to whom he was married on October 13, 1881.

The field work of the Survey being terminated in 1884, he spent the following year at work on the volume embodying its final report. From 1883 to 1886, he was employed from time to time by the New York State Board of Health. Many of his researches and reports are to be found in this Board's annual proceedings; chief among these were studies of the sewerage system of Keene, N. H., and of two localities in Albany where the existing conditions were deleterious to public health.

In 1886 Mr. Andrews obtained a civil service appointment as examiner in the U. S. Patent Office. Before he could accept, he was offered the post of city engineer by the Hon. John Boyd Thacher, then Mayor of Albany; this office he accepted, and began his duties in May of that year. During his tenure of office, a number of public improvements (to which he contributed his professional skill) were initiated and accomplished. Chief of these improvements were the Knox Street, Northern Boulevard, and Southern Boulevard viaducts, and the Beaver Creek Sewer. He was ex officio member of the city Board of Contract and Apportionment, and the Board of Health. He was chairman of the Board of Plumbing Examiners; as well as being a member of the State Board of Examiners for New York State engineering posts for seven years. The writer well remembers that, although every contract called for an inspector, it was very often Mr. Andrews' task to inspect the inspectors! His activities are detailed, although in a way hidden, in the reports of his office from 1891 to 1899. At the close of the latter year, the city passed under a new charter designed to be uniform for cities of the second class (rated by population); this year also marked the close of his work as city engineer.

From 1900 to 1910 he was engaged in practice as consulting engineer, with office in Albany; during this period he designed and constructed under a contractor the sewerage system of Plattsburg, N. Y. He also examined the foundations of some bridges, and was consulted as to pavements in Cohoes, N. Y., and in New Haven. (The first asphalt and brick pavements in Albany were laid during his term as city engineer.)

Much careful literary work, some of it incidental to his professional activities, and some undertaken independently, added to the fruitfulness of these active years. Aside from the annual reports mentioned, the following are from his hand: The chapter on "City Surveying" in Staley's revision of Gillespie's "Higher Surveying" (1897); the final results of the New York State Survey

(1886); a number of papers in the annuals of the American Society of Municipal Improvements; and contributions to the *Transactions* of the American Society of Civil Engineers, to the *Engineering News,* and to numerous other professional publications. An example of his research is his work on a series of maps for Hooper's History of St. Peter's Church in the City of Albany, illustrating the former properties and burial grounds of that old parish. His inquiry into the correct rendering of the seal of the City of Albany shows the same interest.

In 1910 Mr. Andrews retired from active practice, and soon thereafter began to reside for the greater part of the year in Coconut Grove (which was later a part of Miami (Fla.), although he visited Albany every summer until 1941.

He was a fellow of the American Association for the Advancement of Science; for twenty-five years, he was a member of the Deutschen Geometerverein; and, from soon after its organization until 1915, he was a member of the American Society of Municipal Improvements. He was also a member of the Society of Engineers of eastern New York, and of the Yale Alumni Association of that area. He was a member of the ASCE Board of Direction from 1908 to 1910. He had belonged to the Fort Orange Club of Albany since 1887, and was a member of St. Peter's Protestant Episcopal Church in that city since 1893.

In his later years, Mr. Andrews studied bookbinding which, in his methodical and careful way, he followed in the spirit of a true and painstaking artist. Mathematics, photography, historical and genealogical research, and household carpentry were with him more than hobbies, they were means of artistic expression.

By his first wife, Adeleine Louise Downer, who died on December 8, 1893, he had three children, of whom two survive: The Rev. Theodore Andrews and Bertha A., wife of Dr. James E. Maloney. On June 15, 1910, he was married to Frances Eleanor Green, of Hull, England, who died on September 6, 1927; the children of this marriage, Dorothy Andrews and Mrs. Hilary Andrews Pyeatt, survive him.

Mr. Andrews was elected a Member of the American Society of Civil Engineers on April 6, 1887.

ASHER ATKINSON, M. ASCE[1]

Died January 3, 1946

Asher Atkinson, the son of Asher D. and Emma Jane Barnes (Bell) Atkinson, was born on June 10, 1867, in Brooklyn, N. Y. He was of Dutch and English ancestry. When he was about four, he and his family spent about eight months traveling in Europe and then settled in New Brunswick, N. J. Asher celebrated his fourth birthday in Frankfort, Germany. His early education was obtained at Rutgers Preparatory School in New Brunswick; and his technical training at Rutgers University. He received the degree of Bachelor of Science in Engineering in 1885, and the degree of Civil Engineer in 1887.

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[1] Memoir prepared by Willem Rudolfs, M. ASCE.

After graduation, from 1897 to 1902, he was employed by the Middlesex and Somerset Traction Company, constructing about fifty miles of electric railroad, a powerhouse, carhouses, and bridges (some of which, of a new and at that time rather daring design, are still in use). From 1902 to 1904, he was with the East Orange (N. J.) Water Works, designing a reinforced concrete reservoir, concrete standpipe, pumping station, and coal pocket. During this period he experimented with reinforced concrete, and the results of his experiments later were incorporated into a dam for the New Brunswick water reservoir. From 1905 to 1911, he was engineer in charge of the New York (N. Y.) office of the Corrugated Bar Company on the design of concrete structures. From 1911 to 1913, he was employed by the New York office of McArthur, Perks and Company, in charge of work connected with the construction of piers for the Port of Havana Dock Company in Havana, Cuba, principally checking designs and forwarding material for the work.

Most of his life—namely, from 1891 to 1905, from 1913 to 1914, and from 1915 to 1939—was spent as city engineer for New Brunswick. During his tenure, the city grew, requiring the design and construction of streets, paving, sewer system, water works, filtration plant, standpipe, pumping plant, and the Farrington water reservoir and dams. Also a chemical sewage treatment plant including force mains and pumping stations was constructed under his direction. In 1896 he took time out to travel extensively in Europe, and having been engaged in considerable surveying during the previous year in hilly country, was able to wear out a guide while climbing mountains in Switzerland. Mr. Atkinson retired, in 1939, after forty years of service as city engineer, and then became consultant to the city on water supply and water treatment plant operation until his death.

Mr. Atkinson was an enthusiastic golf player and amateur astronomer, especially interested in "shooting stars" which he plotted for many years. During his younger days he was intensely interested in water sports, building rowboats, sailboats, and, finally, a motorboat. He liked all kinds of sport and participated actively in bicycle clubs, taking long-distance pedaling trips. During his entire adult life he was an active member of the Second Reformed Church, being an Elder, Deacon, and chairman of the building committee for a new and beautiful church. In addition, he sang in the choir, did all kinds of committee work, and, when necessary, acted as janitor. He used to state he did everything for his beloved church but preach.

Mr. Atkinson had a fine personality. He was much liked by his associates and many friends for his honesty, loyalty, and genial nature. His kind and unpretentious manners were an inspiration to younger men. He was greatly interested in the progress of younger men, going out of his way to help them and trained a number of men, who did not have an opportunity to go to college, for an engineering career.

Mr. Atkinson was a member of the American Water Works Association, New Jersey Sewage Works Association, American Society of Professional

Engineers, and New Jersey Health and Sanitary Association, and a charter member and former president of the Kiwanis Club. He belonged to the Chi Xi fraternity.

In 1899, Mr. Atkinson was married to Elizabeth Vroom Knox of New Brunswick. Mrs. Atkinson; two sons, Asher, Jr., and William Knox; and four grandchildren survive him.

Mr. Atkinson was elected an Associate Member of the American Society of Civil Engineers on April 5, 1905, and a Member on March 12, 1918.

EDWARD PERRY BABCOCK, M. ASCE[1]

Died November 15, 1944

Edward Perry Babcock was born on October 11, 1887, at Chicago, Cook County, Ill. He was the son of Alfred Perry and Ida May (Winkleplek) Babcock.

After attending the English High School and Manual Training School in Chicago, he entered the Armour Institute of Technology (later part of Illinois Institute of Technology), also in Chicago, from which he was graduated in 1909 with the degree of Bachelor of Science in Civil Engineering.

Mr. Babcock's professional career evidenced special aptitude and skill in both civil and mechanical lines. From 1909 to 1911 he was with George W. Jackson, Inc., of Chicago on design and supervision of construction; from 1911 to 1923 he was employed by the Engineering Department of the City of Chicago as designing engineer on structures for the Chicago Water Supply System, bridges, and other miscellaneous structures in steel, concrete, and timber; and from 1917 to 1919, during World War I, he was chief hull draftsman on steel ships and plant structures for the Pensacola Ship Building Company. At various times from 1923 to 1944 he was associated with the writer, in Chicago, as designer and principal assistant engineer on bridges, buildings, docks, and other structures. During World War II, he was connected with J. W. Beretta, engineers of San Antonio, Tex., on reports, design, valuation, and supervision of construction; with Holabird and Root, architects of Chicago; and with Whitman, Requardt and Smith, consulting engineers of Baltimore, Md., on war projects.

Mr. Babcock had a keen analytic mind, was a hard worker, and devoted himself to the task at hand. His outlook was good, and he possessed not only keen perception of the various phases involved in a problem but also skill in reaching solutions. He was a member of the Chicago Engineers Club.

In 1922 Mr. Babcock was married to Edna Stephenson of Crystal Lake, Ill.

Mr. Babcock was elected a Member of the American Society of Civil Engineers on June 9, 1930.

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[1] Memoir prepared by I. F. Stern, M. ASCE.

ROBERT EDWARD BARRETT, M. ASCE[1]

DIED OCTOBER 13, 1945

Robert Edward Barrett, the son of Frederick Barrett and Laura Maria (Nutting) Barrett, was born on June 28, 1881, in Framingham, Mass.

He attended the public schools of Framingham and after graduation from high school in 1898, entered upon his active career as assistant in the engineering department of the Metropolitan Water Board of Boston, Mass. He did field work on the construction of the Wachusett Dam and reservoir at Clinton, Mass., and the aqueduct and tunnels connecting that reservoir with the Boston metropolitan distribution system.

In 1902, seeking a technical education, he entered the Lawrence Scientific School at Harvard University, at Cambridge, Mass., as a student in civil engineering, completing his work in the class of 1905. He was then employed as an engineer in the designing division of the Charles River Basin Commission of Massachusetts.

Having decided to specialize in hydraulic engineering, he turned from Boston to New York, N. Y., in 1907 where he became associated with the construction of the Catskill water supply system then being constructed for greater New York. He was occupied in this work until 1914. Here he served as civil engineer designer and, in addition to the supervision of the rock tunnel work in the Bronx, he was called upon to design the gatehouses and regulating works for the Ashokan Dam and the regulating works of the Kensico storage dam above White Plains.

In 1913, Mr. Barrett received an appointment as designing engineer from the Massachusetts Commission, then known as the Directors of the Port of Boston, and returned to Massachusetts to begin his new duties. He later became acting chief engineer in charge of all engineering activities of the Commission. This work consisted of dredging ship channels in Boston Harbor and design and construction work for the Commonwealth Pier and the South Boston Dry Dock. It was his plan, for the revision of Boston Harbor lines, which was adopted by Chapter 334 of the Special Acts of the Legislature of 1915, and which created a plan for the future development of both the South Boston and East Boston lands of the Commonwealth.

In 1916, he became associated with Fay, Spofford and Thorndike, consulting engineers of Boston, for whom he designed the foundations of the Memorial Bridge crossing the Connecticut River at Springfield, Mass.

In 1917, he was appointed construction engineer of the Turners Falls Power and Electric Company, which was then engaged in the construction of a 40,000-hp steam electric station on the Connecticut River at Chicopee, Mass. This corporation later became a major unit of the Western Massachusetts Electric Company. Subsequently, Mr. Barrett was employed in hydraulic engineering work at Turners Falls, Mass., and in supervising the construction and maintenance of various water storage dams and reservoirs built by the Connecticut River Conservation Company in Vermont and New Hampshire.

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[1] Memoir prepared by Alexander B. Ferguson, Asst. Treas., and Allin W. Ladd, Hydr. Engr., both of Holyoke Water Power Co., Holyoke, Mass.

In February of 1920, Mr. Barrett accepted the position of treasurer of the Holyoke Water Power Company in charge of the management of the company. In 1923 he was also elected president of the company. For these positions he was eminently qualified by reason of his long engineering experience and executive talents. This company traces its history back to the incorporation in 1792 of the "Proprietors of the Locks and Canals on the Connecticut River" and the completion of the first dam on the Connecticut River in 1795. Since that time many developments of the first magnitude in hydraulic engineering have had their origin in Holyoke.

Since 1920 the Holyoke Water Power Company, under the leadership of Mr. Barrett, has been actively engaged in supplementing its hydraulic power system with an electrical system. Hydroelectric and steam generators have been installed, which supply energy to most of the large industries of Holyoke, as well as to the municipalities of Chicopee and South Hadley, Mass. Electrical energy is also supplied by the company to the United States Army Air Base at Westover Field. Sales of electrical energy by the company increased from 4,380,000 kw-hr in 1920 to 121,357,000 in 1945. An extensive district steam system, supplying steam for heating and process work to a part of the industrial section of Holyoke was developed under his direction.

As president and treasurer of the company, Mr. Barrett, through his engineering experience, was able to lay a firm foundation for future engineering expansion, and through his skill as a financier promoted the conservation of the company's assets and made possible the sound financial structure of the company as it exists today.

He came to the organization at a time of change and growth in the country at large. As a result of his recommendations and under his executive management the plant as a whole and especially the steam and electric departments have been enlarged and strengthened so that it has been possible to serve new territories in new ways through a most modern distribution system. With great energy, courage, and aggressiveness, he advanced the interests of his company and no proper opportunity was lost to build up its resources and extend its usefulness. However, at all times and in all places, his integrity, his attractive personality, his courteous, kindly, and friendly nature won over even his rivals so that at the last the company stands at a peak of physical and financial development of good will in the community.

February 1, 1945, marked the twenty-fifth anniversary of Mr. Barrett's service in the Holyoke Water Power Company, and in March 1945, his fellow directors—aware that the growth of the company through those twenty-five years, the extension of its business and the remarkable increase in its physical assets and facilities were in the very largest measure due to his foresight, wise business judgment, distinguished professional attainments, and unremitting attention to the advancement of the welfare of the company—declared a special commemorative dividend of twenty-five cents a share in his honor. Future annals of the Holyoke Water Power Company and the City of Holyoke will tell of the engineering genius of Robert E. Barrett and his great power for putting his dreams into reality and making them work for those who will follow. It is always the purpose of the engineer to build the bridges for those coming behind to cross over.

> "Might of the roaring boiler,
> Force of the engine's thrust,
> Strength of the sweating toiler,
> Greatly in these we trust.
> But back of them stands the schemer,
> The Thinker who drives things through;
> Back of the Job—the Dreamer,
> Who's making the dream come true."

Mr. Barrett was for many years a director of the Third National Bank and Trust Company of Springfield, and a trustee of the Mechanics Savings Bank of Holyoke. He was also a director of the Holyoke Chamber of Commerce and of the Holyoke Hospital, and a trustee of the Holyoke Public Library. For several years Mr. Barrett served as chairman of the Massachusetts Division of the New England Council. He was a member of the Harvard Club of Western Massachusetts, the Rotary Club of Holyoke, and the Newcomen Society of England.

On June 13, 1908, Mr. Barrett was married to Grace Le Baron Esty of Framingham. He is survived by his widow; two sons, Robert Edward, Jr., and Frederick Park; and one daughter, Dorothy (Mrs. Howard Hollis Allen).

Mr. Barrett was elected a Junior of the American Society of Civil Engineers on January 31, 1905; an Associate Member on December 6, 1910; and a Member on June 16, 1919. He became a Life Member in January, 1945.

GILBERT JAMES BELL, M. ASCE[1]

DIED APRIL 29, 1945

Gilbert James Bell was born in Bainbridge, Putnam County, Ind., on September 9, 1854, and in his early childhood came to Illinois with his parents, John R. and Emily (Gordon) Bell. He was reared on a farm near Farmington, Ill., where he received a public school education.

The Peoria and Farmington Railway, later part of the Minneapolis and St. Louis Railway, was under construction in 1881 near Farmington, with John Findley Wallace (Past-President, ASCE[2]) in charge of location, construction, and operation. Mr. Bell became identified with this construction in September, 1881, as rodman. He was advanced to assistant engineer on construction in 1882, and from January, 1883, to July, 1885, he held the same position on the Iowa Central Railway which included the newly constructed Peoria and Farmington Railway. From July to October of 1885 he was transitman and inspector on the construction of the Iowa Central bridge over the Mississippi River at Keithsburg, Ill., and from January, 1885, to January, 1886, he was assistant engineer on the construction of approaches to this bridge. For the period from January to October of 1886 he was assistant engineer for the Iowa Central in general railroad engineering and on construction of its bridge over Black Hawk Chute, Mississippi River, near Keithsburg.

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[1] Memoir prepared by Frank T. Darrow, Albert F. Reichmann, and R. A. Van Ness, Members, ASCE.

[2] For memoir, see *Transactions*, ASCE, Vol. LXXXV (1922), p. 1635.

Five years of railroad right-of-way and bridge construction experience under the exacting tutelage of Mr. Wallace was an enviable asset to a young engineer and it was on this foundation that Mr. Bell built a monument of constructive engineering during the subsequent fifty years. Mr. Bell spent a few months in 1886 and 1887 as assistant engineer on the construction of the second Union Pacific Railway bridge over the Missouri River at Omaha, serving under the later internationally known bridge engineer, George S. Morison (Past-President ASCE).[8]

Mr. Bell's old chief, Mr. Wallace, had been made chief engineer in 1887 on the construction of Atchison, Topeka and Santa Fe Railway Company ("the Santa Fe") bridge over the Missouri River at Sibley, Mo.; and Mr. Bell joined him there in February of that year as assistant engineer. He continued on this work until its completion in 1889, little suspecting that some twenty-two years later he would have a very responsible part in the construction of a second, heavier structure at the same location.

After the completion of the Sibley Bridge Mr. Bell was transferred from construction to the "Santa Fe" engineering department in charge of building and sinking mattresses, and constructing other forms of protection from the encroachments of the Missouri River against the right of way. He was very active in constructing steel bridges and arch culverts when the second track was built in Missouri and Illinois. He supervised the construction of pneumatic foundations and erection of truss spans over the Canadian River near Purcell, Okla., the pneumatic piers at the Cimarron River near Guthrie, Okla., and a concrete dam for the Thatcher, Colo., reservoir. Culvert and steel bridge construction on the Western and New Mexico divisions was included in the busy period which kept him constantly on the move from 1889 to 1908.

Promotion in January 1908, after nineteen years of traveling from project to project on the "Santa Fe," to division engineer of the Missouri Division at Marceline, Mo., met with the hearty approval of both Mr. Bell and his wife. (He was married to Elizabeth Eddington on May 7, 1885, while stationed at Keithsburg.) At last here was the opportunity to enjoy that settled home life which they had looked forward to for twenty-three years.

As division engineer, until July, 1915, Mr. Bell had a very responsible part in building the second Missouri River Bridge at Sibley. He was also responsible for directing the general engineering work on the division.

Promotion to district engineer of the Western District came in July, 1915;

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[8] For memoir, see *Transactions*, ASCE, Vol. LIV (1905), p. 513.

Mr. Bell's headquarters were moved to Newton, Kans., where he had charge of engineering on three divisions. A transfer in 1918, with the same title, to the Eastern District with headquarters at Topeka, Kans., brought added responsibilities in directing engineering on four divisions. Mr. Bell was intimately connected with many projects until his retirement in July, 1936. Among these were the pneumatic piers and steel erection for a bridge over the Illinois River at Chillicothe, Ill., and for a bridge over the Mississippi River at Fort Madison, Iowa. (The Mississippi River bridge had a swing span which, at the time, was the longest and heaviest in the world.) Other projects were the construction of open piers and the erection of steel for a bridge over the I and M Canal at Joliet, Ill.; and the work of the reinforcing or replacing steel bridges, and protecting the banks of the Mississippi, Missouri, Kaw, North Canadian, and Colorado rivers.

Mr. Bell was of modest and dignified bearing, ever helpful to his associates in suggestion and counsel based on years of broad experience. Because of his diverse knowledge his advice was frequently sought.

His family life was ideal and the success he attained was equally credited by him to his wife through sixty years of loving association. Although there were no children, Mr. and Mrs. Bell gave a well rounded home life and educational advantages to a niece.

When Mr. Bell retired in 1936 at the age of eighty two, he was described by many of his close acquaintances as "among the best known and most beloved railway engineers in the Middle West," a worthy summation of a long and useful life.

Mr. Bell was always active and in good health until April, 1939, when a broken hip curtailed his physical energy and no doubt hastened his death. He is survived by his widow.

He was a life member of the American Railway Engineering Association and had been for three years a director of the National Drainage Congress. He served as a Director of the Society from 1927 to 1929.

Mr. Bell was elected a Junior of the American Society of Civil Engineers on September 7, 1887, and a Member on October 6, 1897.

EDWARD EVERETT BETTS, M. ASCE[1]

DIED JANUARY 18, 1945

Edward Everett Betts was born on August 30, 1863, in Cincinnati, Ohio. He was the son of Charles Suydam and Martha Jane (Lyon) Betts, who were of English stock. Mr. Betts was a dignified, studious, and gracious gentleman; and his high attainments were the result of the scholarly application of a very able mind.

Mr. Betts began his professional career in 1880 as a rodman with the Toledo, Delphas and Burlington Railway Company on tunnel surveying and construction. At the age of seventeen, he moved south and worked for the

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[1] Memoir prepared by C. E. McCashin, M. ASCE.

New Orleans and Northeastern Railroad Company (later part of the Cincinnati, New Orleans, and Texas Pacific Railway Company). During 1884 and 1885 he was assistant engineer and inspector of masonry, trestlework, and pneumatic foundations for Henderson Bridge and Henderson Bridge Railroad companies; assistant engineer for the Springfield, Effingham, and Southeastern Railroad Company; and assistant engineer on Wabash River bridge construction.

In 1886 he moved to Chattanooga, Tenn., and then, from time to time, was assistant engineer on maintenance of way for the Cincinnati Southern Railway Company, Alabama Great Southern Railroad Company, New Orleans and Northeastern Railroad Company, and Vicksburg and Meridian Railroad Company. Mr. Betts was resident engineer on surveys and construction of the Chattanooga and Lookout Mountain Railroad in 1887. In 1888 he became assistant engineer on street paving and sewer construction for the City of Chattanooga.

After these engagements he opened a consulting office in Chattanooga, later becoming assistant engineer in charge of construction of the steel-truss Walnut Street Bridge across the Tennessee River. This bridge was dedicated in 1891.

When the Chickamauga and Chattanooga National Military Park was being built, Mr. Betts acted as topographic and construction engineer. In 1895 he was appointed chief engineer; and, while serving in this capacity, he was detailed by the Secretary of War as special consultant on the development of the Vicksburg (Miss.) National Military Park. On both these jobs he had charge of topography; road surveys; location; construction; and maintenance of roads, bridges, buildings, and observation towers. He resigned from this position in 1911. The part he played in the development of the Chickamauga and Chattanooga National Military Park was the source of Mr. Betts' greatest pride. In 1911 the Hamilton County Highway Commission launched a half million dollar improvement program, and Mr. Betts was made chief engineer.

The Edward E. Betts Engineering Company (later the Betts Engineering Company) was established in the fall of 1912. Mr. Betts headed this company until his retirement in 1930. During this time the firm was responsible for many municipal and land developments in Chattanooga and in neighboring towns. Some of the projects were water supply systems for the towns of Signal Mountain and Lookout Mountain, and the East Brainerd Utility District of Hamilton County, Tennessee; improvements of street paving, sewerage systems etc., for South Pittsburg, Cleveland, and Lookout Mountain, Tenn., and the towns of Riverview and St. Elmo (later a part of Chattanooga); and land developments of Fairyland Estates on Lookout Mountain in Walker County, Georgia, and partly in the corporate limits of Lookout Mountain, and Dallas Heights, Glenwood, and Battery Heights in Chattanooga. He was also city engineer for the towns of Lookout Mountain and St. Elmo.

On April 4, 1889, he was married to Sarah May Aull. After her death, he was married to her sister, Idola Aull, who died in 1929. He is survived by one son, Clifford A. Betts, M. ASCE.

Mr. Betts' hobbies were traveling and modeling ships. He loved new horizons—at the age of seventy five he drove from Greenwich, Conn., to San Francisco, Calif., returning by water through the Panama Canal to New Orleans, La., and by auto to Chattanooga, and enjoyed every mile of the trip. His sincere, quiet, friendly manner, combined with his broad-minded, clear thinking, attracted a host of friends in many parts of the United States. He was an honorary member of the Chattanooga Engineers Club.

Mr. Betts was elected an Associate Member of the American Society of Civil Engineers on April 5, 1899, and a Member on February 4, 1913. He became a Life Member in January, 1934.

JOHN SOULE BUTLER, M. ASCE[1]

Died October 19, 1934

John Soule Butler was born in Rogersville, Tenn., on May 25, 1872. His parents were Francis A. Butler and Sarah F. (Stringfield) Butler. The great-grandparents on both sides of the family were born in the United States prior to the Revolutionary War, and were of Scotch-Irish and Welsh stock. His father's grandfather served in the Revolutionary War with great credit. His mother's great-grandfathers, Col. Joseph Williams of North Carolina and James King of Virginia, were both officers in the Revolutionary War, and served their country with distinction. The ancestors of the Butlers were represented on the Supreme Court Bench, in Congress, and as military officers.

John S. Butler attended the public schools of Knoxville and Nashville, Tenn. On June 20, 1894, he was graduated from Vanderbilt University of Nashville with the degree of Bachelor of Science in Civil Engineering. He was a thorough and conscientious student. In his class was Lytle Brown, M. ASCE, later Major General and Chief of Engineers of the War Department of the United States Army.

After graduation Mr. Butler went directly into the civilian employ of the U. S. Engineer Department and for the succeeding twenty-three years was engaged in the improvement of rivers and harbors, mostly on the Cumberland River, in connection with the Engineer Office at Nashville, Tenn.

During this period he established lines and levels, and laid out work for the construction of temporary buildings, cofferdams, and other necessities at Lock A. He supervised the reinforcement of foundations, the construction of masonry locks, the quarrying and cutting of stone, and the construction of a unique gravity trainway at Lock No. 5. He was responsible for the preparation of plans for new work, including estimates, designs, and the construction supervision of several locks, dams, abutments, and retaining walls. These were the first calculations and designs for steel lock gates, dams, and operating machinery prepared for the Cumberland River.

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[1] Memoir prepared by James O'Sullivan, Irrigation Asst., Columbia Basin Comm., State of Washington, Spokane, Wash.

As junior engineer, Mr. Butler worked on the complete set of plans for Lock and Dam No. 21, the first concrete lock built on the Cumberland River; and on plans for steel gates for the Hales Bar Lock on the Tennessee River. Later he was placed in charge of the construction of Lock and Dam No. 21, the contractor having failed after two years of effort, and completed the work at practically contract prices.

On February 10, 1912, he was made assistant engineer and visited various river and harbor works in the United States and Canada, and the Canal Zone which was then under construction. On August 17, 1912, he was assigned to regular duties in the Nashville Engineer Office and supervised all work on the Cumberland River, including, among other work, preliminary investigations and reports on the advisability of improving various streams.

On July 5, 1917, Mr. Butler received a commission as Major in the Engineer Officers' Reserve Corps, National Army, and was assigned to active duty with the General Engineer Depot, Washington, D. C., where he had supervision of the receiving, storing, and forwarding, or issuing, of all engineer materials and supplies in the United States, as well as the operation of the Port Depot engaged in forwarding engineer materials and supplies overseas. He was also the accountable officer for this property. When the Depot Department was transferred to the Purchase, Storage, and Traffic Division of the General Staff, on November 20, 1918, Major Butler was appointed as special assistant to the Director of Storage. He was then transferred, at the request of the Director of Finance, to the Finance Office, where he did important work in the settlement of Army contracts.

Effective July 18, 1918, Major Butler was promoted to the grade of Lieutenant Colonel of Engineers, National Army, and on August 18, 1919, he was honorably discharged from the Emergency Forces.

On September 30, 1918, Lieutenant-Colonel Butler was commissioned as Major, Corps of Engineers, Regular Army, and assigned to duty as Chief of Construction Division No. 2, in charge of the construction of the powerhouse (estimated cost $19,500,000) at Wilson Dam, Muscle Shoals, Ala., on the Tennessee River. He was in charge of this work until December 31, 1923. On December 27, 1923, Lt.-Col. George R. Spaulding, Corps of Engineers (then District Engineer at Florence, Ala.), wrote Major Butler in regard to his work on Wilson Dam as follows:

> "You came here four years ago. At that time no work of any kind had been done on the Power House section of this dam. Placed in charge of that section, with a very small force of men and under great obstacles, you at once began the pioneer work of plant and railway installation. You have remained in charge of division No. 2 ever since, devoting your entire time and physical and mental strength to the job.
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> "Until the leave which goes into effect December 31, you have had practically no relief from the onerous duties of your position.
>
> "By your energy, leadership, and splendid organizing ability, you have brought the work to such a state of completion and the force engaged upon

> it to such a state of loyal, cooperative efficiency, that our work in carrying on will be simple. You have a right to be proud of what you have accomplished, as we who have worked with you are proud of it."

On February 21, 1924, Major Butler was assigned to duty with the Eleventh Engineers at Corozal, Canal Zone, and from May 12, 1924, to July 1, 1927, was District Engineer in charge of the works for the fortification of the Panama Canal. This work included the preparation of plans, the emplacement and installation of the largest seacoast rifles at that time, the construction of tracks, magazines, switchboards, plotting rooms, fire control systems, and other incidental work. The transportation and emplacement of these heavy guns (the heaviest pieces weighing about 165 tons) in a tropical jungle country involved many difficulties. It was then said: " * * * that this lay-out of modern guns is believed to be the most modern and up-to-date that has ever been undertaken by this, our government, or any other government." On April 2, 1927, Maj.-Gen. C. H. Martin, Department Commander, Canal Zone, wrote Major Butler as follows:

> "The Secretary of War has asked me to express to you his appreciation of arrangements made by you for his inspection of the work at Bruja Point under your charge. He was very favorably impressed by the magnitude of the work that you have accomplished at Bruja Point and by the difficulties that you have overcome in transporting and mounting the heavy armament."

Major Butler was then assigned to the Wilmington, Del., office as District Engineer. Before this assignment became effective, however, he was made District Engineer at Seattle, Wash., where he assumed his duties in July, 1927.

The selection of Major Butler for this highly important post was a tribute to his ability to plan wisely the development of a great river system. Under the River and Harbor Act of January 21, 1927, the Secretary of War was authorized and directed, in accordance with House Document No. 308, 69th Congress, First Session, to survey United States river systems, where power development appeared feasible and practicable, to determine their best uses for navigation, flood control, water power, and irrigation. The survey of the Upper Columbia River in Washington and minor tributaries from the Snake River to the International Boundary, a distance of 425.3 miles, was under the jurisdiction of Major Butler. This river is the greatest potential power stream in the United States.

A comprehensive plan for the best uses of the Upper Columbia River involved the determination of the best means, if any, for the irrigation of 1,200,000 acres of potentially rich but arid land in Grant, Adams, Franklin, and Walla Walla counties in Washington. Grand Coulee Dam and accessory irrigation features were later designed to solve this problem.

After completing the survey of several rivers flowing into Puget Sound, Major Butler and his staff undertook the survey of 425 miles of the Upper Columbia River. A reconnaissance survey (April, 1928, to February 11,

1929) indicated that a detailed survey of the upper river was justified from the power, storage, and irrigation standpoints, navigation being a subsidiary benefit.

The detailed survey, which was begun on June 4, 1929, and completed on July 31, 1931, involved the investigation of hundreds of questions and the solution of many complicated problems. Such investigation, for example, included the various sources of water supply for irrigation and the areas irrigable from them, the extent of storage required for the gravity system, the location and size of canals, the reclassification of the lands, the duty of water for irrigation, a resurvey and core drilling of dam and reservoir sites, designs and estimates of the cost of irrigation works, irrigation and settlement of the land by units and the effects thereof, the economic feasibility of irrigation including the productivity of the land, the market for its produce, transportation facilities, trends in population growth, present industries including the timber industry, the mineral resources of eastern Washington and northern Idaho, and the important question of the feasibility of large-scale pumping at Grand Coulee.

On July 31, 1931, Major Butler submitted to the Chief of Engineers, U. S. Army, his famous report[2] on the Upper Columbia River. To increase the low-water flow of the Columbia River and hence its prime power, he recommended that upstream storage of 4,260,000 acre-ft be created eventually at Hungry Horse dam site, in Flathead Lake, Montana; and in Pend Oreille Lake, Idaho.

As the best plan of power development on the Upper Columbia River, he recommended the construction of dams and power plants on the following sites: Grand Coulee, 151 miles downstream from the International Boundary; Foster Creek, 51 miles downstream from Grand Coulee; Chelan, 42 miles below Foster Creek; Rocky Reach, 30 miles below Chelan; Rock Island (then under construction), 21 miles below Rocky Reach; and Priest Rapids, 56 miles below Rock Island. Beginning at Priest Rapids, each dam would back the water to the next dam upstream. The power plants at these dams would have an installed capacity of 3,880,200 kw, and would utilize for power purposes 87% of the total drop of the river in this section. The cost, with interest during construction, was estimated at $357,000,000. It was estimated that both sections of the Columbia River could produce one third of the electric energy produced in the United States in 1930.

Major Butler also recommended, for future construction, locks and lateral dams at miles 15 and 40 above the Snake River, and navigation locks at Priest Rapids and Rock Island, estimated to cost $8,950,000.

Major Butler found the gravity system of irrigation from the Pend Oreille River economically infeasible. He recommended that the Columbia Basin lands be irrigated by pumping 16,000 cu ft per sec of water from the pool formed by the Grand Coulee high dam into the Grand Coulee, an average height of 280 ft, by using mainly the floodwater or secondary electric energy developed at the Grand Coulee Dam.

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[2] **"Columbia River and Minor Tributaries," by J. S. Butler,** ***House Doc. No. 103,*** 73d Cong., 1st Session, Vol. 2, pp. 565-1402.

It required great vision, daring, and courage on the part of Major Butler to recommend a dam at Grand Coulee of such unprecedented height on a river as large as the Columbia and such a tremendous pumping plant; yet, without such vision and courage, Grand Coulee Dam would probably never be built and the irrigation of the Basin lands would not be possible. In the short time that Grand Coulee Dam has been constructed it has demonstrated its great value to the Pacific Northwest and the United States.

In 1932, the Bureau of Reclamation concurred in the report of Major Butler and in 1933, and 1934, undertook, with certain modifications, the construction of the Grand Coulee high dam recommended by him.

In August 1931, Major Butler was transferred from Seattle to Omaha, Nebr., where he served as U. S. Corps Area Engineer, Seventh Corps Area. About September 1933, he was promoted to Lieutenant Colonel, Corps of Engineers.

On December 22, 1934, Gen. Douglas MacArthur, Chief of Staff, U. S. Army wrote:

> "Lt. Col. Butler was an able officer of quiet, pleasing manner, mature judgment, and wide experience in Engineering projects. Thoroughly reliable, well versed in his profession, and at all times displaying a high sense of responsibility, he performed his various assignments with characteristic zeal and efficiency. His death is deeply regretted by his associates."

Upon the death of Lieutenant-Colonel Butler, the Columbia Basin Commission of the State of Washington, which was created to secure the construction of the Columbia Basin Project, passed the following resolution:

> "The report of Lt. Col. Butler on the Upper Columbia River has in no small measure laid the foundations for the future growth and prosperity of the Pacific Northwest and made possible the reclamation of the Columbia Basin lands by recommending the construction of the Grand Coulee Dam. Lt. Col. Butler, through his broad vision, unswerving integrity, and outstanding courage and ability, has rendered to the Pacific Northwest and the Nation a service that cannot be repaid."

Lieutenant-Colonel Butler contributed an article for *Civil Engineering* in September, 1931, entitled "The Columbia River—for Irrigation and Power." He was a charter member of the Society of American Military Engineers, and a member of the military affairs committees of the Seattle and Omaha Chambers of Commerce. On many occasions, he was called to Washington, D. C., to advise with the Board of Engineers for Rivers and Harbors, and in 1932 appeared before the Irrigation and Reclamation Committee of the House of Representatives in behalf of the construction of the Grand Coulee-Columbia Basin Project.

Lieutenant-Colonel Butler was married on November 16, 1908, to Abbie Holten Throop of Nashville. He is survived by his widow, two sons, and one daughter, namely, Phares Throop Butler, Francis Stringfield Butler, and Clyde Smith Butler (Mrs. Stanley F. DeVore). A third son, John Soule Butler, Jr., died on December 18, 1940.

Lieutenant-Colonel Butler was elected an Associate Member of the American Society of Civil Engineers on March 4, 1908, and a Member on April 2, 1913.

THOMAS HENRY CARVER, M. ASCE[1]

Died December 22, 1944

Thomas Henry Carver was born in Lebanon, Ohio, on November 16, 1871. He was the son of Harrison Newton and Mary (Dickey) Carver who resided for many years at Valparaiso, Ind. His father served as a member of the Valparaiso (Ind.) University faculty, and also as dean of the faculty, for a total service of thirty-nine years.

Mr. Carver attended the public schools at Valparaiso; and, from 1888 to 1892, he was a student at Valparaiso University. From 1893 to 1894, he was a transitman on land surveys; and, from 1894 to 1898, he served as county surveyor of Porter County, Indiana. Together with his duties as county surveyor, he served as city engineer of Valparaiso for three years (1895-1898) and, in the latter capacity, designed and built twelve miles of macadamized highways and ten miles of sewers. He entered the University of Michigan at Ann Arbor, in 1898, and for two years studied special courses in civil engineering. In 1900 he accepted a position as transitman with the city engineer of Salt Lake City, Utah.

From 1901 to 1905, Mr. Carver lived in Utah and was employed, successively, as transitman by the Oregon Short Line Railroad Company, as superintendent on the construction of an irrigation dam, as assistant engineer on construction by the Southern Pacific Railroad Company, and as assistant engineer with the state engineer of Utah on irrigation investigations.

In 1905 he joined the staff of the city engineer department of Seattle, Wash., then directed by R. H. Thomson, Hon. M. ASCE. Because of his general experience, Mr. Carver was assigned to the Water Supply Division of the department. Entering this work in a minor capacity, he made rapid progress through the various positions in the organization and, in 1907, he was appointed engineer in responsible charge of the design and construction of the city's water supply and distribution system. In 1911 his duties were increased to include the direction of enlarging Seattle's municipal Cedar River (Washington) hydroelectric power development project by the construction of a masonry dam and power tunnel, and the reconstruction of the powerhouse, penstocks, and machinery. He also directed the design and construction of an auxiliary steam power plant to serve the city's lighting system.

From 1928 to 1932, his duties included the responsible direction of much of the design and the construction of the Diablo Dam and powerhouse on the Skagit River in Washington and about 100 miles of permanent transmission line from the powerhouse to the city. The Diablo Dam project ranks as the largest municipal hydroelectric power project yet completed in the West.

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[1] Memoir prepared by O. A. Piper, M. ASCE.

Mr. Carver's service with the city engineer department of Seattle extended over a continuous period of thirty-six years (1905-1941). During all this time, coincident with other duties, he was actively engaged in enlarging and extending the city's water supply and distribution system together with its many special features of headworks, aqueducts, storage basins, service reservoirs, subaqueous tunnels, and fire protection. His work involved an increase of more than 500 miles in the pipe lines of the distribution system.

To anticipate the needs of a rapidly growing city and its environs (the population increased from about 80,000 to about 400,000 during his period of service), Mr. Carver's work involved advance planning for a water supply system of sufficient capacity to serve a population of 1,000,000 people, distributed over a disproportionately large area, and also to serve the included industrial activities adequately.

A large measure of appreciation is due him, and the able engineers with whom he served, for the thorough planning in all the projects entrusted to his direction; and also for the high standards of quality required in their construction.

Mr. Carver was the type of engineer who devoted most of his energies to his work. At intervals, however, he found time to take part in the affairs of engineering and allied organizations. These included the Pacific Northwest Society of Engineers (activity discontinued about 1915), the American Association of Engineers, and various water works organizations. Mr. Carver was quiet and retiring by nature, but was always ready to devote time and energy to any efforts that would advance a greater recognition of the engineering profession.

In 1902 he was married to Bertha Pool of Valparaiso. He is survived by his widow and his two sisters, Katherine Carver and Ada Belle (Mrs. Arthur Kelso).

Mr. Carver was elected a Member of the American Society of Civil Engineers on August 31, 1915. He became a Life Member in January, 1942.

CHARLES FRANCIS CHASE, M. ASCE[1]

Died December 28, 1945

Charles Francis Chase, the son of Arthur and Garafelia (Davis) Chase, was born at Claremont, N. H., on February 2, 1864. He was the eldest in a family of three boys and four girls. His grandfather, the Right Reverend Carlton Chase, was the first Episcopal bishop of New Hampshire.

His boyhood was spent in the town of his birth and his early training was in its public schools. He was graduated from Claremont High School in 1881. Following a short period at Lehigh University in Bethlehem, Pa., Mr. Chase transferred to Dartmouth College in Hanover, N. H., and was graduated in 1885, with the degree of Bachelor of Arts. He continued his studies in Hanover at the Thayer School of Civil Engineering, receiving the degree of Civil Engineer in 1889.

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[1] Memoir prepared by George B. Taylor, Assoc. M. ASCE.

The civil engineering curriculum of that day was comprehensive. In addition to general courses in mathematics, surveying, mechanics, and hydraulics, it included such specialized subjects as roofs and bridges, roads and railways, rockwork, mining and tunneling, heat, steam and engines, river and harbor engineering, and sanitary engineering. Mr. Chase's grade in these varied subjects was an over-all average of 94.

The class of 1889 at Thayer was an interesting group, including among its eight members the late Otis E. Hovey,[2] Hon. M. ASCE, long-time Treasurer of the Society, Charles L. Carpenter[3] and Frank B. Sanborn, Members, ASCE, and William R. Michie,[4] Assoc. M. ASCE. Mr. Chase roomed with Mr. Sanborn while they were coguardians of Dartmouth Observatory for some years.

Mr. Chase's first engineering engagement was with the Vermont Construction Company of St. Albans, Vt. In 1890 he joined the engineering staff of Berlin Iron Bridge Company of East Berlin, Conn. This enterprising concern, credited with the introduction of corrugated roofing siding for buildings, and the promotion of the lenticular or "parabolic" bridge on a country-wide scale, was a pioneer fabricator of structural iron and steel. While at East Berlin, Mr. Chase was concerned with the design and construction of the 447-ft span drawbridge over the Connecticut River at Middletown, Conn., reported locally to be the longest in the United States at the time of its erection—about 1890.

When the East Berlin Company was absorbed by the American Bridge Company, of the United States Steel Corporation, at the turn of the century, Mr. Chase joined former associates who "went across the street and opened up another shop," the Berlin Construction Company. He was chief engineer for the young concern, specializing in structural steelwork. Mr. Chase remained with the company throughout his life—during the period covering the change from the use of iron to steel; the transition from the light-riveted bridge and building construction to the use of the heavier, wide-flanged, rolled, sections; and the introduction of welding. As chief engineer of this bridge company, which specialized in both steel and concrete construction in its early years, he was consulted on many railroad, electric railway, and early highway structures in the period from 1900 to 1910. He became one of the elder statesmen group of Connecticut engineers, prominent at this time.

Elected to the presidency of the Berlin Construction Company in 1925, Mr. Chase successfully guided the concern through the depression years which took such a heavy toll among steel fabricators and placed it in a position to serve New England's defense industry in the prewar and later years.

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[2] For memoir, see *Transactions*, ASCE, Vol. 108, 1943, p. 1537.

[3] For memoir, *ibid.*, Vol. 94, 1930, p. 1495.

[4] For memoir, *ibid.*, Vol. XLI, June, 1899, p. 647.

In 1945 failing sight forced him into partial retirement, but as chairman of the company's board of directors, and by almost daily visits to the plant, he maintained his close interest in its affairs until his death.

Mr. Chase was a resident of New Britain, Conn., keenly interested in its welfare and active in public affairs. He served as chairman of the city water board during the enlargement of the supply system (from 1908 to 1912). For twenty-five years he was chairman of the city welfare commission.

Reared in the tradition of the Episcopal Church, Mr. Chase never wavered in his devotion, giving to it his lifelong service as a loyal layman. For more than fifty years he was a communicant of St. Mark's Episcopal Church in New Britain, also serving as vestryman, parish clerk, junior warden and senior warden, filling this last post for many years until his death. On the fiftieth anniversary of his association, the parish expressed its appreciation of his long and faithful service in a public testimonial. As one of Connecticut's leading laymen, he was always a delegate to annual diocesan conventions, taking an active part in the deliberations. He served a term as president of the Church Club (Episcopal) of Connecticut and a term as secretary of the Church Club (Episcopal) of the United States.

Since Mr. Chase outlived so many of his contemporaries, the details of his early tastes and activities are not accessible to his later associates. He was an early and enthusiastic motorist, able to cope with the trials of motor and tire trouble during the infancy of the automobile. He enjoyed touring New England and was thoroughly familiar with the geographical and geological features of this region. Not athletic, at least not in his later years, he found golf "too artificial" but enjoyed the outdoors, picnics, fishing, and gardening. Family, home, and church, in spite of many activities, were his chief interests; and a keen game of bridge, his recreation.

Reserved and unassuming in manner, when the time arrived for him to comment or make a decision, he did so, clearly and forcibly. He had the passion for detail not uncommon in executives, as the junior engineer, bringing in a project not fully worked out, found to his sorrow. It is perhaps characteristic that, although elected to Phi Beta Kappa at Dartmouth, Mr. Chase never wore the key and later associates were unaware of this early honor.

A true son of his native state, there was more than a trace of "the granite of New Hampshire" in his make-up. While no Puritan, he was representative of old New England at its best—in personal rectitude, public service, private benevolence, and hatred of sham.

While his chief business interest was with the Berlin Construction Company, Mr. Chase was prominent in local financial circles. He was president of the City Realty Company, president of the New Britain Real Estate and Title Company, and a director of the New Britain National Bank. He was a charter member of the New Britain Rotary Club and a member of the New Britain Lodge of Elks, the Shuttle Meadow Country Club, New Britain Club, Church Club of Connecticut, and Sons of the American Revolution. Mr. Chase was an early member of the Connecticut Society of Civil Engineers, its president in 1907, and later an honorary member. In 1927, he was president of the Connecticut Section of the Society.

In 1894, Charles Francis Chase was married to Elizabeth Hance Parker of New Britain. They had two children: Eugene P. Chase and Helen (Mrs. Vincent Huertardo). Mrs. Chase died in 1911. In 1914, Mr. Chase was married to Ruth Arnold of Cambridge, Mass. Two daughters were born to them: Margaret (Mrs. W. H. Judd, Jr.) and Alice (Mrs. G. B. Taylor, Jr.). Mr. Chase is survived by his widow; four children; five grandchildren; a brother, the Rev. Arthur Chase; and a sister, Alice Chase.

Mr. Chase was elected a Member of the American Society of Civil Engineers on February 5, 1902.

WALTER SHERMAN LYLE CLEVERDON, M. ASCE[1]

Died March 25, 1945

Walter Sherman Lyle Cleverdon was born on October 11, 1877, in Brooklyn, N. Y., the son of John F. and Catherine A. Cleverdon. His boyhood days were spent in Coram (later Shelton), Conn., where he attended the district school. The family later moved to Tremont in New York, N. Y., where he attended the public schools. In 1896 he entered the College of Engineering of New York University, in New York, and was graduated, in 1900, with the degree of Bachelor of Science in Civil Engineering. He received the degree of Civil Engineer in 1901.

Following his graduation, Mr. Cleverdon became assistant engineer for Woodlawn Cemetery and Associates Land Company, in charge of parties making topographical and city surveys, doing landscape work, and building retaining walls and foundations. From October, 1902, to October, 1904, he was topographical draftsman for the Department of Water Supply, Gas and Electricity in Brooklyn, and chief draftsman from 1905 to 1906, designing pumping stations and equipment, pipe lines, well systems, infiltration galleries, etc. In 1904 he was placed in charge of construction for the same bureau. In 1907 Mr. Cleverdon was appointed assistant engineer (designer) for the New York City Board of Water Supply designing various structures for the Catskill Water Supply System. In 1910 he became personal assistant to the chief engineer of the Bureau of Water Supply of New York City, transferring to the Bronx office as assistant engineer in 1912, and continuing there until 1918. He then resigned to accept an appointment as lecturer in engineering in the College of Engineering of New York University. In 1919 he was appointed supervisor of property for the University at the University Heights campus, and from 1920 to 1925 served as associate professor of sanitary engineering. In 1925 he was placed in charge of all New York University property, including the Medical and Dental School centers, the Trinity Building at the Wall Street center, the fifty-acre campus at University Heights, and the group of buildings at Washington Square. The value of these properties reached a total of $13,000,000 during his incumbency. He retired in 1938, after a serious illness from which he never fully recovered but which did not, however, diminish his interest in his many activities.

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[1] Memoir prepared by William R. Bryans, Asst. Dean, College of Eng., New York Univ., New York, N. Y.

He was the author of many articles on sanitary engineering, and of two books, "The Water Supply of Buildings and Rural Communities," published in 1925 by D. Van Nostrand Company, Inc., and "Plumbing Engineering," published in 1937 by the Pitman Company.

Mr. Cleverdon was active in church work, and served for many years as treasurer of the University Heights Presbyterian Church, and later as an officer of the West Center Church of Bronxville, N. Y. For many years he had been interested in the Alumni Association activities of his alma mater and held the office of treasurer and later president of the Engineering Alumni Association. He was a member of the American Water Works Association and the New York University Faculty Club.

Possessed of a jovial disposition, he had a host of friends and acquaintances in the engineering fraternity, and he assisted many a young graduate to obtain his first position. In recognition of his efforts for alumni welfare the University, in 1933, awarded Mr. Cleverdon the Alumni Meritorious Service Medal.

Mr. Cleverdon was married in September, 1906, to Jessie E. Thomson and to this union two daughters were born, Catherine Lyle and Jessie Thomson (Mrs. Eugene Minteski), all of whom, together with a brother, Frederick R. W. Cleverdon, M. ASCE, survive him.

Mr. Cleverdon was elected a Member of the American Society of Civil Engineers on March 7, 1921.

HARRY EDMOND COTTON, M. ASCE[1]

Died October 12, 1945

The sudden and untimely passing of Harry Edmond Cotton at the age of sixty-four removed a well-known and respected engineer from the field of construction drainage.

Harry, the son of Frank and Sarah Cotton, was born on September 8, 1881, on a farm near Cedar Falls, Iowa. His early life was typical of any Iowa farm boy, helping with the many farm duties and attending country school. After graduating from the Cedar Falls High School, he attended Iowa State Teachers College (Iowa City) for two years, then Iowa State College at Ames, from which he received the degree of Bachelor of Science in Civil Engineering, in 1906. While in college and for several years following, he played amateur and professional baseball. His interest in that sport never ceased to give him much pleasure.

Immediately upon graduation, Harry started on his engineering career as a draftsman for the Paxton and Vierling Iron Works in Omaha, Nebr. In May, 1907, he started working as a draftsman in the Omaha City Engineer's

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[1] Memoir prepared by George E. Shafer, Assoc. M. ASCE.

Office. He interrupted this engagement for two years to work for the Omaha Structural Steel Company; but returned to the city engineer's office in August, 1915, to assume charge of all drawing, design, and estimates for public improvements. In June, 1921, he was appointed assistant city engineer and served in this capacity for six years, the latter two as a member of the City Planning Commission. From November, 1927, to January, 1928, he did estimating work with the Nebraska Department of Public Works. In January, 1928, he left the public service field to enter the commercial field as municipal engineer for the Armco Culvert Manufacturers Association, with headquarters at Middletown, Ohio.

As municipal engineer his early interests were primarily in adapting metal drainage structures to municipal uses. With the increasing interest and activity in airport construction, Mr. Cotton became vitally interested in airport drainage. "Airport Drainage," his first of many papers presented to engineering groups, was given in February, 1930, to the Iowa Engineering Society at Council Bluffs.

His interest spread to highway drainage, and in January, 1931, he transferred to an association member company, the New England Metal Culvert Company, at Boston, Mass. This company was very active in solving troublesome drainage problems throughout the New England States. After five years in New England, gaining valuable field experience, he returned to Middletown where his ideas could be developed on a national scope.

Harry was known throughout the United States and Canada for his investigational work and planning of highway and airport drainage. He was consulted by many highway departments and railroads and his advice was sought on drainage for important military and civilian air bases.

In 1941, Mr. Cotton was invited to discuss drainage at the National Defense Training Course on airport engineering conducted by the State University of Iowa in Iowa City. Later he lectured on drainage at Harvard University, Cambridge, Mass., in connection with special airport drainage classes for the Army Air Forces.

Harry Cotton had many convictions. One of these was that miles of paved surface are pounded into the mud of their subbases because engineers will not take the time or trouble to use a soil auger to find where the water is and where it comes from. With that known, he maintained that drainage, if properly installed, was the best and cheapest insurance for highways and airports.

In addition to his original thinking on drainage, Mr. Cotton was also thoroughly versed in hydraulics, sewer design, and load research problems. His writings included contributions to various engineering journals, and drainage handbooks and bulletins.

He loved nothing better than to work on engineering problems, or possibly for variety, a good competitive game of cards and an occasional fishing trip.

Harry always had a smile to prove his good disposition. He was never heard to speak a word against anyone. He was rugged and dependable. Those who knew him appreciated his forthright honesty and matured judgment. He was persistent, painstaking, logical, and thorough. He was not quick to form a final opinion; but, once the opinion was formed, no one could hold a more rational one or defend it more reasonably.

Mr. Cotton was elected an Associate Member of the American Society of Civil Engineers on September 9, 1919, and a Member on December 23, 1932.

JAMES FRANCIS CULLEN, M. ASCE[1]

Died February 24, 1945

James Francis Cullen, the son of James Francis Cullen and Mary Catherine Cullen, was born in Spruce Creek, Pa., on July 16, 1865.

He attended Mount St. Mary's College, Emmittsburg, Md., and was graduated in 1888 with the degree of Master of Arts. He entered the University of Pennsylvania, at Philadelphia, completing a two-year special course in civil engineering in 1890.

After completing his technical education he entered the service of the Pennsylvania Railroad Company in the chief engineer's department. Here he served as rodman and levelman from 1890 to 1894 on corps engaged on changes of line and grade on the Philadelphia Division and on the construction of stone arch bridges over the Little Juniata River in Pennsylvania.

He was promoted to transitman in 1894; and served in that capacity until 1901 on work in connection with the construction of the Delaware River bridge at Frankford, Pa., and bridge line connecting with the West Jersey and Seashore Railroad at West Haddonfield; also on the construction of a warehouse at Baltimore, Md., surveys for, and the construction of, the Camden Terminal of the West Jersey and Seashore Railroad.

He was appointed assistant engineer in May, 1901, and held that position until September, 1923, when he was appointed engineer of construction. This was an era of rapid growth and expansion of the Pennsylvania Railroad; an extensive program of improvements was carried on almost continuously to keep step with the rapid increase in the volume of freight and passenger traffic. Mr. Cullen was in charge of engineering on many of the important projects completed during this period; namely, the four-track stone arch bridge over the Delaware River at Trenton, N. J., the steel and masonry bridge over the Susquehanna River between Perryville and Havre de Grace, Md., and the steel and masonry bridge over the Raritan River between Perth Amboy and South Amboy, N. J.

He supervised the construction of important buildings, including locomotive construction and repair shops at Altoona, Pa., a 2,000,000-bushel grain elevator and engine terminal facilities in Philadelphia; passenger station improvements at Greensburg, Pa.; and many grade crossing elimination projects.

He was a member of the St. James Roman Catholic Church, Philadelphia.

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[1] Memoir prepared by S. A. Sloan, Asst. to Chf. Engr., Pennsylvania R.R., Eastern Region, Philadelphia, Pa.

Mr. Cullen was a skilful and accomplished engineer and a man of high principles and integrity; he was devoted to his family, and was held in the highest esteem by his many friends and associates.

He was married at Philadelphia, on June 13, 1899, to Anna Beatrice Neville. He is survived by a daughter, Beatrice (Mrs. R. T. Brown).

Mr. Cullen was elected a Member of the American Society of Civil Engineers on May 2, 1906. He became a Life Member in January, 1936.

OSCAR FREDERICK DALSTROM, M. ASCE[1]

Died April 13, 1945

Oscar Frederick Dalstrom was born at Wyanet, Ill., on August 15, 1871. He was the son of Anders John and Anna Christina (Jacobson) Dalstrom. Mr. Dalstrom was graduated from the Fremont (Nebr.) Normal School in 1895. Mr. Dalstrom taught in district and village schools from 1894 to 1897. After studying civil engineering at the University of Nebraska in Lincoln for one year (1897-1898), he later attended Rensselaer Polytechnic Institute in Troy, N. Y., and was graduated on June 19, 1901, with the degree of Civil Engineer.

After his graduation, he began his engineering career as a draftsman for the Pennsylvania Steel Company. From 1902 to 1903 he was a shop inspector for the Pennsylvania Steel Company; from 1903 to 1904, a draftsman for the Scherzer Rolling Lift Bridge Company; from April, 1904, to December, 1904, a draftsman for the Riverside Bridge Company in Martins Ferry, Ohio; and, from 1904 to 1906, a draftsman for the Pennsylvania Steel Company.

In 1906 Mr. Dalstrom saw greater opportunities on the larger railways of the Middle West, and began his career of thirty-five years in the service of the Chicago and North Western Railway Company. He was, successively, draftsman from 1907 to 1909, chief draftsman from 1909 to 1917, and engineer of bridges from 1917 to August 31, 1941, when he retired.

As engineer of bridges, Mr. Dalstrom displayed an untiring zeal and devotion to duty. He was highly respected for his technical ability, as well as for his attention to detail. One of the larger improvements made under his direction was the replacement, in 1923, of the three 330-ft steel truss spans of the railroad bridge across the Missouri River at Blair, Nebr., with steel spans of heavier load-carrying capacity.

Mr. Dalstrom recognized his obligation to the profession, and responded by taking an active interest in the recognized engineering societies. He belonged to the American Railway Engineering Association, the Western Society of Engineers, the American Railway Bridge and Building Association, the Chicago Engineers Club, and the Rensselaer Polytechnic Institute Alumni Association. For the American Railway Engineering Association, Mr. Dalstrom served on Committee No. 15, Steel Bridges, from 1921 until his death (chairman of this committee from 1923 to 1926), taking a keen interest in the study of correct bridge details. He was also on the Editing, Publications, Standardization and Water Proofing Committee. He served on the Special Committee on Live-Load and Impact from 1935 to 1938, and on a joint committee of the American Railway Engineering Association and the American Society of Civil Engineers. He was chairman of this latter committee from 1926 through 1929. He was a trustee of the Western Society of Engineers from 1917 to 1920, and served as trustee and as chairman of the Library Committee. He was a member of the Committee on Bearing Value of Pile Foundations of the American Society of Civil Engineers in 1938, and a director of the American Railway Bridge and Building Association from 1921 to 1923.

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[1] Memoir prepared by A. E. Bechtelheimer, Engr. of Bridges, Chicago and North Western Ry. Co., Chicago, Ill.

When Mr. Dalstrom retired from the Chicago and North Western Railway Company in 1941, he looked forward to a period in his life which he could devote to personal affairs, and went to Princeton, Ill., to reside near his farms. However, after the attack on Pearl Harbor, Hawaii, in 1941, Mr. Dalstrom was called to Washington, D. C., by the War Production Board. He spent two years in Washington from January, 1942, until the latter part of March, 1944. He liked his work because he was happy to have a part in the war effort. In February, 1944, he underwent a major operation and his doctor advised him to return to Princeton and enjoy more leisure. Returning to Princeton, he avoided overexertion and within a few months again felt fit to resume his work as a member of Committee 15 of the American Railway Engineering Association, in addition to supervising and planning improvements for his farm. On March 29, 1945, Mr. Dalstrom suddenly became ill and entered the hospital at Princeton where he died on April 13, 1945. As it was his wish to be cremated, this was done. He was never married.

The friends of Mr. Dalstrom remember him for his fine moral character and his untiring devotion to his chosen profession.

Mr. Dalstrom was elected a Member of the American Society of Civil Engineers on April 25, 1921.

ARTHUR DANIELS, M. ASCE[1]

Died June 27, 1945

Arthur Daniels, the son of Addison L. Daniels and Clara (Hodges) Daniels, was born on December 25, 1884, at Marion, Iowa. He attended the local public schools and high school before entering Iowa State College, at Ames, from which he was graduated in 1907 as a civil engineer. He spent his summer vacations from 1903 to 1907 working as a rodman for the Engineering Department of the Chicago, Milwaukee and St. Paul Railway Company.

His entire professional life was spent with the "Milwaukee" railroad—a period of thirty-eight years. Beginning as an instrumentman in 1907 he was advanced to assistant engineer in 1909 and assigned to making field surveys, and supervising masonry construction. In 1912 he was assigned to valuation work for one year, and from 1913 to 1918, inclusive, he was in charge of extensive track elevation work at Milwaukee, Wis. In September, 1918, he was made district engineer for the Northern District, with headquarters at Minneapolis, Minn., supervising the engineering work on 3,000 miles of main lines. On July 1, 1935, he was made assistant superintendent of track maintenance, with headquarters at Chicago, Ill., which position he held until November, 1937. Then he was made division engineer of the Twin City Terminal and the Iowa and Southern Minnesota divisions, having charge of all construction and maintenance work on the two divisions. He held this position, in Minneapolis until his death.

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[1] Memoir prepared by W. H. Penfield, M. ASCE.

Mr. Daniels was married at Marion, on October 19, 1910, to Jessica Merchant. He is survived by Mrs. Daniels; a son, Lowell Preston Daniels, a Lieutenant Commander in the United States Naval Reserve; and a daughter, Dorothy Harriet Daniels.

Mr. Daniels was very well known in the northwest, especially in Minneapolis and St. Paul, Minn., where for years he participated in many civic affairs. His engineering ability and good judgment were always recognized and respected. He was a member of St. Luke's Episcopal Church in Minneapolis, and a member of the American Railway Engineering Association.

Mr. Daniels was elected an Associate Member of the American Society of Civil Engineers on December 6, 1915, and a Member on May 28, 1923.

FRANKLIN DAVID DAVIS, M. ASCE[1]

Died February 15, 1945

Franklin David Davis, the son of Edwin and Mary Register (Pope) Davis, was born on March 21, 1885, in Baltimore, Md. After private schooling in Baltimore, he attended the Baltimore Polytechnic Institute in 1896, from which he was graduated in 1902.

On September 21, 1902, he entered the service of the Western Maryland Railway Company at Baltimore, as a rodman in the Engineer Corps, assigned to the building of the Tidewater Extension of the railroad around the City of Baltimore. On June 1, 1903, he was employed by the Pennsylvania Railroad Company as a levelman in the Engineering Department; and, after passing through the grades of rodman and transitman, he was appointed an assistant supervisor of track in the Maintenance of Way Department at Media, Pa., in June, 1907. He was transferred to a similar position at Freeport, Pa., in November, 1909; and in May, 1910, he was made acting supervisor of track at Jersey City, N. J. Subsequently, he was supervisor of track at Barnesboro, Pa., Wilkes-Barre, Pa., and Jamesburg, N. J. In April, 1918, he became an assistant freight trainmaster in charge of the Waverly (N. J.) Yard on the

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[1] Memoir prepared by R. W. E. Bowler, Engr., Maintenance of Way, The Pennsylvania R. R., Buffalo, N. Y.

New York Division, Transportation Department. He then received the following promotions:

March 1, 1920, division engineer of the New York Division at Jersey City;
July 1, 1922, superintendent, Norfolk Division, at Cape Charles, Va., in charge of the Transportation, Maintenance of Way, and Maintenance of Equipment departments;
October 24, 1923, superintendent of the New York Division, in Jersey City;
October 16, 1925, acting general superintendent of the New Jersey Division at New York, N. Y.;
April 1, 1926, general superintendent of transportation of the Western Region at Chicago, Ill.; and
June 16, 1927, general superintendent of the Northern Division at Buffalo, N. Y.

On October 15, 1944, he was granted a leave of absence because of illness.

On October 28, 1907, he was married to Muriel Janney, who survives him, together with one son, Edwin Janney Davis.

Mr. Davis was a member of railroad clubs in Buffalo, New York, and Toronto, Ont., Canada; and the Buffalo Club, Buffalo Athletic Club, Buffalo Country Club, Landmark Lodge No. 442, F. and A. M., and Craftsmen's Club of Wilkes-Barre.

Mr. Davis was elected a Member of the American Society of Civil Engineers on July 11, 1921.

GEORGE FREDERICK ECKHARD, M. ASCE[1]

Died December 28, 1945

George Frederick Eckhard, at the time of his death, was dean of the College of Engineering at the University of Vermont, in Burlington, chairman of the Vermont State Board of Registration for Professional Engineers, and a prominent figure among the engineering educators of New England. He was born in Waverly, Iowa, on November 21, 1878, the son of George and Katherine (Freiberger) Eckhard. He received the degree of Master of Didactics from Iowa State Teachers' College in 1900, and was graduated from the University of Iowa (Iowa City) in 1905 with the degree of Bachelor of Science in Civil Engineering. In 1910 he was granted the degree of Civil Engineer by his alma mater.

Soon after his graduation he went to Guantanamo, Cuba, as an assistant engineer on the Cuba Eastern Railroad, and returned early in 1907 to take charge of sewer construction in Cedar Falls, Iowa. Later in 1907 he was appointed professor of civil engineering at the New Mexico School of Mines (Socorro, N. Mex.), a position he held for two years, also serving as city engineer for Socorro and (from 1908 to 1909) as United States Deputy Mineral Surveyor for New Mexico. From 1909 to 1912 he was professor of civil engineering at James Millikin University, Decatur, Ill. The next three years he spent as assistant and associate professor of structural engineering at Pennsylvania State College (State College), where he was in charge of the department of structural engineering. In April, 1915, he came to the University of Vermont as professor of structural engineering, and in 1932 he became dean of the College of Engineering.

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[1] Memoir prepared by Louis B. Puffer, M. ASCE, and James A. Bullard, Prof. of Math. & Mech., Univ. of Vermont, Burlington.

During the years that he served as dean, the college acquired a large amount of valuable new equipment for its laboratories; the space available for its work was greatly increased; and most of its activities were transferred from the old and inadequate Engineering Building to quarters in the spacious new Waterman Building. A lover of perfection in all things, it was Dean Eckhard's constant ambition to improve the physical plant and the scholastic work of the college in every possible way. In his teaching, which he never entirely gave up, he was insistent on quality, accuracy, and completeness, and would never tolerate careless or slipshod work. After his death the president of the University, John S. Millis, said:

> "I take consolation in the fact that the sound condition of the College of Engineering is a worthy memorial to Dean Eckhard's intelligence and energy. He built well, so that even with his passing, the affairs of the College have gone forward."

Although his principal interest was in the field of engineering education, and although he liked best to be known as a structural engineer and a teacher, Dean Eckhard did not limit his activities to teaching and college administration. He gave much time to state and civic affairs, serving for nearly twenty years on the Burlington Street Commission, the City Planning Board, and various other public bodies. He also acted as an expert for the State of Vermont in the final settlement, by the United States Supreme Court, of the century old controversy over the Vermont-New Hampshire boundary. With Mrs. Eckhard he traveled widely through the United States and Canada, mostly by automobile, and visited most of the great engineering works in both countries.

The same love of perfection that characterized his professional relationships was also evident in all departments of his private life. He loved the outdoors, particularly boating, fishing, and gardening; and he was one of those happy persons for whom choice flowers loved to bloom. He enjoyed working with his hands, and whatever he made had always to be of the highest quality, whether a photographic enlargement or a 36-ft cabin cruiser.

He was a fellow of the American Association for the Advancement of Science; a member, for thirty-five years, of the Society for the Promotion of Engineering Education (member of the council from 1937 to 1940); and a member, for nearly thirty years, of the Vermont Society of Engineers (of which he was president in 1932 and 1933). He was also a member of Sigma Xi (Sigma Xi Club) at the University of Vermont. During his residence in Burlington he was a loyal member of the College Street Congregational Church, and served it in various capacities.

Dean Eckhard was a man of highest character and unquestioned integrity. Associates and students alike had great respect and affection for him. The following sentences are from a memoir prepared by a committee of the faculty of the College of Engineering of the University of Vermont:

> "Nearly his whole professional life was devoted to the training of young engineers; and none, we believe, came under his influence who was not made better by it. He was, in the truest sense of the word, a fine man, and in his passing we have lost a dear friend; our college, a trusted leader; and the University of Vermont, a sound and valued counselor."

On Sept. 1, 1908, George Eckhard was married to Henrietta Plock at Burlington, Iowa. Dean Eckhard is survived by his widow; a daughter, Elizabeth (Mrs. Frank McMullan); two grandsons, John and David McMullan; and two sisters, Clara (Mrs. William Langlas) and Lydia (Mrs. John Biggs).

Dean Eckhard was elected an Associate Member of the American Society of Civil Engineers on October 11, 1920, and a Member on April 12, 1937.

THEODORE HORATIO ELLIS, M. ASCE[1]

DIED MAY 26, 1945

Theodore Horatio Ellis, the second son of Prof. John Millot and Minerva Emeline (Tenney) Ellis, was born in Oberlin, Ohio, on February 20, 1867. He was a lineal descendant of Lt. Richard Ellice of Dedham, England, who came to Boston, Mass., in 1632, and settled in Dedham, Mass., where he took a prominent part in the political and social life of the community.

Lieutenant Ellice's grandson, Col. Timothy Ellis, was born in Dedham, Mass., and later (1765) moved to Keene, N. H., with his wife and eight children. He was a soldier and a statesman, taking an active part in the Revolutionary War and holding the office of selectman in Keene for five years. He was a delegate to the Provincial Congress, in 1775, and a representative in the Legislature from 1776 to 1778.

A grandson of Colonel Ellis, Seth Brittun Ellis, moved from Jaffrey, N. H., in 1840, to Oberlin. (Oberlin College had just been founded as the first coeducational school in the United States.) One of his sons was John Millot Ellis. He was educated at Oberlin College and was graduated in 1851. He then attended the Oberlin Theological Seminary, receiving a degree in 1857. John Ellis was a professor at Oberlin College (from 1858 until his death in 1894), Mayor of Oberlin (from 1861 to 1862), a "Delegate to the Battleground" in the United States Christian Commission, associate pastor of the Second Congregational Church of Oberlin (from 1867 to 1874), and United States Commissioner to Europe for the World's Columbian Exposition (in 1891). (In the latter capacity he conferred with representatives of

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[1] Memoir prepared by Alfreda Brennan Ellis, Summit, N. J.

Great Britain, Germany, Denmark, Sweden, Norway, and Russia.) In 1862, Professor Ellis was married to Minerva Emeline Tenney, a graduate of Oberlin College (1858) and a teacher in the college preparatory department.

Theodore Horatio Ellis spent his school and college days in Oberlin. In 1890 he left Oberlin College, after his sophomore year, to begin his business career with the Missouri Lumber and Mining Company of Grandin, Mo. He served in various capacities—as surveyor, railroad construction engineer, and chief of the land department.

In 1892 he became interested in the producing and refining of petroleum; and in 1893 he organized the Tiona Manufacturing Company, building a refinery at Tiona, Pa. He was president and manager of this company until 1901, when the business was absorbed by the Standard Oil Company. During five years of this period Mr. Ellis served as justice of the peace.

From 1901 to 1905, he worked for the Sun Company and assisted in the design and construction of an oil refinery at Marcus Hook, Pa. In 1905, Mr. Ellis organized the Ellis Company, manufacturers of petroleum products. As president of this company he designed and erected a refinery at Curtis Bay, Md., and operated it until 1910 when the Ellis Company was absorbed by the United States Asphalt Refining Company, a subsidiary of the Interocean Oil Company of New York, N. Y. As consulting engineer for the Interocean Oil Company and as director of the United States Asphalt Refining Company, Mr. Ellis designed and constructed an addition to the $1,000,000 refinery at East Brooklyn, Baltimore, Md., and designed and constructed a $500,000 oil refinery at Carteret, N. J. He also designed and constructed a refinery for the Netherlands Petroleum and Asphalt Company at Flushing, Holland; and, for the Interocean Oil Company, the Chesapeake and Curtis Railroad Company, the United States Petroleum Corporation, and the Santo Domingo Development Corporation, designed and supervised installation of a dock and terminal at Tampico, Mexico. He investigated possibilities of obtaining petroleum in the Dominican Republic and started development work there. In 1918, he resigned to accept a commission in the Army.

On November 8, 1918, Captain Ellis was ordered to report to the Commanding General, American Expeditionary Forces, in France, for duty as an "Expert Internal Combustion Engine Lubrication Engineer." The Armistice canceled these orders; and on November 30, 1918, he was assigned to duty as assistant port utilities officer in New York. Shortly afterward he was honorably discharged from the Army.

From 1919 to 1923, Mr. Ellis was employed as consulting engineer for fuel oil stations by the United States Shipping Board. In this capacity Mr. Ellis designed and erected fuel oil stations at the following places: St. Thomas, Virgin Islands; Honolulu, Hawaii; Pago Pago, Samoa; Manila, Philippine Islands; Mobile, Ala.; and Craney Island, Norfolk, Va. (Craney Island was the largest fuel oil station on the Atlantic Coast with storage capacity for a million barrels of oil; at its two wharves, the station could bunker two ships at a rate of 4,000 barrels an hour). The total cost of these plants was $2,250,000. When the refineries were completed, Mr. Ellis resigned and entered private practice as a consulting engineer at Summit, N. J.—his home for thirty years. Mr. Ellis made a number of inventions in connection with refining petroleum on which he secured patents.

Mr. Ellis was married to Harriet Emily Cumings, a daughter of Capt. Henry Harrison and Charlotte Jane (Sink) Cumings, of Tidioute, Pa., on August 27, 1888. They had three children: Bernard Tenney, Charlotte Cumings (Mrs. Stanley W. Fenton), and Minerva Tenney (Mrs. Thomas E. McCracken, Jr.). Mrs. Ellis died on October 9, 1922.

On February 26, 1927, Theodore H. Ellis was married to Alfreda L. Brennan, daughter of Alfred Lawrence Brennan (a noted illustrator of periodicals for over half a century) and Harriet Lucy (Lee) Brennan.

Theodore Ellis was a charter member of the American Legion, Summit Post 138, and of the Old Guard of Summit, organized in 1930. The latter group, composed of retired men of diverse professions and backgrounds, met once a week with a speaker. Mr. Ellis attended all meetings as long as his health would permit. He was also a member of North Star Lodge, F. and A. M. of Warren, Pa.; the Sons of the American Revolution; the Engineers' Club of New York; the Canoe Brook Country Club, Summit, N. J.; the Baltimore Yacht Club; American Society for Testing Materials; and the Society of American Military Engineers.

Among his other interests were golf and fishing and a workshop, where he made some lovely bits of furniture and small items in wood. During the last months of his life he completed several war contracts—handles for kits of tools for the armed forces made by Westinghouse—and he looked forward to repeat orders.

Theodore Ellis is survived by his widow; three children; and six grandchildren, Bernard Tenney Ellis, Jr., John Theodore Ellis, Thomas E. McCracken, Jr., Charlotte Elizabeth Fenton, Virginia Doris Fenton, and Esther Marjorie Fenton.

Mr. Ellis was elected a Member of the American Society of Civil Engineers on April 19, 1920. He became a life Member in January, 1945.

PAUL EMERY, M. ASCE[1]

DIED AUGUST 27, 1945

Paul Emery, the son of William and Elizabeth (Fisher) Emery, was born at Topeka, Kans., on September 27, 1890. He attended the public schools in Topeka. From 1910 to 1912 he was a student of engineering in the University of Illinois at Urbana.

During vacations, and after he left the University of Illinois, in June 1912, he was employed by the Topeka (Kansas) Bridge and Iron Company; here he detailed structural steel, and designed and estimated the cost of bridges. After several years of office work, he was placed in charge of the construction of a number of bridges, with headquarters in San Antonio, Tex. In 1917 he again took up structural steel detailing, this time with the Lackawanna Bridge Company, in Buffalo, N. Y. Shortly thereafter he became associated with the Robert Ingalls Iron Works, at Birmingham, Ala., first designing and detailing structural steel, and later as office and sales engineer in charge of the New Orleans, La., office. In January, 1919, he was engaged by the American Rio Grande Land and Irrigation Company, Mercedes, Tex., to design concrete structures and canals; he remained with this company until August, 1925.

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[1] Memoir prepared by Oscar B. Coblentz, Pres., McLean Contracting Co., Baltimore, Md., with the assistance of Alexander Maitland, Pres., Kansas City Bridge Co., Kansas City, Mo.

In September, 1927, for a period of six months, he worked for the American Bridge Company, at Ambridge, Pa., detailing bridges. From March, 1928, to January, 1929, he was assistant bridge engineer to the County Engineer of Cuyahoga County, Cleveland, Ohio. From January, 1929, to September, 1929, he was engaged in designing and estimating bridges for the Virginia Bridge Company at Roanoke, Va.

He joined the organization of the Kansas City Bridge Company, Kansas City, Mo., in September, 1929, as office engineer in charge of the drafting room. During the eleven years he was part of this organization, his work included the development of construction methods and procedures for various types of substructures; and the construction of the substructures of locks and dams, quay walls, wharves, and bridge foundations. He was instrumental in developing the method that was used in the construction of the foundations of the bridge across the Mississippi River, at Baton Rouge, La. During these years he designed various types of marine equipment used by the company in that period. He was promoted from office engineer to assistant chief engineer on July 15, 1939. He was in charge of all the superstructure work, including the preparation of estimates and proposals, the supervision of the work to completion, and the direction of the company's Kansas City shop.

When the national defense program of the United States became active in the fall of 1940, he resigned his position with the Kansas City Bridge Company to accept a position on December 1, 1940, as project engineer for the McLean Contracting Company. This company had a cost-plus-fixed-fee contract with the Bureau of Yards and Docks, Navy Department, for constructing a supply pier at the Naval Operating Base, Norfolk, Va.—one of the largest such piers on the east coast. The McLean Contracting Company was awarded additional work in the Norfolk area, which, between the date Mr. Emery came with the company and the time of his death, aggregated approximately thirty million dollars. He was in charge of the office engineering, as well as the engineering forces in the field. This work included the construction of piers; the construction of twenty-five 50,000 barrel pre-stressed reinforced-concrete tanks; the reconstruction of the oil handling facilities at the Naval Fuel Depot Annex, Craney Island, Virginia; the construction of a magnetic range in the middle of the Chesapeake Bay; and the construction of a "deperming" station, used in demagnetizing steel ships. After war was declared, the Navy decided to protect the steel fuel tanks at Craney Island, and at the Naval Operating Base in Norfolk, with splinterproofing consisting of reinforced concrete fifteen inches thick. Mr. Emery developed a tongue-

and-grooved reinforced-concrete member, that could be pre-cast and set up to form a circle around these tanks, that could be removed without damage to the tanks, and that at the same time could be used for the construction of bulkheads and similar work. This plan, which he developed, worked out so satisfactorily at Norfolk that it was used elsewhere by the Navy. His duties also included the development of designs and design details in cooperation with the Officer-in-Charge of Construction and his representatives.

Both of Mr. Emery's parents were musicians, and from his childhood until his death he was extremely fond of classical music and was a patron of the Kansas City Philharmonic Orchestra. He was fond of sports and was an ardent baseball fan.

He was physically strong, conscientious, and an untiring worker. He was endowed with the rare ability of reconciling the conflicting views of engineers, so that the desired result could be accomplished in the shortest possible time, and at a reasonable cost. He died from a heart attack, while working at his desk at the Naval Operating Base in Norfolk.

He was married to Grace A. Shimer on September 7, 1912, at Topeka. He is survived by his widow; one daughter, Pauline (Mrs. James P. Biedenharn); one son, James; and a sister Mrs. Lydia McAfee.

Mr. Emery was elected a Member of the American Society of Civil Engineers on April 10, 1939.

OZRO NOWLIN FLOYD, M. ASCE[1]

Died November 30, 1944

Ozro Nowlin Floyd was born on a farm near Hampshire, Maury County, Tenn., on April 12, 1878. He was a son of Henry W. and Celia Jane (Weatherly) Floyd, both members of early Tennessee pioneer families. Mr. Floyd was educated in the public schools of his native county, early attracting attention through his unusual ability in mathematics. He was graduated from the University of Tennessee, at Knoxville, in 1905 as an honor student, receiving the degree of Bachelor of Science in Civil Engineering.

Mr. Floyd's subprofessional engineering work was done in the service of the Louisville and Nashville, Cairo and Thebes, Cairo and Norfolk, and Southern railways. He spent six years in railroad work, rising to the position of chief of party on location and drainage surveys.

In September, 1911, he made a connection which was to fix the trend of all his subsequent career, and which introduced him to the type of work in which he rose to a pre-eminent position. He entered the employ of the Morgan Engineering Company of Memphis, Tenn., as chief of party on drainage surveys and construction, in Arkansas and Mississippi, continuing in this work until May, 1913.

The great Dayton, Ohio, flood of 1913 gave him his first real opportunity to develop and display his genius in flood control work. He arrived in Dayton soon after the waters of that disastrous flood had receded, and at first was in charge of surveys and certain other field investigations. This was two and a half years before the Miami Conservancy District was organized, and long before a comprehensive plan for the control of floods in the Miami valley was perfected. Mr. Floyd was the principal assistant to Arthur E. Morgan, M. ASCE, and had an important part in all of this preliminary work. By 1915, the Ohio Conservancy Act had become law; and a plan for obtaining flood protection, by means of retarding basins and channel enlargement, through centers of population had been officially adopted. Mr. Floyd remained with the District to the end of construction in the latter part of 1922. Particularly efficient was his relocation of steam and electric railways through the District, and his conduct of negotiations with the executives and engineers of the several lines involved.

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[1] Memoir prepared by Frank W. Chappell, M. ASCE.

Through the years from 1923 to 1925, Mr. Floyd served as principal assistant engineer on the design and construction of two large dams for the Wichita Falls (Tex.) Irrigation District, and of a water supply dam for the City of Dallas, Tex. The total cost of these projects was $9,500,000.

He entered a partnership with J. L. Lochridge, M. ASCE, in 1925, and continued this association into 1933. Notable commissions executed by Floyd and Lochridge were a water supply dam for the City of Waco, Tex.; a storage dam for the West Texas Utilities Company near San Angelo, Tex.; consulting service to the City of San Antonio, Tex., in a $1,000,000 river-straightening project through that city; a storage dam for the Dallas Power and Light Company, costing $1,250,000; and special studies and reports for the large utility companies of Texas.

After the dissolution of the partnership, Mr. Floyd confined his practice to personal service consulting work. Back in 1929, he was retained as consultant to the New Orleans District of the Corps of Engineers, U. S. Army, to advise that office on matters relating to condemnation proceedings for acquisition of flowage rights for the Bonnet Carré Spillway, particularly with respect to the rights of way of the Illinois Central Railroad Company, the Yazoo and Mississippi Valley Railroad Company, and the Louisiana and Arkansas Railway Company, all of which cross the floodway. Mr. Floyd acquitted himself with distinction on this work and performed a highly useful service.

During recent years Mr. Floyd's consulting engagements were numerous and important. Among them may be listed the following, although the list is by no means complete:

1. The Federal Power Commission: To determine the legitimate cost of the Conowingo power development on the Susquehanna River.

2. The Tennessee Valley Authority (TVA): After 1934 he was a member of the board of consultants, and devoted much of his time to the many and varied problems of design and construction arising during the comprehensive development of an entire river basin.

3. Red Bluff Water Power Control District, Pecos, Tex., 1935-1937: An irrigation, flood control, and hydroelectric power project.

4. Vicksburg (Miss.) District, Corps of Engineers: On the Sardis Dam, 1935-1940, and the Arkabutla and Wallace Lake dams, 1934-1944.

5. Memphis District, Corps of Engineers: On the design and construction of the Wappapello Dam, 1936-1944.

6. Denison (Tex.) District, Corps of Engineers: On the design and construction of the Denison Dam; "O. N." has been called the "Father of the Denison Dam." (He was a staunch friend of the Hon. Sam Rayburn, long-time representative of the Fourth Texas Congressional District, in which Denison is located, and Speaker of the House of Representatives since 1940. Mr. Floyd had made extensive preliminary investigations into the feasibility of a dam on the Red River, in Texas and Oklahoma, for the dual purposes of flood control and power production. Mr. Rayburn sought his opinion as to the merit of the project. The reply was that, of all similar projects in the Southwest, the Denison project was the soundest. Mr. Rayburn sponsored legislation which authorized the design and construction of the dam and power plant.)

7. Division Engineer, Corps of Engineers, Omaha, Nebr.: On the location of a dam on the Missouri River near Minot, S. Dak., 1944.

8. Division Engineer, Corps of Engineers, Dallas: On soil erosion control studies in Texas and Oklahoma, 1943-1944.

Public bodies did not make up the entire list of O. N.'s clients, however. He enjoyed to a high degree the confidence of contractors and private utility companies. He served the Callahan Construction Company on numerous occasions in the preparation of bids on construction contracts. The first of these engagements occurred in 1923 and 1924, and involved flood control work at Pueblo, Colo., and a water supply dam at Oklahoma City, Okla. Later, he made a study of the Madden Dam project in the Panama Canal Zone, and continued his connection with the project throughout the construction period. He represented the contractors in all their negotiations with the Corps of Engineers relating to the construction of the John Martin Dam on the Arkansas River in Colorado. In 1940 he was the sole arbitrator of disputed claims between Dodds and Wedegartner, contractors, and the Willacy County Water Improvement District No. 1, Raymondville, Tex. In 1944 he made a trip to Alaska, inspected the Canol project, and prepared a full report to the contractors.

The complete confidence which Mr. Floyd enjoyed from cities, state and federal agencies, utility corporations and construction contractors gives a fair measure of his absolute honesty and recognized ability. Most men would consider themselves fortunate to earn the respect and confidence of one or two of these groups of clients. O. N. Floyd had so impressed all of them with his fairness, professional integrity, and technical mastery, that all trusted him and sought his advice. He was one of the outstanding authorities of the United States in the fields of water conservation, flood control, soil erosion, hydroelectric power production, and the construction of large works to make effective these purposes. He frequently was called on by contractors to design their plants for heavy construction, and was considered an authority on earth-moving equipment and concrete-manufacturing equipment. Mr. Floyd was the author of two articles on the construction of earth dams.[2, 3]

Mr. Floyd was a man of unimpeachable character who took a keen interest in helping those in need. He was chairman of the Board of Elders of the Central Christian Church of Dallas, and a director of the Young Men's Christian Association (Y. M. C. A.) of Dallas. As a member of the Camp Committee of the "Y," he sponsored the construction of a swimming pool for boys at Camp Crockett, near Dallas. He made the plans for this small operation with his own hands, with the same meticulous care he gave to the multimillion dollar projects of the TVA; he was also a member of the Property Committee, to which he gave much of his time. His church work and Y. M. C. A. work were labors of love for him, and some measure of his character could be estimated through hearing the glowing tribute paid to his memory by his pastor and the general secretary of the Dallas Y. M. C. A.

O. N. Floyd was married on February 22, 1917, at Cleveland, Ohio, to Alice Cornelia Hofrichter, who survives him. Their only child is Lois Jean Floyd.

Mr. Floyd was an expert marksman with all types of small arms and was a life member of the National Rifle Association. He was also a member of the Technical Club of Dallas; the Texas Society of Professional Engineers; the American Association of Engineers; and the Conservancy Lodge No. 661, Free and Accepted Masons, of Dayton.

An active member of the American Society of Civil Engineers, Mr. Floyd participated in sectional and general activities; he was a member of the Executive Committee of the Construction Division; chairman of the Subcommittee on Cut-Offs and Embankment Slope of the Committee on Earth Dams and Embankments, Soil Mechanics and Foundations Division; and Chairman of the Committee on the State of Art and Science of Hydraulics of the Hydraulics Division. He was a serious student of technological advancement, and greatly enjoyed his work with the Technical Division Committee. Also, he was a member of the Texas Section and of the Dallas Branch, and attended all the Monday monthly luncheons which his busy travel schedules would permit.

Mr. Floyd was elected an Associate Member of the American Society of Civil Engineers on September 3, 1913, and a Member on September 12, 1921.

ALBERT WEBSTER GALBREATH, M. ASCE[1]

DIED OCTOBER 15, 1944

Albert Webster Galbreath was born in Wilmot, Ohio, on October 29, 1883, the son of Charles Burleigh and Ida (Kelley) Galbreath. The families of both father and mother were interested in educational work in the field of literature

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[2] *Civil Engineering*, August, 1940, p. 487.

[3] *Ibid.*, September, 1940, p. 586.

[1] Memoir prepared by H. D. Bruning and Clarence D. Bowser, Members, ASCE.

and the liberal arts. Being of a mechanical turn of mind, Albert, from his early school days, applied himself to scientific matters and chose the engineering profession for his life work.

Mr. Galbreath received his early education in East Palestine, Ohio. In 1896, the family moved to Columbus, Ohio, where Albert entered the Central High School; he spent one year in Otterbein University at Westerville, Ohio. In 1899, he entered the Ohio State University, at Columbus, as a student in civil engineering, which course he pursued into 1904, spending the summer months on surveys and construction work with Wilber A. Ginn, chief engineer of the Columbus, Urbana and Western Traction Company. It was this experience that led him into the field of railroad engineering which he followed up to the time of his retirement, in 1934, on account of a physical ailment.

In January, 1905, he entered the service of the Pennsylvania Railroad Company, at Cincinnati, Ohio. He was connected with this company and its affiliated lines until May, 1917, when he voluntarily entered the service of his country during World War I, receiving his honorable discharge on May 28, 1919. During the aforesaid service on the Pennsylvania Railroad he was engaged as assistant on maintenance and construction. From June 1, 1909, to March 1, 1911, he was in responsible charge of the construction of about twenty-two miles of second main track on the St. Louis Division of the Vandalia Railroad Company between Montrose and Altamont, Ill., including the construction of eleven permanent bridges. The entire work was performed under traffic at a cost of approximately $900,000. After this he was assistant engineer with the Vandalia Railroad in charge of the drafting room of the chief engineer of Maintenance and Way, checking all estimates, preparing plans and specifications of special structures, and doing valuation work. In July, 1919, he resumed his work with the Pennsylvania Railroad as assistant engineer on valuation. In August, 1920, he was appointed valuation engineer on the Missouri, Kansas and Texas Lines with headquarters at Parsons, Kans. This position he held until 1932, when the final report to the Interstate Commerce Commission was accepted and the work of valuation completed. He then followed a private consulting practice in St. Louis, Mo., in railroad work until his retirement.

During World War I, he was stationed in France as First Lieutenant and Captain of the 12th Regiment Railway Engineers on construction, maintenance, and operation of light railways, and some construction of standard-gage lines.

Mr. Galbreath had many friends in both his civilian life and his professional life. He was always sincere in his friendships, and all who knew him and worked with him had the utmost respect for him. Honesty, loyalty, self-sacrifice, and belief in fair play were his outstanding characteristics. He was generous and always willing to help a friend and associate in time of need. He was very active in the Boy Scout movement. As a member of the Parsons Boy Scout Council he was its representative on the National Council at the National Council meeting in New York, N. Y., in 1927.

Mr. Galbreath was a member of the American Railway Engineering Association; the Engineers Club of St. Louis; Circle Club of St. Louis; and the

Engineers Club of Columbus. He was a member of the Masonic Order in Effingham, Ill.

His marriage to Stella Snow occurred on October 14, 1907, in Columbus. He is survived by his widow and his mother.

His recreations were mostly outdoor life and gardening. He had a great interest in photography and in the national game of baseball.

Mr. Galbreath was elected an Associate Member of the American Society of Civil Engineers on April 17, 1917, and a Member on December 3, 1926.

ARTHUR FRANKLIN GORDON, M. ASCE[1]

Died January 29, 1942

Arthur Franklin Gordon was born in Terre Haute, Ind., on October 16, 1875. He was the son of James McClelland and Caroline (Melcher) Gordon. He received his technical training at Rose Polytechnic Institute in Terre Haute, being graduated in 1897 with the degree of Bachelor of Science.

From July, 1897, to June, 1919, he held the following positions:

June to October, 1897—engineer for Townley Brothers in Terre Haute on electrical construction work.

October, 1897, to February, 1899—office engineer for the Standard Wheel Company in Terre Haute.

February, 1899, to March, 1900—rodman and assistant resident engineer on the location of a railroad extension from Marion, Ill., to the Mississippi River, and later electrician in charge of the electrical plant at the Danville (Ill.) shops of the Chicago and Eastern Illinois Railroad Company.

March, 1900, to February, 1902—draftsman with structural engineering firms in Pittsburgh, Pa.

February, 1902, to July, 1907—engineer in charge of a drafting room squad for the McClintic-Marshall Construction Company at Pittsburgh (except for a period as construction engineer in charge of field erection work).

July, 1907, to June, 1911—draftsman, checker, or engineer for various structural engineering firms in Pittsburgh, Pa., Cincinnati, Ohio, Terre Haute, Ind., Chicago, Ill., and Rochester, Pa.

June, 1911, to April, 1912—checker in the drafting office of the American Bridge Company at Ambridge, Pa.

April, 1912, to April, 1916—designer in the Bureau of Bridges of the City of Pittsburgh on the design of the Bloomfield Bridge and numerous other important structures in Pittsburgh.

April, 1916, to October, 1918—assistant chief engineer of the Bureau of Building Inspection, Pittsburgh, in charge of the review and approval of structural work.

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[1] Memoir prepared by the Rev. R. Paul Schearrer, Pastor, Presbyterian Church, Takoma Park, Md., and A. E. McClure, Senior Highway Engr., Public Roads Administration, Washington, D. C.

October, 1918, to January, 1919—concrete designer for the munition plant at Neville Island near Pittsburgh.

January, 1919, to June, 1919—structural engineer in charge of design of mill buildings and concrete work for the plant of the Weirton Steel Company at Weirton, W. Va.

From June, 1919, until his death, Mr. Gordon was with the U. S. Bureau of Public Roads (later the Public Roads Administration) as senior highway bridge engineer in immediate charge of the Bridge Division of District Ten which at that time embraced the states of Ohio, Pennsylvania, Maryland, Delaware, Virginia, North Carolina, and West Virginia, and later the District of Columbia. He had immediate responsibility for supervision of building all federal-aid highway bridges in this district until his death.

During his forty-five years of service in his chosen profession, Mr. Gordon always contributed unselfishly his time, energy, and talent toward the development of local social and civic projects. Mr. Gordon was an active member of the Takoma Park Presbyterian Church. He was elected to serve as an Elder on January 4, 1920, and continued in this service without interruption until April 2, 1936. As a member of the Session, he served as treasurer of benevolences. He was also superintendent of the Senior Department of the Church School. He was a high type of Christian man who lived and practiced the teachings of his religious faith. He was a devoted husband and father. He was also an ardent lover of nature and derived much pleasure from working around his trees, shrubbery, and beautiful flowers. For many years Mr. Gordon lived in Takoma Park, Md., a suburb of Washington, D. C., which was founded by a relative, Benjamin F. Gilbert.

On August 14, 1900, Mr. Gordon was married to Bertha E. Miller, daughter of James B. and Catherine M. Miller. Besides his widow, he is survived by a son, James Miller Gordon (Major, U. S. Army), and three grandchildren, James Miller, Jr., Franklin Lee, and Nancy Ellen.

Mr. Gordon was elected a Member of the American Society of Civil Engineers on December 15, 1924.

LYMAN GRISWOLD, M. ASCE[1]

DIED MAY 10, 1944

Lyman Griswold was born on March 27, 1880, at Fandon, Ill., the son of Dr. John and Zura (Thompson) Griswold.

In January, 1882, the family left Fandon, moving to Helix, Ore., and here Lyman grew to manhood. He acquired his elementary education in the public school at Helix. Gifted with unusual ability and exceptional perseverance, he acquired his technical education from correspondence courses, long and diligent home study, and actual experience.

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[1] Memoir prepared by Harold A. Rands, M. ASCE.

At the age of twenty four he joined the staff of the Oregon Land and Water Company at Irrigon, Ore., as assistant manager. During his connection with that company he made irrigation surveys covering 42,000 acres of company-owned lands. He turned next to railroad engineering, and until March, 1913, he played an active and creditable part in the closing years of the memorable era when the United States was crossed and recrossed by transcontinental and branch-line railroads. His record for this period is as follows:

December, 1905, to December, 1906—resident engineer, Spokane, Portland and Seattle Railroad Company, Spokane and Inland Empire Railway Company, and Spokane International Railway Company;

January, 1907, to December, 1908—locating and resident engineer, Chicago, Milwaukee, St. Paul and Pacific Railroad Company; and

July, 1909, to March, 1913—district and division engineer, Oregon Trunk Railway and Oregon Electric Railway, both subsidiaries of the Great Northern-Northern Pacific System.

Mr. Griswold served the Chicago, Milwaukee, St. Paul and Pacific Railroad Company on the extension of the main line west to Puget Sound. Concerning Mr. Griswold's work on this project, an associate on the same work writes as follows:

> "My first meeting with Lyman Griswold occurred in the summer of 1908. At that time he was Resident Engineer for the Chicago, Milwaukee & St. Paul Railway on a four-mile section of the main line construction just west of the Taft Tunnel, which pierces the main Bitter Root Range at that point. This section comprised some very heavy grading and some high timber trestles, and was considered to include some of the heaviest construction on the entire line."

From September, 1913, to October, 1915, Mr. Griswold was Assistant State Highway Engineer for the State of Oregon. During this period, in addition to assisting in the organization of the highway department, he directed the location of 100 miles of the Columbia River highway and 300 miles of other state roads. Thus, after playing an active and creditable part in the closing drama of steam railroad building in the United States, he became a pioneer in the building of roads for motorized traffic—the Columbia River highway from Portland to Astoria being the first stretch of modern highway to be built in the State of Oregon.

In October, 1915, he opened a consulting office in Portland. For the remainder of his life he continued his practice except for the period from September, 1917, to February, 1919, when he served in France, first as Captain, and later as Major, Corps of Engineers. During the war Major Griswold, from headquarters at Tours, France, was engaged in the construction of roads, light railroads, and other military works; in the operation of quarries; and in the design and construction of pontoons.

After his return from France, Mr. Griswold resumed his consulting practice, serving the State of Oregon and most of the large construction companies of the Pacific Northwest on highway work, particularly tunnels. He also advised numerous towns and cities on various matters. He became recognized as an expert witness in controversies concerning classifications of materials in highway and tunnel excavations.

Later, the business of the firm became largely hydraulic in nature, mostly in connection with proposed power developments. During this period Mr. Griswold was called as a consultant by the U. S. Engineers in connection with the Bonneville Power Project in Oregon, the proposed Umatilla Power and Navigation Dam on the Columbia River in Oregon, and seven proposed navigation dams on Snake River between the mouth and Lewiston, Idaho. Possibly, however, his most outstanding work in the field of hydraulics was the thorough investigation of Lewis River (a tributary of the Columbia River in the State of Washington) which he made for the Northwestern Electric Company of Portland. On this project, for the first and possibly the only time, a practical application was made of the "tree-ring" method for determining past precipitation cycles in the Douglas fir area of the Pacific Northwest. So thorough and satisfactory was Mr. Griswold's work that the power company, as a result of his investigations and recommendations, proceeded with the construction of the Ariel Power Plant. Throughout the construction of this development, Mr. Griswold was retained as consulting engineer and, in addition, had full charge of a material-delivery highway from Woodlawn, Wash., to the site of the plant. Probably, if the federal government had not constructed the large Bonneville and Grand Coulee power plants, one or more of the other sites investigated by Mr. Griswold on Lewis River would have been selected for development. His studies of plant location developed the complete regulation of the annual flow on that river.

The diversion of water for irrigation and the utilization of water for the generation of power were not, however, the only reasons for his interest in the streams of the Pacific Northwest. These were professional reasons. Fishing was his hobby. He gave to this sport the same energy, application, and persistence which enabled him, without benefit of college degree, to win and hold the confidence of great corporations, and to carry through as a Major in France—as is evidenced by the facts that his library contained more than one hundred volumes on fish and fishing; that he tied his own flies; and that his fishing tackle included fly rods of the most approved types and sizes for landing any fish from the "just within limits" brook trout to the largest and strongest Chinook salmon.

By one of those unusual coincidences which occasionally occur in real life, Lyman Griswold's hobby of fishing developed a part of his professional career which became of practical application. His knowledge of the habits of the Columbia River salmon became of real engineering value when he was called upon to solve the problem of caring for the salmon runs which from time immemorial had ascended the Lewis River to spawn in the headwaters. These runs would be destroyed by the building of a dam nearly 200 ft high. Under Mr. Griswold's supervision, a hatchery and accessory facilities were provided —to the satisfaction of the United States and of the authorities on fish in the State of Washington—which have proved to be highly successful. His design of the methods of trapping the adult salmon at the base of the dam and holding them in retaining ponds for artificial spawning proved a model for later problems.

Mr. Griswold was a Past-President of the Oregon Section of the Society. He belonged to the Society of American Military Engineers (first president of the local post) and the Professional Engineers of Oregon. He was also a Mason, and a past-president of the following: The Portland Yacht Club; the Portland Industries Finance Service; and the Meadow Lake Club.

On September 3, 1910, at Portland, he was married to Anna Hansman Aiken, who survives him. He also is survived by his mother; a brother, James Griswold (First Lieutenant, U. S. Army); and a sister, Laura (Mrs. Seymour Doolin).

Mr. Griswold was elected a Member of the American Society of Civil Engineers on October 10, 1921.

FRANK WILLARD HANNA, M. ASCE[1]

Died January 26, 1944

Frank Willard Hanna was born on September 16, 1867, near Geneseo, Ill., the son of James Steel Hanna and Harriet Louise (Hunt) Hanna. He spent his early years at farming, and thus he earned the money that enabled him to attend Highland Park College (later known as Des Moines University) at Des Moines, Iowa, from which he received the following degrees: Bachelor of Science in 1893, Bachelor of Civil Engineering in 1894, Master of Science in 1898, and Civil Engineer in 1902.

He began his professional career as a teacher and dean of mathematics and civil engineering at Highland Park College in 1895, and served there until 1903. During the summer of 1900, he traveled in Europe making studies of schools and of engineering structures for the Governor of Iowa. From 1903 to 1905, he served as hydrographer in the U. S. Geological Survey with headquarters in Chicago, Ill., where he was responsible for stream gaging and power investigations for the government in the upper Mississippi Valley and Great Lakes region. In 1905 he joined the U. S. Reclamation Service and worked, respectively, as designing, project, supervising, and consulting engineer, until 1921. His work included the design or supervision of design on the major reclamation projects which took place in the United States from 1905 to 1921.

In 1921, Mr. Hanna was called to Canada to become manager for the Canada Land and Irrigation Company at Medicine Hat, Alta. He was administrative head for a $16,000,000 business that had 530,000 acres of land and many head of cattle, horses, sheep, and hogs. He returned to the United States in 1924 to take the position of hydraulic and designing engineer for the East Bay Municipal Utility District at Oakland, Calif. From 1929 to 1934 he was chief engineer and general manager of this large utility which was doing a gross business of $4,700,000 annually. Among the important works connected with this position was the design of the Pardee Dam on the Mokelumne River in California. It is one of the highest dams of its type in the world.

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[1] Memoir prepared by L. O. Stewart, M. ASCE.

Mr. Hanna retired from active engineering practice in 1934 and moved to Ankeny, Iowa, where he lived quietly, managing his personal agricultural interests. He moved to Webster City, Iowa, in 1942 and lived there until his death.

He was an inventor and an author. His inventions included an angle multisector, irrigation water meter, and automatic stop and relief valves. He was the author of "Logical Methods in Arithmetic," written in 1900; "Tables for Reinforced Concrete," "Measurement of Irrigation Water," "Agriculture by Irrigation," all written in 1913; "The Agricultural Value of Peat Soils," written in 1919; and "The Design of Dams," written in 1931 and revised in 1938. Also, he wrote many articles on engineering subjects for the technical press.

Mr. Hanna was a member of the American Water Works Association, American Concrete Institute, Cosmos Club of Washington, and a life member of the Lions Club. His religious affiliation was Methodist.

On August 28, 1901, he was married to Frances Grace Gore of Pilot Mound, Iowa. Mrs. Hanna and a son, John A. Hanna, survive.

Mr. Hanna was elected a Member of the American Society of Civil Engineers on January 7, 1913.

ANDREW PHILIP HARTMANN, M. ASCE[1]

Died April 18, 1945

Andrew Philip Hartmann was born in New York, N. Y., on August 27, 1868.

From 1883 to 1886, he was student; field assistant and draftsman with Hamilton Ewen who was civil engineer and city surveyor, New York City, part time; and he attended a scientific course at The Cooper Union for the Advancement of Science and Art, in New York City.

Mr. Hartmann was principal assistant, surveyor, and draftsman to Nathan F. Barret, landscape engineer, New York City, in 1886 and 1887.

In July, 1887, and continuing until 1912, he was in private practice as civil engineer and city surveyor on the following:

1888—principal assistant engineer with the late Horace Crosby, M. ASCE, civil engineer, New Rochelle, N. Y.

1889–1890—engineer and surveyor for the Kensico Cemetery Corporation.

1890—prepared extensive topographical surveys at Big Stone Gap, Va.; made surveys to establish a townsite at Kimball, Tenn.; and was on various municipal engineering jobs in New York City and its surburban sections.

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[1] Memoir prepared by Christopher J. Sheridan, M. ASCE.

1890–1896—engineer for the Yonkers (N. Y.) Park Association.

1893—prepared topographical outline surveys of property of the State Reformatory for Women, near Bedford Station, N. Y.

1895–1896—engineer for the Village of White Plains, N. Y., planning and directing the paving of Railroad Avenue.

1896–1898—engineer to board of trustees of State Reformatory for Women at Bedford Station, designing and supervising sewer system, water supply, and large masonry steam heating conduit.

1900—engineer on preliminary surveys for extension of Riverside Drive from 135th Street to 158th Street in New York City.

1903–1906—engineer in charge of improving Exterior Street, New York City, along the East River from East 64th Street to East 82d Street, including heavy retaining walls.

1903—engineer in charge (under construction engineer) of field work on construction of Riverside Drive Extension from 135th Street to 158th Street (total construction cost was approximately $4,200,000).

1906—engineer on preliminary survey and laying out routes for the extension of Riverside Drive from 155th Street to Spuyten Duyvil.

1908–1909—member of Special Citizen's Commission to deal with settlement of trolley question, and investigate and report on sanitary measures to improve condition of Saw Mill River, in Yonkers.

1910–1911—member of Joint Water Supply Committee to investigate and report on additional water supply for Yonkers.

1911–1912—engineer in charge of construction of a system of private sewers, and of other municipal work of private character in the eastern section of Yonkers; from August, 1911, through 1912, chief engineer of the Bronx Valley Sewer Commission.

In 1920, and until 1922, he was a member of the Zoning Board of Appeals of Yonkers; in 1922 and 1923 he was the city engineer of Yonkers. From January 1, 1924, until his death, Andrew Hartmann was a member of the Zoning Board of Appeals.

Surviving are three children, Walter R. Hartmann, Andrew P. Hartmann, Jr., and Phyllis V. Roe.

Mr. Hartmann was elected a Member of the American Society of Civil Engineers on September 3, 1912.

GEORGE EDWARD HAWTHORN, M. ASCE[1]

Died June 2, 1945

George Edward Hawthorn, the son of Frank and Carrie (Clark) Hawthorn, was born in New Rockford, N. Dak., on November 21, 1889, where he spent his earlier years. The family moved to Seattle, Wash., during his high school days, and he was graduated from the Seattle High School in 1910. In 1915 he received the degree of Bachelor of Science in Civil Engineering from the University of Washington in Seattle and, in 1926, the professional degree of Civil Engineer.

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[1] Memoir prepared by Robert B. Van Horn, M. ASCE.

After a year of teaching mechanical drawing and mathematics at the Mossyrock (Wash.) High School, he began his professional career with the Washington State Highway Department at Olympia, Wash., serving this department from 1917 to 1924, and passing through the positions of draftsman, senior draftsman, chief draftsman, and resident engineer. In October, 1924, he resigned his position with the highway department to accept the appointment of instructor in general engineering at the University of Washington. In 1928 he was appointed assistant professor of civil engineering and, in 1937, associate professor, which rank he held at the time of his death. While in the civil engineering department, Professor Hawthorn was in charge of all courses in highway and railway engineering, and he maintained close contact with the State Highway Department and the problems pertaining to the highway field. During the summers of 1926, 1927, and 1929, he returned to that department serving as assistant resident engineer.

In his twenty-one years of service at the University of Washington, Professor Hawthorn contributed greatly to the advancement of the Department of Civil Engineering, and he was directly responsible for the development of its highway and bituminous laboratories. He gave unstintingly of his time and energy to the various problems submitted by individuals and concerns of the state, and he won the high regard of his students and colleagues as a teacher and organizer of courses. He wrote "Highway and Railway Economics"[2] and "Introduction to Transportation Engineering,"[3] used by the Department of Civil Engineering, and "A Method of Designing Non-Rigid Highway Surfaces,"[4] which brought widespread favorable comment.

On August 16, 1916, he was married to Anne E. Lawson and he is survived by his widow and two daughters, Ruth (Mrs. D. J. Gjesdahl) and Frances (Mrs. Joseph Di Gregorio).

Professor Hawthorn was elected an Associate Member of the American Society of Civil Engineers on October 12, 1925, and a Member on October 20, 1936.

GEORGE TERRY HORTON, M. ASCE[1]

DIED MARCH 19, 1945

George Terry Horton, eldest son of Horace E. Horton,[2] M. ASCE, and Emma (Babcock) Horton, was born in Waupun, Wis., on August 25, 1873, and died in Chicago, Ill., from injuries received in an automobile accident on March 19, 1945.

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[2] "Highway and Railway Economics," by George E. Hawthorn, Univ. Book Store, 1938.

[3] "Introduction to Transportation Engineering," by George E. Hawthorn, Univ. Book Store, 1934.

[4] *Bulletin No. 83,* Eng. Experiment Station, Univ. of Washington, Seattle, Wash.

[1] Memoir prepared by T. L. Condron, I. F. Stern, and Ralph H. Chambers, Members ASCE.

[2] For memoir, see *Transactions,* ASCE, Vol. LXXVI, 1913, p. 2221.

He was a direct descendant of Barnabas Horton of Lancaster, England, who migrated to America in 1665, landing at Hampton, Long Island, N. Y. Barnabas Horton was several times a member of the General Courts of New Haven and Hartford, Conn. Hiram T. Horton, of the sixth generation of Hortons in America, was a carpenter and contractor. He migrated west, first to Ohio and later to Rochester, Minn. His son, Horace E. Horton, was born in Norway, Herkimer County, N. Y., in 1843, but grew up and attended school in Rochester. He later attended Fairfield Academy in Herkimer County for two years. In 1871 he was married to Emma Babcock in Waupun. She was born in Wheeling, Ill., and was a direct descendant of James Babcock, who was born in England in 1612, and settled in Portsmouth, R. I. She was directly descended from the famous Pilgrim, William Bradford, who was baptized in Yorkshire, England, in 1589, and came to Plymouth, Mass., in 1620 on the *Mayflower*. Emily Brown, of the seventh generation of descendants of Governor Bradford, was married to George Babcock. She was the mother of Emma Babcock and grandmother of George Terry Horton.

Horace E. Horton acquired his engineering education and knowledge by personal application and study. Unquestionably he had an engineering and inventive mind, for, when only twenty-three years old, he designed and directed the construction of a timber arch bridge (186-ft span) over the Orinoco River in Minnesota. Later he designed and built the famous iron cantilever bridge over the Mississippi River, at Dubuque, Iowa, and scores of other bridges, including eleven spanning the Mississippi River. By 1887 he had established an enviable reputation as a designer and builder of bridges. His great success in this field, during the twenty-three years he operated from Rochester, was proof of his ingenuity and engineering talent, as well as his business ability. In 1889 he combined his engineering and contracting business with that of the Kansas City Bridge and Iron Company; and established in Chicago, Ill., the Chicago Bridge and Iron Company, to which his sons succeeded.

This same year, 1889, his son George entered Rensselaer Polytechnic Institute at Troy, N. Y., from which he was graduated in 1893 with the degree of Civil Engineer. Immediately following his graduation, George became an employee of the Chicago Bridge and Iron Company, remaining with that company as employee and officer the remainder of his life, nearly fifty-two years.

Like his father, George Horton was a great engineer; he was proud of his profession and delighted in its problems. He had an inventive mind, and he was always striving to improve upon previous designs and methods of construction, applying his unusual originality and imagination with boldness, unconventionality, and skill in the design of steel structures.

He was convinced that any design could be improved. Custom was to him a challenge. He once said, "Standardization is stagnation." "Never, if you can help it, build the same type of structure twice." "There is nothing so good that you can't improve on it."

He had been about a year in the employ of the Chicago Bridge and Iron Company, when, in looking over a book on metal structures,[3] he saw an illustration of a new type of elevated water tank with a hemispherical bottom. He called his father's attention to this and suggested that they build it. At that time the commercial attractiveness of bridge building was at low ebb, and the company was therefore receptive to new ideas. George Horton made some changes to simplify the illustrated design and some experiments on the dishing of plates, and determined to go ahead. The first such elevated tank was built at Fort Dodge, Iowa, in 1904, by the Chicago Bridge and Iron Company. This was the start of the tank business, which grew to great proportions.

Some time later George T. Horton was made chief engineer of the company. In 1912 Horace E. Horton died and George T. Horton was made president. In 1910 the business had grown to such extent that a New York, N. Y., office was warranted, and in the same year a second plant was built at Greenville, Pa., near Pittsburgh, Pa. In 1914 a Canadian plant was built at Fort Erie, Ont., and in 1930 a southern plant at Birmingham, Ala., was purchased. Following World War I, the company withdrew from the construction of bridges and gave its entire attention to tanks and similar structures.

George T. Horton directed the research and development department of his company, where his creative and scientific mind could show its best expression. Under his leadership the Hortonsphere and the Hortonspheroid were developed, as well as a modification of the latter, named the Watersphere, which consists of a spherical tank supported at the top of a tapering cylindrical column, so designed as to give a striking and pleasing appearance. A total of forty-four patents, fifteen of which are design patents, were issued in his name; in addition to these, many other patents have been taken out in the names of his associates working under his jurisdiction.

He was among the first to become interested in, and to make practical application of, electric welding. This opened new opportunities for his creative genius. With the introduction of the coated rod, about 1930, it became possible to build entire structures of steel by welding, and he promptly adopted such welding for all steel storage tank construction designed and built by his company. Although his company had just equipped its plants with the most modern riveting machines, his faith in the future of welding was so great that he did not hesitate to declare this equipment obsolete and proceed with designing for, and fabricating by, welding methods. He was an active supporter of the American Welding Society.

After he became interested in welding, through his influence and benefactions a welding research laboratory was established at the Rensselaer Polytechnic Institute. Before this he had established five permanent scholarships at the Institute, one for graduate study, and four which are available to sons and relatives of employees of his company.

Although the Chicago Bridge and Iron Company was principally engaged in designing and building tanks for the storage of liquids and gases, it broadened its field at the time of World War I, and Mr. Horton interested himself in shipbuilding. His contribution to "winning the war" was to speed up production by developing and prefabricating entire sections of ships to be assembled on launching ways. As assistant general manager of the Submarine Boat Corporation, he was responsible for securing contracts for furnishing of fabricated parts for cargo vessels constructed by that company at its Port Newark, N. J., shipyard.

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[3] "Theory and Practice of Modern Framed Structures," by J. B. Johnson, C. W. Bryan, and F. E. Turneaure, John Wiley & Sons, Inc., New York, N. Y., 1st Ed., 1893; also illustrated description in *Engineering News*, August, 1891.

At the beginning of World War II, he responded to the call of the Navy Department that his company come to the aid of the naval construction program. A shipyard was hurriedly constructed at Seneca, Ill., on the Illinois River for the building and launching of "LST's" (landing ship tanks), and these were launched at the rate of one every five days. Also, other shipyards were established by his company at Newburgh, N. Y., Morgan City, La., and Eureka, Calif., for the building of floating dry docks.

It was natural for one of his active and forward-looking mind to become interested in and participate in aviation. He not only owned his own cabin plane, but he was a licensed pilot. However, in later years he depended upon a professional pilot for operating his plane. When he decided to give up using a private plane, he gave his plane and two additional engines to the aeronautical department of the Rensselaer Polytechnic Institute.

George T. Horton was an unusual looking man of towering height—six feet, four inches. He inherited his heroic figure and his exceptional engineering talent from his father, and from his mother, who was a strikingly beautiful and gracious lady, his handsome appearance and attractive personality. His characteristics were humility, modesty, consideration, generosity, kindness, fearlessness, and basic honesty in thought and deed. His door was always open to employees of the company, to discuss their personal and business problems. His brother, Horace B. Horton, treasurer of the company, said that he was "a great man in intellect and above all in friendship and human sympathy." "No man ever loved his company and his employees more." To this should be added that no man was ever more loved and respected by the employees of the company. His associates talk about the human qualities of the company, as "one big family," and the fixed goal was "doing a better job next time than it did the last."

George T. Horton was philanthropic and civic-minded, and responded to many appeals. He was chairman of the Chicago Plan Commission from June, 1940, to January, 1944. He was a life trustee, from 1925, of the Rensselaer Polytechnic Institute. He was a retired Lieutenant Commander of the Civil Engineering Corps, United States Naval Reserve. He was a member of the New England Historic Genealogical Society, the Chicago Historical Society, the Art Institute of Chicago, the Chicago Galleries Association, the South Shore Country Club, the University Club, the Commercial Club of Chicago, the New York Engineers Club, and an Honorary Member of the Chicago Engineers Club. He was also a member of the Delta Kappa Epsilon fraternity.

He was a member of many professional societies, including the Western Society of Engineers (honorary member, 1944), the American Society for Testing Materials, the American Welding Society (president, 1939-1940), the

Society of Naval Architects and Marine Engineers, the American Water Works Association, the American Petroleum Institute, and the Society of American Military Engineers.

George T. Horton was married to Hazel Hamilton Heath on November 27, 1907. She survives him. Two children were born to them—a son, Ernest Heath Horton, who died on November 1, 1921, and a daughter, Heath (Mrs. Arnold Gillatt). He is survived also by a sister, Susie Mary (Mrs. Robert H. Murray), and two brothers, Horace B. Horton and Hiram T. Horton.

Mr. Horton was elected a Member of the American Society of Civil Engineers on April 6, 1904.

EUGENE CHRISTIAN HULTMAN, M. ASCE[1]

Died April 22, 1945

Eugene Christian Hultman was born in Boston, Mass., on July 13, 1875, the son of Eugene N. and Lydia Crocker (Bacon) Hultman. He received his early education in Adams Academy, Quincy, Mass., and in Thayer Academy, in Braintree, Mass.

He studied civil engineering at the Massachusetts Institute of Technology, then in Boston, and was graduated with the degree of Bachelor of Science in 1896. The first six months after graduation he was engaged on topographic surveys for the United States Geological Survey.

During the next year, until March, 1898, he served as a rodman, transitman, and designer for the West End Street Railway Company and for the Boston Elevated Railway Company. For the next eighteen months, until September, 1899, he was assistant chief engineer of the Barbour Stockwell Company on the design and installation of special street railway work. He continued for several years to do consulting work for this company, and was also engineer for the Board of Directors of the West End Street Railway Company. Then, for four years, ending in 1904, he was engineer and special auditor for the Board of Directors of the Fitchburg Railroad Company. For the next thirteen years, until 1918, he was consulting engineer for the Boston Elevated Railway Company on the design and construction of the Cambridge Subway.

He became interested in politics, and after serving as a member of the Quincy City Council and chairman of a Republican Ward Committee he was elected to the Massachusetts House of Representatives in 1905, 1906, and 1907, serving on its committees on Water Supply and on Public Lighting. He then served two terms in the Massachusetts Senate, on some of its most important committees: On Rules, on Ways and Means, and on Public Lighting. He was made chairman of the Committee on Public Lighting.

In the difficult days of adjustment and equitable distribution of goods following World War I he held a number of important state positions. On

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[1] Memoir prepared by Karl R. Kennison, M. ASCE.

August 4, 1920, Calvin Coolidge then Governor of Massachusetts, appointed him as Commissioner of Necessaries of Life, and later as Fuel Administrator.

He served the City of Boston in many capacities. As Fire Commissioner for about four years beginning July 6, 1926, he modernized the department by installing smoke blowers, searchlights for illumination of fire escapes and alleys in the rear of burning buildings, radio equipment, and small specially equipped one-man boats for fighting fires beneath wharves, by changing drill methods, and by special attention to fire prevention. He served the city for four months early in 1930 as Building Commissioner, and revised the City Building Code. Probably his most notable service to the City of Boston was as Police Commissioner, being appointed by Frank G. Allen, then Governor of Massachusetts, on May 7, 1930. He undertook to reorganize the department and to install a complete coordinated system of records, including a modern fingerprint system in lieu of the obsolete system then in use. The efficiency of the Bureau of Records was greatly increased by the installation of complete and thoroughly modern criminal identification equipment. The signal system was readjusted with flashing lights on the police boxes for prompt contacts with the officer personnel and reliable assistance was available to the public by a system of citizens' alarms. He established a Police School to instruct officers in court procedure, regulations, ordinances, and statutes that they are called on to enforce. He made many radical changes in the organization of Police departments, all in the interest of efficiency and economy. He also discontinued the practice of detaining and confining women prisoners in cells at station houses. With the cooperation of Joseph B. Ely, then Governor of Massachusetts, he established the first two-way police radio in the United States, by the organization on July 11, 1934, of a Bureau of Operations handling all calls for assistance through radio, telephone, teletype, and telegraph.

On December 27, 1934, he resigned to accept an appointment as chairman of the Metropolitan District Commission. As such he was the executive head of the organizations. He maintained and operated the Metropolitan Water District, the Metropolitan Sewerage Districts, the Metropolitan Police Force, and activities in the vast system of Metropolitan Parks. By virtue of this office he was also ex officio chairman of the Metropolitan District Water Supply Commission, a special construction commission which was engaged in extending the sources of metropolitan water supply and which was authorized, during his incumbency, to construct extensions to the Metropolitan Sewerage Works. In this service he gave his best to the state. Following his death, the Metropolitan District Water Supply Commission gave the name of Hultman Aqueduct to the new pressure aqueduct which he had done so much to help construct. This aqueduct extends from the Wachusett Aqueduct terminal chamber of the old Wachusett Aqueduct to the center of distribution at Chestnut Hill, varying in diameter from 11 ft 6 in. to 14 ft, and constructed in tunnel and cut-and-cover pre-cast pipe. At the time of his death the aqueduct was completed except for about five miles of tunnel at the easterly end.

On January 15, 1907, Eugene C. Hultman was married to Elizabeth Blake in Chestnut Hill, Mass. She survives him.

Mr. Hultman was a man of strict probity, he had a keen sense of humor and much political sagacity. He was a man of strong likes and dislikes, and ready at all times to fight for a principle.

In 1935 ouster proceedings were brought against him by James M. Curley, then Governor of Massachusetts. The trial was before the Governor and Council. The contest was a grueling one. There were some one hundred witnesses but none of them was able to produce anything against his character or official conduct. The result was a complete vindication for Mr. Hultman and a severe rebuff to his accusers.

Quoting from a Memorial which was spread upon the records of the Metropolitan Water Supply Commission at the time of Mr. Hultman's death:

> "The Chairman was a man well qualified for the varied duties that he was called to perform. Trained as a civil engineer, proven by active service in the field, he had seasoned experience in construction work which was of great value in the many different problems occasioned by the construction of Winsor Dam, Quabbin Reservoir, and the incidental works. With his experience as an administrator, as Police Commissioner and Fire Commissioner of the City of Boston, he possessed the ability to choose and build up the proper personnel to carry out the multitudinous details of such an extensive project. Honest in his opinions, straightforward in his talk and direct in action, yet he was gentle and sympathetic toward those whose property and lives were necessarily affected by the carrying out of the work. He brought a friendliness and genuineness that never failed to impress the most casual acquaintance."

His readiness to fight for a principle is well illustrated by his carrying to the Supreme Court, in the face of adverse preliminary decisions, two suits brought by the contractors who built the two large hydraulic fill dams of Quabbin Reservoir (Massachusetts). He thus established important legal principles affecting the liabilities of the contractor and the rights of the engineer. The Supreme Court's ruling in the Winsor Dam (Massachusetts) case is well summarized by its decision that the court cannot impose its lay opinion upon the expert opinion of the engineer.

Mr. Hultman was elected a Member of the American Society of Civil Engineers on August 28, 1922.

JOHN PATRICK HURLEY, M. ASCE[1]

DIED DECEMBER 30, 1944

John Patrick Hurley was born in New York, N. Y., on September 15, 1878. He was the son of Michael Hurley and Mary (O'Neill) Hurley.

Following attendance at private schools in Ireland, he entered Cornell University, Ithaca, N. Y., where he received the degree of Civil Engineer in June, 1907. In addition, he studied at St. Kierans College, Kilkenny; Blackrock College, Dublin; and Queen's College, Belfast, Ireland.

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[1] Memoir prepared by Philip T. Coffey, Captain, U. S. Army (Retired), New York, N. Y.

Mr. Hurley was a professional engineer, a military man, and a diplomat.

His first engineering work was in 1907 with the New York State Barge Canal. Shortly thereafter he entered the employ of the New York City Board of Water Supply, where he designed various road crossings, and a street railway bridge. Also, he made working drawings for the proposed Catskill Aqueduct. Later he had charge of the field party of a five-mile section of the Newburgh (N. Y.) Division.

From 1909 to 1915, Mr. Hurley was an assistant engineer with the Bureau of Highways, Borough of Brooklyn, N. Y. During that period he supervised methods of construction of pavements, bulkheads, street railways, retaining walls, etc. From 1915 to 1916, he was an engineer with the Board of Rapid Transit in charge of field work for the construction of a section of the Culver Line Extension of the Brooklyn-Manhattan Transit Division, New York.

From June, 1916, to March, 1917, he served as a Captain in the 69th Regiment of the New York National Guard on the Mexican border. During this tour of military duty, he laid out the regimental camp, supervised the water and sanitary facilities, and instructed in military engineering. Subsequently, he returned to civilian life for four months. He was recalled to active military duty with the 69th Regiment, later known as the 165th United States Infantry Regiment.

His regiment was ordered overseas in December, 1917, and he served with it throughout the Champagne-Marne and Aisne-Marne offensives. He was wounded during the latter action and received the Purple Heart and the Distinguished Service Cross. The citation follows:

> "John Patrick Hurley, Captain, 165th Infantry, 42nd Division. For extraordinary heroism in action at Villers-sur-Fere, France, July 28, 1918. His company having reached its objective he ordered several patrols forward to silence several enemy machine guns which were causing heavy casualties in his own and other companies of his battalion. The patrols meeting heavy casualties from intense enemy fire, Captain Hurley himself organized a patrol, led it forward. While temporarily checked by intense fire he crawled forward and rescued a wounded officer, carrying him to a place of shelter. Resuming the advance of the patrol Captain Hurley was severely wounded, but continued to direct the movement of his men, who, inspired by the great courage and fortitude of their leader, pushed forward and captured the machine-gun nest, killing or capturing the crews thereof."

Shortly after he returned from overseas, he entered the Consular Service; and he was appointed Consul to Reval, Estonia, in September, 1919. The following year he was assigned to Riga, Latvia, where he was stationed for five years. In 1925, he was transferred to Vienna, Austria, and in 1929, after four years of service in that city, he returned to Riga. Two years later he was assigned to Nassau, Bahama Islands; and in 1934 he was appointed Consul General at Marseilles, France, where he was made Dean of the Consular Corps. In 1940 he was assigned to the State Department for special work. He retired in 1942.

Mr. Hurley was a great student of higher mathematics and advanced engineering problems. In his personal relations he was a delightful gentleman. His success in the consular field was a tribute not only to his engineering ability but to his knack of cooperating with other officials. He was a "career man" in every sense of the word.

He was married to Marie Teresa Henry on October 12, 1912. He is survived by his widow.

Mr. Hurley was elected a Member of the American Society of Civil Engineers on July 31, 1933.

JOSEPH FREDERICK JACKSON, M. ASCE[1]

Died February 8, 1945

Joseph Frederick Jackson was born on July 4, 1871, the son of George and Elizabeth (Gallagher) Jackson of New Haven, Conn.

He was graduated from the Sheffield Scientific School of Yale University at New Haven, in 1895, with the degree of Bachelor of Philosophy. For three years prior to graduation, he was employed in the engineering department of the New York, New Haven, and Hartford Railroad Company.

Upon graduation, he was employed in structural drafting for elevated railways of New York, N. Y. From 1896 to 1933, he was principally occupied in sanitary engineering, serving as engineer, consultant, and commissioner for various municipalities, industrial concerns, and the Connecticut State Department of Health. He was an authority in this field and wrote many papers on the subject.

Mr. Jackson was a member of the Engineering Advisory Commission of the War Council of the State of Connecticut and a member of the Evacuation Committee. He had a great interest in engineers in war service. As chairman of the War Effort Committee of the Connecticut Society of Civil Engineers, he advocated, and put into effect, many ideas to bring cheer and encouragement to fellow members in the armed forces. He was an active member of many engineering societies: Member of the New England Water Works Association; fellow and member of the executive committee of the American Public Health Association; member of the New England Sewage Works Association; honorary member, former president, director, and secretary-treasurer of the Connecticut Society of Civil Engineers; and member of the executive committee of the State Sanitary Engineers.

Mr. Jackson served in various capacities, including resident engineer, engineer examiner, and chief engineer for the Rhode Island and the Connecticut region of the Public Works Administration. He was active as an engineer up to the time of his death, being engaged, in 1944, by the City Plan Commission of New Haven to study and report on a six-year, postwar program of capital improvements. In 1944 he read a paper on this subject before the Connecticut Society of Civil Engineers.

Fred Jackson was a credit to the Society and sought to live up to its ideals and to raise, if possible, the standard of its aims. He was ever willing to give his advice and experience. He was one of the organizers and a member of the first executive committee of the Sanitary Division of the Society.

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[1] Memoir prepared by W. Vincent Barry, M. ASCE.

His activities extended beyond the engineering field. He was a member of the Graduates Club of New Haven and of the Meadowbrook Country Club of Hamden, Conn. He was much interested in athletics, especially golfing and swimming. He was president of the Young Men's Institute of New Haven (private circulating library) and president of the Hamden Historical Society.

In religion, Mr. Jackson was a Roman Catholic and was active in many charitable pursuits, being secretary of Highland Heights (a Catholic Orphan Asylum), secretary of the Catholic Cemetery Associations, and trustee of Our Lady of Mount Carmel parish.

On February 3, 1897, he was married to Agnes L. Spencer, daughter of John R. Spencer and Mary O'Holleran Spencer. He is survived by his widow; his daughter, Mary (Mrs. Edward Cowan); and his granddaughter, Maureen Cowan.

Mr. Jackson was elected an Associate Member of the American Society of Civil Engineers on September 3, 1913, and a Member on June 24, 1916.

RUDOLPH GUSTAV KASEL, M. ASCE[1]

DIED JUNE 14, 1945

Rudolph Gustav Kasel was born in St. Louis, Mo., on October 15, 1899, the son of Carl August and Antonia (Ritter) Kasel.

He received his preparatory education in the public schools of New Haven, Mo., and entered the University of Missouri School of Mines and Metallurgy at Rolla, in 1917. After one year his education was interrupted by his enlistment in the Navy during World War I, where he served on the *USS Graf Waldersee* and at Great Lakes Naval Training Station, Great Lakes, Ill. Discharged from active service in 1919, he resumed his studies in January, 1920, at the Missouri School of Mines and Metallurgy. He was graduated with the degree of Bachelor of Science in Civil Engineering in 1923.

Before graduation he obtained practical experience on mine surveying in several mines in Missouri and during his summer vacation, in 1922, he was employed as a draftsman by the Western Electric Company, Chicago, Ill., on automatic telephone switchboard layouts. After graduation he worked for several months as instrumentman and draftsman for the Southern Carbon Company, Fairbanks, La., on plant layout, pipe-line surveys, and townsite maps. From October, 1923, to April, 1924, he was draftsman and computer for the Missouri State Highway Department, Division 10, Sikeston, Mo.

Mr. Kasel was with the U. S. Geological Survey, Water Resources Branch, from April, 1924, until his death on June 14, 1945. He served on numerous and varied assignments, beginning with that of resident engineer at Colorado River gaging stations in the Tucson, Ariz., district. From July, 1925, until March, 1928, he was in the Columbus, Ohio, district on stream measurements, construction of gaging stations, and office computations for gaging stations in Ohio, Indiana, Michigan, and Kentucky. From April, 1928, until September, 1932, he was engaged in the design, construction, and installation of gaging stations and stream-gaging equipment in Idaho, Montana, and British Columbia with headquarters at Boise, Idaho. During this period he was also engaged on special investigations in Kootenai Valley, Idaho, in connection with surface-water and ground-water problems. In October, 1932, he was placed in charge of the district office at Iowa City, Iowa, as acting district engineer, becoming district engineer in 1933. In addition to his duties as district engineer, he also served on the Water Resources Committee of the Iowa State Planning Board and on the staff of the Iowa Institute of Hydraulic Research, University of Iowa, in Iowa City. On April 29, 1940, he was transferred to Washington, D. C., as acting chief, Division of Surface Water, Water Resources Branch, U. S. Geological Survey. He became chief of that division on May 1, 1941.

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[1] Memoir prepared by Charles H. Pierce, M. ASCE.

He was the author of "Water Resources of Iowa, 1873–1932,"[2] and coauthor of numerous Geological Survey water-supply papers and technical reports. He was a member of Sigma Xi fraternity, the Iowa Engineering Society, and various fraternal and civic organizations. Hunting and fishing were his principal hobbies, and his brief vacations were mainly spent in the open.

On December 20, 1926, he was married to Adele Louise Muench of Washington, Mo. He is survived by his widow; a daughter, Helen Louise; and a brother, Archibald Kasel.

Mr. Kasel was gifted as an engineer and possessed exceptional administrative and technical abilities. His clear understanding of the many intricate details of the highly specialized branch of his chosen profession, together with his ability as an organizer and a leader, gave him an enviable record in all his undertakings.

Mr. Kasel was elected a Junior of the American Society of Civil Engineers on December 15, 1924; an Associate Member on May 12, 1930; and a Member on October 17, 1938.

HENRY JAMES KESNER, M. ASCE[1]

Died February 10, 1945

Henry James Kesner was born on October 30, 1883, near the town of Aurora, Ont., Canada. He was the son of Edgar Euryas and Dorcas Ann

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[2] "Water Resources of Iowa, 1873–1932," by R. G. Kasel, Iowa State Planning Board, 1935.

[1] Memoir prepared by Roy M. Green, M. ASCE.

(Hortop) Kesner, who were both teachers by profession. Mr. Kesner studied both broadly and deeply. After obtaining his early education in the public schools of Fairplay, Boulder, and Salida, Colo., he attended the University of Colorado in Boulder from which he received the degrees of Bachelor of Arts in 1905, Bachelor of Science in Civil Engineering in 1907, and Civil Engineer in 1911.

Immediately after graduation in 1907, Mr. Kesner began working for the U. S. Reclamation Service as hydrographer on the Uncompahgre Project. In September, 1907, he accepted a position as an instructor in structural engineering at the University of Minnesota, in Minneapolis.

In June, 1908, he left the teaching profession to become junior engineer in the Technologic Branch of the United States Geological Survey at Pittsburgh, Pa. In March, 1909, he became a draftsman for the Minneapolis Steel and Machinery Company at Minneapolis. In 1909 he returned to teaching, serving from August, 1909, to May, 1913, at the University of California in Berkeley as instructor and later as assistant professor of civil engineering. During the summer of 1912 he acted as director of the Summer School of Surveying at that institution.

From September, 1915, until his death, each academic year Professor Kesner taught in some school of engineering in the United States. During this period he served on the faculty of Purdue University in Lafayette, Ind., from September, 1915, to June, 1921, as instructor, assistant professor, and later as associate professor of structural engineering. From September, 1921, to June, 1923, he was associate professor of civil engineering at Armour Institute of Technology in Chicago, Ill. In September, 1923, he joined the staff of the department of civil engineering at the University of Nebraska in Lincoln. At that institution he held, successively, the ranks of assistant professor, associate professor, and in 1927 professor of civil engineering. From 1925 until his death, he had charge of the courses in structural engineering at the University of Nebraska.

During summer vacations, Professor Kesner worked for the following organizations: From 1910 to 1914, for Charles Derleth, Jr., M. ASCE, consulting engineer, and for Haviland and Tibbets, consulting engineers of San Francisco, Calif.; in 1916 for the Chicago, Burlington and Quincy Railroad Company; in 1917 for the Interstate Commerce Commission in Chicago on valuations; in 1918 for the Foundation Company of New York, N. Y., as a designer; in 1919 and 1921 for the Indiana State Highway Commission as a designer; in 1920 for the Wyoming State Highway Commission on the supervision of concrete bridge construction; in 1922 for H. S. Crocker, Past-President and Hon. M. ASCE, consulting engineer of Denver, Colo., as a designer; in 1923 for the South Park Commission of Chicago as structural engineer; in 1924 for the Chicago Bridge and Iron Works; and in 1926 for W. H. Adams, M. ASCE, consulting engineer of Detroit, Mich., as designer.

In addition to the Society, Professor Kesner was a member of the Society for the Promotion of Engineering Education and the American Association of University Professors. He was a registered structural engineer in the State of Illinois and a registered professional engineer in the State of Nebraska

Besides his professional affiliations, Professor Kesner belonged to Sigma Xi, Phi Beta Kappa, Sigma Tau, Tau Beta Pi, Theta Tau, and Beta Theta Pi fraternities. He was also an active member of the Lincoln chapter of the Interprofessional Institute and served as president in 1943.

On September 3, 1921, Professor Kesner was married to Mary Downer of Denver. He is survived by his widow.

Intensely individualistic, thoroughly competent, habitually studious, and strictly ethical, Professor Kesner was a born teacher who exerted a permanent influence upon the lives of a long line of engineering students.

Professor Kesner was elected a Junior of the American Society of Civil Engineers on December 6, 1910; an Associate Member on March 4, 1913; and a Member on October 10, 1927.

ROBERT VOLANT LABARRE, M. ASCE[1]

DIED SEPTEMBER 27, 1944

Robert Volant Labarre, the son of Clement Labarre and Rosa (Bouligny) Labarre, was born at Gretna, La., on October 20, 1873, at the plantation of his paternal grandparents, who were planters of the old South. In his early youth he was subjected to the hardships of the reconstruction period in New Orleans, La. His education, in the parochial schools at New Orleans and the Jesuit School at Quebec, Que., Canada, with two years at Tulane University in New Orleans, was interrupted by the death of his father. While still in his teens, Robert Labarre entered the employ of the City of New Orleans as draftsman. Later, he worked for the Orleans Levee Board as rodman and draftsman, becoming assistant engineer in 1895. As topographer with a survey party of the U. S. Engineer Department, he did map work on the Mississippi delta region.

Moving to Shreveport, La., in 1901, he was engaged in mechanical design and construction for the Lake End Cotton Mills and the J. L. Means Machinery Company. He was employed on design by the Fred A. Jones Building Company of Houston, Tex., later becoming superintendent of construction on office, apartment, and industrial buildings in Houston, Tex., Birmingham, Ala., and Atlanta, Ga. He then entered the contracting business for himself and erected a number of schools and industrial buildings throughout the South.

During World War I he served as Captain of Engineers, commanding Company F, 606th Engineers, and then Company B, 211th Engineers. After the war, he was employed by the Foundation Company of New York, N. Y., on industrial construction in the Detroit (Mich.) area. He was in charge of port sanitation south of Lima, Peru, and later worked for the Vanadium Corporation in the Andes Mountains in South America. On his return to the

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[1] Memoir prepared by Harry W. Dennis and Charles T. Leeds, Members, ASCE, in collaboration with Mrs. Robert V. Labarre.

United States in 1921, he was employed by the Realty Construction Company on industrial design and construction in Michigan.

Moving to Los Angeles, Calif., in 1923, he worked for the Schofield Engineering Company, and for the P. J. Walker Company on the engineering problems of foundation and underpinning projects, a type of work for which he was well fitted, not only because of his early experience, but also because of his keen ingenuity and ability to reduce the most complex problems to simple elemental concepts. These contractors built the foundations under the Broadway Department Store Building, the Subway Terminal Building and Tunnels, the Southern California Edison Building, the Stock Exchange Building, and Bullocks department stores.

At this time Captain Labarre began to realize that adequate scientific study and thought were not being applied to the structural values of foundation materials—that huge projects were financed and built with little apparent concern for the physical characteristics of the ground to be used as a foundation. Realizing the need for definite knowledge of soil as a structural foundation material, he began to specialize in that field. In 1928 he established a research and consulting foundation engineering office in Los Angeles. Thus, he was one of the pioneers on the West Coast in the consideration of foundation engineering and soil mechanics as a separate and requisite field of engineering. Alert to the necessity of establishing soil mechanics on a scientific basis, he conducted field and laboratory tests on soils, made extensive experiments on the bearing value of various types of piles, and designed and built equipment and apparatus for his experiments, as his knowledge of soil mechanics progressed. His firm continued to practice, constantly enlarging its field of research, until Captain Labarre's retirement because of illness in the latter part of 1942—from 1933 to 1936, with Frederick J. Converse, Assoc. M. ASCE, as associate; and, from 1940 until Captain Labarre's retirement, with William F. Swiger, Jun. ASCE, as associate. During this time, Captain Labarre was foundation consultant for the State of California, municipalities, public utilities, industrial corporations, architects, and engineers. Many jobs of outstanding importance and complexity were brought to him. Among them were the stabilization of the foundations of the Southwest Museum (which, being located upon a high hill of interbedded shale and sandstone, was threatening to slide); the exploration of the foundation material along the Second Boulder-Chino Transmission Line for the Southern California Edison Company; the design of cast-in-place reinforced concrete tower footings; the foundations for the Harbor Steam Plant of the Los Angeles Bureau of Power and Light;[2] a very detailed and comprehensive study and report on the concrete aggregate of the Santa Clara River (which also involved a comprehensive investigation and study of the concrete in which this and other aggregate had been used); facilities for the port of Oakland (Calif.); and foundations of the Geneva Plant of the Columbia Steel Company Defense Plant Corporation at Provo, Utah. He also served as one of the consultants for the Southern California Edison Company on certain problems at the Long Beach (Calif.) steam plant caused by unwatering operations at the near-by U. S. Naval Fleet Operating Base.[2]

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[2] "Foundation Tests for Los Angeles Steam Plant," by W. F. Swiger, *Civil Engineering*, December, 1941, p. 711.

Concurrently with his development of soil mechanics research, Captain Labarre became intensely interested in seismology. About 1930 the United States Coast and Geodetic Survey began a program of measuring both the earth movements during destructive earthquakes and the periods of vibration of buildings and other structures. Captain Labarre cooperated with Frank Neumann, seismologist of the Coast and Geodetic Survey in the development of a vibration meter to measure structural and ground vibrations. Captain Labarre took a very active part in organizing conferences to perfect plans, and especially in creating sentiment in favor of the survey's program. He persuaded building owners to provide sites for instruments and secured permission to make vibration measurements. Largely because of such cooperation, extremely valuable records of the Long Beach earthquake of March 10, 1933, were obtained. Subsequent to this earthquake, several local technical groups and other interested bodies began to make investigations independently. The necessity of correlating these efforts was soon recognized and a permanent committee known as "Joint Technical Committee on Earthquake Protection" was formed under the chairmanship of Robert A. Millikan of the California Institute of Technology at Pasadena. Seven technical groups were represented on the committee, and Captain Labarre was one of the members acting on behalf of the Society. In 1934, while Captain Labarre was president of the Structural Engineers Association of California, his dominant interest in seismology helped to bring the influence of the association to bear on the California Legislature, resulting in the adoption of a revised State Building Code Relating to the Construction of Schools. His also was the driving force which resulted in the Society's Joint Committee on Seismology, organized on July 13, 1936. He was the committee's first chairman, and this committee was well on the way to important accomplishments, when the war, and then Captain Labarre's illness and death, intervened. His accomplishments in the field of seismology are important, especially when it is recalled that his prime interest lay in only one phase of the problem—foundations.

Possessing judgment, ingenuity, and executive ability beyond the average, Captain Labarre accomplished much by his vivid and engaging personality and forcefulness, both in private life and as a member of professional societies. He was affectionately known as "Cap" among his many friends. A fluent speaker, a capable organizer, and a leader, he served on numerous committees for organization as well as on local and national committees of the Society and was Vice-President of the Los Angeles Section of the Society in 1939.

He was a member of the Executive Committee of the Soil Mechanics and Foundations Division from 1937 to 1941, inclusive. He was most active in division affairs and was a member of the Committee on Glossary of Terms and Definitions and on Soil Classification from 1937 to 1944, inclusive; chairman of the Committee to Cooperate with Committee D-18 on Soils for Engineering Purposes of the American Society for Testing Materials in 1938; and chairman of the Joint Committee on Seismology from 1941 to 1944, inclusive. He belonged to the American Society of Mechanical Engineers from 1923 to 1942, American Society for Testing Materials, Engineers Club of San Francisco (Calif.), Engineers Club of New York, University Club of Los Angeles, and the Seismological Society of America. He was a registered civil engineer in the State of California.

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[2] "Novel Construction Plan for Graving Dock Suggested by Soil Studies" (a Symposium), *Civil Engineering*, August, 1944, p. 323.

Captain Labarre keenly enjoyed life. He had many friends and many and diversified interests beyond technical fields. A devotee of music, history, and travel, he also especially liked motoring through the country, driving to many meetings of the Society in his own car. He never tired of seeking out old landmarks of the hardy pioneers, especially in the old mining districts and in Indian country of the Southwest. He was never without an absorbing hobby, which changed along with the years and times. At his home he had a complete machine shop, where he spent many leisure hours. He had keen joy in, and unbounded enthusiasm for, his professional work.

Because of the nature of last illness, he was compelled to retire from active work in the latter part of 1942. He spent his last year in Arizona in and around the haunts of the rugged historical characters with whom he was so familiar. He died at Wickenburg, Ariz. His monument is found in the lives and works of the many younger engineers trained and taught by him with patience and with a deep wisdom of the world and men.

He was married twice—on February 17, 1897, to Alice Lang of New Orleans, and on January 5, 1923, to Gertrude Ward of Los Angeles. He is survived by his widow; a sister, Margaret (Mrs. S. E. Ingles); and three daughters by his first marriage, Anna (Mrs. A. L. Ferguson), Marcelle (Mrs. Laurence Pace), and Gladys (Mrs. George Stewart).

Captain Labarre was elected an Associate Member of the American Society of Civil Engineers on August 9, 1920, and a Member on October 26, 1931.

JOHN ROBERT LAMBERT, M. ASCE[1]

DIED NOVEMBER 12, 1944

John Robert Lambert was born on January 8, 1884, in Niles, Mich. His mother, whose maiden name was Juliette Smith, was a member of one of the numerous families that migrated from New York State to Michigan about the middle of the nineteenth century. His father came from a prominent Virginia family.

John Lambert was graduated from the high school in Niles in 1901, and during the following year worked for a local paper manufacturing and lithographing company. He entered Michigan State College at East Lansing in the fall of 1902, led his class all through college, and had the honor of being valedictorian. He was the first member of his class to be elected to Tau Beta Pi, the engineering honorary fraternity.

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[1] Memoir prepared by Scott B. Lilly, M. ASCE.

His entire professional career was spent in the employ of the Phoenix Bridge Company at Phoenixville, Pa. In 1906, when he began work in the drafting room, the company was engaged in completing the details for the first Quebec Bridge. In August, 1907, the Quebec Bridge collapsed during erection—a failure of an outstanding engineering structure and one that affected engineering design profoundly. The cause of the failure was due partly to inadequate details used in the latticing of the columns, and partly to the use of too high stresses in compression. The weakest members in the bridge were the two anchor-arm bottom-chord sections, one in each truss, adjacent to the main shoes.

The effect of this failure on John Lambert was to intensify his natural inclination to examine everything carefully, to test every conclusion, and to prove every computation.

A succession of important structures passed through the drawing room of the Phoenix Bridge Company, and John Lambert acquired not only an intimate knowledge of design and detail, but he gained a thorough intimate knowledge of shop and erection practice. For years he had a large part of the responsibility for estimating the costs of bridges and buildings.

He remained a student all his life. He was keenly interested in the design, construction, and operation of movable bridges. During this period his company built many such structures, and Mr. Lambert studied the design of every one that was available to him. There was a 320–ft bridge for the New York, New Haven and Hartford Railroad over the Taunton River in Massachusetts. This was a 1,500–ton double track drawbridge. His company built: Scherzer rolling lift bridges over the Seekonk River in Rhode Island and over the Savannah River in Georgia; Strauss trunnion bascule bridges such as that over Bayou Plaquemine, Louisiana, that at Jacksonville, Fla., over St. John's River; the Arlington Memorial Bridge, double-leaf 2,200-ton, central span at Washington, D.C.; vertical lift bridges such as the 341-ft span over the Hudson River near Troy, N.Y.; the 544-ft, 5,000-ton bridge at Buzzards Bay over the Cape Cod Canal, Massachusetts; the Piscataqua River double deck, 224-ft span at Portsmouth, N.H.; and the 240-ft, 2,800-ton bridge over the Reserve Basin at League Island Navy Yard, Philadelphia, Pa.

Then there were cantilever bridges such as the 3,400-ton Smith Street Bridge with a 420-ft main span at Rochester, N.Y.; the 765-ft highway bridge of 990 tons over the Grasse River at Massena, N.Y.; and the 2,400-ft 18,000-ton length of the Pulaski Skyway. There was the 3,400-ton continuous-type truss structure for the Gill-Montague Bridge over the Connecticut River, in Massachusetts; the 1,600-ton Deer Isle suspension bridge over Eggemoggin Reach at Sedgwick, Me., with its 1,080-ft main span; the 540-ft tied arch span of the Chesapeake and Delaware Canal, 7,400-ton crossing at St. Georges, Del. These structures and many others were of absorbing interest to Mr. Lambert. There was something of him in each one of them.

In 1923 he was transferred to the contracting department; in 1926 he became chief engineer; and in March of 1944 he was made general manager, which position he held at the time of his death. He was a director of the Phoenix Bridge Company and also of the Phoenix Iron Company.

Until the last few years of his life, Mr. Lambert gave freely of his time to the young engineers who joined his company. He had classes in structural design, and in the various phases of bridge building, at his home at night. In this way, he not only encouraged these young men to improve their knowledge of structures, but gave them, by his excellent teaching, more than they would have been able to secure in many colleges.

Throughout this time, he was interested in mathematics and developed a method of solving equations of a high order by the use of recurring series. In 1933 he rediscovered a solution of equations by this method; and in 1935 he distributed a monograph covering his solution. In this monograph he had derived sample equations solving, among others, equations of the seventh degree.

He had many other hobbies. He was an enthusiastic stamp collector and was interested in book binding and sun dials. He was interested in the many covered bridges of Pennsylvania, and spent many happy days driving over this part of the state with his wife taking photographs of those that still remained. For years he was an enthusiastic golfer; he competed as a member of the team from his club with similar groups from neighboring courses.

For years, he served on the Board of Health of Phoenixville, and for a much longer period, on the Library Board. He was fond of pictures, music, and books. His outstanding characteristics were his interest in people, his loyalty to his friends, his lifelong conviction of never saying or doing anything that would injure anyone.

In 1936, he was married to Esther Wetherall of Taunton, Mass., who survives him.

Mr. Lambert was elected an Associate Member of the American Society of Civil Engineers on April 19, 1920, and a Member on January 14, 1924.

WILLIAM BARKER LANDRETH, M. ASCE[1]

Died December 9, 1941

William Barker Landreth, the son of the Rev. James Landreth and Sarah (Barker) Landreth, was born in Rushville, N. Y., on August 3, 1857. He was graduated from Union College, Schenectady, N. Y., in 1881, with the degrees of Bachelor of Arts and Civil Engineer. In 1931, the honorary degree of Doctor of Science was conferred on him by his alma mater. He was a member of Sigma Xi and Beta Theta Pi fraternities.

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[1] Memoir prepared by Roy G. Finch, M. ASCE.

Following graduation he was engaged in railroad work in Mexico for several years. Upon his return to New York State he did pioneer work in the construction of sewers and sewage disposal plants in Schenectady, Amsterdam, Tonawanda, White Plains, Jamestown, Cortland, Port Jervis, and other localities. In 1897 he became identified with the Office of State Engineer of New York State, and served in various capacities in that office until 1922 with the exception of four years (1911–1914) when he was engaged in private practice. He served one year (1914) as city engineer of Schenectady. Upon his retirement in 1922 from the State Engineer's office he again engaged in private practice, and continued until he died in 1941.

Professionally his outstanding work was performed during his twenty-two years of state service. He was affectionately known as "Uncle Billie" by hundreds of engineers, many of them youngsters starting on their careers, who were granted the privilege of serving with and under this capable engineer and gentleman. His words of advice and professional guidance will always be gratefully remembered by those fortunate enough to have been associated with him during this period.

There was no one more familiar with, or who played a more important part in, the planning and construction of the New York State Barge Canal than Mr. Landreth. His first experience on canal work was in the deepening of the old system which was the last improvement undertaken prior to the construction of the Barge Canal. Under his direction the first surveys for the Barge Canal were made. The location of the canal was based, in a large measure, on the results of his studies and recommendations. The involved hydraulic computations necessary in providing an adequate water supply for canal operation came under his supervision. Practically all the plans for this great undertaking were prepared under his direction; and the construction of the major part of the system was completed under his general supervision. The history of the Barge Canal reveals the names of many able and distinguished engineers who were identified with the project; but, from its inception to its completion, no one man contributed more to the undertaking than did Mr. Landreth. He believed in the canal, and never doubted that ultimately it would prove its worth and fully justify the expenditure made for its construction.

Mr. Landreth was a friendly man and never lost touch with his former associates of the canal days. After his retirement in 1922, it was his custom to make frequent trips to Albany, N. Y., solely for the purpose of renewing the old friendships with his former associates. These periodical visits were looked forward to by those whom he termed "his boys."

In the death of this kindly and respected gentleman, the profession suffered a great loss.

In 1881 he married Amelia Theresa Fitzgerald who died in 1923. He is survived by two daughters, Mary (Mrs. Thomas S. Bailey) and Grace Louise (Mrs. Elmer L. Anderson), and two grandchildren, the Rev. William L. Bailey and Dorothy L. Anderson (Mrs. James M. Shaw). One daughter, Adeline Sarah Bailey Landreth, died in 1916.

He was a member of the Society of Engineers of Eastern New York, Albany Society of Engineers, Rensselaer Society of Engineers, International Congress of Navigation, Sapphire Masonic Lodge of Camillus, N. Y., and St. George's Church of Schenectady. He served as a Director of the Society from 1905 to 1907.

Mr. Landreth was elected a Member of the American Society of Civil Engineers on October 3, 1888. He became a Life Member in 1923.

ENGBERT A. LEE, M. ASCE[1]

DIED JUNE 7, 1945

Engbert A. Lee was born in Louis Valley, La Crosse County, Wis., on January 29, 1870, the son of Anders Lee and Peronella Lee, who migrated from Norway to Louis Valley in 1869. From there they moved to Minnesota and settled on a farm near Grand Meadow in 1875.

Mr. Lee's preparatory education was obtained in the public schools of Grand Meadow, Decorah, Iowa, and St. Olaf College in Northfield, Minn. He then entered the University of Minnesota in Minneapolis, where he received the degree of Bachelor of Science in Civil Engineering on June 3, 1897. At that time, because of his excellent scholarship, he was awarded the key of Sigma Xi.

After graduation Mr. Lee was employed by the Gillette-Herzog Manufacturing Company in Minneapolis. In January, 1899, he was made assistant engineer of its Denver, Colo., office, which position he held until this company was absorbed by the American Bridge Company in 1900. He was retained by the American Bridge Company until the summer of 1903 at which time he resigned his position to enter private practice in Denver.

In 1905 Mr. Lee was employed by the American Smelting and Refining Company. During this employment he was located in the Denver office as assistant engineer until 1911 and then he was sent to the Perth Amboy plant in New Jersey. He remained there until the end of 1912 when he was transferred to El Paso, Tex. The following year he returned to Denver where he became chief engineer of the Colorado Division. In 1928 he was loaned to the Amarillo, Tex., plant and remained there until his retirement in 1930.

From October, 1935, to July, 1937, Mr. Lee was employed by the Denver Board of Water Commissioners as draftsman engaged in the designing of structural details for the Transmountain Water Diversion from the Frazer River to the City of Denver. Following this and until his death, Mr. Lee was a valued employee of the Climax Molybdenum Company engaged in metallurgical plant design.

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[1] Memoir prepared by Herbert S. Crocker, Past-President and Hon. M. ASCE, and Maynard C. Allen, Mech. Engr., Eng. Dept., Climax Molybdenum Co., Golden, Colo.

Throughout his life Mr. Lee was greatly interested in music, both instrumental and vocal, and enjoyed working in choral societies. With a farming background it is natural that he should be a great lover of the wonders of outdoor life which he enjoyed recording with his camera. Mr. Lee was a man of modest demeanor. His life was purposeful and he was admired for his fine character, competence as an engineer, and loyalty to his employers and his associates. He was a man of whom his family can be justly proud. Would that there were more like him!

In 1911 Mr. Lee was married to Essie R. Lindell of Chicago, Ill., who, with a son, Alfred R., and a daughter, Martha Ann (Mrs. Gerald G. Christensen), survives him.

Mr. Lee was elected an Associate Member of the American Society of Civil Engineers on October 7, 1903, and a Member on October 11, 1920. He became a Life Member in 1938.

HENRY ALEXANDER LEEUW, M. ASCE[1]

Died October 18, 1945

Henry Alexander Leeuw, the son of Charles Richard and Henrietta (Jungman) Leeuw, was born in New York, N. Y., on November 5, 1882. His early education was obtained in the city public schools. In 1901 he entered The Cooper Union for the Advancement of Science and Art, in New York, taking the evening course in general science.

While continuing his evening courses in The Cooper Union, he was engaged during the day in survey work for the Atlantic Avenue improvement, Brooklyn, N. Y., and later in building subaqueous tunnels under the Hudson River; he was made general superintendent of the Washington Street shaft of the Hudson Tunnels in 1908. In 1910 he was superintendent in charge of the construction of five miles of narrow gage railroad for the Degnon Construction Company at New Paltz, N. Y. From 1910 to 1912 he was general superintendent on tunnel work for Glyndon Contracting Company at Yorktown Heights, N. Y., and for Condon Graham and Milner at Tallaluh Falls, Ga.

Because of his experience in building tunnels and placing concrete linings in them, he turned his attention to developing pneumatic methods of placing concrete; and from 1912 to 1915 he was vice-president and general manager of the Pneumatic Concrete Placing Company.

In 1916 he was superintendent for the New York and New Jersey Construction Company building concrete-lined tunnels under Newark Bay for the Passaic Valley Sewerage Commission. From 1916 to 1920 he was general superintendent on the construction of a sewerage disposal plant at Skillman, N. J.; railroad yards at Pittcairn, Pa.; and a dirigible hangar at Lakehurst, N. J. Mr. Leeuw again left the construction field from 1920 to

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[1] Memoir prepared by F. R. W. Cleverdon, M. ASCE.

1923 to enter the employ of the New Methods Utilities Company of Newark, N. J.; as sales engineer. In 1923 he was engaged by Allen N. Spooner and Son, Inc., as general superintendent on the construction of a tunnel under the Passaic River at Belleville, N. J.; from then until his death, he was associated with that corporation, as construction engineer, and directed innumerable projects in waterfront construction and subaqueous work.

Mr. Leeuw held the unique distinction of having been a member of the small group of engineers (who formally worked with the New York and New Jersey Construction Company) that met at a dinner on October 29, 1936, at the Masonic Club, Jersey City, N. J. At that time he moved that they might be formed into a permanent organization. At a second dinner, on April 29, 1937, he was appointed chairman of a committee to select a name and prepare a set of by-laws for the new organization. At Mr. Leeuw's suggestion the name of "The Moles" was adopted. The first dinner of the organization under the name of "The Moles" was held on January 15, 1938; Mr. Leeuw was elected its first president. From that time until October 1945, a period of seven years, "The Moles" enjoyed the most phenomenal growth, both in members and the quality of membership, ever experienced by any organization of its kind. Its membership, increasing from 26 members, in 1938, to 316 members, in 1945, consisted of the most prominent engineers and contractors in civil life, and the most prominent officers in construction work in both the Army and Navy of the United States.

Mr. Leeuw's death occurred at the peak of his engineering ability and when his qualities of character were well defined. Thoroughness of detail, accurate judgment of men, ability to secure the loyal cooperation of subordinates, absolute devotion to any responsibility he had once accepted—these were the qualities that made him a credit to his profession and an inspiration to all who enjoyed his personal friendship. He died of a heart attack at the Harkness Pavilion, Medical Center in New York City, on October 18, 1945.

On September 22, 1918, in Baltimore, Md., he was married to May V. Bergen. He is survived by his widow and a daughter, Virginia (Mrs. Louis Ragonetti).

Mr. Leeuw was a licensed professional engineer in both New York and New Jersey. He was a member of the National Society of Professional Engineers.

Mr. Leeuw was elected a Member of the American Society of Civil Engineers on August 15, 1938.

OLIVIER ODILON LEFEBVRE, M. ASCE[1]

Died March 25, 1944

Canada has lost a great engineer and above all a good citizen in the death of Olivier Odilon Lefebvre. Mr. Lefebvre was a leader in the field of stream regulation. Largely through his efforts, the primary power output of most of the great water power plants in the Province of Quebec, Canada, has been greatly increased; the output of some has been more than doubled. Mr. Lefebvre interested himself not only in the engineering but in the economic features of these developments. He was largely instrumental in combining public and private interests in such a way that the facilities were paid for in the form of advance rentals by the owners of the existing or possible future plants that benefit by the increased low-water flow, although ownership remained with the province. This plan contrasts favorably with developments elsewhere in which the public have been saddled with the initial costs of plants that compete with existing privately-owned works.

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[1] Memoir prepared by James H. Brace, M. ASCE.

Olivier Odilon Lefebvre was born at St. Hugues, Bagot County, Que., Canada, on November 14, 1879. His father was Olivier Lefebvre and his mother was Justine (Lefebvre) Lefebvre. He had five brothers and six sisters. His family typified the best in French-Canadian farm life as it existed three quarters of a century ago. He was fond of telling of his early life and stated that the family produced nearly everything it required. Mr Lefebvre was educated in the rural school at St. Hugues and then attended Mount St. Louis College and the École Polytechnique, both in Montreal, Que., Canada. He was graduated with honors from the latter institution in 1902 with the degree of Bachelor of Applied Science in Civil Engineering. In 1929 the University of Montreal at Montreal conferred on him the degree of Doctor of Science.

For eleven years Mr. Lefebvre was employed by the Dominion Government on laboratory work, surveys, and construction work at Ottawa, Ont., and on the west coast in the vicinity of Vancouver, B. C. His real life work began in February, 1913, when he was appointed chief engineer of the Quebec Streams Commission. He continued this work throughout the remainder of his life—as chief engineer of the Streams Commission from 1913 to 1935, as vice-chairman of the Quebec Electricity Commission from 1935 to 1937, as member of the Provincial Electricity Board from 1937 to 1940, and as vice-chairman of the Streams Commission from 1940 until his death in 1944. During this period Mr. Lefebvre was largely responsible, among other projects, for the initiation, design, and construction of the following important works: Gouin Dam and other storage dams on the St. Maurice River by which minimum flow was increased from 6,000 cu ft per sec to 16,000 cu ft per sec and the Mercier Dam, built about 1927, which increased the minimum flow of the Gatineau River from about 2,000 cu ft per sec to 10,000 cu ft per sec. The increased flow made economically practicable the three power developments on the Gatineau River at Chelsea, Farmers Rapids, and Paugan Falls. These plants have an aggregate installed capacity of 504,000 hp. His last large work was the construction of the Cadillac Power Development, on the Ottawa River, completed in 1941.

Mr. Lefebvre was a member of the Joint Board of Engineers on the St. Lawrence Waterways Project which reported in 1927. He represented Canada at the World Power Conference in Berlin, Germany, in 1930 and again at Washington, D. C., in 1936. At the time of his death he was a member of the Canadian Temporary Great Lakes-St. Lawrence Basin Committee. In

1933 and 1934, as chief engineer, he had charge of the construction of the Mercier Bridge over the St. Lawrence River near Montreal.

Throughout his entire professional career, Mr. Lefebvre took an active interest in the Engineering Institute of Canada, serving as councillor, vice-president, and finally in 1933 as president. He was prominent in the formation of the Corporation of Professional Engineers of the Province of Quebec (president in 1941).

He was also prominent in the educational affairs of the City of Montreal and of the Province of Quebec, having been a member of the administrative board of the École Polytechnique and of the board of administration of the University of Montreal at Montreal. When he died, he was chairman of the Catholic School Board of Outremont. Mr. Lefebvre spent much time in connection with many charitable organizations, among which were the Federation of French Charities, the Red Cross, and the Society for Crippled Children. He was a member of the Rotary Club, the Cercle Universitaire, and the Knights of Columbus. He was fond of reading, greatly enjoyed a game of bridge or billiards, and had a large circle of personal friends. Mr. Lefebvre did more through his personal influence than any other engineer to promote and develop cordial relations between the English-speaking and French-speaking population of Quebec. He possessed in an unusual degree the confidence of those in both groups with whom he came in contact.

On September 5, 1905, at St. Hugues, Mr. Lefebvre was married to Marie-Anna Comtois. He is survived by his widow and two daughters, Louise and Eliane. In fulfilment of a request he had often made, he was buried at his native place, St. Hugues.

Mr. Lefebvre was elected a Member of the American Society of Civil Engineers on April 12, 1926.

THEODORE ALFRED LEISEN, M. ASCE[1]

Died July 18, 1944

Theodore Alfred Leisen was born in Philadelphia, Pa., on February 17, 1864, the son of Charles Francis Leisen and Katherine (Murray) Leisen. After his education was completed, his entire life was spent in the practice of civil and hydraulic engineering. His father was a mechanical engineer and was granted numerous patents on mechanical devices; and his maternal grandfather, John Murray, was an engineer and architect in the British Government Service.

After graduating from a Philadelphia high school Theodore Leisen took special courses in engineering under the guidance of two professors of engineering in the Polytechnic College of Philadelphia. Also during that time he obtained valuable practical experience on general surveys and railway location.

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[1] Memoir prepared by George H. Fenkell, Hon. M. ASCE, and W. C. Hoad, M. ASCE.

On April 27, 1892, he was married to Sarah G. Watson, of Wilmington, Del. Mrs. Leisen died in 1938.

Mr. Leisen spent the years from 1884 to 1890 in the Republic of Colombia, South America, first on the location and construction of the Ferrocarril de Antioquia, and then as chief engineer of the Barranquilla Railway and Pier Company, Ltd. This work—the building of the first railroads from the coast—was a good beginning for a young engineer, a beginning which aided him through his entire career.

In March, 1890, Mr. Leisen resigned from the office of chief engineer of the railway and pier company and, after spending a vacation of a few months in England and France, returned to the United States to become principal assistant engineer in the Street and Sewer Department of Wilmington. In 1892 he became chief engineer of the Department of Parks at Wilmington, and he served the city, in this capacity, and as consultant to the water department, until 1903. From 1904 to 1908 he was chief engineer of the water department. The Rockford Water Tower, a landmark at Wilmington, was his work.

In 1908 he accepted an offer from the Louisville Water Company, Louisville, Ky., to become its chief engineer and superintendent. In addition to carrying the responsibilities of this position, he was active in civic affairs in Louisville and was an honorary member and past president of the Engineers and Architects Club of that city. During this period also he served as consulting engineer to the water company at Frankfort, Ky. He remained at Louisville until 1914, when he accepted the offer of the Board of Water Commissioners of the City of Detroit, Mich., to become chief engineer and general manager of that city's expanding water department.

In 1917 Mr. Leisen was commissioned as Major in the Construction Division of the Quartermaster Corps and was assigned to the post of Officer in Charge of Utilities at Camp Custer, Battle Creek, Mich. When it was decided to double the size of Camp Custer, Major Leisen was appointed Constructing Quartermaster and, in addition to his duties as Utilities Officer, was put in charge of the enlargement program and of the construction of a new base hospital. In this assignment he set records for economy and efficiency that were unsurpassed in the Construction Division.

In 1919, immediately after his release from the United States Army, he returned to Detroit as consulting engineer to the Board of Water Commissioners and had entire charge of the design and construction of Detroit's first filtration plant. At the time of its completion, in 1923, this was the largest filtration plant in the world.

While in Detroit Mr. Leisen organized the Detroit Local Section of the Society and was its president. He was awarded a life membership in 1932. From 1936 to 1940 he was a member of its Board of Direction. He was also an honorary member and past president of the American Water Works Association.

In November, 1923, Mr. Leisen was appointed general manager of the Metropolitan Utilities District of Omaha, Nebr., in charge of the water, gas, and municipal ice plants. For some time previous to this appointment he had served the District as consultant on the design and construction of its filtration plant. Here again he established records of economy and efficiency which were outstanding. In Omaha he was also active in the movement to improve the Missouri River. He held this post until his retirement from active professional work at the end of 1940.

After his retirement he resided with his son, Frederic, in Detroit. He spent the winters in Florida, where he was able to indulge in his favorite recreation of golfing.

He maintained an active interest in military affairs, remaining in the Reserve Corps after the end of the World War I. In 1924 he was commissioned a Colonel in this organization and he held that rank in the inactive Reserve at the time of his death.

He is survived by two sons, Theodore A. Leisen, Jr., and Frederic A. Leisen.

Theodore Alfred Leisen's name among engineers is chiefly associated with the design, construction, and operating management of public water supply systems. His eminence in that field will always be recognized. He also devoted much of his time and energy to other and related civic problems in the communities, and to the interests of young engineers. There are many men in responsible posts throughout the United States who will always remember his friendly and helpful counsel.

Mr. Leisen was elected a Member of the American Society of Civil Engineers on June 2, 1897. He became a Life Member in January, 1932.

LEON RODERICK MACKENZIE, M. ASCE[1]

DIED JUNE 26, 1944

Deeply rooted in the formative period of the West, the traditions of Leon Roderick MacKenzie stem from the poineer era of Omaha, Nebr., of which townsite his grandfather, William David Brown, was the original owner, registering his claim in 1853. Although in later years Mr. MacKenzie became identified with activities which took him far afield, the commencement and broadening of his career lay within the western regions which were his lifelong attachments. Without doubt, the qualities of fairness and resourcefulness in his professional life, as well as his habitual energy and refusal of discouragement, were inherited from his pioneer forbears and were the result of his western environment.

Born on April 23, 1883, at Omaha, Leon MacKenzie was the second child of Alexander F. MacKenzie, a native of Gairlock, Scotland, and Emma N. (Brown) MacKenzie of Omaha. He received his fundamental education in the public schools of Omaha and acquired his technical training through instruction by, and association with, the well-known engineers by whom he was employed. Continuous application to study in the fields he had selected provided the basis for a comprehensive technical knowledge, utilized in later life in the particularized practices in which he became engaged.

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[1] Memoir prepared by Norman D. Dean, Assoc. M. ASCE.

His first assignment, in 1900, found him a member of field parties of the Union Pacific Railroad Company and the Choctaw, Oklahoma and Gulf Railway Company in Wyoming, Oklahoma, and Texas. After two years in these activities, he decided to prepare for entry into the United States Military Academy at West Point, N. Y., and months were devoted to study. Toward the end of 1904 he abandoned this in favor of the career he had so auspiciously begun. The possibilities of the Cheesman Dam in Colorado appeared sufficiently compelling, and he became assistant to the construction engineer and, later, assistant superintendent of maintenance on the project. After about a year and a half on this project, he went to Mexico as assistant superintendent and engineer of surface operations for the Coahuila Coal Company at Hondo, Coahuila.

In 1907 he returned to Omaha as construction engineer for the Union Telephone Construction Company on the erection of exchange buildings and conduit installations. This work led to his employment by a prominent Omaha organization which was engaged in municipal engineering and construction.

Thus far, the young man had followed the not unusual path of many who enter the realm of engineering and are forced by circumstance, or desire for change, to seek other and greener fields. At the beginning of 1912, however, he organized his own business, under the firm name of Jackson and MacKenzie, consulting and contracting engineers, in Omaha. A year later he went to Chicago, Ill., to carry out research in street paving materials and methods. This led to the position of manager, and, soon after, chief engineer, of the California Brick Company at San Francisco, Calif., the largest paving brick plant on the West Coast.

The years from 1915 to shortly after the commencement of World War I were devoted to Mr. MacKenzie's well-developed specialization—street and highway pavements, research, development, and materials. As field and consulting engineer for firms of national repute in the uses of concrete and asphalt pavements, his activities were distributed throughout the United States. These abruptly ended when he became commissioned a Captain of Engineers in the National Army in 1918, and he was assigned to the engineering staff of the Commanding General of the Port of Embarkation at Newport News, Va.

The end of the war brought new ideas and stimulated new efforts, but also created delays and necessitated adjustments. Various surety companies, at this time, had discovered the desirability of utilizing engineers in contract completion, and in providing data prior to the issuance of surety bonds on important works to be undertaken. In the capacity of surety engineer, Mr. MacKenzie assumed charge for the Southern Surety Company of Iowa at Des Moines. This company had broad interests in many states. During tenure of this office he introduced many original procedures and practices still in use. He tendered his resignation in 1922.

By then, the years of application in the fields of street and highway paving materials and methods brought results. The uses of concrete for surfaces had increased steadily and were given added impulse following the war. After much experiment in the effects of vibration on concrete, he, with old friends and associates, founded the American Vibrolithic Corporation of Des Moines, Iowa, an organization which, by the end of a few years, had become nation-wide, extending its activities to Cuba and its patents to other foreign states. For ten years, as president of the corporation, Mr. MacKenzie became a well-known figure among engineers and contractors, and in municipalities all through the United States.

Concurrent with his work in the perfecting of vibrated concrete techniques, he commenced long and exhaustive experiment and research in the development of asphalt emulsions. The laboratory at Des Moines and work in the field, at last, yielded a product which added much to the advancement of asphaltic surfaces, resulting in the creation of his own organization, L. R. MacKenzie, Inc., at Des Moines. He was president of this organization until his death. In 1934 he was invited by the Soviet government to partake in a series of conferences in Moscow, U.S.S.R., concerned with highway planning and organization. In preparation for this event, he consumed months in study and gathering data calculated to be of practical value to the Russian government. His ideas and organizational plans were highly regarded by the Russians, who, at that time, were intent upon modernization of their road system, and upon the establishment of a road network which could be economically built and adapted to the vast road mileages involved. The mission to Moscow allowed Mr. MacKenzie the opportunity to visit Poland, Finland, France, and England in the interest of his patents.

Through a lifetime of activity he ran the gamut of struggle, progress, disappointment, and eventual success. In later years, although engaged in highly competitive production, he never relaxed his staunch principles of professional ethics and fair dealing. In private life he was recognized for his kindness and unfailing willingness to help others. He disliked display, or any manner of ostentation, preferring his rewards in the form of successful accomplishment. From Mexico to Canada he was fondly known as "Mac" and by his Russian friends as "Mr. Macansky." His hobby was his family of which there remain his widow, Ann (Amundson) MacKenzie; one daughter, Margaret Ann (Mrs. Robert B. Ash); a granddaughter, Margaret Randall Ash; his mother, Mrs. Alexander MacKenzie; a sister, Norma MacKenzie; and a brother, Lyle MacKenzie.

Mr. MacKenzie was elected an Associate Member of the American Society of Civil Engineers on September 12, 1916, and a Member on March 9, 1920.

LESTER CHIPMAN McCANDLISS, M. ASCE[1]

Died October 31, 1945

Lester Chipman McCandliss, the son of Harry E. and Harriet (Barr) McCandliss, was born in Anderson, Ind., on June 16, 1886. He was educated in the Anderson Public Schools and at Purdue University, in Lafayette, Ind., from which he received the degree of Bachelor of Science in Civil Engineering in 1909.

His first engineering experience was acquired with the Chicago, Burlington and Quincy Railroad. He then was employed by the Carmichael Construction Company of Akron, Ohio, and as office building superintendent.

In 1912 he joined the faculty of the School of Engineering of the University of Pittsburgh in Pittsburgh, Pa., and thus began his long career as an educator. He became assistant professor in 1915 and in 1931 was made professor and head of the department of civil engineering of the University, which position he held at the time of his death. During this period, at various times he taught most of the courses included in the civil engineering curriculum, including mechanics, strength of materials, geodetic surveying, and hydraulics.

For more than twenty-five years Professor McCandliss was in charge of the summer engineering camp maintained by the University of Pittsburgh at Windber, Pa., where he supervised the development of an outstanding surveying camp and the design and erection of an unique hydraulics laboratory. He was also largely responsible for the design and construction of the highway and automotive laboratory of the University on the campus.

During the operation of the service and management war training classes at the University before and during World War II, Professor McCandliss had charge of the engineering drawing and civil engineering classes in which several thousand men were trained.

During World War I, from May, 1917, to April, 1919, Professor McCandliss served as First Lieutenant and Captain of the Fifteenth Engineers, United States Army, spending twenty months in France in responsible charge of construction of railroads, warehouses, barracks, and highways. He had the unique distinction of leading the first company of armed American soldiers ever to set foot on British soil in wartime.

Professor McCandliss was active in the civic affairs of the community, being a member of the Pittsburgh Better Traffic Committee and at the time of his death, president of the Pittsburgh Motor Club. He was past-president of the Pitt Faculty Club and had served as Director of the Pittsburgh Section of the Society. He was a member of the Society for the Promotion of Engineering Education, Society of Military Engineers, and of the following fraternities: Scabbard and Blade, Phi Gamma Delta, and Signa Tau. He also belonged to the Bellefield Lodge, F. and A.M., and the Pennsylvania Consistory. He was a registered engineer in the State of Pennsylvania.

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[1] Memoir prepared by Lewis W. McIntyre and John M. Rice, Members, ASCE, and Donald McNeil, Assoc. M. ASCE.

Professor McCandliss' outstanding characteristic was his devotion to his family and students. Although his main concern during the greater part of his active professional life was the development and improvement of the technique of teaching civil engineering subjects, he had a deep conviction that guidance in character building was of equal importance if the engineer graduate was to assume his place in society properly. The impression of his own high character and the earnestness of his teaching upon the thousands of his students made them not only good engineers, but also better men. The high plane of friendliness in student relations which he achieved is best illustrated by the affectionate title of "Cappy" with which he was universally greeted on the campus. Most of all, his hundreds of past and present students loved him. His office door was always open to them; and, if they were in trouble, they always left with a feeling of new courage and determination after having talked with him. He believed that every man and every student, if properly adjusted, would give out the best that was in him. His departmental faculty respected and recognized him as a leader. In the words of a prominent alumnus, "He was a great teacher." His kindly nature, loyal friendship, and judicious counsel endeared him in the highest degree to his colleagues, associates, friends, and students. All have suffered an irreparable loss by his death.

He was married, in Anderson, on October 12, 1910, to Abby W. McNamee. He is survived by his widow; two sons, John Henry and Alfred Neal McCandliss; and two brothers, Bernard B. McCandliss and Donald A. McCandliss.

Professor McCandliss was elected an Associate Member of the American Society of Civil Engineers on November 28, 1916, and a Member on September 9, 1935.

RALPH STECK McCORMICK, M. ASCE[1]

DIED MARCH 18, 1946

Ralph Steck McCormick, the son of Franklin S. and Elizabeth (Moore) McCormick, was born in Quaker City, Guernsey County, Ohio, on June 22, 1873. The family moved to Saginaw, Mich. (about 1883) where Ralph was graduated from high school. He attended the University of Michigan at Ann Arbor, for two years.

On leaving the University he was employed by the Pere Marquette Railway, from 1890 to 1894, as assistant engineer in Saginaw. Then, in 1894, he became draftsman and assistant engineer for the Grand Trunk Railroad at Battle Creek and Detroit, Mich., until 1900, when he went with the Algoma Central and Hudson Bay Railway at Sault Ste. Marie, Ont., Canada, as locating engineer. In 1903, Mr. McCormick returned to the Pere Marquette Railway, at Saginaw, as division engineer. In 1905, he became construction engineer for the New York Barge Canal at Rochester, N. Y.; and from 1905 to 1909 he was construction and locating engineer for the Grand Trunk Railroad (later the Canadian National Railway), at Fort William, Ont., Canada. In 1909, as chief engineer and general superintendent, he returned to the Algoma Central Railroad at Sault Ste. Marie. From 1916 until he retired, in 1945, he was general superintendent and chief engineer for the Algoma Central.

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[1] Memoir prepared by L. C. Maxwell, Esq., Engr. Maintenance of Way (Retired), Algoma Central R.R., Sault Ste. Marie, Minn.

Ralph McCormick belonged to the American Railway Engineering Association, the Engineering Institute of Canada, and the Toronto Railway Club. He was a member of the Presbyterian Church (president of the board of trustees for several years), the Boy Scout Council, and the Masons.

On June 21, 1921, he was married to Eva Maxwell. They had four children: Two sons, Harry and Ralph McCormick, and two daughters, Gene and Dorothy Ann McCormick. He is survived by his widow and children.

He was a devoted husband, and, as a father, took a keen interest in the education and welfare of his children.

Mr. McCormick was elected a Member of the American Society of Civil Engineers on October 3, 1911.

JOHN GILMORE McMILLAN, M. ASCE[1]

DIED FEBRUARY 14, 1935

John Gilmore McMillan was born in Bristol, R. I., on July 11, 1851. He was the son of William Wallace McMillan and Sarah (McCaughy) McMillan, the second child in a family of four sons and two daughters. His father first arrived in California in 1849 and engaged in mining in Butte County, at Union Bar on the Feather River. After returning to Rhode Island, he again went to California by way of Cape Horn. Mrs. McMillan and two small boys crossed the Isthmus of Panama early in 1856, proceeding to San Francisco, Calif., by sailboat—an arduous journey. They were finally met by an ox team which conveyed them to the mines.

After the mine at Union Bar was worked out, the family moved to Hurleton, Calif., in the Feather River section of the Mother Lode country. Mr. McMillan and his elder brother were sent to a school held in a little cabin—a walk of six miles through the rough wooded country. With the benefit of the tutelage of his father, a graduate of the University of Glasgow in Glasgow, Scotland, and with the encouragement of the late Smith Hurles, Mr. McMillan extended his studies far beyond the curriculum of the remote country school; and, at the age of sixteen, he was teaching. His aptitude for mathematics, together with the influence of his contact with the mining activities of the Mother Lode country, led him to devote his spare hours to the study of civil engineering.

His cousin, the late Sarah Daley, wrote of the early life of Mr. McMillan:

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[1] Memoir prepared by Earl C. Thomas, Assoc. M. ASCE.

> "His childhood days * * * were not made of luxuries, as so much depended upon the mines. He was always a good studious boy and the pride and joy of a good mother's heart * * *. He won the admiration of not only his own relatives but very many friends who yet remember him."

In an old scrapbook Mr. McMillan cited an extract from a letter written by his father just after leaving home:

> "I will allow nothing on the outside to draw my attention from my daily duties, my firm resolution is to go slow, be modest, agreeable and dignified and avoid affectation in reading, speaking or acting. Be natural now and forever, it is much easier to be what we would like to appear to be, than to appear to be what we are not. The more we read and the more we see, the more are we inclined to take a common sense view of all that surrounds us and a veneration of all that is above or beyond our comprehension."

Apparently this advice had great influence on the young man since the sense of the quotation comprehends the best description of his character. Mr. McMillan's good judgment, sustained by his mathematical and inherent technical ability, was the groundwork for his engineering achievements.

In 1878 Mr. McMillan was elected county surveyor of Sutter County, California, serving for nearly three years. He was forced to resign because of ill health and then entered the employ of the Southern Pacific Railroad Company. In 1883–1884 he was engaged as assistant construction engineer on the Market Street System of cable railways in San Francisco. The following year he was in Central America, as assistant engineer on the location and construction of the railroad from San José to Guatemala City, Guatemala.

Returning to California he was engaged by a French syndicate, in charge of location and construction of reservoirs and flumes for the development of hydraulic mining in the northeastern part of the state. In 1886 Mr. McMillan was engaged by Leland Stanford, then U. S. Senator from California, as chief engineer for the Stanford Properties. The greater part of his activities was centered at the Palo Alto (Calif.) Farm on the surveys for, and early construction of, Leland Stanford Junior University.

In 1890 Mr. McMillan became county surveyor of Santa Clara County, California. He began his career in the field of highway bridges in 1891 by designing one of the earliest part concrete bridges in the United States over Penetencia Creek. (The plans were adopted by the board of supervisors in that year but work did not commence until 1892.) During this early period, Mr. McMillan was not sure of the action of concrete in an arch and constructed this 36-ft arch of four rowlock brick with concrete side-walls and abutments. In 1896 he built two similar bridges. In the fall of 1897, construction began on a 24-ft concrete arch bridge reinforced with fabricated steel ribs, over Canoas Creek, near San Jose, Calif. A search by the Engineering Societies Library reveals that not more than nine concrete arch highway or railroad bridges were constructed in the United States prior to 1897. Thus, Mr. McMillan was among the pioneers in the design and construction of reinforced concrete bridges.

He continued to specialize in brick arch and fabricated steel reinforced concrete arch bridges until 1903 when he used ⅞-in. rods to reinforce a 40-ft bridge over Los Gatos Creek at Alma, Calif. Notable among the bridges constructed by Mr. McMillan are Long Bridge near Saratoga, Calif. (a two-span arched masonry bridge constructed of native rock) and the municipal bridge at Los Gatos, Calif. In 1910 Mr. McMillan constructed a three-span (one span of 70 ft) bridge over the Coyote River on the highway leading to Oakland, Calif. This bridge was outstanding as an economical type of spandrel construction which later became commonly recognized. All Mr. McMillan's work shows that he was resourceful and inventive. In 1899 he devised a bridge flooring composed of timbers laid on edge longitudinally to form a solid and continuous deck, covered with asphalt. Numerous similar bridge floors were built in California, several of which were covered with concrete instead of asphalt. Descriptions of this type of construction have recently reappeared in engineering literature.

During his tenure of office and during his residence in San Jose until his death, Mr. McMillan was engaged in various engineering consultation and construction, including the location and construction of the San Jose and Alum Rock Railroad, many sewer and sewage disposal plants, water works, and other municipal projects.

Mr. McMillan was married in 1887 to Elizabeth D. Weisshaar. He is survived by his widow; two sons, W. Bruce McMillan, M. ASCE, and Percy W. McMillan; and a daughter, Doris A. McMillan.

He became a Mason in Forbestown Lodge No. 50 (Butte County) in 1872. He subsequently joined the Knights Templar and the Scottish Rite. He was also a member of several other fraternal organizations, the Technical Society of the Pacific Coast, and the Sons and Daughters of Pioneers.

Mr. McMillan was elected a Member of the American Society of Civil Engineers on October 5, 1909. He became a Life Member in January, 1934.

LEON SOLOMON MOISSEIFF, M. ASCE[1]

Died September 3, 1943

With the death of Leon S. Moisseiff, the engineering profession lost a man of great knowledge, who had a keen analytical mind, professional honesty, and moral integrity. His knowledge of languages, combined with his intellectual curiosity, made him one of the best informed of engineers. He placed the foundation of his professional career when the engineering science as it is known today was still in its infancy and he grew with engineering science, always remaining abreast of it.

Leon Solomon Moisseiff, the son of Solomon and Anna (Bloch) Moisseiff, was born in Riga, Russia, on November 10, 1872. He studied at the Emperor

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[1] Memoir prepared by O. H. Ammann and Frederick Lienhard, Members, ASCE.

Alexander Gymnasium and the Baltic Polytechnic Institute in Riga. His activities in liberal groups forced him to leave Russia in 1891. On his arrival in the United States he worked as a draftsman in New York, N. Y., until he entered Columbia University in 1892. In 1895 he was graduated with the degree of Civil Engineer. During his college studies he was privately employed by W. H. Burr,[2] M. ASCE, one of his professors.

In those early days the public's conception of an engineer was an outdoor man in puttees. A college degree was not part of this picture and subtracted from, rather than added to, the concept. After two years of holding modest positions with private concerns, in 1897 Mr. Moisseiff entered the service of the City of New York, remaining for the next eighteen years. He began as draftsman and bridge engineer in the Borough of the Bronx, but was soon transferred to the Department of Bridges of the Borough of Manhattan and became chief draftsman in 1900. About this time he began to work on designs for the Manhattan Bridge and the Queensborough Bridge, as well as on reports for the projected reconstruction of the Brooklyn Bridge. He was the first to recognize the significance of the deflection theory, which had been outlined by J. Melan in 1888.[3] Mr. Moisseiff proved that this theory allows important savings in material. His complete development and application of the deflection theory became a standard work of reference for later designers of suspension bridges.[4]

Mr. Moisseiff was put in charge of the Williamsburgh Bridge investigation in 1909. His studies led to reinforcing the main span and strengthening the side spans. Two years later he was charged with the design of the proposed Hudson Memorial Bridge across the Harlem River. Because of lack of funds, this reinforced concrete arch with an unprecedented span of 725 ft was not built. In 1910 the City of New York established a Division of Design of which Mr. Moisseiff became head, with the title of engineer of design.

In 1915 he left the service of the City of New York and opened an office as consulting engineer. He did little professionally for the next few years. World War I caused a cessation of bridge building, and the use of the automobile had not reached the proportions which were to demand the building of large bridges. The first large suspension bridge to be built after the war was the Delaware River Bridge connecting Philadelphia, Pa., with Camden, N. J. In 1920 Mr. Moisseiff was appointed engineer of design for this structure.

From that time until his death he was actively engaged in the design of most of the large bridges in the United States—The Ambassador Bridge at Detroit, Mich.; the Maumee River Bridge at Toledo, Ohio; the Golden Gate and the San Francisco-Oakland Bay bridges at San Francisco, Calif.; the Triborough, the Bayonne Arch across the Kill Van Kull, the Bronx-Whitestone, and the George Washington bridges at New York, N. Y.; and the Tacoma Narrows Bridge at Tacoma, Wash.

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[2] For memoir, see *Transactions*, ASCE. Vol. 100, 1935, p. 1617.

[3] "Theorie der eisernen Bogenbruecken und der Haengebruecken," by J. Melan, 2d Ed., 1888.

[4] "Report to the City of New York, Department of Bridges, 'Manhattan Bridge," by Ralph Modjeski, 1909.

In addition, Mr. Moisseiff was retained by the Commissariat of Railroads of the Union of Soviet Socialist Republics for two and one-half years, from 1929 to 1932. The work involved engineering advice in the broadest sense on existing bridges as well as on standards for new structures. The studies necessitated an extensive trip in Russia. Two years later his advice was again sought for a design audit of the proposed Palace of Soviets in Moscow.

The planners of the Chicago (Ill.) World's Fair (1933) will remember Mr. Moisseiff as the responsible designer of the Travel and Transport Building. The basic conception for this structure was like that of a suspension bridge, the main sustaining members being cables.

In 1935 the City of New York requested a complete investigation of its East River crossings—the Manhattan, Williamsburgh, and Queensborough bridges—with recommendations for strengthening the structures as a whole and improving the roadways. Mr. Moisseiff was a member of a two-man board and all work was conducted in his office.

From 1937 to 1940 the Aluminum Company of America engaged Mr. Moisseiff to make a thorough study of aluminum as a structural material for bridges. The results of his investigations are summarized in "Design Specifications for Bridges and Structures of Aluminum Alloy 27S-T."[5] This research may, in due time, contribute to rewriting present specifications for steel.

In 1939 the Edward G. Budd Manufacturing Company of Philadelphia requested Mr. Moisseiff to make a study of stainless steel—particularly of its application to passenger railway cars. In addition, Mr. Moisseiff was active on many projects which did not materialize, notably the Mackinac Straits Bridge in Michigan. His activities in recent years may be summarized by the statement that Mr. Moisseiff was a consultant to consulting and executive engineers.

Throughout his career Mr. Moisseiff was very active in professional societies. He reviewed many papers, took part in discussions, and prepared a number of notable papers including "The Deflection Theory as Applied to Suspension Bridges with Suspended Trusses,"[6] "The Towers, Cables and Stiffening Trusses of the Bridge Over the Delaware River Between Philadelphia and Camden,"[7] "Manufacture and Use of High Strength Structural Steel for Bridges,"[8] "George Washington Bridge: Design of the Towers,"[9] "The Design, Materials and Erection of the Kill Van Kull (Bayonne) Arch,"[10] "Investigation of Cold Drawn Bridge Wire,"[11] and "Evolution of High-Strength Steels Used in Structural Engineering."[12] In collaboration with Frederick Lienhard, M. ASCE, Mr. Moisseiff wrote "Suspension Bridges Under Action of Lateral Forces,"[13] and "Theory of Elastic Stability Applied to Structural Design."[14] In collaboration with E. C. Hartmann and R. L. Moore, Assoc. M. ASCE, Mr. Moisseiff wrote "Riveted and Pin-Connected Joints of Steel and Aluminum Alloys."[15]

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[5] "Design Specifications for Bridges and Structures of Aluminum Alloy 27S-T," by Leon S. Moisseiff, Aluminum Co. of America.

[6] "The Bridge Over the Delaware River," Final Rept. of the Board of Engrs. of the Delaware River Bridge Comm., June 1, 1927.

[7] *The Journal of the Franklin Institute,* October, 1925.

[8] *Transactions,* Am. Soc. for Steel Treating, Cleveland, Ohio, 1928, p. 941.

[9] *Transactions,* ASCE, Vol. 97, 1933, p. 164.

[10] *The Journal of the Franklin Institute,* Vol. 213, No. 5, May, 1932.

[11] *Proceedings,* A.S.T.M., Vol. 30, Pt. II, 1930.

[12] *Transactions,* ASCE, Vol. 102, 1937, p. 1333.

[13] *Ibid.,* Vol. 98, 1933, p. 1080.

Mr. Moisseiff received the Norman Medal and the James Laurie Prize from the Society, the Egleston Medal from Columbia University, and the Modern Pioneer Award from the National Association of Manufacturers. He was on many committees of the Society and gave freely of his time and experience. He was a member of the American Society for Testing Materials, American Railway Engineering Association, and American Welding Society, serving in an executive capacity and on committees for some of these organizations. He belonged to Sigma Xi and Zeta Beta Tau fraternities.

Mr. Moisseiff was married to Ida Assinovski in 1894 in New York City. Surviving are his widow; a son, Siegfried Moisseiff; and two daughters, Mrs. Liberty Weiss and Mrs. Grace Smith.

Mr. Moisseiff was elected a Junior of the American Society of Civil Engineers on December 3, 1895; an Associate Member on September 5, 1900; and a Member on December 3, 1907.

PHILLIPS BATHURST MOTLEY, M. ASCE[1]

Died January 13, 1946

Phillips Bathurst Motley was born in Calcutta, India, on March 22, 1871. His parents were John Phillips Motley, a shipmaster in the East India Company, and Annie Eliza Carey, granddaughter of the great Dr. William Carey. Dr. Carey, who lived from 1761 to 1834, was the first missionary to India, and worked in the then hostile land for forty years. He translated the Bible into forty languages and was instrumental in abolishing the custom of "suttee," the burning to death of widows.

Phillips Motley received his early education in England, notably at Padcroft School, West Drayton, London. He received his engineering training at the School of Practical Engineering, Sydenham, England, and then worked for one year (1891) for a firm of mining engineers in London, as draftsman and engineer.

Mr. Motley left England for Montreal, Que., Canada, in 1892, to join the engineering department of the Canadian Pacific Railway, and he was instrumental in the construction of many of the large bridges built in the Rockies, along the north shore of Lake Superior, and in the Maritime Provinces. Among them were: Lethbridge Viaduct; Edmonton High Level Bridge; Stoney Creek Arch Bridge in the Rockies; the cantilever bridge at Saint John, N. B., Canada; the double-leaf bascule bridge at Fort William, Ont., Canada; and the double-track bridge over the St. Lawrence River between Ville LaSalle and Caughnawaga, Que., Canada.

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[14] *Transactions*, ASCE, Vol. 106, 1941, p. 1052.

[15] *Ibid.*, Vol. 109, 1944, p. 1359.

[1] Memoir prepared by Press Relations Dept., Canadian Pacific Ry., Montreal, Que., Canada.

In his forty-five years with the Canadian Pacific, Mr. Motley, successively, occupied the positions of draftsman, bridge inspector, assistant engineer, and assistant engineer of bridges before becoming engineer of bridges for the entire system, in 1911—overseeing all design, construction, and supervision of bridges from the Atlantic to the Pacific.

He was married on December 24, 1901, to Mary Helena Scott, daughter of William Scott, picture and fine art dealer of Montreal. He is survived by his widow and a son, Phillips C. Motley.

He was a life member of the Engineering Institute of Canada; and a member of the American Railway Engineering Association, the Institution of Civil Engineers of Great Britain, and the Professional Engineers of the Province of Quebec.

Accurate and conscientious to a degree, Mr. Motley carried the weight of great responsibility which his position involved with consistent faithfulness and success. On his retirement he was able to follow his many hobbies, chiefly photography and the collection and repairing of antique clocks and furniture, and other wide interests.

Mr. Motley was elected a Member of the American Society of Civil Engineers on April 7, 1915. He became a Life Member in January, 1942.

EVERETT BODMAN MURRAY, M. ASCE[1]

DIED JULY 10, 1945

Everett Bodman Murray, the son of Emma (Atkins) and Albert D. Murray, was born in Chicago, Ill., on September 26, 1885. He was graduated from the University of Illinois at Urbana, with the degree of Bachelor of Science in Civil Engineering in 1908. He was a member of Phi Gamma Delta fraternity.

While still attending school, he worked part time from 1902 to 1908, including three summers (1904–1907), as a rodman and instrumentman for the Chicago, Milwaukee and St. Paul Railway Company. From 1908 to 1909 he was in charge of a survey party for the Tallahatchie Drainage District in Mississippi. From 1909 to 1910 he was field superintendent on drainage, reinforced concrete, and power-plant construction work for the Harmon Engineering Company of Peoria, Ill. From 1910 to 1911 he was field superintendent of construction for the Federal Engineering Company of Milwaukee, Wis. From 1911 to 1912, he was chief engineer for the Kaw Valley Drainage District at Kansas City, Kans. From 1912 to 1917 he engaged in private practice—E. B. Murray and Company in Kansas City, Mo.—as consulting engineer on drainage, water works, sewerage, pumping plants, water purification, sewage disposal, etc.

With the entrance of the United States into World War I in 1917, Mr. Murray was appointed a Captain in the Corps of Engineers, U. S. Army, and served in France with the Three Hundred and Fourteenth Engineers, attaining the rank of Major. In 1924 he resigned from the regular army and returned to private practice as consulting engineer on management of income producing properties, office buildings, etc. He was president of E. B. Murray and Company, Inc., at the time of his death.

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[1] Memoir prepared by E. A. Blanpied and G. H. Frieling, Assoc. Members, ASCE.

Mr. Murray, familiarly known to his friends as "Mike," made building management his profession but was deeply interested in the human side of business and civic affairs connected with his real estate and housing projects. His contagious interest in life brought him a host of friends and his counsel and leadership in civic affairs made him one of Kansas City's most valued citizens.

He was president of the Missouri Bank and Trust Company, a director of the Missouri Land and Loan Company, and national president of the National Association of Building Owners and Managers. He belonged to the American Association of Engineers, American Society of Mechanical Engineers, and National Real Estate Board. Mr. Murray's civic activities were numerous, including service as the chairman of the Kansas City Housing Authority and of the Jackson County Citizens Reassessment Committee. For two years he was president of the University Club of Kansas City, Mo.

In 1910 he was married in Kansas City, Mo., to Clara Lee Webb. She died in 1943. He is survived by a son, Everett W. Murray; his father, A. D. Murray; and a brother, Donald Murray.

Mr. Murray was elected an Associate Member of the American Society of Civil Engineers on April 2, 1913, and a Member on October 3, 1922.

EGBERT STEPHEN NEEDHAM, M. ASCE[1]

Died March 13, 1945

Egbert Stephen Needham, the son of an American father and an Irish mother, was born on April 11, 1887, at St. Paul, Minn., and attended high school there.

At the age of seventeen, he took his first job, as a draftsman in the mechanical and engineering departments of the Chicago Great Western Railroad Company. Here he was engaged on bridge plans, starting an association with bridge work which he was destined to follow almost uninterruptedly for the next quarter of a century. After five years with the Chicago Great Western Railroad Company, he joined the bridge department of the Northern Pacific Railway Company, in 1909, as a designing draftsman; he remained there until 1913. This period included approximately a year with the Strauss Bascule Bridge Company of Chicago, Ill., in a similar capacity. From 1913 to 1915 he was a designer in the departments of the Engineer of Structures and Grand Central Terminal Improvements of the New York Central Railroad Company;

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[1] Memoir prepared by Leroy A. Beaufoy, King's College, London Univ., London, England.

then, in 1915, he went to the West Indies as resident engineer to the Department of Public Works, Santo Domingo, Dominican Republic, where he had charge of the construction of the Ozama River Highway Bridge, 700 ft long. When this job was finished, in 1917, he rejoined the Northern Pacific Railway Company as an assistant engineer supervising bridge reconstruction. In 1918 he joined the Army as a private in the 43d Engineer Corps, 20th Engineers, and spent a year in France with the American Expeditionary Force.

After World War I he remained in Europe. On July 26, 1923, he was married at Folkestone, England, to an English girl, Ivy Maud Dexter, the daughter of an Admiralty official stationed at Devonport, England. For a short period immediately after the war he was in private practice as a structural engineer in London, England, working on the design of a railway bridge over the Santa Lucia River in Montevideo, Uruguay. He entered the civil engineering and contracting department of Sir W. G. Armstrong Whitworth and Company, Ltd., general contractors, Westminster, London, in 1921, and was soon at work on an arch design of the accepted type for the Sydney Harbour Bridge, Australia, together with details and weights sufficient for a preliminary price. He also reported on the port works at Valencia, Spain. Then he was sent by the firm to Ireland, the land of his mother's birth, on the design and construction of the Bann River Bridge for the London, Midland and Scottish Railway at Coleraine. The opening span of the bridge is a Strauss trunnion bascule with underhung counterweight. It was probably the first of its type in the United Kingdom, and doubtless reflected his earlier experience with the Strauss Bascule Bridge Company at Chicago. Mr. Needham subsequently prepared a report on this work for the technical press.[2]

In 1925, his firm sent him to South America as their engineer and agent in charge of the construction, for the Colombian National Railways, of the Girardot Bridge over the Magdalena River. Shortly after he had settled down in Colombia, Mr. Needham was joined by his wife, who traveled from England with their infant son and made the last stage of the journey from Barranquilla, on the Atlantic side of Colombia, to Girardot, 700 miles up the Magdalena River, by plane. The Girardot Bridge, the largest bridge in Colombia, is of the cantilever type, 1,530 ft long, and the river span between centers of main piers is 426 ft. The work involved heavy foundations and anchorages, and construction under severe tropical conditions. Transport also was difficult. By the successful termination of this assignment in 1930, Mr. Needham displayed his ability to work in harmony with governmental bodies.

There followed a short engagement on harbor works and water supply problems in the Near East. During the slump period (1932–1933) in Great Britain, Mr. Needham took the examinations of the Institution of Civil Engineers, London, and began a special study of electric welding and its structural applications. This led to his being engaged (1933–1935) by Messrs. Murex, Ltd. on the design and construction of extensions to their works at Rainham, Essex, England. This work involved heavily reinforced-concrete raft foundations on piles, and steel-framed, all-welded factory buildings. He also designed the first all-welded Vierendeel truss to be erected in the United Kingdom, and was joint author of a paper[3] on the subject presented to the Institution of Civil Engineers in London. In 1936 he transferred to Messrs. Murex Welding Processes, Ltd. and, as their construction engineer, was responsible for the design and construction of new factory buildings at Freezywater, Waltham Cross, Herts, England, the main building alone being a 1,000-ton, all-welded structure covering four and one half acres. This work inspired two papers presented by him to the Institute of Welding.[4,5] The Freezywater factory finished, Mr. Needham, in 1939, opened a private practice in London as consulting engineer for structural welding; with the imminence of World War II, the occasion was not favorable. He was, however, invited to inquire into the then recent collapse of the Hasselt Bridge over the Albert Canal in Belgium, an all-welded structure of the Vierendeel type; and, in an article in *Engineering*,[6] he gave a critical review of the known facts and made an examination of the design of the bridge.

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[2] "Strauss Bascule Bridge over the Bann River, Ireland," by E. S. Needham, *Engineering* (London), Vol. 118, 1924, pp. 758–759.

In 1940 he went to Turkey to represent Sir Alexander Gibb and Partners, Consulting Engineers, Westminster, London, and he remained there on fortification works in the Dardanelles until 1943. On his return to England, he was for a short time in charge of a Ministry of Supply establishment for training welders, of which the country was then in great need. Then, in 1944, with the invasion of Europe by the allied forces impending, he accepted a post with the U. S. Army as a civilian engineer with the assimilated rank of Major. After a brief period at U. S. Army headquarters in London, he moved over to France shortly after the allied invasion began, and was concerned in the restoration of port facilities, and later with Rhine bridging problems. The strain, however, proved to be great and toward the end of the year he was invalided back to England. Soon, as was usual with him on his return from trips overseas, he plunged into the professional activities of the engineering societies in London, and it was actually while he was making a contribution to the discussion of a paper at the Institution of Civil Engineers, London, that he suddenly collapsed and shortly afterward died without regaining consciousness. He thus died "in harness," as doubtless he would have wished had the choice been his, for he was exceptionally devoted to his profession, and was a frequent contributor to the discussions at meetings of the leading institutions. It was perhaps fitting that his last act should have been that of contributing to a paper on bridge construction, a subject in which he was well versed.

He always maintained close contact with his Colombian friends, some of whom visited at his home. Those who were privileged to join his family circle were charmed by it. Mr. Needham always seemed to fit into it, on his return from a trip abroad, just as though he had never been away.

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[3] "Analysis of a Vierendeel Truss by Moment Distribution and Deformeter Methods," by E. S. Needham and L. A. Beaufoy, *Journal*, Inst. C. E., Vol. 10, 1939, pp. 253–272.

[4] "Design and Construction of a Large All-Welded Factory," *Transactions*, Inst. of Welding, Vol. I, 1938, pp. 139–186.

[5] "Design of Knee Joints for Portal Frames," paper presented to the North Eastern (Teesside) Branch of the Inst. of Welding, London, on April 17, 1940.

[6] "The Hasselt Bridge Collapse," *Engineering* (London), Vol. 147, 1939, pp. 395–397.

His stay in Turkey, during World War II, had endeared him to it. He thought it a country of great promise, and was looking forward to going back after the war to open a practice there. On his return from Turkey he enthusiastically availed himself of courses in the Turkish language given under Turkish Government auspices at the Turk Halkevi in London.

He was a remarkably self-contained man and in general would go out on a social occasion only if there were a scientific or engineering flavor to it. Then he would welcome and enjoy it. He was keen on the game of patience, a habit probably acquired during his trips abroad, and was a competent pianist who sought and found intense pleasure in the relaxation that music provides.

He is survived by his widow; two children, James Christopher and Alicia Margaret (Néna); a brother, John; and a sister, Eleanor.

In addition to his membership in the Society, Mr. Needham was a member of the Institution of Civil Engineers and the Institute of Welding, both in London.

Mr. Needham was elected an Associate Member of the American Society of Civil Engineers on November 27, 1917, and a Member on July 6, 1931.

CHARLES MARSHALL NEELD, M. ASCE[1]

Died December 23, 1945

Charles Marshall Neeld was born on December 8, 1875, at Banksville, near Pittsburgh, Pa., the son of Capt. John R. and Harriett (Robinson) Neeld. After preparation in the public schools and the Pittsburgh Academy, he entered the Western University of Pennsylvania (later the University of Pittsburgh) in Pittsburgh, from which he received the "degree in Civil Engineering" in 1896.

For the next two years he was draftsman and designer with the Fort Pitt Bridge Works in Pittsburgh, after which he was sales engineer, designer, and vice-president with the Columbia Bridge Company of Pittsburgh (later the Columbia Steel Shafting Company).

In 1902 Mr. Neeld was married to Alice McNeilly, to whom three children were born—Charles R., Alice (Mrs. H. D. McMurray), and Ruth (Mrs. A. W. Baisler). Mrs. Neeld died in 1923 and two years later Mr. Neeld was married to her sister, Roberta, who, with his son and two daughters, survives him.

From 1900 to 1911 he was with McClintic-Marshall Construction Company of Pittsburgh, as, successively, designer and estimator, manager of erection, superintendent of shops, contract manager, and, for five or six years, general manager of the Pittsburgh plant which had a capacity of 8,000 tons of fabricated steel per month. The Pittsburgh and Lake Erie Railroad bridge over the Ohio River at Beaver, Pa., and the Panama Canal lock gates, together with hundreds of important and similar projects, were constructed under his direction.

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[1] Memoir prepared by a Committee of the Pittsburgh Section, consisting of Vernon R. Covell, Clarke K. Harvey, and James C. Jordan, Members, ASCE.

From 1911 to 1934 he was engaged in the construction business as president of the C. M. Neeld Construction Company (later the Neeld Construction Company) of Pittsburgh. The company designed and constructed many bridges, reservoirs, and buildings, and conducted coal stripping operations. Outstanding examples of his work were the relining of the Eden Park Reservoir at Cincinnati, Ohio; the construction of a reinforced concrete bridge in Wilkes Barre, Pa., spanning three important railroad yards and many tracks, at a cost of $300,000; the reconstruction and double-decking of the Havre de Grace (Md.) highway bridge over the Susquehanna River (approximately a mile long, involving change of elevation of spans to make the runoffs, the reconstruction of the draw span and change of the machinery—all without interfering with traffic at a cost of $450,000); the reconstruction of the Chester-East Liverpool interstate highway bridge spanning the Ohio River at East Liverpool, Ohio, with a new, lightweight, permanent floor, designed and patented by Mr. Neeld; the reconstruction of the West Penn Railways bridges at Wheeling, W. Va.; and the reconstruction of fifteen bridges of the Pittsburgh Railways Company under traffic.

In 1934 Mr. Neeld entered consulting work and later often served as an expert witness in cases involving bridge design, construction, and maintenance. Some of his more important contributions were revision of the layout of the three plants of the Dominion Bridge Company in Montreal, Canada; design and construction of bridges for the State of Maryland; and many engagements with the Pennsylvania Railroad Company.

Mr. Neeld was a lifelong member of the Mount Lebanon United Presbyterian Church in the South Hills of Pittsburgh, and for many years was superintendent of the Sabbath School and an Elder in the church. He always stood for his high ideals and unflinchingly for what he believed to be right.

He was a registered professional engineer in Pennsylvania.

Mr. Neeld was elected a Member of the American Society of Civil Engineers on October 7, 1908.

JAMES CLARENCE OGDEN, M. ASCE[1]

DIED SEPTEMBER 10, 1945

James Clarence Ogden, the son of John W. Ogden and Susanna (Rhoads) Ogden, was born at Springfield, Pa., on September 21, 1867. He took a special course in civil engineering followed by one year of postgraduate work at the University of Pennsylvania, Philadelphia.

Following completion of his college course, in 1887, he was assistant to the superintendent of construction of the Atlantic Refining Company, Philadelphia, where from June, 1887, to November, 1888, he was in charge of laying out and constructing pipe lines—designing and in charge of erecting buildings, tanks, and stills. From November, 1888, to May, 1889, he was engineer in charge of field surveys with Joseph Young, engineer and contractor. From May, 1889, to March, 1893, he was mill and shop inspector for the Pittsburgh Testing Laboratory, part of the time in Philadelphia checking details of bridges. In March, 1893, Mr. Ogden entered the employment of the Robert W. Hunt Company. From 1893 to 1902 he was mill and shop inspector in charge of the Philadelphia District. From 1902 to 1923 he was manager of the New York office in charge of mill, shop, and erection inspection, preparation of specifications, and design and inspection of safe deposit vaults. On June 28, 1923, when the company was incorporated, Mr. Ogden was elected a director. On July 24, 1930, he became vice-president; and, on March 25, 1935, he was elected president of the company. On April 17, 1945, he was made chairman of the board and held that position to the time of his death.

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[1] Memoir prepared by W. L. Cooper, Pres., Robert W. Hunt Co., New York, N. Y.

During 1914 to 1918 Mr. Ogden directed the inspection of large quantities of ordnance and materials of all kinds for war purposes used by the Allied Nations.

He had extensive experience with foreign engineering contracts and made several visits to Europe in that connection.

Throughout his career he played an important part in obtaining improvements in specifications and methods of production and construction of engineering materials and equipment.

Mr. Ogden had high standards of personal integrity and retained the respect and loyalty of all who came in contact with him.

He was a member of the American Society for Testing Materials, the American Society of Metals, the Union League Club of Chicago, the Bankers Club of New York, and the Plainfield Country Club. He was, for many years, a resident of Plainfield, N. J.

On June 1, 1893, Mr. Ogden was married to Elizabeth Morton Lewis who died on February 24, 1930. He is survived by his daughters, Edith (Mrs. Monroe H. Long) and Elizabeth (Mrs. Will McLain); and by four grandchildren

Mr. Ogden was elected a Member of the American Society of Civil Engineers on October 3, 1911. He became a Life Member in January, 1938.

ARTHUR KERSHAW OLSEN, M. ASCE[1]

Died November 17, 1944

Arthur Kershaw Olsen was born at Salt Lake City, Utah, on February 17, 1904. He was the son of Haakon Kershaw Olsen and Fredricca (Rystrom) Olsen. After his early education in the public schools of Salt Lake City, he attended the University of Utah also in Salt Lake City, from which he was graduated in June, 1929, with the degree of Bachelor of Science in Civil Engineering. In 1935 he was awarded the degree of Bachelor of Laws by the LaSalle Extension University in Chicago, Ill.

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[1] Memoir prepared by A. Diefendorf, M. ASCE.

Prior to his graduation, from July, 1925, to May, 1926, he was employed as a rodman in the division engineer's office, Chicago, Milwaukee, St. Paul and Pacific Railroad Company, at Deer Lodge, Mont., on service, acting as instrumentman on a four-man party. From May, 1926, until May, 1929, he was with the Iowa Highway Commission in Ames, Iowa, on construction of approximately fifty miles of concrete paving (including grading, culverts, and bridges) as instrumentman and inspector, responsible to the resident engineer for proportioning, mixing, and placing concrete for paving slabs and structures.

After he completed his engineering course, Mr. Olsen was employed for four years (from 1929 to 1935) as county highway engineer for Union County at Creston, Iowa, on construction and maintenance of a 750-mile highway system (expenditures, $130,000 annually, supplemented by grants of funds, material, labor, and equipment from the Public Works Administration and the Works Progress Administration). He had full charge of administration and engineering (under him were from 50 to 400 men), and was responsible to a board of supervisors and the State Highway Commission for surface design, construction, and maintenance. In 1930 he reorganized the county highway department to conform to the new highway law.

From February, 1935, to December, 1941, Mr. Olsen served as general manager and engineer, for the Creston (Iowa) city water works, on the rehabilitation and reorganization of the water system. He was engineer in charge of design and construction of a $200,000 improvement program, involving earth dams, concrete reservoirs, modernization of the filtration plant, an elevated tank, and extensive main replacement. He was also general manager in charge of operation and maintenance, responsible to the board of trustees for administration (including the payment of interest and the retirement of $269,000 worth of bonds).

In December, 1941, he was made assistant chief of operations, Construction Section, Special Engineering Division, Panama Canal, on the construction of a third set of locks (estimated to cost $350,000,000). He was responsible to the chief engineer for coordination of all construction operations, procurement and delivery of materials, cost estimates, engineering reports, construction planning, review of all plans and specifications, and the supervision of safety.

In August, 1943, he joined the United States Army and was commissioned a Captain, serving as a public works and utilities specialist in the division of military government. He first reported to Camp Custer, Mich., and then went to Yale University in New Haven, Conn., for military government training. He was sent overseas in January, 1944. When he died, he was in Alsdorf, Germany, installing a sanitary water system.

Concerning his work, Maj.-Gen. L. S. Hobbs said the following of Captain Olsen in awarding him, posthumously, the Bronze Star Medal:

> "Captain Olsen distinguished himself by outstanding performance of duty as a public utilities specialist during a difficult period of military operations. His soldierly attributes and demonstrated fortitude in the face of handicaps have been decided factors in successfully accomplishing in a superior manner all tasks assigned him. His coolness under fire, de-

> votion to duty, and loyalty to his leaders have contributed to the combat achievements of his unit and are worthy of emulation by his associates."

Captain Olsen also received the Purple Heart on January 22, 1944.

On August 7, 1926, in Iowa City, Iowa, he was married to Lora Bernice Duncan. He is survived by his widow; a son, Arthur K. Olsen, Jr.; his mother; and a sister, Mrs. Lucy Augustson.

Captain Olsen had the faculty of drawing people around him. He had a great sense of humor and was well liked by every one who met him. The men who worked with him had a great deal of admiration and respect for him and they found him always ready and willing to listen to their problems and help out wherever he was able.

While Captain Olsen resided at Creston, he was president of the Rotary Club and the Chamber of Commerce, chairman of civilian defense, district leader of the Boy Scouts, and a member of the First Presbyterian Church. He belonged to Scabbard and Blade, Theta Tau, and Phi Alpha Upsilon.

Captain Olsen was elected a Member of the American Society of Civil Engineers on May 18, 1943.

SHEPARD BROWN PALMER, M. ASCE[1]

Died August 17, 1945

Shepard Brown Palmer, the son of Benjamin H. and Eliza (Brown) Palmer, was born in Norwich, Conn., on January 23, 1871. He went to the local schools and was graduated from the Norwich Free Academy. He attended Worcester Polytechnic Institute at Worcester, Mass., being graduated in June, 1894, with the degree of Bachelor of Science in Civil Engineering.

Following his graduation he was employed by Charles E. Chandler of Norwich, in a general engineering capacity. In 1897, the firm of Chandler and Palmer was formed; he was a partner in the firm until the time of his death. In the early days of the firm, Mr. Palmer had charge of design and construction of sewer systems for Taftville, Jewett City, Danielson, and Wauregan—all in Connecticut. He also made preliminary surveys for trolley lines from Norwich to Jewett City and from Norwich to Westerly, R. I. For many years he acted as engineer for the Water Department of Norwich; and under his direction the dam and pipe line to Stony Brook reservoir was constructed in 1909. In 1926, the Deep River reservoir and pipe line was constructed for the department; this represented an investment of about $1,000,000. As city engineer for many years he had charge of important municipal work including the widening of many important and congested city streets.

During World War I, Mr. Palmer acted as engineer for a Federal Housing Project at Newport and at Jamestown, in Rhode Island.

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[1] Memoir prepared by Benjamin H. Palmer, M. ASCE.

He was considered an authority in the hydraulic field, and he served the state of Connecticut for many years as a member of the board having supervision of dams and structures. All dams and hydraulic structures in eastern Connecticut were inspected and approved, as to detail, by him.

A man with many civic, as well as business interests, Mr. Palmer gave much time to outside affairs. He served as director and chairman of the board of The Dime Savings Bank. He was a member of the Advisory Board of The Hartford-Connecticut Trust Company, director of The Norwich Water Power Company, and president of The New London County Mutual Fire Insurance Company.

Mr. Palmer served as president of The Norwich Free Academy Board for fifteen years, member of the Backus Hospital Advisory Board, and president of The Johnson Home. He was elected a member of the Connecticut Society of Civil Engineers in 1895, was president in 1910, and made an honorary member in 1935. He was a member of the Central Baptist Church of Norwich.

He is survived by his widow; two daughters, Sybil C. Palmer (Mrs. George P. Bellos) and Clara Palmer (Mrs. Harold Kranz); and a son, Shephard B. Palmer, who is also an engineer.

Mr. Palmer was elected a Member of the American Society of Civil Engineers on April 4, 1911. He became a Life Member in January, 1942.

HARRY THOMAS PATERSON, M. ASCE[1]

Died January 1, 1943

Harry Thomas Paterson was born in Newport, Ky., on September 7, 1874. He was the son of William Alexander and Alice (Clemens) Paterson. After graduation from high school in his native city he studied engineering from 1892 to 1895 at Rose Polytechnic Institute in Terre Haute, Ind., where he became a charter member of the Beta Upsilon Chapter of the Sigma Nu fraternity. Ill health forced him to leave college before graduation, but in 1896–1897 he took a special course at the Missouri School of Mines and Metallurgy at Rolla, Mo. From November, 1895, to January, 1898, with exception of the period spent at Rolla, he was employed in Colorado Springs, Colo. Here Mr. Paterson worked in the office of E. A. Sawyer, city engineer, first as draftsman (November, 1895, to March, 1896) and then as superintendent of construction on eighteen miles of ditch for placer mining (May, 1896, to November, 1896). From May, 1897, to December, 1900, he was principal assistant, assistant engineer, assistant superintendent, and chief engineer of the Colorado Springs Rapid Transit Railway Company. From January 1, 1901, to January 1, 1902, he was deputy county surveyor, El Paso County, Colorado, and from February, 1902, to October, 1902, draftsman, leveler, and transitman for the Denver and Rio Grande Railroad Company.

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[1] Memoir prepared by George H. Maurice, M. ASCE.

In October, 1902, he entered the United States Government Service, and remained in government employment, under various departments, until his death. His first assignment, under the U. S. Reclamation Service, was as assistant engineer, afterward as topographer until May, 1903. During this time he traveled more than two hundred miles along the Colorado River in a row boat, from a point near the present Boulder Dam, in Arizona-Nevada, to Yuma, Ariz. Mr. Paterson was also in charge of parties on surveys near the towns of Reno and Fallon in Nevada, and from May to October, 1904, of road construction in Shoeshone Canyon, Wyoming. From November, 1904, to January, 1907, he was resident engineer in charge of location and construction of 114 miles of laterals and ditches and the construction of Hondo Reservoir, New Mexico. Mr. Paterson had the responsibility for the completion of the Hondo Reservoir after May, 1905, and design and construction of three headgates and takeouts on the distribution system. The month of January, 1907, and subsequent winter months he spent in a tent camp near Malta and Great Falls, Mont., with the thremometer frequently reaching 30° below zero. He decided to seek a warmer climate, and in July, 1907, obtained a transfer to the U. S. Engineer Department with headquarters at Wilmington, N. C. His first assignment was the survey and construction of the Inland Waterway from Neuse River to Beaufort Harbor, including triangulation, survey, location, plans, estimates, and specifications for the waterway and a steel drawspan highway bridge. In 1910 he was placed in charge of eleven river and harbor works in the northern portion of the district, with headquarters at New Bern, N. C. With the closing of the New Bern office in 1932, he was recalled to the Wilmington office, and resided in Wilmington until his death on January 1, 1943.

On December 11, 1909, he was married to Loula Burroughs McConchie, of Fredericksburg, Va. Mr. Paterson had three sons, Harry T., Jr., William Alexander, II, and Robert Clemens, who, with Mrs. Paterson, survive him.

The outstanding feature of Mr. Paterson's nonprofessional life was his love of fraternal activities and the resulting associations and interests. His brilliant Masonic career began on November 10, 1900, when he became a member of El Paso Lodge No. 13, F. and A. M., at Colorado Springs, Colo. His early work in sparsely settled sections of the West prevented much activity in Masonry until he moved to North Carolina in 1907. Since then he had advanced through all degrees to the Thirty Third having filled many honorable offices in all of them, and was Grand Master in North Carolina in 1938. He became a member of Oasis Temple in 1911 and was a charter member of Sudan Temple in 1914. He was also active in the Benevolent and Protective Order of Elks which he joined in 1901, and was past president of the North Carolina State Elks Association. He was a member of the North Carolina Society of Engineers, which he joined in 1939, and a member of the Wilmington Engineers Club.

Among his associates he was known as a man of wide experience, marked ability, and high integrity. He was modest and retiring and of a cheerful disposition, making friends easily. He was a Presbyterian and had long been a member of the First Church in Wilmington. His early religious training was influenced by his paternal grandmother who had been a childhood playmate of David Livingston, the great Scottish explorer and missionary.

Mr. Paterson was elected an Associate Member of the American Society of Civil Engineers on February 3, 1904, and a Member on May 4, 1909.

JASPER MARION PHILLIPS, M. ASCE[1]

Died December 22, 1944

Jasper Marion Phillips was one of a family of nine children. He was the son of Thomas Wiley and Quintinia (Lane) Phillips and was born in St. Clair, Hawkins County, Tenn., on September 25, 1863. He received his high school education at St. Clair, and was graduated in 1880. He attended the military academy at Knoxville, Tenn., in 1881 and the Tusculum College at Jonesboro, Tenn., in 1882.

He then spent a year wandering about in the western states doing miscellaneous work, and, in 1884, he returned to Tusculum, entering the employ of the city engineer. About a year later he was appointed city engineer. In 1886 he worked for the Charleston, Cincinnati and Chicago Railway, first as leveler, then as transitman, and, in 1888, in charge of some location and construction. In 1889 he entered private practice in Johnson City, Tenn. Then from 1890 to 1892 he was with the Embreeville (Tenn.) Freehold Land and Iron Company. He left the Embreeville Freehold Land and Iron Company to serve with various southern and midwestern railways and, from 1899 to 1902, inclusive, he was chief engineer of the Louisiana Railway and Navigation Company. During this period he was in charge of the construction of 160 miles of railway. He then served on location and reconnaissance, and was in charge of construction on the Chicago Great Western, The Tidewater, and the Alberta and Great Western railways. He was employed by the Great Northern Railway in 1909, remaining most of the time until May, 1913. During this period he was engaged on location, construction, and reconnaissance, including surveys for extensive line revisions. Before and after working with the Great Northern Railway he was, at various times, with the firms of Waddell and Hendrick, and Howard and Ash. In 1915 he was engaged in exploration and reconnaissance for a railway in the Dominican Republic and Haiti, and then did miscellaneous railway work in the south and west. In 1920 he practically retired to make his home with his father and mother in Wichita, Kans.

During the next few years he worked at a number of short jobs but spent most of the time with his parents. He was extremely fond of them and devoted his time to them until they died in 1928 and 1929. In 1930 he moved to Port Isabel, Tex., where he lived until just before his death. During his residence in Port Isabel he did some local work, but his health was failing to such an extent that he lived in practical retirement.

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[1] Memoir prepared by Oscar S. Bowen, M. ASCE.

Mr. Phillips spent a roving life. He was passionately fond of the out of doors, and a large part of his active life was spent in camp, mainly on railway work. In his wanderings he covered a good part of the United States. He was a great reader, had a splendid memory, was very fond of people, made friends wherever he was, and, with his wide travel experience, he was a most interesting companion. He contributed liberally to the training of younger men working under his supervision. He took a great interest in, and was very popular with, young people, enjoyed associating with them and helping them with their problems. Mr. Phillips never married and is survived by only one near relative, a brother, H. G. Phillips. He was a Thirty-Second Degree Mason and was very active in the lodge.

Mr. Phillips was elected a Member of the American Society of Civil Engineers on July 2, 1913.

STEUART PURCELL, M. ASCE[1]

Died April 18, 1946

Steuart Purcell, the son of William H. Purcell of Dublin, Ireland, and Mary Steuart Purcell of "Holly Hill," Anne Arundel County, Maryland, was born in Dublin on February 22, 1878. Although his father was Irish, through his mother, he was a direct descendant of the Lords Baltimore. When he was three years old, his parents came to the United States and settled in Baltimore, Md. He was educated in the public schools of Baltimore, being graduated from Baltimore City College in 1897. In 1901 he received the degree of Civil Engineer from Cornell University in Ithaca, N. Y.

Shortly after graduation, he entered the employ of the Factory Mutual Fire Insurance Company of Boston, Mass., where he remained for approximately one year. Subsequently, he was employed as office engineer by the S. D. Brady Engineering Company, mining engineers of Clarksburg, W. Va., where he remained for a few months. In the early part of 1903 he entered the employ of the Pennsylvania Railroad Company, which was at that time just beginning its improvement program in Washington, D. C. He served as levelman, transitman, and as assistant to the resident engineer on the construction of tunnels, bridges, freight yards, and brick and steel warehouses until the completion of this program in 1910. In November, 1910, he entered the employ of the City of Baltimore, where he was employed in various capacities, from draftsman to chief engineer. His service to the City of Baltimore covered a total of thirteen years.

Just prior to World War I, he served as superintendent of assembly for the Gathmann Ammunition Plant in Texas, Md. In June, 1917, he enlisted in the U. S. Army Engineers, serving for two years as Captain. After the war, he was in the Reserve Officers Corps, earning his commission as Major.

In 1927 he was appointed a member of the Public Service Commission of Maryland, on which he remained until his death—for the last five years as chairman. No other member of the Maryland commission ever served for such a length of time.

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[1] Memoir prepared by Paul L. Holland, M. ASCE.

Few men have been appointed to regulatory commissions who have had more varied and suitable engineering training than had Steuart Purcell, who was known throughout the state as "Major." Few men have enjoyed greater respect and esteem from utilities and their customers alike.

On September 14, 1901, in Baltimore, Major Purcell was married to Julia Eichelberger. He is survived by his widow; two daughters, Penelope Purcell and Mary (Mrs. G. Richard Purdy); two sisters, Eleanor Purcell and Mary Steuart Purcell; and a granddaughter, Cary Glenn Bruton. A daughter, Hannah Cary (Mrs. C. Glenn Bruton), died in 1929, and a son, William H. Purcell, died in 1945.

Mr. Purcell was elected an Associate Member of the American Society of Civil Engineers on October 10, 1916, and a Member on October 21, 1924.

RAMON RAMOS CASELLAS, M. ASCE[1]

Died August 3, 1944

Ramon Ramos Casellas was born on October 6, 1880, at Manati, Puerto Rico. He was the son of Judge Juan Ramon Ramos and Rosa (Casellas) Ramos. He attended local schools and later the Institute of Secondary Education of Puerto Rico at San Juan, Puerto Rico, from which he was graduated with the degree of Bachelor of Arts in 1898.

Since Puerto Rico had become a possession of the United States in 1898 as a result of the Spanish-American War, to become proficient in the language of his new country, Mr. Ramos attended Matawan Military Institute at Matawan, N. J., during 1900 and 1901, where he studied special subjects in the English language. In 1905 he received the degree of Civil Engineer from Rensselaer Polytechnic Institute at Troy, N. Y.

Soon after graduation he was appointed city engineer of Arecibo, Puerto Rico, continuing in this position until 1915. For that city he built the first concrete streets in Puerto Rico. In 1915 Mr. Ramos became connected with the Insular Department of the Interior where he was, successively, resident engineer in charge of road maintenance for the district of Arecibo, chief of the Division of Roads and Bridges, and chief of the Division of Municipal Works.

His executive ability in government service attracted attention; and, in 1920, he was asked to take charge, as general manager, of the interests of Mario Mercado e Hijos, an association owning a sugar mill, Central Rufina, and a large area of some of the best sugar cane lands in the southern part of Puerto Rico. During his management, which lasted until 1927, the organization prospered considerably. From 1927 to 1930 he was general manager for Benitez Sugar Company and J. J. Benitez e Hijos, two concerns owning and operating a sugar mill and an extensive sugar cane estate in Vieques, a small island just east of the main island of Puerto Rico.

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[1] Memoir prepared by Rafael A. González, M. ASCE.

Management of the two mills and the attached cane lands gave Mr. Ramos the opportunity to acquire a thorough knowledge of the sugar industry. This, coupled with his intense devotion to the interests confided to him, no doubt led to his selection as secretary of the Association of Sugar Producers of Puerto Rico, which has as members most of the sugar mill owners of the island. He held this position from 1930 until his death. During this period the sugar industry, by far the most important source of income to Puerto Rico, was attacked strongly by a large section of the public and became subject to increasing restrictive legislation aimed at obtaining an equitable distribution of the sugar income between the mill owner, the cane planter, and the laborer and, thus, at helping to solve the distressing economic situation of the great mass of the people. In 1941 the movement culminated in the passage by the legislature of the "Land Law of Puerto Rico," whereby the ownership by a sugar mill of more than 500 acres of land was declared illegal. It became Mr. Ramos' duty to present the case and to argue in favor of the sugar industry, in the press, at public hearings, before the insular government, and in Washington, D. C. This he did nobly, with intense loyalty, great courage, and thorough knowledge of the economic and operating phases of the industry as well as of its relative position in the economic and social structure of the island. However, his was a losing battle, as the wave of social and economic reforms moved forward. A prolonged illness gradually undermined his health during the last few years.

Mr. Ramos was noted for his personal honesty and straightforwardness. He was kind to his fellow engineers, particularly to the young members of the profession whom he liked to encourage.

He was a member of the Sigma Xi and Phi Sigma Alpha fraternities, Sociedad de Ingenieros de Puerto Rico, American Chemical Society, Society of American Military Engineers, American Society for the Advancement of Science, International Association of Sugar Technologists, and Association of Sugar Technologists of Puerto Rico.

In October, 1906, he was married to Mary Alice Plunkett, who survives him, together with a son, Ramon. Another son, Jorge Eduardo, died on February, 24, 1934.

Mr. Ramos was elected a Member of the American Society of Civil Engineers on October 10, 1927.

JOHN MARSHALL RIBBLE, M. ASCE[1]

DIED JUNE 23, 1945

John Marshall Ribble was born at Culpepper, Va., on May 9, 1900. He was the son of Dr. Frederick Goodwin Ribble, Dean of the Bishop Payne Divinity School, and Caroline Stribling (Marshall) Ribble, great-granddaughter of Chief Justice John Marshall of the United States Supreme Court. The antecedents of both families came to America before the Revolutionary War and, in both instances, settled in Virginia.

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[1] Memoir prepared by Howard L. King, M. ASCE.

John Ribble received his grammar school and high school education in Virginia. He attended Virginia Military Institute at Lexington and was graduated with the degree of Bachelor of Science in 1921. He received the degree of Civil Engineer in 1928, after completion of the requirements for the practice of engineering and after the submission of a thesis based on the subway construction on which he had been engaged.

His first five years after leaving college were spent on highway construction in his native state where he worked first as a laborer, then as an instrumentman with the State Highway Department, and, later, as junior highway bridge engineer with the United States Bureau of Public Roads. Turning from employment under a state or federal body, he took a position as engineer and concrete superintendent with the Robert Martin Company, a Virginia railroad contractor, thereby acquiring some knowledge of the contractor's viewpoint as distinguished from that of the owner.

At Virginia Military Institute, John Ribble had been a classmate of Sam A. Mason, a nephew of Silas B. Mason, the head of a company that had done much construction in Virginia, West Virginia, and Kentucky. Sam Mason later became president of the Silas Mason Company and, in 1926, invited Mr. Ribble to come to New York, N. Y., to take a position as field engineer on a contract to build a section of the new city subway, deep in the rock of upper Manhattan Island.

At the time that this work was nearing completion, the Silas Mason Company and its affiliate, the Mason and Hanger Company, were starting the construction of two subway tubes from the vicinity of St. Paul's Churchyard in lower Manhattan, east on Fulton Street to the river, under the East River and east on Cranberry Street, Brooklyn, to Borough Hall. This was a $22,000,000 contract; much of the work was to be done by the shield method under compressed air. The scope, hazard, and difficulty of the undertaking called for contracting ability and courage. The contractors chose John Ribble as field engineer on the Brooklyn half of the work.

The underriver part of the work was done under compressed air, and, in accordance with the custom of subaqueous work, it was prosecuted twenty-four hours a day, six days a week. The challenge of the magnitude of the work and the exacting standards of engineering and construction that were necessary to secure safety in the work and permanence and quality in the completed structure appealed to John Ribble and he gave his best to the job. He reported directly to George B. Montgomery—he could have had no better teacher. From the early and primitive days of the Hudson and Manhattan tubes, Mr. Montgomery had been a foreman or superintendent on nearly every subaqueous tunnel job under the Hudson River or the East River—he knew the way of a shield as few men ever have.

A construction engineer has to move to new locations as new work presents itself. John Ribble's next task was as superintendent on the construction of a large sewer tunnel in the city of Philadelphia, Pa.—also a compressed-air job. After this, he went to Boston, Mass., where the Silas Mason Company had obtained a contract to build a vehicular tunnel under the harbor connecting the north end of the City of Boston with the City of East Boston, Mass. On this work, he stepped over from the field of engineering to that of actual supervision of the construction force; he was the superintendent on one of the three shifts, this job, like most tunnel jobs, having been carried through all the hours of the day. Each tunnel job has its own peculiar quality; the Boston-East Boston tube is the only compressed-air tunnel job on record where the excavated material was conveyed on a belt from the face back through the locks to the construction shaft.

In 1933 and 1934, there was an interlude in tunnel work and Mr. Ribble returned to Virginia as chief engineer for the state on the program of the Civil Works Administration, working with Dean James A. Anderson, M. ASCE, of Virginia Military Institute, who was on leave from the Institute to guide the state's participation in the federal government's program.

Returning to New York in 1934, Mr. Ribble rejoined the Mason and Hanger Company. This company was about to build the south tube of the Lincoln Tunnel through the silt that underlies the Hudson River. When the excavation of this tunnel was completed, he went to work as chief engineer for the Rosoff Tunnel Corporation on the construction of six miles of the Wards Island Sewer.

In 1939, he returned to the employ of Mason and Hanger. The company was just beginning work on the construction of Merriman Dam near Ellenville, N. Y., an important part of the new Delaware Aqueduct project of the Board of Water Supply of the City of New York. The Merriman Dam undertaking included a rock shaft, one thousand feet deep, two miles of rock tunnel, and a long line of concrete caissons to be sunk under compressed air to form a cutoff wall under the rolled earth dam. Mr. Ribble began as job engineer and was later advanced to the position of job manager. After the attack on Pearl Harbor, Hawaii, on December 7, 1941, the construction of the dam was slowed down and later stopped altogether to give preference and priority to war projects. In September, 1942, Mr. Ribble who had had a commission in the Reserve joined the Army as a Captain of Engineers. In June, 1943, he was promoted to the rank of Major and in September, 1944, he was made a Lieutenant Colonel.

Captain Ribble was assigned to the Army base in Anchorage, Alaska, where he was Assistant to the Chief of Operations and was concerned with the building of airfields, docks, and other facilities for the use of the rapidly increasing armed forces in Alaska and the Aleutians. The secrecy of these undertakings forbade his writing home much about his activities. Ordinarily a good letter writer, he was restrained from telling much of a story. The long winter darkness and the incessant fog were more of a hardship than the cold. In March, 1945, he was assigned to the Office of the Chief of Engineers in Washington, D. C., where he was engaged in the handling of engineering equipment for lend-lease to our allies. Colonel Ribble died suddenly on June 23, 1945, at Glendale, Markham, Va.

John M. Ribble's engineering career did not encompass many years but he played a leading part on a number of large contracts. Probably his highest ability lay in supervising the work of others in actual construction. When he went into the Army, he had already acquired something of a reputation as a tunnel superintendent. He commanded the respect of those who worked under him because he was courageous and forceful by nature and straightforward in his dealings. He had learned the fundamentals of his chosen field by many years of intimate contact and association with the men who actually did construction work; from their fund of experience he was always ready to learn. He was the possessor of a playful and unobtrusive sense of humor that did not invite attention to itself but made him a good companion.

John Ribble was married to Anne Jeter in Jacksonville, Fla., on September 26, 1941. He is survived by his widow; his brother, Frederick Deane Goodwin Ribble; and four sisters, Mildred S. Ribble, Frances Le B. Ribble, Elsie Sylvester Ribble (Mrs. J. D. McCarty), and Caroline Marshall Ribble (Mrs. J. R. Green).

Mr. Ribble was elected an Associate Member of the American Society of Civil Engineers on November 15, 1926, and a Member on February 13, 1945.

HOLTON DUNCAN ROBINSON, M. ASCE[1]

Died May 7, 1945

Holton Duncan Robinson, the son of Ichabod Harvey and Isabelle (McLeod) Robinson, was born on February 7, 1863, at Massena, N. Y., of Scotch-English ancestry. One of his ancestors was Sir Alexander Mackenzie, a pioneer Canadian explorer, for whom the Mackenzie River was named. The Robinson farm was on the shore of Robinson Bay in the St. Lawrence River. In the bitter winters of the "North Country," with their icy winds and the resulting frozen ears, the boy developed a wiry physique and a hardihood to physical exposure. His one difficulty was a shyness in meeting people; in his childhood days, when his cousins came to visit him, he would hide under the bed. He was outstandingly shy, modest, and retiring; and he carried these characteristics through life.

Following country schooling in the district school and high school at Massena, he studied hard to pass the entrance examinations for St. Lawrence University at Canton, N. Y., from which he was graduated with the degree of Bachelor of Science in 1886. He always retained a certain modesty about his college education, because he specialized in liberal arts and sciences and not strictly in engineering. However, he obtained a sound foundation for his later career.

Mr. Robinson received his start and his direction in engineering through his uncle, George W. McNulty, bridge builder, who was associated with Leffert L. Buck,[2] both Members, ASCE. These men in turn had received their start and training under Col. Washington Roebling on the construction of the Brooklyn Bridge in New York, N. Y. In August, 1886, directly after graduation, young Robinson entered the employ of Buck and McNulty, civil engineers, in New York, as a chainman and rodman, and began studying engineering at home. Until July, 1887, he was employed on the construction of ferry terminals, docks, elevated railroad extensions, etc. He was then sent by his employers, Buck and McNulty, to the Rochester Bridge and Iron Works, at Rochester, N. Y., to learn shop inspection and to inspect the shopwork on a ten-span bridge for the Northern Pacific Railway Company (Columbia River Bridge at Pasco, Wash.). While at Rochester, young Robinson also inspected the shopwork on the Niagara Falls (N. Y.) and Clifton Highway Suspension Bridge (erected in 1888).

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[1] Memoir prepared by D. B. Steinman, M. ASCE.

[2] For memoir, see *Transactions*, ASCE, Vol. LXXIII, September, 1911, p. 493.

He returned to New York City in May, 1888, and entered the employ of Mr. Buck—the association of Buck and McNulty having been terminated. He was employed on computations and drawings for various bridges, including the plans for the Driving Park Avenue Bridge over the Genesee River at Rochester. This bridge (still standing after fifty-six years) included a main spandrel-braced, three-hinged arch of 416-ft span. In July, 1889, he went to Rochester for Mr. Buck and took charge of construction of the arch bridge as assistant engineer. His duties included masonry inspection and shop inspection of the structural work at the Rochester Bridge and Iron Works. Mr. Robinson returned to New York in March, 1890, and was employed on the design of bridges and other structures until the end of that year. Then he was sent to Suspension Bridge (near Niagara Falls), N. Y., to take charge of repairs on the stiffening truss of the Niagara Railway Suspension Bridge built by John A. Roebling,[3] M. ASCE, and opened to traffic in 1855. He served about six weeks on the repairs and then, in February, 1891, he returned to New York City and entered the employ of the Broadway and Seventh Avenue Cable Railway Company, on surveys and design of special trackwork under Mr. McNulty, who was chief engineer. In September, 1891, Mr. Robinson was offered a position as draftsman and assistant engineer in the bridge department of the chief engineer's office of the New York Central and Hudson River Railroad Company. In August, 1895, he resigned to accept a position as chief draftsman, under Mr. Buck (the chief engineer), on the New East River Bridge Commission, preparing the plans for the Williamsburg Bridge in New York.

In 1899 he was appointed assistant engineer in charge of cable construction on the Williamsburg Bridge, and remained in that position until July, 1903, when he was appointed engineer in charge of all construction work on the bridge by the Commissioner of Bridges for the Greater City of New York, Gustav Lindenthal,[4] Hon. M. ASCE. Mr. Robinson remained in charge of the construction of the Williamsburg Bridge until it was opened for highway traffic in December, 1903, and thereafter until March, 1904, when he was transferred and placed in charge of design and construction of the Manhattan Bridge, reporting to the chief engineer of the Department of Bridges of the City of New

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[3] For memoir, see *Transactions*, ASCE, Vol. 98, 1933, p. 1614.

[4] For memoir, *ibid.*, Vol. 105, 1940, p. 1790.

York, O. F. Nichols,[5] and his consulting engineer, R. S. Buck, both Members, ASCE. He remained in charge of this work until July, 1907, when he resigned to accept the position as chief engineer of the Glyndon Contracting Company, a subsidiary of the Carbon Steel Company, contractor for the construction of the main cables of the Manhattan Bridge in New York. He designed the footbridge, machinery, and plant used in the construction of the four 21-in. cables, which were larger than any previously built.

During the three years with the Glyndon Contracting Company, Mr. Robinson made a proposed suspension bridge design for the crossing at Quebec, Que., Canada, working for the Dominion Bridge Company of Montreal, Que., Canada. He made plans for a combined aqueduct and highway suspension bridge for the proposed crossing of the Hudson River at Storm King, and also was employed part of the time (for the contractors) on the construction of a section of the new Catskill Aqueduct, including one rock and earth tunnel, 6,150 ft long and a rock tunnel, 400 ft long.

He resigned as chief engineer of the Glyndon Contracting Company in June, 1910, and took the contract on his own design to build a highway suspension bridge with a main span of 400 ft and side spans of 100 ft each for his native town, over the Grasse River at Massena Center, N. Y. When the lowest bid was $60,000, Mr. Robinson stated that he could build the bridge himself for $40,000—and he did it. He completed this work in January, 1911, and returned to New York. In 1911 and 1912 he estimated and bid for various contracts, and received the contract for the installation of nine miles of 1,000,000-cm weather-proofed feeder cable and accessories on the Manhattan Bridge; this work was completed in the summer of 1912.

In October, 1912, he entered the employ of Mackenzie and Mann as engineer of design of the Mount Royal Tunnel and Terminal Company. In the spring of 1913 he was also appointed consulting bridge engineer of the Canadian Northern Railway Company on the Eastern Lines from Port Arthur, Ont., to St. Johns, Que., Canada. He resigned from the service of Mackenzie and Mann in the spring of 1914 to take charge of the work of third-tracking and other improvements for the contractor on the Manhattan Elevated Railway in New York City. This work included the replacement of the old Harlem River double-track bridge by a new four-track, double-deck bridge, all floated into place on scows and lowered by sand jacks devised by Mr. Robinson—a method that had never before been used. He completed this work at the end of 1916.

During World War I, in October, 1917, Mr. Robinson entered the employ of the Bureau of Yards and Docks, U. S. Navy, as a supervising engineer on plant extension work for war purposes, and remained with the department until December, 1919.

In the spring of 1920, when the writer was head of the engineering school at the College of the City of New York, he received a telephone call from a man who modestly introduced himself as H. D. Robinson. In the subsequent conference, Mr. Robinson invited the writer to join him in preparing a competitive design (in a world-wide competition) for the proposed bridge at Florianópolis, Brazil. This joint effort, which later (1926) culminated in successful achievement, was the beginning of an association that lasted through the years. With no agreement in writing, the simple and flexible relationship developed into a lasting friendship and partnership, based on mutual confidence and esteem.

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[5] For memoir, see *Transactions*, ASCE, Vol. LXI, December, 1908, p. 564.

In 1920 Mr. Robinson prepared for Daniel E. Moran,[6] and William H. Yates, both Members, ASCE, the plans for a 705-ft suspension bridge over the Rondout River, at Kingston, N. Y. This was the first suspension bridge designed to carry a concrete floor and to have continuous (hingeless) stiffening trusses. The writer was retained independently by Mr. Yates to calculate the stresses and the sections. In 1920 and 1921 Mr. Robinson was in charge of construction as consulting engineer to the contractor on this work.

During 1921 the writer opened his own engineering office and, subsequently, invited Mr. Robinson to share it. Previously Mr. Robinson's office had consisted of a drafting table in his home, where he did all computing and drafting himself. He would solve difficult three-span catenary problems by suspending a fine chain against the wall and measuring the ordinates.

Effective July, 1922, Mr. Robinson was appointed consulting engineer on cable construction by the Joint Commission on the Delaware River Bridge (between Camden, N. J., and Philadelphia, Pa.). He was asked whether it would be possible to build cables 30 in. in diameter so the construction could be simplified from a four-cable layout to a two-cable design. The largest cables (on the Manhattan Bridge) previously built had been only 21 in. in diameter. Mr. Robinson replied that 30-in. cables could be built and that he was prepared to build them. In August, 1923, after the plans were completed, he resigned to work for the contractors, the Keystone State Construction Company, first on the preparation of the bid and then as consulting engineer on the construction of the cables. All the equipment for the cable construction (including the footbridges, storm cables, spinning machinery, squeezing machines, and wrapping machines) was designed in the Robinson and Steinman office. On this job were used, for the first time, the "flat-band seizings" invented by Mr. Robinson to shorten the time required for the construction of large bridge cables. In March, 1926, at a joint meeting of the Franklin Institute and the Philadelphia Section of the Society, he presented a paper on "Construction of the Cables of the Delaware River Bridge."[7] This was Mr. Robinson's first technical paper; he suffered excruciatingly from stage fright and the experience so unnerved him that he vowed he would never repeat it.

From 1923 to 1924 Mr. Robinson was retained as consulting engineer on the cable construction of the Bear Mountain Bridge at Bear Mountain, N. Y., and on the preliminary plans (by Ralph Modjeski,[8] M. ASCE, and Mr. Moran) for the Mid-Hudson Bridge at Poughkeepsie, N. Y.

Beginning in 1920, the firm of Robinson and Steinman completed many interesting and notable bridge engagements during the twenty-five years of partnership, including the design and construction of the Florianópolis Bridge in Brazil; the General U. S. Grant Bridge over the Ohio River at Portsmouth, Ohio; the Mount Hope Bridge in Rhode Island; the Sparrows Point (bascule) Bridge in Baltimore, Md.; the bridges of the Cochabamba-Santa Cruz Railway in Bolivia; the Bayhead (movable draw) Bridge at Point Pleasant, N. J.; the Grande Mère Bridge across the St. Maurice River in Quebec Province, Canada; the Waldo-Hancock Bridge across the Penobscot River in Maine; the Verona Island Bridge in Maine; the St. Johns Bridge in Portland, Ore.; the Sky Ride and Observation Towers at Chicago, Ill.; the Henry Hudson Bridge in New York, N. Y.; the Thousand Islands International Bridge (five bridges) over the St. Lawrence River; the Sullivan-Hutsonville Bridge over the Wabash River between Indiana and Illinois; the Madison Avenue Bridge at Elmira, N. Y.; the Marine Parkway Bridge (continuous truss approach spans) at Rockaway, N. Y.; the Paerdegat Basin Bridge in Brooklyn, New York; the Deer Isle Bridge in Maine; and the Charter Oak Bridge over the Connecticut River at Hartford, Conn. The firm was engaged on the strengthening or reconstruction of the Steubenville Bridge, the East Liverpool Bridge, the Wheeling Bridge, the Marietta Bridge, and other bridges over the Ohio River; the Hackensack River Bridge in New Jersey; the Southside Bridge over the Kanawha River at Charlestown, W. Va.; the McGregor Bridge at Manchester, N. H.; and many others. It was also retained on the construction of the Ambassador Bridge at Detroit, Mich.; the Triborough, the George Washington, and the Bronx-Whitestone bridges at New York, N. Y.; the Rip Van Winkle Bridge over the Hudson River at Catskill, N.Y.; the Transbay Bridge at San Francisco, Calif.; the Lions' Gate Bridge at Vancouver, B. C., Canada; the Maumee River Bridge at Toledo, Ohio; the Ile d'Orleans Bridge over the St. Lawrence River in Canada; and others. In 1923–1924 the firm was retained to prepare an alternate design for the Sydney Harbor Bridge in Australia; in 1927 for the Cologne-Mühlheim Bridge over the Rhine River in Germany; and in 1928, for the Lillebaelt Bridge in Denmark. The firm served as consultants on the design of the Topeka Avenue Bridge in Topeka, Kans.; the Coolidge Memorial Bridge at Northampton, Mass.; and the bascule bridge at Bilbao, Spain. It also had numerous engagements on inspection, strengthening, traffic analysis, and valuation of various bridges.

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[6] For memoir, see *Transactions,* ASCE, Vol. 103, 1938, p. 1840.

[7] "Construction of the Cables of the Delaware River Bridge," by H. D. Robinson, *Journal of the Franklin Institute,* Vol. 201, January-June, 1926, p. 735.

[8] For memoir, see *Transactions,* ASCE, Vol. 106, 1941, p. 1624.

Mr. Robinson's inventions in the field of suspension bridge construction include the (hydraulically-operated) cable-squeezing machine, the (electrically-operated) cable-wrapping machine, the flat-band seizings, and a simplified type of cable anchorage. Jointly with the writer, Mr. Robinson developed the Florianópolis type of suspension bridge and a method of preventing aerodynamic instability. As the result of Mr. Robinson's inventions and improvements in the art of cable construction, the time required for building suspension bridges has been enormously reduced. When the Brooklyn Bridge was built, during Mr. Robinson's boyhood, it took twenty-one months to construct the cables. (Each contained only 900 tons of wire.) The required time was progressively reduced, for successively larger cables, to seven months for the Williamsburg Bridge, and four months for the Manhattan Bridge; and in

1939 the Bronx-Whitestone Bridge cables, with 2,000 tons of wire per cable, were completed in only two months.

In 1926 his alma mater, St. Lawrence University, conferred on Mr. Robinson the degree of Doctor of Science, in recognition of his character and achievements. In his modesty, Mr. Robinson always disclaimed this degree as "not earned" and protested when his friends addressed him as "Doctor."

From 1935 to 1937 Mr. Robinson was retained as consultant on the roadway construction for the Williamsburg, Manhattan, and Queensborough bridges over the East River, New York, and during 1944-1945 (to the time of his death) he was consultant on the physical survey of the condition of the Brooklyn Bridge preparatory to its roadway reconstruction.

Even in his last years, Mr. Robinson was active, agile, and fearless in his outdoor work on bridges. He would climb on high steelwork or walk the cables of a suspension bridge with greater ease than most younger engineers. In 1941, during the investigations that followed the Tacoma Narrows (Washington) Bridge disaster, he was retained by the insurance companies, and made a personal examination of the cables of the wrecked structure. Although seventy-eight years old at the time, he calmly walked out over the 17½-in. cables, each 5,900 ft long and 450 ft high at each tower, to examine the condition of the wires and to cut out samples of the wire at midspan. His feat was rendered more difficult and hazardous by the fact that the hand ropes in the main span had been wrecked.

On a bridge construction job, out in the field where the actual erection was being done, Mr. Robinson was happier than anywhere else. He knew the bridgemen and the steel erectors by name, and they knew him. In fact, he felt more at home in their company than in the company of public officials, bankers, and clients. He was thoroughly democratic, and enjoyed the affection and esteem of all who came to know him. He loved his work. A bridge inspection was, to him, a holiday. He would accept such engagements without pay, for the pleasure of doing them; and he would grant the use of his patents for a nominal sum, for the satisfaction of adding another suspension bridge to his list. He was professionally connected with the construction of almost every notable suspension bridge built during his lifetime. This fact was his chief pride.

In 1912, when plans were being prepared for a new suspension bridge over the Rhine River at Cologne, Germany, the authorities called on Mr. Robinson, in the United States, to secure the benefit of his advice and judgment on the most efficient design for that structure, which was completed in 1915. In 1927, when another notable suspension bridge was being planned to cross the Rhine River at Cologne, Robinson and Steinman were retained to prepare a design for that bridge. In 1938 Mr. Robinson made a trip to Sweden to give his consulting services on the construction of an interesting suspension bridge north of the Arctic Circle. The bridge at Florianópolis is the largest bridge in South America and the largest eyebar suspension bridge in the world. The Transbay Bridge connecting San Francisco with Oakland, completed in 1937, at a cost of $78,000,000, the world's greatest bridge achievement, was constructed with the aid of Mr. Robinson's inventions and the firm's consulting services.

Mr. Robinson was enthusiastic about flying. His first air experience was in 1931 when the two partners were taken by Tex Rankin, northwest flying ace, in a stunting plane (with open cockpit) to circle the towers and to fly over and under the newly completed St. Johns Bridge at Portland. This was probably the first time bridge construction received its final inspection from the air, and (on a gusty, rainy day) it was a thrilling experience. Thereafter, Mr. Robinson missed no opportunity to take a plane for his professional trips around the United States. He made numerous transcontinental flying trips in connection with his professional engagements on the Pacific Coast, and he looked forward to them with boyish delight.

Holton Duncan Robinson had many of the finest traits of character, which endeared him to all who had the pleasure of working with him. He was kind, considerate, and generous—almost to a fault. Above all, he was characterized by that genuine innate modesty which is the mark of a truly great man. He was at his office, working, on his eighty-second birthday. The next day he did not feel well—his first illness in a long and active life. Three months later, on May 7, 1945, he passed away.

On June 15, 1893, Mr. Robinson was married to Frank Brown. He is survived by his widow; a daughter, Mary Olivia Robinson; a son, Allan McLeod Robinson; and two granddaughters, Barbara Jean Robinson and Ann Catherine Robinson.

Mr. Robinson was elected a Junior of the American Society of Civil Engineers on March 1, 1892; an Associate Member on January 3, 1894; and a Member on January 7, 1913. He became a Life Member in January, 1929.

ELMER WAYLAND ROSS, M. ASCE[1]

DIED MAY 20, 1944

Elmer Wayland Ross, the son of Erastus Wellington Ross and Hannah (Paine) Ross, was born in Glocester, R. I., on March 3, 1861. His father was a descendant of the Ross family which was one of the early families in Connecticut. His mother was a descendant of Stephen Paine who came to Hingham, Mass., from Great Ellingham, England, on the good ship *Dilligent* in 1638. Later Stephen Paine was one of the men who, under the leadership of Elder Newman, founded Rehoboth, Mass., in 1643.

Erastus Ross was an authority on the subject of weaving and was connected with various textile mills in Rhode Island and Pennsylvania.

Elmer Wayland Ross was educated in the town schools of Warren and Smithfield, R. I., and in the Mowry and Goff Preparatory School in Providence, R. I. He was graduated from the latter school and in June, 1879, entered the City Engineer's Office in Providence as a student engineer.

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[1] Memoir prepared by Robert L. Bowen, Assoc. M. ASCE.

In March, 1881, Mr. Ross resigned from the City Engineer's Office and was employed by the Conant Thread Company to do engineering work on the construction of a new mill, boilerhouse, and auxiliary buildings by which it was greatly expanding its plant.

In January, 1882, Mr. Ross returned to the City Engineer's Office in Providence where he was destined to spend the remaining years of his professional career. He served in the various departments until May, 1887, when he was appointed assistant engineer in the Bridge Department under William D. Bullock,[2] M. ASCE. He continued in this position for thirty-six years and then in April, 1923, upon the death of Mr. Bullock, he was appointed engineer in charge. Two years later, in April, 1925, he resigned from the City Engineer's Office and retired to private life.

Providence is located at the head of Narragansett Bay. It is bounded on the east by the Seekonk River and is traversed by the Moshassuck and the Woonasquatucket rivers which, flowing together at the center of the city, form the Providence River. These topographic conditions result in a relatively large amount of bridge and harbor work, all of which was designed, constructed, and maintained by the Bridge Department of the City Engineer's Office.

Mr. Ross designed and supervised the construction of twenty fixed-span steel bridges, two swing drawbridges, and two stone arch bridges. A third swing draw span, which he designed, was constructed after he resigned. He also designed considerable lengths of granite masonry river wall, some wharf work, and the first section of the Municipal Dock at Fields Point, which includes a heavy granite quay wall and auxiliary structures.

After retiring to private life Mr. Ross moved to North Foster, R. I., to live on his farm, the ancestral home of the Paine family. Here he took up the work of commercial apple growing and in so doing carried out plans toward which he had been working since boyhood. In this venture he was joined by his son, Donald. They progressively enlarged the orchards which had been started in earlier years, and at the time of Mr. Ross's death were producing, not the largest crops in the state, but some of the finest.

Mr. Ross was in good health, in full possession of all his faculties, and was a delightful companion to the end of his days. He had a keen appreciation of life: He thoroughly enjoyed living and possessed the rare faculty of awakening that enthusiasm in those with whom he was in contact.

In his early days he found time to study art, with particular attention to painting. He was a bicycle enthusiast when the high bicycle was in style and toured many of the eastern states on his "wheel." Genealogy and beekeeping were also among his hobbies.

Mr. Ross was a member of the Boston Society of Civil Engineers, the Roger Williams Association of Rhode Island, and the First Baptist Church in Providence. He was also a member and trustee of the North Foster Baptist Church.

He was married to Adelaide Virginia Drowne, a great, great granddaughter of Dr. Solomon Drowne, famous as a surgeon in the Continental Army during the Revolutionary War and later Professor of Materia Medica and Botany at

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[2] For memoir, see *Transactions*, ASCE, Vol. 88, 1925, p. 1356.

Brown University in Providence. She died in 1941. Surviving are his daughters, Margaret Drowne (Mrs. Lakes Gill), Esther Winifred (Mrs. William H. Sweetland, Jr.), and Virginia Paine (Mrs. Godfrey Goff); and his son, Elmer Donald Ross.

Elmer Wayland Ross was an able engineer, a good citizen, and above all a true Christian gentleman. His courtesy, his thoughtfulness of others, and his eminent fairness were outstanding characteristics.

Mr. Ross was elected a Junior of the American Society of Civil Engineers on March 5, 1890; an Associate Member on June 1, 1892; and a Member on April 3, 1906.

ARTHUR VALENTINE RUGGLES, M. ASCE[1]

Died October 21, 1945

Arthur Valentine Ruggles was born at Clinton, Mass., on January 23, 1883, the son of Marcus Morton Ruggles and Mary Elizabeth (Burdett) Ruggles. He was a descendent of George Ruggles who was in Boston, Mass., by 1632.

Mr. Ruggles received his early education at Clinton and was graduated from Dartmouth College, Hanover, N.H., in 1902 with the degree of Bachelor of Science.

During his summer vacations from college, Mr. Ruggles received his first engineering experience when he worked as a rodman with the Metropolitan Water Board of Boston on the construction of the Wachusett reservoir. After his graduation from college in 1902, he went to Broklyn, N.Y. From July, 1902, to May, 1904, he served, for short periods of time, as draftsman with the American Bridge Company, E. B. Hyde Company (map publisher), and the Brooklyn Edison Company, all of Brooklyn, and *Engineering News* of New York, N.Y. From May, 1904, until February, 1906, he was associated with Hazen and Whipple, consulting engineers of New York, on the design of filtration plants and on other water supply work.

In March, 1906, Mr. Ruggles became an assistant engineer with the Board of Water Supply, City of New York. His first work with the Board was on the design of the Catskill Aqueduct and Eastview filters. In 1910 he was appointed first assistant to the section engineer on the construction of the Bryn Mawr Steel Pipe Siphon and Yonkers Pressure Tunnel. In December, 1911, he was made assistant engineer for the Department of Water Supply, Gas, and Electricity, City of New York, on the design of a proposed filtration plant at Jerome Park Reservoir for the Croton supply. Mr. Ruggles left the Department of Water Supply, Gas, and Electricity in June, 1913, and for six months was draftsman for the Public Service Commission, 1st District, State of New York.

In January, 1914, Mr. Ruggles went to Cleveland, Ohio, as assistant engineer with the Division of Water, Department of Public Utilities. His first work there was on the design and construction of the $4,000,000 Division Avenue filtration plant and pumping station. Before its completion he was promoted to the position of engineer of construction for this work.

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[1] Memoir prepared by Ralph E. Boyce, Civ. Engr., Board of Water Supply, City of New York, N. Y.

From September, 1917, to January, 1919, Mr. Ruggles served as Captain in the Engineering Corps of the United States Army. He was with the American Expeditionary Forces in France for one year. For nine months of this time he was first assistant to Maj. J. B. Hawley, in charge of all water supply and sanitation in Base Section 1 at St. Nazaire.

In February, 1919, Mr. Ruggles returned to Cleveland, serving until January, 1925, as engineer of construction and surveys, Division of Water, Department of Public Utilities, and as Commissioner of Water from July, 1922, to February, 1924. During this period as the chief civil engineer he planned, and completed the design and construction of, the Baldwin Fairmount Project (including the 130,000,000 gal covered Baldwin Reservoir, the 165 million gal per day rapid sand Baldwin filtration plant, the 300 million gal per day Fairmount pumping station and several million dollars worth of water mains). The total cost of the project was $15,000,000.

For a short time early in 1925, Mr. Ruggles was associated with R. F. MacDowell, M. ASCE, consulting engineer, Cleveland. From June, 1925, to December, 1927, he was with the United States Cast Iron Pipe and Foundry Company of Burlington, N.J., on sales and engineering work. In February, 1928, he came to New York as civil engineer for the Community Water Service Company.

From October, 1928, until April, 1936, Mr. Ruggles was assistant to the secretary of the American Water Works Association, New York. While serving in this position, he compiled an index to the *Journal* and other publications of the American Water Works Association from 1881 to 1933. During this time he also acted as executive assistant to the chairman of the American Standards Association's Sectional Committee A-21 on Specifications for Cast Iron Pipe and Fittings, of which committee he was one of the most active members. He remained active on the work of the committee until his death.

In November, 1936, Mr. Ruggles returned to the Board of Water Supply, City of New York as assistant engineer designer. From June, 1938, until his death, he was assistant hydraulic engineer of the Water Bureau in the New York Office of the Public Service Commission, State of New York.

Mr. Ruggles was a member of the New England Water Works Association. On September 28, 1943, the Association awarded him the Dexter Brackett Memorial Medal for 1942 for his paper,[2] "Determination by Statistical Methods of the Average Service Life of 6-Inch Cast-Iron Water Mains in Brooklyn, New York." This paper is one of the few on this subject which is one of the bases in the estimating of depreciation of water works properties. Mr. Ruggles was also a member of the Thayer Society of Engineers (Dartmouth College), the Chi Phi fraternity, the Masons, the National Sojourners, and the American Legion.

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[2] "Determination by Statistical Methods of the Average Service Life of 6-Inch Cast-Iron Water Mains in Brooklyn, New York," by A. V. Ruggles, *Journal* New England Water Works Assn., September, 1942, p. 361.

Those who knew Mr. Ruggles will always remember him for his never failing courtesy, his kindly manner, and his absolute dependability. He never spared himself in his efforts to help others, with the result that associates and subordinates alike knew him to be their friend.

In 1902, he was married to Marion Chellis Nye, and is survived by her.

Mr. Ruggles was elected an Associate Member of the American Society of Civil Engineers on April 30, 1912, and a Member on March 9, 1920.

WILFORD BATES RUSSELL, M. ASCE[1]

Died May 28, 1945

Wilford Bates Russell, the son of Wilbur C. and Roah (Shiffler) Russell, was born on May 30, 1898, at Amboy, Mich. After completing his elementary and high school education at Litchfield, Mich., he entered the College of Engineering at the University of Michigan, at Ann Arbor, and was graduated in 1920 with the degree of Bachelor of Science in Civil Engineering.

After his graduation Mr. Russell entered the employ of the Fargo Engineering Company, at Jackson, Mich., as a draftsman, designer, and surveyor. While affiliated with this firm he was in charge of surveys for the Consumers Power Company on the St. Joseph River and at Niles, Mich. He was also assistant superintendent of construction on the Alcona Dam and Power House built for the same utility.

In June, 1924, he went to Syracuse, N. Y., where he joined David R. Cooper in conducting surveys and making economic analyses of proposed hydraulic engineering projects for the Niagara, Lockport and Ontario Power Company along the Moore and Beaver rivers in the Adirondack Mountains. In January, 1927, Mr. Russell entered private practice as a consulting engineer. In this capacity he continued his field surveys and reporting for the Niagara, Lockport and Ontario Power Company until the job was completed in August, 1928.

Returning to the midwest he accepted a position as engineer with the R. D. Baker Construction Company of Royal Oak, Mich., where he was placed in charge of the estimating and construction of sewers, highways, and dams. Mr. Russell remained with this organization until January, 1933; and then returned to private practice and was engaged by the Peerless Cement Corporation to conduct an appraisal of its plants and properties at Detroit and Port Huron, Mich. In November, 1935, he joined the Peerless organization as chief engineer in charge of all plant engineering and labor relations. In these matters he revealed such outstanding ability that, in April, 1942, he was appointed to the office of vice-president. Quietly and without ostentation he accomplished important tasks. He devoted his fine mind to the betterment of his fellow workers. This was tangibly evidenced by the adoption of the Peerless plan of adjusted compensation, of which he was the author; by his intensive studies of a company pension plan; and by a more effective and satisfactory relationship between the employees and management.

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[1] Memoir prepared by F. N. Menefee, M. ASCE.

Many outside activities held his absorbed interest. Mr. Russell was an active member of the Society and the Michigan Engineering Society (for several years on its Board of Directors and president during 1943). He was a director and chairman of the Committee on Promotion, on Audits, and on Relationships with the National Society of Professional Engineers, and a member of the Committee on Post Graduate Engineering Studies as well as a trustee of its fund. To the success of each of these organizations he made substantial contributions. He also belonged to the Red Run Golf Club, the Birmingham Golf Club, and the Economic Club of Detroit.

In addition to his professional and business activities, Mr. Russell displayed great interest and talent in other fields. His deep love and appreciation of beauty found expression in the collecting, restoring, and refinishing of fine antique furniture. A splendid craftsman, he took great pride in restoring rare pieces and was considered a local authority in this field. His collection of antique furniture was a source of never-ending enjoyment to him. His other hobbies included stamp collecting and golf.

To those with whom he worked Mr. Russell showed a strength of character and depth of mind not found in ordinary men. His unquestionable ability to perform the tasks to which he set his mind well, his complete honesty in all matters, and his profound respect for honorable conduct—all inspired great trust and confidence among his associates. Capable, efficient, and keenly analytical, he was nevertheless shy and self-effacing. However, so warm and friendly was his personality that it won for him the love and admiration of all who were privileged to know him.

On October 15, 1921, in Ann Arbor, Mich., Mr. Russell was married to Edith Apfel. He is survived by his widow; a daughter, Phyllis; his mother; a sister, Helen Russell; and two brothers, Walter C. and Robert L. Russell.

Mr. Russell was elected a Member of the American Society of Civil Engineers on March 11, 1940.

GEORGE HENRY SAGER, JR., M. ASCE[1]

DIED APRIL 23, 1942

George Henry Sager, Jr., the son of Arthur N. Sager and Elizabeth (Scott) Sager, was born on January 8, 1895 in Yellville, Ark. In 1901 he moved to St. Louis, Mo., where he attended grammar and high schools. He continued his education at Phillips Academy at Andover, Mass. (scientific course), and the Sheffield Scientific School of Yale University at New Haven, Conn. (civil engineering). In 1938 he also studied political science, administration, and city management at the University of Louisville at Louisville, Ky.

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[1] Memoir prepared by H. E. Whitaker, Vice-Pres. & Chf. Engr., Ford, Bacon & Davis, Inc., New York, N. Y.

Mr. Sager's first employment was with James Stewart and Company, Inc., New York, N. Y., as a junior engineer. In January, 1917, he went into private practice in Kentucky, surveying and mapping for oil companies and others.

During World War I Mr. Sager joined a Red Cross unit formed in Louisville, serving as an ambulance driver in France and later as Sergeant in charge of construction for the U. S. Army Base Hospital No. 40, which included construction of a 60,000-gal activated sludge, sewage-disposal plant near Southampton, England.

From 1919 to 1923 Mr. Sager was field manager of the Allen-McLean Oil Company at Frankfort, Ky. During the following year he was in private practice as general manager of the Sager Construction Company, Monroe, La., supervising the construction of dwellings and two small industrial plants. In 1923 he was employed by the George F. Hardy Company, New York, N. Y., as assistant resident engineer on the construction of the Brown Paper Mill, West Monroe, La. In December, 1924, he joined the forces of Ford, Bacon and Davis, Inc., engineers of New York. From 1924 to 1932 Mr. Sager served in the following capacities on many important commitments:

(*a*) Until December, 1925—field engineer, assistant resident engineer, and superintendent in the construction of a 30,000-kw gas-steam-electric station for the Louisiana Power and Light Company, Sterlington, La.

(*b*) Until April, 1927—assistant engineer to vice-president in charge of surveys, foundation tests, design approach plans, and start of construction on the Lake Pontchartrain Bridge for the New Orleans Pontchartrain Bridge Company, New Orleans, La.

(*c*) Until August, 1928—resident engineer on a 60,000-kw gas-steam-electric station addition to the Louisiana Power and Light Company plant.

(*d*) Until June, 1930—assistant engineer, main office of Ford, Bacon and Davis, Inc., in connection with business reports, valuations, appraisals, estimates, specifications, and design.

(*e*) Until July, 1932—resident engineer in charge of general construction of a 60,000-kw coal-steam-electric station for the West Virginia Hydro-Electric Company, Alloy, W. Va.

(*f*) Until November, 1932—resident engineer on appraisals, estimates, and new business in West Virginia and Kentucky.

In 1932 Mr. Sager went into private practice, and for the following year was affiliated as a consulting engineer with the Blanton Stone Company, Frankfort.

In September, 1933, he entered the government service as engineer-examiner for the Federal Emergency Administration of Public Works, Louisville, and was promoted rapidly to the positions of assistant state engineer, state engineer, acting state director, and state director, successively.

After the liquidation of the state offices of the Public Works Administration in 1937, Mr. Sager became associated with the City of Louisville as engineer, and secretary-treasurer of the City Planning and Zoning Commission; and, as secretary-treasurer, technical adviser, and executive director of the Municipal

Housing Commission, on the construction and operation of two municipally owned slum clearance housing projects, costing approximately $10,000,000.

In June, 1940, he returned to the firm of Ford, Bacon and Davis, Inc., and was engineer in charge of the construction of a large chemical plant for the Carbide and Carbon Chemicals Corporation at Texas City, Tex. During the period from 1940 to 1942 Mr. Sager concurrently was in charge of another very important project—a tin smelter—built by the Defense Plant Corporation at Texas City.. He also acted as consulting engineer for the City of Galveston (Tex.), on improvements to water supply, which included general supervision of construction, and other engineering activities in the Galveston area.

George H. Sager, Jr., was a member of the Phi Gamma Delta fraternity, the National Society of Professional Engineers, the Kentucky Society of Professional Engineers, and the Kentucky and Texas Sections of the American Society of Civil Engineers, as well as being a registered professional engineer in the states of Kentucky and Texas. He was also a member of the Galveston Country Club and the Rotary Club.

Mr. Sager is survived by his widow, the former Nellie Cheatham Rodman, to whom he was married on November 10, 1917, in Frankfort, and a son, Lt. Arthur Rodman Sager. Another son, the late Lt. George Henry Sager, III, a pilot, was killed in action over Luzon, Philippine Islands, in April, 1945.

Mr. Sager was elected an Associate Member of the American Society of Civil Engineers on August 29, 1927, and a Member on January 13, 1936.

ALBERT LYON SCOTT, M. ASCE[1]

Died March 2, 1946

Albert Lyon Scott was born on June 21, 1878, at Cleveland, Ohio, the son of John H. and Florence Madeline (Davis) Scott. His father was a Baptist minister and at one time a college president, which makes significant Mr. Scott's lifetime participation in religious and educational activities. His early years were spent in Ypsilanti, Mich., Minneapolis, Minn., and Chicago, Ill. In the latter city during his high school years he supported himself by selling newspapers and eggs.

He worked his way through Brown University, Providence, R. I., and was graduated in 1900 with the degree of Bachelor of Arts. Although he was preparing for a career in engineering he felt that too many engineers gave insufficient time to cultural subjects.

Immediately after his graduation, Mr. Scott became an assistant resident engineer for the architectural-engineering firm of Lockwood Greene and Company, then a Boston (Mass.) organization, which specialized in the design of textile mills. All Mr. Scott's business life was spent with this company. He became director in 1903, treasurer in 1905, vice-president in 1920, president in 1926. In 1928 the firm name was changed to Lockwood Greene Engineers, Inc.; the executive offices were established in New York, N. Y., and Mr. Scott continued as president until his death.

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[1] Memoir prepared by David Elwell, Vice-Pres., Lockwood Greene Engrs., Inc., New York, N. Y.

While the design of textile plants has always been a specialty of the firm, during Mr. Scott's career, and considerably due to his stimulation, the company expanded the scope of its work widely through the entire industrial field. He participated in the company's work in an engineering as well as an administrative capacity. From his first employment as assistant engineer at Greenville, S. C., he became resident engineer in charge of construction of a new factory for the American Cigar Company at Richmond, Va.

From 1902 through 1905 Mr. Scott was in charge of the company's southern projects with headquarters at Greenville, having responsible engineering supervision. This included preliminary studies, reports, and cost estimates for cotton mills and water powers, including Yadkin River, Parr Shoals, and Greenville-Carolina Power Company (20,000 hp, 15,000 hp, and 93,500 hp, respectively). He was local supervising engineer in charge of Dan River Power and Manufacturing Company, Danville, Va., and Pacolet Manufacturing Company and No. 5 Mill, Lockhart Mills (mill and water power), both in South Carolina.

In 1905 Mr. Scott was called to headquarters then at Boston, as treasurer of the company; and until 1917, in addition to administrative duties, selected sites and designed and supervised the construction of industrial plants for automobile companies, knitting, woolen, cotton mills, etc.

From 1913 to 1926 he served as his company's representative and general consulting engineer, supervising all engineering projects for the U. S. Rubber Company and its subsidiaries. This work involved the study of plants, new construction, and plant changes and improvements.

An important assignment, in 1931, was a study of operations and value of twenty-four plants of the American Woolen Company, Inc.

In 1933 Mr. Scott was the engineering member of the construction committee for the Cité Universitaire, Paris, France.

His interest in religious and educational affairs is exemplified by the following: He was president of the board of trustees of Riverside Church, New York; secretary of The Corporation, and of the Board of Fellows, Brown University; and a trustee of the University of Chicago, Chicago, Ill., of Spelman College, Atlanta, Ga., and of Lingnan University, Canton, China.

Mr. Scott was a member of the Laymen's Inquiry Commission which wrote "Rethinking Missions." In collaboration with Raymond B. Fosdick[2] he made a study of liquor control systems in the United States, Canada, and Europe in the hope of finding an ideal plan for use in the United States after the repeal of the prohibition amendment.

Mr. Scott was a member of the committee on supplies of the Council of National Defense, 1917; and chief of Supply and Equipment Division, Quartermaster Corps, U. S. Army, from 1917 to 1918. He was also a director for the Textile Research Institute, a trustee for the Textile Museum; and a member of the New York State Society of Professional Engineers, the

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[2] "Toward Liquor Control," by Albert Lyon Scott and Raymond B. Fosdick, Harper & Bros., New York, N. Y., 1933.

Society of American Military Engineers, the American Historical Association and the Delta Upsilon fraternity. He was a member of the Merchants Club, the University Club of New York, and the University Club of Boston.

On June 12, 1906, at Geneva, Ohio, he married Alice May Chamberlin, of that city, who survives him. He is also survived by a daughter, Alice C. (Mrs. Robert H. Dods); and four sons, David H., Albert L., Jr., Richard C., and Robert L. Scott.

Mr. Scott was elected an Associate Member of the American Society of Civil Engineers on November 7, 1906, and a Member on January 11, 1937.

CHARLES SMITH SHELDON, M. ASCE[1]

Died July 25, 1945

Charles Smith Sheldon was born at Lexington, Mich., on October 1, 1870. His father, Wilbur Carlos Sheldon, was born in Vermont and at the close of his service in the Civil War settled in Lexington, where he married Elizabeth Roach. In 1872 the family moved to Marlette, Mich., where Charles was graduated from high school in 1888. When an appointment to the United States Naval Academy in Annapolis, Md., became void because of his failure to meet the visual requirements for entrance, he spent a winter teaching in a district school. He was then employed for about a year on railroad surveys and construction, another year on land and drainage surveys, and two years as assistant to the city engineer of Port Huron, Mich. In 1893, his ability in that field brought him an offer of the position of topographical draftsman with the Chicago and West Michigan Railway Company and its affiliate, the Detroit, Grand Rapids and Western Railroad Company. At that time these railroads were starting to make complete resurveys of their properties. The maps that he prepared in connection with these surveys are still in daily use and serve as an effective monument to his skill.

Not content with being a first-class surveyor and draftsman, he enrolled in 1899 in the College of Engineering of the University of Michigan at Ann Arbor, Mich., and was graduated in 1902. After a year with the Canadian Bridge Company as draftsman and designer, he served the Cincinnati, Hamilton and Dayton Railway for ten years as assistant engineer and bridge engineer. He became bridge engineer for the Pere Marquette Railroad in 1913 and engineer of bridges and structures in 1929, in which capacity he served until his retirement in December, 1940. Between 1913 and 1925, most of the many timber bridges on the Pere Marquette Railway were replaced by permanent structures; and after 1925 the grades at a large number of highway crossings were separated.

Although many bridges in Michigan and Ohio, as well as several car-ferry terminals on the Pere Marquette Railway, attest his ability as a designer and constructor, Mr. Sheldon looked on himself as primarily a maintenance man and took quite as much interest in prolonging the life of a structure, by adequate strengthening and repair, as in making plans for a new one. His work was characterized by his integrity, and a conservatism tempered by readiness to adopt the new when its merit was well proved.

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[1] Memoir prepared by Paul Chipman, M. ASCE.

His work was his chief concern, but throughout a busy life he cultivated other interests. In the realm of sport, he engaged in hunting, trout fishing, and golf. Gifted with unusual manual dexterity, he became an expert cabinetmaker and woodcarver, and furnished his home with many beautiful pieces. He was interested in all branches of natural science and had a very good knowledge of local geology. An engaging conversationalist, his terse and piquant observations were the delight of his associates. Life with Charlie Sheldon was never dull!

In his later years, his other hobbies were largley supplanted by travel and color photography. In these activities he had an interested and helpful companion in his wife, the former Stella May White. She was a schoolgirl sweetheart in Marlette, to whom he was married in 1902. Her death soon after his retirement shattered a dream of spending their declining years together in the search for beauty.

Mr. Sheldon is survived by a brother, Lemuel C. Sheldon; and a sister, May Sheldon Wood.

Mr. Sheldon was elected an Associate Member of the American Society of Civil Engineers on June 7, 1905, and a Member on May 5, 1915.

CARLTON MANSON SOULE, M. ASCE[1]

DIED JUNE 28, 1945

Carlton Manson Soule was born in Boston, Mass., on June 10, 1884, the son of John Manson Soule and Ida Waterman (Bates) Soule. He was graduated from Dartmouth College, at Hanover, N. H., in 1906, with the degree of Bachelor of Science, and from the Thayer School of Civil Engineering, in 1907, with the degree of Civil Engineer.

Following three years spent, successively, with the American Bridge Company at Edgemoor, Del., the Hamilton Bridge Works Company, Hamilton, Ont., Canada, and the J. E. Greiner Company, Baltimore, Md., in 1910 Mr. Soule became a bridge designer and, later, assistant chief draftsman of the bridge department of the Baltimore and Ohio Railroad Company in Baltimore, where he remained until March, 1917. During World War I, his activities were devoted to industrial developments directly connected with the production of chemicals essential for the prosecution of the war against the Central Powers. During this period he was associated with Deverell, Spencer and Company, consulting engineers at Baltimore, Richard K. Meade, consulting engineer at Baltimore, and the War Industries Board.

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[1] Memoir prepared by H. M. Brown, M. ASCE.

From April, 1919, until October, 1925, Mr. Soule was chief engineer of the Spencer Construction Company at Baltimore. This company, under his supervision, constructed many large industrial projects, including a 120-in. plate mill for Bethlehem Steel Company at Sparrows Point, Md., and cement plants for several Portland cement manufacturers. This association developed into the formation, in 1925, of the partnership of Soule and Zepp, consulting engineers, specializing in the design and construction supervision of industrial plants. During this practice, plants for the Pittsburgh Plate Glass Company, Century Cement Company, and others were designed and constructed. As an individual, at this time, Mr. Soule was also associated with J. E. Greiner Company, supervising the design and construction of an $8,500,000 port terminal for the Western Maryland Railway Company at Baltimore. The partnership of Soule and Zepp was dissolved in 1935, when Mr. Soule became director of operations for a district of the Works Progress Administration embracing the city of Baltimore and surrounding counties.

In October, 1938, Mr. Soule became an associate engineer of J. E. Greiner Company, with executive supervision of design. Under his direction many important projects, including $5,000,000 bridge structures over the Susquehanna River at Havre de Grace, Md., and the Potomac River at Dahlgren, Va., were completed. At the time of his death, Mr. Soule was engaged upon the production of construction plans and specifications for the northern portion of a six-lane, divided superhighway between Washington, D. C., and Baltimore.

Mr. Soule's light shone steadfastly and true. Without flash, but with steady brilliance, his friendships were many and warm. His quiet dignity and understanding humor were outstanding attributes of his character, which quietly guided many younger engineers who sought his advice and guidance.

On April 15, 1911, he was married to Mary Ethel Woodrow at Philadelphia, Pa. He is survived by his widow; two sons, John Woodrow Soule and William Carlton Soule; one daughter, Ethel Ida (Mrs. Ralph C. Johnson); and four grandchildren, Carol Ann Johnson, David Soule Johnson, Sara Charlotte Johnson, and Douglas Carlton Soule.

Mr. Soule was elected a Member of the American Society of Civil Engineers on April 10, 1940.

LINFORD SPEARING STILES, M. ASCE[1]

DIED MAY 5, 1943

The sudden death of Linford Spearing Stiles in his seventieth year was a shock to his many friends. He had been construction engineer for the Brooklyn Union Gas Company since 1908, having been employed by that company since 1903.

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[1] Memoir prepared by Harry F. Crofton, an associate of Mr. Stiles, with The Brooklyn Union Gas Co., Brooklyn, N. Y.

Linford Spearing Stiles, the son of John Edward Stiles and Margaret Cecelia (Walsh) Stiles, was born in Philadelphia, Pa., on October 26, 1872.

After receiving his primary education in "the little red schoolhouse" of those days, he completed four years of grammar school before he was fourteen, the eligible age for high school admission. In the interim between graduation and attaining the age of fourteen he took a senior course in the grammar school provided for such students. He attended the Manual Training High School which prepared students for college entrance in scientific courses. One of his high school subjects was elementary electrical engineering. During these years, Mr. Stiles was a grocery boy after school hours in true Horatio Alger style. On one occasion the proprietor was away sick all summer, and he was in charge, going to market at four in the morning, opening the store at seven, and closing at nine in the evening.

An electrical engineer, the father of one of his school chums, procured a job for Mr. Stiles on the erection of a building with markets on the ground floor and ballrooms and assembly halls upstairs. Although only in his teens, he was in charge of a squad on electric wiring. When the building was completed, he was employed in the boiler room, firing the boilers and assisting the engineer.

He had saved money toward paying his tuition at the University of Pennsylvania in Philadelphia; but his family suffered from financial reverses and he had to forego college. Thwarted in his desire for a higher engineering education, a less indomitable youth might have resigned himself to fate. Instead, Mr. Stiles apprenticed himself for four years to the I. P. Morris Company of Philadelphia, going through all the shops, foundries, etc., and finally reaching the engineering offices. He studied mechanical drawing at night in the Young Men's Christian Association. His teacher was the chief engineer of the Morris works, who, no doubt, spotting the dogged insistence of the boy, took him into the engineering offices. Mr. Stiles attended night classes in Spring Garden Institute, Franklin Institute, and Drexel Institute of Technology in Philadelphia and even paid a private tutor to teach him for a few hours in the evenings.

The William Cramp Shipbuilding Company was working at a fast tempo at that time and absorbed the adjoining Morris works. The merger was accompanied by an expansion of the shipyard facilities. On several miles of waterfront along the Delaware River, new dry docks, shipways, foundries, shops, mold lofts, etc., were erected. Mr. Stiles was assistant to the chief engineer on this construction.

When times became dull, shipyards were avid for any kind of work, and Mr. Stiles often recollected with a chuckle the ambulance-chasing tactics employed to obtain business. The *Maritime News* reported the impending arrival of incoming ships at the Delaware breakwater. He would be one of a group from the Cramp Company that went down the bay to the breakwater, boarded the ship, and accosted the chief engineer about needed repairs. Sketches would be made on the spot, and the men returned posthaste to the office to work on the data. Almost by the time the ship docked in Philadelphia, plans, bills of material, and estimates would be ready.

Mr. Stiles was friendly with one of William Cramp's sons, Benjamin H. Cramp, who often spoke of the desirability of a pneumatic machine that would replace the manual tamping of sand against the molds and the cores for the castings. Mr. Stiles worked on this idea after hours and developed and patented such a machine. Mr. Cramp was pleased with it and financed Mr. Stiles' trip through a number of states to introduce the machine to the trade. The effort brought some success and the Cramp Company manufactured a number of the machines. However, since the machine was a labor-saving device, the opposition of the labor unions was an obstacle to commercial feasibility. Mr. Stiles evidently found the inventive field intriguing for he later developed and patented a number of ideas—a collapsible crate, a non-refillable bottle, and a synchronized phonograph and projecting machine which contained the germ of the present sound, motion pictures. In the manufactured gas field he patented a number of improvements on Williamson generators to increase their capacity.

About this time the toss of a coin might have decided whether Mr. Stiles would remain in the shipbuilding business or transfer to other fields. A friend in the Cramp Company, Joseph W. Powell (later Special Assistant to the Secretary of the Navy), who was studying naval architecture and later became president of the Fore River Shipbuilding Company, left to go with Admiral Francis T. Bowles in the U. S. Navy Department. Because of his friend's interest, the Navy Department made Mr. Stiles an attractive offer, but he stayed with the Cramp Company. Shortly afterward, the father of a school friend was instrumental in Mr. Stiles' employment as a draftsman by the United Gas Improvement Company. The United Gas Improvement Company had leased the Philadelphia Gas Works and was replacing the manufacturing and distributing apparatus with modern equipment. Mr. Stiles worked on the design and construction of the Point Breeze and Twenty-Fifth Ward plants. Later he was transferred to the main office on the design of all types of gas plants.

In 1903 he was employed by the Brooklyn Union Gas Company. Although apparently only a "rookie" and filling what were ostensibly a draftsman's shoes, he was doing real engineering work from the start. His basic training in fundamentals was a fine background in the design and construction phases of the gas business and he soon won recognition. Plant extensions were in prospect, and Mr. Stiles could not resist the temptation to lay out a tentative scheme on his own initiative at home. His plan came to the attention of the president of the company, the late Gen. James Jourdan, who was so impressed by its engineering logic that he stopped the preliminary work which had been done and adopted the plan in full. Shortly after this, the Construction Department was created, and Mr. Stiles was appointed construction engineer. During his thirty-five years with the company, manufacturing capacity expanded from seven billion to twenty odd billion cubic feet each year. Five large plants were entirely rebuilt, and in 1929 the largest project of all, a combination coke oven and water gas plant with a capacity of about fifty million cubic feet a day, was completed. His department also designed and contracted for the erection of numerous office buildings, shops, pumping stations, etc.

His work brought Mr. Stiles in contact with many borough, city, and state department heads and with the War Department engineers of the old Harbor Line Board in connection with waterfront improvements, building permits, etc. His contagious enthusiasm cemented lifelong friendships, and his deep fundamental knowledge impressed his associates.

Mr. Stiles gave much of his time to civic matters. In 1930 he moved from Brooklyn, N. Y., to Great Neck, Long Island, N. Y., but his interest in Brooklyn's welfare never ceased. Perhaps his most noteworthy contribution was as chairman of a fact-finding committee in connection with the proposed railroad tunnel from Greenville, N. J., to Bay Ridge, Brooklyn. The late Frederick L. Cranford, M. ASCE, asked Mr. Stiles to accept this assignment, and he worked on it for two years with the officials and engineers of the Pennsylvania Railroad Company and The Port of New York Authority, submitting a report with accompanying cost estimates.

Mr. Stiles was a member of the American Society of Mechanical Engineers; the Engineers' Club of New York, N. Y.; the Brooklyn Engineers' Club (president for two terms); the National Society of Professional Engineers; and the New York State Society of Professional Engineers (director of this society and president of the Kings County Chapter for two terms). He belonged to the Brooklyn Chamber of Commerce, the Long Island Association, the Pennsylvania Society of New York, and the Crescent Athletic Club (later the Huntington-Crescent Club). He was a registered architect as well as a licensed professional engineer and a member of the Long Island Society of Architects and the New York Council of Architects. He was a trustee and chairman of the administration committee of the House of St. Giles the Cripple, a charitable institution.

In 1914 Mr. Stiles presented a paper before the Brooklyn Engineers' Club, describing a gas holder with a capacity of ten million cubic feet and an attendant pumping station which used gas engines as prime movers.[2] In 1926, before the same organization, he read a paper on a coal handling system erected at one of the gas plants.[3] Each of these papers was awarded the Alfred T. White Prize. Also in 1926 he presented a paper before the Society describing a pipe tunnel under the Gowanus Canal in Brooklyn, built by the trench method without the use of compressed air and shield. This paper was awarded the Thomas Fitch Rowland Prize of the Society in 1927.[4] Subsequently, he accepted an invitation to present the tunnel paper [4] before the Franklin Institute in Philadelphia. He confessed it was with a tingle of pride that he appeared before his native folks on that occasion, for he had left Philadelphia to join the Brooklyn Union Gas Company as a young man who had not yet made his way in life.

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[2] "Gas Engines as Prime Movers for Power and Electric Stations, and Their Applications to the Modern Gas Pumping Station," by Linford S. Stiles, *Proceedings*, Brooklyn Engrs.' Club, 1914, p. 189.

[3] "The Installation of a Coal Handling, Screening and Storing System at the Nassau Works of the Brooklyn Union Gas Co.," by Linford S. Stiles, *ibid.*, January, 1926, p. 31.

[4] "Pipe Tunnel Under Gowanus Canal, Brooklyn, New York," by L. S. Stiles, *Transactions*, ASCE, Vol. 90, 1927, p. 444.

In 1900 at Philadelphia, Mr. Stiles was married to Katherine E. Kelly. He is survived by his widow; a son, Linford Edward Stiles; two grandchildren, Linford Edward, Jr., and Elizabeth Anne; and a sister, Adeline M. (Mrs. William Foster).

Mr. Stiles was elected a Member of the American Society of Civil Engineers on May 28, 1923.

GEORGE LEWIS SWENDSEN, M. ASCE[1]

DIED DECEMBER 31, 1944

Louis Peter and Ellen Maria Swendsen, two pioneers of the west, came to the United States in early youth, from Denmark and Wales, respectively, and found their way to Utah via the Emigrant Trail in prairie schooners. Their son, George L. Swendsen, was born at Richland, Utah, on December 21, 1869.

George's early training was on the farms and ranches and in the public schools of that area. He entered Harvard University at Cambridge, Mass., in 1890, and, in 1894, received the degree of Bachelor of Science in Civil Engineering. He returned west to head the Engineering Department of the Utah State Agricultural College, at Logan, where he remained until 1903. During this period, he also engaged in private practice.

The field of his operations, Utah and southern Idaho, was then generally undeveloped or in the pioneer state. Except in the larger towns or centers of population, kerosene lamps and candles provided illumination. Culinary water found its way to the kitchen and bath from open wells, via hand-operated pumps or improvised lifts and the old-fashioned bucket brigade. Sewer systems, septic tanks, and other sanitary conveniences were not yet available.

Agriculture, the then dominating industry, was successful only under irrigation and was limited to areas where water was made easily available by means of simple, and easily constructed canals, aqueducts, and irrigation works. Its further expansion called for the construction of more intricate structures, reservoirs, and dams.

Much of his time, therefore, during that period, was devoted to the design, financing, and construction of irrigation works, culinary water systems, and small hydroelectric power plants. His compensation for this early effort was the satisfaction of rendering service to humanity under trying circumstances rather than from pride in professional accomplishment and monetary return.

Times were "tough" in this "hang-over" period, following a depression then called the "Panic of '93." Money was almost unobtainable and, for the most part, engineering designs had to contemplate the use of available native raw materials, fabricated and placed largely by hand labor. Notwithstanding these improvisions and compromises, necessary under the circumstances, the resulting works are still, with limited expansions and improvements, serving the area in question. Much of the relief from the 1893 panic conditions came from such efforts on the part of the engineering profession as it then existed throughout the west and in many parts of the United States.

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[1] Memoir prepared by Warren G. Swendsen, Assoc. M. ASCE.

In 1903, Mr. Swendsen left the college, and private practice, and accepted a position as district engineer for the U. S. Reclamation Service. He was placed in charge of work and investigations in Utah and southern Idaho. He inaugurated the first construction on what is known as the Strawberry Valley project, the principal structure involving a 20,000-ft tunnel through the Wasatch Range, which serves to divert water from the upper reaches of the Colorado River basin to the Utah lake area. He left the Reclamation Service in about 1908 to engage in hydroelectric power construction, and other work for the Telluride Power Company—which company was building and operating power plants in Colorado, Utah, and southern Idaho. Leaving this company in 1912, he engaged in private engineering practice, with headquarters at Boise, Idaho, devoting his time largely to the design and construction of irrigation work. In 1915, he entered the service of the Utah Power and Light Company, successor to the Telluride Power Company, with headquarters at Salt Lake City, Utah, where he remained until about 1918, when he went to Fresno, Calif., to serve as manager and chief engineer for the Fresno Irrigation District. He held this position until about 1938, at which time he retired and took up his residence at Long Beach, Calif.

He engaged in consulting engineering work until the beginning of World War II. Notwithstanding his age and physical limitations, and, out of a sense of patriotism, he accepted employment in the shipyard industry, having charge of tool inventories. He was employed in this position at the time of his death.

Mr. Swendsen was married in 1896 to Mary Mathews, deceased, and was married to May Douglas Loughney in 1906. He is survived by his widow; a son, Harold Swendsen; two brothers, Myron Swendsen, M. ASCE, and the writer of this memoir; and two sisters, Meda Swendsen and Mrs. R. E. Herrick.

Mr. Swendsen was elected a Member of the Society of Civil Engineers on June 7, 1905.

VALENTINE VASILIEVISH TCHIKOFF, M. ASCE[1]

DIED OCTOBER 7, 1945

Valentine Tchikoff was born in Vladikavkaz, Russia, on October 30, 1883. He was the son of a prominent Russian General, Vassili Tchikoff, and Anna (Shushpahou) Tchikoff. He was graduated in 1909 from the Institute of Ways and Communications of St. Petersburg (later Leningrad), Russia. This institute is considered the best civil engineering college in Russia and is comparable to our finest American universities. He was graduated among the first of his class and his thesis was left to be used as a text for the classes to follow.

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[1] Memoir prepared by R. Burchuk, Secretary to Mrs. Tchikoff, Greenbelt, Md.

Upon his graduation he was connected with the construction of the North Donetz Railway. Following this, and for the next five years, he was employed as assistant and chief engineer in the Russian Ministry of Agriculture in Tashkent, Asia. He was sent to Germany by the Russian Government tc make a survey of the Carl Zeis factory studying stereophotographic instruments. He then went to British India for a six-month stay. There he visited Punjab, United Provinces, and Calcutta where he studied water power and reclamation projects, including the economic developments. From May, 1915, to April, 1916, he was chief engineer in Kherson, Russia, and while there he published a report[2] of 208 pages on the feasibility of irrigation of a large area adjacent to the lower part of the Dnieper River. The investigation was made in connection with utilization of the power of a large hydroelectric plant, "Dnieperstroy," then under design and later constructed (destroyed in World War II by the retreating Russian Army). Besides various alternative designs of irrigation systems, the publication includes a complete description and consideration of economic, climatological, hydrogeological, and agricultural factors of this semi-arid region. Among the interesting features of this report, a "Diagram of Probable Droughts" can be mentioned. The importance of this diagram is explained in another of Mr. Tchikoff's articles.[3]

Mr. Tchikoff was always a great admirer of the methods and progress of American engineering and did much to promote friendly relations among the engineers of Russia and America. It was always a dream and secret ambition to come to America for study and research. Because of his interest in furthering the career of fellow engineers and students, both in America and in Russia, he did much to introduce the American methods and technique to Russian students. To further carry out his hope for a more cultural relationship between the two, he required all his assistants to study English so that they would better understand American engineering methods. He also founded and was the first editor of the American Engineering and Industry publication which is now owned and published by the Amtorg Trading Corporation. This publication helped to familiarize the Soviet Union with the latest developments in the field of American engineering and industrial practice. It was devoted to the promotion of closer technical and trade relations between the United States and the Soviet Union.

Mr. Tchikoff came to the United States in 1916, as a member of the Russian Military Purchasing Committee, and was assigned as chief inspector for the committee at the Remington Firearms plant in Bridgeport, Conn., and Hoboken, N. J. After completing this work he took a postgraduate course at the University of California, in Berkeley, studying water power, reclamation, and history of education. At the time he prepared a report for the United States Department of State on water power in Russia. This was written for the American members of the Peace Commission at the request of

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[2] "Irrigation of Taurida Region, South Russia," by V. V. Tchikoff, Russian, Dept. of Agriculture, Bureau of Soil Conservation, Petrograd, Russia, 1916.

[3] Method of Constructing a Diagram to Show Probable Droughts," by V. V. Tchikoff, *Engineering News-Record,* September 18, 1919, p. 554.

O. C. Merrill, M. ASCE, who was the chairman of the American Committee of the World Power Conference.

The outbreak of the Revolution in Russia, while Mr. Tchikoff was in America with his wife and baby, prevented his return. Following this, Mr. Tchikoff entered private practice for several years as a consulting engineer. Later he did engineering service with the Arcos Trading Corporation of London (New York Branch) and the Amtorg Trading Corporation of New York in the technical departments. As chief of this department he interpreted plans, specifications, and other data prepared in Russian for the use of engineers associated with Col. Hugh L. Cooper, M. ASCE, who was employed by the government of the Union of Soviet Socialist Republics (U. S. S. R.) as a consulting engineer for construction of the Dnieper River power development.

In 1929 he returned to Russia for two years as assistant engineer to the late Arthur P. Davis (Past-President, ASCE), chief consulting engineer for the General Cotton Committee. This assignment consisted of a review of all important reclamation projects in the U. S. S. R., Turkestan, and Transcaucasia. In addition to plans for utilizing power resources in individual drainage basins of rivers, he specialized in the growing of cotton and other crops under irrigation. In conjunction with this work he wrote "Irrigation in Agriculture"[4] for use in the schools in Russia. At this time also we mention the many publications, books, and articles that Mr. Tchikoff wrote in both English and Russian on various engineering subjects. He wrote several discussions in *Proceedings*, ASCE, and *Engineering News-Record* published several of his articles. The *Asia* magazine published one of his important articles on cotton in Turkestan.

Mr. Tchikoff, always the student and pioneer, felt the need of further study and went to France for a year. There he studied water power, reclamation, and economics. He then returned to America and spent some time in writing. One of the reports[5] (written and published for the Soviet Union Middle Asia Water Economy Service) which Mr. Tchikoff wrote after working with Mr. Davis was printed in Russia.

In 1935, he entered public service with the Farms Security Administration as project analyst. His work consisted of making reports on the resettlement projects, water rates, homestead associations, maintenance of structure, and disposition of projects. Later in this same bureau he worked in the capacity of a social economist.

He transferred to the Federal Public Housing Administration and it was there that his health started to fail. He served as senior building and maintenance engineer. However, because of his failing health, he came to Greenbelt, Md., to work, in the same capacity, in a more countrylike and peaceful atmosphere.

A tireless and energetic worker, his character, honesty, and consideration for his fellow men created loyalty in, and left an indelible impression on, all those who had the privilege of working with him.

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[4] "Irrigation in Agriculture," by V. C. Tchikoff, Donskoy Inst. Novocherkassk, on Don, U. S. S. R., 1932.

[5] "Duty of Water and Duty of Rivers as Determined by Russian and American Specialists," by V. V. Tchikoff, Russian Govt., Taskent, U. S. S. R., March, 1931.

Surviving him are his widow, Mary (Russin) Tchikoff, who was his constant companion and friend wherever his travels and work took him; a daughter, Alla (Mrs. Robert Ford, Jr.); and a granddaughter, Barbara Ford, aged five.

Mr. Tchikoff was elected an Associate Member of the American Society of Civil Engineers on September 9, 1919, and a Member on January 15, 1934.

DANIEL LAWRENCE TURNER, M. ASCE[1]

DIED MARCH 12, 1942

Daniel Lawrence Turner, who retired from an active career in planning and constructing urban and suburban transportation systems in 1933, died on March 12, 1942, at his home, "The Wayne" in Greenway Court, Norfolk, Va. He was born in Portsmouth, Va., on October 25, 1869, the son of Daniel James and Mary Elizabeth (Lawrence) Turner. His early education was received at Norfolk Academy in Norfolk, after which he attended Rensselaer Polytechnic Institute, in Troy, N. Y. He was graduated in 1891 with the degree of Civil Engineer.

After nine years of varied activities, including a year as assistant in mathematics at Rensselaer Polytechnic Institute and seven years as instructor of surveying at Lawrence Scientific School, Harvard University, Cambridge, Mass., in December, 1900, Mr. Turner became an assistant engineer on the staff of the Rapid Transit Railroad Commission of New York, N. Y. In this capacity, he prepared plans for draining New York City's first subway, worked on construction, and finally was in charge of surveys for the Brooklyn extension of the subway.

Then, for thirty-three years, he was continuously associated with the transportation problems of New York City, including the planning and development of the rapid transit system from forty-five miles of elevated track to five hundred miles of subway and elevated track, at a total expenditure of $800,000,000. During this period Mr. Turner advanced through the positions of assistant engineer and general inspector of stations (1903), chief of Bureau of Transit Inspection (1907), division engineer (1912), deputy engineer of subway construction (1913), acting chief engineer (1916), to chief engineer, Public Service Commission (April, 1917). On May 21, 1921, he resigned to become consulting engineer to the Transit Commission and later to the Board of Transportation of New York City. Mr. Turner was also retained in a consulting capacity on transit matters by the cities of Detroit, Mich., and Pittsburgh, Pa., and by the North Jersey Transit Commission. In 1933 he retired from active practice.

An outstanding characteristic of Mr. Turner's personality was his devotion to his chosen profession. He was never too busy to turn down a call to serve on a committee of one of his engineering societies, and the many requests he received from students or others for information related to his work always received sympathetic consideration. Next to his profession came his alma mater, Rensselaer Polytechnic Institute, which in 1933 conferred upon him the honorary degree of Doctor of Engineering. He served several terms as president of the Rensselaer Alumni Association and was elected an alumni trustee of the Institute in 1928.

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[1] Memoir prepared by Henry Heins, Public Service Comm., New York, N. Y.

Mr. Turner was appointed a United States delegate to the World Engineering Congress meeting in Tokyo in 1929 by President Herbert Hoover. He was a member of the University Club, the Engineers Club of New York, and Theta Xi fraternity; also the Century Association, American Institute of Consulting Engineers, and Sigma Xi fraternity.

On February 3, 1896, Mr. Turner was married to Eva Barcine Denby.

Mr. Turner was elected an Associate Member of the American Society of Civil Engineers on February 2, 1898, and a Member on January 3, 1905.

CHARLES DAVIS VAIL, M. ASCE[1]

Died January 8, 1945

Charles Davis Vail was born on September 11, 1866, on a farm near Lonetree, Ill. He was the son of Richard Randolph and Catherine (Brokaw) Vail. His early education was received in the rural schools of Bureau County, and in the Henry (Ill.) High School. After being graduated from high school, he taught school for a year and then enrolled in the University of Illinois, at Urbana. While still in college, he began his practical engineering career as an engineering assistant on a branch railroad being built from Whitewood, S. Dak., into the Belle Fourche country.

In 1891 "Charlie" received the degree of Bachelor of Science in Civil Engineering from the University of Illinois, and in 1936 the degree of Civil Engineer. He continued his "railroading" by joining a crew of the Union Pacific Railroad Company at Cheyenne, Wyo. Beginning as instrumentman, he soon worked up to the position of assistant to the division engineer, where he remained until the panic of 1893. Then the position was abolished. Nothing daunted by being out of a job in a panic year, he went to Denver, Colo. There he was married to Jessie Paden on December 7, 1893, and they went to the World's Fair in Chicago, Ill.

Returning to the West, he stopped briefly in Denver, and then returned to Cheyenne as a draftsman in the U. S. Surveyor General's office, soon becoming chief draftsman. As might be expected by those who knew him, Mr. Vail's temperament would not permit him to stay long on a strictly office job, so, in 1894, he turned again to railroading and became locating engineer for the

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[1] Memoir prepared by a Committee of the Colorado Section consisting of Oliver T. Reedy, M. ASCE, Chairman, Arthur V. Williamson, M. ASCE, Dan W. Ormsbee, Assoc. M. ASCE, and Albion H. Vickery, formerly City Engr., Denver, Colo. (deceased).

Trinity, Cameron and Western Railway Company, near Georgetown, Tex., and for a few months was assistant engineer for the Mexican International Railway Company.

Evidently, his association with the Union Pacific proved mutually satisfactory, for, in 1895, he returned to that railroad as assistant engineer. Later, he held the same position with the Union Pacific subsidiary, the Oregon Short Line.

However, during the next eight years, 1897 to 1905, Mr. Vail turned to another engineering field—water—and during that period he was engaged as chief engineer of the Creeklyn Ditch Company, in Jefferson County, Montana; and later as principal assistant engineer, Butte (Mont.) Water Company. He installed a water and lighting system in St. Anthony, Idaho; surveyed and designed water works for East End Irrigation Company, East End, Assinaboia, Canada; designed and installed water works at Basin, Mont., including water supply to concentrator for Montana Ore Purchasing Company; did expert hydraulic work for Missoula (Mont.) Water Company and Helena (Mont.) Water Works; designed water works for Glasgow, Mont., and improvements for water works for Fort Benton, Mont., with numerous reports and estimates on power development.

On account of his experience in railroad engineering, in 1905 the Chicago, Milwaukee, St. Paul and Pacific Railroad Company engaged Mr. Vail to turn aside from his irrigation and water-works practice to become locating engineer over the Continental Divide near Butte. When this work was finished, he became chief engineer of the Ely (Nev.) Townsite Company, and for two years engaged in municipal work, surveying, and platting the town of East Ely, designing and installing water works for both Ely and East Ely, superintending the building of an electric light plant for Ely, as well as making reports on water supplies for mining companies.

In 1908 Mr. Vail organized the consulting firm of Vail, Wolbran and Read, with offices in Denver. This location was the center of his activities until his death, thirty-seven years later. During this period as a consulting engineer, his most important works were: Construction of concrete diversion dam across the Colorado River, near Palisade, Colo.; installation of water works at De Beque, Colo.; construction of the Grand Mesa Irrigation project; chief valuation engineer for the Uintah Railroad Company; construction of the narrow gage railroad from Marble, Colo., into the quarries above the town. (It is from these quarries and over this railroad that the marble was obtained for the Lincoln Memorial in Washington, D. C.)

In 1917 Mr. Vail was appointed railroad and hydraulic engineer for the Public Utilities Commission of Colorado, a position he held until September, 1923, when he was given the position of manager of improvements and parks for the City of Denver. He was responsible for a balanced program of prominent improvements, the most outstanding of which were an extensive system of street and alley paving projects, the construction of the City and County Municipal Building, and the development of Denver's park system, especially that of the Denver Mountain Parks. Scores of these parks, miles beyond the city limits, dot the rugged ranges west of Denver. These are maintained by the city and made easy accessible by splendid roads. Mr. Vail brought this development to such a point that it has attracted nation-wide attention as the "playground of the Colorado Rockies."

With such a background of professional experience, and such evidence of his vision relating to development of public utilities, it is natural that he should be sought after to take charge of Colorado's highways. He was appointed state highway engineer in November, 1930, and from then until his death he had but one goal—developing and improving the highway system of his adopted state. Mr. Vail assumed his new task and great responsibilities with characteristic energy. He analyzed the problems and then outlined a comprehensive, far-reaching program. His chief problem lay in the fact that Colorado then had only 539 miles of hard-surfaced highways and thousands of miles of roads that had to be surfaced to meet modern motor traffic, with a very limited highway income. In view of this fact, he decided to eliminate concrete paved roads, except where definitely indicated by excessive heavy traffic, and specialize in oil-mat surfaced roadways. Thus, Colorado became one of the pioneers in this type of construction, and provided a network of almost 5,000 miles of hard-surfaced highways in a few years, extending throughout the state's vast expanse and through the great mountain empire that makes up almost half of the state's area.

Colorado's road-building program is not finished. It never will be. This is equally true of the other forty-seven states; but the tremendous progress in highway improvement, the system that enables motorists to travel from one corner of the state to the other on hard-surfaced, dustless highways, over every type of terrain, will ever be a monument to the indefatigable energy, the breadth of vision, and the tenacity of purpose of one man—Charles Davis Vail.

One of Mr. Vail's severest newspaper critics describes him as a tough-minded man, and agrees that he had to be since he was in a tough job. He measured up fully to the requirements, and a bit more. At the first meeting of the State Association of County Commissioners after his appointment to the highway job, Mr. Vail stated in an address he had been invited to make,

> "I am often said to be hard-boiled. I'm not hard-boiled, but I don't see any sense in telling you the answer perhaps may be 'yes' when I know positively it cannot be anything other than 'no.'"

He was blunt, undiplomatic, and unruffled by opposition; he never dodged a fight or an issue. Yet with it all, he had a kindly understanding of human relations, and an appreciation of the rights of minorities. To have him for a friend was to be backed by an army.

Mr. Vail was a leader in Colorado Democratic circles, a Thirty-Second Degree Mason, an ardent hunter, and fisherman. He was a member of the Colorado Society of Engineers, the American Road Builders' Association, the Western Association of State Highway Officials, and the American Association of State Highway Officials.

He is survived by three children, Allen P., Kenyon C., Assoc. M. ASCE, and Vera Mary (Mrs. Willis M. Winslow); and one brother, Richard Randolph.

Mr. Vail was elected an Associate Member of the American Society of Civil Engineers on June 6, 1900, and a Member on August 31, 1909. He became a Life Member in January, 1935.

JOHN LANE VAN ORNUM, M. ASCE[1]

DIED NOVEMBER 6, 1943

John Lane Van Ornum, the son of Adoniram Judson and Sarah Josephine (Lane) Van Ornum, was born at Hartford, Vt., on May 14, 1864. His ancestors on his father's side were among the Dutch pioneers of the Hudson River valley, particularly from the area near Albany, N. Y. On his mother's side his ancestors came from England as early as 1635, and were Colonial settlers in Massachusetts.

In his early childhood his parents left Vermont and settled in Racine, Wis. He obtained his engineering training at the University of Wisconsin in Madison, being graduated with the degree of Bachelor of Science in Civil Engineering in June, 1888. The professional degree of Civil Engineer was granted to him three years later in June, 1891.

Mr. Van Ornum began his professional career with the Municipal Engineering Department of Milwaukee, Wis., immediately after graduation. During the next three years he served consecutively as surveyor and inspector of the United States Harbor Works on the coasts of Georgia and Florida; with Army Engineers on plans for improvement of Ocmulgee River, Georgia; assistant engineer of the Milwaukee, Lakeshore and Western Railroad Company (later the "Northwestern"); and United States assistant engineer on river surveys and river improvements in Georgia and Tennessee.

In November, 1891, in compliance with previous arrangements between the United States and Mexico, the International Boundary Commission was organized and immediately made plans for a resurvey and remarking of that part of the boundary between these two countries which is not determined by river courses. Mr. Van Ornum became the chief topographer for this survey and served in that capacity from December, 1891, until May, 1894.

Lt. D. D. Gaillard, one of the commissioners, reported in a letter regarding Mr. Van Ornum's work:

> "His duties embraced reconnaissance, and topographic work, and supplying a large party with food, water and forage in one of the most arid and least traveled regions in our country. His work was always of the highest grade and he showed great capacity for surmounting successfully the numerous difficulties encountered in work of the character described, and marked ability to direct and control employees. He is a man of strong character and great integrity."

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[1] Memoir prepared by E. O. Sweetser, M. ASCE, and Luther Ely Smith, Secretary of the 3d Volunteer Engrs., U. S. Army (War with Spain), St. Louis, Mo.

In June, 1895, Mr. Van Ornum read a paper on "Topography on the Survey of the Mexico-United States Boundary"[2] at the Annual Convention of the Society. This paper not only tells of the unusual physical conditions surrounding the field work and maintenance of field parties, but also discusses in detail the importance of transit and stadia methods for such work, giving important data regarding the speed and accuracy which were obtained by this system of mapping. He also wrote "Geology in Its Relation to Topography" in 1898,[3] and several discussions.[4]

In June, 1894, Mr. Van Ornum was appointed instructor in the civil engineering department at Washington University, St. Louis, Mo. He served in that capacity until 1897 when he took a leave of absence and, accompanied by his wife and small son, traveled extensively in Europe investigating foreign engineering works and methods.

In July, 1898, Mr. Van Ornum, a man of lofty patriotism, volunteered for service in the War with Spain and was commissioned a Captain in the Third United States Volunteer Engineers, serving under Col. D. D. Gaillard, with whom he had been in close professional and personal association on the Mexican Boundary Survey.

In a memorial tribute in the 1944 *Yearbook* of the Third United States Volunteer Engineers the following appears:

> "Capt. Van Ornum was an excellent officer, solicitous of the real needs of the men under him, insistent upon discipline, yet not unmindful of the human equation in every problem. His men were devoted to him. The exceptional quality of his service as Company Commander and the high estimate placed upon him by Col. Gaillard were clearly indicated when a vacancy occurred among battalion commanders. Capt. Van Ornum was promoted to Major. The men of 'B' Company and of the 2nd Battalion remember him with deep affection.
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> "Major Van Ornum was entrusted with important special duties both in the United States and Cuba, and he discharged them all with distinction.

A paper[5] by Major Van Ornum published in 1899 describes in some detail the work of this regiment in Cuba from December, 1898, to April, 1899. In addition to the building of docks, establishing of military camps, extensive military road reconnaissance, and the preparation of adequate maps and harbor surveys, principal attention was given to the improvement of sanitary conditions in various Cuban cities. Of particular interest was the case of Cienfuegos where, in a period of fifteen weeks, the weekly death rate of one hundred and thirty eight was reduced to twenty nine. The Army Engineers did this by the consistent attention to sanitary improvement. The Third United States Volunteer Engineers was mustered out of service in May, 1899, and Major Van Ornum returned to civil life.

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[2] *Transactions*, ASCE, Vol. XXXIV, 1895, p. 259.

[3] *Ibid.*, Vol. XXXIX, 1898, p. 90.

[4] *Ibid.*, Vol LIV, Pt. A, 1905, pp. 369 and 497; Pt. B, 1905, p. 455; and Pt. E. 1905, pp. 207 and 555.

[5] "Work of the Third Regiment U. S. Volunteer Engineers in Cuba," *Engineering News*, Vol. XLI, No. 25, June 22, 1899, pp. 400–401.

In June, 1899, he was appointed professor of civil engineering and head of the department at Washington University, where he began a career of thirty-four years in the field of engineering education.

At the beginning of his appointment as professor of civil engineering he was placed in charge of plans and estimates for grading and developing the new Washington University campus. This new campus was located on a hill in what was then open country just beyond the city limits and required, in its preliminary development, the moving of more than 1,000,000 cu yd of earth, the establishment of roads, and an adequate surface drainage system, as well as an elaborate construction program. The distance of the campus from established sewer systems at that time made it necessary to install an unusually large septic tank. An "all-concrete grandstand," which received its inauguration at the Olympic Games of 1904, was built on the campus and described by Professor Van Ornum.[6] He obviously occupied a position of leadership in the expanding program of the university and demonstrated foresight and engineering skill at a critical period of its development.

In 1903, Professor Van Ornum served as president of the Engineers' Club of St. Louis and took a leading part in preparing the groundwork for an International Engineering Congress which, during the period of the Louisiana Purchase Exposition of 1904, held its meetings in what later became the library reading room of Washington University. This room has on its wall a large bronze plaque, inscribed, "This tablet is erected by the American Society of Civil Engineers to commemorate the International Engineering Congress held under its auspices in these buildings of Washington University, October 3rd to October 8th 1904." The initial plant of the University buildings was, in fact, rented in its entirety by the World's Fair Committee, thereby postponing for one year the removal of the university itself from a "downtown" location. Professor Van Ornum took an active part in the International Engineering Congress participating frequently in the discussions of that body.[7]

His services were frequently sought in major engineering problems of St. Louis. In 1903 he took charge of an investigation of the flow of the Illinois River from Chicago to Grafton, Ill., and its effect upon the Mississippi River from Grafton to the intake of the St. Louis water supply at Chain of Rocks.

In 1907 he served as chairman of a special Lighting Committee appointed by the Civic League to investigate the question of street lighting in St. Louis. After more than a year of study an extensive report was published in May, 1908, which received wide publicity through the St. Louis newspapers and frequent comments by engineering magazines.

In May, 1908, Professor Van Ornum was appointed by the Mayor of the City of St. Louis, to examine and report upon "the progress of preparation for the construction" of the municipal bridge across the Mississippi River. His report resulted in the clarification of responsibilities on the municipal bridge program and promoted progress in the early engineering work of this important project.

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[6] "The Proposed Septic Tank and Sewage Purification Studies at Washington University," by J. L. Van Ornum, *Engineering News*, Vol. XLIV, No. 20, November 15, 1900, p. 329.

[7] "Transactions of the International Engineering Congress in 1904," *Transactions*, ASCE, Vols. LIV (see Cumulative Index, 1920, p. 260).

During this same period, Professor Van Ornum served on the building committee for Pilgrim Congregational Church, of which he was a member. His contribution of engineering skill and sound judgment, generously given, promoted wise decisions in the design and construction of this impressive building.

He was a frequent contributor to magazines and journals, demonstrating a wide range of interest in the fields of economic problems, technical developments, and engineering education. The following examples from a long list of publications confirm this statement: "Theory and Practice of Special Assessments,"[8] "Some Engineering Lessons from the San Francisco Disaster"[9] (which followed a personal inspection of the area), and "Elements of Effective Education."[10]

Among his many papers published in the *Transactions* of the Society, three in particular represent research of a "pioneer" character: "The Fatigue of Concrete Products,"[11] "The Fatigue of Concrete,"[12] and "The Effect of Saturation on the Strength of Concrete."[13] These papers attracted wide attention and discussion. References to them may be noted frequently in America and foreign textbooks.

In 1914 his book, "The Regulation of Rivers,"[14] which was widely used as a text for classwork, was published. Extensive and favorable comments appeared in the technical magazines. The *Engineer* of London stated in part, "It is a pleasure to note the sound literary style," and an American reviewer spoke of it as "a noteworthy addition to American engineering literature."

Professor Van Ornum's long career as a teacher was devoted to the development of his students as individuals, trained in clear reasoning, sound judgment, and integrity of purpose. Uncompromising in his attitude toward the superficial, he was prompt to discern and encourage the earnest and sincere student or co-worker. Outstanding records among alumni in later professional life bear witness to the soundness of his instruction and his ideals.

Throughout his life his concern was manifest for the health and welfare of those who served with him. His kindly sympathy and practical help for those in trouble knew no limitations of social position but were given generously without thought of acclaim or reward. He consistently shunned publicity in all matters pertaining to the finer attributes of life and his sterling character and sincere convictions served as a worthy example to his colleagues and students.

In June, 1933, Professor Van Ornum withdrew from active teaching because of poor health and was given a year's leave of absence. In June, 1934, at the age of seventy, he was permitted to retire and was appointed professor emeritus of civil engineering. He lived a very quiet and secluded life for the next ten years, but finally succumbed to a severe heart attack and died at his home on November 6, 1943.

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[8] *Transactions*, ASCE, Vol. 38, 1897, p. 336.

[9] *Journal of the Association of Engineering Societies*, Vol. 38, 1907.

[10] *Bulletin of Washington University*, April, 1911.

[11] *Transactions*, ASCE, Vol. 51, 1903, p. 443.

[12] *Ibid.*, Vol. 58, 1907, p. 294.

[13] *Ibid.*, Vol. 77, 1914, p. 438.

[14] "The Regulation of Rivers," by J. L. Van Ornum, McGraw-Hill Book Co., Inc., New York, N. Y., 1914.

He was a member of many scientific, technical, and educational societies.[15,16]

He was married to Carrie Beattie Scott, of Newburgh, N. Y., on July 25, 1894. He is survived by his widow and a son, Capt. Thurwood Van Ornum (retired).

Professor Van Ornum was elected an Associate Member of the American Society of Civil Engineers on October 3, 1894, and a Member on October 4, 1899.

EVERETT PINE WEATHERLY, M. ASCE[1]

DIED FEBRUARY 22, 1944

Everett Pine Weatherly was born on March 12, 1877, on a farm near Shannondale, a small town in Chariton County, in the central part of Missouri. It was a farming community and his parents, William James and Mary (Elizabeth) Weatherly, raised a family of two sons on what was some of the best farm land in Missouri. William was of French-English descent, and Mary, of Welsh, the families of both having settled in Missouri about 1850.

Everett attended grade and high schools in Salisbury, Mo. He entered Missouri University at Columbia in 1893 and received the degree of Bachelor of Arts in 1897 and the degree of Bachelor of Science in Civil Engineering in 1899. The vacation periods during his college work were spent with railroad field surveying parties.

When Everett left the university he went to the Chicago, Burlington and Quincy Railroad Company as chainman. Viewed in retrospect, the romantic history of railroading had reached its peak, but in 1899 that peak was so close it served only to thrill the imagination, and it is probable that most of Everett's classmates hoped for just such a connection as he made. In this company he gathered experience and knowledge of his abilities as well as the confidence of superiors and associates. Mr. Weatherly progressed from chainman, inspector, and instrumentman, to resident engineer (1900) in charge of maintenance at Brookfield, Mo., and then at Hannibal, Mo., until 1903. He later became assistant engineer at St. Joseph, Mo., in charge of maintenance and construction.

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[15] "Who's Who in America," The A. N. Marquis Co., Chicago, Ill., Vol. 14, 1926–1927, p. 1932.

[16] "Who's Who in Engineering," Lewis Historical Publishing Co., Inc., New York, N. Y., 3d Ed., 1931, p. 1346.

[1] Memoir prepared by A. B. Taylor, Assoc. M. ASCE.

In July, 1909, he was sent to southern Illinois. The construction of the Panama Canal was well under way, and there was much talk of the loss of traffic by railroads to ocean ships. This loss was to be felt particularly by railroads without tidewater terminals on the East Coast or Gulf Coast. The Burlington Line was built into the southern Illinois coal fields, and Mr. Weatherly's new assignment was as engineer of construction for a single-track line from Herrin, Ill., to Metropolis, Ill., which, according to rumor current at the time, was the first step in a line to the Gulf. The new line was forty miles long not counting a gap of semimountainous construction near the upper end. The line was put into service in the latter part of 1910. It is probable that this job gave Mr. Weatherly more satisfaction than any other of his career. He was a young man, and this, his first major work, was a considerable job—$1,000,000 worth of grading, bridging, and stations, and he had handled it in a manner which had pleased T. C. Calvert, chief engineer, and others of his superiors.

Kansas City, Mo., having survived its boom of the nineties, the flood of 1903, and the 1907 depression, was experiencing solid growth. The Kansas City Terminal Railway Company, owned jointly by several of the railroads entering the city, was to be provided with new tracks, the elimination of many grade crossings, and a Union Station, all estimated at $30,000,000. The late J. V. Hanna and Arthur C. Everham, Members, ASCE, chief engineer and assistant chief engineer, respectively, were in charge of the construction. Mr. Weatherly was sent in by the Burlington Line as a division engineer. After a short time on the east division, he was moved to the middle or second division. This division included the Union Station grounds and tracks, and extended east to Kansas Avenue, Kansas City. He had responsibility for the excavation of 630,000 cu yd of earth and rock on the station grounds. The waste from the excavation was hauled east or west over main-line tracks to build outlying yards. This job, being complicated by the use of railroad operating facilities, was done by company forces under the direction of the division engineer. Excavation was by steam shovel and the waste was hauled away first on flat cars and later in side dump cars. To keep the tracks in condition to carry these loads, large repair units were needed at both ends. The widening and deepening of the heavy main line cut to the east was handled in the same manner. This cut, two miles long, already two tracks wide and a maximum of 40 ft deep when the improvement started, was deepened to a maximum of 78 ft and widened to four tracks with many side cuts for industry. The quantities totaled 1,000,000 cu yd, all work being done under traffic. Concurrent with the foregoing was the construction by contract of seventeen street viaducts spanning the cut and retaining walls which ran almost continuously on one or both sides of the cut.

In 1914 most of the terminal work was finished, the Union Station was put in service, and the fine engineering organization which had been assembled for the purpose was disbanded. Mr. Weatherly considered returning to the Burlington Line, but abandoned the idea, and decided that the contracting field was more attractive. Early in 1915 he went out as superintendent on contract work for William M. List of Kansas City. In 1916 he entered into partnership with Mr. List, forming the firm of List and Weatherly, and under that name he continued in business until his death. Mr. List had at the time a successful business as a railroad contractor, and the new partnership was to handle concrete and steel structures on such work. Mr. Weatherly was vice-president and general manager of the new firm and was responsible for the conduct of its work.

Most of the firm's business was in the western part of the Mississippi Valley and was confined to heavy construction, such as bridges, intakes, viaducts, paving, etc. In 1934 the partnership was broken and Mr. Weatherly became president and owner of the business, as well as its active head. The List and Weatherly contracts cover 335 engagements ranging from small jobs to the Long-Allen Bridge over the Red River at Shreveport, La. This bridge was a $750,000 state highway project undertaken in 1930. The completion was delayed on account of shortage of state funds. The successful completion of this contract in 1933 was a matter in which Mr. Weatherly took great pride.

Mr. Weatherly was a member of the Associated General Contractors of America. He was treasurer of the Missouri Branch of the organization in 1929, and its president in 1930. E. C. L. Wagner, M. ASCE, manager of the branch, advises that Mr. Weatherly interested himself in and forwarded the work of the contractors in the matters of "Prequalifications of Bidders" and a depreciation rate schedule for construction equipment. The former was a practice adopted by the Missouri State Highway Department in 1930, whereby a bidder on a contract satisfied the owner—in advance of making a bid—that the contractor was qualified by experience and resources, etc., to undertake the work. The latter was an authoritative rate schedule to be used by bonding companies and in tax matters. This resulted in the adoption in 1931, by the Bureau of Internal Revenue, of a schedule set up by the Associated General Contractors.

Mr. Weatherly got much out of his university work. In addition to his degree, he was elected a member of Tau Beta Pi. His extracurricular interests included membership in Kappa Sigma fraternity. He achieved some prominence in amateur tennis circles while at the university, and continued to enjoy an occasional game until late in life. As his tennis game slowed, he took up bridge. At this game he was excellent, and he was an indulgent partner and opponent. Mr. Weatherly was a soft-spoken man of friendly, nonaggressive demeanor, who made his way through life with a minimum of upsets to an even-tempered disposition. It was extremely seldom that he found sufficient cause for an angry remark or harsh argument. He was a tall, well-built man, who enjoyed the out of doors. A love of the land survived from his early training, and found outlet in supervision of his farm investments, a country summer home, and in the last year of his life—although a semi-invalid—in a victory garden. He was a member of the Christian Church. In politics, he was an Independent Democrat. He was a member of the Masonic Order, Blue Lodge, having taken his degrees at St. Joseph.

On September 23, 1903, Everett Weatherly was married to Rose Drescher of Hannibal, who, with two sons, Edward H. and Everett P., survives him. To two other boys, Edward and Joseph Bounds, orphaned nephews of his wife, he was almost a father. He took them to his home, looked after their estate, and guided them to manhood.

Beginning early in 1909, Mr. Weatherly's home and business headquarters were in Kansas City. Mr. Weatherly was a member of the American Railway Engineering Association.

Mr. Weatherly was elected a Member of the American Society of Civil Engineers on April 30, 1912.

EINAR WEIDEMANN, M. ASCE[1]

Died November 12, 1941

Einar Weidemann was born near Trondheim, Norway, on May 29, 1877, the son of Einar and Johanna (Tonder) Weidemann. He received his preparatory and technical education in Norway, and was graduated from the Technical Institute at Trondheim in 1897 with the degree of Civil Engineer.

Mr. Weidemann served as an engineer for the Norwegian State Railroads from 1897 to 1902. In the spring of 1903, he came to Pittsburgh, Pa.; and in that and the following year served as draftsman for the American Bridge Company, and other structural steel companies. In 1904 he became assistant to Hermann Laub, consulting engineer of Pittsburgh, remaining with Mr. Laub until 1909. During this period his work included design of several river bridges and design and supervision of construction of industrial buildings.

In 1909 he was employed by the Pennsylvania Railroad Western Lines in the office of the chief engineer, Maintenance of Way, at Pittsburgh. Here he devoted practically all his energy to the design of powerhouses and enginehouses, shops, docks, highway bridges, and a multitude of railroad facilities during the period of expansive railroad construction. In 1914, in acknowledgment of his excellent performance of these duties, he was placed in responsible charge, under the chief engineer, of structural designs of the Pennsylvania Freight Terminal in Chicago, Ill., which, in view of its massive and novel development, was a major undertaking.

During the period of development and construction of the Chicago Union Station (1916 to 1926), in which the Pennsylvania Railroad Company participated, Mr. Weidemann was granted leave from his normal duties. On January 1, 1916, he was appointed structural engineer; and, on January 1, 1923, engineer of buildings and structures for the Station Company. He served in these capacities until the completion of the new railroad terminal. Mr. Weidemann was responsible for the preparation of plans and specifications for the various structures, including, besides the essential railroad facilities, several large street viaducts and bridges. Special features of this work included investigation of foundation problems; design of a reinforced concrete bed for supporting the station and approach tracks; and design of the heavy structural steel, concrete, and glass train sheds. The sheds were a new development, with columns centered on the passenger platforms, and roof structure spanning the intermediate tracks and separate trucking platforms.

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[1] Memoir prepared by Joshua D'Esposito, M. ASCE.

When the Chicago Union Station was in satisfactory operation, Mr. Weidemann returned to the Pennsylvania Railroad Company, being appointed engineer of bridges on April 1, 1927; and on January 1, 1928, he was appointed engineer of bridges and buildings of the Western Region which position he held at the time of his death.

Mr. Weidemann was thorough and accurate in engineering matters, preferring to weigh each problem carefully; but, having made his decision, he was positive in upholding it. No matter how perplexing a situation might become, he was sincere and pleasant, yet he had a becoming dignity that would not permit encroachment upon his authority. It was natural for him to give assistance within his field of knowledge, and to welcome advice from those in associated work. He had that rare ability of not offending those who might temporarily disagree with his opinions; and his quiet reserve and understanding sympathy resulted in the formation of lasting friendships.

The writer, who was associated with Mr. Weidemann for nearly twenty years, cannot remember a single incident when Mr. Weidemann expressed himself unkindly about any person or any organization. All respected him profoundly as an engineer, and valued him as a comrade and fellow man. In fact, Mr. Weidemann was one of nature's gentlemen, always kind and considerate, quiet and unassuming, but possessing a sense of personal dignity without the least vestige of arrogance. He was a lover of nature; and, like a good son of Norway, he enjoyed winter sports, and participated in some of them.

Mr. Weidemann was married on August 10, 1910, to Mary Louise Weidemann, daughter of Mr. and Mrs. Robert H. Weidemann. He is survived by his widow.

He was a member of the Western Society of Engineers and of the Chicago Engineers Club.

Mr. Weidemann was elected a Member of the American Society of Civil Engineers on January 17, 1927.

DETHIC HEWITT WOOD, M. ASCE[1]

Died November 3, 1945

Dethic Hewitt Wood, the son of Col. Gustavus Adolphus Wood and Lydia Caroline (Carpenter) Wood, was born on February 23, 1871, in Chattanooga,

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[1] Memoir prepared by J. E. Morelock, M. ASCE.

Tenn. Colonel Wood was in command of the 15th Indiana Regiment during the Civil War in and around Chattanooga. He was so impressed with the beautiful scenery of Chattanooga and its environs that, shortly after the close of the war, he resided there permanently.

Dethic Hewitt Wood was named for his great-great-grandfather, Capt. Dethic Hewitt of the 24th Connecticut Regiment, Revolutionary War, killed in action in 1778 at the battle of Wyoming (Wilkes Barre, Pa.).

Mr. Wood was educated in the public schools of Chattanooga and at the University of Chattanooga. He was one of the first graduates from that institution after it qualified as a full college. He took a very active interest in the University during the remainder of his life—serving as chairman of the Building Committee for the construction of the football stadium and always being an active worker and contributor in all campaigns for raising endowment or building funds. He was a member of the Board of Trustees until his death.

He began his work in civil engineering in the employ of the City of Chattanooga, under the late Robert Malcolm Hooke, who was then city engineer. Mr. Wood served as rodman in 1888, as inspector on sewer construction and paving in 1889 and 1890, and as assistant city engineer in 1891 and 1892. During 1892 and 1893 he was engaged in the general practice of engineering for various parties, on railroad location, land subdivision, and mining engineering. During 1894 he was assistant engineer in charge of location and construction of a sewage system for the City of Meridian, Miss. In Meridian he met Miss Alnita Hasselle to whom he was married in 1897.

In 1895 he was engaged as assistant engineer on the survey of Shiloh National Military Park, near Corinth, Miss. On January 1, 1896, he became associated with the late William Hassen Converse, acting first as designing engineer, and later as chief engineer, for the Converse Bridge Company of Chattanooga. When this company was reorganized in 1914, as the Converse Bridge and Steel Company, Mr. Wood became president of the company, a position he held until retirement from active work in 1943. Under his management, this company grew to be one of the leading industries of its kind in the South. During his long connection with the company, he designed and erected many large steel bridges and buildings throughout the South.

About 1925 Mr. Wood became interested in the Southern Chemical Cotton Company of Chattanooga. This company was just beginning the processing of cotton linters for use in the new field of plastics. Mr. Wood served as president of the company from 1925 until 1943—during which time the company became one of Chattanooga's leading and most prosperous industries.

During the nineteen twenties Mr. Wood became interested in banking. He was associated with, and was very active in, the management of various banks for the remainder of his life. At the time of his death he was a director of the American Trust and Banking Company of Chattanooga.

Mr. Wood's vital energy appeared in all his activities both business and social. Although, in later years, executive and financial duties took most of his attention, he was always intensely interested in any engineering problem that might arise. Subordinates might hold the nominal title and responsibility; but Mr. Wood, through his engineering talent and intensive energy, was the chief engineer of any project with which he was connected. In all human relationships his sympathy and understanding of problems were very keen.

In 1943 Mr. Wood retired from active work, becoming chairman of the Board of Directors of the Converse Bridge and Steel Company, and director and vice-president of the Southern Chemical Cotton Company. He also retained his position as director of the American Trust and Banking Company. Mr. Wood served as president of the Chattanooga Manufacturers' Association for two terms—1918 to 1919 and 1919 to 1920. He belonged to the Rotary Club, from 1915 to 1935, Chattanooga Lodge, F. and A.M., John B. Nicklin Chapter, R.A.M., Lockout Commandery Knights Templar, and Alhambra Shrine. He was a life member of Park Lodge, Knights of Pythias, a charter member of the Chattanooga Engineers' Club, and a member of the Mountain City Club. He was an active church member, and had been a member of the Board of Deacons of the Central Presbyterian Church since 1910.

He is survived by his widow; a son, Percy H. Wood; a daughter, Merle (Mrs. B. C. Holtzclaw); and five grandchildren.

Mr. Wood was elected an Associate Member of the American Society of Civil Engineers on February 6, 1907, and a Member on November 1, 1910. He became a Life Member in January, 1942.

AUGUST ZIESING, M. ASCE[1]

DIED FEBRUARY 16, 1942

August Ziesing, the son of Henry and Katherine (Brenneman) Ziesing, was born on February 19, 1858, in Peru, Ill., where he spent the early years of his life. His father was a country physician and surgeon, and one of August's duties was to drive his father about the countryside to the homes of patients. From this early association with horses and their care undoubtedly came his later interest in blooded driving horses, which lasted through most of his life until, as he put it, "The automobile drove me off the road." Since his birthplace was the center of a farming country, he spent his summer vacations on his father's farm as well as on the farms of other relatives in the vicinity. Probably because of this early experience he cultivated a liking for agricultural pursuits.

His early education was obtained by private tutoring and was rather intensive since he entered the University of Illinois in Urbana at the age of sixteen. He was graduated with a Bachelor of Science degree in 1878. The degree of Civil Engineer was conferred on him by the University of Illinois in 1905. He was also an honorary member of Tau Beta Pi, an engineering honor society.

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[1] Memoir prepared by Albert F. Reichmann, M. ASCE.

After graduation, he was employed for two years (1878 to 1880) as a draftsman with M. Lassig, a bridge builder of Chicago, Ill. From 1880 to 1882, he was employed by the Pennsylvania Railroad Company at Pittsburgh, Pa., as bridge engineer and inspector of iron material and structures. Later he became assistant engineer, in charge of a party relocating a line for double tracking, straightening out curves, and reducing grades on the Pittsburgh, Fort Wayne and Chicago Railway, between Smithsville and Shreeve in Ohio, and in charge of the construction of this line as located. He also supervised parties on preliminary surveys of several connecting lines in Ohio.

In 1882, he became chief draftsman of the Lassig and Alden Bridge Company of Chicago. He was made chief engineer when Mr. Lassig secured control of the company. Later he was appointed general manager and chief engineer of the Lassig Bridge and Iron Works, remaining until August, 1897.

From 1897 to 1901, he was engaged in private practice as a consulting engineer, specializing in the design of iron and steel structures. In this capacity he was employed as consulting engineer of bridges by the Chicago Rock Island and Pacific Railroad, the Chicago and Alton Railroad, the Chicago, St. Paul, Minneapolis and Omaha Railroad, and the Rock Island and Peoria Railroad companies. Also he was employed by five other western railroads, and he designed track elevation at Sixteenth Street in Chicago. Probably the most outstanding structure which he designed was the Missouri River Bridge for the Chicago and Alton Railroad at Glasgow, Mo.

During the time that Mr. Ziesing was engineer for the Lassig Bridge and Iron Works, he directed the designing, construction, and sales of the products of the company. Many of the prominent railroads in the Middle West did not have their own bridge designing departments and depended upon contractors to furnish complete designs with their bids. This all around experience in the three major departments of a structural concern gave him an unique position which undoubtedly influenced his appointment as vice-president and western manager of the American Bridge Company in 1901. He was elected president on September 31, 1905. He held this office until his retirement on April 1, 1927.

About the time he assumed the presidency of the Bridge Company, which had just previously become a subsidiary of the United States Steel Company, a great forward movement in the country's economic development was beginning. It was characterized by extended and improved rail lines, new terminals, heavier locomotives and hence heavier bridges, increased spans (required by the government) over navigable streams, taller buildings, and the rapidly expanding automobile industry.

As modern structures increased in both size and weight, steel of greater strength and larger section became economical. At first a high carbon steel was used and later silicon and other alloys were introduced. The wide flange beam was an even more economical innovation, both as to weight and fabrication cost. These changes necessitated the installation of improved and heavier machinery. During this development period a modernization plan was started, directed, and brought to a successful conclusion by Mr. Ziesing. The Ambridge Plant near Pittsburgh, Pa., the Ambridge Barge Yard, the Elmira (N. Y.) Plant, the Gary (Ind.) Plant, and the Shiffler Tower Plant were all built under his immediate direction and according to his ideas of modernization. Mr. Ziesing also supervised the construction of the Williamsburg Bridge and the Hellgate Bridge (connecting railway), both over the East River in New York.

During his presidency the company fabricated and built the steel structure of many of the largest buildings in the United States. Among them were the Chicago Union Station, the Stevens Hotel, the Palmer House, and the Pure Oil and Bankers buildings in Chicago; and the Metropolitan Towers, the Woolworth Building, and the American Telegraph and Telephone Building in New York, N. Y.

Thus, his most important professional accomplishments were interrelated with a natural expansion in the steel industry and a great growth in the American Bridge Company.

In 1909 Mr. Ziesing bought a large farm in Deerfield, Ill., where he concentrated on the breeding of fine Guernsey cattle. He worked out his own ideas of crossbreeding to produce large stock with heavy production strains but without loss of the quality of the milk. Because of his success in this line he was awarded, by the American Guernsey Cattle Club, a silver pitcher on which were inscribed these words: "May 11, 1938—In recognition of his constructive work in breeding Guernsey cattle over a period of 30 years." His herd at one time was adjudged as fine in quality as any Guernsey herd in the United States.

Although, during his business life, his duties frequently took him away from home, he was, nevertheless, a home lover and maintained a beautiful place in Glencoe, Ill. He surrounded himself with his closest friends. His home life was very simple and his tastes modest. In his later years, besides continuing to superintend the raising of cattle, he spent most of his time reading books, classics in English, French, and German, which he had not had the opportunity to read during his active business life.

Mr. Ziesing was married to Alice C. Hanna of Wooster, Ohio, on February 27, 1884. They had five children: Mabel, Henry Hanna, Margaret (Mrs. Mark D. Rector), Gertrude (Mrs. James S. Kemper), and Katherine (Mrs. Katherine Van Cleave).

He joined The Western Society of Engineers on June 5, 1889, and served as a trustee from 1899 to 1902. He was a member of the American Railway Engineering Association, University Club, Union League Club, Chicago Engineers' Club, and the Skokie Country Club.

Mr. Ziesing was elected a Member of the American Society of Civil Engineers on October 5, 1898.

EARL CURTIS ALEXANDER, Assoc. M. ASCE[1]

DIED FEBRUARY 9, 1942

Earl Curtis Alexander was born in Frankfort, Ind., on March 2, 1883, the son of Wilkenson and Ida (Thompson) Alexander.

His early education was received in the schools of Frankfort. In 1902 he began his civil engineering studies at Purdue University, Lafayette, Ind.; these were continued until 1905, at which time he was employed in the engineering department of the Chicago, Rock Island and Pacific Railway Company.

In 1906 he became a member of the engineering staff of the Philippine Railway Company, and for two years was employed by the railway company and J. G. White and Company, contractors, on location and construction of bridges. Returning to the United States in 1909, he was associated with the bridge department of the Illinois Central Railroad Company as a draftsman; then for a period of five years, from March, 1910, to October, 1915, he was assistant engineer of the Northern Pacific Railway Company in charge of bridge construction, including considerable work on heavy foundations and the installation of drawbridges. In 1915 he returned to the Illinois Central Railroad Company to devote his training and experience to the design of concrete and steel structures. In 1918 he joined the staff of the Massey Concrete Products Corporation as an estimator, and served subsequently from 1921 until 1928, as assistant chief engineer. All his engineering and business experience subsequent to 1918 was devoted to the far-flung activities of the Massey organization. In 1928 he was designated manager of factory operations and, from then until 1940, had complete charge of all plant construction and all production problems. In 1940 he was appointed chief engineer, and, until his death, served in that capacity, continuing at the same time his previous function as manager of operations.

Mr. Alexander was well liked by all his associates; was ever faithful to his duties; and, as one of the pioneers in pre-cast concrete construction, had a large part in developing this field for the railroads, municipalities, and the construction field in general. He had a wide acquaintanceship among the membership of the Society and others engaged in similar work. His untimely death is a distinct loss to his industry and to the profession.

For many years, he was a member of the Chicago Engineers Club, the Western Society of Engineers, and several railway engineering organizations.

On December 27, 1911, Mr. Alexander was married to Hazel Israel. His widow; one son, Maj. William Alexander, U. S. Army; and two daughters, Mary (Mrs. D. F. Hunzilie, Jr.) and Peggy Jean Alexander survive him.

Mr. Alexander was elected an Associate Member of the American Society of Civil Engineers on June 19, 1922.

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[1] Memoir prepared by G. H. Redding, Pres., Massey Concrete Products Co., Chicago, Ill.

CHARLES NEWTON BAINBRIDGE, Assoc. M. ASCE[1]

Died July 1, 1941

Charles Newton Bainbridge was born in Philadelphia, Pa., on March 29, 1887, the son of George M. and Mary Malinda (Coulson) Bainbridge. In 1907 he received the degree of Bachelor of Science in Civil Engineering from Pennsylvania State College at State College. For a year thereafter he was a draftsman with the American Bridge Company, and then he returned to his alma mater as an instructor in civil engineering. He received the professional degree of Civil Engineering from the college in 1910.

In 1909 Mr. Bainbridge entered the engineering department of the Chicago, Milwaukee, St. Paul and Pacific Railroad Company, under Charles F. Loweth (Past-President, ASCE), and remained in its service until 1933, with the exception of a six months' interval with the Baltimore and Ohio Railway Company in 1911. While with the Chicago, Milwaukee, St. Paul and Pacific Railroad Company, he worked in the positions of draftsman, squad foreman, office engineer, and engineer of design. As office engineer he had charge of detail, design, and special studies for grade separation projects in Minneapolis, Minn., Milwaukee, Wis., Chicago, Ill., and Evanston, Ill. As engineer of design he was responsible for the design of bridges and structures on the following track separation projects of the railroad company: Mayfair track elevation in Chicago from Irving Park Boulevard to Forest Glen; North Side track depression in Milwaukee, Wis., with reinforced concrete overhead highway crossings; and track elevation north of Church Street in Evanston, Ill.

After leaving the railroad company, Mr. Bainbridge was engaged by the Public Works Administration on various projects.

In 1910 Mr. Bainbridge was married to Lillian Schwab of Chicago. He is survived by his widow; two children, Mary Katherine (Mrs. G. W. Stephens) and Charles Newton Bainbridge; two grandchildren, Glenn and David Stephens; and a brother, Edmond Bainbridge.

The home of Mr. Bainbridge was at Lombard, Ill. He took an active interest in the affairs of his community, being a Village Trustee for ten years, president of the Zoning Board, and president of the First Trust Bank of Lombard. At all times he was ready to give the village the benefit of his engineering ability and experience. He was a loyal friend and was held high in the esteem of all those who came in contact with him. He was a Royal Arch Mason.

Mr. Bainbridge became an Associate Member of the American Society of Civil Engineers on October 9, 1917.

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[1] Memoir prepared by Theodore Lovel Donner Hadwen, M. ASCE.

WALTER CHRISTMAS BODYCOMB, Assoc. M. ASCE[1]

Died December 1, 1944

Walter Christmas Bodycomb, the son of Rosser and Mary (Christmas) Bodycomb, was born at Olyphant, Pa., on June 5, 1875. Both parents were of Welsh descent. The family moved to Wilkes-Barre, Pa., in 1882, where Walter obtained his education in the grade schools and high school. From the Wilkes-Barre High School he went to Wyoming seminary, at Kingston, Pa.

He was employed first by I. E. Hartwell and later by the Crescent Coal Mining Company, both of Wilkes-Barre, as rodman, chainman, and transitman. In 1898 and 1899, he worked on railroad locations for the El Paso Northern Railroad and with the New Mexican Fuel Company, prospecting and surveying for coal lands in New Mexico.

For two years he was employed by Westinghouse Church Kerr Company, as inspector of bridge construction on the Lackawanna and Wyoming Railroad. In 1903 and 1904, he was with G. W. Milnes as chief of party on railroad, land, and mine surveying, with headquarters at Clarksburg, W. Va.

In 1904, he returned to Westinghouse Church Kerr Company, continuing with that firm and its successors until his death. For about six years he was field engineer on location and construction for the Lackawanna and Wyoming Valley Railroad at Scranton, Pa.; on location of B. F. and H. Railway at Frederick, Md.; on hydroelectric reports, appraisals, and surveys; and on the construction of the Chicago, Burlington and Quincy Railroad Company at Havelock, Nebr.

Beginning in 1910, for thirty-four years, Mr. Bodycomb was employed as superintendent of construction on heavy industrial and railroad work—his ability in handling the many problems of management being demonstrated repeatedly on important jobs. He supervised the building of the Westinghouse Electric and Manufacturing Company plant, consisting of heavy reinforced concrete factory buildings and power plant, at Newark, N. J. In 1912 the J. I. Case Threshing Machine Company built the South Works Plant at Racine, Wis., under his direction, including foundry, machine, and erection shops, with sewers, water supply, and power distribution. He then rebuilt the shops of the Chesapeake and Ohio Railroad Company, at Peru, Ind. (after they had been destroyed by fire and flood). A notable feat of engineering-construction work was accomplished under his direction in correcting the uneven foundation settlement of the Empress Hotel in Victoria, B. C., Canada. To obtain relief of an overloaded area some 25,000 tons of earth were removed and replaced with concrete decking, thus preventing further settlement of the south wing.

In 1914, Mr. Bodycomb supervised improvements for the Canadian Pacific Railroad Company at Winnipeg, Man., Canada, including track elevation, main street subway, and remodeling the station and Royal Alexandria Hotel.

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[1] Memoir prepared by Wallace E. Belcher, M. ASCE.

During World War I (in 1915), he was field engineer on a rush plant for the Aetna Explosives Company, at Drummondville, Que., Canada, where a complete powder manufacturing plant with seventy-five buildings covering six hundred and forty acres, was completed in twenty weeks. Also, in 1915, he supervised construction of the East St. Louis Cotton Oil Company plant, and, in 1916, the Lehigh Valley Railroad engine terminal at Niagara Falls, N. Y., with the usual powerhouse, roundhouse, coaling station, turntable, and shops.

The construction and equipment installation of the Artillery Ammunition Assembly Plant, at Rock Island Arsenal, Rock Island, Ill., in 1917, was remarkable not only for the magnitude of the work, and the speed with which it was accomplished, but also for certain methods developed to meet conditions, such as:

(*a*) The pouring of mass concrete in subzero weather by a system of steam piping to protect the concrete from the time it entered the feeding chutes;

(*b*) Rock excavations under winter conditions where vast quantities of water were encountered; and

(*c*) The construction of the first modern igloos for storage of explosives—accomplished without removing any of the natural forest by offsetting lines to avoid clearing.

After this war work came the building of the Baltimore and Ohio Railroad Company shops at Cumberland, Md.; the construction of a fifteen-stall roundhouse for the Pittsburgh and West Virginia Railway Company at Rook, Pa., the remodeling of the locomotive shops of the Pennsylvania Railroad at Fort Wayne, Ind.; and the construction of the new shops for the Westinghouse Electric and Manufacturing Company extension at Essington, Pa.

The Westinghouse Church Kerr Company became part of the organization of Dwight P. Robinson and Company in 1920. Mr. Bodycomb's first assignment was superintendent on a four-story, reinforced concrete, manufacturing plant for the Westinghouse Lamp Company at Indianapolis, Ind., followed by the construction of a five-story, reinforced concrete, hotel building for the Middletown Hotel Company, at Middletown, Ohio.

At the American Rolling Mill Company plant in Ashland, Ky., Mr. Bodycomb supervised blast furnace improvements and general mill construction including the installation, under secret processes, of the first continuous strip steel mill in 1922. Later for the same company, at Middletown and at Butler, Pa., he constructed blooming mills, open hearth furnaces, a rolling mill, other buildings, and equipment.

In 1924, he built the locomotive repair shops for the Southern Railway at Birmingham, Ala. At Bessemer, Ala., the construction of car shop, machine shops, lumber storage, dry kilns, power plant, and office—a project costing five million dollars—was handled in record speed for the Pullman Car Manufacturing Company. The United Engineers and Constructors, Inc., of which Dwight P. Robinson and Company became an integral part, was formed in 1928.

In 1930, for the St. Joseph Lead Company plant at Josephtown, Pa., Mr. Bodycomb built an electric furnace zinc oxide smelter. Then followed the construction of improvements for the Public Service Gas and Electric Company at the Kearny and Burlington power plants. Additional buildings for the United States Pipe and Foundry Company were completed at Chattanooga, Tenn., in 1935 and 1936. A complete plant for making asbestos pipe was constructed for the Keasbey and Mattison Company at Ambler, Pa.

From 1940 to 1942, Mr. Bodycomb supervised the erection of the United States Navy Power Plant at Parris Island, N. C. His last work as construction superintendent was the steam power plant for the Republic Steel Corporation at Cleveland, Ohio, finished on May 1, 1944. He then retired to St. Petersburg, Fla., where he lived until his death.

Mr. Bodycomb was a lifelong member of the Westminster Presbyterian Church of Wilkes-Barre, and a devoted member of the Masons and of the Shrine. He took an active interest in the Westinghouse-Church-Kerr Alumni Association.

In 1902, Mr. Bodycomb was married to Caroline Booth at Forty Fort, Pa. She died in 1915. In 1917 he was married to Helen B. Smith at St. Louis, Mo. He is survived by his widow; his daughter, Irma Joyce Bodycomb; and three sons, Frederick, Walter C., Jr., and Capt. Rosser I. Bodycomb.

Mr. Bodycomb was elected an Associate Member of the American Society of Civil Engineers on September 2, 1914.

CHARLES BARTO BROWN, Assoc. M. ASCE[1]

Died November 2, 1944

Charles Barto Brown, the son of Charles Edgar and Henrietta (Hitchcock) Brown, was born on September 22, 1873, in New Haven, Conn. He received his early education in the schools of that city, and was graduated from Sheffield Scientific School in 1894 with the degree of Bachelor of Philosophy. He received the degree of Civil Engineer from Yale in 1896.

After graduation he was employed on general city engineering work in and around New Haven; later going to Binghamton, N. Y., where he became a member of the Chenango Engineering Company, and was engaged in preliminary surveys, estimates of cost, and estimates of traffic for thirty-two miles of steam road. From July, 1898, to May, 1903, he left engineering and became connected with Connecticut State Department of Insurance, and, during the last year, was in charge of actuarial work. In the succeeding twenty months he suffered an illness, after which, in March, 1905, he became connected with the New Haven Gas Light Company as inspector on heavy foundations and building construction on contracts involving several millions of dollars. In August, 1906, he went to the University of Maine, at Orono, as instructor in civil engineering. He soon won promotion, and in 1908 became professor of railroad engineering. In 1916 his title was changed to professor of civil engineering.

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[1] Memoir prepared by Harold S. Boardman, M. ASCE.

Although Professor Brown was an excellent teacher and successful in his work at the University, he never lost his desire for a business life. He resigned in 1919 and returned to New Haven, where he became associated with Frederick M. Ward Company, real estate and mortgages. He remained until 1928. The following year he was state appraiser for the United States Bond and Mortgage Corporation of Connecticut. In 1929 and 1930 he was state appraiser for the Connecticut Mortgage and Title Guaranty Company of New Haven. In 1930 he went into the mortgage department of the Connecticut Savings Bank of New Haven, and he was connected with this organization when he passed away.

As a teacher, Professor Brown was dignified, courteous, and sympathetic; he was keenly aware of the thoughts of those about him, and oftentimes took the troubles of the students as his own. In his business life his chief desire was to be of assistance to his clients without thought of reward. To those who knew him well he was a true friend, and his word was "as good as his bond."

He was interested in music, played the piano with more than average ability, and also was trained in vocal music.

He was a charter member of the Yale chapter of Sigma Xi, and a member of Tau Beta Pi, Phi Kappa Phi, and the Connecticut Society of Civil Engineers.

On October 21, 1896 he was married to Maude Manwaring; he is survived by his widow, and a daughter, Madeleine Brown.

Mr. Brown was elected an Associate Member of the American Society of Civil Engineers on February 4, 1914. He became a Life Member in January, 1944.

LEONARD DEMPSTER BROWNELL, Assoc. M. ASCE[1]

Died June 12, 1945

Leonard Dempster Brownell was born in Lassellsville, N. Y., on September 23, 1877, the son of Ezekiel Milton Brownell and Julia (Dempster) Brownell. He attended schools in Lassellsville and Gloversville, N. Y., and, in 1903, was graduated from Purdue University, Lafayette, Ind., with a degree of Bachelor of Science in Civil Engineering. He was awarded a Master's degree in Civil Engineering in 1910.

Mr. Brownell began his engineering career in 1899, as rodman with the Cincinnati, Hamilton, and Dayton Railroad Company at Cincinnati, Ohio. During vacations of his college years, he worked also as rodman for the

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[1] Memoir prepared by William Robinson, Dist. Engr., State Dept. of Public Works, Syracuse, N. Y.

Delaware, Lackawanna and Western Railroad Company at Binghamton, N. Y. Following graduation from Purdue, he was, from June, 1903, to July, 1904, local manager of the Badger Coal Company at Beloit, Wis. In August, 1904, he entered the engineering service of New York State and remained in the state service until his death. His first assignment was at Syracuse, N. Y., as leveler in the department of the state engineer and surveyor. From March, 1906, until January, 1912, he held the title of assistant engineer and from that date until May, 1915, he was resident engineer in the same department. During these years his work was in connection with highway surveys, construction, preliminary studies for the Barge Canal structures, and hydraulic investigations connected with the Barge Canal improvement.

In May, 1915, he was transferred to the newly organized New York State Highway Department, and was in charge of construction and maintenance on various highways of the state system. His headquarters were at Syracuse. In February, 1920, he was appointed district engineer with headquarters at Utica, N. Y. After 1924 his duties were enlarged to include supervision of both highways and canals in the six counties comprising district No. 2. During this period he had an important part in the development of the present New York State Highway System.

In 1906 Mr. Brownell was married to Mabel Duane. He is survived by a son, Leonard Dempster Brownell; and two daughters, Jane (Mrs. Kenton Flint) and Emily (Mrs. Russel Dunmore). Mr. Brownell was a Thirty-Second Degree Mason and a member of Ziyara Temple. He was a member of the City Club of Utica and of the Sigma Alpha Epsilon fraternity. He was also an active member of the North Atlantic States Association of Highway Officials, and of the New York State Association of Highway Engineers.

Mr. Brownell was elected an Associate Member of the American Society of Civil Engineers on November 6, 1907.

WALLACE LYON CALDWELL, Assoc. M. ASCE[1]

DIED APRIL 2, 1945

Wallace Lyon Caldwell was born at Indianapolis, Ind., on July 23, 1888, the son of pioneer Indiana parents, William Caldwell and Elizabeth (Braden) Caldwell.

After completion of his education he taught for two years in the Manual Training High School of Indianapolis and was for three years engineering chemist for the City of Indianapolis. He left this position in 1912 to come to Birmingham, Ala., as paving chemist for the Pittsburgh Testing Laboratory. In 1913 he was advanced to manager of the Birmingham office. During this assignment he was the laboratory's paving consultant for the entire

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[1] Memoir prepared by a Committee of the Alabama Section consisting of J. H. Mayer, M. ASCE, Chairman, R. D. Jordan, M. ASCE, and J. H. Conzelman, Assoc. M. ASCE.

United States. In 1921 he went to Louisville, Ky., as president of the Kentucky Rock Asphalt Company where he remained until 1927. Through his knowledge of the paving industry he greatly improved the technique of control and production of this native rock asphalt; this work had far-reaching effects. From 1927 to 1928, he was vice-president of the Peoples Light and Power Company. His duties with this company took him to all parts of the United States. He returned to Birmingham, in 1928, as president of the Alabama Asphaltic Limestone Company and served in this capacity until his death. He was widely known in Birmingham and the United States. At the time of his death he was also president of the Southern Vacuum Concrete, Inc., the Aerocrete Western Corporation of Chicago, Ill., vice-chairman of the Southern Research Institute, director of the National Association of Manufacturers, and past-president of the Associated Industries of Alabama.

In 1940 he was awarded the Modern Pioneer Award of the National Association of Manufacturers in recognition of distinguished achievement in the field of science and invention which advanced the American standard of living. His work in perfecting machinery which improved the quality and production of light aggregates did much for the industry.

Aside from his membership in the Society he was a member of the American Chemical Society, American Society for Testing Materials, American Society of Municipal Engineers, American Concrete Institute, Association of Asphalt Paving Technologists, and Engineers Club of Birmingham. Mr. Caldwell organized and was the first president of the Boys Club of Birmingham. He was always active in civic affairs and always ready to give of his time and talents to any project for the benefit of Birmingham or the State of Alabama. He was a deacon of the Independent Presbyterian Church and a member of the Rotary Club of Birmingham and the Mountain Brook Country Club.

On November 30, 1911, Mr. Caldwell was married to Leona Templeton of Indianapolis. He is survived by his widow; two daughters, Elizabeth Jane (Mrs. Glenn Means Tindall) and Lenore Lyon Caldwell; two grandchildren, Richard Caldwell and William Lyon Tindall; and two brothers, Dumont and Braden Caldwell.

Mr. Caldwell was a man of exceptionally fine character and moral worth. His attainments as a chemist, engineer, and scientist were widely known and recognized. His advice was freely sought and given in a great variety of scientific and industrial endeavors. This, coupled with his fine executive ability, and his long and varied professional experience and wide culture, made him a consultant of the first order in his chosen life work. The engineering profession has lost one of its most earnest and outstanding members in his death.

Mr. Caldwell was elected an Associate Member of the American Society of Civil Engineers on June 1, 1920.

BENJAMIN LEFEVRE COULSON, Assoc. M. ASCE[1]

DIED FEBRUARY 24, 1946

Benjamin LeFevre Coulson, the son of William and Martha (LeFevre) Coulson, was born on December 21, 1870, near Mason, Ohio. His parents died during his childhood, and his education was guided by an uncle, Benjamin LeFevre. He first attended the public schools near Mason and later Phillips Exeter Academy at Exeter, N. H. He was graduated from Rensselaer Polytechnic Institute at Troy, N. Y., in 1893.

Mr. Coulson was city engineer of Sidney, Ohio, from 1894 to 1897. In this capacity he designed and supervised the sanitary system for the city; and, serving as chief engineer of the Sidney Sewer Commission, from 1921 to 1923, he supervised the enlargement and modernization of the system. Mr. Coulson was an instructor in the Columbia Summer School of Surveying, at New York, N. Y., in 1897.

He moved to Hodgenville, Ky., in 1898, where he built and managed the Hodgenville Manufacturing Plant and the Hodgenville Electric Plant until 1901. In 1901 he supervised special track layouts for the Paige Iron Works, Harvey, Ill. From April, 1902 to March, 1906, Mr. Coulson was assistant engineer for the Louisville and Nashville Railroad Company. His important assignments, while serving in this capacity, were: Engineer in charge of improvements at Columbia, Tenn.—constructing new passenger and freight depots and a coaling plant, eliminating grade crossings, and enlarging and rearranging yards; and engineer in charge of construction of the new terminal at Knoxville, Tenn. From March, 1906, to May, 1907, he was employed as assistant engineer by the Erie Railroad Company in charge of the Mansfield and Ontario (Ohio) improvements, consisting of double tracking and rearrangement of yards and switching facilities at Mansfield. From May to December, 1907, Mr. Coulson was associated with the late John F. Stevens,[2] Past-President and Hon. M. ASCE, as division engineer on the valuation of the New York, New Haven, and Hartford Railroad. In 1907 and 1908, he served in the same capacity for the Erie Railroad.

Mr. Coulson was appointed professor of civil engineering at the University of the South at Sewanee, Tenn., teaching there from 1919 to 1924. During this time he designed and supervised the construction of a new Science Hall, to complete the quadrangle of the campus. He returned to valuation of railroad properties from 1914 to 1916, and was in charge of a division of such work for the Louisville and Nashville Railroad Company.

He was associated with the Jennings Lawrence Engineering Company of Columbus, Ohio, from 1916 to 1920—supervising the layout and construction of streets and sanitary systems for subdivisions and in charge of the construction of the sanitary system for the cantonment at Chillicothe, Ohio, in 1917. From 1923 to 1930, he was engaged in private practice at Cincinnati, Ohio. Mr. Coulson retired from active service in 1930, and in 1932 moved to Lakeland, Fla., where he had business and grove interests, remaining until his death. He was a member of All Saints Parish of the Episcopal Church at Lakeland, and F. and A. M. Lodge of Mason, Ohio.

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[1] Memoir prepared by W. H. Coulson, County Extension Agent, Co-operative Extension Work, State of Ohio, Woodsfield, Ohio.

[2] For memoir, see *Transactions*, ASCE, Vol. 109, 1944, p. 1440.

Benjamin LeFevre Coulson was married to Mary E. Leadley in June, 1893. To this union two sons were born, William Henri and James. He is survived by both sons and a sister, Nettie (Mrs. F. J. Canty).

He retained an active interest in, and prided himself on keeping posted on, engineering techniques. He was possessed of a poised, well-balanced, and active mental capacity and a cheerful disposition to the end.

Mr. Coulson was elected an Associate Member of the American Society of Civil Engineers on May 6, 1908. He became a Life Member in January, 1941.

BERNARD REUVELT CROWLEY, Assoc. M. ASCE[1]

Died November 28, 1944

Bernard Reuvelt Crowley, the son of Jess Crowley and Maude (Davis) Crowley, was born on September 10, 1905, at Paris, Tex. From 1922 to 1926 he was a student in the Agricultural and Mechanical College of Texas, at College Station, Tex., studying textile engineering.

He began his civil engineering career in 1928, when he was employed by the Texas Highway Department at Paris, as chainman and rodman on highway location and construction projects. His entire professional career was in connection with this department where he was promoted through the various grades to resident engineer. In these various capacities he handled all types of surveys, plans, and construction incident to the development of a modern highway system.

Mr. Crowley's outstanding characteristics were honesty, industry, and loyalty. He was a good student and had the courage of his convictions, which enabled him to defend his decisions stoutly.

On December 24, 1933, he was married to Willa Merle Johnson, daughter of Mr. and Mrs. Era Johnson of Avinger, Tex. He is survived by his widow and one daughter, Marilyn Johnson Crowley.

Mr. Crowley was elected an Associate Member of the American Society of Civil Engineers on July 8, 1941.

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[1] Memoir prepared by James E. Pirie, M. ASCE.

ERNEST BUEL DAY, Assoc. M. ASCE[1]

Died February 28, 1945

Ernest Buel Day was born in the Town of Jericho, in the State of Vermont, on April 14, 1884. His father, Hiram Byron Day, was also born in Jericho, and his mother, Persis (Goodwin) Day, in Milton, Vt. His ancestors settled in Vermont. A monument in Underhill, Vt., commemorates members of the Brown branch of his family who were twice kidnapped by the Indians, but escaped each time. Mr. Day, himself, served five years with the Seventh Regiment of the New York National Guard.

Orphaned at the age of nine, he went to live with an uncle in Fort Worth, Tex. He later attended the Manual Training School of St. Louis, Mo., where he was the valedictorian of his graduating class. In 1906 on his graduation from the evening course in general science of Cooper Union for the Advancement of Science and Art, in New York, N. Y., he received the degree of Bachelor of Science; in 1919, the postgraduate degree of Civil Engineer was conferred on him.

From July, 1901, to August, 1906, he was a draftsman with the Manhattan Elevated Railroad Company of the City of New York; and, from August, 1906, to February, 1909, he served in a similar capacity with the Board of Education of the City of New York.

On February 26, 1909, he became a draftsman in the engineering department of the Hudson and Manhattan Railroad Company, New York City; a designer in September, 1909; an assistant engineer in December, 1911; and engineer of design in March, 1921, holding that position with this company until his demise on February 28, 1945. He was a licensed professional engineer in the State of New York.

While in charge of the design department of the Hudson and Manhattan Railroad Company, he was given many important assignments. He designed a number of this company's stations and buildings above and below ground surface as well as additions and changes to tunnel structures, improvements to rolling stock, and other numerous problems relating to railroad practice.

On the day of his death he went to his office apparently in good health, but passed away suddenly in the afternoon. He is survived by his widow, the former Eleanor Hubbard Miller, to whom he was married on April 2, 1915; and a daughter, Marjorie.

A man of tireless energy, and of pleasant instant response, he was greatly respected and esteemed by the management and the employees of the Hudson and Manhattan Railroad Company, which it will be noted he served for a period of thirty-six years.

Mr. Day was elected an Associate Member of the American Society of Civil Engineers on April 2, 1913.

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[1] Memoir prepared by James Forgie, M. ASCE.

HARRY KERCHEVER DEVLIN, Assoc. M. ASCE[1]

Died May 8, 1945

Harry Kerchever Devlin was born in Toronto, Ont., Canada, on July 23, 1891, the son of James Kerchever and Minnie (Carney) Devlin.

Mr. Devlin received his training as civil engineer at McGill University, Montreal, Que., Canada. On January 11, 1915, he enlisted in the Sixth Canadian Engineers in World War I. Upon arriving in England he was transferred to the British Field Engineers, with whom he served throughout the war with rank of First Lieutenant. While in France, as field engineer, his work covered such matters as army bases, water supply, railheads, casualty clearing stations, hospitals, roadwork, and narrow gage railroads. On returning to the United States he became field engineer with the U. S. Structural Company of Detroit, Mich., and was in charge of the layout for the construction of the Hudson motor car plant.

In recent years, and until his last illness, he was chief estimator with Gibbs and Hill, Inc., of New York, N. Y., for foundations, abutments, overhead bridges, catenary structure, and substations in the electrification of the Pennsylvania Railroad from New York to Washington, D. C. He was also estimator in connection with construction of power stations, transmission lines, and various industrial projects. These included, for example, a power station at San Antonio, Tex., the Lake Charles (La.) power plant, the Dilles Bottom plant, and the power station of the Jones-Oslow Power Corporation; a transmission line of the Consolidated Edison Company in Brooklyn, N. Y.; and industrial plants of the Detroit Sulphite Pulp and Paper Company, the American Viscose Company, and the Utica Knitting Company.

Of his earlier connections may be mentioned the U. S. Structural Company (1919-1923), where he was field engineer in charge of the layout of the Dayton Power and Light Company's station, and of the Hellgate power station; he was estimator and job runner on the construction of the extensive Coral Gables development in Miami, Fla. He was also engaged on the construction of schools and hospitals in New York State in the employ of Deriso Brothers, Inc. For Starrett Brothers and Eken, he worked on the Williamsburg housing project. As estimator for A. L. Hartridge of New York, he was associated, among other things, with seven projects at the New York World's Fair. Among his first activities in 1940–1941 was the preparation of estimates for competitive bidding on Curtiss-Wright Corporation plants; this was followed by estimates for the Trinidad Naval Base prepared on behalf of the James Stewart Associates of New York.

For avocations Mr. Devlin turned to hunting, fishing, and boating. He built his own sailboats and motorboats. In his high school days and at McGill University he played on the hockey team, and in one season won a special prize.

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[1] Memoir prepared by Alfred J. Lotka, Asst. Statistician, Metropolitan Life Insurance Co., New York, N. Y.

He was a member of Keystone Lodge, of the Masonic Order in Sault Ste. Marie, Ont., Canada; a member of the Society of American Military Engineers; and a member of the Technical Valuation Society. A mere list of Mr. Devlin's activities, such as here recited, although testifying to his varied experience and his excellence in his profession, gives only a very incomplete picture of the man in his total human relations. His was the modesty that so often accompanies high competence; and the personal worth that made him a friend firm and true. The writer deeply appreciates the privilege of having been chosen to pay these last respects to him.

Mr. Devlin is survived by his widow, Florence Perkins Devlin.

Mr. Devlin was elected an Associate Member of the American Society of Civil Engineers on May 11, 1936.

EMERSON DOLLIVER, Assoc. M. ASCE[1]

Died December 2, 1944

Emerson Dolliver was born in Milwaukee, Wis., on April 19, 1897, the son of John Walter and Marth (Fernekes) Dolliver. His grandfather, Capt. John Sargent Dolliver, sailed around Cape Horn in 1850, was a pilot on the Yangtze River in China until 1864, and for forty-six years thereafter was a bar pilot at San Francisco, Calif. One of his most noted exploits was the rescue of the English ship, *Jessomene,* during the great hurricane of February, 1891. Mr. Dolliver's father likewise followed the sea, but forsook it to study in Germany, afterward returning to the United States and settling in Milwaukee. Six months after Emerson was born, he moved to San Francisco where he practiced architecture until his death in June, 1927.

Emerson Dolliver obtained his early schooling in Marin County, California, being graduated from the Tamalpais High School in 1916. Entering the service of his country on October 16, 1917, he was first assigned to the 28th Engineers. He left the United States on January 3, 1918, assigned to the Transportation Headquarters Troop, 2d Army Corps, and saw active service with the American Expeditionary Forces in the Somme Offensive, the Ypres-Lys Defensive Sector, and other operations in Belgium. He also took part in the attack on the Hindenburg Line (Bellicourt, Nauroy, and Bony) from September 29 to October 2, and in the attack on Brancourt, St. Souplet, Selle River, and Mazingham (with Corps troops) from October 7 to 21. As a Private First Class, he landed in the United States on February 19, 1919, and received his honorable discharge at the Presidio in San Francisco on March 18, 1919.

He entered the College of Engineering of the University of California at Berkeley, Calif., and was graduated in 1923 with the degree of Bachelor of

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[1] Memoir prepared by Stewart Mitchell, M. ASCE, and William H. Holmes and Theodore Neuman, Assoc. Members, ASCE.

Science in Civil Engineering. He was a member of the Alpha Chi Rho fraternity. After graduation he was employed by the county engineer of Marin County on highway location and bridge design. As a member of the contracting firm of Dolliver and Cuffe, in San Rafael, Calif., he was engaged in building roads and small structures for the county and for the town of Larkspur, Calif. In 1927, he was employed by the Wallace and Tiernan Company, of Newark, N. J., as sanitary engineer, and later became their district manager with headquarters at Seattle, Wash. At the close of 1932, he went on active duty as a Second Lieutenant in the U. S. Army Reserve Corps serving with the Civilian Conservation Corps, and later was employed as an engineer by the Federal Land Bank. From August, 1936, to March, 1938, he was employed by Charles H. Lee, M. ASCE, as assistant hydraulic engineer on the construction of works on Treasure Island, San Francisco, for the Pan-Pacific Exposition.

On April 1, 1938, Mr. Dolliver was appointed an assistant hydraulic engineer in the Division of Water Resources of the California Department of Public Works. Four years later, he advanced to the grade of associate hydraulic engineer and served as office engineer for the Department of Dams until his death.

Emerson Dolliver had many interests outside of his daily work. He was an active member of the Sacramento Section of the Society, serving on the Committee on Juniors in 1939, on the Program Committee in 1940, and as Secretary in 1941. He was a delegate to the Local Section Conference in San Diego, Calif., that same year. Intensely interested in the subject of mapping, he spoke before the Local Section in February, 1939, on the subject of "A Plane Coordinate System for California." At the time of his death he was chairman of the Aerial Mapping Project Committee of the State Reconstruction and Reemployment Commission.

His interest in the welfare of boys was outstanding as shown by his activities in scouting. He had been a boy scout himself and had served as an assistant scoutmaster. After becoming a resident of Sacramento, Calif., he assumed a leading part in this group, first as a troop committeeman and then as the promoter and organizer of Cub Packs. In January, 1943, he was made a board member on the Golden Empire Council of the Boy Scouts of America and held this position until his death. In addition to scouting, Mr. Dolliver found time to take part in many other civic activities such as civilian defense work following Pearl Harbor. Those with whom he worked will miss the friendly nature and sense of humor that lightened many tasks.

On September 19, 1925, he was married to Kathrine Green of Berkeley. Mrs. Dolliver and their three sons, Robert Emerson, Richard Sargent, and Peter Clarke, survive him.

Mr. Dolliver was elected a Junior of the American Society of Civil Engineers on January 14, 1924, and an Associate Member on March 11, 1929.

THOMAS JONES HENDRIX, Assoc. M. ASCE[1]

Died December 31, 1944

Thomas Jones Hendrix, the son of Clyde and Estelle Hendrix, was born in Decatur, Ala., on February 8, 1909. After completing his high school course in the local public schools, he entered Alabama Polytechnic Institute at Auburn, from which he was graduated in 1930 with the degree of Bachelor of Science. He was a member of the Pi Kappa Alpha fraternity and his college career was distinguished both by his scholastic attainments and the wide range of his extracurricular activities.

In June, 1930, he was engaged by the Alabama State Highway Department as an inspector and draftsman on all stages of highway and bridge construction. In 1933, he was promoted to the position of assistant office engineer, in charge of laying out, planning, and supervising highway engineering and construction. In May, 1939, his outstanding ability was recognized by his elevation to the position of executive head of division shop and warehouse, in charge of repair, maintenance, and proper dispatching of all equipment and materials allocated to the State Highway Department. Mr. Hendrix occupied this position until he entered the military service of the United States in 1943.

During the early part of 1942, the shipbuilding division of the Decatur Iron and Steel Company in Decatur, which had been awarded contracts for the construction of landing craft for the United States Navy, offered Mr. Hendrix the position of chief engineer, in charge of all hull construction and mechanical installations on such vessels. Mr. Hendrix declined this offer with the statement that he felt compelled to give his services to the Armed Forces.

In January, 1943, Mr. Hendrix accepted an appointment as Second Lieutenant, Corps of Engineers, Army of the United States, and proceeded to Camp Claiborne, Louisana, for his initial training. Following the successful completion of this course, he was transferred to Camp Grant, Illinois, for additional training. In July, 1943, he was promoted to the rank of First Lieutenant and received a letter of commendation for the superior manner in which he performed his duties at this installation.

Lieutenant Hendrix proceeded to an overseas destination in March, 1944, with the 1301st Engineers (General Service Regiment), as engineer officer. In July, 1944, he was promoted to the rank of Captain and thereafter served as planning and training officer (S-3) of his regiment, which was assigned to the Third Army, during battles through France, Belgium, and Germany. In recognition of the heroic manner in which he performed his duties in the course of these engagements against the enemy, he was awarded the Bronze Star Medal.

In January, 1945, his family was notified that Captain Hendrix had been killed in action on December 31, 1944. He had served the United States faithfully as an officer in the Corps of Engineers and died, as he would have wished, serving his country in time of war.

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[1] Memoir prepared by R. T. Harris, Pres., Southeastern Metals Co., Decatur, Ala., assisted by John A. Caddell, Attorney at Law, Decatur, Ala., and Seybourne H. Lynne, U. S. District Judge, Birmingham, Ala.

Captain Hendrix was a leader in every phase of community life. His outstanding characteristic was his unselfish love of his fellow man. His death in the line of duty both shocked and inspired all who knew him.

In 1935, Mr. Hendrix was married to Helen Wallace, who survives him.

Captain Hendrix was elected an Associate Member of the American Society of Civil Engineers on November 13, 1944.

HARRY LEGRAND HILTON, Assoc. M. ASCE[1]

DIED MAY 20, 1945

Harry LeGrand Hilton was born in Omro, Wis., on July 13, 1876, the son of William Harley and Mattie Hilton.

He was educated in the Omro Public Schools, and after graduation he entered the State Normal School at Oshkosh, Wis. Two years later, he entered Lawrence Scientific School, Harvard University, at Cambridge, Mass., where he majored in civil engineering, completing his education at that institution in 1902.

Mr. Hilton was first employed by J. H. Wallace and Company at the Portsmouth, N. H., plant, first as a stenographer, then testing cement, running transit and level, inspecting steel erection, etc. In 1903 he was transferred to the New York, N. Y., office of that firm as a draftsman.

Late in 1903, Mr. Hilton was appointed assistant city engineer of Bangor, Me. He served in this capacity for four years, during which time he completed design, plans, and specifications; and he had direction of construction of sewer extension lines, water lines, heavy building foundations and retaining walls, and the design and erection of steel trestle sidewalks.

From 1907 to 1910, Mr. Hilton was employed as a civil engineer for the Quartermaster's Corps of the U. S. Army. During this time, under Maj. J. M. Carson, Jr., Mr. Hilton was superintendent of construction on the $10,000,000 enlargement project of the U. S. Military Academy at West Point, N. Y.

In June, 1910, Mr. Hilton transferred to the United States Navy department where, until March, 1917, he served as civilian assistant and expert aide to the Public Works Officer of the Navy Yard at Portsmouth. It was during these years, when he served as chief draftsman, head of survey parties, chief inspector on the Bureau of Yards and Docks contracts, and administrative assistant that Mr. Hilton gained the background of wide experience in naval administration and procedure which served him so well during his long career as an officer in the U. S. Navy.

On April 11, 1917, Mr. Hilton was commissioned a Lieutenant in the Civil Engineer Corps of the U. S. Navy. He was, until 1920, assistant to the Public Works Officer of the Navy Yard, Portsmouth, and in this capacity he assisted in the construction and administration of extensive facilities which included the naval hospital and the railway and highway bridge at Portsmouth.

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[1] Memoir prepared by Alden K. Fogg, Assoc. M. ASCE.

In 1920, Lieutenant Hilton was transferred to the Navy Yard at Boston, Mass., where he served as assistant to the Public Works Officer at that station until December, 1922. He was then transferred to the Bureau of Yards and Docks in Washington, D. C., to assist in the preparation of the Bureau manual.

This important work was completed in May, 1923, at which time Lieutenant Hilton was assigned to duty as assistant to the Public Works Officer of the Eleventh Naval District at San Diego, Calif. From this duty, Lieutenant Hilton was ordered to the Naval Air Station at Pensacola, Fla., in 1926. In the capacity of Assistant Public Works Officer at that station, he supervised the construction of the Bayou Grande Bridge at Pensacola. It was in June, 1930, that he was transferred as Assistant Public Works Officer of the Navy Yard, Pearl Harbor, Territory of Hawaii, where he served until June, 1932.

Lieutenant Hilton was transferred in July, 1932, to the post of Public Works Officer of the Submarine Base, New London, Conn., where he was stationed until September, 1935. During this tour of duty, he supervised the design and construction of the sewage disposal plant at that station.

In 1935 Lieutenant Hilton was transferred to the Third Naval District in New York, where he served as Assistant Public Works Officer until June, 1939, at which time he received his promotion to the rank of Lieutenant Commander. He was then given duty as Public Works Officer of the Naval Air Station, Terminal Island, Calif. He was instrumental in negotiating part of the initial leases of land which marked the expansion of the Terminal Island Air Base to its present proportions. Lieutenant-Commander Hilton remained at this post until he was retired from active duty on August 1, 1940, having reached the statutory age limit.

Upon being recalled to active duty on October 25, 1940, Lieutenant-Commander Hilton was assigned to duty in the War Plans Section of the Eleventh Naval District, San Diego. In this assignment Lieutenant-Commander Hilton, with his broad and comprehensive knowledge of the naval shore establishment, was of great assistance in advanced planning for facilities and camps throughout Southern California.

After completing his assignment in the War Plans Section, he was made Executive Assistant Public Works Officer of the Eleventh Naval District. He was promoted to the rank of full Commander on April 20, 1942.

In 1940 he had returned from a well-earned retirement to take up the service of his country. He died in the performance of his duty on May 20, 1945, at the U. S. Naval Hospital in San Diego. He had spent a lifetime in the service of his nation and had done his work well.

He had aided in the design and construction of vast facilities for the training and equipping of our fighting men. He had materially aided the expansion of the Naval Shore Establishment from its peacetime standards to a mighty war machine.

Commander Hilton was a member of the Society of American Military Engineers. Always interested in the welfare of handicapped children, he was active in supporting the Shrine Crippled Childrens Home at San Francisco, Calif. He was keenly interested in Society affairs and was a frequent speaker at Society meetings.

On October 19, 1903, Commander Hilton was married to Helen C. Locke of Portsmouth, who survives him.

Commander Hilton was elected an Associate Member of the American Society of Civil Engineers on June 1, 1909.

ALBERT HOWARD HORTON, Assoc. M. ASCE[1]

DIED MARCH 4, 1945

Albert Howard Horton, the son of Elmer and Christiana (Thompson) Horton, was born near Fairview, Butler County, Pa., on July 24, 1875. He was a descendant of Roger Williams, and his ancestors were closely connected with the early development and settlement of New England and New York. Shortly after his birth his parents moved to Silver Creek, N. Y., where they were pioneers in the development of the later renowned vineyards of western New York. After receiving his preparatory education in the high school at Silver Creek, he attended the College of Civil Engineering, Cornell University, Ithaca, N. Y., from which he was graduated with the degree of Civil Engineer in June, 1898.

His first engineering employment was with the U. S. Board of Engineers on Deep Waterways. During the period of about one year with the Board of Engineers, he was engaged on field surveys in central and northern New York. These surveys were in connection with improvements to the old Erie Canal, over which his grandfather, Albert Horton (for whom he was named) had migrated westward.

From 1899 to 1903 he was with the U. S. Lake Survey. During this period he was identified with the early stream-flow measurements on the St. Clair, Niagara, and St. Lawrence rivers. During his four years with the U. S. Lake Survey, he developed at least two characteristics which remained with him through life—namely, a passion for accuracy in stream-flow measurements and a capacity for giving written and oral orders which he expected to be followed implicitly.

In May, 1903, he was appointed an assistant engineer in the U. S. Geological Survey and was engaged on surveys and estimates for the Shoshone and Utah Lake projects of the U. S. Reclamation Service, then a part of the Geological Survey but later a bureau of the Department of the Interior. Except for the two years or less spent on those early western irrigation projects in Wyoming and Utah, all Mr. Horton's work was in the central and eastern portions of the United States.

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[1] Memoir prepared by William G. Hoyt, M. ASCE.

In 1905 he was engaged in making discharge measurements in the Central Atlantic States. During the same year he was in charge of a field party which surveyed the Roanoke River from the mouth to its headwaters at the request of, and in cooperation with, the state geologist of Virginia.

In January, 1906, he became district hydrographer in charge of the Upper Mississippi River District, consisting of the states of Minnesota, Wisconsin, Michigan, and Illinois, with headquarters in Chicago, Ill., where he worked in cooperation with the states and the Chicago Sanitary District. One of his important contributions during this period was an intensive investigation of errors resulting from the use of dams as gaging stations. He determined that, as a result of leakage, obstruction to gates, and other causes, the records based on computations of flow through openings in dams were likely to be grossly in error. In 1907, when it became necessary to close the Chicago office because of lack of funds, Mr. Horton was transferred to the Washington, D. C., office and later to Newport, Ky., where he was district engineer in charge of surface-water investigations in the Ohio River basin.

In connection with flood measurements of the Ohio River, he developed a reel and crane for handling the heavy weights needed to hold the current meter in its proper position during the observations of velocity. Although reels and cranes are used extensively, Mr. Horton was the first to use them in Geological Survey work and took an active part in developing them as efficient tools in stream gaging. His earlier experiments on the U. S. Lake Survey in measuring deep rivers "stood him in good stead." After August, 1913, he supervised the Ohio River work from headquarters in Washington, D. C. While district engineer of the Ohio River District, he made an important contribution to hydrologic literature through the analysis of fragmentary records of stage and discharge of the Ohio River and its tributaries, which led to the compilation of long-time records of these important waterways.

At the beginning of World War I, no one had reliable information as to the installed power resources of the United States, and there was no over-all record of electric power output or of fuel consumption. To meet the war needs the Division of Power Resources was organized in the Geological Survey on July 1, 1918. In August, 1918, Mr. Horton was detailed to this division, and on July 1, 1919, he became its chief. One of the first tasks that was undertaken was the compilation of a mailing list of all concerns engaged in business as electric utilities. Thereupon was started a collection of data pertaining to the installed capacities, fuel consumption, and output of all electric power plants in the United States. The records collected by the Division of Power Resources were of inestimable value in connection with the prosecution of World War I. Following the Armistice the Survey continued the collection and compilation of statistics and the dissemination of information concerning power output and fuel consumption. Mr. Horton continued this work until July, 1936, when the activities relating thereto were transferred to the Federal Power Commission.

The studies and analyses by Mr. Horton focused attention for the first time on the wide variations in the consumption of fuel per kilowatt hour and to the wide differences in fuel consumption between efficient and inefficient equipment and methods. Mr. Horton must be given some of the credit for the remarkable increase in the efficiency of power plants obtained during the decade from 1920 to 1930. On July 1, 1921, he became district engineer in charge of surface water investigations in the Middle Atlantic District, with headquarters at Washington, D. C., until June, 1940, when they were moved to College Park, Md. During this period his position as chief of the Division of Power Resources was continued.

Bert, as he was affectionately called by his friends, was a keen student and an enthusiastic worker. His wit and dry humor and his outstanding personality were recognized at all gatherings of Survey engineers. This was especially so at the conferences of district engineers, which he attended for many years. Many of the improvements in the technique of stream gaging, especially in construction methods and procedures were brought about through his advice and suggestions. One of his most recent suggestions, for which he received a substantial cash award from the Board of Suggestions Awards, Department of the Interior, related to a method of marking the number of days remaining on supply rolls of paper for water stage recorders.

Of his nearly forty-seven years in federal employment, forty-two years were with the Geological Survey. Mr. Horton trained a considerable number of the present district engineers of the Survey, and many of the engineers in charge of responsible work. W. E. Wrather, director, Geological Survey, wrote of him as follows:

> "Mr. Horton for many years has been one of the highly respected engineers of the Geological Survey on whom the Directors, Dr. Smith, Dr. Mendenhall, and I have relied for information, especially in relation to power matters, not only in this country but throughout the world."

Harold L. Ickes, Secretary of the Department of the Interior, wrote that:

> "* * * in his service with the Geological Survey for over 40 years, he was always faithful and conscientious in the performance of his many and varied duties. His technical skill and judgment will be greatly missed."

Although Mr. Horton had not been in the best of health for several years, he had been critically ill for only a short time prior to his death at Garfield Hospital, Washington, D. C., on March 4, 1945. Few had realized the seriousness of his illness.

Mr. Horton was married to Althea Wilbor Briggs on October 25, 1905. Their marriage was an extremely happy one. Surviving Mr. Horton are his widow; a daughter, Adah (Mrs. Thomas Wriggins); a son, John Horton, associate engineer, U. S. Geological Survey (John is following in his father's footsteps); a brother, Judge Clinton Horton, referee judge of the State Supreme Court, Buffalo, N. Y.; and four grandchildren, Natalie Ann Wriggins, Adah Virginia Wriggins, Thomas Wriggins, III, and John Michael Horton.

He was a member of the Cosmos Club, Washington, D. C., and the Washington Society of Engineers.

Mr. Horton was elected a Junior of the American Society of Civil Engineers on June 3, 1902, and an Associate Member on March 6, 1907.

CHARLES PORTERFIELD KAHLER, Assoc. M. ASCE[1]

Died February 25, 1946

Charles Porterfield Kahler, the son of Charles P. and Harriet Agnes (Moran) Kahler, was born on November 25, 1880, in Baltimore, Md. He attended the Baltimore City College and the Maryland Institute and was graduated from the University of Notre Dame, Notre Dame, Ind., with the degree of Bachelor of Science in Civil Engineering, in 1904.

He then entered the employ of the Cleveland, Cincinnati, Chicago and St. Louis Railway Company, as assistant engineer, and in 1906 became assistant engineer for the Oregon Short Line Railroad Company at Salt Lake City, Utah. He was appointed electrical engineer in 1912.

In 1931 Mr. Kahler was appointed electrical engineer—system—with headquarters at Omaha, Nebr., having jurisdiction over all Union Pacific Railroad lines. In April, 1945, he became general electrical engineer.

During his railroad service Mr. Kahler participated in the planning of many important projects involving not only electrical works, but also steam power plants, locomotive boiler washing plants, and oil development projects.

Mr. Kahler was married to Mary Heinekamp in January, 1908. He is survived by his widow and daughter, Mary Agnes.

He was a member of the American Association of Railroads (Electrical Section), the American Institute of Electrical Engineers, and the Nebraska Engineering Society.

Mr. Kahler was elected an Associate Member of the American Society of Civil Engineers on March 13, 1917.

MARTIN WILLIAM LAUTZ, Assoc. M. ASCE[1]

Died April 23, 1945

Martin William Lautz, the son of Martin F. and Elizabeth (Bank) Lautz, was born at Buffalo, N. Y., on July 13, 1886.

He studied civil engineering at Princeton University in Princeton, N. J., from 1904 to 1907, and at the Massachusetts Institute of Technology in Cambridge, Mass., from 1907 to 1908, where he was a member of Sigma Chi fraternity.

During summers he worked on surveying parties for the Pennsylvania Railroad; Lackawanna Steel Company; and C. C. Conkling and Company, consulting engineers.

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[1] Memoir prepared by B. H. Prater, Chf. Engr., Union Pacific R.R. Co., Omaha, Nebr.

[1] Memoir prepared by R. J. McIntosh, Mgr. of Sales, Bethlehem Steel Co., Bethlehem, Pa.

After college he spent six years, from 1908 to 1914, as assistant engineer with the Ferederick K. Wing Company of Buffalo, on field work as chief of party on surveys for sewer systems, townsites, electric railways, etc. He was resident engineer on the East Aurora, N. Y., sewer and disposal plant construction.

From 1915 to 1922, he was assistant engineer for the Lackawanna Steel Company working on the development and use of steel sheet piling when the product was just starting to be used generally.

In 1922 he became designing engineer for the Bureau of Water, Filtration Division, City of Buffalo, on concrete design and later was in charge of inspection of the $5,000,000 filtration plant. After completion of this job, he became associated with the John W. Cowper Company, general contractors, as engineer on the construction of the structural Grey Mill for the Bethlehem Steel Company at Lackawanna, N. Y., and as superintendent of construction on a Harrison Radiator Company job at Lockport, N. Y.

In 1930 he became sales engineer for the Bethlehem Steel Company at Bethlehem, Pa., on steel sheet piling and structural steel bearing piles. In this work he collaborated with engineers and contractors on the use of steel piling on many important engineering projects throughout the United States.

He was a member of the Piling Sub-Committee of the American Iron and Steel Institute Committee on Building Codes and of the Committee on Bearing Value of Pile Foundations of the Waterways Division of the American Society of Civil Engineers. At the 1945 annual meeting of the Society in New York, N. Y., Mr. Lautz presented a discussion[2] of Karl Terzaghi's paper on "Cellular Cofferdams."

For many years Mr. Lautz was a member of the Guido Chorus of Buffalo and sang at many of the concerts given by this famous men's chorus including the competitions of the Associated Glee Clubs of America in New York. He was also a member of the Bach Choir of Bethlehem. He was licensed as a professional engineer and surveyor by the State of New York in 1925.

In 1913, Mr. Lautz married Helen Hyde Tobin of Syracuse, N. Y. Mrs. Lautz and two daughters, Helen (Mrs. Albert Weinrich) and Elizabeth, survive.

Mr. Lautz was a huge man physically with a forceful personality and keen intellect. In his work he combined a thorough knowledge of engineering with a sound practical sense. His greatest interest was his family to whom he was devoted. His cheerful and kindly disposition made friends of all who knew him.

Mr. Lautz was elected an Associate Member of the American Society of Civil Engineers on December 15, 1924.

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[2] *Transactions*, ASCE, Vol. 110, 1945, p. 1083.

RANSOME TEDROWE LEWIS, Assoc. M. ASCE[1]

DIED MAY 17, 1945

Ransome Tedrowe Lewis was born at Indianapolis, Ind., on January 22, 1868, the son of George W. and Sarah J. (Tedrowe) Lewis.

He was graduated from Purdue University at Lafayette, Ind., in 1888 with the degree of Bachelor of Civil Engineering, after working his way through college. In 1892, after four years of practical experience, he received the professional degree of Civil Engineer from Purdue.

Like most civil engineers of those days his first engagement was with a railroad. He was in the Maintenance of Way Department of the Corps of Engineers of the Pittsburgh, Cincinnati, Chicago and St. Louis Railroad Company from August, 1888, to March, 1890, at Pittsburgh, Pa. This led to specialization in the field of railway and other structures and later into fabrication of structural steelwork for all types of construction.

After March, 1890, he spent six years with G. W. G. Ferris and Company, engineers and inspectors, of Pittsburgh, as assistant and principal inspector, inspecting structural steelwork at various shops and mills in Pittsburgh and throughout the Middle West. Some of the more important works for which he inspected the steel during this period were: Ohio Connecting Railway Bridge at Pittsburgh; Metropolitan Elevated Railroad at Chicago, Ill.; Halsted Street Lift Bridge at Chicago; Section 3, Park Avenue Improvement at New York, N. Y.; portions of the Boston (Mass.) subway; and the Ferris Wheel and the Administration Building at the Chicago World's Fair.

From March, 1896, to December, 1899, Mr. Lewis was with the Osborn Engineering Company of Cleveland, Ohio, in charge of steel inspection for the Northwestern Elevated Railroad of Chicago, the Elmira Bridge Company, Ltd., Elmira, N. Y., and the Union Bridge Company, Athens, Pa. He then worked at the home office in Cleveland as draftsman, assisting on the design of several Buffalo grade crossing structures, buildings, and railroad structures. In April, 1899, he went back to Elmira in charge of inspection of steel for the Buffalo grade crossing work and also finished the inspection for the Northwestern Elevated of Chicago. From December, 1899, to November, 1900, he was in charge of mill and shop inspection of steel for bridges as chief inspector for the New York Central and Hudson River Railroad Company. He then returned to the Osborn Engineering Company, representing this concern in the Philadelphia District until September, 1901, when he became manager of the Athens Plant of the American Bridge Company, which was then engaged on the steelwork for a New York City subway. In February, 1902, he was transferred to the Elmira plant of the company as manager, a position which he held until his retirement in 1933, almost a third of a century.

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[1] Memoir prepared by G. J. Watson, M. ASCE, with the assistance of F. A. Barnes, M. ASCE.

Even after nearly forty-five years of active practice, his retirement did not mean cessation of activities to Mr. Lewis. In November, 1935, he was made president of the Elmira Precision Tool Corporation, and later was retained in a consulting capacity when the company was absorbed by Remington-Rand, Inc. He retained this position until his death on May 17, 1945.

In 1894, he was married to Etta L. Hillard, who survives him, together with their two children, Joseph H. Lewis, vice-president and general manager of the Blue Ridge Glass Corporation, Kingsport, Tenn., and First Lieutenant Kathleen Lewis, a dietitian in the United States Army.

During his forty-three years of residence at Elmira, Mr. Lewis performed many acts of kindness and helpfulness known only to the recipients. He was particularly interested in the welfare of those working under his direction and they were always at liberty to consult him at either office or home. His keen interest in the community gained him a wide circle of friends and acquaintances in all walks of life.

One of his primary interests was scouting. He served as a Boy Scout Commissioner for many years, and was president of the Elmira Area Council of Boy Scouts for ten years. The esteem in which he was held by his associates in this work was attested by his election as a Silver Beaver Scouter, the highest honor in the gift of any local council.

Other community activities included membership on the Elmira Board of Education, services as chairman of the Board of Trustees of the Steele Memorial Library, and member of the Board of Elders of the First Presbyterian Church from 1915 until his death. He was also a Mason, a Rotarian, and a member of the Flat Tires and the Torch clubs. He was a charter member of the Ithaca Section of the Society and served a term as its President. He was always keenly interested in its activities and faithfully attended its meetings whenever possible.

Mr. Lewis was elected an Associate Member of the American Society of Civil Engineers on June 6, 1906. He became a Life Member in January, 1939.

CLARE SLOAN McARDLE, Assoc. M. ASCE[1]

DIED JANUARY 2, 1942

Clare Sloan McArdle, the son of Lucian and Mittie (Sloan) McArdle, was born on February 9, 1892, at Ferry, Mich., where he obtained his early education. In 1910 he entered Michigan State College of Agriculture and Applied Science at East Lansing, Mich., and was graduated with the degree of Bachelor of Science in Civil Engineering in 1914.

From 1914 to 1917 he served the Department of Highways of the State of Illinois, in various capacities from transitman to assistant division engineer in charge of construction and surveys.

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[1] Memoir prepared by Harry E. Frech, Assoc. M. ASCE.

When World War I was declared, Mr. McArdle went on active duty as a First Lieutenant, Corps of Engineers, at Fort Sheridan, Ill. Later he was transferred to Fort Leavenworth, Kans. He was promoted to Captain and then to Major. As Captain, Corps of Engineers, he was assigned to the American Expeditionary Force, North Russia Expedition, in July, 1918. While serving on the Archangel front in north Russia, he distinguished himself as a capable and resourceful officer as indicated by the Distinguished Service Order for Conspicuous Gallantry and Ability awarded to him by the British Government. The men who served under his command had high praise for his valor and leadership.

On his return from Europe in August, 1919, he was employed by the Illinois State Highway Department, and was placed in charge of four large construction plants operated by the state. As general superintendent of day labor construction, Mr. McArdle built more than 100 miles of concrete roads. During the next five years he was engaged in private engineering and contracting practice.

In 1927 he became western technical service manager for the Atlas Portland Cement Company and later was division sales manager at Birmingham, Ala. Until 1941 he was employed by the Missouri Portland Cement Company of St. Louis, Mo., as general sales manager and as vice-president.

Mr. McArdle's ability as a civil engineer in both military and civilian assignments was held in high esteem by his colleagues. He believed in close application to his work, but found time to take part in outdoor sports. Golf was one of his favorite pastimes and a place in his foursome was much sought after. His ability to make friends and hold them was a great part of his success in his business and professional career.

On April 15, 1924, in Kingsport, Tenn., Mr. McArdle was married to Lucile Dorsey. He is survived by his widow and daughter, Dorothy Ann.

Mr. McArdle was elected an Associate Member of the American Society of Civil Engineers on October 21, 1924.

DONALD GRANT McKIM, Assoc. M. ASCE[1]

Died July 29, 1945

Donald Grant McKim, the son of John R. and Jennie (Grant) McKim, was born on January 9, 1887, at Pickrell, Nebr. He was graduated from Manual Training High School at Kansas City, Mo., in 1906.

From 1909 to 1918, Mr. McKim was connected with several railroad companies in the midwest and southwest on location, construction, and maintenance. He served in the U. S. Army from July, 1918 to March, 1919; and, upon his release, became construction superintendent for the Western Oil Corporation at Burkburnett, Tex., handling the construction of casing-head gasoline plants until 1921. From 1921 to 1927 he was supervisor of bridges and buildings for the Kansas, Oklahoma and Gulf Railroad Company with headquarters at Muskogee, Okla.

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[1] Memoir prepared by W. J. Van London, Engr.-Mgr., Houston Urban Expressways, Houston, Tex.

From 1928 until his death Mr. McKim was resident engineer for the Texas Highway Department in charge of construction of many of the most important bridges constructed in the state during that period.

Mr. McKim was a man of unquestionable integrity and high professional ethics. He was a member of the Episcopal Church, the Masonic Lodge, and the Houston Engineers Club.

In 1920, at Fort Worth, Tex., Mr. McKim was married to Katharine D. Brown of Culpeper Court House, Virginia. He is survived by his widow; his mother; and a sister, Elizabeth McKim.

Mr. McKim was elected an Associate Member of the American Society of Civil Engineers on November 11, 1929.

JOHN DONALD MITSCH, Assoc. M. ASCE[1]

Died January 18, 1946

John Donald Mitsch was born in Boston, Mass., on March 2, 1898. He was the son of John A. Mitsch and Mary (Joyce) Mitsch, both of Pittsburgh, Pa. His early education was obtained in Boston, Mass. He enrolled for one year in the Engineering School of Tufts College in Medford, Mass., and then transferred to the Massachusetts Institute of Technology (M.I.T.) in Cambridge, Mass. Mr. Mitsch was graduated, in 1920, with the degree of Bachelor of Science in Civil Engineering.

After graduation Mr. Mitsch remained at M.I.T. as assistant instructor in civil engineering for two years. He then spent one year with the Barney-Ahlers Construction Corporation of New York, N. Y., as estimator and field engineer on construction in reinforced concrete. In September, 1923, he returned to M.I.T. and remained on the faculty until his death. At M.I.T. he held, successively, the positions of instructor in civil engineering from September, 1923, to June, 1934, assistant professor in civil engineering from July, 1934, to June, 1940, and associate professor of civil engineering from July, 1940, until his death.

Professor Mitsch's major field was structural engineering with emphasis on theory of structures and design of reinforced concrete structures. He also devoted much time to the teaching of surveying with teaching assignments at the M.I.T. Summer Mining Camp at Dover, N. J. (from 1924 to 1934), and at the M.I.T. Summer Surveying Camp at East Machias, Me. (since 1927). In addition to his regular daytime work, he conducted evening classes in surveying problems for the Massachusetts State Department of Education, from 1930 to 1935, and, from 1935, in reinforced concrete design for the

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[1] Memoir prepared by Eugene Mirabelli, M. ASCE.

Lowell Institute at Cambridge. During his career he was engaged at various times as estimator on building construction, as structural designer of buildings and bridges, as consultant for insurance companies, and as expert witness in court cases on surveying and structural design.

His popularity with both faculty and student body led to his appointment on various faculty committees and also to various special administrative duties. In 1935 he was made administrative officer in charge of the Surveying Camp at East Machias. During the war he was liaison officer and adviser to military and naval students at M.I.T. With the approach of the end of the war, he was appointed chairman of the M.I.T. Surplus Property Committee. The duties of this committee were disposal of property on terminated government contracts with M.I.T. and procurement of government surplus property to accommodate M.I.T.'s postwar expansion program.

He took an unusual interest in M.I.T. alumni affairs and served on the Alumni Council and the Executive Committee of the Alumni Association, and for several years was chairman of the Alumni Day Committee.

He was a very active member of the Boston Society of Civil Engineers and served as chairman of many committees. He was also a member of the American Concrete Institute and the Society for the Promotion of Engineering Education. For a number of years Professor Mitsch was faculty adviser to the Student Chapter of the American Society of Civil Engineers. He belonged to Phi Kappa fraternity, held an honorary position in Chi Epsilon, and was a member of the Knights of Columbus.

On January 18, 1946, while returning from a business trip for M.I.T., during which he attended the Society's Meeting in New York, he met his death in an airplane crash in Cheshire, Conn.

Professor Mitsch had a remarkably good understanding of human nature and was always interested in the welfare of both young and old. This characteristic permitted him to make friends easily and to maintain them. He was admired by his students not only as a teacher but as one who took a friendly, personal interest in their affairs. His associates will miss his quiet, good-fellowship, and his dependable counsel and readiness to be of service.

In June, 1938, Professor Mitsch was married to Frances McFaul of Machias, Me. He is survived by his widow; a daughter, Mary Regina; a son, John Donald, Jr.; his mother; three brothers, Walter A., Paul F., and Arthur E.; and a sister, Cyrilla R. (Mrs. Joseph G. Green).

Professor Mitsch was elected an Associate Member of the American Society of Civil Engineers on August 12, 1935.

WILLARD AVERELL POLLARD, JR., Assoc. M. ASCE[1]

DIED FEBRUARY 20, 1942

Commander Willard Averell Pollard, Jr., an officer of the Civil Engineer Corps of the United States Navy, was born in Washington, D. C., on August

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[1] Memoir prepared by R. E. Bakenhus, M. ASCE.

15, 1895. He was the son of Willard A. Pollard who was a native of Washington, D. C., and of Olivia (Guest) Pollard, a native of Baltimore, Md. He lived in a period when the art of design and construction of concrete in various structures was undergoing its greatest development, and he contributed materially to this development. His outstanding work in this line was recognized when the U. S. Maritime Commission named one of its concrete ships in his honor, namely, *The Willard A. Pollard,* on July 30, 1944.

Willard A. Pollard, Jr., chose civil engineering as his profession; he was graduated from Lafayette College, Easton, Pa., in June, 1916, with the degree of Civil Engineer. From that date until January, 1918, he was a concrete designer with Westinghouse, Church, Kerr and Company—Dwight P. Robinson Company. In the meantime, the United States Navy held competitive examinations for appointments, from civil life, of officers in the Civil Engineer Corps. This corps conducts a great variety of work in connection with public works of the Navy, and the examinations are correspondingly broad and difficult. Mr. Pollard stood second in a group of distinguished engineers who were appointed from the large number of competitors. He was commissioned Lieutenant (Junior Grade), Civil Engineer Corps (CEC), U. S. Navy, as of January 12, 1918. He was promoted to Lieutenant as of July 1, 1920; to Lieutenant Commander as of October 2, 1925; and reached the rank of Commander on June 30, 1935.

Upon first reporting he was assigned to temporary duty under instruction at the United States Naval Academy in Annapolis, Md. Upon completion of the course in February, 1918, he was assigned to the Bureau of Yards and Docks and served first as assistant project manager and then as project manager for public works at ordnance stations and the U. S. Helium Production Plant. This assignment included responsible charge of designs, plans, specifications, and estimates for public works, including the technical features of construction.[2] From October, 1921, until 1924, he was Public Works Officer at the Navy Mine Depot and Fuel Oil Station, Yorktown, Va. As plant maintenance superintendent he had duty at the Navy Yard, Philadelphia, Pa., until 1926.

In 1926 he was assigned to the Virgin Islands where, although he was Public Works Officer of the Naval Station, his duties were principally with the civil governor. This was due to the fact that the Public Works Officer was brought into more immediate contact with people of all classes than any other officer under the civil government. In addition, he had jurisdiction over the Civil Engineer Officer at St. Croix, Virgin Islands.

Next he stood duty from 1929 to 1934 at the Naval Operating Base at Norfolk, Va. At this time two concrete barges designed, constructed, and tested under his direction were used with some success as crane barges.

During 1934 and 1935 he attended the Army Industrial College at Washington, D. C., under instruction. This course was intended to establish close liaison and understanding on the part of the Army with civilian industry, and, by assignment of naval officers to these courses, to establish even closer liaison between the Army and Navy, as well as with industry.

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[2] "U. S. Helium Plant," by Willard A. Pollard, Jr., *Public Works of the Navy,* 1920.

Another tour of duty in the Bureau of Yards and Docks occurred from 1935 to 1938. Here as Assistant War Plans Officer and then War Plans Officer, he was instrumental in the conception and development of all types of advance base equipment including barges.

As Public Works Officer of the 15th Naval District, Commander Pollard went to Panama in 1939. Here in conjunction with the Bureau he did outstanding work in the preparation of studies for dry docks which later formed the basis for getting appropriations for construction of dry docks at Pearl Harbor, Hawaii, Puget Sound, Washington, and Mare Island, California.

In January, 1940, his physical condition was such as to require hospitalization in the Gorgas Hospital at Panama. In March he was returned to the United States to the Naval Hospital at Washington, D. C.; and in September, 1940, he was placed on the retired list and relieved of all active duty.

During his career in the Navy he served on numerous boards some of which were the following: Member of Board for Examination of Sites in Southern States to select sites for Marine Corps air stations (1918); member of board to select sites for free-balloon stations; senior member of board to report upon suitability of site at Lakehurst, N. J., for a dirigible construction station (1919); member of Naval Examining Board for Civil Engineers, examining four groups of candidates (1921); member of board to recommend tropical and winter clothing for enlisted personnel (1936); recorder of selection board for officers of CEC, U. S. Navy; and member of board to make studies of naval needs and interests in connection with strategic raw materials.

Commander Pollard belonged to the Phi Beta Kappa and the Tau Beta Pi fraternities. He was a member of the Army and Navy Club of Washington, D. C., and of the Army and Navy Country Club of Arlington County, Virginia. He was a man of an obliging and friendly disposition, always ready to serve a friend. He was essentially modest as to his own attainments, yet positive and vigorous in the execution of his duties and in meeting his responsibilities. His work was always well done—efficiently, promptly, and without ostentation. He gave his country outstanding service during World War I and World War II, and in the preparatory period between.

He is survived by his wife, Julia (Bottum) Pollard, a native of South Dakota, and by three children, Lt. (jg) Willard A. Pollard, III, U. S. Navy, Anne S. Pollard, and Philip H. Pollard.

Commander Pollard was elected an Associate Member of the American Society of Civil Engineers on November 21, 1921.

EDWARD WILLIAM ROBINSON, Assoc. M. ASCE[1]

DIED MAY 5, 1942

Edward William Robinson was born in Hamburg, Germany, on May 13, 1874, the son of the American Consul General at Hamburg and of Rose Weisman. Mr. Robinson attended school in New York, N. Y., Stevens Preparatory School in Hoboken, N. J., and was graduated from Stevens Institute of Technology in Hoboken (class of 1895) with the degree of Mechanical Engineer.

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[1] Memoir prepared by Frederic R. Harris, M. ASCE.

From 1900 to 1905, he was a member of the firm of Smith and Robinson, constructors of river and harbor works in the New York area, and then he joined the Snare and Triest organization, contractors for harbor works. From 1910 to 1917, he was vice-president of the Ruggles and Robinson Company, building constructors in New York. In 1917 he attended the Officers' Training School at Plattsburg, N. Y., being graduated with the rank of Captain, thereafter becoming a Major in the Corps of Engineers. In 1920 he was released from the Army to become vice-president and general manager of the Triest Contracting Company of New York, on subway work, at Queens Plaza, New York; on bridge foundations of the Goethals Bridge; and on work for port improvements at Philadelphia, Pa., and in the Delaware River. In 1935 he became associated with the consulting engineering organization of Frederic R. Harris, Inc., New York, and later became president of Construction Management, Inc., New Jersey, an affiliated company, in which he was active on national defense for the Navy Department and the Maritime Commission. He was also a director of the Federal Water and Gas Corporation and the New York Water Company, both of New York. He was a well-known construction engineer and director of utility companies.

Mr. Robinson was for some time an alumnus-trustee of Stevens Institute of Technology. He belonged to the Engineers Club of New York. He was a member of the Mayflower Society, the New York Chapter of the Sons of the American Revolution, and the Smithtown Country Club.

Edward W. Robinson was first married to Georgia Brock, after whose death, many years ago, he was married to May Olivia Andrews of New Orleans, La. She died about a year before his death. Mr. Robinson is survived by his brother, Herman Robinson. Mr. Robinson's son, Edward Robinson, was a private in the Army and died in France during World War I.

Mr. Robinson was elected an Associate Member of the American Society of Civil Engineers on October 7, 1908.

LESTER CHAUNCEY SMITH, Assoc. M. ASCE[1]

DIED NOVEMBER 17, 1944

Lester Chauncey Smith was born in Montgomery, Ala., on February 9, 1889, the son of Lester C. Smith and Annie (Jackson) Smith. He died on November 17, 1944, at Mobile, Ala. Mr. Smith was educated in the primary schools of Montgomery, and was graduated from the Alabama Polytechnic Institute at Auburn, with the degree of Bachelor of Science in Civil Engineering in the class of 1909.

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[1] Memoir prepared by J. P. Ewin, Doullut and Ewin, Inc., New Orleans, La.

Subsequent to his graduation he spent approximately one year in railroad surveying and construction, approximately one year with the Alabama Power Company on hydroelectric development on the Coosa and Tallapoosa rivers in Alabama, and approximately six years as a junior engineer with the United States Engineer Department on various surveys and construction projects for river improvement.

During World War I, he was a First Lieutenant and subsequently a Captain in the Second Division, Second Engineers of the regular army. His company participated in eight major battles during the war, the principal ones being the First and Second Battles of the Marne, Soissons, Chateau Thierry, and Belleau Woods. He received the Croix de Guerre for gallantry in action.

After the war Mr. Smith returned to the practice of engineering. From 1920 until 1925, he worked as a construction engineer for the firm of Doullut and Williams, Inc. During that time he was principally engaged in the construction of piers and other river terminals, the principal one of which was a wharf for the Standard Oil Company at Baton Rouge, La. Subsequent to that engagement, and up until a short time prior to his death, Mr. Smith was a member of the firm of Doullut and Ewin, Inc. During approximately twenty years with this firm, he had charge of design and construction of many projects, including piers, bulkheads, shipbuilding ways, bridges, and other similar structures. He was in charge of the field work on the construction of the Sugar Bowl Stadium in the City of New Orleans, La.

He was a member of the Louisiana Engineering Society, New Orleans.

In 1919 Mr. Smith was married to Mary Baldwin Williams of Montgomery. In addition to his many friends, he is survived by his widow and four children, Lester Chauncey Smith, Jr., Mary Baldwin Williams Smith, Etheldra Graham Smith, and Martin Baldwin Smith. His oldest son, Lester C. Smith, Jr., served in the United States Army, as a gunner and engineer, on a B-17 in flights over Germany.

Mr. Smith was elected an Associate Member of the American Society of Civil Engineers on July 9, 1923.

RALPH CARL SMITH, Assoc. M. ASCE[1]

Died September 2, 1945

Ralph Carl Smith was born February 14, 1894, at Grant, Iowa, the son of Frank E. Smith and Annie (Gray) Smith. In his childhood he moved with his parents to the State of Washington and located in Stevens County.

Mr. Smith obtained his primary education and part of his high school education in the public schools of Kettle Falls. This is a small community about ten miles northwest of Colville, the county seat of Stevens County. He completed his high school course at Vancouver, Wash., working to pay his way, and being graduated in 1914. In 1922 he entered Washington State College at Pullman, Wash., and completed his sophomore year; during 1925 and 1926 he finished his junior year; and in 1941 he received the degree of Bachelor of Science in Civil Engineering with high honors. His persistent determination to have an education was outstanding and most commendable.

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[1] Memoir prepared by H. J. Doolittle, M. ASCE.

He started his engineering career in 1915 as a helper to a land surveyor. In 1917 he went with the Washington State Highway Department, working at intervals to 1940 through the various positions of computer, inspector, instrumentman, resident engineer, and location engineer. From March to August, 1940, he was the principal inspector on a Naval Air Base at Kodiak, Alaska. He held a similar position with the United States Army Engineers at the Galena (Wash.) air depot, ten miles west of Spokane, Wash., from February to August, 1942, when illness forced him to resign.

Throughout his life Mr. Smith showed unusual ambition; he was always exceedingly honest; and, he always put his work first, regardless of personal inconvenience, or personal gain. Integrity of character was his most outstanding characteristic in both his professional and personal life. He was an asset to any community, for this reason as well as for his virile, colorful personality. He exemplified the true engineer in his vitality, strength, and love of the great outdoors. His sincerity and warm personality won him friends among all classes of people. He was a veteran of World War I and a member of the Masonic Lodge.

In 1928 Mr. Smith was married to Esther L. Johnson of Rice, Wash., and in 1940 they adopted a little boy, Alan Smith. He is survived by his wife and son.

Mr. Smith was elected an Associate Member of the American Society of Civil Engineers on October 9, 1941.

JAMES HARGRAVES TEBBETS, Assoc. M. ASCE[1]

DIED JUNE 12, 1945

James Hargraves Tebbets, the son of James Edward Tebbets and Harriet B. (Hargraves) Tebbets, was born on June 20, 1882, in Somersworth, N. H., where his father was in the banking business. His grandfather was born in England but lived in the United States and was in the woolen mill business in Maine. Mr. Tebbets went to the local schools in Somersworth until 1901 when he entered Massachusetts Institute of Technology at Boston, with the class of 1905.

After a little more than a year in the offices of Lockwood, Greene and Company, and Stone and Webster in Boston, he was employed by the firm of Charles T. Main, Inc. He was in the Boston office for two years and then transferred to the office at Great Falls, Mont. Here he worked on the Rainbow Falls hydroelectric development for the Montana Power Company.

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[1] Memoir prepared by Howard M. Turner, M. ASCE.

During the next four years he had further experience in power plant design, both steam and hydroelectric, with Byllesby in Chicago, Ill., and with Stone and Webster on the Falls Village, Conn., hydro plant. In 1915 he again went out to Great Falls for Charles T. Main, Inc., on the Montana Power Company's Volta and Thompson Falls hydroelectric developments where he was engineer on mechanical equipment (except water wheels). He returned later to work in the Boston office until 1918.

For the next three years he was in charge of mechanical engineering, Central Plant Engineering Department, of the Bethlehem Shipbuilding Company at Bethlehem, Pa. This work was on the mechanical equipment of shipyards, docks, and floating dry docks. After two years at the American Woolen Company in Lawrence, Mass., he spent five years with Vaughan Engineers in Boston on hydroelectric projects in the United States, Canada, and the West Indies. This work was on valuations, rate cases, and industrial work, and on the design and construction of the Jackman hydro development at Hillsboro, N. H. He spent two more years with Charles T. Main, Inc., in charge of design and field inspection on the 2-mgd filtration plant and the boiler plant for the Slaterville Finishing Company, returning to the Vaughan Engineers office for another year as principal assistant. From 1935 to 1940 he did various engineering work around Boston, generally not of long duration, until 1941, when he assumed charge of the layout of the water distribution system at Camp Edwards.

From 1941 until his death he was with the writer as general engineering assistant on various consulting work in connection with utility valuation, hydroelectric studies, and design. During this time he spent a month in Charleston, S. C., and another in Puerto Rico on consulting work for Jackson and Moreland of Boston.

He had a serious operation in the summer of 1944 but after that worked for as much of the time as his health would permit.

Mr. Tebbets was married to Mary Mathieson in 1907 in Dorchester, Mass. He is survived by his daughter, Mrs. Helen M. Tolman.

Except for the periods when he was away from Boston he made his home in Wollaston, Mass., with his family. Golf and bowling on the green were his forms of recreation, and his chief hobby was making ship models.

It is apparent that Mr. Tebbets was a man of wide experience in general engineering work, particularly in the field of power plants, hydraulic work, and engineering costs and valuations. Everything he did was characterized by an honest thoroughness that left no loose ends. This quality, coupled with his experience, made him an extremely capable and reliable engineer. His work was an expression of a loyal and conscientious character which made it impossible for him to compromise on what he thought was right, any more than it would allow him to do sloppy or careless work. He was always interested in his work and apparently derived much satisfaction from it.

He was elected an Associate Member of the American Society of Civil Engineers on April 14, 1919.

FREDERICK WAZNEY TOWNROW, Assoc. M. ASCE[1]

DIED JANUARY 5, 1946

Frederick Wazney Townrow, the son of Frederick W. and Sarah (Thompson) Townrow, was born in Philadelphia, Pa., on August 9, 1873.

At the age of nineteen, while working as a rodman with Samuel H. McElroy of Brooklyn, N. Y., Mr. Townrow took the initial step of his formalized engineering career by beginning a correspondence course in bridge engineering. The eight years between 1894 and 1902 Mr. Townrow spent working as a tracer and a draftsman for the Union Bridge Company of Athens, Pa., and also as a division engineer, performing routine field work in the Pennsylvania and New York Division of the Lehigh Valley Railroad. He also performed the duties of a detailer for the Elmira Bridge Company of Elmira, N. Y., and then served as a draftsman and checker for the McClintic-Marshall Construction Company of Pittsburgh, Pa.

In 1902 Mr. Townrow began his 22-year relationship with the Fort Pitt Bridge Works of Canonsburg, Pa., as a squad leader, designer, and assistant engineer, acting as plant engineer and designer. Until 1908 he directed the detailing and checking of railroad bridges, office buildings, and numerous other projects. He was responsible for accurate design, engineering judgment, and correspondence. In 1909 he very capably designed the open-hearth plant for the Jones and Laughlin Steel Company of Aliquippa, Pa. The following years found him actively engaged in designing and building the following projects: The Cleveland and Indianapolis track elevation for the Pennsylvania Lines West; buildings for the Massillon Rolling Mills Company in Massillon, Ohio, and the Mark Manufacturing Company of Indiana Harbor, Ind. He redesigned the Syria Temple, in Pittsburgh, as well as the falsework for the erection of the roof. He also redesigned the New Statler Hotel in Buffalo, N. Y.; and the steel design, falsework, and method of erection for part of the Pennsylvania Railroad's Ohio Connecting Bridge at Pittsburgh. During this time, while under agreement with the Fort Pitt Bridge Works, he also assisted S. Diescher and Sons, Engineers, Pittsburgh, on the redesign and investigation of existing structures, including an inclined railroad, for the Wheeling Steel and Iron Company. He also spent several months designing, inspecting, and repairing county bridges with the engineer of Mercer County, Pennsylvania.

In 1924 Mr. Townrow became associated with the Koppers Company as a designer of steelwork for the various by-product plant structures, particularly the material handling equipment. This association continued intermittently until 1945, when Mr. Townrow retired from the company. During this time he also served as the assistant staff engineer of the County Emergency Relief Board of Washington County, Pennsylvania.

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[1] Memoir prepared by James R. Hess, M. ASCE.

Prior to his death, Mr. Townrow was engaged in the designing of coal washing equipment for the McNally-Pittsburgh Manufacturing Corporation of Pittsburgh.

He was married to Helen Josephine Jones on May 30, 1898, in Brooklyn.

Mr. Townrow's inherent honesty, unstinted generosity, diligence, and conscientiousness in work, together with his keen wit and warm humor were but a few of the very fine characteristics that brought Mr. Townrow the respect and friendship of all those who were associated with him both professionally and socially.

He is survived by his widow; a daughter, Helen (Mrs. Louis D. Donaldson); a sister, Ione (Mrs. M. W. Macready); and four grandchildren.

Mr. Townrow was elected an Associate Member of the American Society of Civil Engineers on December 15, 1924.

ALBERT OTIS TRUE, Assoc. M. ASCE[1]

DIED MARCH 9, 1945

Albert Otis True, the son of Col. Edward A. and Jane (Milnes) True, was born on December 26, 1879, in Lowell, Mass. He was a descendant of Capt. Henry True who settled in Salem, Mass., in 1654. His father was colonel of the 8th Maine Volunteers, during the Civil War, and his grandfather, Edward True, was a captain in the Revolutionary War and the War of 1812.

Mr. True was graduated from the public schools in Newton, Mass., and from the Massachusetts Institute of Technology (Cambridge) in 1905 with the degree of Bachelor of Science in Sanitary Engineering. For two years after graduation he was assistant to the chief engineer, draftsman and designer for the Pittsburgh Filter Manufacturing Company, Pittsburgh, Pa. During the summer of 1906 he inspected a number of sanitary works in England. In May, 1907, he joined the staff of the late Rudolf Hering and George W. Fuller, Members, ASCE, hydraulic and sanitary engineers, New York, N. Y., as assistant engineer and worked there until May, 1910. He was employed on the design of mechanical filtration plants for Bangor, Me., and Burlington, Vt., and was in charge of design and plans for a pumping station and mechanical filter plant at Botany Worsted Mills, Passaic, N. J., costing $98,000. In March, 1908, he made a series of tests on hydraulic performance of large aeration nozzles. He was in charge of the plans and details of various works in connection with reports of the firm as chief engineer of the Passaic Valley Sewerage Commission. In 1908 Mr. True and E. G. Manahan, M. ASCE, designed water sterilization works for Jersey City, N. J., (50 mgd) and worked on surveys and designs of various sewage-disposal plants. The summer of 1909 he was acting resident engineer on three miles of pipe sewers and a small disposal plant at Kings Park, N. Y.

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[1] Memoir prepared by George H. Maurice, M. ASCE.

Following this, in June, 1909, he was consulting engineer on revision and extension of sewerage and sewage-disposal works for the Massachusetts State Hospital for Epileptics at Monson, Mass.

From 1910 to 1913, he was an assistant sanitary engineer for the New York State Department of Health, and, from 1913 to 1916, he was sanitary engineer for the New York State Department of Architecture. In 1916, he became the junior member of the firm True and McClelland, of Boston, Mass.

It was natural for Mr. True to follow the family military tradition and, on September 26, 1917, he was commissioned Captain of the Corps of Engineers of the United States Army. He held this commission until November 8, 1920. His tour of duty included: Command of the Water Supply School at Camp A. A. Humphreys, Va. (April, 1918); second in command of the 78th Engineers (October, 1918); an assignment in the Historical Data Section, Office of the Chief of Engineers (November, 1918, to October, 1919); and, from November, 1919, to November, 1920, Utilities Officer at Newport News, Hospital No. 43, Hampton, and Fort Monroe, all in Virginia. He was also in charge of army water supplies on the lower Virginia peninsula. During the summer of 1921, he was resident engineer on construction of a reservoir for the City of Wauseon, Ohio. From October, 1921, to April, 1922, Mr. True was assistant engineer for the Pennsylvania State Department of Health, and in April, 1922, he became sanitary engineer for the Proximity Manufacturing Company, Greensboro, N. C., and continued in that capacity until his death. During his engineering career he had many papers published in technical journals.

On June 18, 1913, at Rensselaer, N. Y., Mr. True was married to Charlotte Elizabeth Bishop who survives him.

At a meeting of the North Carolina Section, on December 11, 1944, at Winston-Salem, Mr. True was presented with a Life Membership certificate at which time he spoke with feeling of his deep appreciation and how much his membership in the Society had meant to him during the many years he had been a member.

Mr. True was a member of the American Water Works Association, and served as chairman of the North Carolina Section; the National Geographic Society; the Henry K. Burtner Post of the American Legion at Greensboro; and the Forty and Eight Club.

He was a member of Holy Trinity Episcopal Church of Greensboro for many years. On March 11, 1945, the Reverend Robert E. Roe, Rector Emeritus of that church held the funeral services of the late Albert Otis True. The internment was at Albany, N. Y.

Mr. True's recreations had been, during earlier life, tennis and volleyball, and in later years, walking and motoring. He was of an unusually quiet and retiring disposition. Company officials have words of highest commendation for Mr. True, his department being so well and capably handled that it never gave them a moment's concern; it functioned perfectly.

Mr. True was elected a Junior of the American Society of Civil Engineers on February 5, 1907, and an Associate Member on May 31, 1910.

WILLIAM HOWELL WOOD, Assoc. M. ASCE[1]

Died May 7, 1945

William Howell Wood was born in Waco, Tex., on November 16, 1896, the son of William Howell Wood and Annie (Battle) Wood. From his earliest years, when he secured his first job at the age of ten, he became self-educating and self-supporting. Upon his graduation from Waco High School he started learning engineering the hard, but thorough, way—by practical experience.

Practically all Mr. Wood's professional work was performed in Texas, covering a very large part of its area. His engineering work began in the railroad field in 1917, with the Panhandle and Santa Fe Railroad Company, and then with the North Texas and Santa Fe Railroad Company. Later he accepted positions with the Morgan Engineering Company, the Texas Power and Light Company, the Reclamation Service, U. S. Department of the Interior, and the City Engineer's Office of Waco.

In 1919 Mr. Wood entered the highway engineering field with the firm of Bryant and Huffman, at Denton, Tex., supervising field parties and preparing estimates for construction. During 1922 and 1923, he was superintendent for the F. P. McElwrath Construction Company of Corsicana, Tex., in charge of highway construction projects in Fannin, Kaufman, Freestone, and Navarro counties. For the next fifteen months he conducted preliminary locations of highways in Freestone County for J. E. Johnson, county highway engineer.

In the fall of 1925, when he was twenty-nine years of age, he realized the need for technical training to advance himself in his chosen profession, and he entered the College of Engineering of the University of Texas at Austin, Tex. From June, 1927, to December, 1928, while a student at the University, he was also assistant city engineer of Austin. In 1929 he received the degree of Bachelor of Science in Civil Engineering from the University of Texas.

Nearly half Mr. Wood's career was spent with the Texas State Highway Department. In August, 1924, he was appointed maintenance engineer in the Waco division, and served in that capacity until November, 1925. Shortly after receiving his engineering degree from the University, he returned to the Texas State Highway Department as engineer of materials and tests. He was administrative head of that division, including the department's laboratories, and continued in that capacity until August, 1936, when he was made asphalt research engineer. In September, 1937, he returned to his former position as engineer of materials and tests.

One of the outstanding accomplishments of Mr. Wood's professional life was planning, organizing, and supervising the building up of the Texas State Highway Department's Materials and Tests Laboratories until they ranked among the best in the United States.

In February, 1940, he resigned from his position as engineer of materials and tests to enter business for himself at Austin, Tex., selling materials for the construction of highways and airports. Here his long years of practical experience, which had engendered self-confidence, gave him the ability to act quickly in the war emergency, and his reputation for reliability among engineering friends and associates secured for him large and remunerative contracts in connection with the airfields of the United States.

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[1] Memoir prepared by Henry C. Porter, M. ASCE.

Mr. Wood's illness was of short duration. A man of simple and temperate habits, he was expected to have a long and useful future. A typical Texan, product of one of her smaller cities, schooled from early boyhood by hard work and duty's stern demands, he was, nevertheless, totally lacking in the egotism or grimness which sometimes characterize self-made men; instead, an overwhelming optimism and zest for life characterized all his undertakings and constituted the mainspring of his personality. Quiet and unassuming in manner, he possessed a sense of humor and tolerance which made him very successful in handling men. He loved his work and devoted practically all his time to engineering matters. At the same time he enjoyed the companionship of his many friends and he was always a genial and entertaining host.

On May 7, 1931, Mr. Wood was married to Elise Burnett of Austin. He is survived by his widow and two sons, William Howell Wood, III, and Charles Burnett Wood.

Mr. Wood was a licensed professional engineer of the State of Texas, a member of the Texas Society of Professional Engineers, the Austin Rotary Club, and the Baptist Church.

Mr. Wood was elected an Associate Member of the American Society of Civil Engineers on June 13, 1938.

ROBERT TYRIE BENTON, Jun. ASCE[1]

Died March 24, 1945

Robert Tyrie Benton, the son of the late Dean John Robert Benton, of the College of Engineering, University of Florida, at Gainesville, and Maybelle (Williams) Benton, was born on June 24, 1915, in Williston, Fla.

His preparatory education was received in the Gainesville High School. He was graduated, magna cum laude, from the University of Florida with the degree of Bachelor of Science in Civil Engineering in June, 1935.

During the winter of 1935–1936, he assisted in surveying the route for the proposed Florida Cross State Canal. After the discontinuance of the canal project in 1936, he was employed in the office of the United States Government Engineers at Vicksburg, Miss., on the work of Mississippi River flood control.

In the fall of 1937, he resigned this position in order to take part in a sailboat race between St. Petersburg, Fla., and Havana, Cuba, in which he and two friends sailed a thirty-three-foot boat. Later, with a crew of four, they spent several months cruising around the Bahama Islands.

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[1] Memoir prepared by Mrs. John Robert Benton, Gainesville, Fla.

From August, 1938, until January, 1939, he worked as surveyor with a land company in New Smyrna, Fla. In February, 1939, he went to Venezuela to work as a gravity meter operator for the Socony-Vacuum Oil Company. He continued in this work until he returned home, in 1942, to enter the United States Army.

He began his basic training at Fort Leonard Wood, Missouri, in October, 1942. From there he was sent to Grand Rapids, Mich., for a nine months' course in meteorology. The *Forecaster,* weather school publication, chose him "Cadet of the Day" to represent the activities of a typical day's routine there. He finished second in a class of several hundred, and received his commission as Second Lieutenant in September, 1943. He was about to be sent out on foreign duty as weather forecaster in the Air Forces, when, because of a shortage of navigators, he was selected with fourteen other members of his class to become a navigator. He completed the four and a half months of navigation training at Monroe, La., with the highest record in the history of that navigation school. He was retained there as an instructor for about six months.

He was sent to England in August, 1944, as a navigator on a C–47 Skytrain transport plane. From his base in southern England he took part in missions over Europe until his squadron was transferred to France for the greatest air offensive of the war. A letter from his commanding officer, dated April 20, 1945, states that he had received his promotion to the rank of First Lieutenant, effective April 1.

Lieutenant Benton's C–47 departed from France on a glider tow mission on March 24. The plane ran into flak two miles southeast of Wesel, Germany, after having completed its mission, and caught fire. All crew members parachuted over enemy territory. Lieutenant Benton was shot down and killed. The other crew members were taken prisoners and later liberated. He was awarded, posthumously, the Purple Heart, the Air Medal, and two Oak-Leaf Clusters.

Lieutenant Benton was scholarly and lovable, always ready to serve or befriend those with whom he came in contact. Just before leaving England, Lieutenant Benton visited a relative whom he had never met before. In her letter to his mother after his death, she wrote:

> "I have a vivid picture of Robert standing tall on the hearthrug. He seemed so lighthearted I hardly thought of the danger he was running. I was struck with the accord between him and his air corps friend. They just looked like boys who did serious work together."

Interested in his profession to the end, Lieutenant Benton wrote a friend that he had developed some improved methods of navigation.

His commanding officer wrote:

> "Robert was respected and admired for his quiet and efficient performance of work, and liked by all for his pleasant amiability."

Lieutenant Benton is survived by his mother and three brothers. The brothers are Sgt. Charles R. Benton, Lt. (jg) Hugh Herbert Benton, and John J. Benton, M.D.

Lieutenant Benton was elected a Junior of the American Society of Civil Engineers on November 10, 1941.

BERLIN FRANK CODAY, JR., Jun. ASCE[1]

Died April 9, 1945

Berlin Frank Coday, Jr., was born in Commerce, Okla., on February 14, 1921, the son of Berlin F. Coday and Ruby C. (Latimer) Coday. He attended public schools in Tulsa, Okla.; San Antonio, Tex.; and Springfield and Kansas City, Mo. He was graduated from Southwest High School in Kansas City.

He attended Kansas City Junior College for two years, and then entered the University of Utah at Salt Lake City, Utah, where he received his engineering degree in June, 1944. He was affiliated with Beta Theta Pi fraternity.

During his summer months in Salt Lake City, Frank worked in the engineering department of the Intermountain Contractors on the building of Camp Kearns, Utah. He also worked on various engineering projects for the Illinois Powder Manufacturing Company, making surveys and estimates on large quarry blasts for the Ideal Cement Company's quarry at Devil Slide, Utah.

After being inducted into the Army in July, 1944, Frank was sent overseas the following December and assigned to Company A, 383d Infantry, 96th Division of the Tenth Army. He was killed in action on Okinawa Island on April 9, 1945, while acting as a personal messenger for his company commander. He had previously been awarded the Infantryman's Combat Badge for excellence in infantry combat.

Frank enjoyed outdoor life to the fullest extent. His hobbies were music, golf, and hunting. He was a clean sportsman, a true American of high character, integrity, and ideals. He was loved by all who knew him. He is survived by his parents; a sister, Betty Jean Coday; and an aunt, Mrs. Maude Davis.

Mr. Coday was elected a Junior of the American Society of Civil Engineers on March 12, 1945.

JUAN CRISOSTOMO DORIA PAZ, Jun. ASCE[1]

Died December 28, 1944

Juan Crisostomo Doria Paz was born in Linares, Nuevo León, Mexico, on June 24, 1911. He was a son of Juan C. Doria and Leonarda Paz de Doria.

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[1] Memoir prepared by Frank E. Bodine, San Francisco Mgr., Westinghouse Elec. Corp., San Francisco, Calif.

[1] Memoir prepared by Lisandro Peña, Jr., Architect, Monterrey, N.L. Mexico.

His grandfather, a doctor from Villagrán in the same state, came to reside in Linares when quite young and there Juan C. Doria, Sr., was married and started practice as a civil engineer. Later they changed their residence to Monterrey, Nuevo León, Mexico, where the family lived while Juan, Sr., worked continuously on the development of railroads which were then in their infancy. The line from Monterrey to Tampico, Mexico, was pioneered, planned, and executed by Juan C. Doria, Sr.

After graduating at the old Colegio Civil in Monterrey, Mr. Doria followed his father's footsteps and studied at the Agricultural and Mechanical College of Texas at College Station, Tex., from which he was graduated on June 2, 1934.

Immediately after graduation he started to work for Compañia Fubdidora de Fierro y Acero in Monterrey designing steel structures and eighteen months later he went into partnership with Luis F. Flores, architect and college friend. He was then registered as civil engineer in the Office of Public Works of the State of Nuevo León. This partnership lasted for three years. During this time he initiated his work in residential construction, to which he dedicated most of his time after the partnership was dissolved in 1939.

In the following six years his office handled more than two and one half million pesos which went into such construction jobs as: A swimming pool for the Círculo Mercantil, structures of Florida theater, a nail factory, Banco Popular, Molinos El Fenix, Facultad de Ciencias Quimicas, and many residences.

On the 28th of December, 1944, he made his usual visit to the Cristo Rey Temple job where the concrete roof was being poured. He climbed the wooden forms and started to check the pouring. The vertical posts collapsed, and he and seventeen workmen on the job fell to their deaths from a height of sixty feet. They were buried in thirty tons of concrete and steel.

He left, besides the Cristo Rey Temple, such unfinished jobs as the million peso bridge to Del Valle suburban development, the Santos Office Building, another bridge across the Santa Catarina River, Industrias Thermoplásticas Factory, and numerous other factories and residences.

Mr. Doria is survived by his wife Idolisa Salazar; his two daughters, Idolsia Margarita and Maria Eugenia; his father, Juan C. Doria; his brother, Attorney Francisco Doria Paz; his sister, Elena; and numerous other relatives.

Juan C. Doria Paz will never be replaced either professionally or in the hearts of his friends, including his colleagues and many members of other professions. The vacant spot he left as chairman of the Engineering Faculty of the University of Nuevo León will be difficult to fill. His energy and "drive" in local society and public work will be greatly missed. His Christianity, charm, and gentleness will long live in the memories of all his friends.

Mr. Doria Paz was elected a Junior of the American Society of Civil Engineers on February 11, 1935.

EDWARD FRY, JR., Jun. ASCE[1]

DIED NOVEMBER 23, 1944

Edward Fry, Jr., the son of Edward Fry, and Ethel (Neale) Fry, was born in Brownsville, Tex., on November 10, 1918. He attended high school and junior college in that city, and was graduated from the junior college in 1938.

In 1939, he entered the Agricultural and Mechanical (A. and M.) College of Texas, at College Station. In 1942, he was made First Lieutenant in the cadet corps at A. and M., and was second in command of Company "A" Engineers. Lieutenant Fry distinguished himself as a student, and received the degree of Bachelor of Science in Civil Engineering in January, 1943. During his four years at Texas A. and M., Lieutenant Fry was a member of the Student Chapter of the American Society of Civil Engineers, the Society of American Military Engineers, and a member of the rifle team.

Immediately after he received his Bachelor of Science degree, he was inducted into the army. He went to Fort Belvoir, Va., where he was commissioned a Second Lieutenant on May 12, 1943. His first assignment was at Camp Swift, Texas. He was sent overseas in November, 1943, with the 82d Combat Engineers, being stationed first in North Africa and later in England. In March, 1944, he received his promotion to First Lieutenant.

Lieutenant Fry was with the First Army under Gen. Courtney H. Hodges, Jr., and left England on D-Day, moving through France, Holland, Belgium, and into Germany. On November 23, 1944, he was killed in action, in Germany, and was buried in a United States Military Cemetery in Holland. He was awarded, posthumously, a Presidential Citation and the Purple Heart, for military merit and for wounds received in action which caused his death. These awards were presented to his mother, Mrs. Jack Hirschmann, who survives him.

Lieutenant Fry was elected a Junior of the American Society of Civil Engineers on June 14, 1943.

RICHARD ALDEN KEASEY, Jun. ASCE[1]

DIED FEBRUARY 4, 1945

Richard Alden Keasey, the son of Dorr E. and Florence Anne Keasey, was born on July 21, 1911, in Portland, Ore. He was educated in Portland, being graduated from Lincoln High School in June, 1929. In September, 1929, he entered the University of Oregon at Eugene, majoring in business administration. In June, 1931, he was awarded membership in Alpha Kappa Psi (Kappa Chapter).

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[1] Memoir prepared by C. E. Sandstedt, Assoc. M. ASCE.

[1] Memoir prepared by Mrs. Florence A. Keasey, Portland, Ore.

He entered the service of the Oregon State Highway Commission in 1931, serving as head chainman until 1935, when he was employed by the Bonneville Dam Administration at Bonneville, Ore. In September, 1938, he entered Oregon State College at Corvallis, majoring in civil engineering. He served in the Engineering Corps of the Reserve Officers Training Corps, receiving the commission of Second Lieutenant. He belonged to the Phi Delta Theta fraternity (Oregon Beta), being manager of his house in 1940. He was graduated from Oregon State College, with the degree of Bachelor of Science.

Lieutenant Keasey was a member of the following societies; Tau Beta Pi (Alpha of Oregon); Sigma Tau (Zeta Chapter, Oregon State College); Pi Mu Epsilon (Oregon Beta Chapter); and the national military society, Scabbard and Blade.

In April, 1941, he was called to active duty in the United States Army and ordered to report in Manila, Philippine Islands. On April 17, 1941, he went to Fort Mason in San Francisco, Calif. He left San Francisco on April 22, 1941, on the Army transport *Washington,* and arrived in Manila on May 8, 1941. In Manila he was assigned to the Philippine Department Headquarters, Corps of Engineers, Fort Santiago. Fort Santiago being headquarters for all the Army posts and air corps fields in the Islands. The day following his arrival, he took over supervision of construction of a new runway for Nichols Field, about five miles from Manila. His office was at headquarters where his duties included procurement work, all requisitions to the engineer depots, estimates, and requisitions to the depots of supply. Later he was sent around on payrolls, inspections, supervisions, and other duties.

In November, 1941, he was sent to Cabanatuan, Nueve Eciji Province, Luzon. He was in charge of construction of airplane runways, roads, railroad sidings, warehouses, barracks, etc. He was the only officer there, and the only other white man in the immediate country was a merchant in Cabanatuan.

Sometime in November, 1941, he flew by Army plane and Navy amphibian plane to the south, on engineering duties, covering such places as Baler, Tayabes, Bolinao, Pangasinan, Lubang Island, Mindora, and as far south as Mindinao. In the Philippines he served as adjutant to his Commanding Officer, Major Findley, until the latter part of October, 1941, when Major Findley died. Lieutenant Keasey was commissioned a First Lieutenant effective January, 1941.

After December 7, 1941, two letters were received—one from somewhere in the Philippine Islands, on March 16, 1942, and the other from somewhere in Bataan. On Easter morning, April 2, 1942, he sent a telegram from the Island of Cebu. His last letter was dated April, 1942, from somewhere in Bataan. After the "Death March," he was taken to Cabanatuan Camp No. 1, remaining there until the latter part of October, 1944, when the Japanese moved able-bodied prisoners of war to Bilibid Prison Camp. On December 15, 1944, he was put aboard a prison ship that was torpedoed a short way out from Manila Bay. However, he was rescued and taken to Honshu Island, dying of acute colitis on February 4, 1945, in Fukuoka Prisoner of War Camp No. 22, in Japan.

Richard Keasey early showed the characteristics of a student. In the elementary grades he ranked first in his school. He also won a bank account for an essay on "Savings," written at the age of twelve. In high school he was graduated with honor. During this period, he won six medals and the silver champion loving cup, in one night's performance, for junior athletics at the Multnomah Athletic Club. He was an honor student at University of Oregon and Oregon State College.

His personality could best be summed up by an expression of his professor, Charles A. Mockmore, M. ASCE, head of the civil engineering department who called him a "Christian gentleman" and stated that he had never had a student for whom he had so much regard. The different activities of his life showed that Lieutenant Keasey was always striving to attain perfection in whatever he undertook. Lieutenant Keasey was not married. He was an only child.

Lieutenant Keasey was elected a Junior of the American Society of Civil Engineers on October 9, 1941.

ROBERT AUGUSTUS LEWIS, Jun. ASCE[1]

Died September 20, 1944

Robert Augustus Lewis, the son of Lt.-Col. Augustus T. and Blanche (Deloison) Lewis, was born at Quantico, Va., on October 26, 1920. He was graduated from the San Diego (Calif.) Army and Navy Academy, in 1938; and he immediately enrolled in the Virginia Military Institute (V. M. I.) at Lexington, being graduated at the top of his class in 1942. He was on the roll of honor every month during his four years at college, and his diploma shows him to be a distinguished graduate of V. M. I. During his vacation from college, in 1941, he was employed as a civil engineer by the Angle Construction Company at Parris Island, S. C.

Mr. Lewis received his commission as Second Lieutenant of Infantry in the Army of the United States upon his graduation from V. M. I., and reported for duty immediately at Fort Benning, Ga. He served as commanding officer of Company B, 15th Armored Infantry Battalion, Fifth Armored Division, from the time of its organization until his death.

In January, 1944, he arrived in England; and on August 6, 1944, he distinguished himself by personally wiping out a pocket of Germans in France for which he was awarded the Bronze Star medal. He was also awarded the Infantryman's Combat Badge, and the Purple Heart.

On September 19, 1944, his division received orders to withdraw from Germany, and Company B was assigned to cover the Americans' withdrawal. His company remained at its post and was sacrificed. Lieutenant Lewis was in the vicinity of Wallendorf, Germany, when he was killed by a direct hit from artillery. His body was buried in the United States Military Cemetery at Foy, Belgium.

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[1] Memoir prepared by Lt.-Col. Augustus T. Lewis, U. S. Marine Corps, San Diego, Calif.

Lieutenant Lewis was elected a Junior of the American Society of Civil Engineers on November 9, 1942.

STEPHEN TENER LOWRY, Jun. ASCE[1]

Died September 9, 1944

Stephen Tener Lowry, the son of Swift Lowry and Katharine (Tener) Lowry, was born in Cleveland, Ohio, on October 16, 1919. He was graduated from Shaw High School in East Cleveland. He was a member of the National Honor Society, and became an Eagle Scout. In September, 1937, he entered Lehigh University in Bethlehem, Pa., from which he was graduated, with honors, in June, 1941, with the degree of Bachelor of Science in Civil Engineering. At Lehigh his principal extracurricular activity was intramural and intercollegiate debating, and he was elected a member of Delta Omicron Theta, honorary debating society.

A member of the Reserve Officers Training Corps, Stephen was commissioned a Second Lieutenant upon his graduation and was called to active duty on July 6, 1941. After eight months at Fort Custer, Mich., he went to Iceland as a member of the Eleventh Infantry Regiment of the Fifth Division. While there he was promoted to First Lieutenant. In July, 1943, the Division moved to England, then to Ireland, and in July, 1944, became part of the Third Army under the late Gen. George S. Patton, Jr., in the drive across France. Early in September, Lieutenant Lowry was made a Company Commander and recommended for a Captaincy. He was killed while leading one of the units crossing the Moselle River south of Metz, France. The Purple Heart and the Silver Star medals were awarded to him posthumously. A fellow officer wrote, "His first thoughts were always for his men and he was protecting them when he was killed."

He is survived by his father, a veteran of World War I; his mother; two brothers, William and Robert, both veterans of World War II; and a sister, Katharine (Mrs. M. C. Dietrich, Jr.).

Mr. Lowry was elected a Junior of the American Society of Civil Engineers on November 10, 1941.

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[1] Memoir prepared by Swift and Katharine Lowry, Cleveland, Ohio.

HAROLD LESTER ODOM, Jun. ASCE[1]

Died January 7, 1946

Harold Lester Odom, the son of George A. and Alma (Deaton) Odom, was born at Many, La., on July 2, 1917. He attended the public schools of Jackson Parish, Louisiana, and was graduated from Jonesboro (La.) High School in 1934. In September, 1934, he entered Louisiana State University at Baton Rouge. In 1936 he went to work for the Soil Conservation Service in Ruston, La. He subsequently entered Louisiana Polytechnic Institute at Ruston, as an engineering student, and continued to work on part-time basis for the same agency for two years. At the same time, he engaged in the publication of a newspaper with his brother, Roy, in the capacity of reporter, typist, and salesman. In 1938 he was employed in Baton Rouge by the U. S. Bureau of Agricultural Economics on map drafting and statistics, and a few months later he was transferred to the Little Rock, Ark., office. In 1939 he accepted a position with H. V. Howe, head of the Lousiana State University School of Geology, as a draftsman, resuming his college career in the School of Engineering and working part time in the geology drafting room. Mr. Odom was awarded the degree of Bachelor of Science in Civil Engineering in 1941. In July, 1941, he enlisted in the United States Marine Corps and was sent to Quantico, Va., for training as an officer.

In January, 1942, he was commissioned a Second Lieutenant in the United States Marine Corps Reserve and was selected for further training in the Field Artillery. He was then attached to the Second Defense Battalion and joined the Second Marine Brigade on the American mandate island of Samoa. At that time this was the furthest outpost of our defense system and he was actively engaged in preparing the defenses on the island. In September, 1942, he was promoted to the rank of First Lieutenant and transferred to the Eighth Marine Regiment, in command of a heavy and light machine gun platoon.

In November, the Eighth Marines were assigned to the First Marine Division on Guadalcanal where they were engaged in the first offensive action against the enemy. Lieutenant Odom's platoon was actively engaged in combat for more than three months and was several times cited for repulsing enemy "Banzai" attacks.

Eventually the Eighth Marines were sent to New Zealand with the Second Division. Lieutenant Odom was promoted to Captain and given command of Company "K," Third Battalion. After an extensive training period, the Second Division went into action in the Gilbert Islands on the atoll of Tarawa and Captain Odom's Company was one of the first to land. He was personally responsible for the reorganizing of two rifle companies and for leading the attack against some of the enemy's strongest emplacements. He was wounded slightly but refused to leave his troops—his actions at all times were an inspiration to his men. For his actions, bravery, and leadership during this attack, he was awarded the Silver Star Medal personally presented by Admiral Chester W. Nimitz in Hawaii.

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[1] Memoir prepared by Maj. Scott Corbett, Jr., Nelson Equipment Co., Portland, Ore.

The Second Division was reorganized and retrained on Hilo, Hawaii, where Captain Odom was primarily responsible for development of some new infantry tactics—tank cooperation and the use of demolitions against pillboxes. In May, 1944, he again took his company into action against the enemy on the Island of Saipan and, at one time, was acting Battalion Commander. In a long extensive campaign he was responsible for leading his men time and again against the enemy and again his actions and leadership carried his battalion through. He was severely wounded during this campaign; but, after spending several weeks in the hospital, rejoined his troops in time for the Tinian offensive action.

After thirty months overseas and four major campaigns, he was returned to the United States and promoted to the rank of Major. He was assigned command of an Officers' Training Battalion at Camp Pendleton, Calif., and served in that capacity until the end of the war and his release from the Marine Corps.

While in the service, Major Odom had become interested in construction machinery and chose his civilian career along those lines. He became associated with a concern in Portland, Ore., with branch offices in the states of Washington and Idaho. With his engineering and military background, a very brilliant future was predicted for him. While en route along the Columbia River Highway to his new office in Boise, Idaho, the car in which he was riding was struck by a rolling rock and thrown off a steep cliff. He was killed instantly—this man who had offered up his life time and again for his country in overseas combat—in pursuit of his civilian profession on the highways of the United States.

His death was an extreme shock and mystery to those who loved him. His future life promised nothing but success and service to a constructive country—yet, in his short span of years, he did fulfill the duties and promises of a lifetime. All who associated with him will bear the sorrow of his death, but they will remember with pleasure the privilege of being associated with him, his sincerity and honesty, his clarity of purpose, his personal character and courage, and his love of life.

Major Odom is survived by his widow, Nedra Foster; his three-month-old son, Tom; five brothers, Leo M., M. ASCE, Roy H., Thomas D., George E., and John S. Odom; and two sisters, Elsie (Mrs. Frank B. Brown) and Nell (Mrs. Sydney Chalmers).

Major Odom was elected a Junior of the American Society of Civil Engineers on November 10, 1941.

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